Extended Abstract 1

71st EAGE Conference & Exhibition — Amsterdam, The Netherlands, 8 - 11 June 2009
6129

Improved Characterization of Fault Zones by Quantitative Integration of Production and Seismic AVAZ Data

A. Ali* (University of Bergen), A. Shahraini (University of Bergen) & M. Jakobsen (University of Bergen)

SUMMARY

This paper proposes a method for the parameterization and quantification of reservoirs which consist of fault core along with a damaged zone using integration of seismic and production data. We will try to represent the faulted reservoir models with relatively small number of parameters and then focus on the inverse problem; that is how to estimate the parameters of the fault core and damaged zone from the production data and seismic attributes. This method is based on a model for the effective hydraulic and poroelastic properties of faulted/fractured porous media and a Bayesian method of inversion (Monte Carlo Markov Chain), which provides information about uncertainties as well as mean values. The two parameters inverted for are the transmissibility and fracture density. The initial inversion results are based on a simplified model (involving a fault core with fractured damage zone in which fractures are aligned in the same direction as the fault core), which is both heterogeneous and anisotropic. An application to the synthetic data suggests that one may obtain a significantly better estimate of the parameters of fault and fractures within the reservoir using the seismic AVAZ data in addition to the reservoir production data in history matching process.
Introduction

The impact of faults on the flow of fluids in hydrocarbon reservoirs has been recognized long time ago, but workers are still struggling with the parameterization and quantification of fault properties. Traditionally, reservoir engineers have modelled faults as planar features or discontinuities characterized by one or more fault transmissibility parameters. Fault controls the flow behaviour in a reservoir and these flow properties have traditionally been treated as a history matching parameter by applying a transmissibility multiplier (Edris et al., 2008). In reality, however, faults are complex 3D objects typically consisting of a fault core surrounded by a heterogeneous damaged zone. The fault damaged zone typically consists of fractures and/or deformation bands that may or may not have a preferred orientation (depending on the stress history of the formation, etc.). Researchers have developed quite sophisticated fault facies models that incorporates information from geological outcrop studies, core and laboratory data. However, such detailed fault facies models are described by a wide range of parameters and may not be suitable for inversion or history matching, unless some simplifications are introduced. Remote physical measurements such as seismic measurements play a central role in helping to simulate the flow of fluids in fractured/faulted reservoirs, but this requires a good understanding of the relevant rock physics and scaling issues. Rock physics models can be used to map the effects of sub-grid fractures/heterogeneities to grid-cell scale (see Jakobsen and Shahraini, 2008 a, b).

Rock physics models are very useful when trying to perform an inversion of production and/or seismic data. To estimate the effective transport properties like anisotropic permeability we can use seismic or production data alone in principal, but by using them alone we are in situation where we cannot get the desired solution because of non-uniqueness of the solution (see Jakobsen et al., 2008 a, b). So it may be a good idea to perform some kind of joint inversion of production and seismic data (See Figure 1). Some of the efforts for joint inversion using seismic and production data are given by (Edris et al., 2008). In this study we have employed a very simple model (see Figure 2) for the fault core, and focus more on the fault damaged zone. More specifically, we shall represent the fault core by a single transmissibility parameter and use anisotropic effective medium theory to estimate the overall properties of the fault deformation zone under the assumption that it consists of a single set of fractures that have a preferred orientation parallel to the fault core. Our aim is to generate synthetic seismic and production data for this model of a fault zone, and to see if we can recover the parameters of the fault core and the damaged zone under various conditions of random noise. The present study represents an extension of the fracture characterization system developed by Jakobsen and Shahraini (2008a, b) in the sense that we introduced the fault core as an additional complication. We have followed Jakobsen and Shahraini (2008a, b) in using a Bayesian inversion method, but we here use the Markov Chain Monte Carlo method (McMc) rather than the Randomized Maximum Likelihood method. We have found that the McMc method works fine for the relatively small and simple models we have considered so far; and it seems to be well suited for more complex models involving a higher number of model parameters.

Monte Carlo inversion method

The non-linear forward problem defined by:

\[ \mathbf{d} = \mathbf{G}(\mathbf{m}) \]  

(1)

Here, \( \mathbf{d} \) is a vector of observable quantities (e.g., seismic AVAZ data and/or production data) and \( \mathbf{m} \) is a vector of model parameters related with the fault core and fractured damage zone (e.g., the fault transmissibility and fracture density). The operator \( \mathbf{G} \) is based on a combination of the rock physics model (see Jakobsen and Shahraini, 2008a, b) and tools for fluid flow simulation (Eclipse 100) and seismic attribute generation. The seismic attributes or seismic AVAZ data (reflection coefficients) has been obtained using the weak anisotropy approximation of Ruger (2002) for HTI media.
The inverse problem consists of estimating the model parameters $\mathbf{m}$ related with the fault core and fractured damage zone from the production and/or reflection coefficient data $\mathbf{d}$. The solution to an inverse problem in a Bayesian setting is given by the posterior probability distribution $\sigma(\mathbf{m})$ over the model space $M$. $\sigma(\mathbf{m})$ basically carries all the information about the model originating from two sources. The first source is the data and the information is given by the likelihood function $L(m)$. The likelihood of a given model $\mathbf{m}$ is measured through its misfit or objective function $J(\mathbf{m})$. In case of the Gaussian data, objective function $J(\mathbf{m})$ can be expressed as,

$$ J(\mathbf{m}) = -(1/2)(\mathbf{G}(\mathbf{m}) - \mathbf{d})^T \mathbf{C}_d^{-1} (\mathbf{G}(\mathbf{m}) - \mathbf{d}), $$

where the covariance matrix $\mathbf{C}_d$ contains information about the measurement errors. The second source is the prior information, which is expressed through a probability density $\rho(\mathbf{m})$. These densities may be simple Gaussian PDF’s or they may be more complicated ones. In our synthetic test case, we have considered a constant prior distribution. Thus our posterior probability distribution is same as the likelihood function.

$$ L(\mathbf{m}) = N \times \exp (-J(\mathbf{m})), $$

where $N$ is normalization constant that can be found by the fact that the integral of the posterior over $M$ is equal to 1. To quantify the uncertainty in the inverted model parameters (the fault transmissibility and fracture density) the exploration of the posterior PDF can be done by the Monte Carlo sampling (since the forward model is not given by simple mathematical expression). In Bayesian setting the Metropolis-Hasting algorithm or Randomized maximum likelihood (RML) method is used to generate samples from $\sigma(\mathbf{m})$. We have adapted the Metropolis algorithm to the problem of sampling the posterior probability density. Suppose that at a given step, the random walker is at point $\mathbf{m}_i$, and the application to rules would lead to a transition to point $\mathbf{m}_j$. Sometimes we reject this proposed transition by using the following rule (Tarantola, 2005):

- If $L(\mathbf{m}_j) \geq L(\mathbf{m}_i)$, then accept the proposed transition to $\mathbf{m}_j$.
- If $L(\mathbf{m}_j) \leq L(\mathbf{m}_i)$, then decide randomly to move to $\mathbf{m}_j$, or to stay at $\mathbf{m}_i$, with the following probability of accepting to move to $\mathbf{m}_j$.

$$ P_i \rightarrow j = L(\mathbf{m}_j)/ L(\mathbf{m}_i), $$

Then the random walker samples the posteriori probability density $\sigma(\mathbf{m})$. For obtaining the independent posterior samples, there is no general rule and it strongly depends on the particular problem at hand. The other important point to emphasize is the acceptance rate of the Metropolis criterion, which should not be too small and too large. (Tarantola, 2005).

**Numerical Results and discussion**

We consider a simple fault facies model described by a fault core and fractured damaged zone (fractures oriented in the same direction as fault core) with one production and one injection well on one compartment (having known fracture density equal to 0.04) of the fault (Figure 2). We assume that the true fracture density within the second compartment of the damaged fracture zone and true fault transmissibility of the fault core is equal to 0.05 and 0.5 respectively; and our goal is to investigate how accurate one can recover the fault transmissibility and fracture density from production and/or seismic data under various amounts of noise conditions. An example of synthetic production and seismic AVAZ data is shown in Figures 3 and 4, respectively. After ten years of production, the saturation data is obtained from reservoir simulator and used for obtaining the elastic model. We obtained 50,000 samples of the posterior PDF on the basis of Monte Carlo Markov Chain
method (described earlier) using a constant (uninformative) prior PDF. The standard deviation of the measured seismic data is equal to 5% and production data is equal to 2%. The histograms (samples of the marginal posterior PDF) obtained using the McMc from the seismic data alone and joint inversion of seismic AVAZ and production data as shown in figures 5, 6, 7 and 8. Clearly, joint inversion of seismic and production data helps to reduce the uncertainty of the estimated fault transmissibility and fracture density.

Conclusions and recommendations

Improved characterization for this kind of reservoir model with fault core and fractured damaged zone can be obtained via a joint inversion of the production and seismic AVAZ data, and is demonstrated by using a simple model (Figure 2). Quantitative integration of seismic attributes with production data leads to improved certainty and better management of fractured reservoirs. Seismic data away from the fault core is sensitive to the fault transmissibility. Fracture aperture in the effective permeability model is assumed to be known through well log data and geologic outcrop studies. The next step will be to include for the impacts of fracture aperture on the effective permeability model and also include some pressure effects. For oil recovery programs with this kind of anisotropic systems, seismic anisotropy data should be considered.

References


Jakobsen, M. and Shahraini, 2008a. Improved characterization of fractured reservoirs by joint inversion of seismic AVAZ and production data. 70th EAGE meeting, Rome, Italy.


Figure 1. Workflow diagram.

Figure 2. A simple reservoir model consisting of a fault core and fractured damage zone used for synthetic modelling and inversion.
Figure 3. Production data without noise. Green: bottomhole pressure. Red: oil production.

Figure 4. Seismic AVAZ data; i.e., PP reflection coefficients vs. polar angle for different azimuth angles.

Figure 5. Samples of the marginal posterior PDF for the transmissibility obtained by using seismic data only.

Figure 6. Samples of the marginal posterior PDF for the fracture density using seismic data only.

Figure 7. Samples of the marginal posterior PDF for the transmissibility obtained by using seismic and production data.

Figure 8. Samples of the marginal posterior PDF for the fracture density obtained by using seismic and production data.
Extended Abstract 2

72nd EAGE Conference & Exhibition incorporating SPE EUROPEC 2010
Barcelona, Spain, 14 - 17 June
ON THE ACCURACY OF RUGER’S APPROXIMATION FOR REFLECTION COEFFICIENTS IN HTI MEDIA: IMPLICATIONS FOR THE DETERMINATION OF FRACTURE DENSITY AND ORIENTATION FROM SEISMIC AVAZ DATA

Aamir Ali¹ and Morten Jakobsen¹, ²

¹Department of Earth Sciences, University of Bergen, Allegaten 41, 5007 Bergen, Norway. ²Centre for Integrated Petroleum Research, University of Bergen, Allegaten 41, 5007 Bergen, Norway.

Summary

We here investigate the accuracy of Ruger’s approximation for PP reflection coefficients within the context of seismic AVAZ analysis in HTI media; that is simple reservoir model with a single set of vertical fractures. An individual comparison of forward and inverse modelling results have been done for the interface between an isotropic and HTI medium. The comparison is performed for different contrast (small and large) and anisotropy by changing the fracture density. The elastic stiffness tensor of the fractured reservoir of HTI medium was calculated using a combination of T-matrix with Brown-Korringa relations. We have calculated the percentage error for the Ruger’s approximation(s) with respect to the exact formula for reflection coefficients. The inversion of this non-linear forward model with respect to fracture density and azimuthal fracture orientation was done in a Bayesian setting, which provide information about uncertainties as well as the most likely values. The maximum percentage error for large contrast and large anisotropy (fracture density = 0.2) may reach up to 70%. An application to the synthetic case of characterizing a fractured damage zone, where fracture density and orientation depend on distance from the fault core was also provided to elaborate the workflow.
Introduction

The detailed study of reflection and transmission coefficients is the basic part of amplitude-versus-offset (AVO) or amplitude-versus-angle and azimuth (AVAZ). This study becomes complicated in the case of a reflector surrounded by anisotropic media, but can be simplified substantially if the anisotropy and velocity contrast across the reflector are weak. Several approximate forms exist for anisotropic reflection coefficients but the most widely used in the industry are Ruger’s approximations for a medium with horizontal axis of symmetry (Ruger, 1997; Ruger, 1998; Ruger, 2002). The reason for their wide use is that with the help of simple analytic approximations the effect of anisotropy on reflection signature can be analyzed. The usefulness of Ruger’s approximation is that it gives direct and straightforward insight into the azimuthal signature, while the other available approximations (e.g. Psencik and Martin, 2001) involve more parameters and a less straightforward interpretation (Ruger, 2002). So it may be useful idea to investigate the accuracy of Ruger’s approximation for reflection coefficients for an HTI media in both forward and inverse modelling using a suitable rock physics model.

The advantage of using a suitable rock physics model for a fractured medium is that we are left with relatively small number of parameters to be inverted for using reflection seismic data (see Bakulin et al., 2000a). Bakulin et al., (2000a) verified the accuracy of AVO gradient term involved in linearized P-wave reflection coefficients given by (Ruger, 1997; Ruger and Tsvankin, 1997) between isotropic and HTI medium for varying Vp/Vs ratio for different crack models with moderate fracture density (0.07) and small contrast. The main motivation of this study is to go beyond these so called moderate fracture densities and small contrasts and check the performance of Ruger’s approximation(s). This is also because of the fact as suggested by Thomsen (1985) that the rocks of interest to the petroleum industry may commonly have porosity as high as 40% and/or crack/fracture density greater than 0.3. For finding the estimates of fracture parameters (fracture density and azimuthal fracture orientation) using approximation(s) (Ruger, 1998; Ruger, 2002) and exact method (Schoenberg and Protazio, 1992) in order to implicate the accuracy of Ruger’s approximation(s) for the determination of these parameters we have followed a simple workflow. This simple workflow involves generating an effective elastic model form a discrete fracture network model, on which seismic modelling (reflection coefficients form top of the reservoir) can be done. This scheme devised for testing the Ruger’s approximation(s) will reveal the model errors produced, if we use approximation(s) instead of exact method. Also it will identify the limit on anisotropy (fracture density) and contrast (small and large), below which the approximations(s) will give satisfactory results.

Bayesian inversion method

The non-linear forward problem defined by:

$$\mathbf{d} = \mathbf{G}(\mathbf{m}).$$  \hspace{1cm} (1)

Here, $\mathbf{d}$ is a vector of observable quantities (e.g., seismic AVAZ data) and $\mathbf{m}$ is a vector of model parameters related with fractures (e.g., the fracture density and azimuthal fracture orientation). The operator $\mathbf{G}$ is based on a combination of the rock physics model (T-matrix approach with the formulae of Wood and Brown-Korringa) and seismic attribute generation. The seismic attributes or seismic AVAZ data (reflection coefficients) has been obtained using the weak anisotropy approximation given by Ruger (Ruger, 1998; Ruger, 2002) for HTI media or the exact solution given by Schoenberg and Protazio (1992).

The inverse problem consists of estimating the model parameters $\mathbf{m}$ related with the fractures from reflection coefficient data $\mathbf{d}$. The solution to an inverse problem in a Bayesian setting is given by the posterior probability distribution $q(\mathbf{m}|\mathbf{d})$ over the model space $M$. $q(\mathbf{m}|\mathbf{d})$ basically carries all the information about the model originating from two sources. The first source is the data and the information is given by the likelihood function $f(\mathbf{d}|\mathbf{m})$, while the second source is the prior
information, which is expressed through a probability density $p(m)$. The solution for posterior PDF based on Gaussian statistics is also given by (Aster et al., 2005)

$$q(m|d) = N \cdot e^{-J(m)}$$

where $N$ is a constant and $J(m)$ is the objective or cost function and for Gaussian statistics given by (Aster et al., 2005)

$$J(m) = \frac{1}{2} \left[ (G(m) - d)^T C_D^{-1} (G(m) - d) + (m - m_0)^T C_M^{-1} (m - m_0) \right].$$

where $m_0$ is defined as the mean value of the a priori distribution, and $C_D$ and $C_M$ are the covariance matrices for the data and for the model respectively. The posterior PDF represents the degrees of belief about the possible values of $m$ (the fracture density and azimuthal orientation) before and after observing the data $d$. If we have an uninformative prior then equation (3) reduces only to term describing the likelihood function. To quantify the uncertainty in the inverted model parameters (the fracture density and azimuthal fracture orientation), a calculation of marginal PDFs is required. In general, marginal PDFs can be obtained via numerical integration or Monte Carlo simulation. In this study, we have used the numerical integration method, because the number of unknown model parameters was only 2. Since the posterior PDF represents the probability density that the parameters of the fractures have certain values, its integral over the model space $M$ must necessarily equal to unity; that is

$$\int_M q(m|d)/m = 1.$$  \hspace{1cm} (4)

Equation (4) can be used to find the constant $N$ in Equation (2). For the inversion of distribution of fracture density or orientation on a map of reservoir, we have only found the maximum of equation (2) for each grid block (maximum a posteriori (MAP) solution) via a systematic search through all allowed points within a discretized version of the model parameters.

**Numerical Results and discussion**

For investigating the accuracy of Ruger’s approximation, we have used a simple rock physics model for a fractured medium (HTI) and compared the results of forward and inverse modelling based on Ruger’s approximation(s) and exact relations for reflection coefficients with small and large contrast (isotropic overburden and reservoir) and anisotropy changing only with fracture density (maximum fracture density up to 0.2). The elastic properties of overburden(s) used for small contrast were $V_p = 5.4$ (km/sec), $V_s = 3.09$ (km/sec) and $\rho = 2.6$ (g/cm$^3$), while for large contrast $V_p = 4.32$ (km/sec), $V_s = 2.54$ (km/sec) and $\rho = 2.44$ (g/cm$^3$), respectively. The elastic properties of the reservoir changing with fracture density (from 0 to 0.2) were $V_p = 5.45$ to 5.34 (km/sec), $V_s = 3.0963$ to 3.0959 (km/sec) and $\rho = 2.6322$ to 2.6306 (g/cm$^3$). For forward modelling, reflection coefficients (synthetic seismic AVAZ data) have been obtained from the top of the reservoir (HTI symmetry) as function of incidence angle and azimuth using approximate form and exact relations and compared for different azimuths, with incidence angle in the range of 0° to 40°. Different plots of Figure 1 show the percentage error analysis with respect to exact for small and large contrast with small (0.01 and 0.05), moderate (0.1) and large (0.2) fracture density. The relation used for obtaining the percentage error analysis is as follows,

$$\Delta = \left( \frac{R_e - R_a}{R_a} \right) \times 100 \%$$

where $\Delta$ is the error in percentage, $R_e$ are the reflection coefficients obtained from the exact method of Schoenberg and Protazio (1992), and $R_a$ are the reflection coefficients obtained from the Ruger approximations (Ruger 1998, Ruger 2002). This clearly shows the difference in percentage between exact and approximate relation for reflection coefficients for different contrasts and different levels of anisotropy (fracture density). For inverse modelling, the true data has been generated using the exact formula for reflection coefficients, and inversion has been done using exact and approximate solutions for reflection coefficients in a Bayesian setting. These results are then compared for the model...
parameters (e.g. fracture density and azimuthal fracture orientation). The standard deviation of measured seismic data was 10% for small contrast case and 5% for large contrast case. The marginal posterior PDFs for azimuthal fracture orientation and fracture density for different true set of model parameters along with small and large contrast are shown in Figure 2. Clearly the Ruger’s approximation(s) tend to underestimate the fracture density more for large contrast case even for small fracture density (0.05). For orientation we have an acceptable solution with small uncertainty in both the cases of small and large contrast. We have also considered a synthetic case of characterizing a fractured damage zone in terms of fracture density and orientation for one compartment of a vertically faulted reservoir using exact method and approximation(s). The fractured damage zone represents an HTI medium. We have assumed that the fracture density and the azimuthal fracture orientation vary exponentially with distance from the fault core (Upper plots of Figure 3 & 4). Ruger’s approximation(s) tend to underestimate the value of fracture density near to the fault core, but at least it recovers the trends of fracture density away from the fault core though the numerical values were different (Lower plot of Figure 3). For the case of orientation the Ruger’s approximation(s) perform satisfactorily with small uncertainty (Lower plot of Figure 4).

Conclusions and recommendations

The accuracy of Ruger’s approximation(s) has been investigated for different contrasts and different levels of anisotropy or fracture density on synthetic reflection coefficient data. For small contrast and small anisotropy (fracture density = 0.01) the maximum percentage error was 6%, which increase several orders of magnitude if large fracture density (0.2) is used. The maximum percentage error for large contrast and large anisotropy or fracture density may reach up to 70%. For inversion with small contrast case, the approximation(s) gives satisfactory solution with small uncertainty for moderate fracture density (0.1). In the case of a large contrast it underestimates the value of fracture density even with small fracture density (0.05). For orientation, we always had an acceptable solution with small uncertainty for different contrasts and different fracture densities. So it can be concluded, that for inversion with small contrast and moderate fracture density (0.1), Ruger’s approximation(s) for reflection coefficients can work satisfactorily. For all other cases such as small contrast with large fracture density (> 0.1) or large contrast with small fracture density (0.05), it always underestimates the value of fracture density and one must be careful while using approximation(s) instead of exact formula for reflection coefficients.

References

Figure 1: Plots showing the percentage error for the Ruger’s approximation(s) with respect to exact formula for different fracture density and contrast.

Figure 2: Marginal posterior PDFs for the fracture density and azimuthal orientation. Dashed and solid lines show the approximation(s) and exact formula for reflection coefficients, respectively.

Figure 3: True distribution of fracture density (top plot), inverted distribution using exact (middle), and inverted distribution using approximation(s) (bottom) for a vertically faulted reservoir case.

Figure 4: True distribution of fracture orientation (top plot), inverted distribution using exact (middle), and inverted distribution using approximation(s) (bottom) for a vertically faulted reservoir case.
Extended Abstract 3

72nd EAGE Conference & Exhibition incorporating SPE EUROPEC 2010
Barcelona, Spain, 14 - 17 June
K035
Seismic History Matching in Fractured Reservoirs Using a Consistent Stiffness-permeability Model - Focus on the Aperture
A. Shahraini* (University of Bergen/CIPR), M. Jakobsen (University of Bergen/CIPR) & A. Ali (University of Bergen)

SUMMARY
This paper proposes a method for characterization of naturally fractured reservoirs by quantitative integration of seismic and production data. The method is based on unified (T-matrix) model for the effective hydraulic and elastic properties of fractured porous media and a (nonlinear) Bayesian method of inversion which provides information about uncertainties as well as mean (or maximum likelihood) values. We consider a fractured reservoir as a porous medium containing a single set of vertical fractures characterized by an unknown fracture density, azimuthal orientation and aperture. We then look at the problem of fracture parameter estimation as a nonlinear Bayesian inverse problem and try to estimate the unknown fracture parameters by joint inversion of seismic AVAZ data and dynamic production data. Once the fracture parameters have been estimated the corresponding anisotropic stiffness and permeability can be estimated using consistent models. A synthetic example is provided to explain the workflow. It shows that seismic and production data complement each other, in the sense that the seismic data resolve a non-uniqueness in the fracture orientation and the production data helps to recover the true fracture aperture and permeability, because production data are more sensitive to the fracture aperture rather than the seismic data.
Introduction

A considerable percentage of hydrocarbons are trapped in fractured reservoirs. A proper characterization of fractured reservoirs can contribute to a better field development strategy. Historically many methods have been used for the characterization of fractured reservoirs. These methods include, well test, production data analysis, logs and seismic data analysis. Production data which are available in well locations have good time resolution, whilst seismic data have good areal resolution. This is one reason that seismic and production data are complementary to each other. Another reason is that the seismic data are not very sensitive to the fracture aperture whilst the production data are highly influenced by the fracture aperture. Therefore joint inversion appears to be necessary to estimate the fracture parameters including the fracture aperture and subsequently the fracture permeability. One of the first multidisciplinary approaches which integrates both seismic and production data for the fractured reservoir characterization was developed by Will, et al. (2005), which used a gradient based inversion method to invert for the fracture intensity and the fracture trend. Jakobsen et al. (2007b) and Jakobsen and Shahraini (2008) modified and extended Will et al.’s (2005) method in different ways for application in different faulted and/or fractured systems. The main improvement in the method was made by introducing unified rock physics models for the estimation of the fracture effective permeability and stiffness tensors based on the T-matrix approach which takes into account the effects of fracture-fracture interaction as well as fracture shape, orientation and density (Jakobsen et al. 2007b). In this study, we also use T-matrix method, but now we invert for the fracture aperture in addition to the fracture density and orientation. In all mentioned methods the authors assumed a known fracture aperture, and this of course could introduce some uncertainty on the inversion results, especially on the estimated effective permeability.

Jakobsen et al. (2007a) presented a workflow for estimating the upper bound anisotropic permeability of fractured reservoirs from seismic AVAZ analysis but a significant uncertainty is associated to the estimated permeability, due to the fact that the effective permeability tensor of a fractured porous medium is much more sensitive to the aperture of the fractures than the stiffness tensor. In present work we try to fill the gap by performing a joint inversion of production and seismic data with respect to the fracture density, aperture and orientation.

Methodology

Figure 1 show the workflow for forward and inverse modelling. The characterization method implemented in this study starts by building a fractured reservoir model, and then we use a consistent rock physics model to translate the fracture density, aperture and orientation to the effective stiffness and permeability tensors of the fractured reservoir model. The effective medium theory has been used to homogenise (or upscale) the heterogeneous grid (Figure 2). The effective stiffness tensor $C_d^{\ast}$ of the fractured porous medium for the dry case is given by (Jakobsen, et al. 2007b):

$$C_d^{\ast} = C^{(0)} + C_1 : (I_4 + C_1^{-1} : C_2)^{-1},$$

Here, $C^{(0)}$ is the background stiffness tensor, $I_4$ is the identity for fourth-rank tensors; $C_1$ is a of first order corrections for the effect of isolated fractures and $C_2$ is the second order correction for the effects of fracture-fracture interaction (Jakobsen et al. 2007b). Similarly the effective permeability can be obtained:

$$k^{\ast} = k^{(0)} + k_1 : (I_2 + k_1^{-1} : k_2)^{-1}.$$
(equation 3) has been introduced into equation (2) to make the effective permeability of the fractured reservoir as a function of fracture aperture as well as fracture density.

\[
k^{(r)} = \frac{(a^{(r)})^2}{12},
\]

where, \(a^{(r)}\) is the fracture aperture and \(k^{(r)}\) is fracture permeability for fractures of type \(r\).

Figures 3 and 4 show the effective stiffness and permeability tensors as a function of fracture density and aperture, using these models.

In the next step we use the generalization of Gassman relation (Brown and Korringa 1975) to obtain the saturated effective stiffness tensors. Then we use Ruger’s approximation (Ruger 2002) for HTI media to obtain PP reflection coefficients. Similar to the seismic modelling we perform fluid flow modelling. To do the fluid flow simulation we use the calculated effective permeability tensors for every grid block as an input permeability map to the reservoir simulator in order to calculate the production data such as well oil production and well bottomhole pressure as a function of the fracture parameters.

These calculated production and seismic attributes alongside with the observed data will be used in a Bayesian method of inversion based on Markov chain Monte Carlo (Tarantola 2005) method (which is adapted for the current problem) to estimate the fracture parameters (fracture aperture, density and orientation) and finally the fracture permeability of the reservoir.

**Numerical example and discussion**

We consider a vertical fractured reservoir model consists of one production and one injection well in two corners of the reservoir. The fluid flow model is a simple black oil model with 15*15*1 grid blocks in x, y and z directions, respectively (Figure 2). We assume a true fracture density, aperture and orientation of 0.09, 0.002 cm and 60°, respectively. We then try to estimate the true values using calculated AVAZ and production data. The overburden is assumed to be isotropic and the reservoir is transversely isotropic with horizontal axis of symmetry.

The reservoir is assumed to have a background (matrix) permeability of 100 mD and the porosity of 17%. Based on true fracture parameters and using the rock physic model (T-matrix approach) the true permeability tensor of the reservoir is found to be 112.37, 104.12, 116.49 and 7.15 mD corresponding to \(k_{11}^*, k_{22}^*, k_{33}^*\) and \(k_{12}^*\), respectively.

Figures 5-12 show some of the results using seismic AVAZ data and/or production data. A measurement error of 5 and 10 % was assumed for the production and seismic data, respectively. As expected due to the low sensitivity of seismic data to the fracture aperture, the aperture could not be estimated using only seismic data (Figure 5). On the other hand fracture density and orientation were recovered with low standard deviation (Figures 6-7). Since seismic data are not very sensitive to the fracture aperture, the posterior PDFs of the effective permeability obtained with the inversion of seismic data do not give a good estimation of the true permeability. For example Figures 8 shows \(k_{33}^*\) which does not give good estimation of the true \(k_{33}^*\) which is 116 mD.

Due to high sensitivity of fracture permeability to the fracture aperture (equation 3) production data gives good estimation of fracture aperture, but it gives non-unique results for the fracture orientation, and it fails to estimate the fracture density. It gives non-unique result for the off diagonal term \(k_{12}^*\).

The best results were obtained when doing a joint inversion of seismic and production data. This example shows that the joint inversion gives fairly good estimation for all fracture parameters (Figures 9-11) as well as the effective anisotropic permeability (e.g., \(k_{33}^*\) in Figure 12).
Conclusions

By using a simple model of a fractured reservoir which is anisotropic (due to a single set of vertical fractures), we have demonstrated that a joint inversion of seismic and production data is necessary to estimate the fractured reservoir parameters and the effective permeability of the reservoir. Previous studies show that there is a high uncertainty in the estimated effective permeability using only seismic data due to low sensitivity of seismic data to the aperture. In this study we show that this problem can be solved if we integrate the production data with seismic data in the inversion workflow. Since the effective permeability tensor (and production data) is highly influenced by the aperture, the estimated effective permeability by joint inversion of seismic and production data is more accurate. This study also shows that the seismic data not only can help to reduce the uncertainty of estimated fracture orientation and density but also they can help to solve the problem of non-uniqueness of the fracture orientation obtained with only production data. The next step may be to account for the pressure effect. Pressure changes can have important effect on the aperture and subsequently on the permeability of the reservoir. Also further improvement in the method can be achieved with incorporating percolation effects into rock physic part for calculation of effective permeability. Assimilation methods such as Ensemble Kalman Filter (EnKF) can also be used to efficiently invert for the high number of unknown fracture parameters which can vary from grid block to the grid block.

References


Figure 1: Work flow diagram. Figure 2: Upscaling- Reservoir model. Figure 3: Effective stiffness as a function of fracture density and aperture for a fixed orientation.
Figure 4: Effective permeability as a function of fracture density and aperture for a fixed orientation.

Figure 5: Samples of posterior pdf of fracture aperture using seismic data only.

Figure 6: Samples of posterior pdf of fracture density using seismic data only.

Figure 7: Samples of posterior pdf of fracture orientation using seismic data only.

Figure 8: Samples of posterior pdf of $k_{33}$ using seismic data only.

Figure 9: Samples of posterior pdf of fracture aperture using joint inversion.

Figure 10: Samples of posterior pdf of fracture density using joint inversion.

Figure 11: Samples of posterior pdf of fracture orientation using joint inversion.

Figure 12: Samples of posterior pdf of $k_{33}$ using joint inversion.
10981. On the Relative Importance of Global and Squirt Flow in Cracked Porous Media

A. Ali* (University of Bergen) & M. Jakobsen (University of Bergen)

Summary

In acoustic or seismic modelling, a cracked porous medium can be replaced by a long wavelength, equivalent homogenous medium that can be both anisotropic and viscoelastic due to microstructural alignments and global or squirt flow, respectively. We here investigate the relative importance of global and squirt flow in cracked porous media. We use the unified theory of global and squirt flow of Jakobsen and Chapman in cracked porous media. The investigation has been done for the implication of using the correct wave number for the relative importance of global and squirt flow in cracked porous media saturated with different fluids characterized by different viscosities and for the observations of negative velocity dispersion dealing the phenomenon of wave-induced fluid flow in models of cracked porous media where global flow effects dominates. Our numerical results suggest that the observations of negative velocity dispersion in Jakobsen and Chapman theory still remain, even if we use the correct effective wave number. The peak of attenuation always moves towards relatively lower frequencies in the case of solution with correct wave number as compared to the solution with approximate wave number for different viscosities.
Introduction

In acoustic or seismic modelling, a cracked porous medium can be replaced by a long wavelength, equivalent homogenous medium that can be both anisotropic and viscoelastic due to microstructural alignments and wave induced fluid flow, respectively. Wave induced fluid flow can occur in the form of global or squirt flow. Global flow, also known as Darcy flow, is caused by pressure gradients at the scale of the acoustic wavelength and in the direction of wave propagation, whereas squirt flow is caused by the pressure gradients at the microscopic or mesoscopic scale and in the direction potentially different from that of the wave propagation. There have been some attempts to develop special (phenomenological or microstructural) theories of global flow (Biot, 1962; Hudson et al. 1996), special (microstructural) theories of squirt flow (Mavko and Nur, 1975; O’Connell and Budiansky, 1977; Chapman, 2003), and unified (phenomenological and microstructural) theories of global and squirt flow (Dvorkin and Nur, 1993; Hudson et al. 1996; Chapman et al. 2002; Jakobsen et al. 2003). The unified theory of Jakobsen et al. (2003), which contains the theory of interconnected cracks developed by Hudson et al. (1996) as a special case, originally had an error related to fluid mass conservation, but this error was recently corrected by Jakobsen and Chapman (2009). Unlike the unified theory of Chapman et al. (2002), the corrected version of the theory of Jakobsen et al. (2003) presented by Jakobsen and Chapman (2009) can deal with effects of anisotropy as well as attenuation. In the unified theories for global and squirt flow presented by Hudson et al. (1996), Jakobsen et al. (2003) and Jakobsen and Chapman (2009), the effective stiffness tensor \( C^* \) depends on the effective wave vector \( k \) and as well as on the angular frequency \( \omega \). Since we are dealing with theories of the coupled physical process of wave-induced fluid flow, this is perhaps not surprising. However, it means that we are effectively dealing with seismic examples of non-local elasticity. In all previous studies, these non-local effects have been avoided simply by replacing the effective wave vector by the unperturbed wave vector associated with the waves in the solid reference medium; that is, by using the approximation \( k = \omega / \nu^{(0)} \), where \( \omega \) is the angular frequency, \( \nu^{(0)} \) is the speed of the wave mode under consideration in the solid matrix and \( k \) is the length of \( k \). This approximation for effective wave vector \( k \) was considered to be one of the possible explanations for some rather counter-intuitive observations of negative velocity dispersion in numerical experiments dealing with the phenomenon of wave-induced fluid flow in models of cracked porous media where global flow effects dominates (Jakobsen and Chapman, 2009). An important aim of this study is to investigate if this velocity dispersion will remain negative in these models, if we use the effective wave vector rather than the unperturbed wave vector; that is, if we implement the global flow part of the theory in a proper manner. A further aim is to investigate the implication of using the correct wave number for the relative importance of global and squirt flow in cracked porous media saturated with different fluids characterized by different viscosities. A change in the viscosity may lead to a shift of the attenuation peak towards lower or higher frequencies, depending on the mechanism of wave-induced fluid flow.

Effective stiffness tensor and T-matrix for a communicating cavity

We consider a model in which a solid contains inclusions or cavities characterized by different shapes, orientations and different spatial distributions. The effective stiffness tensor \( C^* \) with the assumption that the distribution of inclusions is same for all pair of inclusions is given by Jakobsen et al. (2003),

\[
C^* = C^{(0)} + C_1 : (I_4 + G_d : C_1)^{-1}, \quad C_1 = \sum_r \nu^{(r)} G(r).
\]

Here, ‘:\’ denotes the double scalar product, \( C^{(0)} \) is the stiffness tensor of the solid matrix material, \( I_4 \) is the (symmetric) identity for second-rank tensors, \( \nu^{(r)} \) is the volume concentration for inclusions of type \( r \) and \( G_d \) tensor is given by the strain Green’s function integrated over an ellipsoid determining the symmetry of the correlation function for the spatial distribution of inclusions or cavities (see Jakobsen et al. 2003).
The t-matrix for an inclusion of type \( r \) fully saturated with a homogenous fluid is given by (Jakobsen and Chapman, 2009)

\[
\mathbf{t}^{(r)} = \mathbf{t}^{(r)}_d + \mathbf{t}^{(r)}_s : \mathbf{S}^{(0)} : (\mathbf{I}_2 \otimes \mathbf{\psi}^{(r)}) : \mathbf{C}^{(0)},
\]

and,

\[
\mathbf{t}^{(r)} = -\mathbf{C}^{(0)} : (\mathbf{I}_4 + \mathbf{G}^{(r)}) : \mathbf{C}^{(0)}^{-1},
\]

where, \( \mathbf{S}^{(0)} = (\mathbf{C}^{(0)})^{-1} \) is the compliance tensor of the solid matrix material; \( \mathbf{I}_2 \) is the identity tensor for second-rank tensors; \( \otimes \) is the dyadic tensor product (see Jakobsen et al. 2003), \( \mathbf{G}^{(r)} \) is a fourth-rank tensor given by the strain Green’s function (for a material with properties given by \( \mathbf{C}^{(0)} \)) integrated over a characteristic spheroid having the same shape as inclusions of type \( r \) (see Jakobsen and Chapman 2009) and \( \mathbf{\psi}^{(r)} \) is a second-rank tensor (fluid polarization tensor) that relates the fluid pressure to the applied stress. The fluid polarization tensor \( \mathbf{\psi}^{(r)} \) is given by (Jakobsen and Chapman, 2009)

\[
\mathbf{\psi}^{(r)} = -\Phi \sum_{r} \phi^{(r)} I_2 : \mathbf{K}^{(r)}_d \frac{1 - i \omega \tau}{1 + i \omega \tau} - i \omega \tau \kappa_f I_2 : \mathbf{K}^{(r)}_d,
\]

with

\[
\gamma^{(r)} = 1 + \kappa_f I_2 : (\mathbf{K}^{(r)} - \mathbf{S}^{(0)}) I_2, \quad \mathbf{K}^{(r)}_d = (\mathbf{I}_4 + \mathbf{G}^{(r)} : \mathbf{C}^{(0)})^{-1} : \mathbf{S}^{(0)},
\]

and

\[
\hat{\Theta} = \frac{\Theta_f}{\kappa_f} \left[ \sum_{r} \phi^{(r)} \gamma^{(r)} \frac{i \Gamma_f k_i k_j}{\eta_f \omega (1 - \Delta)} \right]^{-1}, \quad \Delta = \frac{\Gamma_f k_i k_j}{\phi \eta_f \omega} \tau.
\]

Here \( \Gamma_f \) are the components of the permeability tensor of the reservoir and \( k_i \) and \( k_j \) are components of the wave number vector \( \mathbf{k} \), where \( i, j = 1, 2, 3 \). \( \kappa_f \) is the bulk modulus for fluid, \( \eta_f \) is the viscosity of the fluid, \( \phi \) is the total porosity, \( \omega \) is the angular frequency of the propagating plane-harmonic wave and \( \tau \) is the relaxation time constant.

### Numerical Experiments

We performed numerical experiments to investigate the relative importance of global and squirt flow in idealized models for cracked porous media. For the background elastic properties, we take \( \kappa = 37 \text{ GPa}, \mu = 44 \text{ GPa}, \) and \( \rho = 2.5 \text{ g/cm}^3 \) to simulate the properties of quartz. The characteristic time scale constant or squirt flow relaxation time for micro-porosity with water as a saturating fluid was taken to be \( \tau_w = 10^{-5} \text{s} \). For other fluids (oil or gas), the squirt flow relaxation time constant was calculated using the relation \( \tau_f = \eta_f (\tau_w / \eta_w) \), where \( \tau_f \) is the relaxation time for the fluid (can be oil or gas) and \( \eta_f \) is the viscosity of fluid. The viscosity of water, oil and gas was set to \( 10^{-3} \text{ Pa s}, 3 \times 10^{-7} \text{ Pa s} \) and \( 2 \times 10^{-2} \text{ Pa s} \), respectively. Figures 1 and 2 show the comparison of iterative (with the use of correct wave number and the starting point for iterations being the wave-number used for the solid matrix) and approximate (the approximation for wave-number \( \omega / V^{(0)} \)) solutions for different models in the form of T-matrix estimates of velocity and attenuation spectra of plane wave propagation to higher order in porosity and crack density for different fluids characterized by different viscosities.

For the model with randomly oriented cracks (Figure 1), we observed positive dispersion (squirt flow/local pressure gradients) at relatively low frequencies due to different orientations of randomly oriented cracks along with negative dispersion (global flow/Darcy flow) at relatively higher frequencies, when water and oil were used as the saturating fluids in porous matrix. The squirt and
global flow parts change their positions i.e. we observed global flow at relatively lower frequencies and squirt flow at relatively higher frequencies, when gas was used as a saturating fluid in porous matrix. There were large differences in the approximate solution with respect to iterative solution only for the global flow part. The peak of attenuation for the global flow part always shifts towards relatively low frequencies in the case of iterative solution as compared to the approximate solution for different viscosities. The magnitude of global flow part dominates over the squirt flow part. Increase in permeability from 1 Darcy to 10 Darcy also shifts the attenuation peak for both global and squirt flow to lower frequencies.

For the model consisting of pores and randomly oriented cracks (Figure 2) we observed both squirt and global flow with the dominance of squirt flow part. No global flow was observed when gas was used as a saturating fluid in porous matrix. Again the peak of attenuation for the global flow part shifts towards relatively low frequencies in the case of iterative solution as compared to the approximate solution, when water and oil were used as the saturating fluids in porous matrix. The squirt and global flow attenuation peaks shifts to lower frequencies when permeability is increased from 1 to 10 Darcy.

Conclusions

We have investigated the relative importance of global and squirt flow in cracked porous media. Our numerical results suggest that the observations of negative velocity dispersion in Jakobsen and Chapman (2009) theory still remain, even if we use the correct effective wave number, when dealing with the phenomenon of wave-induced fluid flow in models of cracked porous media where global flow effects dominates. The comparison between iterative and approximate solution show differences in terms of a shift in the dispersion part of the velocity spectra, while for attenuation spectra it shows a shift in the attenuation peak. The peak of attenuation moves always toward relatively lower frequencies for global flow part in the case of iterative solution as compared to the approximate solution for different viscosities. For a model of pores and randomly oriented cracks, the magnitude of squirt flow dominates over global flow and global flow occurs at relatively higher frequencies. We may also conclude from the experiments that at seismic frequencies global flow effects are not so important and needs high permeability and low viscosity to have an effect.

References


**Figure 1** Comparison of iterative and approximate solution for T-matrix estimates of velocity and attenuation spectra of the plane wave propagation for a model consisting of randomly oriented cracks with different fluids (water, oil, and gas). The aspect ratio of randomly oriented cracks was set to 1/1000, while fracture density was set to 0.4. Panels (a) and (b) show results with a permeability of 1 Da. Panels (c) and (d) show the same as panels (a) and (b) but with permeability of 10 Da.

**Figure 2** Same as Figure 1 but for a model of pores and randomly oriented cracks. Panels (a) and (b) shows result with permeability of 1 Da. Panels (c) and (d) shows same as panels (a) and (b) but with permeability of 10 Da.
Extended Abstract 5

73rd EAGE Conference & Exhibition incorporating SPE EUROPEC 2011
Vienna, Austria, 23-26 May 2011
10650. Seismic Characterization of Reservoirs with Multiple Fracture Sets Using Velocity and Attenuation Anisotropy Data

A. Ali* (University of Bergen) & M. Jakobsen (University of Bergen)

Summary

The successful management of fractured reservoirs depends upon improved characterization of fractured systems which often provide pathways for fluid flow during production. Knowledge about spatial distribution of fracture density and azimuthal fracture orientation can greatly help in optimizing production from fractured reservoirs. Frequency dependent measurements of seismic anisotropy or seismic velocity and attenuation can potentially give important information about the fractures present in the reservoir. In this study, we use frequency dependent velocity and attenuation data to infer information about the multiple fracture sets present in the reservoir. We model a reservoir containing two sets of fractures oriented in different directions characterized by unknown azimuthal fracture orientations and fracture densities. The method is based on a viscoelastic T-matrix approach and a Bayesian method of inversion that provides information about uncertainties as well as the mean values. We then try to estimate the unknown azimuthal fracture orientations and fracture densities from velocity and attenuation data as a function of frequency and azimuth. Our synthetic example shows that velocity data alone cannot recover the unknown fracture parameters. It also shows how joint inversion of velocity and attenuation data greatly reduces the uncertainty in the unknown fracture parameters.
Introduction

The successful management of fractured reservoirs depends upon improved characterization of fracture systems which often provide pathways for fluid flow during production. Alignment of these fracture systems to preferred orientations will lead to direction dependent velocities and significant permeability anisotropy in the reservoir. This suggests the use of seismic anisotropy to determine the orientation of fractures (Sayers, 2009). Measurements related to frequency dependence of seismic anisotropy or seismic velocity and attenuation can potentially give important information about the fracture systems and fluid saturation (see Liu et al. 2006; Chapman, 2009).

Wave induced fluid flow and multiple scattering are believed to be the main driving mechanisms behind the attenuation of seismic waves. Scattering attenuation can be safely ignored in the long wavelength domain i.e. when fractures are much smaller than the seismic wavelength. This is due to the fact that the propagating seismic wave or flowing fluid only sees a homogenized structure and not the individual pores, micro-cracks or fractures. Wave induced fluid flow can occur at microscopic scale of pores/micro-cracks, the mesoscopic scale of fractures or macroscopic scale of seismic wavelengths. As discussed by Jakobsen (2004), at the scale of wavelength the effective permeability of the fractured reservoir becomes the most important factor controlling the wave induced fluid flow.

The objective of this study is to infer more information about multiple fracture systems using frequency dependent velocity and attenuation data as a function of azimuth. This has been done by some authors before in the context of forward modelling (see Liu et al. 2006; Chapman, 2009), but here we perform this in the inverse modelling case. We use the viscoelastic T-matrix approach of Jakobsen et al. (2003) and Jakobsen and Chapman (2009) representing the most general model among the inclusion models, because it allows for non-dilute concentration of cavities characterized by different shapes, orientations and spatial distributions. Numerical example is presented about the inverse problem of finding the fracture parameters (azimuthal fracture orientations and the fracture densities) for the case of open (supporting fluid communication) fracture sets. This also shows how joint inversion of velocity and attenuation data helps to reduce the uncertainty in the estimation of fracture parameters.

The forward problem

The non-linear forward problem is defined by:

\[ d = G(m). \]  

Here, \( d \) is a vector of observable quantities (e.g., seismic velocity and attenuation data as a function of frequency and azimuth) and \( m \) is a vector of model parameters related with fractures (e.g., the azimuthal fracture orientations (\( \psi_1 \) and \( \psi_2 \)) and the fracture densities (\( \epsilon_1 \) and \( \epsilon_2 \))). The function \( G \) is based on a combination of the viscoelastic rock physics model and seismic attribute generation (seismic velocity and attenuation data as a function of frequency and azimuth).

We consider a model in which a solid contains inclusions (fractures) characterized by different shapes, orientations and spatial distributions. The effective stiffness tensor \( C^* \) of a fractured porous medium corresponding with the assumption that the distribution of fractures is same for all pair of fractures is given by Jakobsen et al. (2003),

\[ C^* = C^{(0)} + C_1 : (I_4 + G_d : C_1)^{-1}, \quad C_1 = \sum_r \nu^{(r)} t^{(r)}. \]  

Here, ‘:’ denotes the double scalar product, \( C^{(0)} \) is the stiffness tensor of the dry porous matrix, \( I_4 \) is the (symmetric) identity for second-rank tensors, \( \nu^{(r)} \) is the volume concentration for fractures of type \( r \) and \( G_d \) tensor is given by the strain Green’s function integrated over an ellipsoid determining the symmetry of the correlation function for the spatial distribution of fractures (see Jakobsen et al. 2003).
The t-matrix for an inclusion of type $r$ fully saturated with a homogenous fluid can be written as (see Jakobsen and Chapman, 2009)

$$
t^{(r)}(\Omega, \alpha, \Omega, \alpha, \Omega, \alpha, \kappa_s, \tau); 
$$

where, $v = (v^{(1)}, ..., v^{(n)})$ symbolizes the volume concentration for each inclusion set, $\Omega = (\Omega^{(1)}, ..., \Omega^{(n)})$ symbolizes the angles determining the orientation of each inclusion set relative to the crystallographic axes of the material with properties given by $c^{(0)}$, $\alpha = (\alpha^{(1)}, ..., \alpha^{(n)})$ symbolizes the aspect ratio for each inclusion set, $k_f$ is the bulk modulus of the saturating fluid, $f$ is the viscosity of the fluid, $\eta_f$ is the effective permeability tensor and $\tau = (\tau^{(1)}, ..., \tau^{(n)})$ symbolizes the relaxation time constant for each cavity set. For a porous medium consisting of two fractures (inclusions) sets (as assumed in this study) with all other parameters known, we can write $t^{(r)}$ explicitly as

$$
t^{(r)} = t^{(r)}(\psi_1, \psi_2, \epsilon_1, \epsilon_2). 
$$

Here volume concentration of each fracture set is represented in terms of its fracture density ($\epsilon_1$ and $\epsilon_2$) and ($\psi_1$ and $\psi_2$) represent the azimuthal fracture orientations.

The real-valued phase velocities and attenuation factors can be obtained by inserting the viscoelastic effective stiffness tensor $C^*$ into the Christoffel equation, which can be solved by using eigenvalue/eigenvector method (see Carcione, 1995). The phase velocity is the reciprocal of the slowness and is given in the component form as

$$
\frac{1}{V_p} = \Re\left\{\frac{1}{V} \right\}^{-1} 
$$

and the quality factor $Q$ is defined as the ratio of the peak strain energy to the average loss energy density, and is given as

$$
\frac{\Re(V^2)}{\Im(V^2)} 
$$

(see Carcione, 1995).

The inverse problem

The inverse problem consists of estimating the model parameters $m$ related with fractures ($\psi_1$, $\psi_2$, $\epsilon_1$, $\epsilon_2$) from velocity and attenuation (as a function of frequency and azimuth) data $d$. The solution to an inverse problem in a Bayesian setting is given by the posterior probability distribution $\sigma(m)$ over the model space $M$. $\sigma(m)$ basically carries all the information about the model originating from two sources. The first source is the data and the information is given by the likelihood function $L(m)$. The likelihood of a given model $m$ is measured through its misfit or objective function $J(m)$. The objective or cost function $J(m)$ in case of Gaussian statistics for the inversion example (two fracture sets) used in this study can be written as

$$
J(\psi_1, \psi_2, \epsilon_1, \epsilon_2) = \frac{1}{2} \sum_{m=1}^{10} \sum_{l=1}^{12} \left[ \frac{V^{\text{calc}}(\psi_1, \psi_2, \epsilon_1, \epsilon_2) - V^{\text{obs}}}{\Delta V^{\text{obs}}_{mn}} \right]^2 + \frac{1}{2} \sum_{m=1}^{10} \sum_{l=1}^{12} \left[ \frac{Q^{\text{calc}}(\psi_1, \psi_2, \epsilon_1, \epsilon_2) - Q^{\text{obs}}}{\Delta Q^{\text{obs}}_{mn}} \right]^2. 
$$

The first term in the objective function is the misfit between the calculated and observed velocities summed over 10 frequencies and 12 azimuths. Similarly, the second term is the misfit between calculated and observed attenuation data summed over 10 frequencies and 12 azimuths. The velocity and attenuation data are weighted according to uncertainties in the measurements. The second source is the prior information, which is expressed through a probability density $\rho(m)$. These densities may be simple Gaussian PDF’s or they may be more complicated ones. In our synthetic test case, we have considered a constant prior distribution. Thus our posterior probability distribution is same as the likelihood function and for the inversion example used in this study, it can be written as

$$
\frac{1}{2} \sum_{m=1}^{10} \sum_{l=1}^{12} \left[ \frac{V^{\text{calc}}(\psi_1, \psi_2, \epsilon_1, \epsilon_2) - V^{\text{obs}}}{\Delta V^{\text{obs}}_{mn}} \right]^2 + \frac{1}{2} \sum_{m=1}^{10} \sum_{l=1}^{12} \left[ \frac{Q^{\text{calc}}(\psi_1, \psi_2, \epsilon_1, \epsilon_2) - Q^{\text{obs}}}{\Delta Q^{\text{obs}}_{mn}} \right]^2.
$$
\[ L(\psi_1, \psi_2, \varepsilon_1, \varepsilon_2) = N \cdot e^{-J(\psi_1, \psi_2, \varepsilon_1, \varepsilon_2)}. \]  

(6)

Where, \( N \) is normalization constant that can be found by the fact that the integral of the posterior over \( M \) is equal to 1. To quantify the uncertainty in the inverted model parameters the exploration of the posterior PDF can be done by the Monte Carlo sampling (since the forward model is not given explicitly). We have adapted the Metropolis algorithm to the problem of sampling the posterior probability density in a Bayesian setting (see Tarantola, 2005).

**Numerical Experiment**

We now perform an inverse numerical experiment to recover the true fracture parameters (the azimuthal fracture orientations and fracture densities) using the velocity and attenuation data as a function of frequency and azimuth. We consider the case for two fracture sets, when both fracture sets are open (supporting fluid communication). The tensor of the viscoelastic stiffness constants can be viewed as a function of parameters related with fractures. We let the background stiffness tensor \( C^{(0)} \) to simulate the properties of calcite and a porosity of 9\% was used. The rock was considered to be fully saturated with water. The viscosity of water (saturating fluid) was set to \( 10^{-3} \) Pa s. The aspect ratio of randomly oriented cracks was set to 0.001, while crack density of randomly oriented cracks was set to 0.1. The parameters related with fracture geometry like fracture lengths and aspect ratios were assumed to known from geologic outcrop data/well log data and set to 20 cm and 0.001, respectively. Following the analysis of Agersborg et al. (2007) we assume that the relaxation time constant \( \tau \) will be same for the pores and randomly oriented cracks in the micro-porosity and can be found from core plug velocity and attenuation measurements. The relaxation time of fractures \( \tau_f \) can be calculated according to their size from the relation \( \tau_f = (a_f / \xi) \tau_m \), where \( a_f \) is the radius of fractures, \( \xi \) is the size of the grains (\( 200 \times 10^{-6} \) m assumed in this study) and \( \tau_m \) is the relaxation time for the micro-porosity (\( 2 \times 10^{-7} \) s assumed in this study).

The true azimuthal fracture orientations (\( \psi_1 \) and \( \psi_2 \)) and fracture densities (\( \varepsilon_1 \) and \( \varepsilon_2 \)) were set to (35°, 108°) and (0.03, 0.05), respectively. Figure 1 shows an example of velocity and attenuation data as a function of frequency within seismic range (< 100Hz) and azimuth used for inversion in this study. Figure 2 represent the result of Monte Carlo Markov Chain inversion for the parameters related with fractures using only velocity data as a function of frequency and azimuth. Clearly, velocity data alone cannot recover the fracture parameters. Figure 3 represent the result of Monte Carlo Markov Chain inversion for the fracture parameters, using both velocity and attenuation data as a function of frequency and azimuth producing the best results.

![Figure 1 Seismic velocity and attenuation data as a function of frequency (seismic range < 100Hz) and azimuth.](image-url)
Conclusions

For fractured reservoir containing multiple (two) sets of fractures, we found that in principle we can estimate the azimuthal fracture orientations and fracture densities from seismic velocity and attenuation data as a function of frequency and azimuth, if we have knowledge about the porous matrix, saturating fluid(s) and fracture geometry. Seismic velocity data alone may contain some information about the fracture densities, but that information alone produce highly uncertain estimates and also it is unable to differentiate between azimuthal fracture orientations. Integration of attenuation data leads to improved estimates of fracture parameters and better management of fractured reservoirs. These results may help in obtaining improved estimates of anisotropic permeability in complex fractured reservoirs systems. A satisfactory characterization of complex fractured reservoirs requires a model accounting for frequency-dependent anisotropy.

References


