Modelling of immiscible WAG with emphasis on the effect of capillary pressure

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Dissertation for the degree philosophiae doctor (PhD) at the University of Bergen

2008
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2008
Preface

This dissertation was submitted for the degree Philosophiae Doctor at the University of Bergen. The dissertation consists of seven papers and an introduction including theoretical background. The papers are based on work done in the period 2004 to 2007 at the Centre for Integrated Petroleum Research at the University of Bergen.

The subject of this thesis is parameters governing three-phase flow in porous media with a special emphasis on the effect of capillary pressure. The effects of different parameters on flow and the relation between these parameters have been studied. The effect of capillary pressure on flow has been studied for both two- and three-phase cases. How three-phase capillary pressure differs from two-phase capillary pressure was investigated, and representations of three-phase capillary pressures were described.
Acknowledgements

I would like to thank my supervisor Professor Arne Skauge for helpful discussions and guidance.

I would also like to thank family, friends and colleagues for their support.

The financial support for this project was provided by StatoilHydro ASA.
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Papers
1. Introduction

Three-phase flow of gas, oil and water is common in petroleum reservoirs, especially because of use of water-alternating-gas injection (WAG). It is important to describe three-phase flow behaviour in porous media as accurately as possible. This will lower the risk of making wrong decisions.

Identifying the parameters governing three-phase flow in porous media is essential. The relation between these parameters is also important to find. These issues are addressed in this work.

Capillary pressure is in many cases neglected in reservoir simulators. This could be due to lack of measured data or the belief that the capillary pressure only has insignificant effect on the flow behaviour. This work tries to show the effect of capillary pressure on flow. The effect of capillary pressure on estimation of relative permeability and differential pressure has been studied.

The effect of three-phase characteristics and capillary pressure on the size of the three-phase area, breakthrough time of the injected fluids and oil recovery has been investigated. The features considered are three-phase representations of relative permeability, effect of trapped gas on residual oil, and capillary pressure effects.

Three-phase capillary pressure is poorly understood. It is very difficult to measure three-phase capillary pressure. Two-phase capillary pressure or models for the three-phase capillary pressure is often used as input to three-phase flow simulators. In this work the three-phase capillary pressure was predicted using a network model anchored to experimentally measured two-phase data.
This thesis consists of two parts; the first part presents a theoretical background for the work, and the second part is a collection of papers.

Chapter 2 to 7 of the theoretical background includes definitions and descriptions of wettability, interfacial tension, spreading, capillary pressure, hysteresis and relative permeability. Background for the network model used is presented in chapter 8. Chapter 9 describes three-phase modelling in the simulator Eclipse 100. A summary of the main results are given in chapter 10. Further work is discussed in chapter 11.

Seven papers are included. The titles of the papers are “Fluid Flow Properties of WAG Injection Processes”, “Features Concerning Capillary Pressure and the Effect on Two-Phase and Three-Phase Flow”, “Influence of Capillary Pressure on Estimation of Relative Permeability for Immiscible WAG Processes”, “Effect of implementing three-phase flow characteristics and capillary pressure in simulation of immiscible WAG”, “Prediction of Three-Phase Capillary Pressure Using a Network Model Anchored to Two-phase Data”, “Creating Three-Phase Capillary Pressures by Parameter Matching using a Modified Ensemble Kalman Filter” and “Progress in Immiscible WAG Modelling”.
2. Wettability

2.1 Definition

Wettability describes the fluids tendency to spread on a surface in the presence of other immiscible fluids. It is a measure of which phase preferentially adheres to a surface. When two immiscible phases, i.e. oil and water, is in contact with a rock surface the contact angle between the two fluids determines the wettability of the rock. The contact angle, $\theta_{ow}$, is by convention measured through the denser fluid, in this case the water phase, see figure 2.1. If the angle is between 0° and 60° to 75° the rock is defined as water-wet, if the angle is between 105° to 120° and 180° the rock is defined as oil-wet and if the angle is between these ranges, close to 90°, the rock is considered to have neutral or intermediate wettability.\textsuperscript{1-5}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.1.png}
\caption{A drop of water spreading on a solid surface.}
\end{figure}

2.2 Intermediate wettability classes

The intermediate wettability state is often divided into two classes; fractionally-wet and mixed-wet. Fractionally wet porous media have oil-wet and water-wet pores randomly distributed in the different pore sizes, see figure 2.2 a). In the mixed-wet case the oil-wet and water-wet pores are distributed by pore size. When the large pores are oil-wet and the small pores are water-wet it is called a mixed-wet large
state, see figure 2.2 b). Mixed-wet small is the case when the small pores are oil-wet and the large pores are water-wet, see figure 2.2 c).

![Diagram](image)

**Figure 2.2:** Distribution of oil-wet pores for a) fractionally-wet, b) mixed-wet large and c) mixed-wet small systems.
3. Interfacial tension

3.1 Definition

When two immiscible fluids, like oil and water, are in contact there exists a surface between them. The molecules near this surface have different degrees of attractions to their neighbouring molecules. This produces a free surface energy per surface unit, called interfacial tension. If the fluids are a gas and a liquid the forces acting on the interface is called surface tension. The interfacial or surface tension is usually denoted using the symbol $\sigma$. The interfacial tension between oil and water, $\sigma_{ow}$, is shown in figure 2.1. The energy barrier produced by interfacial tension prevents one liquid from becoming emulsified into the other.$^{2,3,5,6}$

3.2 The presence of a solid surface

The Young-Dupre equation follows from the force balance and is given by$^{2,5}$

$$\sigma_{os} - \sigma_{ws} = \sigma_{ow} \cos \theta_{ow},$$

where $\sigma_{os}$ is the interfacial tension between the oil and the solid, $\sigma_{ws}$ is the interfacial tension between the water and the solid, $\sigma_{ow}$ is the interfacial tension between the oil and the water and $\theta_{ow}$ is the angle between the interface and the solid.

In three-phase flow three contact angles are involved, the angle between the oil-water surface and the solid, between the gas-water surface and the solid and between the gas-oil surface and the solid. An expression for the relationship between the different contact angles in the three-phase case can be found by combining the force balance for the three cases seen in figure 3.1.
\[ \sigma_{os} - \sigma_{ws} = \sigma_{ow} \cos \theta_{ow}, \quad (2) \]

\[ \sigma_{gs} - \sigma_{ws} = \sigma_{gw} \cos \theta_{gw} \quad \text{and} \quad (3) \]

\[ \sigma_{gs} - \sigma_{os} = \sigma_{go} \cos \theta_{go}. \quad (4) \]

Figure 3.1: Force balance of fluid-solid combinations for three-phase flow.
When these three relations are combined we get the Bartell-Osterhof equation

\[ \sigma_{gw} \cos \theta_{pw} = \sigma_{ow} \cos \theta_{ow} + \sigma_{go} \cos \theta_{go}. \]  

(5)

3.3 Forces in a capillary tube

If a tube is lowered into water the surface tension and wettability will cause the water to rise inside the tube to a higher level than outside. The water will rise to the height where the capillary forces are balanced by the weight of the water column, see figure 3.2.

The upward force is given by

\[ F_{up} = (2\pi r) \sigma_{aw} \cos \theta_{aw}, \]  

(6)

where \( \sigma_{aw} \) is the surface tension between air and water, \( \theta_{aw} \) is the contact angle and \( r \) is the radius of the tube. The downward force is given by

\[ F_{down} = \pi r^2 h (\rho_w - \rho_a) g, \]  

(7)

where \( h \) is the height the water rises in the capillary tube, \( \rho_w \) is the density of water, \( \rho_a \) is the density of air and \( g \) is the gravity acceleration.

If we assume that the density of air is negligible when compared to the density of water the surface tension is given by

\[ \sigma_{aw} = \frac{r h \rho_w g}{2 \cos \theta_{aw}}. \]  

(8)
Figure 3.2: Capillary tube (Adapted from Ahmed).
4. Spreading

4.1 Definition

When three fluids are in contact there may be formed a three-phase contact line between the fluids. In figure 4.1 a) a lens of one of the fluids is formed on the contact surface of the two other fluids. In other cases one of the fluids will spread as a layer separating the two other fluids, as seen in figure 4.1 b).

![Figure 4.1](image)

\textit{Figure 4.1: a) Non-spreading system \hspace{1cm} b) Spreading system}

4.2 Force balance

Figure 4.2 shows a contact line formed between gas, oil and water. A lens of oil is resting on the gas-water interface. The gas-water interfacial tension is denoted $\sigma_{gw}$, the gas-oil interfacial tension is denoted $\sigma_{go}$ and the oil-water interfacial tension is denoted $\sigma_{ow}$. The angle between the gas-oil and oil-water interface is denoted $\theta_{go}$ and the angle between the gas-water and oil-water interface is denoted $\theta_{ow}$.
To get equilibrium the equation

$$\sigma_{gw} = \sigma_{ow} \cos \theta_{ow} + \sigma_{go} \cos \theta_{go}$$

(9)

has to be satisfied. This is the case when $\sigma_{gw} < \left( \sigma_{ow} + \sigma_{go} \right)$. A lens of oil will then be formed in the gas-oil-water system. If $\sigma_{gw} > \left( \sigma_{ow} + \sigma_{go} \right)$ the condition for equilibrium is not satisfied, and the oil will spread as a layer between the gas and the water.²

The spreading coefficient is defined as

$$C_{s,o} = \sigma_{gw} - \sigma_{ow} - \sigma_{go}.$$  (10)

If $C_{s,o}$ is equal to zero the oil is spreading and if the coefficient is less than zero the oil is non-spreading.
5. Capillary pressure

5.1 Definition

The difference in pressure across a curved boundary between two immiscible fluids is called capillary pressure. The capillary pressure is defined as the pressure in the non-wetting phase minus the pressure in the wetting phase\cite{1,5,8}

\[ P_c = P_{\text{non-wetting}} - P_{\text{wetting}}. \]  

(11)

The capillary pressure between e.g. oil and water, where water is the wetting phase, is thus defined as

\[ P_c = P_{\text{oil}} - P_{\text{water}}. \]  

(12)

The capillary pressure is dependent on the interfacial tension between the fluids and the rock, the geometry of the pores and the wettability\cite{6}.

5.2 Capillary pressure in a capillary tube

The capillary pressure between air and water at static conditions in a capillary tube can be expressed as

\[ P_{c,aw} = \frac{F_{\text{down}}}{A} = h(\rho_w - \rho_a)g, \]  

(13)

where \( F_{\text{down}} \) is the downward forces in the capillary tube, \( A \) is the cross section area of the tube, \( h \) is the height the water rises in the capillary tube, \( \rho_w \) is the density of water, \( \rho_a \) is the density of air and \( g \) is the gravity acceleration.

In general the capillary pressure is given as\cite{6}
\[ P_c = \Delta \rho \, g \, h. \] (14)

### 5.3 The Young-Laplace equation

An expression for calculating the capillary pressure across curved surfaces is provided by the Young-Laplace equation\(^9\)-\(^10\)

\[ P_c = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right), \] (15)

where \( P_c \) is the capillary pressure, \( \sigma \) is the interfacial tension, and \( R_1 \) and \( R_2 \) is the main radii of curvature for the surface. A curved surface is described by the two radii of curvature, \( R_1 \) and \( R_2 \), seen in figure 5.1.\(^3\),\(^5\)

\[ R_2 \]
\[ R_1 \]

*Figure 5.1: The principal radii of curvature, \( R_1 \) and \( R_2 \) (Adapted from Zolotukhin and Ursin\(^5\)).*
5.4 Cylindrical tube

If we consider a pore to be a straight cylindrical tube with radius $r$, the surface between the two fluids is approximated by a sphere surface, see figure 5.2. (This is not entirely accurate because the surface curves more near the walls of the tube.) The radius of curvature between the two fluids is then $R_1 = R_2 = R$, and

$$R = \frac{r}{\cos \theta}.$$  \hspace{1cm} (16)

![Figure 5.2: A straight cylindrical tube containing two fluids.](image)

The capillary pressure for a cylindrical tube of radius $r$ can therefore be written as

$$P_c = \frac{2\sigma \cos \theta}{r}.$$  \hspace{1cm} (17)
5.5 Capillary pressure curve - connection with saturation

It is evident from equation 15 that the capillary pressure is proportional to the inverse of the radii of curvature. The saturation of the different fluids will affect the radii of curvature. If we consider a water-wet reservoir we find that if the water saturation declines the radii of curvature is also reduced, as seen from figure 5.3. The conclusion is therefore that there must be an inverse connection between the capillary pressure and the water saturation for a water-wet reservoir. For reservoirs with other wettabilities the argumentation will be similar.\textsuperscript{3,5}

\textbf{Figure 5.3: Water between two sand grains in a water-wet reservoir (Adapted from Zolotukhin and Ursin\textsuperscript{5}).}

When the saturation of the wetting fluid is decreasing it is called a drainage process. A typical capillary pressure curve for drainage in the case of two-phase flow of oil and water is seen in figure 5.4. The minimum capillary pressure needed to start the invasion of oil in the porous medium is called the capillary entry pressure, $P_e$, and is
seen in figure 5.4. The capillary pressure increases as the water saturation decreases and goes towards infinity at the irreducible water saturation, $S_{wi}$.

![Figure 5.4: Capillary pressure curve.](image)

### 5.6 Drainage and imbibition - capillary pressure hysteresis

Drainage is the process where the non-wetting fluid displaces the wetting fluid, and imbibition is the process where the wetting fluid displaces the non-wetting fluid. Primary drainage is the process where oil is forced into the oil-water system by increasing the oil pressure, between point 1 and 2 in figure 5.5.

When decreasing the oil pressure after primary drainage the capillary pressure will decrease, and the water will spontaneously enter the rock, between point 2 and 3. This spontaneous imbibition will stop when the capillary pressure reaches zero, point 3. In order to increase the water saturation further the pressure in the water phase has
to be increased. The capillary pressure will get a more and more negative value, between point 3 and 4. This is called forced imbibition. When the residual oil saturation is reached the capillary pressure goes towards minus infinity.

Figure 5.5: Drainage and imbibition.

Because of hysteresis effects, the capillary pressure is different for drainage and imbibition\(^1\), see figure 5.5. The imbibition capillary pressure is lower than the drainage capillary pressure at the same saturation value. The cause of hysteresis effects is discussed in chapter 6.
5.7 Influence of pore size distribution

The pore size distribution will have a large effect on the shape of the capillary pressure curve. If the capillary pressure is given by

$$P_c = \frac{2\sigma \cos \theta}{r},$$

(18)

it is evident that a large pore radius will give low capillary pressure and a small pore radius will give high capillary pressure. In a drainage situation the largest pores will be invaded first and then the invasion will continue through smaller and smaller pores.

The entry pressure is given by

$$P_e = \frac{2\sigma \cos \theta}{r_{max}},$$

(19)

where $r_{max}$ is the maximum pore size in the porous medium. A high maximum pore radius, as seen in case 2 in figure 5.6, will give a low capillary entry pressure as seen in figure 5.7. If the maximum pore radius is higher, as in case 1, the entry pressure will be higher.

If the porous medium is well sorted, case 1 in figure 5.6, the capillary pressure will be almost flat in the middle section of the curve. In this case many pores are invaded at the same time because many pores have approximately the same size.\textsuperscript{11} If the porous medium is poorly sorted as in case 2 the capillary pressure will have a fairly steep slope for the entire saturation range\textsuperscript{11}, as seen in figure 5.7.
Figure 5.6: Different pore size distributions (Adapted from Selley\textsuperscript{11}).

Figure 5.7: Capillary pressure for different pore size distributions (Adapted from Selley\textsuperscript{11}).
5.8 Influence of wettability

The wettability of the rock will have a large effect on the capillary pressure curve.

When the rock is strongly water wet, a large amount of work is required for the drainage process where the wetting fluid is displacing the non-wetting fluid, see figure 5.8. Little or no work is required during imbibition, when the wetting fluid is displacing the non-wetting fluid. The wetting fluid will spontaneously imbibe into the strongly water-wet rock.

Also for a strongly oil-wet rock a large amount of work must be used to drain the wetting fluid, see figure 4.8. In this case the wetting fluid is oil, and the capillary pressure is often defined as negative when the water has the highest pressure, see eq. 12. Oil imbibes spontaneously into the strongly oil-wet rock until zero capillary pressure is reached.

For intermediate-wet cases some work is required in both the drainage and imbibition process, because the rock has no preference for either one of the fluids, see figure 5.8. Some of the water imbibes spontaneously into the core. When increasing the water pressure more water can be forced into the core. The forced imbibition has a negative capillary pressure value.\textsuperscript{12}
Figure 5.8: Capillary pressure for different wettability (Adapted from Killins).}

5.9 Capillary pressure correlations for two-phase flow

Correlations for capillary pressure describe the relationship between capillary pressure and saturation. One of the best-known correlations is the Brooks and Corey equation, where the capillary pressure is a function of the effective saturation

\[ P_c = P_c(S_e), \quad (20) \]

where the effective saturation is expressed as

\[ S_e = \frac{S_w - S_{wi}}{1 - S_{wi}}. \quad (21) \]
$S_w$ is the water saturation and $S_{wi}$ is the irreducible water saturation. The correlation is given as

$$S_e = \left( \frac{C}{P_c} \right)^2,$$  
(22)

where $C$ is a constant. Based on this work Standing\textsuperscript{14} formulated the expression

$$S_e = \left( \frac{P_e}{P_c} \right)^{\lambda},$$  
(23)

where $P_e$ is the capillary entry pressure and $\lambda$ is a pore size index.

Skjæveland et al.\textsuperscript{15} found an expression for mixed wettability based on the Brooks and Corey correlation. The capillary pressure is given as

$$P_c = \frac{c_w}{\left( S_w - S_{wi} \right)^{a_w}} + \frac{c_o}{\left( S_o - S_{or} \right)^{a_o}},$$  
(24)

where $a$ and $c$ are constants.

### 5.10 Capillary pressure correlations for three-phase flow

Killough’s method is often used to model three-phase capillary pressure\textsuperscript{23}. The three-phase capillary pressure is constructed as a weighted average between the two-phase drainage and imbibition curves\textsuperscript{24}

$$P_c = P_{cd} + F(P_{ci} - P_{cd})$$  
(25)

and $F$ given by
where $P_{Cd}$ is the drainage capillary pressure, $P_{Ci}$ is the imbibition capillary pressure, $S_{w,hyz}$ is the water saturation at the hysteresis reversal point, $S_{w,max}$ is the maximum water saturation attainable on the scanning curve when trapping of the other phases is subtracted and $E$ is a curvature parameter. Figure 5.9 show the three-phase scanning curves and the two-phase drainage and imbibition curves.

Helland and Skjæveland\textsuperscript{25} proposed a correlation for three-phase capillary pressure. This analytical function depends on the direction of saturation change. The input parameters have to be determined by matching to three-phase experiments.

$$F = \frac{\left(\frac{1}{S_{w} - S_{w,hyz} + E} - \frac{1}{E}\right)}{\left(\frac{1}{S_{w,max} - S_{w,hyz} + E} - \frac{1}{E}\right)}, \quad (26)$$

\textit{Figure 5.9: Capillary pressure for three-phase flow (Adapted from Eclipse technical description\textsuperscript{26}).}
Very few measurements of three-phase capillary pressure exist.\textsuperscript{27-28} Kalaydjian\textsuperscript{27} measured three-phase capillary pressure on an outcrop water-wet core and on unconsolidated material. The capillary pressure was a function of all three saturations for both drainage and imbibition. The three-phase capillary pressure had a higher value than the two-phase capillary pressure. This would indicate that Killough’s correlation\textsuperscript{23} could not be used to predict three-phase capillary pressure.

5.11 Three-phase capillary pressure from network modelling

A network model is a representation of the pore space geometry, and incorporates the physical rules of flow inside the pores. As we are getting closer to describing the microscopic flow satisfactorily the development of truly predictive models is possible. A large range of transport phenomena can be studied by use of network models.\textsuperscript{29}

A few attempts have been made to estimate three-phase capillary pressure using network models.\textsuperscript{30-33} The network models are usually anchored to measured two-phase capillary pressure data and three-phase capillary pressure is predicted. For more details on network modelling see chapter 8 and 10.
6. Hysteresis

6.1 Definition

The irreversibility of the fluid flow processes is called hysteresis. Capillary pressures and relative permeabilities are history dependent. They have different values depending on whether a fluid saturation is increasing or decreasing.

Several things may cause hysteresis. The main causes are contact angle hysteresis, pore geometry and trapping of fluids.

6.2 Contact angle hysteresis

The interfacial tensions and the contact angles between the fluids may be different for advancing, $\theta_a$, and receding cases, $\theta_r$, as seen in figure 6.1. Advancing contact angles are larger than the receding contact angles.

![Figure 6.1: Contact angle Hysteresis (Adapted from Zolotukhin and Ursin\textsuperscript{5}).]
Contact angle hysteresis could be caused by contamination of the liquid or the solid, surface roughness, or surface immobility on a macromolecular scale.\textsuperscript{34}

### 6.3 Trapping

Hysteresis can also be caused by trapping. During a flooding process phases can be trapped in the porous medium causing the process to be irreversible. Two models explaining the trapping mechanism is the pore doublet model and the snap-off model.

The pore-doublet model explains how fluids can be trapped when the fluids flow at different speeds through pores of different sizes.\textsuperscript{35-36} The capillary forces will pull the wetting fluid into the small pores, but the viscous forces will give a higher speed through the large pores. The competition between these two forces will lead to trapping in either the small or large pores. Figure 6.3 illustrates a process with low injection rate dominated by capillary forces. The wetting phase flows at a larger speed through the smaller pores, and the non-wetting phase is trapped in the larger pores.

![Figure 6.3: Pore doublet model; a) before trapping and b) after trapping.](image)
The snap-off model explains how fluid can be trapped if the aspect ratio, the relation between the size of the pore body and the pore throat, is high. In water-wet pores the oil can be trapped if the collar of water in the pore throat expands and meets in the middle, as seen in figure 6.4. The oil in the pore is then no longer connected to the rest of the oil-phase and cannot escape from the pore.

Figure 6.4: Snap-off model (Adapted from Chatzis et. al36).
7. Relative permeability

7.1 Definition

The permeability of a porous medium quantifies the ability of the medium to transport fluids through the pores. In Darcy’s law the permeability is considered to be a constant value, \( k \). This is true if the porous media is completely saturated with one fluid. This parameter is called the absolute permeability. The Darcy equation for horizontal flow of an incompressible fluid is given as

\[
q = \frac{kA}{\mu} \left( \frac{dP}{dx} \right),
\]  

(27)

where \( q \) is the flow rate, \( k \) is the absolute permeability, \( A \) is the cross-section area of the porous medium, \( \mu \) is the viscosity of the fluid, \( dP \) is the differential pressure and \( dx \) is the differential length of the porous medium.

If more than one fluid is present the flowing capability of each fluid is described by an effective permeability. The sum of the effective permeability values is often less than the absolute permeability, and sometimes the sum is higher than the absolute permeability. The relative permeability for each of the fluids is expressed as the relation between effective permeability and the absolute permeability. The relative permeability for fluid \( i \) is written as

\[
k_{ri} = \frac{k_i}{k},
\]  

(28)

where \( k_i \) is the effective permeability for fluid \( i \) and \( k \) is the absolute permeability.

The relative permeability of oil and water plotted against water saturation is seen in figure 7.1. At water saturations below the irreducible value, \( S_{wi} \), the water is immobile and the relative permeability to water is zero. The relative permeability of
water increases towards the maximum water saturation. The relative permeability of oil is zero at the residual oil saturation and increases towards lower water saturation i.e. higher oil saturation.

![Relative Permeability Curves](image)

**Figure 7.1:** Relative permeability curves for oil and water.

### 7.2 Relative permeability hysteresis

Hysteresis effects are also found for relative permeability as well as for capillary pressure. Relative permeability is also direction dependent; it has different values for a saturation depending on whether the saturation increases or decreases.

Figure 7.2 shows hysteresis for oil and water relative permeability in a water wet rock. At point A we have 100 % water saturation. Between point A and B primary drainage, oil flooding, is performed. The solid lines are the relative permeability curves for the drainage process. After drainage an imbibition process, water flooding,
occurs between point B and C. The dashed lines are the relative permeability curves for the imbibition process. The relative permeability curves for the imbibition process are different from the curves from the drainage process. The initial state of 100 % water saturation is not reached after imbibition due to capillary trapping of oil. The hysteresis effect is in general more substantial in the non-wetting phase.

The two dominating causes for hysteresis are, as described in chapter 6, differences in the contact angles and trapping of phases.

Figure 7.2: Hysteresis in relative permeability (Adapted from Bennion et. al).

7.3 Relative permeability correlations for two-phase flow

Empirical correlations for relative permeability are an important tool for creating relative permeability curves when no laboratory data is available. If relative permeability is used as a history matching parameter in reservoir simulation, the
correlations make it easier to change the relative permeability between each run. One of the most well known relative permeability correlations is the Corey model\textsuperscript{39}.

For a gas-oil system he expressed the relative permeability of oil as

\[ k_{ro} = \left(1 - S_g^*\right)^4 \]  \hspace{1cm} (29)

and the relative permeability of gas as

\[ k_{rg} = (S_g^*)^4 \left(2 - S_g^*\right) \]  \hspace{1cm} (30)

where

\[ S_g^* = \frac{S_g}{1 - S_{wi}} \]  \hspace{1cm} (31)

\( S_g \) is the gas saturation and \( S_{wi} \) is the irreducible water saturation.

For an oil-water system he expressed the relative permeability of oil as

\[ k_{ro} = \left(\frac{1 - S_w}{1 - S_{wi}}\right)^4 \]  \hspace{1cm} (32)

and the relative permeability of water as

\[ k_{rw} = \left(\frac{S_w - S_{wi}}{1 - S_{wi}}\right)^4 \]  \hspace{1cm} (33)

where \( S_w \) is the water saturation and \( S_{wi} \) is the irreducible water saturation.

Corey’s equations can only be applied to homogeneous well-sorted rocks. For other types of rocks the exponent must be changed. A more general expression for an oil-water system the water relative permeability is calculated by

\[ k_{rw} = (S_w)^{n_w} \]  \hspace{1cm} (34)
where $S_e$ is the effective saturation given by

$$S_e = \frac{S_w - S_{wi}}{1 - S_{wi}}$$  \hspace{1cm} (35)$$

and $a_w$ is a constant. The oil relative permeability is given by

$$k_{ro} = (1 - S_e)^{a_o},$$  \hspace{1cm} (36)$$

where $a_o$ is a constant.

Corey-type equations are simple and easy to use. In numerical simulation analytical expressions for the relative permeabilities are often used. For an oil-water system the relative permeabilities of oil and water are given as

$$k_{ro} = (k_{ro})_{S_{ow}} \left( \frac{1 - S_w - S_{orw}}{1 - S_{wi} - S_{orw}} \right)^{n_{ow}}$$  \hspace{1cm} (37)$$

and

$$k_{rw} = (k_{rw})_{S_{orw}} \left( \frac{S_w - S_{lw}}{1 - S_{lw} - S_{orw}} \right)^{n_{w}},$$  \hspace{1cm} (38)$$

where $(k_{ro})_{S_{ow}}$ is the oil relative permeability at irreducible water saturation, $(k_{ro})_{S_{orw}}$ is the water relative permeability at residual oil saturation after water injection, $S_w$ is the water saturation, $S_{wi}$ is the irreducible water saturation, $S_{orw}$ is the residual oil saturation after water injection, $n_{ow}$ is the oil relative permeability exponent and $n_{w}$ is the water relative permeability exponent.

For gas-oil systems the following expressions are used for oil and gas relative permeabilities

$$k_{ro} = (k_{ro})_{S_{og}} \left( \frac{1 - S_g - S_{lg}}{1 - S_{lg} - S_{lc}} \right)^{n_{og}}$$  \hspace{1cm} (39)$$
\[ k_{rg} = (k_{ro})_{S_{ro}} \left( \frac{S_g - S_{gc}}{1 - S_{lc} - S_{gc}} \right)^{n_o}, \]  

where \((k_{ro})_{S_{ro}}\) is the oil relative permeability at critical gas saturation, \((k_{rg})_{S_{org}}\) is the water relative permeability at residual oil saturation after gas injection, \(S_g\) is the gas saturation, \(S_{gc}\) is the critical gas saturation, \(S_{org}\) is the residual oil saturation after gas injection, \(S_{lc}\) is the total critical liquid saturation given as \(S_{lc} = S_{wi} + S_{org}\), \(n_{og}\) is the oil relative permeability exponent and \(n_g\) is the gas relative permeability exponent.

### 7.4 History dependent relative permeability correlations for two-phase flow

Hysteresis is neglected in the Corey equation, but in many cases significant hysteresis effects have been seen. Trapping of the phases is one of the important parameters to describe. Land\(^{40}\) suggested a relationship between the maximum saturation of the non-wetting phase and the trapped saturation of the non-wetting phase

\[ \frac{1}{S_{nw,max}} - \frac{1}{S_{nw,t}} = C, \]

where \(S_{nw,max}\) is the maximum saturation of the non-wetting fluid in an imbibition process, \(S_{nw,t}\) is the trapped saturation of the non-wetting fluid after the imbibition process and \(C\) is called the Land constant. The Land constant will be dependent on wettability and process history and must be determined from experimental data.

Killough\(^{41}\) and Carlson\(^{42}\) also suggested models for two-phase drainage and imbibition hysteresis.
7.5 Relative permeability correlations for three-phase flow

When the three phases oil, water and gas are present in a porous media at the same time three-phase flow parameters must be used to describe the behaviour.

Three-phase relative permeability is difficult to measure and correlations of two-phase data are usually used to estimate the three-phase data. It is often assumed that the water and gas relative permeability is a function of its own saturation only, and two-phase relative permeabilities for water and gas can therefore be used for three-phase cases. Three-phase oil relative permeability is assumed to be a function of both gas and water saturation.

Many correlations for the oil relative permeability exists. Several authors have made summaries of correlations for three-phase relative permeability.

The Stone I and II models for estimation of three-phase relative permeability are widely used. Stone introduced the normalised saturations

$$S^*_o = \frac{S_o - S_{orm}}{1 - S^*_{wi} - S_{orm}},$$

$$S^*_w = \frac{S_w - S^*_{wi}}{1 - S^*_{wi} - S_{orm}},$$

and

$$S^*_g = \frac{S^*_g}{1 - S^*_{wi} - S_{orm}},$$

where $S_{orm}$ is the minimum oil saturation after gas and water injection.

In the Stone I model the relative permeability of oil is defined as

$$k_{ro} = S^*_o \beta_w \beta_g.$$

$\beta_w$ and $\beta_g$ are given by
\[
\beta_w = \frac{k_{\text{row}}}{1 - S_w^*} \quad \text{and} \quad \beta_g = \frac{k_{\text{rog}}}{1 - S_g^*},
\]

where \( k_{\text{row}} \) is the two-phase oil relative permeability in a oil-water system and \( k_{\text{rog}} \) the two-phase oil relative permeability in a gas-oil system.

The Stone II model\(^{45}\) defines the oil relative permeability as

\[
k_{ro} = (k_{\text{row}})_{S_w} \left[ \frac{\frac{k_{\text{row}}}{(k_{\text{row}})_{S_w}} + k_{rw}}{\frac{k_{\text{rog}}}{(k_{\text{row}})_{S_w}} + k_{rg}} \right] - (k_{rw} + k_{rg}),
\]

where \((k_{\text{row}})_{S_w}\) is the relative permeability of oil at the irreducible water saturation from the two-phase oil-water system.

It is difficult to conduct a three-phase flow experiment. The number of experiments concerning three-phase flow is therefore limited, but some reported cases exist.\(^{60-71}\) The experiments show a very complex behaviour. Several different results for the hysteresis behaviour of gas, water and oil have been reported, where wettability plays an important role. The relative permeability correlations often fail to predict what is seen in the experiments.

### 7.6 History dependent relative permeability correlations for three-phase flow

In order to describe the experimental data correctly many mechanisms must be included in the model. It must be possible to use different relative permeabilities for gas depending on if the gas saturation is decreasing or increasing. It should be possible to use a different gas relative permeability for three-phase flow when compared to two-phase flow, that is gas relative permeability dependent on both gas
and water saturations. The gas modelling must include gas trapping, and the relative permeability should be able to vary with gas trapping history.

It should be possible to use different water relative permeability with increasing versus decreasing water saturation. The water relative permeability should be able to depend on gas saturation in addition to water saturation, i.e. different water relative permeability for three-phase flow when compared to two-phase flow.

The residual oil is often lower when trapped gas is present (see chapter 10). A three-phase relative permeability model for oil must take this into account. The oil relative permeability should be modelled with dependence on gas and water saturation, as in the earlier discussed models. The oil relative permeability should also be history dependent.

Three-phase correlations with hysteresis and trapped gas have been introduced to take the complex three-phase behaviour into account. Larsen and Skauge\textsuperscript{72} have made a history dependent model for three-phase relative permeability, which have been included in the Eclipse 100 simulator.\textsuperscript{26} The model takes into account the effect of trapped gas on the residual oil saturation. The oil relative permeability is therefore process dependent. It is also possible to use different water relative permeabilities for two- and three-phase flow. The model allows reduced gas mobility in the three-phase case with gas trapping described by Carlson’s\textsuperscript{42} two-phase hysteresis model.

Egermann et. al\textsuperscript{73} suggested an analytical expression for the hysteresis behaviour. The model was included in a reservoir simulator. The simulations of WAG experiments were successful.

### 7.7 Three-phase relative permeability from network modelling

Network models can also be used to predict three-phase relative permeability curves.\textsuperscript{74-83} These models incorporate the physical rules of flow and the fluid placement in the pores. These models can be used to simulate the processes taking
place inside the pores. See chapter 8 or the paper by Blunt\textsuperscript{29} for more details on network modelling.
8. Three-phase modelling in a network model

Van Dijke et al.\textsuperscript{84-89} have developed a process based model for three-phase behaviour. Pore geometry and wettability are input parameters to the network model.

The procedure for prediction of three-phase parameters is to first anchor the network model to the experimentally measured two-phase data. This determines the pore properties and wettability parameters in the network. The three-phase behaviour is then estimated by using this preconditioned network.\textsuperscript{87-88}

8.1 Two-phase drainage in a network

Two-phase drainage can be illustrated by using a very simple pore size distribution, see figure 8.1. The frequency of the different pore sizes is evenly distributed between the maximum and minimum pore size.

![Figure 8.1: Primary drainage process (Adapted from Sorbie and van Dijke\textsuperscript{85}).](image-url)
The condition for entry of an invading fluid into a pore is that the pressure has reached a value, which is higher than the entry pressure of that pore,

\[ P_c > P_{c,\text{entry}} . \]  

As seen in figure 8.1, the pressure has to reach the value

\[ P_{c1} = \frac{2\sigma}{R_{\text{max}}} \]  

before the oil starts invading the largest pore with radius \( R_{\text{max}} \).

Accessibility is also an important parameter. Even though a pore satisfies the entry conditions, it cannot be invaded if the invading fluid is not yet connected to the pore. There can be pores with higher entry pressures standing in the way.

If the invading fluid has surrounded the initial fluid, the initial fluid can become trapped. In the case of a water-wet rock the initial fluid is water. If the water is trapped it has no connection to the outlet. It is however possible for the water to escape through films if the conditions for films are present in the system.

### 8.2 Two-phase imbibition in a network

Imbibition can happen by two different processes. The fluid can invade by piston-like displacement, see figure 8.2 a). The invading fluid pushes the initial fluid, out of the pore with a clearly defined front. The other process is invasion by snap-off, see figure 8.2 b). The invading fluid is then flowing along the walls of the pores and when the film of the invading fluid becomes thick enough it connects in the middle of the pore. In the case of a water-wet rock the invading fluid is water.
The capillary pressure for snap-off is lower than the capillary pressure for piston-like displacement. The capillary pressure for piston-like displacement is

$$P_c \sim \frac{2\sigma}{r},$$  

(51)

and the capillary pressure for snap-off for a strongly water-wet case is

$$P_c \sim \frac{\sigma}{r}.$$  

(52)

Remember that capillary pressure is defined as

$$P_c = P_{\text{non-wetting}} - P_{\text{wetting}}$$  

(53)

If the pressure of the non-wetting phase occupying the pore is lowered the capillary entry pressure for piston-like displacement is reached first. If the front of the wetting fluid has not reached the pore, piston-like displacement will not take place. Snap-off can still occur if there are films of the wetting fluid present in the pore. Snap-off will
then happen if the pressure of the non-wetting phase gets even lower, or the pressure of the wetting fluid is increased. In a fully accessible capillary bundle model only piston-like displacement will occur.

8.3 3D network model

A 3D network model for describing three-phase behaviour was constructed at Heriot-Watt\textsuperscript{84-86}. A picture of the software which shows the network can be seen in figure 8.3.

![Network model](image)

*Figure 8.3: Network model.*
The pore radius, \( r \), can be used to quantify the capillary pressure, volume of the pore and the conductance of the pore. All three parameters are functions of the pore radius

\[
P_C \propto \frac{1}{r},
\]

(54)

\[
V \propto r^n \quad \text{and}
\]

(55)

\[
g \propto r^\lambda,
\]

(56)

where \( V \) is the volume, \( n \) is the volume exponent, \( g \) is the conductance and \( \lambda \) is the conductance exponent.

In order to describe the network several parameters are necessary

- Minimum and maximum pore size (\( R_{\text{min}} \) and \( R_{\text{max}} \)),
- Pore size distribution (if exponential distribution, the exponent \( n \) must be specified),
- Volume exponent (\( n \), typically between 0 and 4),
- Conductivity exponent (\( g \), typically between 2 and 4),
- Coordination number, how well the network is connected (\( z \), typically between 2.5 and 6),
- Wettability (wettability type, \( \cos \theta_{ow} \), fraction of oil-wet and water-wet pores etc.) and
- Existence of films and layers.

8.4 Contact angle relations for weakly wetted pores

The Bartell and Osterhof\textsuperscript{7} equation states that

\[
\sigma_{gw} \cos \theta_{gw} - \sigma_{go} \cos \theta_{go} - \sigma_{ow} \cos \theta_{ow} = 0.
\]

(57)

For weakly wetted pores Sorbie and van Dijke\textsuperscript{89} proposed the linear relationships for non-spreading oil
\[
\cos \theta_{go} = \frac{1}{2} \left\{ -1 + \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{go}} \cos \theta_{ow} + 1 + \frac{\sigma_{gw} - \sigma_{ow}}{\sigma_{go}} \right\} \quad \text{and} \quad (58)
\]
\[
\cos \theta_{gw} = \frac{1}{2} \left\{ -1 - \frac{\sigma_{go} - \sigma_{ow}}{\sigma_{gw}} \cos \theta_{ow} + 1 + \frac{\sigma_{go} - \sigma_{ow}}{\sigma_{gw}} \right\} . \quad (59)
\]

Figure 8.4 illustrates the contact angle relationship.

\[\text{Figure 8.4: Contact angle relations for non-spreading oil (Adapted from Sorbie and van Dijke\textsuperscript{89}).}\]

For spreading oil they proposed

\[
\cos \theta_{go} = 1 \quad \text{and} \quad (60)
\]
\[
\cos \theta_{gw} = \frac{\sigma_{ow}}{\sigma_{gw}} \cos \theta_{ow} + \frac{\sigma_{go}}{\sigma_{gw}} . \quad (61)
\]
Figure 8.5 shows the contact angle relationship for spreading oil.

\[ \cos \theta_{go} = \frac{1}{2\sigma_{go}} \left\{ C_{S,o} \cos \theta_{ow} + C_{S,o} + 2\sigma_{go} \right\} \]  
\[ \cos \theta_{gw} = \frac{1}{2\sigma_{gw}} \left\{ \left( C_{S,o} + 2\sigma_{ow} \right) \cos \theta_{ow} + C_{S,o} + 2\sigma_{go} \right\} , \]  

where
\[ C_{S,o} = \begin{cases} 
\sigma_{gw} - \sigma_{go} - \sigma_{ow} & \text{if } \sigma_{gw} - \sigma_{go} - \sigma_{ow} < 0 \\
0 & \text{if } \sigma_{gw} - \sigma_{go} - \sigma_{ow} > 0 .
\end{cases} \]
The wetting order of water, oil and gas can be determined for a pore with any value of \( \cos \theta_{ow} \).

### 8.5 Pore filling sequence

The entry condition, eq. 50, will decide the pore filling sequence. The invasion will start in the pores with the smallest entry pressure.

One example of a pore filling sequence is shown in figures 8.6 to 8.9. Water is invading oil-filled pores. Figure 8.6 illustrates a mixed-wet porous medium with large pores oil-wet and small pores water-wet. The pores are initially filled with oil.

\[
\begin{align*}
  P_{entry,ow}(r_{\min}^w) &= P_o - \frac{2\sigma_{ow}}{r_{\min}^w}.
\end{align*}
\]  

(65)

The ordering of the capillary entry pressures can be found from

\[
\text{Figure 8.6: Mixed-wet medium with oil filled pores (Adapted from Sorbie and van Dijke).}
\]
All the water-wet pores are filled first because the entry pressures for all the water-wet pores are lower than any entry pressure in the oil-wet pores, see figure 8.8.

\[
\frac{2\sigma_{\text{ow}}}{r_{\text{min}}^w} > \frac{2\sigma_{\text{ow}}}{r_{\text{wet}}^w} > -\frac{2\sigma_{\text{ow}}}{r_{\text{max}}^o} > -\frac{2\sigma_{\text{ow}}}{r_{\text{wet}}^o}.
\]
After all the water-pores are invaded the water starts invading from the largest oil-wet pores, see figure 8.9. The largest oil-wet pore has a lower entry pressure than the smallest oil-wet pore, seen from eq. 67.

![Figure 8.9: Water is invading oil filled pores (Adapted from Sorbie and van Dijke\textsuperscript{89}).](image)

### 8.6 Pore occupancy for three phases

The pore filling can result in a number of pore occupancies. Three types of occupancies can occur. When oil is intermediate wet, oil has boundaries with both water and gas, we have a type I occupancy. Type II occupancy occurs if gas is intermediate wet and type III if water is intermediate wet\textsuperscript{84}.

![Figure 8.10: Type I; oil is intermediate wetting (Adapted from Sorbie and van Dijke\textsuperscript{89}).](image)
In the type I case, see figure 8.10, the gas-water capillary pressure and the relative permeability of oil are functions of two saturations:

\[ P_{Cgw}(S_w, S_g) = P_{Cgo}^{2-phase}(1 - S_g) + P_{Cow}^{2-phase}(S_w) \]  

\[ k_{ro}(S_w, S_g) = 1 - k_{rg}^{2-phase,go}(S_g) - k_{rw}^{2-phase,ow}(S_w). \]  

The other capillary pressures and relative permeabilities are functions of one saturation value:

\[ P_{Cgo}(S_g) = P_{Cgo}^{2-phase}(1 - S_g), \]  

\[ P_{Cow}(S_w) = P_{Cow}^{2-phase}(S_w), \]  

\[ k_{rg}(S_g) = k_{rg}^{2-phase,go}(S_g) \]  

\[ k_{rw}(S_w) = k_{rw}^{2-phase,ow}(S_w). \]  

---

Figure 8.11: Type II; gas is intermediate wetting (Adapted from Sorbie and van Dijke).
For type II, see figure 8.11, gas is intermediate wetting i.e. has boundaries with both water and oil. The oil-water capillary pressure and the relative permeability of gas are functions of two saturations\textsuperscript{84}

\[ P_{Cw} (S_w, S_o) = P_{Cgw}^{2-phase} (S_w) - P_{Cgo}^{2-phase} (S_o) \]  
and

\[ k_{rg} (S_w, S_o) = 1 - k_{rw}^{2-phase, gw} (S_w) - k_{ro}^{2-phase, go} (S_o). \]

The rest of the capillary pressures and relative permeabilities are functions of one saturation value\textsuperscript{84}

\[ P_{Cgo} (S_o) = P_{Cgo}^{2-phase} (S_o), \]  

\[ P_{Cgw} (S_w) = P_{Cgw}^{2-phase} (S_w), \]  

\[ k_{ro} (S_o) = k_{ro}^{2-phase, go} (S_o) \]  
and

\[ k_{rw} (S_w) = k_{rw}^{2-phase, gw} (S_w). \]

\[ r_{min} \quad r_{wet} \quad r_{wet} \quad r_{max} \]

\textbf{Figure 8.12: Type III; water is intermediate wetting (Adapted from Sorbie and van Dijke\textsuperscript{89}).}
In the type III case, see figure 8.12, the water is intermediate wetting and the gas-oil capillary pressure and the relative permeability of water are functions of two saturations

\[
P_{Cgw} (S_g, S_o) = P_{Cgw}^{2-\text{phase}} (1 - S_g) - P_{Cow}^{2-\text{phase}} (1 - S_o) \quad \text{and} \quad (79)
\]

\[
k_{ro} (S_o, S_g) = 1 - k_{ro}^{2-\text{phase,ow}} (S_o) - k_{rg}^{2-\text{phase,gw}} (S_g). \quad (80)
\]

The rest of the capillary pressures and relative permeabilities are functions of one saturation value

\[
P_{Cow} (S_o) = P_{Cow}^{2-\text{phase}} (1 - S_o), \quad (81)
\]

\[
P_{Cgw} (S_g) = P_{Cgw}^{2-\text{phase}} (1 - S_g), \quad (82)
\]

\[
k_{ro} (S_o) = k_{ro}^{2-\text{phase,ow}} (S_o) \quad \text{and} \quad (83)
\]

\[
k_{rg} (S_g) = k_{rg}^{2-\text{phase,gw}} (S_g). \quad (84)
\]
9. Three-phase modelling in Eclipse 100

9.1 Modelling of lower residual oil for three-phase

The residual oil saturation is usually lower for three-phase flow than for two-phase flow. The SOMWAT-keyword\(^{90}\) specifies the minimum oil saturation as a function of water saturation, see figure 9.1. The first column is the water saturation and the second column is the residual oil saturation. The first row lists the minimum water saturation and the residual oil after two-phase gas injection, and the last row lists the maximum water saturation and the residual oil after two-phase water injection. This is a static change of the minimum residual oil saturation, \( S_{orm} \).

![Figure 9.1: SOMWAT; Minimum oil saturation as a function of water saturation.](image)

Trapping of gas can lead to lower minimum oil saturation. This is often described by the equation

\[ S_{orm} = \frac{S_{org} - S_{orm}}{S_{org} - S_{orm}} \]
\[ S_{orm} = S_{orw} - R \cdot S_{gt}, \]  

where \( S_{orm} \) is the minimum residual oil saturation, \( S_{orw} \) is the residual oil after water injection, \( R \) is a constant between 0 and 1 and \( S_{gt} \) is the trapped gas saturation.

The amount of gas trapped is often described by the Land constant, see equation 41. The effect of gas trapping can be modelled using the WAGHYSTR-keyword\(^90\), see figure 9.2. The first number is the Land constant, the third item is the flag for the modification of residual oil by trapped gas and the last parameter is the fraction of the trapped gas which is modifying residual oil (the \( R \)-constant in equation 85). This is a dynamic change of \( S_{orm} \).

![Figure 9.2: WAGHYSTR; trapped gas is modifying residual oil.](image)
9.2 Modelling of double displacement

When gas is injected into an oil and water filled core the gas can displace oil directly, but a double displacement could also occur. Double displacement takes place when the gas displaces oil, which again displaces water. This leads to less oil production and more water production when compared to direct displacement. Figure 9.3 shows an illustration of direct displacement on top and double displacement below.

![Direct displacement and double displacement.](image)

Double displacement is more likely to occur at the beginning of the gas injection period, when the water saturation is high. At the end of the gas injection period the water saturation is low and direct displacement will dominate. This can lead to lower residual oil at the end of the gas injection when compared to the early period of gas injection. This behaviour can be modelled by the SOMWAT-keyword\textsuperscript{90}. The double displacement, at high water saturation, has higher residual oil saturation, as seen in figure 9.4.
Figure 9.4: Double displacement period \((Sw \geq 0.45)\) has higher residual oil saturation, \(Sor(dd) = 0.11\), than the residual oil at the end of the G2 injection \(Sorm = 0.08\).

9.3 Modelling of hysteresis and three-phase relative permeability

Three-phase relative permeability is difficult to measure experimentally. Different models are used to estimate three-phase relative permeability from two-phase relative permeability. One example is the Stone I correlation. The relative permeability of oil is a function of the three saturation values, \(S_o\), \(S_w\) and \(S_g\), the two-phase relative permeability of oil in presence of water, \(k_{row}\), and in presence of gas, \(k_{rog}\).

Hysteresis refers to the fact that relative permeability is directional dependent; it has different values for a saturation value depending on whether the saturation is increasing or decreasing.
The three-phase relative permeability of oil is modelled by requesting the Stone I model using the STONE1-keyword\textsuperscript{90}. Other three-phase models are also available and have been compared by Kossack\textsuperscript{91}.

The WAGHYSTR-keyword\textsuperscript{90} can be used to model hysteresis see figure 9.5. The second value is the factor for reduction of gas mobility in three-phase flow, the third entry is the flag for use of the WAG Hysteresis model for the gas relative permeability, entry four is the flag for use of trapped gas saturation to modify the residual oil used in Stone I, and the fifth item is the flag for use of the WAG Hysteresis model for the water phase.

\begin{verbatim}
WAGHYSTR
  1.0  1.3   'YES'   'YES'   'YES'   0.1   0.001   0.19 /
\end{verbatim}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9.5.png}
\caption{STONE1 and WAGHYSTR; modelling of hysteresis and three-phase relative permeability.}
\end{figure}
10. Summary of main results

10.1 Three-phase features

The parameters governing three-phase flow and the relationship between these parameters have been described in several papers. A summary of the most important findings will be presented here.

It has been observed that the three-phase residual oil saturation is lower than after two-phase flow. This has been linked to the trapped gas saturation. Trapped gas will often result in lower residual oil. The effect that the trapped gas saturation has on residual oil has been connected to wettability. The amount of trapped gas has been associated with wettability, process and initial or maximum gas saturation.\textsuperscript{92-104}

The wetting phase has generally shown little hysteresis in relative permeability. For strongly water-wet cases the relative permeability of water has often been found to be a function of its own saturation. The relative permeability of oil in oil-wet cores is usually a function of its own saturation.\textsuperscript{40,92-96} The intermediate-wetting phase has shown some hysteresis in most cases. Water-wet cores show three-phase hysteresis for oil relative permeability, and oil-wet cores show hysteresis for the water relative permeability.\textsuperscript{92-93,95-96} The gas is usually the non-wetting phase, and the gas relative permeability generally shows strong hysteresis for all wettabilities.\textsuperscript{40,93-95} In neutrally-wet cases the behaviour is complex. In a mixed-wet case all the phases showed hysteresis in the relative permeability.\textsuperscript{95}

In this work a large number of three-phase core flooding experiments were collected in a database and analysed. The three-phase effects and connections between different parameters were investigated. The results are presented in paper 1.

The three-phase residual oil saturation was significantly lower than the residual oil after only gas injection or only water injection. In a water-wet core the residual oil saturation appears to be lower when gas is injected first in a WAG-sequence, and for
an oil-wet core the residual oil seems to be lower when water is injected first in a WAG-sequence. In the case of initial gas injection the residual oil becomes lower for more oil-wet cores than for water-wet cores.

Trapped gas leads to lower residual oil. The effect is usually quantified by the R-factor in the equation

\[ S_{orm} = S_{orw} - R \times S_{gt}, \]  

(86)

where \( S_{orm} \) is the minimum residual oil saturation, \( S_{orw} \) is the residual oil after water injection and \( S_{gt} \) is the trapped gas saturation.

The effect of trapped gas on residual oil was strongest in water-wet cores when compared to more oil-wet cores. The impact of trapped gas was higher for WAG-flooding than for tertiary water injection (after initial water injection and secondary gas injection), which had a higher impact of trapped gas than secondary water injection (after initial gas injection).

There was a strong connection between the initial or maximum gas saturation and the trapped gas saturation. This can be quantified by the Land constant\(^{40}\)

\[ C = \frac{1}{S_{gt}} - \frac{1}{S_{gi}}, \]  

(87)

where \( S_{gi} \) is the initial or maximum gas saturation

The average trapped gas saturation was higher for secondary water injection after gas injection than for WAG-injection, which has short slugs of gas and water.

Hysteresis in relative permeability was also studied. In general significant hysteresis effects are found for the gas relative permeability in almost all cases. Hysteresis was found for the water relative permeability in some cases.
Multivariate analysis showed that the three-phase residual oil saturation was correlated with the residual oil after water injection and negatively correlated with the trapped gas saturation, in agreement with equation 86.

The trapped gas saturation was related to the initial gas saturation, in agreement with equation 87. Trapped gas was also correlated with absolute permeability and the endpoint water relative permeability.

### 10.2 The effect of capillary pressure in history matching

One of the important parameters influencing three-phase flow is capillary pressure. Relative permeability has usually been calculated by analytical methods\(^{105-107}\), where the effect of capillary pressure has been neglected. More recent inverse methods\(^{108-109}\) have included capillary pressure in the estimation of relative permeability. The inverse method was used in this work.

The effect of capillary pressure on two- and three-phase flow was studied by use of a simulation model, Eclipse 100. The results are discussed in paper 2 and in more detail in paper 3. A history match of a core flooding experiment with and without capillary pressure was performed. Two-phase, gas-oil and oil-water, capillary pressure curves were used as input. The capillary pressure had a significant effect on the shape of the oil production curve, the total oil recovery and the differential pressure for both two- and three-phase flow.

A history match without capillary pressure was first done by tuning the relative permeability curves. Then capillary pressure was included and the result was that the total oil recovery was much lower. The capillary pressure clearly had a large effect on the flow. This was seen for both initial gas injection and initial water injection, and also for three-phase flow.
In order to get a new match with capillary pressure the relative permeability curves had to be changed. The relative permeability of oil had to be increased and the relative permeability of the injected fluids had to be decreased.

The new history match with capillary pressure was closer to the experimental data than the match without capillary pressure. The shape of the total oil production curve and the differential pressure was better matched with the experimental data.

Three-phase flow was modelled both with the two-phase capillary pressure and with Killough's model\(^\text{23}\) for three-phase capillary pressure. The match of the experimental data was better when two-phase capillary pressure was used for three-phase flow, than when using Killough's correlation\(^\text{23}\).

### 10.3 Impact of three-phase characteristics and capillary pressure

The size of the three-phase area has a large effect on the total oil recovery, because the three-phase zone has lower residual oil.\(^{93,100}\) The analytical methods for estimating the three-phase zone may strongly underestimate the size.\(^{110-111}\) Simulation studies have shown larger three-phase zones.\(^{112-116}\)

It has been shown that the three-phase relative permeability is significantly different from the two-phase relative permeability. The relative permeabilities of the injected fluids, gas and water, is reduced in the three-phase zone.\(^{67}\)

The effect of three-phase features and capillary pressure on field scale was investigated. The effect on the size of the three-phase zone, breakthrough time of the injected fluids and oil recovery was studied by using a black oil simulator. The results are presented in paper 4.

In three-phase flow the relative permeability of the injected fluids are lower and this delays the segregation of gas and water. The three-phase zone is therefore bigger when these effects are taken into account. Another important three-phase effect is gas trapping. Gas trapping often leads to lower residual oil. Using three-phase
representations of relative permeability and including gas trapping effects gave a larger three-phase zone, later breakthrough of injected fluids and higher oil recovery.

In paper 2 and 3 it was found that the capillary pressure had a significant effect on history matching of a core flooding experiment. It was therefore important to find out how capillary pressure influenced the flow on field scale. The work described in paper 3 indicated that using two-phase capillary pressure for three-phase flow gave better results than using Killough’s three-phase capillary pressure correlation. Two-phase capillary pressure curves were therefore used in the field scale study.

The results showed that capillary pressure also had a significant effect on field scale. Including capillary pressure delayed the segregation of gas and water even more than when capillary pressure was neglected. In paper 3 it was found that the relative permeability of the injected fluids is lower when capillary pressure is included. Inclusion of this effect further delayed the segregation of gas and water. The three-phase zone was even bigger and the oil recovery was higher.

### 10.4 Three-phase capillary pressure

Two-phase capillary pressure is often measured experimentally and can be included in reservoir simulations. Three-phase capillary pressure is very difficult to measure, and methods for estimation of three-phase capillary pressure must be used.

When studying the effect of capillary pressure on three-phase flow it is important to model the three-phase capillary pressure correctly. In simulations the two-phase capillary pressure is often used also for three-phase flow, or Killough's model\(^\text{23}\) can be used to describe the three-phase capillary pressure. Paper 3 indicated that using two-phase capillary pressure gives better results than using Killough’s correlation. A network model was used to try and predict the three-phase capillary pressure and compare it with the two-phase capillary pressure and Killough’s model.

The concept of using a network of pores to model flow in porous media was first described by Fatt\(^\text{117-119}\) in 1956. The concept was not further investigated before the
1980s, when percolation theory was incorporated in network models. The network models can be used to explain and predict many pore scale phenomena. Network models have been used to predict parameters like three-phase relative permeabilities and three-phase capillary pressures.

In this work a network model developed at the Heriot-Watt University was used to investigate three-phase capillary pressure. The work is discussed briefly in paper 2 and in more detail in paper 5. The network model was anchored to the experimentally measured two-phase, gas-oil and oil-water, capillary pressure curves. The pore geometry and wettability was established in the anchoring process. This was done partly by trial and error. A more automatic method for matching the two-phase data using an ensemble Kalman filter is demonstrated in paper 6.

After anchoring the network model to the two-phase data, the three-phase capillary pressure was predicted using the same geometry and wettability. The three-phase gas-oil capillary pressure had a higher positive value than for two-phase, and the three-phase oil-water capillary pressure had a higher negative value than for two-phase. The three-phase capillary pressure loop was outside the two-phase capillary pressure loop.

Killough’s model assumed that the three-phase gas-oil and oil-water capillary pressure was a weighted average of the two-phase curves. In this model the three-phase capillary pressure curves were inside the two-phase capillary pressure loop. The conclusion was therefore that Killough’s model could not be used to describe the three-phase capillary pressure predicted by the network model.

Paper 7 gives a summary of important progress in modelling of immiscible WAG. The main issues discussed are relative permeability hysteresis and capillary pressure. The results from this work are discussed together with earlier work in this research area.
11. Further work

In paper 6 a method for generating three-phase capillary pressure surfaces from a network model anchored to measured two-phase data is described. The three-phase data is defined for all saturation values. Tables are written from the three-phase capillary pressure surfaces. These tables can be included in a reservoir simulator.

Eclipse 100 has an option for including two-dimensional tables for capillary pressure, but the tables are too restricted for our case. The capillary pressure values in Eclipse have to be lower for three-phase than for two-phase. This is not the case for the three-phase capillary pressure generated by the network model.

Other simulators have been considered for simulation with two-dimensional tables for the capillary pressure, but several problems occurred when trying to implement the tables. An in-house simulator, Athena, has been used with some success. The early results show slower segregation and thus a larger three-phase zone, when three-phase capillary pressure tables are included when compared to using two-phase capillary pressure.
References


# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>cross section area</td>
</tr>
<tr>
<td>$a_o$</td>
<td>constant in the Corey type and Skjæveland et. al correlation</td>
</tr>
<tr>
<td>$a_w$</td>
<td>constant in the Corey type and Skjæveland et. al correlation</td>
</tr>
<tr>
<td>$C$</td>
<td>constant in the Corey correlation and Land correlation</td>
</tr>
<tr>
<td>$c_o$</td>
<td>constant in the Skjæveland et. al correlation</td>
</tr>
<tr>
<td>$C_{s,o}$</td>
<td>spreading coefficient</td>
</tr>
<tr>
<td>$c_w$</td>
<td>constant in the Skjæveland et. al correlation</td>
</tr>
<tr>
<td>$dP$</td>
<td>differential pressure</td>
</tr>
<tr>
<td>$dx$</td>
<td>differential distance</td>
</tr>
<tr>
<td>$E$</td>
<td>curvature parameter</td>
</tr>
<tr>
<td>$F$</td>
<td>factor in Killough’s correlation</td>
</tr>
<tr>
<td>$F_{down}$</td>
<td>downward force</td>
</tr>
<tr>
<td>$F_{up}$</td>
<td>upward force</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity acceleration and conductance</td>
</tr>
<tr>
<td>$h$</td>
<td>height</td>
</tr>
<tr>
<td>$k$</td>
<td>permeability</td>
</tr>
<tr>
<td>$k_i$</td>
<td>effective permeability for fluid $i$</td>
</tr>
<tr>
<td>$k_{rg}$</td>
<td>relative permeability for gas</td>
</tr>
<tr>
<td>$k_{ri}$</td>
<td>relative permeability for fluid $i$</td>
</tr>
<tr>
<td>$k_{ro}$</td>
<td>relative permeability for oil</td>
</tr>
<tr>
<td>$k_{reg}$</td>
<td>two-phase oil relative permeability in a gas-oil system</td>
</tr>
</tbody>
</table>
\( k_{row} \)  two-phase oil relative permeability in a oil-water system

\( k_{rw} \)  relative permeability for water

\( (k_{rg})_{S_{row}} \)  water relative permeability at residual oil saturation after gas injection

\( (k_{ro})_{S_{gc}} \)  oil relative permeability at critical gas saturation

\( (k_{ro})_{S_{wi}} \)  oil relative permeability at irreducible water saturation

\( (k_{rw})_{S_{row}} \)  water relative permeability at residual oil saturation after water injection

\( k_{2-phase,go} \)  gas relative permeability for the 2-phase gas-oil case

\( k_{2-phase,gw} \)  gas relative permeability for the 2-phase gas-water case

\( k_{2-phase,go} \)  oil relative permeability for the 2-phase gas-oil case

\( k_{2-phase,ow} \)  oil relative permeability for the 2-phase oil-water case

\( k_{2-phase,gw} \)  water relative permeability for the 2-phase gas-water case

\( k_{2-phase,ow} \)  water relative permeability for the 2-phase oil-water case

\( n_{g} \)  gas relative permeability exponent

\( n_{og} \)  oil relative permeability exponent in presence of gas

\( n_{ow} \)  oil relative permeability exponent in presence of water

\( n_{w} \)  gas relative permeability exponent

\( P_{c} \)  capillary pressure

\( P_{c,aw} \)  capillary pressure between air and water

\( P_{Cd} \)  drainage capillary pressure

\( P_{c,entry} \)  capillary entry pressure

\( P_{Cgo} \)  gas-oil capillary pressure
\( P_{Cgw} \) gas-water capillary pressure

\( P_{C_{im}} \) imbibition capillary pressure

\( P_{Cow} \) oil-water capillary pressure

\( P_{Cgo}^{2-phase} \) gas-oil capillary pressure for 2-phase

\( P_{Cgw}^{2-phase} \) gas-water capillary pressure for 2-phase

\( P_{Cow}^{2-phase} \) oil-water capillary pressure for 2-phase

\( P_c \) capillary entry pressure

\( P_{entry,w} \) capillary entry pressure for water wet pores

\( P_{non-wetting} \) pressure in non-wetting phase

\( P_{oil} \) pressure in oil

\( P_{water} \) pressure in water

\( P_{wetting} \) pressure in wetting phase

\( q \) flow rate

\( R \) radius and residual oil reduction factor

\( r \) radius

\( R_{max} \) maximum radius

\( r_{max} \) maximum radius

\( R_{min} \) minimum radius

\( r_{min} \) minimum radius

\( r_{max}^o \) maximum radius for oil wet pores

\( r_{min}^w \) minimum radius for water wet pores

\( r_p \) radius pore
$r_t$ radius throat

$r_{wet}^o$ radius for oil wet pores near wettability change

$r_{wet}^w$ radius for water wet pores near wettability change

$S_e$ effective saturation

$S_g$ gas saturation

$S_g^*$ effective gas saturation

$S_{gc}$ critical gas saturation

$S_{gi}$ initial gas saturation

$S_{gt}$ trapped gas saturation

$S_{lc}$ total critical liquid saturation given as

$S_{nw,max}$ maximum saturation of the non-wetting fluid

$S_{nw,t}$ trapped saturation of the non-wetting fluid

$S_o$ oil saturation

$S_o^*$ effective oil saturation

$S_{or}$ residual oil saturation

$S_{org}$ residual oil saturation after gas injection

$S_{orm}$ minimum residual oil saturation

$S_{orw}$ residual oil saturation after water injection

$S_w$ water saturation

$S_w^*$ effective water saturation

$S_{w,hys}$ water saturation at the hysteresis reversal point
\( S_{wi} \) irreducible water saturation

\( S_{w,max} \) maximum water saturation attainable on the scanning curve

\( V \) volume

\( \beta_g \) gas factor in the Stone I correlation

\( \beta_w \) water factor in the Stone I correlation

\( \Delta \rho \) difference in density

\( \theta_a \) advancing contact angle

\( \theta_{aw} \) angle between the air-water interface and solid

\( \theta_{go} \) contact angle gas-oil

\( \theta_{gw} \) contact angle gas-water

\( \theta_{ow} \) contact angle oil-water

\( \theta_r \) receding contact angle

\( \lambda \) pore size index and conductance exponent

\( \mu \) viscosity

\( \nu \) volume exponent

\( \pi \) constant approximately 3.14

\( \rho_{a} \) density of air

\( \rho_{w} \) density of water

\( \sigma \) interfacial tension

\( \sigma_{aw} \) interfacial tension between air and water

\( \sigma_{go} \) interfacial tension between the gas and oil

\( \sigma_{gs} \) interfacial tension between the gas and solid
\(\sigma_{gw}\) interfacial tension between the gas and water

\(\sigma_{os}\) interfacial tension between the oil and solid

\(\sigma_{ow}\) interfacial tension between oil and water

\(\sigma_{ws}\) interfacial tension between the water and solid