The true fluence distribution of terrestrial gamma flashes at satellite altitude

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In this paper we use the fluence distributions observed by two different instruments, RHESSI and Fermi GBM, corrected for the effects of their different orbits, combined with their different daily TGF detection rates and their relative sensitivities to make an estimate of the true fluence distribution of TGFs as measured at satellite altitudes. The estimate is then used to calculate the dead-time loss for an average TGF measured by RHESSI. An independent estimate of RHESSI dead-time loss and true fluence distribution is obtained from a Monte Carlo (MC) simulation in order to evaluate the consistency of our results. The two methods give RHESSI dead-time losses of 24–26% for average fluence of 33–35 counts. Assuming a sharp cut-off the true TGF fluence distribution is found to follow a power law with \( \lambda = 2.3 \pm 0.2 \) down to \( \sim 5/600 \) of the detection threshold of RHESSI. This corresponds to a lowest number of electrons produced in a TGF of \( \sim 10^{12} \) and a global production rate within ±38° latitude of 50000 TGFs/day or about 35 TGFs every minute, which is 2% of all IC lightning. If a more realistic distribution with a roll-off below 1/3 (or higher) of the RHESSI lower detection threshold with a true distribution with \( \lambda \leq 1.7 \) that corresponds to a source distribution with \( \lambda \leq 1.3 \) is considered, we can not rule out that all discharges produce TGFs. In that case the lowest number of total electrons produced in a TGF is \( \sim 10^{12} \).


1. Introduction

With the discovery of terrestrial gamma flashes (TGFs) above thunderstorms [Fishman et al., 1994] by the Burst and Transient Source Experiment (BATSE) a new mechanism of the coupling between the lower atmosphere and space was found. The phenomenon involves both gamma photons, relativistic electrons and positrons. Charged particles are accelerated in extremely strong electric fields (>300 kV/m sea level equivalent) associated with lightning discharges and initiate a relativistic runaway process [Gurevich et al., 1992]. Through interaction with the neutral atmosphere bremsstrahlung is produced, resulting in the escape of electrons [Dwyer et al., 2008], positrons [Briggs et al., 2011] and gamma photons into space. There are still many open questions related to TGFs, and one of them will be addressed in this paper: How common are TGFs? Or more specifically: What is the true fluence distribution of TGFs as measured from satellite altitude?

From the first observations it was believed that the TGFs are produced above 40 km and that they were related to transient luminous events [Fishman et al., 1994; Nemiroff et al., 1997], a reasonable suggestion given the relatively few observations of about 10 TGF/year by BATSE (78 TGFs in 9 years according to http://www.batse.msfc.nasa.gov/batse/misc/triggers.html). However, results from Reuvan Ramaty High Energy Solar Spectroscopic Imager (RHESSI) ten years later indicated that their production altitude is most likely around 15–21 km [Dwyer and Smith, 2005]. While BATSE had an on-board trigger algorithm with a 64 ms search window, the data from RHESSI were downloaded and a more sophisticated, but still rather conservative, search algorithm with a search window of 1 ms was applied. For more details about the search algorithm we refer to Grefenstette et al. [2009]. Having a trigger window significantly longer than the typical duration of a TGF(<1 ms), like BATSE had, only events with high count rates that exceed the statistical fluctuations of background counts will be classified as TGFs. However, RHESSI had a search window comparable to the duration of a TGF and could identify much weaker TGFs. Thus, RHESSI was able to report more than 100 TGFs/year (975 TGFs in 8.5 years according to http://scipp.ucsc.edu/~dsmith/tgflib_public/). Reanalyses of the BATSE data have also confirmed a production altitude of TGFs below 20 km [Carlson et al., 2007; Østgaard et al., 2008; Gjesteland et al., 2010]. Consistent with this production altitude and general lightning physics, Williams [2006] speculated that TGFs are related to positive intracloud lightning, a suggestion that has been supported by a few studies comparing TGFs with
electromagnetic characteristics of lightning [Cummer et al., 2005; Shao et al., 2010; Cummer et al., 2011]. As intra-cloud lightning accounts for about 75% of all the lightning [Boccippio et al., 2001] and most of these are positive intracloud lightning bringing negative charges upward, this may imply that TGFs are a rather common phenomenon. X-ray bursts have been observed from negative leader steps in cloud-to-ground (CG–) lightning [Dwyer et al., 2005] and from dart leaders in rocket triggered lightning [Dwyer et al., 2004] before the return strokes of the CG– lightning. Discharge experiments in the laboratory [Nguyen et al., 2008] have also shown that bursts of X-rays are observed slightly before (∼1 μs) the discharge return stroke. All these studies give some hints that TGFs might be more common than observations from space have indicated so far. On the other hand, Smith et al. [2011] suggested that the non-detection of TGFs by the Airborne Detector for Energetic Lightning Emissions (ADELE) may indicate the opposite, that there are very few TGFs with intensities two-three orders of magnitude weaker than those observed by RHESSI.

[4] Measurements from space have been hampered by the loss of counts due to dead-time in the electronics, limited instrument sensitivity and limitations due to the on-board trigger window. In this paper we will use the fluence distributions observed by two different instruments, RHESSI and Fermi GBM, corrected for the effects of their different orbit, combined with their different daily TGF detection rates and their relative sensitivities to make an estimate of the true fluence distribution of TGFs at satellite altitudes. This estimate is then used to calculate the dead-time loss for an average TGF fluence measured by RHESSI. Independent estimates of RHESSI dead-time loss and true fluence distribution are obtained from a Monte Carlo (MC) simulation in order to evaluate the consistency of our results. Finally, we discuss our results in the context of ADELE’s sensitivity and the non-detection of TGFs by this aircraft.

2. The Measured TGF Fluence Distributions and Average Duration

[5] The fluence distribution of the 591 TGFs measured by RHESSI (March 4, 2002–December 31, 2005) and the first 53 TGFs measured by Fermi (Aug 7, 2008–March 10, 2010) are shown in Figure 1. The RHESSI TGFs were downloaded from http://scipp.ucsc.edu/~dsmith/tgflib_public/ and are the same as used in the quantitatively analysis by Grefenstette et al. [2009] obtained before the degradation of the instrument’s sensitivity when the effective detector area was still 256 cm². The Fermi TGFs are taken from Fishman et al. [2011, Table 2]. The three double peaks in that table are treated as separate TGFs giving a total of 53 TGF pulses. All these TGFs were detected when an on-board 16 ms trigger window was used. A power function with the form

\[
\frac{dN}{dn} = A_0n^{-\lambda}
\]

\[(1)\]

\((dN\) is the number of TGFs with fluence within \(dn\) and \(A_0\) is a scaling factor) has been fitted to each of the distribution, giving \(\lambda\) of 3.5 and 1.4, for RHESSI and Fermi, respectively. The fit is based on 14 (4) bins from the peak using bin size of 2 (50) counts for the RHESSI (Fermi) distribution. A power function was chosen because the measured RHESSI fluence distribution could be fairly well fitted with such a function. The accuracy of the fit will be discussed in section 5. We interpret the very soft fluence distribution (meaning relatively many low fluence TGFs) from RHESSI to be caused by dead-time losses that are most significant for high photon fluxes. Although Fermi also has dead-time losses, the very hard fluence distribution (meaning relatively many high fluence TGFs) from Fermi can probably be explained by the long trigger window of 16 ms, which favors high fluence TGFs. For these reasons we believe that the true fluence distribution is somewhere in between these two distributions.

[5] The durations of the 591 RHESSI TGFs have a mean of 374 μs and a median of 299 μs. The duration of a TGF is defined as the ±2σ of a Gaussian function fitted to the light curve of total counts. The majority of the first 53 TGF pulses measured by Fermi have durations between 100 μs and 400 μs [Fishman et al., 2011]. For comparison Gjesteland et al. [2010] reported 5 TGFs measured by BATSE to have a production duration of 200–250 μs.

3. Differences in Sensitivity and Total Number of Observed TGFs

[7] For the 591 RHESSI TGFs observed before January 1, 2006 the average time between TGFs was 2.35 day or 0.42 TGFs/day using a lower threshold cut-off of 17 counts [Grefenstette et al., 2009]. For the first 53 TGFs measured by Fermi they observed 0.03 TGFs/day when a 16 ms on-board trigger window was applied to the NaI scintillators, which increased to 0.3 TGFs/day when the same window was applied to the BGO detectors [Fishman et al., 2011]. However, after the Fermi team started downloading most of the data obtained over regions where TGFs are produced, Fishman [2011] reported that more than 1 TGF/day has been
observed. According to Briggs [2011; M. Briggs, personal communication, 2011] their ground search found 234 TGFs in 591.8 hours of data over regions which are expected to have a high TGF rate. Over the same hours and from the same regions, they found 23 triggered TGFs, a 10.2 times increase in detection rate. According to Fishman et al. [2011] 35 TGFs were observed after the trigger algorithm change (from NaI to BGO) in at least 141 days of data. Of the 35 triggered TGFs 21 were inside the regions where all the data have been downloaded [Briggs, 2011] and the scaling factor of 10.2 should apply. We do not know if this ratio is also valid for the areas outside the boxes which are mostly over ocean. Although there are fewer thunderstorms over ocean the ratio of IC/CG and the fluence distribution of TGFs might be the same. As we are not aware of any studies that give any information whether the TGF distribution over ocean is softer or harder than over land, we will apply an uncertainty of ±50% for the triggered-to-search ratio for the regions outside the boxes. This uncertainty also accounts for any seasonal biases in the downloaded data. This gives us a daily detection rate of 2.5 ± 0.5 TGFs/day (35/141 × 10.2 and 21/141 × 10.2 + 14/141 × (15.3 or 5.1)).

[9] From the RHESSI data we know that TGFs have a strong latitudinal dependence with fewer TGFs produced at higher latitudes. As Fermi, due to its inclination of 25.6° spends more time over regions with more TGFs than RHESSI (38° inclination), Fermi should see more TGFs than RHESSI. As we want to derive a relative daily detection rate that only depends on sensitivity differences we need to correct for this effect. This correction is performed as follows: First, we consider the RHESSI TGF fluence distribution (Nq) versus latitude (θ), dNq/dθ, corrected for the latitudinal cosine effect on area. Then we calculate the fraction of the orbit RHESSI (Oq) spends at various latitudes, dOq/dθ, when the orbit is given as a sine function with amplitude of 38° + 3° latitude. A similar calculation is performed for Fermi, dOq/dθ, but with an amplitude of 25.6° +3° latitude. The extra 3° is to account for a field of view of about 400 km. The expected Fermi TGF distribution is then given as

\[
\frac{dN_F}{d\theta} = \frac{dN_R}{d\theta} \frac{dO_F}{d\theta} / \frac{dO_R}{d\theta}
\]

By integrating \( dN_R/d\theta \) and \( dN_F/d\theta \) over latitudes we estimate that Fermi, just due to orbital differences between the two spacecraft, is expected to see 65% more TGFs than RHESSI. This means that the relative detection rate between Fermi and RHESSI due to sensitivity differences only, Y, is given by \( 2.5 \pm 0.5 / 1.65 / 0.42 = 3.6 \pm 0.7 \). It should be noted that this is what Fermi would have seen if they downloaded data similar to RHESSI and is what we will use as the relative detection rate between the two instruments. However, the real detection rate for Fermi is 1.6 TGFs/day (21 × 10.2/141 + 14/141).

[9] Even if the photon flux of a TGF has a rapid rise, the decay, due to Compton scattering, is usually slow [Ostgaard et al., 2008] and there is no reason to believe that RHESSI, due to dead-time losses, should miss TGFs with high fluence. Dead-time losses would only lead to underestimating the fluence of strong TGFs. When Fermi sees more TGFs than RHESSI it implies that its sensitivity is better. Although Fermi BGO detectors have a slightly larger effective detector area than RHESSI, that is 320 cm² [Meegan et al., 2009; Briggs et al., 2010] compared to 256 cm² [Grefenstette et al., 2009] flying at practically the same altitude, the most important reason for the higher sensitivity is that a more efficient trigger algorithm for the on-ground analysis has been developed for Fermi. According to Briggs [2011] the on-ground trigger algorithm requires ≥4 counts in each of the two BGO detector, ≥4 in all the 12 NaI detectors and with a probability less than 10^-11 giving a lower threshold of 19 counts in all detectors. For the comparison with the 591 RHESSI TGFs for which a lower cut-off threshold of 17 counts (before background subtraction) have been used we use the ≥8 counts (also before background subtraction) in the two BGO detectors with an energy averaged effective detector area of 160 cm² × 2 = 320 cm² to obtain the relative sensitivity, X, between Fermi and RHESSI as

\[
X = \frac{17}{8} \frac{320}{256} = 2.7
\]

This is equivalent to Fermi having a lower threshold of 6.3 on the RHESSI scale as visualized in Figure 2. Although there are uncertainties related to this estimate we will show that it provides results that converge with the rest of the information we have and are consistent with an independent MC simulation of RHESSI dead-time. Uncertainties related to the relative sensitivity will be discussed.

4. The True Fluence Distribution and RHESSI Dead-Time Losses

[10] In the search algorithm to find the 591 RHESSI TGFs with the daily detection rate of 0.42 TGFs/day a lower threshold cut-off of 17 counts was used. However, our MC simulations of dead-time loss indicates that RHESSI only has a one-to-one response up to 10 counts (see Figure 4a). However, between 10 and 20 counts the errors of the estimated true counts are still overlapping the one-to-one response. We will therefore use a fluence of 15 counts as the threshold where the RHESSI results start to be affected by dead-time losses, but also show the effect of using 10 and 20 counts.
Given that both RHESSI and Fermi are measuring from a true fluence distribution that follows a power law with an unknown exponent, $\lambda$, but with different lower detection thresholds, we have the following expression for the total number of TGFs detected by Fermi:

$$N_F = \int_{n_0}^{\infty} A_0 n^{-\lambda} dn = \frac{A_0}{\lambda - 1} n_0^{1-\lambda}$$

where $n$ is fluence and $n_0$ is the lower threshold of detection. The total number of TGFs detected by RHESSI, $N_R$, can be expressed similarly, but with a different lower threshold, $n_0$. We can then express the relative total number of detected TGFs, which is equivalent to the relative daily detection rate, $Y$, as a function of the two lower thresholds

$$Y = \frac{N_F}{N_R} = \left(\frac{n_0}{n_\text{th}}\right)^{1-\lambda} = \left(\frac{1}{X}\right)^{1-\lambda}$$

With relative sensitivity, $X = 2.7$, and relative daily detection rate, $Y = 3.6 \pm 0.7$, this can be solved to get an exponent

$$\lambda = 2.3 \pm 0.2$$

Knowing the distribution of TGFs measured by RHESSI, with $\lambda = 3.5$ and the estimated true TGF distributions, with $\lambda = 2.3$ we can calculate RHESSI dead-time losses as a function of incoming photons. For a specific number of TGFs within a fluence interval, $dN/dn$, in Figure 3a the dead-time loss is the difference between the true fluence, $n_T$, and the measured fluence $n_M$ divided by $n_T$. This is shown in Figure 3b where we have used a fluence of 15 (solid line), with 10 and 20 as uncertainties (dotted lines).

![Figure 3](image)

**Figure 3.** (a) The distribution measured by RHESSI (thick grey) and the estimated true TGF distribution at RHESSI altitude based on the two instrument’s different photon detection sensitivities and their relative daily TGF detection rate. (b) The loss due to dead-time in the RHESSI electronics as a function of true counts (incoming photon fluence). Solid line is for 15 counts used as the threshold where RHESSI experiences dead-time losses. Dotted lines are for lower threshold of 10 counts (upper) and 20 counts (lower). The grey cross is the average dead-time loss determined by the MC simulations described in section 5.

![Figure 4](image)

**Figure 4.** (a) Monte Carlo simulation of the TGF observed May 2, 2005, with a duration of 361 ms, with increasing true fluence from 0 to 100. Vertical line denotes the measured counts and the true counts can be read out from the intersection between MC values and horizontal line, here 45. The diagonal line indicates that RHESSI has no dead-time losses up to about 15 counts. (b) Grey histogram is the measured fluence distribution of the 591 RHESSI TGFS, while black histogram is the true fluence distribution running the MC model on each of the 591 TGFs. Due to background subtraction there are TGFS with less than 17 counts. The black, grey and red lines show the fitted power distributions for the measured ($\lambda = 3.5$) true ($\lambda = 2.6$) and the lower bins of the true ($\lambda = 1.7$).
5. Monte Carlo Simulation of RHESSI Dead-Time Losses

[12] To obtain an independent estimate of RHESSI dead-time losses a MC simulation was performed. For this MC simulation we used the characteristic times of the RHESSI electronics [Grefenstette et al., 2009] to determine the dead-time in each of the 8 detectors. Then, for each TGF the following two steps are performed: 1) The duration of the TGF is calculated as within 2 standard deviations of a Gaussian fit to the TGF light-curve. 2) By increasing the number of photons distributed randomly within the duration of each TGF a detection efficiency curve is obtained. As this was performed hundred times for each number of photons we obtain the statistical error due to the random distribution of photons within the duration, which is shown as vertical lines in Figure 4a. The black horizontal line at 34 counts is what RHESSI measured for this specific TGF and the true counts can be read out from the intersection between the MC values and the horizontal line, here 45 ±7. This curve would have been identical to the one shown in Figure 3b if both the measured counts and duration were equal to the averages, 34 counts and 374 μs. When this MC scheme is applied to all the 591 RHESSI TGFs a true fluence distribution of TGFs can be obtained, as shown by the black histogram in Figure 4b. Using the average fluence of the true distribution and the measured distribution we get the dead-time losses for an average RHESSI TGF of 26%, as shown as a grey cross in Figure 3b. Power functions can be fitted to the distributions. Depending on how many bins from the peak value that are used for the fit we find that the measured distribution before dead time correction (grey histogram) can be fitted with power exponents ranging from 3.2 (11 bins) to 3.7 (17 bins). A χ²-test (reduced) of these fits is equally good (χ²/R ≤ 0.15). Similarly, for the dead time corrected distribution (black histogram) we find power exponents ranging from 2.3 (10 bins) to 3.0 (19 bins), which are equally good with χ²/R ≤ 0.2. In Figure 4b we have chosen to show exponents in the middle of the intervals, λ of 3.5 for the non-corrected distribution, that was used for estimating the RHESSI dead time losses in section 4. For the corrected distribution we show a λ of 2.6 for the entire distribution and a λ of 1.7 for the lower part, indicating a roll-off, as will be discussed in section 6.

6. Discussion

[13] Fermi also has a dead-time loss up to 50% for intense TGFs [Briggs et al., 2010]. Because Fermi is seeing 3.6 ± 0.7 times more TGFs than RHESSI, we believe that Fermi due to its more sophisticated search algorithm, is seeing the weaker part of the TGF fluence distribution. We can not rule out that Fermi may lose some counts due to dead-time even for these weak TGFs, but we will argue that the lower threshold of TGF detection for Fermi is most likely determined by the signal-to-noise ratio rather than dead-time losses.

[14] There are two important values that our estimated true fluence distribution depends on: 1) the relative sensitivity (X) of the two instruments and 2) the relative daily detection rate (Y), where we have used X = 2.7 and Y = 3.6 ± 0.7. To examine how uncertainties in these two estimates may influence our result we can rewrite equation (5) to obtain

$$\lambda = 1 + \frac{\ln(Y)}{\ln(X)}$$  

(7)

[15] In Figure 5a we keep the relative daily TGF detection rate fixed at Y = 3.6 and let the relative sensitivity (X) vary from 1 to 5. One can see that if Fermi is more sensitive relative to RHESSI than we have estimated (moving to higher values) the true distribution will be slightly harder. On the other hand, if the two instruments have almost similar sensitivities the true fluence distribution quickly becomes very soft. The dashed lines show the same dependence when the upper and lower limits of Y are used. We have based our estimate of relative sensitivity on information presented by

![Figure 5](image-url)
Grefenstette et al. [2009], Meegan et al. [2009], Briggs et al. [2010], and Briggs [2011, also personal communication, 2011]. For the RHESSI data we have only used the 591 TGFs before the degradation of the instrument occurred. The average effective detection area is adopted from Briggs et al. [2010], but looking at Figure 11 of Meegan et al. [2009] one could argue that the average is closer to 170 cm². This would have given us a λ of 2.2, but introduces an uncertainty too small to affect the ±0.2 used in equation (6).

[16] In Figure 5b we keep the relative sensitivity fixed at \( X = 2.7 \) and let the daily TGF detection rate (\( Y \)) vary from 1 to 6. The daily TGF detection rate for RHESSI is fairly well established by Grefenstette et al. [2009], while Fermi’s daily detection rate is given as approximately 1 [Fishman, 2011]. As described above, based on the information given by Briggs [2011, also personal communication, 2011] we found that the equivalent (to RHESSI) daily detection rate for Fermi after downloading data, due to sensitivity differences only, is 1.5 ± 0.3 TGFs/day, with 1.2 (1.8) TGFs/day corresponding to TGFs with higher (lower) fluence over ocean than land. The grey shaded box in Figure 5b shows the range spanned by the two extreme values and indicates that the true fluence distribution of TGFs as measured from satellite altitude follows a power law with \( \lambda = 2.3 ± 0.2 \). This is in good agreement with the estimated power distributions with \( \lambda \) ranging from 1.9 to 2.5 reported by Gjesteland et al. [2011], using geolocation and energy spectra of RHESSI TGFs.

[17] The two methods we have used give converging \( \lambda \)-values. Furthermore, if 10 to 12 bins were used for the fit to dead-time corrected distribution in Figure 4b we would get \( \lambda = 2.3 \). As we in our first approach focus on extending the distribution down to fluences below the RHESSI lower threshold, we conclude that both methods support a distribution with \( \lambda = 2.3 ± 0.2 \).

[18] What we have estimated is the true TGF distribution as measured from satellite altitude, which is not necessarily the same as the true TGF source distribution. Flying much closer to the source, an experiment like ADELE is probably exposed to a distribution more similar to the latter. In a recent paper Carlson et al. [2012] have calculated the relationship between the two and for hard distributions the differences are significant. For a distribution with \( \lambda = 2.3 ± 0.2 \) the true source distribution would have \( \lambda = 2.0 ± 0.2 \). As reported by Smith et al. [2011] ADELE, flying at 14 km altitude, saw only one TGF when passing 1213 lightning discharges less than 10 km away. However, ADELE was closer than 4 km to 133 discharges and according to the model results presented in that paper the sensitivity of ADELE is increased about two-to-three orders of magnitude from 10 km to 4 km.

[19] It has been suggested that TGFs are associated with IC lightning bringing negative charges upward [Cummer et al., 2005; Williams, 2006; Shao et al., 2010; Cummer et al., 2011]. As this type of lightning accounts for about 75% of all lightning [Boccippio et al., 2001] this would imply that almost all lightning discharges have an associated TGF. We will now discuss this hypothesis in the context of the power distributions we have found and the non-detection of TGFs by ADELE as well as the sensitivity of ADELE versus RHESSI.

[20] First, we estimate the relative sensitivity between ADELE at 10 km and RHESSI. We use 400 km as the radius of the effective detection area below RHESSI [see Collier et al., 2011, Figure 6] and notice that RHESSI detects TGFs produced within ±38° latitude. Then, the global production rate of TGFs within this latitude range and with strength larger than the RHESSI threshold of 17 counts is about 260 TGFs/day. The global lightning rate is 3.8 × 10⁶/day [Christian et al., 2003], but within ±38° latitude it is 3.5 × 10⁶/day. If we only consider the IC lightning (75% of total) we get a RHESSI-TGF/lightning ratio of 9.8 × 10⁻⁵. Of 1213 lightning RHESSI would have seen 0.1 TGF, while ADELE saw 1. Solving equations (5) or (7) with \( Y = 10 \) and \( \lambda = 2.3 \) gives \( X = 6 \) indicating that ADELE’s sensitivity at 10 km is about 6 times better than RHESSI and 2 times better than Fermi. If the source distribution with \( \lambda = 2.0 \) were used these number would be larger.

[21] In Figure 6a we show the integrated distribution of TGFs, \( N \), as a function of lower detection threshold, \( n_0 \), (equation (4)) from 1213 and 133 lightning discharges assuming that they all make TGFs with a fluence distribution following a power law with \( \lambda = 2.0 \) (solid lines). The two values of \( n_0 \) denote the lower threshold (relative scale) for detecting 1 TGF (\( N = 1 \)). For \( \lambda = 2.0 \) the sensitivity has to increase by a factor of ~10 (1/0.1) to see 1 TGF from a distribution of 133 given that 1 TGF was detected from a distribution of 1213. ADELE’s sensitivity is modeled to be 100–1000 times better at 4 km compared to 10 km [Smith et al., 2011] and corresponds to having a lower threshold of \( n_0 = 1/100 \) to 1/1000 (Figure 6a). This would imply that ADELE should have seen about 10 (at \( n_0 = 1/100 \)) TGFs from the 133 lightning discharges if they all produce TGFs, and the probability of non-detection is very low.

[22] It should be noticed that the modeling of ADELE’s sensitivity is based on certain assumptions. The model is only valid for IC+ discharges, while at least 50% of the subset shown in Figure 2 (top and middle) of Smith et al. [2011] are CG– discharges. A fixed 87 g/cm² is used for the avalanche region, which might be reasonable for charge top below 16 km (3 km charge separation), but is very large (5 km) for the higher charge tops.

[23] Assuming that ADELE’s sensitivity is indeed 1000 times better at 4 km compared to 10 km our results indicate that there is a cut-off (or roll-off) in the TGF distribution. Such a cut-off is implicit in the analysis of a fixed number of lightning discharges: the lower limit must be chosen such that the integral of the distribution matches the number of events. ADELE’s single observation at a relative intensity of \( n_0 = 1 \) out of 1213 lightning discharges implies a minimum intensity threshold of \( n_0 \sim 1/1000 \), the minimum value on the x axis in Figure 6a. We can estimate at which fluence value relative to the lower threshold of RHESSI detection this cut-off might be, assuming that the TGFs follow Poisson statistics. The probability, \( p \), of non-detection when predicted number of detection is \( N_0 \), is given by

\[
p(p = 0|N = n_0) = e^{-n_0}
\]
Figure 6. (a) The distribution of TGFs if all the 1213 and 133 lightning discharges can produce TGFs with a power law distribution with $\lambda = 2.0$ (solid). The vertical dotted line is the highest number of observed TGFs given a sharp cut-off in the distribution. The dashed lines are for a power distribution with $\lambda = 1.3$. (b) The probability of non-detection as a function of relative sensitivity for ADELE at 10 km and 4 km given that one TGF was detected at 10 km. Probabilities are shown for distributions with $\lambda = 2.0$ (solid) and $\lambda = 1.8$ and 2.2 (dotted) and $\lambda = 1.3$ (dashed). The horizontal dotted line indicates a probability of 1 out of 10.

given by the relative lower thresholds of detection, $n_a/n_{10}$. Given that 0.1 ($N_{pe} = 2.6$) from the 133 distribution is a reasonable probability of non-detection (marked with a dotted horizontal line in Figure 6b) this cut-off is at a sensitivity level of 5/100 of ADELE at 10 km, which is 5/600 of the weakest TGF observed by RHESSI (RHESSI has 1/6 of ADELE sensitivity at 10 km), or $\sim$3/600 if one compares with the average RHESSI TGF, which is a factor of 2 larger than the RHESSI lower threshold. If the increase of ADELE’s sensitivity is less than three orders of magnitude (from 10 km to 4 km) this cut-off would move to lower values. If all the lightning discharges produces TGFs, the modeling results of Smith et al. [2011] would have to be off by a little less than one order of magnitude.

We can relate this cut-off in the TGF distribution to the lowest number of electrons that can be produced in a TGF and what the global TGF production rate would be. Our modeling results, using the model described by Østgaard et al. [2008], indicate that the total number of photons produced in an average RHESSI TGF ranges from $10^{16}$ (21 km production altitude) to $10^{18}$ (15 km production altitude) in agreement with others [e.g., Smith et al., 2011]. The probability of bremsstrahlung production increases non-linearly with energies and is about 10% for 2 MeV electrons [Berger and Seltzer, 1972] and approaches 100% at higher electron energies. Measured photon energies $>20$ MeV indicate that we are in this energy range, which implies that the number of electrons is also ranging from $10^{16}$ to $10^{18}$. With a cut-off in the TGF distribution at 5/600 of the RHESSI threshold the lowest possible number of electrons produced in a TGF would be $\sim 10^{14}$.

From Figure 6a one can see that a cut-off at $n_0 = 5/100$ which corresponds to $\sim$5/100 of ADELE at 10 km and 5/600 of the RHESSI lower threshold would give 20 TGFs from the 1213 lightnings from which RHESSI would have seen 0.1 TGF. This implies that the global production rate of TGFs within $\pm 38^\circ$ latitude is about 200 (20/0.1) times what we estimated from RHESSI TGF detection. This gives 50000 TGFs/day or about 35 TGFs every minute and compared to the IC lightning occurrence frequency within the same latitude range of $2.7 \times 10^4$/day, the ratio of TGF/lightning is about 2%. These numbers are slightly larger than estimated by Smith et al. [2011].

We should emphasize that these estimates are based on only one single TGF observation from 10 km. Furthermore, they are based on the assumption of having a sharp cut-off in the TGF distribution. In reality there is probably a roll-off which would decrease the lowest number of electrons and increase the global TGF production rate. Our estimates are consistent with the non-detection by ADELE and depend strongly on these results. If future aircraft or balloon missions find slightly different results our estimates need to be recalculated.

Finally, we will discuss the implication of a roll-off instead of a sharp cut-off in the TGF distribution which is a more realistic distribution. Our results indicate that the power law with $\lambda = 2.3$ is valid at least down to the Fermi threshold, which is 1/3 of RHESSI. Looking at the black histogram in Figure 4b one can argue that there is indeed a roll-off which would decrease the lowest number of electrons and increase the global TGF production rate. Such a roll-off would move the Fermi threshold to lower energies. Measured photon energies $>20$ MeV indicate that we are in this energy range, which implies that the number of electrons is also ranging from $10^{16}$ to $10^{18}$. With a cut-off in the TGF distribution at 5/600 of the RHESSI threshold the lowest possible number of electrons produced in a TGF would be $\sim 10^{14}$.
a radius of 400 km. The lowest number of total electrons produced in a TGF would then be \( \sim 10^{12} \).

7. Summary

[29] To summarize, we have used two independent methods to find the RHESSI dead-time losses and an estimate of the true fluence distribution of TGFs as measured from satellite altitude. The two methods give dead-time losses of 24% and 26% for an average RHESSI TGF 33–35 counts. Assuming a sharp cut-off the true TGF fluence distribution is found to follow a power law with \( \lambda = 2.3 \pm 0.2 \) down to \( \sim 5/600 \) of the detection threshold of RHESSI. This corresponds to a lowest number of electron produced in a TGF to be \( \sim 10^{11} \) and a global production rate within \( \pm 38\% \) latitude of 5000 TGFs/day or about 35 TGFs every minute, which is 2% of all IC lightning. If a more realistic distribution with a roll-off below 1/3 (or higher) of the RHESSI lower detection threshold with a true distribution with \( \lambda \leq 1.7 \) that corresponds to a source distribution with \( \lambda \leq 1.3 \) is considered, we can not rule out that all discharges produce TGFs. In that case the lowest number of total electrons produced in a TGF is \( \sim 10^{12} \).

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References


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