Department of Physics and Technology

Particle Physics

Master Thesis

Mass Ordering of Third Generation Sleptons in a Trilinear-Augmented Gaugino Mediated Supersymmetry-Breaking Scenario

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Abstract

In this thesis, we are exploring the particle spectrum of trilinear-augmented gaugino mediation. In particular, we calculate the masses of the stau and the tau sneutrino in a section of the parameter space in which one is found to be the NLSP, while being very nearly mass-degenerate to the other. We attempt to understand the mechanisms behind this small mass difference at tree-level, and discuss how the situation changes by going up to 1-loop level.

Contents

1 Introduction

The standard model of particle physics (SM) is often hailed as one of the greatest achievements of modern physics. One illustrative example of its astonishing predictive power lies in the measurement of the electromagnetic coupling α, as inferred from measurements of the magnetic moment of the electron [1]: the difference between theory and experiment is currently measured to be less than 10^{-12} [2]. Nevertheless, the theory is also generally viewed as incomplete, the most glaring omission being the gravitational force. It also lacks any particle matching the characteristics of the observed mass abundance in the universe, often dubbed Dark Matter [3], and its prediction of the cosmological constant is infamously 120 orders of magnitude too large [4]. In addition, a general uneasiness resides regarding the arbitrariness of the theory: it has some 20 free parameters, including the masses of most of its particle contents, and in order to work, cancellations between huge numbers are needed to produce the observed values. This latter problem is dubbed the Hierarchy Problem [5], and as we will see, provides one of the main motivations for considering Supersymmetry (SUSY) as an extension to the standard model.

Figure 1: The Bullet Cluster, providing striking evidence for Dark Matter [6].

However, if Supersymmetry were to hold exactly, we would see a lot more particles all around us than we do. As such, if the universe is fundamentally supersymmetric, something must break the symmetry, and in the process give the new particles higher masses, making them more difficult to observe. There are many suggestions to how this might

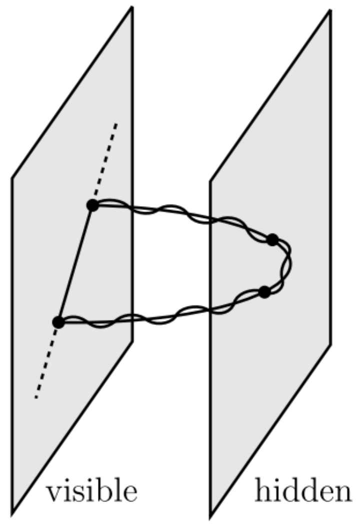

Figure 2: A gaugino loop, giving mass to a scalar particle through interactions on the hidden brane, (figure taken from [8]).

happen, and one such suggestion is the theory we've come to know as *gaugino-mediation* [7, 8].

Here, we embed the visible universe on a four-dimensional "brane" living in a higher-dimensional "bulk", with the extra spatial dimensions compactified. The fermions and their superpartners are confined to the brane, while the gauge and Higgs bosons and their superpartners are allowed to propagate in the bulk. Supersymmetry is then broken by interactions with fields on other branes, spatially seperated from ours, and carried back to the visible brane-fields through loop interactions (see figure 2).

In this thesis, we will explore some of the phenomenological consequences of such a setup. The superpartner of the graviton, the gravitino, is found to be the lightest supersymmetric particle (LSP), and a viable dark matter candidate[1]. Due to the weakness of gravity, this makes the next-to-lightest supersymmetric particle (NLSP) long-lived, and a good candidate for beyond-the-standard-model searches in colliders like the LHC [11].

[1] Such an explanation for the perceived mass abundance is usually referred to as WIMP dark matter, for Weakly Interacting Massive Particles; other possible explanations include Primordial Black Holes (PBHs) [9] and Modified Newtonian Dynamics (MOND) [10].

After defining our conventions in Chapter 2, we give in Chapter 3 a general introduction to supersymmetry, supersymmetry breaking, and the Minimal Supersymmetric Standard Model (MSSM). In chapter 4, we discuss the process needed for making precision corrections in Quantum Field Theory (QFT) finite, known as Renormalisation. After describing gaugino-mediated SUSY-breaking in Chapter 5, we provide in Chapter 6 an analysis of a part of parameter space in which the NLSP changes from the neutrino superpartner, the sneutrino, to the tau superpatner, the stau. Reconstructing the masses at tree-level and at one-loop level, we then identify the processes most important in determining the difference in mass between them. Concluding remarks are given in Chapter 7, and some details omitted in the main text are provided in the Appendices.

2 Conventions and notations

Before we start exploring the many aspects of supersymmetry and gaugino mediation, it is necessary to define some conventions; in this thesis, we will mainly follow those of references [12] and [5].

We work in a relativistic framework wherein three-dimensional space is combined with time to form four-dimensional spacetime, described by the Minkowski metric[2]

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{2.1}$$

Here, we follow the standard convention of using letters from the middle of the Greek alphabet to denote space-time indices, and letters from the middle of the Roman alphabet to denote three-dimensional space indices. Vectors now have four components, $a^\mu = (a^0, a^1, a^2, a^3)$, with the zeroth component denoting the time part and the other three denoting the usual three-dimensional space parts.

The inner product of vectors is given by

$$g_{\mu\nu} a^\mu b^\nu \equiv a_\nu b^\nu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 . \tag{2.2}$$

Note that here we use Einstein's summation convention, wherein summation over repeated indices is implied. We also state for the record that consistency requires that only an upper and a lower space-time index be contracted with one another, and that in the first equality above we have defined the metric's ability to raise and lower space-time indices.

$$g_{\mu\nu} a^\nu = a_\mu , \qquad g^{\mu\nu} a_\nu = a^\mu . \tag{2.3}$$

[2]Here we find a slight abuse in notation, as technically the left hand side of this equation is a single number, while the right hand side is a matrix. We will allow ourselves this in order to write the indices explicitly, which has become the standard notation for relativistic theories.

Position vectors combine with time to become

$$x^{\mu} = (t, x, y, z) \; , \tag{2.4}$$

while momentum vectors combine with energy to become

$$p^{\mu} = (E, p_x, p_y, p_z) \; . \tag{2.5}$$

To simplify our equations, we work in natural units, where the speed of light and the reduced Planck constant are both set to unity:

$$c = \hbar = 1 \; . \tag{2.6}$$

In several of the definitions to come, we will employ the *commutator*, defined as

$$[A, B] = AB - BA \; , \tag{2.7}$$

as well as the *anti-commutator*, defined as

$$\{A, B\} = AB + BA \; . \tag{2.8}$$

Particles are described as excitations of fields of different spins, where we separate between fields of integer spin, called bosons (due to them obeying Bose-Einstein statistics), and fields of half-integer spin called fermions (obeying Fermi-Dirac statistics). Spin 0 fields are called scalar fields, and they carry no index, while spin 1 fields are called vector fields, as they carry one space-time index. Spin 1/2 fields are known as spinor fields, and they are described by four-dimensional objects called Dirac spinors, named after the legendary physicist who developed them.

$$\Psi_{\alpha} = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix} \; . \tag{2.9}$$

As illustrated, we will denote spinor indices by letters from the beginning of the Greek alphabet, in order to separate them from space-time indices. These objects obey equations of motion given by the Dirac equation,

$$(i\gamma^\mu\partial_\mu - m)\Psi \equiv (i\partial\!\!\!/ - m)\Psi = 0\,, \tag{2.10}$$

where $m \equiv m\,\mathbb{1}$, and where we've employed the Feynman slash $\gamma^\mu a_\mu = a\!\!\!/$. The γ^μ are matrices obeying

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\,, \tag{2.11}$$

and can be given in several representations. In this thesis, we operate with the so-called *chiral* or *Weyl* representation, wherein they're given as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}\,, \tag{2.12}$$

where

$$\begin{aligned} \sigma^0 = \bar{\sigma}^0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\,, \qquad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,, \\ \sigma^2 = -\bar{\sigma}^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,, \qquad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \end{aligned} \tag{2.13}$$

A useful definition, which has come to be known as the fifth gamma matrix, is given by

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{2.14}$$

With this, we can now define the chiral projection operators:

$$P_L = \frac{1}{2}(1 - \gamma^5)\,, \qquad P_R = \frac{1}{2}(1 + \gamma^5)\,, \tag{2.15}$$

which, as the names imply, return the left- and right-handed[3] parts, respectively, of any spinor they act upon.

$$\Psi_L = P_L\Psi\,, \qquad \Psi_R = P_R\Psi\,. \tag{2.16}$$

These operators now take on a particularly simple form, giving the chiral representation

[3]Technically left- and right-*chiral*; there is a slight difference in these definitions which can be summarised as to say that chirality is Lorentz-invariant for all particles, while handedness, or helicity, isn't. We will follow the standard nomenclature and make it implicit that whenever we talk about handedness, we are referring to chirality.

its name.

$$P_L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \qquad P_R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{2.17}$$

Thus, the left- and right-handed parts of a Dirac field simply become

$$\Psi_L = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ 0 \\ 0 \end{pmatrix} , \qquad \Psi_R = \begin{pmatrix} 0 \\ 0 \\ \Psi_3 \\ \Psi_4 \end{pmatrix} . \tag{2.18}$$

Chirality turns out to play a major role in formulating the laws of nature, as left-handed fields transform differently under the various gauge transformations of the standard model than right-handed fields. As such, it is useful to define the fields in terms of left- and right-handed *two*-component spinors, called Weyl spinors:

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} , \tag{2.19}$$

where

$$\psi_L = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} , \qquad \psi_R = \begin{pmatrix} \Psi_3 \\ \Psi_4 \end{pmatrix} . \tag{2.20}$$

Now we need to be a bit more careful with the spinor indices. The conventions are as follows: left-handed Weyl spinors carry undotted indices $\alpha \in \{1, 2\}$, while right-handed Weyl spinors carry dotted indices $\dot{\alpha} \in \{1, 2\}$. Also note that the Hermitian conjugate of any left-handed Weyl spinor is a right-handed Weyl spinor (and vice versa), such that

$$\psi^{\alpha} = (\psi^{\dagger\dot{\alpha}})^{\dagger} . \tag{2.21}$$

This means that any right-handed field may be defined in terms of a left-handed Weyl spinor. Following convention, we define *all* fields in terms of left-handed spinors and their conjugates, and as such, we decompose the four-component Dirac spinor as

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \xi_{\alpha} \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} . \tag{2.22}$$

The Weyl spinor indices are raised and lowered by the antisymmetric symbol $\epsilon_{\alpha\beta}$, given by

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1 \,, \qquad \epsilon_{11} = \epsilon^{22} = \epsilon^{11} = \epsilon^{22} = 0 \,, \tag{2.23}$$

satisfying $\epsilon_{\alpha\sigma}\epsilon^{\sigma\beta} = \epsilon^{\beta\sigma}\epsilon_{\sigma\alpha} = \delta^{\beta}_{\alpha}$, and likewise with dotted indices. Note that this implies that

$$\begin{aligned} \xi_\alpha \chi^\alpha &= \epsilon_{\alpha\sigma}\xi^\sigma \epsilon^{\alpha\beta}\chi_\beta \\ &= \xi^\sigma(-\epsilon_{\sigma\alpha})\epsilon^{\alpha\beta}\chi_\beta \\ &= -\xi^\sigma \delta^{\beta}_{\sigma}\chi_\beta \\ &= -\xi^\beta \chi_\beta \,. \end{aligned} \tag{2.24}$$

This means that, as opposed to with spacetime and Dirac indices, we now need to keep track of which way the indices are contracted. We choose the definitions

$$\xi^\alpha \chi_\alpha = \xi\chi \,, \qquad \xi^\dagger_{\dot\alpha}\chi^{\dagger\dot\alpha} = \xi^\dagger\chi^\dagger \,. \tag{2.25}$$

σ^μ and $\bar\sigma^\mu$ each carry two indices, one dotted and one undotted.

$$(\sigma^\mu)_{\alpha\dot\alpha} \,, \qquad (\bar\sigma^\mu)^{\dot\alpha\alpha} \,. \tag{2.26}$$

The Dirac equation can then be written as [13]

$$\begin{pmatrix} m & -i\sigma^\mu\partial_\mu \\ -i\bar\sigma^\mu\partial_\mu & m \end{pmatrix} \begin{pmatrix} \xi \\ \chi^\dagger \end{pmatrix} = 0 \,. \tag{2.27}$$

Finally, we note the rather nice result that now easily follows:

$$\xi\chi = \xi_\alpha\chi^\alpha = -\xi^\alpha\chi_\alpha = \chi_\alpha\xi^\alpha = \chi\xi \,, \tag{2.28}$$

where we have used the fact that the components of a fermion field are anti-commuting[4].

[4] Such variables are called *Grassmann numbers* — though, of course, they are not numbers in the ordinary sense. Nevertheless, it is possible to define algebraic operations on them, and their study has become especially important in the wider field of supersymmetric theories. For a review, see e.g. [5, Ch. 4].

3 Supersymmetry as an extension to the Standard Model

3.1 Motivation: the hierarchy problem

As was mentioned in the introduction, the so-called hierarchy problem has provided one of the chief motivations for supersymmetry ever since its inception [5]. It pertains to a problem one encounters in general extensions to the standard model, and in particular in attempting to make predictions to the Higgs mass[5]. The problem is that any analytical expression for the Higgs mass would receive enormous corrections from every other massive particle in the theory. This happens through so-called loop corrections, which after renormalisation affects all masses and couplings of the theory[6].

For example, for any (Dirac) fermion f, coupling to the Higgs field with a Lagranigan term $-\lambda_f H\bar{f}f$, where λ_f is the Yukawa coupling of the particle, the Feynman diagram in figure 3b yields a correction

$$\Delta m_H^2 = -\frac{\lambda_f^2}{8\pi^2}\Lambda_{UV}^2 + \mathcal{O}(m_f)\ . \tag{3.1}$$

Here, the cut-off scale Λ_{UV} is to be interpreted as the scale above which new physics enters to alter the high-energy behaviour of the theory, ensuring renormalisability. In addition to this, if there are heavier, undiscovered Dirac fermions, we see from equation (3.1) that the contribution would be even larger, demonstrating that the *largest* contribution to the Higgs mass comes from the *heaviest* of the particles it is interacting with.

Similarly, if there were an undiscovered scalar particle S, coupling to the Higgs via the term $-\lambda_S H^2 S^2$, a diagram such as figure 3a would give us

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\Lambda_{UV}^2 + \mathcal{O}\left(m_S^2\right)\ . \tag{3.2}$$

[5]This is often talked of as being a problem with the standard model itself, though with the Higgs mass simply being a free parameter, it would be more accurate to call it a *worry* than a problem.

[6]For a detailed discussion, see section 4.

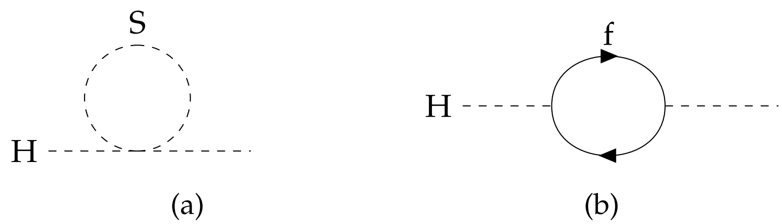

Figure 3: One-loop corrections to the Higgs mass due to (a) a scalar, and (b) a fermion.

One might picture a situation where such an undiscovered particle zoo extends all the way to the Planck scale[7], $M_P = \frac{1}{\sqrt{8\pi G}} = 2.4 \times 10^{18}$ GeV, above which the complete and as-of-yet undiscovered theory of quantum gravity enters to take care of the high-energy behaviour. Even though we see from equations (3.1) and (3.2) that terms of opposite signs would enter the complete Higgs-mass correction, the cancellations needed between terms of order $\mathcal{O}\left(M_P^2\right) = 10^{36}$ GeV^2 to produce the observed Higgs mass parameter $m_H^2 = -(92.9 \text{ GeV})^2 = -\mathcal{O}\left(10^4\right)$ GeV^2 would be miraculous indeed[8]. Even if one were to assume that *none* of the undiscovered particles interacted directly with the Higgs, similarly large corrections would still enter through higher loop orders as long as the particles in question shared *any* of the gauge interactions of the Higgs.

If there were a *symmetry*, however, pairing every fermion with two scalars in such a way that $\lambda_S = \lambda_f^2$, we see that the quadratic divergences in (3.1) and (3.2) would cancel exactly. This is the kind of symmetry we have come to know as *supersymmetry*, and it has been shown that imposing it not only makes the quadratic divergences of the renormalised Higgs mass cancel, but *all* its divergences, to *arbitrary* loop order [5].

3.2 The symmetries of nature

In order to understand supersymmetry, then, we must first understand symmetry. Let's start with the obvious question[9].

[7] G is Newton's gravitational constant.

[8] Such cancellations might be described as being *unnatural*, giving the subject its alternate name of *the naturalness problem*.

[9] In the following, we will be quoting mainly from [14].

3.2.1 What is a symmetry?

The dynamics of a system are in field theories determined by the Lagranigan density of the system, $\mathcal{L}$, hereafter simply referred to as the Lagrangian. It is defined as

$$S = \int d^4x \mathcal{L}(\phi_r(x^\mu), \partial_\nu \phi_r(x^\mu), x^\mu) \ , \tag{3.3}$$

where S denotes the action, and the ϕ_r denote the dynamical variables of the system. The equations of motion can then be derived by demanding that the action take on stationary values.

$$\delta S = 0 \ . \tag{3.4}$$

A symmetry of the system, then, is a transformation of the dynamical variables that leaves the Lagrangian invariant, at least up to a total spacetime derivative[10].

$$\begin{aligned} &\phi_r, x \longmapsto \phi_r', x' \ , \quad \mathcal{L} \longmapsto \mathcal{L}' \ + \partial_\mu K^\mu , \\ &\mathcal{L}'(\phi_r'(x'), \partial_\mu \phi_r'(x'), x') = \mathcal{L}(\phi_r'(x'), \partial_\mu \phi_r'(x'), x') \ , \end{aligned} \tag{3.5}$$

where we have suppressed the space-time index of x in order to avoid clutter. As an example, consider the theory of a massive complex scalar field:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \ , \tag{3.6}$$

where ϕ denotes the field, ϕ^* its complex conjugate, and m denotes the mass. It is trivial to see that this Lagrangian is invariant under the transformation

$$\begin{aligned} \phi &\longmapsto \phi' = -\phi \ , \\ \phi^* &\longmapsto \phi'^* = -\phi^* \ . \end{aligned} \tag{3.7}$$

This transformation represents a typical example of a discrete internal symmetry, and is known as a $\mathbb{Z}_2$ symmetry.

[10] The argument goes as follows [12]: since the Lagrangian is to be integrated over all space and time, one may use the divergence theorem to convert the volume integral of the total derivative to a term integrated over the boundary of the volume — and since the volume is infinite, the surface is infinitely far away, and we may without loss of generality set the fields to vanish there. In other words, we set any *surface terms* that show up in our Lagranigans, of the general form $\partial_\mu K^\mu$, to be zero.

Another class of symmetries, particularly important to field theories, are *continuous* symmetries. Consider the transformation

$$\begin{aligned} \phi &\longmapsto \phi' = e^{i\varepsilon}\phi \ , \\ \phi^* &\longmapsto \phi'^* = e^{-i\varepsilon}\phi \ , \end{aligned} \tag{3.8}$$

where ε denotes a real, continuously variable parameter. Applying the transformation to our Lagrangian, we get

$$\begin{aligned} \mathcal{L}' &= \partial_\mu(e^{-i\varepsilon}\phi^*)\partial^\mu(e^{i\varepsilon}\phi) - m^2(e^{-i\varepsilon}\phi^*)(e^{i\varepsilon}\phi) \\ &= \partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi \\ &= \mathcal{L} \ . \end{aligned} \tag{3.9}$$

This particular transformation is known as a *global* U(1) transformation, for reasons that will become apparent shortly, and is a generalisation of the previous $\mathbb{Z}_2$ symmetry.

An even larger class of symmetry is achieved if one allows the continuous parameter to depend on space-time.

$$\varepsilon = \varepsilon(x^\mu) \ . \tag{3.10}$$

These are then called *local*, or *gauge*, transformations, and invariance under them implies the existence of *gauge bosons* — spin-1 fields carrying a space-time index. All known forces of nature can be described as being mediated by such bosons belonging to different *gauge groups*[11].

In fact, it can be easily seen that the above Lagrangian (3.6) would *not* be invariant under such transformations. In order to achieve invariance, we need to make the replacement $\partial_\mu \longmapsto D_\mu$,

$$\mathcal{L} = (D_\mu\phi)^* D^\mu\phi - m^2\phi^*\phi \ , \tag{3.11}$$

where D_μ now includes a term proportional to a new vector field, A_μ.

$$D_\mu = \partial_\mu + igA_\mu \ . \tag{3.12}$$

[11] With the notable exception of gravity — its force carrier, the graviton, is a spin-2 field, meaning it carries two space-time indices: $g^{\mu\nu}$; its symmetry group is that of a general coordinate transformation, known as a diffeomorphism [15].

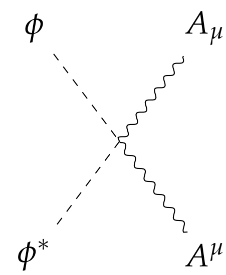

Figure 4: Quartic interaction between a complex scalar and a gauge boson.

In order to complete the Lagrangian, the vector field will need a kinetic term too. This is easiest expressed in terms of the field strength tensor

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{3.13}$$

Adding the so-called Maxwell term, we then get the complete Lagrangian of a gauged complex scalar field.

$$\mathcal{L} = (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \tag{3.14}$$

This will be invariant under the gauge transformation

$$\begin{aligned} \phi(x) &\longmapsto \phi'(x) = e^{ig\varepsilon(x)} \phi(x) \ , \\ \phi^*(x) &\longmapsto \phi'^*(x) = e^{-ig\varepsilon(x)} \phi(x) \ , \\ A_\mu(x) &\longmapsto A'_\mu(x) = A_\mu(x) - \partial_\mu \varepsilon(x) \ . \end{aligned} \tag{3.15}$$

We now have terms proportional to $\sim A^\mu \phi$, $A_\mu \phi^*$, which we interpret as being interactions between the scalars and the gauge boson — g is then interpreted as the coupling strength. For example, from the term $-g^2 A_\mu \phi^* A^\mu \phi$, we would get an interaction vertex like the one in figure 4.

Note that we did not add a mass term for A_μ — as can easily be checked, such a term would break the gauge symmetry of the Lagrangian (3.14).

For a more detailed discussion on the subject, see e.g. [12, Ch. 11].

3.2.2 Lie groups

In defining a framework to parametrise different classes of continuous symmetries, it is useful to introduce the notion of a group, G, defined to be a set of elements $g \in G$ and an operation $\circ$, satisfying

1. Closure: $g, h \in G \Rightarrow g \circ h \in G$
2. Contains neutral element e: $g \circ e = g \quad \forall \quad g \in G$
3. Contains inverse elements g^{-1}: $g \circ g^{-1} = e \quad \forall \quad g \in G$
4. Association: $f, g, h \in G \Rightarrow f \circ (g \circ h) = (f \circ g) \circ h$

As an example, consider the set of all real 3 by 3 matrices O that are orthogonal,

$$O^T O = 1 \,, \tag{3.16}$$

and have the "special" property of unit determinant,

$$\det O = 1 \,. \tag{3.17}$$

One can easily verify that this set, with the addition of the operation of matrix multiplication, satisfies the definition of a group. This group is called SO(3), for *special*, *orthogonal*, and consisting of *3-dimensional* matrices. It contains rotations in three-dimensional euclidean space, with vectors transforming as

$$v^i \to v'^i = O^{ij} v^j \,, \tag{3.18}$$

and tensors transforming as

$$T^{ij\ldots k} \to T'^{ij\ldots k} = O^{il} O^{jm} \ldots O^{kn} T^{lm\ldots n} \,. \tag{3.19}$$

Being orthogonal (and therefore square), the rotation matrices may be written as

$$O = e^A \equiv \sum_{n=0}^{\infty} \frac{1}{n!} A^n \,. \tag{3.20}$$

Our conditions (3.16) and (3.17) then imply that A must be real and anti-symmetric; we choose to parametrise it as the sum of three anti-symmetric matrices, such that

$$O = e^{i(\theta_1 J_1 + \theta_2 J_2 + \theta_3 J_3)} . \tag{3.21}$$

Here, the J_1 represents a rotation around the first axis by an angle θ_1, J_2 around the second axis, and so on. Note that we have defined the J's to be imaginary, and being anti-symmetric, they are therefore Hermitian.

$$J_i^\dagger = J_i , \tag{3.22}$$

The J's will now satisfy the commutation relation

$$[J_i, J_j] = i\varepsilon_{ijk} J_k , \tag{3.23}$$

where ε_{ijk} is the totally anti-symmetric Levi-Civita symbol. Equation (3.23) defines the *algebra* of the rotation group in three dimensions. Any three-dimensional theory claiming to be isotropic should possess a Lagrangian invariant under the action of this group.

In general, a Lie group[12] is any group whose elements may be written:

$$g = e^{i\alpha^a T^a} , \tag{3.24}$$

where the T^a are $N \times N$ matrices common for every element, called the *generators* of the group, with $\alpha^a \in \mathbb{R}$ determining the element. If the elements of the group are real, orthogonal $N \times N$ matrices with unit determinants, the group is called SO(N), and will have $a \in \{1, 2, \ldots, \frac{1}{2}N(N-1)\}$; if we instead allow the elements to be complex $N \times N$ matrices satisfying unitarity,

$$U^\dagger U = 1 , \tag{3.25}$$

as well as having unit determinant, the group is called SU(N), and has $a \in \{1, 2, \ldots, N^2 - 1\}$.

The generators of the group will follow the Lie algebra given by

$$[T^a, T^b] = i f^{abc} T^c . \tag{3.26}$$

[12] Named after the Norwegian mathematician Sophus Lie.

The f^{abc} are known as the structure constants, and are for example for SU(2) simply given by the three-dimensional Levi-Civita symbol ε^{abc} again.

It's this latter class of symmetry groups that turns out to be realised in nature through the various gauge groups of the standard model: the strong interaction of quarks and gluons follows an SU(3) gauge symmetry, and the unified theory of electromagnetic and weak interactions, known as electroweak theory, follows an SU(2) chiral gauge symmetry in addition to a U(1) hypercharge symmetry. The full gauge group of the standard model is then:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \,. \tag{3.27}$$

3.2.3 Broken symmetries

If the Lagrangian contains terms that are not invariant under a transformation of the fields, we say that the symmetry in question is explicitly broken by the terms. However, it is possible that the Lagrangian is completely invariant, but the resulting dynamics *still* possess the features of a broken symmetry. This happens when the Lagrangian is invariant under the transformation, but the lowest energy state, known as the *vacuum state*, isn't. We then say the symmetry is *spontaneously* broken. Consider the Lagrangian:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - g(\phi^* \phi)^2 \,. \tag{3.28}$$

Again, it is easy to verify that the Lagrangian is invariant under the $U(1)$ symmetry (3.8). If m^2 and g are both positive, the potential given by $V(\phi) = m^2\phi^*\phi + g(\phi^*\phi)^2$ will have its minimum energy at $\phi = 0$, which trivially possesses the same $U(1)$ symmetry.

If, on the other hand, $m^2 < 0$,[13] the potential possesses a continuous range of minima given by

$$\phi = \sqrt{\frac{-m^2}{2g}} e^{i\theta} \,, \tag{3.29}$$

none of which are invariant under (3.8). We say that the $U(1)$ symmetry has been *sponta-*

[13] and $g > 0$, of course; theories with both parameters negative contain an unstable vacuum state, and are therefore never considered.

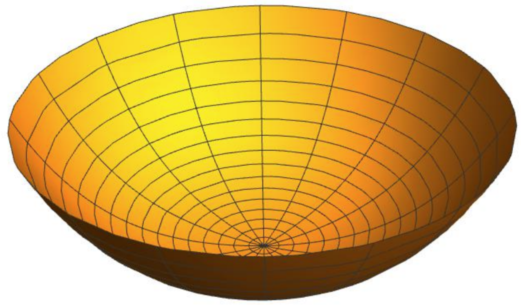

Figure 5: Potential $V(\phi)$ with $m^2, g > 0$. The lowest energy state is at $\phi = 0$.

neously broken by the non-zero vacuum expectation value, or *vev*:

$$v^2 = -\frac{m^2}{2g} \, . \tag{3.30}$$

This is exactly the kind of mechanism responsible for breaking the electroweak symmetry: the Lagrangian is invariant under an $SU(2)_L \otimes U(1)_Y$ gauge transformation, but the vacuum state is *not*. In the process, all Standard Model fermions, as well as the W and Z bosons, are given their masses, and a new particle arises from the symmetry-breaking scalar field. The mechanism is known as the *Brout-Englert-Higgs mechanism*, and the particle is known as the *Higgs boson* [16, 17]. Its 1964 prediction and subsequent 2012 discovery [18, 19] represents one of the greatest successes of the standard model of particle physics.

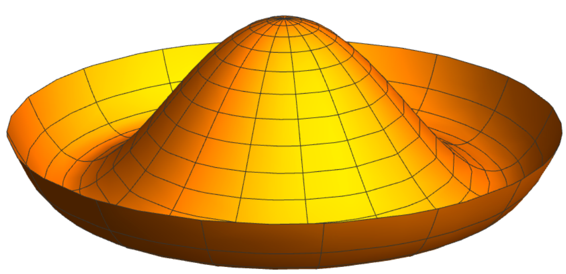

Figure 6: Potential $V(\phi)$ with $m^2 < 0$, $g > 0$, affectionately known as the "Mexican hat potential".

3.3 A different kind of symmetry

Simply put, supersymmetric theories are theories in which the Lagrangian is invariant under a transformation of the bosonic fields into fermionic fields, and vice versa.

$$Q\,|Boson\rangle = |Fermion\rangle\,, \qquad Q\,|Fermion\rangle = |Boson\rangle\,. \tag{3.31}$$

Fermions being fields with half-integer spin (1/2 or 3/2), and bosons fields with integer spin (0, 1, or 2)[14], we can already sense a connection between this symmetry and the symmetries of space-time, which will be made manifest shortly.

The simplest supersymmetric Lagrangian consists of a single chiral supermultiplet (ϕ, ψ) with just a kinetic term for each:

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi\,. \tag{3.32}$$

This is known as the massless, non-interacting Wess-Zumino model [21]. Here, ϕ is again a scalar field, and ψ a left-handed Weyl spinor (see section 2 for a discussion on the latter).

We impose the transformation rule for scalar fields to be

$$\phi \longmapsto \phi' = \phi + \varepsilon\psi\,, \qquad \phi^* \longmapsto \phi'^* = \phi^* + \varepsilon^\dagger\psi^\dagger\,. \tag{3.33}$$

Here, ε^α is a constant[15], infinitesimal Weyl fermion object, with spinor index $\alpha \in \{1,2\}$. Correspondingly for a fermion,

$$\begin{aligned} \psi_\alpha &\longmapsto \psi'_\alpha = \psi_\alpha - i(\sigma^\mu\varepsilon^\dagger)_\alpha \partial_\mu\phi\,, \\ \psi^\dagger_{\dot\alpha} &\longmapsto \psi'^\dagger_{\dot\alpha} = \psi^\dagger_{\dot\alpha} + i(\varepsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^*\,. \end{aligned} \tag{3.34}$$

The hope now is that the Lagrangian (3.32) will be invariant under these transformations.

[14] Any interacting theory containing a finite number of particles with spin > 2 will be inconsistent and such theories are therefore never considered; see [20, Ch. 9.5] for an argument why.

[15] In this thesis, we study *global* supersymmetry, meaning we only consider transformations satisfying $\partial_\mu\varepsilon^\alpha = 0$. *Local* supersymmetry, in which the ε^α are allowed to depend on the spacetime variables x^μ, leads to the possibility of curved backgrounds, and poses a possible path towards understanding gravity in a quantum field theoretic context. The resulting theory is known as *supergravity*, (see e.g. [22] for an introduction on the topic).

Let's check. Inserting for our transformations, we get

$$\mathcal{L} \longmapsto \mathcal{L}' = \partial^\mu(\phi + \varepsilon\psi)\partial_\mu(\phi^* + \varepsilon^\dagger\psi^\dagger) + i(\psi^\dagger + i\varepsilon\sigma^\nu\partial_\nu\phi^*)\bar{\sigma}^\mu\partial_\mu(\psi - i\sigma^\rho\varepsilon^\dagger\partial_\rho\phi)\ . \tag{3.35}$$

Expanding, we have to first order in ε

$$\mathcal{L}' = \mathcal{L} + \varepsilon^\dagger\partial^\mu\phi\partial_\mu\psi^\dagger + \varepsilon\partial^\mu\psi\partial_\mu\phi^* + \psi^\dagger\bar{\sigma}^\mu\sigma^\rho\varepsilon^\dagger\partial_\mu\partial_\rho\phi - \varepsilon\sigma^\nu\bar{\sigma}^\mu\partial_\mu\psi\partial_\nu\phi^*\ . \tag{3.36}$$

Now we can rewrite the latter two terms, such that we end up with

$$\begin{aligned}\mathcal{L}' = \mathcal{L} &+ \varepsilon^\dagger\partial^\mu\phi\partial_\mu\psi^\dagger + \varepsilon\partial^\mu\psi\partial_\mu\phi^* - \varepsilon^\dagger\partial^\mu\phi\partial_\mu\psi^\dagger - \varepsilon\partial^\mu\psi\partial_\mu\phi^* \\ &- \partial_\mu(\varepsilon\sigma^\nu\bar{\sigma}^\mu\psi\partial_\nu\phi^* - \varepsilon\psi\partial^\mu\phi^* - \varepsilon^\dagger\psi^\dagger\partial^\mu\phi)\ .\end{aligned} \tag{3.37}$$

Finally, cancelling and setting the surface term to zero, we get what we wanted:

$$\mathcal{L}' = \mathcal{L}\ . \tag{3.38}$$

The details in going from equation (3.36) to (3.37) actually involves some rather clever tricks — the interested reader will find a step-by-step calculation in appendix C.

In order for us to be able to conclude that this theory is indeed supersymmetric, we also need to show that the supersymmetry algebra closes, i.e. that two supersymmetry transformations is another symmetry of the Lagrangian. We will merely quote the results here; for a detailed discussion, see [5].

Given two supersymmetry transformations, parametrised by ε_1 and ε_2, their commutator yields

$$[\delta_{\varepsilon_2}, \delta_{\varepsilon_1}]\phi = i(-\varepsilon_1\sigma^\mu\varepsilon_2^\dagger + \varepsilon_2\sigma^\mu\varepsilon_1^\dagger)\partial_\mu\phi \tag{3.39}$$

for the bosonic field, and

$$[\delta_{\varepsilon_2}, \delta_{\varepsilon_1}]\psi_\alpha = i(-\varepsilon_1\sigma^\mu\varepsilon_2^\dagger + \varepsilon_2\sigma^\mu\varepsilon_1^\dagger)\partial_\mu\psi_\alpha + i\varepsilon_{1\alpha}\varepsilon_2^\dagger\bar{\sigma}^\mu\partial_\mu\psi - i\varepsilon_{2\alpha}\varepsilon_1^\dagger\bar{\sigma}^\mu\partial_\mu\psi \tag{3.40}$$

for the fermionic field.

Remarkably, we see from (3.39) that the commutator of two supersymmetry transformations gives back a *spacetime translation of the field* — the generator of such transformations is the space-time momentum operator P, generating translations on the fields X according to

$$[P^{\mu}, X] = -i\partial^{\mu} X \; . \tag{3.41}$$

We get the same result from (3.40), given that the equation of motion for the field, as derived from the Lagrangian (3.32), is employed.

$$\bar{\sigma}^{\mu}\partial_{\mu}\psi = 0 \; . \tag{3.42}$$

So here, then, we see the connection between supersymmetry and space-time that was hinted at earlier[16].

From this, we conclude that the supersymmetry algebra indeed closes, but only *on-shell*, i.e. when the equations of motion are satisfied. This might be a bit worrisome, as quantum mechanically there exist *virtual* states that don't obey these equations. Of course, we would very much like supersymmetry to hold quantum mechanically as well as classically, so to remedy this situation we introduce a complex scalar field F, known as an *auxiliary* field, with the Lagrangian

$$\mathcal{L}_{auxiliary} = F^{*}F \; . \tag{3.43}$$

From this, one might derive its equations of motion, finding the rather unexciting result $F = F^{*} = 0$. However, they turn out to be useful in ensuring that supersymmetry also closes off-shell, which is achieved if we revise our initial transformations (3.33), (3.34) to

[16]This is especially remarkable given that there exists a theorem, the Coleman Mandula Theorem, that states that "space-time and internal symmetries cannot be combined in any but a trivial way" [23]. Supersymmetry being an internal symmetry, and clearly combining with space-time symmetries in a non-trivial way, one would think that this means our construction is bust. Supersymmetry circumvents this theorem, however, by being not a Lie algebra but a Lie *superalgebra*. The corresponding, generalized theorem is known as the Haag Lopuszanski Sohnius Theorem [24].

include the auxiliary field. The transformation rules then become:

$$
\begin{aligned}
\phi &\longmapsto \phi' = \phi + \varepsilon\psi \; , \\
\phi^* &\longmapsto \phi'^* = \phi^* + \varepsilon^\dagger\psi^\dagger \; , \\
\psi_\alpha &\longmapsto \psi'_\alpha = \psi_\alpha - i(\sigma^\mu\varepsilon^\dagger)_\alpha\partial_\mu\phi + \varepsilon_\alpha F \; , \\
\psi^\dagger_{\dot\alpha} &\longmapsto \psi'^\dagger_{\dot\alpha} = \psi^\dagger_{\dot\alpha} + i(\varepsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\phi^* + \varepsilon^\dagger_{\dot\alpha}F^* \; , \\
F &\longmapsto F' = F - i\varepsilon^\dagger\bar\sigma^\mu\partial_\mu\psi \; , \\
F^* &\longmapsto F'^* = F + i\partial_\mu\psi^\dagger\bar\sigma^\mu\varepsilon \; .
\end{aligned} \tag{3.44}
$$

With these, one can verify that the supersymmetry algebra indeed closes for every field, and for arbitrary field configurations.

$$
[\delta_{\varepsilon_2}, \delta_{\varepsilon_1}]X = i(-\varepsilon_1\sigma^\mu\varepsilon_2^\dagger + \varepsilon_2\sigma^\mu\varepsilon_1^\dagger)\partial_\mu X \; , \tag{3.45}
$$

with $X \in \{\, \phi, \phi^*, \psi, \psi^\dagger, F, F^*\}$.

According to Noether's theorem [25], invariance of the action under a continuous symmetry implies a conserved current; for supersymmetry, we get the supercurrents J^μ_α, $J^{\dagger\mu}_{\dot\alpha}$. Perhaps unsurprisingly, they carry spinor indices as well as the usual spacetime index — this is one of the unique features of SUSY, and a consequence of the symmetry having fermionic generators. Their form may be derived from the definition,

$$
\varepsilon J^\mu + \varepsilon^\dagger J^{\dagger\mu} \equiv \sum_X \delta X \frac{\delta\mathcal{L}}{\delta(\partial_\mu X)} - K^\mu \; , \tag{3.46}
$$

where K^μ is defined by $\delta\mathcal{L} = \partial_\mu K^\mu$ — in practice, just the surface term again, though it can be redefined as $K^\mu \mapsto K'^\mu = K^\mu + k^\mu$ for any k^μ satisfying $\partial_\mu k^\mu = 0$. Up to this ambiguity then, one then ends up with

$$
J^\mu_\alpha = (\sigma^\nu\bar\sigma^\mu\psi)_\alpha\partial_\nu\phi^* \; , \qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger\bar\sigma^\mu\sigma^\nu)_{\dot\alpha}\partial_\nu\phi \; . \tag{3.47}
$$

These will be separately conserved, $\partial_\mu J^\mu_\alpha = \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0$, and from them one is now able to construct the conserved charges,

$$
Q_\alpha = \sqrt{2}\int \mathrm{d}^3\vec{\mathbf{x}}\, J^0_\alpha \; , \qquad Q^\dagger_{\dot\alpha} = \sqrt{2}\int \mathrm{d}^3\vec{\mathbf{x}}\, J^{\dagger 0}_{\dot\alpha} \; . \tag{3.48}
$$

Here, finally, we have the *generators* of supersymmetry, satisfying *the supersymmetry algebra*:

$$\begin{aligned} \{Q_\alpha, Q^\dagger_{\dot\alpha}\} &= 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \,, \\ \{Q_\alpha, Q_\beta\} &= 0\,, \qquad \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0\,. \end{aligned} \tag{3.49}$$

The fact that the generators satisfy *anti*-commutation relations rather than the usual commutation relations (3.26) is another one of the unique features of supersymmetry, truly making it a *different kind of symmetry*.

Finally, we note that, as can be seen from (3.41) and the fact that we are talking about *global* supersymmetry, these transformations will commute with space-time translations.

$$[Q_\alpha, P^\mu] = 0\,, \qquad [Q^\dagger_{\dot\alpha}, P^\mu] = 0\,. \tag{3.50}$$

3.4 General supersymmetric Lagrangians

In this section, we will quote the most general form the Lagrangian of a consistent supersymmetric field theory can take; to see how the form of such Lagrangians is constrained to give the following results, see [5].

The general form of such Lagrangians turns out to be

$$\begin{aligned} \mathcal{L} = {} & D^\mu\phi^{*i} D_\mu\phi_i + i\psi^{\dagger i}\bar\sigma^\mu D_\mu\psi_i - \frac{1}{2}(W^{ij}\psi_i\psi_j + W^*_{ij}\psi^{\dagger i}\psi^{\dagger j}) - W^i W^*_i + F_i F^{*i} + W^i F_i + W^*_i F^i \\ & - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\bar\sigma^\mu D_\mu\lambda^a + \frac{1}{2}D^a D^a \\ & - \sqrt{2}g(\phi^* T^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a\phi) + g(\phi^* T^a\phi)D^a\,. \end{aligned} \tag{3.51}$$

Here, ψ^i represent the fermions, ϕ^i their scalar partners, and F^i their auxiliary fields; A^a_μ (see eq. (3.57)) are the gauge bosons, λ^a their Weyl-fermionic partners, and D^a their bosonic auxiliary fields[17], with the index a enumerating the generators of the gauge

[17] Not to be confused with D_μ, the covariant derivative.

group[18]. The *superpotential* W is defined as

$$W = L^i \phi_i + \frac{1}{2} M^{ij} \phi_i \phi_j + \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k \,, \tag{3.52}$$

where the term linear in ϕ_i is typically excluded, as it requires ϕ_i to be gauge singlets. In terms of this, we have the definitions

$$W^i = \frac{\delta}{\delta \phi_i} W \,, \qquad W^{ij} = \frac{\delta^2}{\delta \phi_i \delta \phi_j} W \,. \tag{3.53}$$

The covariant derivative D_μ is given for the various fields as

$$D_\mu \phi = \partial_\mu \phi - i g A^a_\mu (T^a \phi) \,, \tag{3.54}$$

$$D_\mu \psi = \partial_\mu \psi - i g A^a_\mu (T^a \psi) \,, \tag{3.55}$$

$$D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c \,, \tag{3.56}$$

and similarly for their conjugate fields. The field strength tensors $F^a_{\mu\nu}$ are defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu \,. \tag{3.57}$$

g now stands for a general gauge coupling. Of course, for theories with more than one gauge symmetry, the Lagrangian and covariant derivatives should be extended to include terms for each coupling separately.

Now the equations of motion for the auxiliary fields are no longer trivial, though they *are* expressible algebraically (i.e. without derivatives) in terms of the scalar fields only.

$$F_i = -W^*_i \,, \qquad F^{*i} = -W^i \,, \qquad D^a = -g(\phi^* T^a \phi) \,. \tag{3.58}$$

Finally, we quote the full transformation rules for these fields, where we for the sake of

[18]Fields carrying such an index are said to be in the *adjoint representation* of the gauge group; see e.g. [14, Appendix B] for a discussion on representations.

brevity have excluded the conjugate fields:

$$\delta\phi_i = \varepsilon\psi_i \,, \tag{3.59}$$

$$\delta\psi_{i\alpha} = -i(\sigma^\mu\varepsilon^\dagger)_\alpha D_\mu\phi_i + \varepsilon_\alpha F_i \,, \tag{3.60}$$

$$\delta F_i = -i\varepsilon^\dagger\bar{\sigma}^\mu D_\mu\psi_i + \sqrt{2}g(T^a\phi)_i\varepsilon^\dagger\lambda^{\dagger a} \,, \tag{3.61}$$

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}(\varepsilon^\dagger\bar{\sigma}_\mu\lambda^a + \lambda^{\dagger a}\bar{\sigma}_\mu\varepsilon) \,, \tag{3.62}$$

$$\delta\lambda^a_\alpha = -\frac{i}{2\sqrt{2}}(\sigma^\mu\bar{\sigma}^\nu\varepsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\varepsilon_\alpha D^a \,, \tag{3.63}$$

$$\delta D^a = \frac{i}{\sqrt{2}}(-\varepsilon^\dagger\bar{\sigma}^\mu D_\mu\lambda^a + D_\mu\lambda^{\dagger a}\bar{\sigma}^\mu\varepsilon) \,. \tag{3.64}$$

3.5 The Minimal Supersymmetric Standard Model

Having understood the basics of supersymmetry, it's time to see how it may be applied as an extension to the standard model. In the minimal version, the particle content is essentially doubled, with a superpartner for every particle. In addition, we will see that an extended Higgs sector is needed. Since no superpartners are observed at low energy, the particle and sparticle masses must be non-degenerate, and supersymmetry must therefore be broken. Instead of committing to one of the many proposed models of how this happens, it is useful to simply parametrise our ignorance by introducing explicitly supersymmetry-breaking terms into the Lagrangian. The resulting theory is known as the Minimal Supersymmetric Standard Model, or MSSM. Let us tackle each of these subjects in order[19].

3.5.1 Particle contents

In the MSSM, every particle is joined with a superpartner in a supermultiplet: the fermions get scalar superpartners, called *sfermions* (for *scalar* fermions); the gauge bosons get fermion superpartners, called *gauginos*; while the several Higgs bosons (see section 3.5.2) also get fermion partners, called *higgsinos*. They are distinguished by a tilde (see

[19] In this section we are once again quoting [5], unless otherwise noted.

tables 1, 2), and they transform identically as their standard model partners under the gauge groups of the SM.

More technically, the two degrees of freedom in a Weyl fermion (from the two spin states) are grouped together with the two degrees of freedom of a complex scalar field in a supermultiplet. Thus, the left-handed and right-handed parts of a particle get separate superpartners: for instance, the superpartners of the electron will be the selectrons $\tilde{e}_L$ and $\tilde{e}_R$. Such supermultiplets are then called *chiral* supermultiplets. Likewise, the two degrees of freedom of a massless gauge boson are grouped with the two degrees of freedom of its massless, Weyl-fermionic partner. These are then known as *gauge* supermultiplets.

Only the fields needed to make the standard model supersymmetric are added. In particular, right-handed neutrinos are absent, and thus right-handed sneutrinos are too; likewise for the graviton and *its* superpartner, the gravitino. Thus, we get a *minimal* supersymmetric extension to the standard model.

In terms of these supermultiplets, then, the superpotential for the MSSM can be expressed as:

$$W_{MSSM} = \bar{u}\lambda_u Q H_u - \bar{d}\lambda_d Q H_d - \bar{e}\lambda_e L H_d + \mu H_u H_d \ . \tag{3.65}$$

where the $\lambda_{u,d}$ are the Yukawa couplings — in general, they will be unitary 3×3 matrices in family space. Applied to equation (3.51), one is able to deduce the full, unbroken Lagrangian of the MSSM; this can be found in [26].

In order to explain the conservation of baryon and lepton number, which is a "happy accident" in the standard model (meaning no renormalisable terms can be written down that violate them) but not in the MSSM, a new, discrete $\mathbb{Z}_2$ symmetry is introduced, called *R-parity*. Each particle is assigned a new quantum number, defined as

$$P_R = (-1)^{3(B-L)+2s} \ , \tag{3.66}$$

where B and L are the baryon and lepton numbers, respectively, and s is the spin of the particle. Now, all standard model particles, as well as the scalars of the extended Higgs sector, have even R-parity ($P_R = 1$), while all superpartners have odd R-parity ($P_R = -1$). We then demand that any Lagrangian term to be included must have the product of its R-parities be $+1$, meaning that there must be an even number of $P_R = -1$ particles in any

Supermultiplet		spin 0	spin 1/2
squarks, quarks (×3 generations)	Q	$(\widetilde{u}_L, \widetilde{d}_L)$	(u_L, d_L)
	$\bar{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$
	$\bar{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$
sleptons, leptons (×3 generations)	L	$(\widetilde{\nu}, \widetilde{e}_L)$	(ν, e_L)
	$\bar{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$
Higgs, higgsinos	H_u	(H^+_u, H^0_u)	$(\widetilde{H}^+_u, \widetilde{H}^0_u)$
	H_d	(H^0_d, H^-_d)	$(\widetilde{H}^0_d, \widetilde{H}^-_d)$

Table 1: Chiral supermultiplets in the MSSM.

interaction vertex.

This has the consequence that any decaying superpartner must decay to at least one other superpartner, the lightest of which will be stable. This particle, then, is known as the *lightest supersymmetric particle*, or LSP. If it is electrically neutral, it will be an attractive candidate for Dark Matter.

3.5.2 Higgs in the MSSM

As the Higgs resides in an SU(2) doublet, it seems clear that so must the higgsinos, together forming a chiral supermultiplet. However, as is seen in table 1, it is necessary to introduce not just one but *two* chiral supermultiplets. The reasons are two-fold: firstly, the introduction of higgsinos with hypercharge $Y = 1$ introduces gauge anomalies (i.e. loop diagrams breaking gauge symmetry — see figure 7) that can only be cancelled by introducing a similar higgsino with $Y = -1$; this, of course, means introducing a $Y = -1$ Higgs doublet as well.

Specifically, in order to have an anomaly-free theory, the conditions $\mathrm{Tr}\left[T^a\{T^b, T^c\}\right] = 0$ must be satisfied, with the T's being the generators of the interacting currents in the

Supermultiplet	spin 1/2	spin 1
gluino, gluon	$\widetilde{g}$	g
winos, W bosons	$\widetilde{W}^{\pm}$ $\widetilde{W}^{0}$	$W^{\pm}$ W^{0}
bino, B boson	$\widetilde{B}^{0}$	B^{0}

Table 2: Gauge supermultiplets in the MSSM.

anomalous diagrams [1, ch. 19.4]. Applied to the electroweak $SU(2) \otimes U(1)$ symmetry, these include $\mathrm{Tr}\left[I_3^2 Y\right] = \mathrm{Tr}\left[Y^3\right] = 0$, with the trace taken over all left-handed Weyl fermionic degrees of freedom — these, then, are the conditions violated by the single higgsino multiplet.

Secondly, it turns out to be necessary for giving masses to both up- and down-type quarks. In the standard model, these are given by interactions with the Higgs field (with $Y = 1$) and its conjugate (with $Y = -1$), respectively. However, one can show quite generally that interaction terms with conjugate scalar fields will not be supersymmetric (i.e., their contribution to $\delta\mathcal{L}$ cannot cancel against any other term, nor can they be formulated as a total derivative). Therefore, the only way to make this work is to have two Higgs supermultiplets, with $Y = 1$ and -1 respectively, designated H_u and H_d after the families of quarks they give mass to. Their supersymmetry-conserving mass terms are

$$\mathcal{L}_{\text{supersymmetric Higgs mass}} = -|\mu|^2 \left(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2\right) . \qquad (3.67)$$

We see here that the mass cannot be negative. Therefore, electroweak symmetry-breaking cannot be understood without additional SUSY-breaking Higgs mass parameters. For this to happen without too much cancellation, μ should be roughly on the order of 10^2–10^3 GeV — or, in other words, roughly the order of the soft masses. Why this should be the case, with these being in principle completely independent mass scales, is dubbed "the μ problem"; it is, in a sense, the final remnant of the original hierarchy problem.

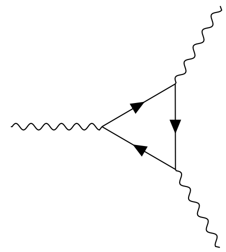

Figure 7: A loop diagram breaking gauge symmetry, arising from fermion-gauge boson interactions. In the standard model, all such diagrams end up cancelling out; in order for the MSSM to be a viable theory, they must be cancelling out here too. This necessitates an extended Higgs sector.

3.5.3 Breaking the symmetry

As mentioned in the introduction, supersymmetry needs to be broken somehow. In the MSSM, this is simply done explicitly. We restrict ourselves to *soft* SUSY-breaking terms, meaning terms that lead to at most logarithmic divergences[20]. In the MSSM, the possible such terms are

$$\begin{aligned}\mathcal{L}_{soft}^{MSSM} = &-\frac{1}{2}(M_3\tilde{g}\tilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + c.c.)\\ &-(\tilde{\bar{u}}\mathbf{a_u}\widetilde{Q}H_u - \tilde{\bar{d}}\mathbf{a_d}\widetilde{Q}H_d - \tilde{\bar{e}}\mathbf{a_e}\widetilde{L}H_d + c.c.)\\ &-\widetilde{Q}^\dagger\mathbf{m_Q^2}\widetilde{Q} - \widetilde{L}^\dagger\mathbf{m_L^2}\widetilde{L} - \tilde{\bar{u}}\mathbf{m_{\bar{u}}^2}\tilde{\bar{u}}^\dagger - \tilde{\bar{d}}\mathbf{m_{\bar{d}}^2}\tilde{\bar{d}}^\dagger - \tilde{\bar{e}}\mathbf{m_{\bar{e}}^2}\tilde{\bar{e}}^\dagger\\ &-m_{H_u}^2H_u^*H_u - m_{H_d}^2H_d^*H_d - (B\mu H_uH_d + c.c.)\,. \end{aligned} \tag{3.68}$$

Here, M_1, M_2 and M_3 are the Bino, Wino, and Gluino soft masses, respectively; $\mathbf{a_u}$, $\mathbf{a_d}$, and $\mathbf{a_e}$ are complex 3×3 matrices in family space, containing the trilinear (scalar)^3 couplings; $\mathbf{m_Q^2}$ and $\mathbf{m_L^2}$ are similar 3×3 mass matrices for the left-handed sparticles, while $\mathbf{m_{\bar{u}}^2}$, $\mathbf{m_{\bar{d}}^2}$, and $\mathbf{m_{\bar{e}}^2}$ are the right-handed mass matrices. Finally, the $m_{H_u}^2$, $m_{H_d}^2$, and $B\mu$ terms are the SUSY-breaking contributions to the Higgs potential.

In order to satisfyingly solve the hierarchy problem, these parameters should all be on the order of a characteristic mass scale m_{soft}, not much larger than a TeV.

The MSSM is infamous for having a tremendously large number of free parameters, and here is where they enter. The supersymmetry-preserving part of the Lagrangian gets

[20]Otherwise, we would just be trading one hierarchy problem for another, up to the scale of the SUSY-breaking.

all its parameters from the standard model, and thus have just the same amount of free parameters. Supersymmetry-*breaking*, however, appears to be very arbitrary: no less than 105 new, free parameters are introduced [27]!

But this is, of course, just a consequence of our ignorance when it comes to exactly how supersymmetry is broken — as we will see in section 5, by restricting ourselves to a particular model of SUSY-breaking, one is typically able to reduce this to just a handful of free parameters.

3.5.4 Mixing sparticles and Higgses

Any scalars with the same electric charge, R-parity, and colour quantum numbers may mix with each other. In order to find the mass eigenstates, it is therefore necessary to express interactions between such particles as a mass matrix to be diagonalised. As a simplified example, consider two such scalars χ and ξ, with the Lagrangian

$$\mathcal{L} \;=\; \partial^\mu \chi^* \partial_\mu \chi + \partial^\mu \xi^* \partial_\mu \xi - m_\chi^2 \chi^* \chi - m_\xi^2 \xi^* \xi - g\chi^* \xi + c.c. \tag{3.69}$$

Expressing the fields in terms of an array $\phi = (\chi, \xi)^T$, these terms are then expressible in terms of the squared-mass matrix $\mathbf{m^2}$.

$$\mathcal{L} \;=\; \phi^\dagger \mathbf{m^2} \phi = \begin{pmatrix} \chi^*, \xi^* \end{pmatrix} \begin{pmatrix} m_\chi^2 & g \\ g^* & m_\xi^2 \end{pmatrix} \begin{pmatrix} \chi \\ \xi \end{pmatrix} \;. \tag{3.70}$$

In order to find the mass eigenstates, then, one must "rotate" the field ϕ into a basis in which $\mathbf{m^2}$ becomes diagonal. Rotating by an angle θ, one ends up with two mass eigenstates:

$$\phi_1 = \cos\theta\, \chi + \sin\theta\, \xi\;, \qquad \phi_2 = \cos\theta\, \xi - \sin\theta\, \chi\;. \tag{3.71}$$

In the MSSM, then, one would have to diagonalise a 6×6 squared-mass matrices (3 generations $\times$ 2 helicities) for the squarks and sleptons, and a 3×3 sneutrino matrix. However, the general hypothesis of flavour-blind soft parameters (as needed to suppress large

FCNCs and CP-violations [5] — see Chapter 5),

$$\mathbf{m}^2_{\mathbf{Q}} = m^2_Q \mathbb{1} \;, \quad \mathbf{m}^2_{\bar{\mathbf{u}}} = m^2_{\bar{u}} \mathbb{1} \;, \quad \mathbf{m}^2_{\bar{\mathbf{d}}} = m^2_{\bar{d}} \mathbb{1} \;, \quad \mathbf{m}^2_{\mathbf{L}} = m^2_L \mathbb{1} \;, \quad \mathbf{m}^2_{\bar{\mathbf{e}}} = m^2_{\bar{e}} \mathbb{1} \;,$$
$$\mathbf{a_u} = A_{u0}\lambda_u \;, \quad \mathbf{a_d} = A_{d0}\lambda_d \;, \quad \mathbf{a_e} = A_{e0}\lambda_e \;. \tag{3.72}$$

where the $\mathbb{1}$s are 3×3 unit matrices in family space, leads to most of these mixing angles being negligible; the only non-negligible mixing will be in the pairs $(\tilde{t}_L, \tilde{t}_R)$, $(\tilde{b}_L, \tilde{b}_R)$, and $(\tilde{\tau}_L, \tilde{\tau}_R)$. The mass eigenstates are denoted by 1, for the lightest, and 2, for the heaviest. For example, we have for the $\tilde{\tau}$,

$$\begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} c_{\tilde{\tau}} & s_{\tilde{\tau}} \\ -s_{\tilde{\tau}} & c_{\tilde{\tau}} \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix} \;, \tag{3.73}$$

where $c_{\tilde{\tau}} \equiv \cos\theta_{\tilde{\tau}}$ and $s_{\tilde{\tau}} \equiv \sin\theta_{\tilde{\tau}}$.

The situation is similar for the gauginos and higgsinos, although here the interactions that lead to mixing only arises after the Higgses develop vevs. The neutral gauginos, $\widetilde{B}$ and $\widetilde{W}^0$, mix with the neutral higgsinos, $\widetilde{H}^0_u$ and $\widetilde{H}^0_d$, to form the four *neutralinos*[21] $\widetilde{\chi}^0_i$, while the positively charged gaugino, $\widetilde{W}^+$, mixes with the charged higgsino $\widetilde{H}^+$ to produce the two *charginos* $\widetilde{\chi}^+_i$, (and similarly for their conjugates). The neutralino masses are then found by acting on the fields with the unitary matrix N such that $N^* M_{\widetilde{\chi}^0} N^\dagger$ becomes a real, diagonal matrix containing the neutralino masses $m_{\widetilde{\chi}^0_i}$. The chargino masses are similarly found by a biunitary transformation, such that $U^* M_{\widetilde{\chi}^+} V^\dagger$ is a diagonal matrix containing the chargino masses $m_{\widetilde{\chi}^+_i}$.

After electroweak symmetry-breaking, the two Higgs doublets will acquire vacuum expectation values v_u and v_d. The masses of the various standard model particles are then given in terms of these as [1]

$$m_u = \frac{1}{\sqrt{2}}\lambda_u v_u \;, \qquad m_d = \frac{1}{\sqrt{2}}\lambda_d v_d \;, \tag{3.74}$$

$$M_W^2 = \frac{1}{4} g^2 (v_u^2 + v_d^2) \;, \qquad M_Z^2 = \frac{1}{4}(g'^2 + g^2)(v_u^2 + v_d^2) \;, \tag{3.75}$$

[21] In models with minimal particle contents, the lightest of these is often assumed to be the LSP, as it is the only dark matter candidate in the MSSM that has not been ruled out by observation [5].

where the λ are the usual Yukawa couplings, with subscript u designating $I_3 = \frac{1}{2}$ fermions and d designating $I_3 = -\frac{1}{2}$ fermions, and where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively.

In light of equation (3.75), one might want to define v_u and v_d in terms of the standard model vev as

$$v_u = v \sin\beta\,, \qquad v_d = v\cos\beta\,, \tag{3.76}$$

where β is a free parameter. In particular, their ratio is then expressed as

$$\frac{v_u}{v_d} = \tan\beta\,. \tag{3.77}$$

Letting $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, and with s_W, c_W being the sine and cosine of the weak mixing angle, the mass matrices of the MSSM are as follows[22]:

For the neutralinos, we get the mass terms $-\widetilde{\chi}^{0T} M_{\widetilde{\chi}^0}\widetilde{\chi}^0 + h.c.$, where $\widetilde{\chi}^0 = (-i\widetilde{B}^0, -i\widetilde{W}^0, \widetilde{H}_d, \widetilde{H}_u)^T$, and

$$M_{\widetilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & \mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & \mu & 0 \end{pmatrix}\,. \tag{3.78}$$

For the charginos, we get the mass terms $-\widetilde{\chi}^{-T} M_{\widetilde{\chi}^+}\widetilde{\chi}^+ + h.c.$, where $\widetilde{\chi}^+ = (-i\widetilde{W}^+, \widetilde{H}_u^+)^T$, $\widetilde{\chi}^- = (-i\widetilde{W}^-, \widetilde{H}_d^-)^T$, and

$$M_{\widetilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & -\mu \end{pmatrix}\,. \tag{3.79}$$

For up- and down-type squarks, we have terms like $\tilde{u}^\dagger \mathbf{m}^2_{\tilde{\mathbf{u}}}\tilde{u}$ and $\tilde{d}^\dagger \mathbf{m}^2_{\tilde{\mathbf{d}}}\tilde{d}$, with $\tilde{u} = (\tilde{u}_L, \tilde{u}_R)^T$, $\tilde{d} = (\tilde{d}_L, \tilde{d}_R)^T$, and

$$\mathbf{m}^2_{\tilde{\mathbf{u}}} = \begin{pmatrix} M_Q^2 + m_u^2 + \Delta_{\tilde{u}_L} & \frac{1}{\sqrt{2}}\left(v_u \mathbf{a}_\mathbf{u}^* - \mu\lambda_u v_d\right) \\ \frac{1}{\sqrt{2}}\left(v_u \mathbf{a}_\mathbf{u} - (\mu\lambda_u)^* v_d\right) & M_U^2 + m_u^2 + \Delta_{\tilde{u}_R} \end{pmatrix}\,, \tag{3.80}$$

[22] All but the sfermion mass matrices are taken from reference [28] — these are instead quoted here in the form of reference [29].

$$\mathbf{m}^2_{\tilde{\mathbf{d}}} = \begin{pmatrix} M_Q^2 + m_d^2 + \Delta_{\tilde{d}_L} & \frac{1}{\sqrt{2}}\Big(v_u \mathbf{a}_\mathbf{u}^* - \mu\lambda_u v_d\Big) \\ \frac{1}{\sqrt{2}}\Big(v_d \mathbf{a}_\mathbf{d} - (\mu\lambda_d)^* v_u\Big) & M_D^2 + m_d^2 + \Delta_{\tilde{d}_R} \end{pmatrix} , \tag{3.81}$$

$\Delta_{\tilde{f}}$ represents a "hyperfine splitting" arising from quartic interactions with the Higgs of the form $\tilde{f}^2 H^2$ — it is model-independent for a given choice of $\tan\beta$, and is given by [5]

$$\Delta_\phi = (I_3^\phi - e_\phi s_W^2) M_Z^2 c_{2\beta} , \tag{3.82}$$

where e_{ϕ_L}, e_{ϕ_R} are the electric charge of the left-handed field and the conjugate of the right-handed field, respectively, in units of the elementary charge.

The gluino mass, meanwhile, is simply given by the soft mass M_3, while the sneutrino masses are given by

$$m_{\tilde{\nu}}^2 = M_L^2 + \Delta_{\tilde{\nu}} . \tag{3.83}$$

As for the Higgses, the gauge eigenstates $(H_u^+,\ H_u^0)$, $(H_d^0,\ H_d^-)$ mix to produce four mass eigenstates: a charged Higgs, H^+, a CP-odd pseudoscalar, A^0, a heavy neutral Higgs, H^0, and a light neutral Higgs, h^0, the latter of which is identified with the standard model Higgs observed at $\sim$ 125 GeV. Treating the pseudoscalar Higgs mass $m_{A^0}^2$ as a free parameter, the charged Higgs mass is given by

$$m_{H^+}^2 = m_{A^0}^2 + M_W^2 , \tag{3.84}$$

while the neutral Higgs masses is found through diagonalising the mass matrix [30]

$$M_{H^0}^2 = \begin{pmatrix} m_{A^0} s_\beta^2 + m_Z^2 c_\beta^2 & -(m_{A^0}^2 + m_Z^2) s_\beta c_\beta \\ -(m_{A^0}^2 + m_Z^2) s_\beta c_\beta & m_{A^0}^2 c_\beta^2 + m_Z^2 s_\beta^2 \end{pmatrix} . \tag{3.85}$$

This is achieved by a rotation through the Higgs mixing angle, α.

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H_u \\ H_d \end{pmatrix} . \tag{3.86}$$

3.6 A comment on the current status of experiments

The Large Hadron Collider has been actively searching for signs of supersymmetry since it began operations in 2008 — indeed, besides the prospect of producing the Higgs particle, this was one of the main motivations for building the machine. It was said that either the LHC must discover at least *some* superpartners, or the case for low-energy supersymmetry would be "significantly weakened" [31]. This was based on the argument that we don't expect the cancellations in the Higgs mass parameter, as explained in section 3.1, to occur at more than one order of magnitude.

Well, the LHC has looked, and the situation seems dire: no unambiguous evidence for physics beyond the standard model has been found[23], and in particular, *no superpartners*. Though it will continue collecting data for decades to come, a general pessimism regarding the prospects of supersymmetry seems to have taken root in the field [33].

It is important to emphasise, however, that we are talking about *low-energy* supersymmetry here, as needed to satisfyingly solve the hierarchy problem. Even besides the hierarchy problem, there remain several very good reasons to study supersymmetry. For one, it ensures the unification of the gauge couplings at higher energies, something the standard model fails to do (see figure 8)[24]. Second, it still readily provides attractive candidates for Dark Matter, as discussed in section 3.5.1. Third, and perhaps most importantly, it is required by the only known candidate for a renormalisable theory of quantum gravity, namely (super)string theory [36].

As for the hierarchy problem, one might have to accept that at least *some* fine-tuning is involved, but if that cancelling happens at a couple of orders of magnitude, as would be the case if the SUSY breaking occurs just beyond the LHC limit, that would still be a lot less disturbing than the 36 orders of magnitude we began with. If so, there are good prospects for discovering superpartners in future colliders like the ILC [37] and the FCC [38, 39][25].

[23]Other than the non-zero neutrino masses that is, as inferred from the observed mixing of solar neutrinos [32].

[24]The resulting theories, commonly referred to as a Grand Unified Theories, or GUTs, typically combine the gauge groups of the standard model $SU(3) \otimes SU(2) \otimes U(1)$ into the unified gauge group $SU(5)$ [34], or more commonly $SO(10)$; see [35] for a review.

[25]In particular, assuming the standard model of cosmology, the observed dark matter relic abundance

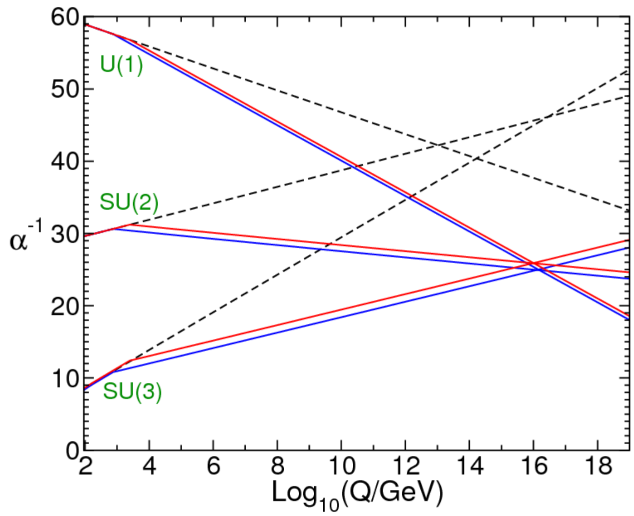

Figure 8: Running couplings in the SM (dashed lines), in the MSSM for sparticles at $\sim$ 750 GeV (blue lines), and in the MSSM for sparticles at $\sim$ 2.5 Tev (red lines). Figure taken from [5].

Finally, even if the breaking were to take place at the GUT scale of $\mathcal{O}\left(10^{16}\right)$ GeV (and thus implying fine-tuned cancellations between terms of order $\mathcal{O}\left(10^{32}\right)$), it *still* wouldn't imply any explicit inconsistencies of the theory — this is fundamentally a problem of aesthetics, rather than mathematical consistency[26].

bounds the mass of WIMP dark matter from above by $M_{DM} < 1.8\text{TeV}(g_{eff}^2/0.3)$, with g_{eff} the effective coupling determining their annihilation rate, strongly suggesting new physics at the TeV scale [39].

[26]In order to make sense of such a situation, though, one might find it necessary to resort to anthropic reasoning, since in order to support complex structures, and thus life, the Higgs mass parameter cannot differ much from its observed value. Talking about "naturalness" and "likelihoods" in the face of this may make little sense, as otherwise we wouldn't be here to discuss it. Indeed, such arguments have seen a rise in popularity with the advent of multiverse theories, wherein it is hypothesised that we live in one of an infinitude of different "bubble universes", each with slightly different physical parameters [40]. For a detailed discussion of such "fine-tuned" SUSY, including its phenomenology, see [41].

4 Renormalisation

One of the main successes of QFT as a theoretical framework over the older framework of relativistic quantum mechanics, is the prediction of precision corrections to the interactions of the theory. In addition to the more precise predictions of masses and cross sections associated with going to higher orders in perturbation theory, this leads to scale-dependent, or "running", gauge couplings [12]. Such higher-order interactions are represented in the usual perturbative framework by Feynman diagrams with loops.

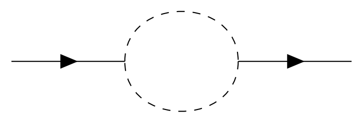

Figure 9: A one-loop Feynman diagram. This particular diagram yields a correction to the mass of the propagating fermion through interaction with a virtual scalar.

In the early development of quantum field theory, these kinds of diagrams represented a major difficulty. The problem lies in the fact that the momentum of the inserted particle is undetermined, and must therefore be integrated over all possibilities. Schematically,

$$\text{(undetermined loop momentum } q) \quad \longrightarrow \quad \int_{-\infty}^{\infty} \frac{d^4 q}{(2\pi)^4} \,. \tag{4.1}$$

Not only are these integrals divergent, but there would be, in the non-perburbative limit, infinitely many of them for any given process. The situation was finally resolved in the sixties by the development of the method known as *renormalisation*. Roughly speaking, the process is as follows:

First, introduce a *regulator* such that the integral becomes finite. The simplest way to do this is to simply introduce a cut-off Λ to the integral.

$$\int \frac{d^4 q}{(2\pi)^4} \quad \longrightarrow \quad \int_{|\mathbf{q}|<\Lambda} \frac{d^4 q}{(2\pi)^4} \,, \tag{4.2}$$

where, at the end of the calculation, Λ is to be taken to infinity again. This method has the benefit of being both simple and intuitive[27]. On the other hand, it breaks both gauge symmetry and Lorentz invariance, leading to computational difficulties.

The more popular method of regularisation is *dimensional* regularisation, in which the dimensionality of the integrals is altered instead.

$$\int \frac{d^4 q}{(2\pi)^4} \quad \longrightarrow \quad \int \frac{d^D q}{(2\pi)^4}\,, \qquad D \equiv 4-\epsilon\,, \tag{4.3}$$

where ϵ is to be taken to zero at the end of the calculation. With this method, gauge invariance is preserved, and the divergences show up as terms proportional to $1/\epsilon$.

After having regularised the theory, the next step is then to absorb the divergent terms into redefinitions of the gauge couplings and masses — or, put another way, make the parameters of the Lagrangian infinite in just such a way as to cancel the divergences of the integrals[28].

$$g_0 \quad \longmapsto \quad g \;=\; g_0 - \frac{1}{\epsilon}\,, \tag{4.4}$$

where g_0 would be the "bare" coupling of the tree-level theory, and g the renormalised coupling. Here we see what was stated above: in order for g to be finite in the $\epsilon \to 0$ limit, g_0 must be infinite. This isn't a problem, however, as g_0 is not an observable parameter.

If all divergences of the theory can be absorbed by such redefinitions, the theory is said to be *renormalisable*. We are left with finite predictions, but now in terms of the *renormalised* parameters. These are running parameters, meaning they now depend on the energy scale, a dependence which may be derived from the *renormalisation group equations*

[27] It is also instrumental to the formulation of so-called "effective" field theories. These are to be thought of as non-fundamental approximations, correct up to an energy scale Λ, above which a more fundamental (possibly unknown) theory enters.

[28] For many years, this was thought to be an improper mathematical procedure, and doubts lingered as to whether the procedure was even mathematically consistent. Consider for example this comment by Paul Dirac, from as late as 1975 [42, p. 184]:

> Most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory and we do not have to worry about it any more.' I must say that I am very dissatisfied with the situation, because this so-called 'good theory' does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small — not neglecting it just because it is infinitely great and you do not want it!

Today, however, renormalisation is generally accepted to be a consistent procedure, and integral to the study of QFT.

(RGEs):

$$\mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = \beta(g) . \tag{4.5}$$

Here, g is a coupling constant, μ is the energy scale, and $\beta(g)$ is a function of the coupling constant that may be computed from the theory. For example, at one-loop order, the coupling constant of quantum electrodynamics has $\beta(\alpha) = 2\alpha^2/3\pi$ [1, ch. 12.2].

At a more technical level, we have the following integrals appearing at one-loop level in self-energy calculations [28]:

$$A_0(m) = 16\pi^2\mu^{4-D} \int \frac{d^D q}{i\,(2\pi)^D} \frac{1}{q^2 - m^2 + i\varepsilon} , \tag{4.6}$$

$$B_0(p, m_1, m_2) = 16\pi^2\mu^{4-D} \int \frac{d^D q}{i\,(2\pi)^D} \frac{1}{\left[q^2 - m_1^2 + i\varepsilon\right]\left[(q-p)^2 - m_2^2 + i\varepsilon\right]} , \tag{4.7}$$

$$p_\mu B_1(p, m_1, m_2) = 16\pi^2\mu^{4-D} \int \frac{d^D q}{i\,(2\pi)^D} \frac{q_\mu}{\left[q^2 - m_1^2 + i\varepsilon\right]\left[(q-p)^2 - m_2^2 + i\varepsilon\right]} , \tag{4.8}$$

$$\begin{aligned} p_\mu p_\nu B_{21}(p, m_1, m_2) &+ g_{\mu\nu} B_{22}(p, m_1, m_2) , \\ &= 16\pi^2\,\mu^{4-D} \int \frac{d^D q}{i\,(2\pi)^D} \frac{q_\mu q_\nu}{\left[q^2 - m_1^2 + i\varepsilon\right]\left[(q-p)^2 - m_2^2 + i\varepsilon\right]} . \end{aligned} \tag{4.9}$$

A_0 can be integrated to give

$$A_0(m) = m^2 \left(\frac{2}{\epsilon} - \gamma_E + \ln 4\pi + 1 - \ln \frac{m^2}{\mu^2} \right) , \tag{4.10}$$

where γ_E is the Euler-Mascheroni constant [43], while the function B_0 can be rewritten as

$$B_0(p, m_1, m_2) = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi - \int_0^1 dx \, \ln \frac{(1-x)\, m_1^2 + x\, m_2^2 - x(1-x)\, p^2 - i\varepsilon}{\mu^2} . \tag{4.11}$$

Here we see explicitly the divergent terms that are to be subtracted. If one uses redefinitons like (4.4) to subtract only the $2/\epsilon$ poles, the renormalisation scheme is called MS, for minimal subtraction; if one instead chooses to subtract $2/\hat{\epsilon} = 2/\epsilon - \gamma_E + \ln 4\pi$, the scheme is known as $\overline{MS}$ (pronounced MS-bar). Clearly, the resulting predictions will depend somewhat upon the scheme one chooses, though this dependence must necessarily disappear in the non-perturbative limit.

All the other integrals can now be written in terms of A_0 and B_0. For example,

$$B_1(p, m_1, m_2) = \frac{1}{2p^2}\left[A_0(m_2) - A_0(m_1) + (p^2 + m_1^2 - m_2^2)B_0(p, m_1, m_2)\right]. \qquad (4.12)$$

Similar expressions for the other functions can be found in reference [44].

5 Gaugino-mediated supersymmetry-breaking

The MSSM, in its most basic form, comes with several problems. For one, it predicts large flavour-changing neutral currents (FCNCs); no such currents, beyond those arising from higher-order interactions in the standard model, have been observed. Second, it predicts large CP-violating phases. Again, no CP-violation have been observed beyond that predicted by the standard model [2]. Therefore, for a particular supersymmetric model to be phenomenologically viable, it needs to predict these terms and phases to be negligible[29].

One particularly promising scenario is that of gaugino mediated supersymmetry breaking [7, 8]. In this setup, the matter fields of the MSSM are confined to a 4-dimensional *brane*, embedded in a d-dimensional *bulk*[30]. The gauge and Higgs superfields are allowed to propagate in the field, and so is gravity (see figure 2). The size of the extra dimensions determines the compactification scale $M_C \equiv (1/V_{d-4})^{\frac{1}{d-4}}$, where V_{d-4} is their collected volume — this will be the scale needed to resolve the compact dimensions.

We work in a model with one additional dimension and one additional brane, spatially separated from the MSSM brane by a distance R_5. This brane contains at least one chiral superfield, S, which is a singlet under the SM gauge groups. Here, SUSY is broken by the vacuum expectation value of its auxiliary field F_S. The gauginos and Higgses acquire soft masses through couplings to this field proportional to the vev, while the soft sfermion masses are suppressed by factors of order $e^{-R_5 M_P}$. Thus, even if R_5 is on the order of $M_U^{-1} = \mathcal{O}(10^{-16})$, they will be suppressed by a factor $e^{-10^2} = \mathcal{O}(10^{-44})$!

In order for the additional FCNCs to be negligible, it is sufficient to assume that the mass matrices in eq. (3.68) are diagonal:

$$\mathbf{m^2_Q} = m_Q^2 \mathbb{1}\,, \quad \mathbf{m^2_{\bar{u}}} = m_{\bar{u}}^2 \mathbb{1}\,, \quad \mathbf{m^2_{\bar{d}}} = m_{\bar{d}}^2 \mathbb{1}\,, \quad \mathbf{m^2_L} = m_L^2 \mathbb{1}\,, \quad \mathbf{m^2_{\bar{e}}} = m_{\bar{e}}^2 \mathbb{1}\,, \tag{5.1}$$

where the $\mathbb{1}$s are 3×3 unit matrices in family space. Since this theory predicts the sfermion soft masses to be 0, this requirement is trivially satisfied.

[29] In particular, just *assuming* them to be small is seen to be unsatisfactory. This is because the one place where CP-violation can show up in the SM, namely in the CKM matrix [1], it appears at order one [2].

[30] Now with $d \in \mathbb{N}$, of course.

We assume gauge unification to occur at a scale $M_U \sim 10^{16}$ GeV, such that

$$M_1 = M_2 = M_3 = m_{1/2} \, . \tag{5.2}$$

From interactions between S and the Higgs, we also get SUSY-breaking trilinear couplings proportional to the Yukawa matrices[31]:

$$\mathbf{a_u} = A_{u0}\mathbf{y_u} \, , \quad \mathbf{a_d} = A_{d0}\mathbf{y_d} \, , \quad \mathbf{a_e} = A_{d0}\mathbf{y_e} \, . \tag{5.3}$$

Making the additional assumption that CP-violation only occurs on the visible brane ensures that the only CP-violating phase in the theory is the usual CKM phase of the SM [8]. The free parameters of the theory are then just $m_{1/2}$, $m^2_{H_u}$, $m^2_{H_d}$, A_{u0}, A_{d0}, and $B\mu$.

The sfermions acquire masses from loop-interactions with the gauginos, as illustrated in figure 2. The compactification scale, where $m_{\tilde{f}} = 0$, is taken as the input scale, and the low-energy parameters of the theory are evaluated by backwards running down to an energy scale appropriate for phenomenology[32]. This is done using RGEs analogous to that in equation (4.5), which are listed in [5]. In most of the parameter space, the gravitino will be the LSP [7], with a mass $m_{3/2} \gtrsim 10$ GeV [46]; this can then be a viable Dark Matter candidate[33]. The next-to-lightest supersymmetric particle (NLSP) then becomes stable on collider time-scales, and is found to be either the lightest neutralino, the stau, or the tau sneutrino [7, 8, 45].

The μ problem, meanwhile, is solved by the Giudice-Masiero mechanism [8, 49], wherein it is given its value through interactions with the auxiliary field vev — just like the soft masses!

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[31] These couplings were absent in the original versions of the theory [7, 8]. However, their absence would require $m_{1/2} \gtrsim 3$ TeV in order to reproduce the observed Higgs mass of ~ 125 GeV, and thus very heavy sparticles; the addition of trilinear couplings alleviates this situation [45].

[32] This scale is usually taken to be $\mu_{SUSY} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$.

[33] Though, many would argue, an unfortunate one; being a singlet under all the Standard Model gauge groups, it is all but impossible to detect directly. However, as direct detection experiments keep producing null-result after null-result [47, 48], this scenario is increasingly becoming one worth taking seriously.

6 Our work

In [45], an extensive parameter search was conducted for the nature of the NLSP given different high-scale input parameters. This was done at 1-loop level[34] using the spectrum generator SPheno [29]. The goal of our work is to achieve a greater understanding of the mass ordering in a particular part of the parameter space, in which it was found that either the stau or the tau sneutrino is the NLSP — this while being nearly mass-degenerate. In particular, we attempt to identify what process is most important in determining this small mass difference.

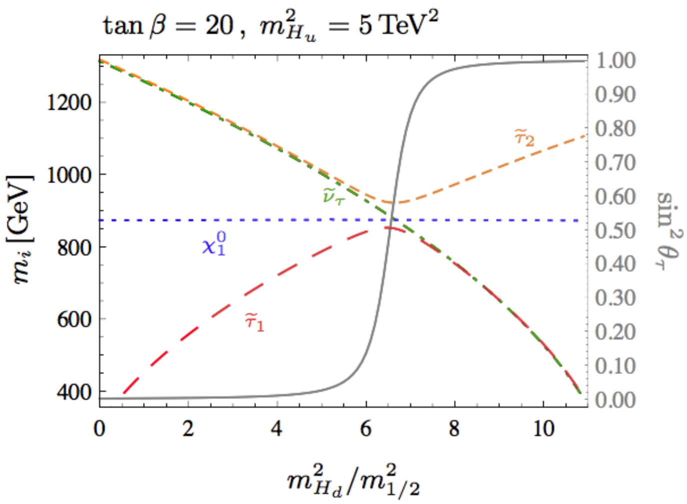


Figure 10: Plot taken from [45]. The dashed lines show the masses of the possible NLSPs in a slice of parameter space, while the solid line shows the stau mixing angle. Here, $B\mu$ has been traded for $\tan\beta$ as a free parameter, as explained in [5, ch. 8.1]. The other input values are $m_{1/2} = 2$ TeV, $A_0 = -2$ TeV, and $\mu > 0$.

Though it can be hard to see in the above plot, the identity of the NLSP changes at about $m^2_{H_d}/m^2_{1/2} \approx 8.5$, from the stau to the tau sneutrino, (see figure 11).

This, then, is the point of interest, motivating our investigation into the driving forces behind the mass splitting.

[34]Or rather "1.5"-loop level: in addition to the complete one-loop corrections to the sparticle masses, SPheno incorporates the most important two-loop corrections to the neutral Higgs sector. The RGEs are numerically solved at two-loop order. [29]

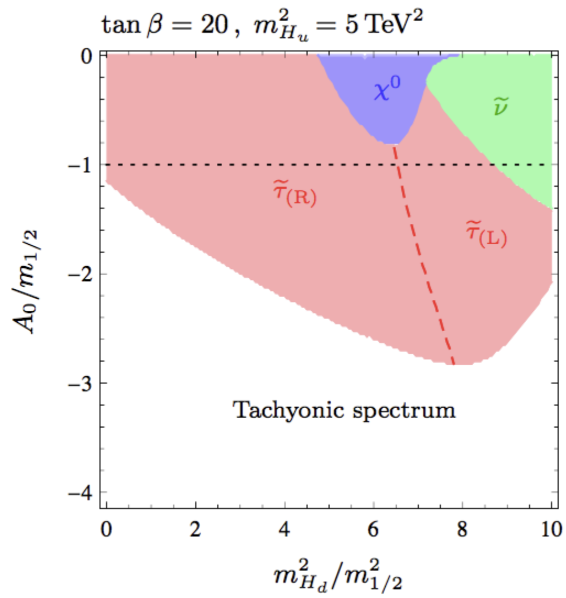


Figure 11: Also from [45], a plot showing regions of different NLSPs. In the white region, the spectrum becomes tachyonic, i.e., containing negative soft squared-masses. The black dotted line is the region considered in figure 10.

We will follow reference [45] and constrain ourselves to the case $A_{u0} = A_{d0} \equiv A_0$[35]. In addition, we take $\mu > 0$, $A_0 < 0$,[36] and let the input scale to be the scale of grand unification, $M_C = M_U \approx 10^{16}$ GeV.

6.1 Tree-level calculations

The first step in the process was to determine whether the results could be recreated at tree-level. The tree-level stau mass matrix is [28]

$$\mathbf{m}^2_{\tilde{\boldsymbol{\tau}}} = \begin{pmatrix} M^2_{\tilde{L}_3} + m^2_\tau + \Delta_{\tilde{\tau}_L} & m_\tau(A_{\tilde{\tau}} + \mu\tan\beta) \\ m_\tau(A_{\tilde{\tau}} + \mu\tan\beta) & M^2_{\tilde{e}_3} + m^2_\tau + \Delta_{\tilde{\tau}_R} \end{pmatrix}\,, \tag{6.1}$$

where we've now taken $\mathbf{a}_{\tilde{\boldsymbol{\tau}}} = A_{\tilde{\tau}}\lambda_\tau$, and where the hyperfine splitting Δ_ϕ is given in equation (3.82).

[35]Not to be confused with the *divergent integral* A_0, as given in equation (4.6) — which of these we are referring to shall always be clear from context.

[36]Changing the sign of both leads to similar phenomenology [45].

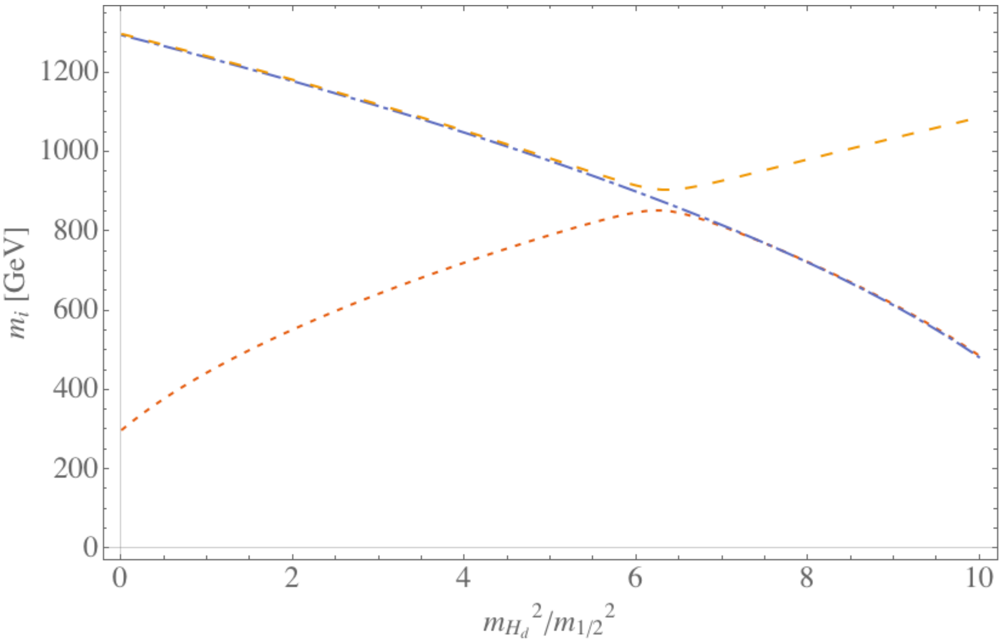


Figure 12: Tree level masses of the sneutrino (blue, dash-dotted), the heavy stau (orange, long-dashed), and the light stau (red, short-dashed) as a function of the high-energy input parameter $m^2_{H_d}/m^2_{1/2}$. The other input values are $\tan\beta = 20$, $m^2_{H_u} = 5\,\text{TeV}^2$, $m_{1/2} = 2\,\text{TeV}$, and $A_0 = -2\,\text{TeV}$.

Diagonalising yields the tree-level mass formulas

$$m^2_{\tilde{\tau}_1} = \frac{1}{2}\Bigg(M^2_{L_3} + M^2_{\bar{e}_3} + (I_3^{\tau_L} - e_{\tau_L}s^2_W - e_{\tau_R}s^2_W)M^2_Z c_{2\beta} - \sqrt{(M^2_{L_3} - M^2_{\bar{e}_3} + (I_3^{\tau_L} - e_{\tau_L}s^2_W - e_{\tau_R}s^2_W)M^2_Z c_{2\beta})^2 + 4m^2_\tau(A_{\tilde{\tau}} + \mu\tan\beta)^2}\Bigg), \tag{6.2}$$

$$m^2_{\tilde{\tau}_2} = \frac{1}{2}\Bigg(M^2_{L_3} + M^2_{\bar{e}_3} + (I_{3\tau_L} - e_{\tau_L}s^2_W - e_{\tau_R}s^2_W)M^2_Z c_{2\beta} + \sqrt{(M^2_{L_3} - M^2_{\bar{e}_3} + (I_3^{\tau_L} - e_{\tau_L}s^2_W - e_{\tau_R}s^2_W)M^2_Z c_{2\beta})^2 + 4m^2_\tau(A_{\tilde{\tau}} + \mu\tan\beta)^2}\Bigg). \tag{6.3}$$

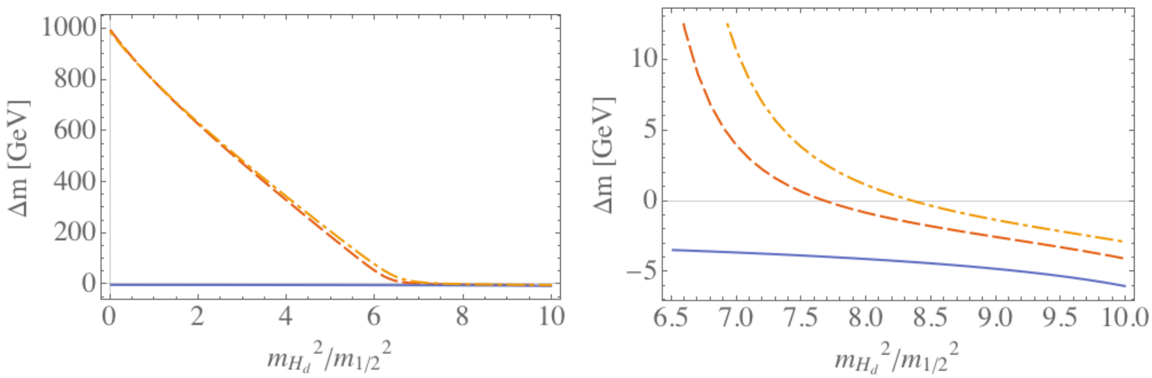
There being no right-handed sneutrinos in our model, the sneutrino mass is meanwhile simply given by

$$m^2_{\tilde{\nu}_\tau} = M^2_{L_3} + \Delta_{\tilde{\nu}_\tau}\ . \tag{6.4}$$

We wrote a Mathematica script that ran SPheno over a range of values, extracting the relevant parameters and calculating the tree-level stau and tau sneutrino masses for each one in order to recreate the results in figure 10. This was done in steps of $\Delta(m^2_{H_d}/m^2_{1/2}) =$

Tree-level	SPheno
$m_{\tilde{\tau}_1} = 791.16$ GeV	$m_{\tilde{\tau}_1} = 797.88$ GeV
$m_{\tilde{\tau}_2} = 983.59$ GeV	$m_{\tilde{\tau}_2} = 1011.70$ GeV
$m_{\tilde{\nu}_\tau} = 977.12$ GeV	$m_{\tilde{\nu}_\tau} = 1003.65$ GeV

Table 3: Comparison between the tree-level masses and SPheno's (1-loop level) output. These values are taken from the point $m^2_{H_d}/m^2_{1/2} = 5$ in figures 10, 12.



(a)

(b)

Figure 13: (a) The mass difference $\Delta m = m_{\tilde{\nu}_\tau} - m_{\tilde{\tau}_1}$ at tree-level (red dashed curve) compared to the same mass difference in the SPheno output (yellow dash-dotted curve). The solid blue line is the unmixed tree-level mass difference, $\Delta m = m_{\tilde{\nu}_\tau} - m_{\tilde{\tau}_L}$. (b) The same plot zoomed in to the region in which the NLSP changes identity.

0.1, making it a total of 100 data points. The results are given in figure 12.

The first thing one notices is that the tree-level masses are consistently lower than SPheno's results — see table 3 for a comparison at the point $m^2_{H_d}/m^2_{1/2} = 5$. This is not too surprising, as one expects the loops to push them upwards somewhat. Secondly, the amount of stau mixing clearly depends a great deal on the value of $m^2_{H_d}$, with $\tilde{\tau}_1$ going from being almost entirely right-handed to almost entirely left-handed over the course of the plot. This is because the Higgs mass parameter μ, which shows up in the off-diagonal terms in (6.1), shares a linear dependence with m_{H_d}. (Specifically, in unbroken SUSY, $m^2_{H_d} = |\mu|^2$, as shown in eq. (3.67)).

But in order to get a clearer view of the situation, it is helpful to look only at the mass *difference*, $\Delta m = m_{\tilde{\nu}_\tau} - m_{\tilde{\tau}_1}$ — wherever it crosses zero will of course be the point where the NLSP changes identity. Our result is compared to the mass difference in the SPheno output in figure 13, together with the unmixed mass difference $\Delta m = m_{\tilde{\nu}_\tau} - m_{\tilde{\tau}_L}$.

From these plots, the situation starts to become clear: the unmixed stau $\tilde{\tau}_L$ is always heavier than the sneutrino, staying just above it at a mass difference of just a few GeV. Comparing equation (6.1) and (6.4), we see that this must be due to the terms m_τ^2 and $\Delta_{\tilde{\tau}_L}$. The first term is of course just the tau mass squared, with $m_\tau \approx 1.777$ GeV [2] — but it is the latter term, the hyperfine splitting, that dominates[37]. Up to the (negligible) tau mass, then, this squared-mass difference is just

$$\Delta m \approx \Delta_{\tilde{\nu}_\tau} - \Delta_{\tilde{\tau}_L} = M_Z^2 c_{2\beta} c_W^2 \,. \tag{6.5}$$

This is found to be negative for $\tan\beta = 20$. And so, as the lightest stau goes from being predominantly right-handed to more and more left-handed, it is only natural that it should end up the heavier of the two.

One might wonder, in looking at equation (6.5), how it is that this mass difference *also* seems to depend on the value of $m_{H_d}^2$, given that all of those parameters should be fixed. The reason is that the Z boson squared-mass M_Z^2 is here a running parameter[38], evaluated using eq. (3.75), and the scale μ at which SPheno evaluates the parameters we are using in these calculations is defined as

$$\mu_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \,, \tag{6.6}$$

where $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are the tree-level masses of the light and the heavy stop, respectively. These *will* depend on the value of $m_{H_d}^2$, and therefore, so will the scale at which we evaluate M_Z^2.

Looking at figure 13, we find that the identity of the NLSP changes from the stau to the sneutrino somewhat early compared to SPheno, with $m_{H_d}^2 / m_{1/2}^2$ around 7.5 versus SPheno's ~ 8.5. The desire to close this gap prompted us to investigate the situation at 1-loop level.

[37] $m_\tau^2 = \mathcal{O}(1)$ GeV^2, $\Delta_{\tilde{\tau}_L} = \mathcal{O}(10^3)$ GeV^2.

[38] As opposed to the pole mass, $M_{Z,\,\text{pole}} \approx 80.3$ GeV [2].

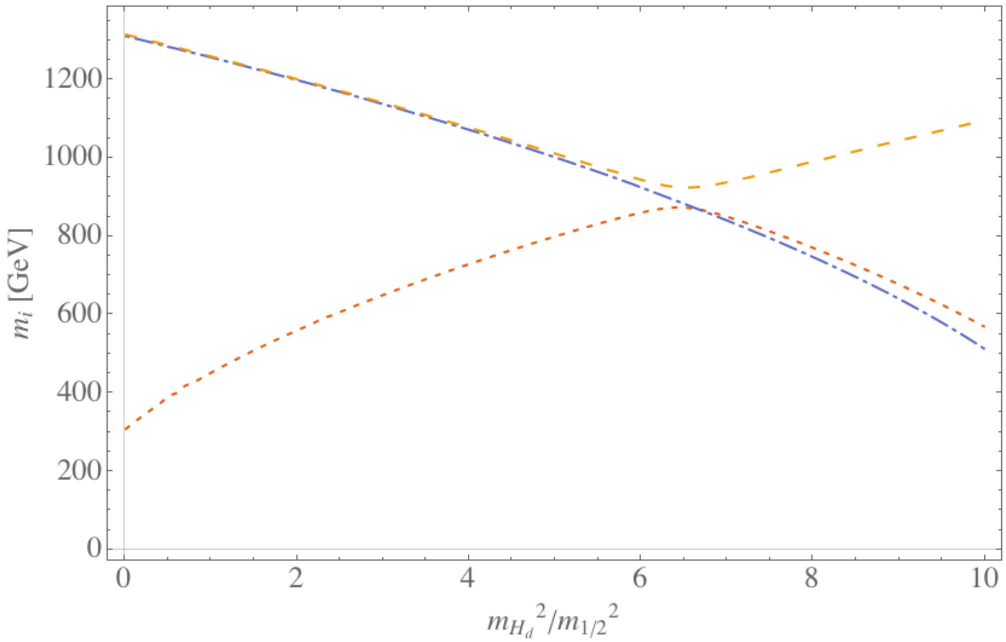


Figure 14: Masses of the sneutrino (blue, dash-dotted), the heavy stau (orange, long-dashed), and the light stau (red, short-dashed) as a function of the high-energy input parameter $m^2_{H_d}/m^2_{1/2}$, now evaluated at one-loop level; the high-energy input parameters are the same as those in figure 12.

6.2 In the loop

Initially, we thought we could save ourselves some time by only studying the mass difference $m^2_{\tilde{\nu}_\tau} - m^2_{\tilde{\tau}}$, as due to cancellations this would mean we could ignore large parts of the loop corrections the individual particles receive, and because precisely this mass difference had already been studied in detail in reference [50]. This turned out to be more tedious than expected, however, as it lead to a seemingly never-ending rabbit hole of referenced formulas, many with conflicting conventions, and with increasingly archaic notation. Upon realising that these formulas also completely neglected sparticle mixing, and in particular loop-corrections to the off-diagonal terms of the mass matrix, this path was abandoned in favour of doing the full one-loop calculations, based on the delightfully self-contained appendices of reference [28].

In order to find the pole masses of the staus, one needs to solve

$$\det\left[p_i^2 - \mathbf{m}^2_{\tilde{\boldsymbol{\tau}}}(p_i^2)\right] = 0\,, \qquad m^2_{\tilde{\tau}_i} = \mathrm{Re}(p_i^2)\,, \tag{6.7}$$

One-loop level	SPheno
$m_{\tilde{\tau}_1} = 799.13$ GeV	$m_{\tilde{\tau}_1} = 797.88$ GeV
$m_{\tilde{\tau}_2} = 1010.98$ GeV	$m_{\tilde{\tau}_2} = 1011.70$ GeV
$m_{\tilde{\nu}_\tau} = 1001.29$ GeV	$m_{\tilde{\nu}_\tau} = 1003.65$ GeV

Table 4: Comparison between the one-loop calculated masses and SPheno's output. These values are taken from the point $m_{H_d}^2/m_{1/2}^2 = 5$ in figures 10, 14.

where

$$\mathbf{m}^2_{\tilde{\tau}}(p^2) = \begin{pmatrix} m^2_{\tilde{\tau}_L\tilde{\tau}_L} - \Pi_{\tilde{\tau}_L\tilde{\tau}_L}(p^2) & m^2_{\tilde{\tau}_L\tilde{\tau}_R} - \Pi_{\tilde{\tau}_L\tilde{\tau}_R}(p^2) \\ m^2_{\tilde{\tau}_R\tilde{\tau}_L} - \Pi^*_{\tilde{\tau}_L\tilde{\tau}_R}(p^2) & m^2_{\tilde{\tau}_R\tilde{\tau}_R} - \Pi_{\tilde{\tau}_R\tilde{\tau}_R}(p^2) \end{pmatrix} , \tag{6.8}$$

and similarly for the sneutrino,

$$\left| p^2 - m^2_{\tilde{\nu}_\tau}(p^2) \right| = 0, \qquad m^2_{\tilde{\nu}_\tau} = \mathrm{Re}(p^2) , \tag{6.9}$$

with

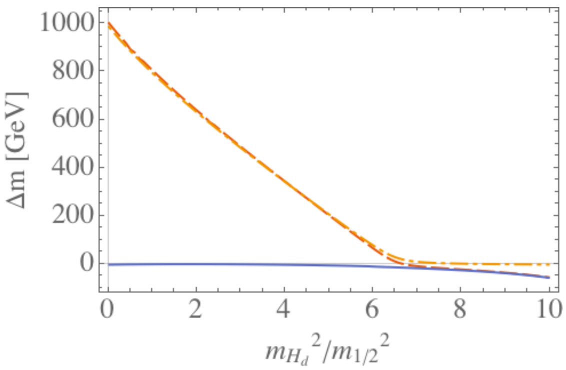
$$m^2_{\tilde{\nu}_\tau}(p^2) = M^2_{L_3} + \Delta_{\tilde{\nu}_\tau} - \Pi_{\tilde{\nu}_\tau\tilde{\nu}_\tau}(p^2) . \tag{6.10}$$

As mentioned, we based our calculations on the Πs listed in [28], but adapted to the present case of staus and sneutrinos; the resulting formulas can be found in appendix A.

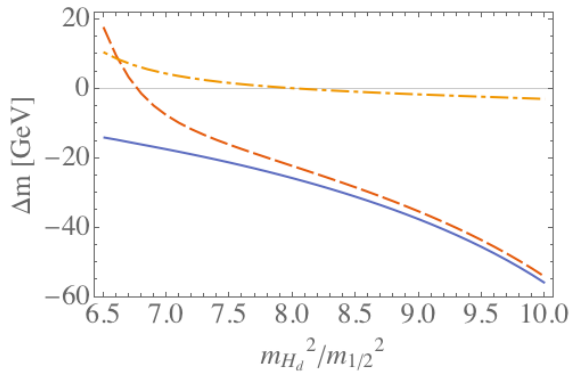
The calculations were done by first running SPheno, extracting the relevant parameters, and doing the tree-level calculations for $m_{\tilde{\tau}_i}$, $m_{\tilde{\nu}_\tau}$ again. These values were then used in evaluating eq. (6.7) (with the other parameters again taken from SPheno), before being updated to their new, one-loop corrected values. The new values were *then* used to re-evaluate (6.7), before being updated *again*. This process was repeated a total of four times for each point, as we found this to be sufficient for the result to have stabilised ($|m_n - m_{n-1}| < 10^{-2}$ GeV). See Appendix B for a flowchart of our code.

Doing this calculation over the same range of high-energy input parameters as in section 6.1, we get the results presented in figure 14. We see that, rather than become smaller, as one might have expected from figures 10 and 11, the loops seem to have *enhanced* the mass difference between the stau and sneutrino.

This result is, by and large, the opposite of what we expected — the point at which the NLSP changes identity is further from SPheno's result than ever before, now at $m_{H_d}^2/m_{1/2}^2$ below 7!



(a)



(b)

Figure 15: (a) The mass difference $\Delta m = m_{\tilde{\nu}_\tau} - m_{\tilde{\tau}_1}$ calculated at one-loop level (red dashed curve) compared to the same mass difference in the SPheno output (yellow dash-dotted curve). The solid blue line is the unmixed one loop-level mass difference, $\Delta m = m_{\tilde{\nu}_\tau} - m_{\tilde{\tau}_L}$. (b) The same plot zoomed in to the region in which the NLSP changes identity — note that the range of the y-axis is now four times larger than in figure 13b, reflecting the large enhancement of the mass splitting.

As for the overall mass scale, we did indeed come very close to SPheno's output by going up to 1-loop level, as can be seen in table 4. In this region, the difference is on the order of 1 GeV, or $\sim 0.1\%$. This is of course *not* the case for higher values of $m^2_{H_d}$, as discussed above.

6.3 Understanding the enhanced mass splitting

Though sparticle mixing still affects the results, the most striking feature of the above plots is how the mass difference only becomes larger as $\tilde{\tau}$ becomes more left-handed. Ignoring the mixing, then, we delved into the individual terms of the $\tilde{\tau}_L$ and $\tilde{\nu}_\tau$ loop corrections. We found three terms to be of the greatest importance, several orders of magnitude above the rest. The first two are

$$\Pi_{\tilde{\tau}_L\tilde{\tau}_L}(p^2) \supset \sum_{i=1}^{4}\Big[f_{i\tau\tilde{\tau}_{LL}} G(m_{\tilde{\chi}^0_i}, m_\tau) - 2\, g_{i\tau\tilde{\tau}_{LL}}\, m_{\tilde{\chi}^0_i} m_\tau B_0(m_{\tilde{\chi}^0_i}, m_\tau) \Big] + \sum_{i=1}^{2} f_{i\nu_\tau\tilde{\tau}_{LL}} G(m_{\tilde{\chi}^+_i}, 0)\,, \tag{6.11}$$

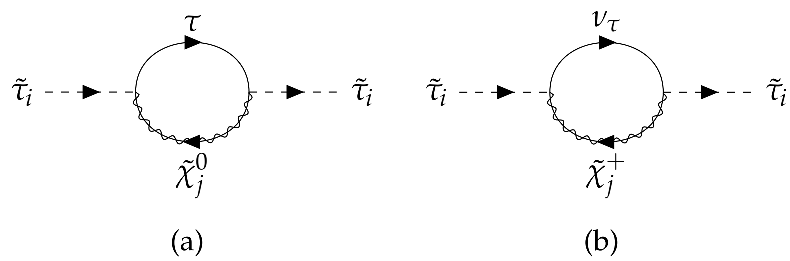


Figure 16: Examples of corrections to the stau mass through interactions with the (a) neutral gauginos, and (b) charged gauginos. Making the replacement $\tau \longleftrightarrow \nu$, $\tilde{\chi}_j^+ \longmapsto \tilde{\chi}_j^-$ would yield the corresponding diagrams for the sneutrino.

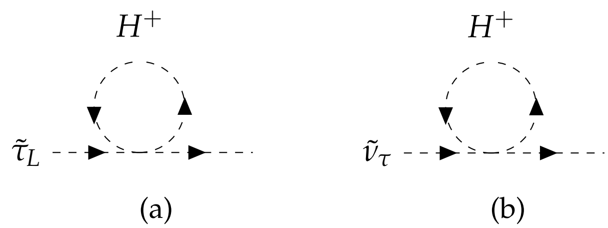


Figure 17: A loop arising from quartic interactions between the left-handed sleptons and the charged Higgs, giving the mass correction terms (6.13), (6.14).

for the stau, and

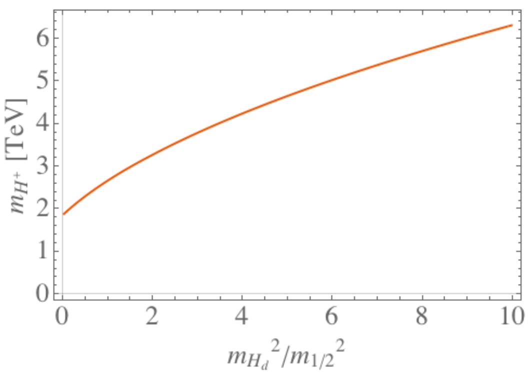
$$\begin{aligned}\Pi_{\tilde{\nu}_\tau \tilde{\nu}_\tau}(p^2) \supset & \sum_{i=1}^{4} f_{i\nu_\tau \tilde{\nu}_\tau LL} G(m_{\tilde{\chi}_i^0}, 0) \\ & + \sum_{i=1}^{2} \left[f_{i\tau\tilde{\nu}_\tau LL} G(m_{\tilde{\chi}_i^+}, m_\tau) - 2\, g_{i\tau\tilde{\nu}_\tau LL}\, m_{\tilde{\chi}_i^+} m_\tau B_0(m_{\tilde{\chi}_i^+}, m_\tau) \right] ,\end{aligned} \tag{6.12}$$

for the sneutrino — the definitions of the various functions and parameters appearing here are listed in reference [28]. Though these terms differ both numerically and aesthetically for the different particles, they sum up to give both an approximately equal upwards push. They correspond to diagrams like those in figure 16, and, in being the most important contributions, are the kinds of corrections that give gaugino mediation its name [8].

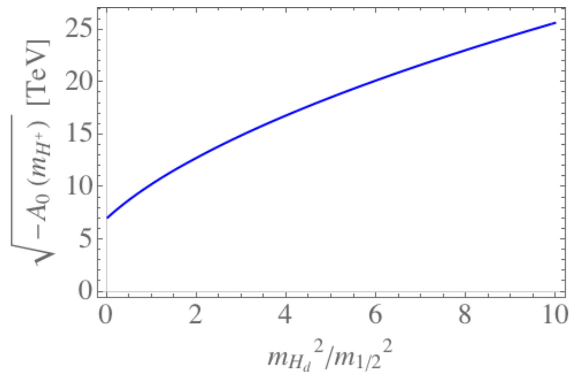
The third term identified to be of importance was:

$$\Pi_{\tilde{\tau}_L \tilde{\tau}_L}(p^2) \supset g^2 I_3^\tau \cos(2\beta) A_0(m_{H^+}^2) \, , \tag{6.13}$$

$$\Pi_{\tilde{\nu}_\tau \tilde{\nu}_\tau}(p^2) \supset g^2 I_3^\nu \cos(2\beta) A_0(m_{H^+}^2) \, . \tag{6.14}$$



(a)



(b)

Figure 18: (a) The mass of the charged Higgs as a function of $m_{H_d}^2/m_{1/2}^2$. (b) The divergent integral $A_0(m_{H^+})$ after $\overline{MS}$-subtraction, as a function of $m_{H_d}^2/m_{1/2}^2$ (we take minus the square root in order to get the same units in both plots). Note the differing ranges on the y-axes.

These correspond to the diagrams shown in figure 17.

It now becomes clear how the enhanced mass difference comes about — as we see from equations (6.13), (6.14), the left-handed stau and the tau sneutrino receive exactly the same correction to their masses by interactions with the charged Higgs, only of opposite sign. In addition, we find that the reason for the increasing mass difference with increasing $m_{H_d}^2$ is that the mass of the charged Higgs increases as well, as shown in figure 18a. This is again unsurprising, given the relationship between m_{H_d}, μ, and m_{H^+}. Finally, as demonstrated in figure 18b, the finite part of the divergent integral $A_0(m_{H^+})$ only acts to boost the effect of the changing mass, (in addition to a change of sign).

7 Conclusions and future prospects

In this thesis, we have given a short introduction to supersymmetry and its phenomenological prospects. Specialising to the theory of gaugino mediation, we then proceeded to analyse the processes determining the NLSP in a region of the parameter space where it is one of the third-generation sleptons. After discussing the situation at tree-level, where it is largely determined by the mixing of the stau, we proceeded to redo the calculations at one-loop level. Here, we found an unexpected enhancement of the mass-splitting, in direct contradiction to SPheno. We found that this could be traced back to a single term in the loop-corrected mass formulas, corresponding to a loop interaction with the charged Higgs boson.

The disagreement between our results and SPheno's remains a mystery; one wonders if the two-loop corrections that SPheno includes for the neutral Higgs sector, as noted in the footnote on page 41, really can be enough to close the gap. In the future, it would be interesting to identify these two-loop corrections and incorporate them into our calculations too, though it might well prove necessary to engage in a study of the SPheno source code in order to identify the real culprits.

One might also want to redo the analysis with other spectrum generators, e.g. SOFTSUSY [51], in order to get an impression of the uncertainties typically involved in such calculations. A more careful analysis of the theoretical uncertainties, and to what degree the respective predictions remain within them, might also prove valuable.

Finally, it would be interesting to investigate how the situation changes once one allows for differing values in A_{u0} and A_{d0}.

Acknowledgements

The author would like to thank his supervisor for invaluable instruction and guidance; Inga Strümke, Daniel Alvestad, Nick Murphy, Werner Porod, and Stephen Martin for helpful discussions; and his parents, for their continued support throughout.

Feynman diagrams were created using the package TikZ-Feynman [52], while the divergent integrals (4.6)–(4.9) appearing in (A.1)–(A.4) were evaluated using LoopTools [53].

A One-loop formulas

Following are the complete 1-loop corrections to the squared-mass parameters of the staus and the tau sneutrino. They are adapted from similar formulas listed in the appendices of [28]; the definitions of the various functions and parameters appearing in these formulas can be found there.

$$
\begin{aligned}
16\pi^2\,\Pi_{\tilde\tau_L\tilde\tau_L}(p^2) &= \lambda_\tau^2\bigg(s_\tau^2 A_0(m_{\tilde\tau_1}) + c_\tau^2 A_0(m_{\tilde\tau_2})\bigg)\\
&+ \frac{1}{2}\sum_{n=1}^{4}\bigg(\lambda_\tau^2\,D_{nd} - \frac{g^2 g_{\tau_L}}{2\hat c^2}C_n\bigg)A_0(m_{H_n^0}) \;+\; \sum_{n=3}^{4} g^2\left(\frac{g_{\tau_L}}{2\hat c^2} - I_3^\tau\right)C_n A_0(m_{H_{n-2}^+})\\
&+ \sum_{n=1}^{4}\sum_{i=1}^{2}(\lambda_{H_n^0\tilde\tau_L\tilde\tau_i})^2 B_0(m_{H_n^0}, m_{\tilde\tau_i}) \;+\; \sum_{n=1}^{2}(\lambda_{H_n^+\tilde\tau_L\tilde\nu_\tau})^2 B_0(m_{\tilde\nu_\tau}, m_{H_n^+})\\
&+ \frac{4g^2}{\hat c^2}(g_{\tau_L})^2 A_0(M_Z) + 2g^2 A_0(M_W) \;+\; (e_\tau e)^2\bigg(c_\tau^2 F(m_{\tilde\tau_1},0) + s_\tau^2 F(m_{\tilde\tau_2},0)\bigg)\\
&+ \frac{g^2}{\hat c^2}(g_{\tau_L})^2\bigg[c_\tau^2 F(m_{\tilde\tau_1}, M_Z) + s_\tau^2 F(m_{\tilde\tau_2}, M_Z)\bigg] \;+\; \frac{g^2}{2}F(m_{\tilde\nu_\tau}, M_W)\\
&+ \frac{g^2}{4}\bigg[c_\tau^2 A_0(m_{\tilde\tau_1}) + s_\tau^2 A_0(m_{\tilde\tau_2}) + 2A_0(m_{\tilde\nu_\tau})\bigg]\\
&+ g^2\sum_f N_c^f I_3^\tau I_3^f\bigg(c_f^2 A_0(m_{\tilde f_1}) + s_f^2 A_0(m_{\tilde f_2})\bigg) \;+\; \frac{g'^2}{4}(Y_{\tau_L})^2\bigg(c_\tau^2 A_0(m_{\tilde\tau_1}) + s_\tau^2 A_0(m_{\tilde\tau_2})\bigg)\\
&+ \frac{g'^2}{4}Y_{\tau_L}\sum_f N_c^f\bigg[Y_{f_L}\bigg(c_f^2 A_0(m_{\tilde f_1}) + s_f^2 A_0(m_{\tilde f_2})\bigg) \;+\; Y_{f_R}\bigg(s_f^2 A_0(m_{\tilde f_1}) + c_f^2 A_0(m_{\tilde f_2})\bigg)\bigg]\\
&+ \sum_{i=1}^{4}\bigg[f_{i\tau\tilde\tau_{LL}} G(m_{\tilde\chi_i^0}, m_\tau) - 2\,g_{i\tau\tilde\tau_{LL}}\, m_{\tilde\chi_i^0} m_\tau B_0(m_{\tilde\chi_i^0}, m_\tau)\bigg]\\
&+ \sum_{i=1}^{2} f_{i\nu_\tau\tilde\tau_{LL}} G(m_{\tilde\chi_i^+}, 0)\,. \qquad\qquad (\text{A.1})
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2\,\Pi_{\tilde\tau_R\tilde\tau_R}(p^2) &= \lambda_\tau^2\Big(c_\tau^2 A_0(m_{\tilde\tau_1}) + s_\tau^2 A_0(m_{\tilde\tau_2}) + A_0(m_{\tilde\nu_\tau})\Big)\\
&+ \frac{1}{2}\sum_{n=1}^{4}\Big(\lambda_\tau^2\,D_{nd} - \frac{g^2 g_{\tau_R}}{2\hat c^2}C_n\Big)A_0(m_{H_n^0}) \;+\; \sum_{n=3}^{4}\Big(\lambda_\tau^2 D_{nu} + \frac{g^2 g_{\tau_R}}{2\hat c^2}C_n\Big)A_0(m_{H_{n-2}^+})\\
&+ \sum_{n=1}^{4}\sum_{i=1}^{2}(\lambda_{H_n^0\tilde\tau_R\tilde\tau_i})^2 B_0(m_{H_n^0}, m_{\tilde\tau_i}) \;+\; \sum_{n=1}^{2}(\lambda_{H_n^+\tilde\tau_R\tilde\nu_\tau})^2 B_0(m_{\tilde\nu_\tau}, m_{H_n^+})\\
&+ \frac{4g^2}{\hat c^2}(g_{\tau_R})^2 A_0(M_Z) + (e_\tau e)^2\Big(s_\tau^2 F(m_{\tilde\tau_1},0) + c_\tau^2 F(m_{\tilde\tau_2},0)\Big)\\
&+ \frac{g^2}{\hat c^2}(g_{\tau_R})^2\Big[\,s_\tau^2 F(m_{\tilde\tau_1},M_Z) + c_\tau^2 F(m_{\tilde\tau_2},M_Z)\,\Big]\\
&+ \frac{g'^2}{4}(Y_{\tau_R})^2\Big(s_\tau^2 A_0(m_{\tilde\tau_1}) + c_\tau^2 A_0(m_{\tilde\tau_2})\Big)\\
&+ \frac{g'^2}{4}Y_{\tau_R}\sum_f N_c^f\Big[\,Y_{f_L}\Big(c_f^2 A_0(m_{\tilde f_1}) + s_f^2 A_0(m_{\tilde f_2})\Big) \;+\; Y_{f_R}\Big(s_f^2 A_0(m_{\tilde f_1}) + c_f^2 A_0(m_{\tilde f_2})\Big)\Big]\\
&+ \sum_{i=1}^{4}\Big[\,f_{i\tau\tilde\tau_{RR}} G(m_{\tilde\chi_i^0}, m_\tau) - 2\,g_{i\tau\tilde\tau_{RR}}\,m_{\tilde\chi_i^0} m_\tau B_0(m_{\tilde\chi_i^0}, m_\tau)\,\Big]\\
&+ \sum_{i=1}^{2} f_{i\nu_\tau\tilde\tau_{RR}} G(m_{\tilde\chi_i^+},0)\;. \qquad\qquad (\mathrm{A}.2)
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2\,\Pi_{\tilde\tau_L\tilde\tau_R}(p^2) &= \sum_{n=1}^{4}\sum_{i=1}^{2}\lambda_{H_n^0\tilde\tau_L\tilde\tau_i}\lambda_{H_n^0\tilde\tau_R\tilde\tau_i} B_0(m_{H_n^0}, m_{\tilde\tau_i}) \;+\; \sum_{n=1}^{2}\lambda_{H_n^+\tilde\tau_L\tilde\nu_\tau}\lambda_{H_n^+\tilde\tau_R\tilde\nu_\tau} B_0(m_{\tilde\nu_\tau}, m_{H_n^+})\\
&+ \frac{\lambda_\tau}{2}\sum_{f_d} N_c^f \lambda_d s_{2\theta_d}\Big(A_0(m_{\tilde d_1}) - A_0(m_{\tilde d_2})\Big) \;+\; \frac{g'^2}{4}Y_{\tau_L}Y_{\tau_R}s_\tau c_\tau\Big(A_0(m_{\tilde\tau_1}) - A_0(m_{\tilde\tau_2})\Big)\\
&+ (e_\tau e)^2 s_\tau c_\tau\Big(F(m_{\tilde\tau_1},0) - F(m_{\tilde\tau_2},0)\Big) \;-\; \frac{g^2}{\hat c^2}g_{\tau_L}g_{\tau_R}s_\tau c_\tau\Big(F(m_{\tilde\tau_1},M_Z) - F(m_{\tilde\tau_2},M_Z)\Big)\\
&+ \sum_{i=1}^{4}\Big[\,f_{i\tau\tilde\tau_{LR}} G(m_{\tilde\chi_i^0}, m_\tau) - 2\,g_{i\tau\tilde\tau_{LR}}\,m_{\tilde\chi_i^0} m_\tau B_0(m_{\tilde\chi_i^0}, m_\tau)\,\Big]\\
&+ \sum_{i=1}^{2}\Big[\,f_{i\nu_\tau\tilde\tau_{LR}} G(m_{\tilde\chi_i^+},0)\Big]\;. \qquad\qquad (\mathrm{A}.3)
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2\, \Pi_{\tilde{\nu}_\tau\tilde{\nu}_\tau}(p^2) = {} & \lambda_\tau^2 \Big(s_\tau^2 A_0(m_{\tilde{\tau}_1}) + c_\tau^2 A_0(m_{\tilde{\tau}_2}) \Big) \\
& - \frac{1}{2} \sum_{n=1}^{4} g^2 \frac{g_{\nu_\tau}}{2\hat{c}^2} C_n A_0(m_{H_n^0}) \; + \; \sum_{n=3}^{4} \Big(\lambda_\tau^2 D_{nu} + g^2 \left(\frac{g_{\nu_\tau}}{2\hat{c}^2} - I_3^\nu \right) C_n \Big) A_0(m_{H^+_{n-2}}) \\
& + \sum_{n=1}^{4} (\lambda_{H_n^0 \tilde{\nu}_\tau \tilde{\nu}_\tau})^2 B_0(m_{H_n^0}, m_{\tilde{\nu}_\tau}) \; + \; \sum_{i,n=1}^{2} (\lambda_{H_n^+ \tilde{\nu}_\tau \tilde{\tau}_i})^2 B_0(m_{\tilde{\tau}_i}, m_{H_n^+}) \\
& + \frac{4 g^2}{\hat{c}^2} (g_{\nu_\tau})^2 A_0(M_Z) + 2 g^2 A_0(M_W) \\
& + \frac{g^2}{\hat{c}^2} (g_{\nu_\tau})^2 F(m_{\tilde{\nu}_\tau}, M_Z) \; + \; \frac{g^2}{2} \Big[c_\tau^2 F(m_{\tilde{\tau}_1}, M_W) + s_\tau^2 F(m_{\tilde{\tau}_2}, M_W) \Big] \\
& + \; \frac{g^2}{4} \Big[A_0(m_{\tilde{\nu}_\tau}) + 2 \Big(c_\tau^2 A_0(m_{\tilde{\tau}_1}) + s_\tau^2 A_0(m_{\tilde{\tau}_2}) \Big) \Big] \\
& + g^2 \sum_f N_c^f I_3^\nu I_3^f \Big(c_f^2 A_0(m_{\tilde{f}_1}) + s_f^2 A_0(m_{\tilde{f}_2}) \Big) \; + \; \frac{g'^2}{4} (Y_\nu)^2 A_0(m_{\tilde{\nu}_\tau}) \\
& + \frac{g'^2}{4} Y_\nu \sum_f N_c^f \Big[Y_{f_L} \Big(c_f^2 A_0(m_{\tilde{f}_1}) + s_f^2 A_0(m_{\tilde{f}_2}) \Big) \; + \; Y_{f_R} \Big(s_f^2 A_0(m_{\tilde{f}_1}) + c_f^2 A_0(m_{\tilde{f}_2}) \Big) \Big] \\
& + \sum_{i=1}^{4} f_{i \nu_\tau \tilde{\nu}_\tau \tilde{\nu}_\tau LL} G(m_{\tilde{\chi}_i^0}, 0) \\
& + \sum_{i=1}^{2} \Big[f_{i \tau \tilde{\nu}_\tau LL} G(m_{\tilde{\chi}_i^+}, m_\tau) - 2\, g_{i \tau \tilde{\nu}_\tau LL}\, m_{\tilde{\chi}_i^+} m_\tau B_0(m_{\tilde{\chi}_i^+}, m_\tau) \Big] \, . \qquad \text{(A.4)}
\end{aligned}
$$

B A flowchart of our code

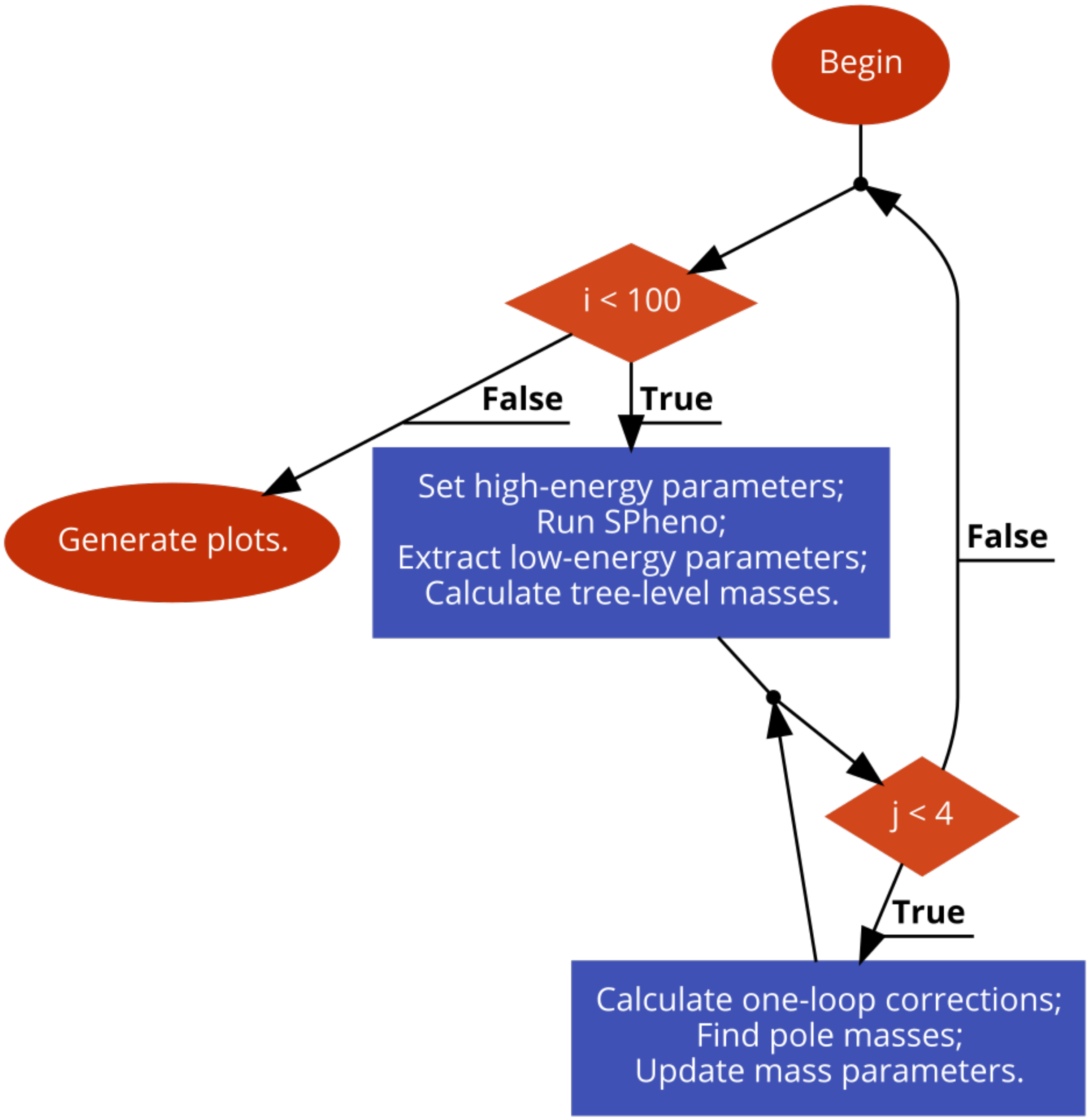


Figure 19: A flowchart of our code. The program was written and executed in Wolfram Mathematica 11.0 [54]. This flowchart was made using code2flow [55].

C SUSY-transforming the massless non-interacting Wess-Zumino model

In this appendix, we will show explicitly how to go from equation (3.36) to (3.37).

First, we need some additional identities [5]:

$$(\chi^\dagger \bar{\sigma}^\nu \sigma^\mu \xi^\dagger)^* = \xi \sigma^\mu \bar{\sigma}^\nu \chi = \chi \sigma^\mu \bar{\sigma}^\mu \xi = (\xi^\dagger \bar{\sigma}^\mu \sigma^\nu \chi^\dagger)^*, \tag{C.1}$$

and

$$(\sigma^\mu \bar{\sigma}^\nu + \sigma^\nu \bar{\sigma}^\mu)_\alpha^{\ \beta} = 2g^{\mu\nu} \delta_\alpha^\beta \,, \tag{C.2}$$

$$(\bar{\sigma}^\mu \sigma^\nu + \bar{\sigma}^\nu \sigma^\mu)^{\dot{\beta}}_{\ \dot{\alpha}} = 2g^{\mu\nu} \delta^{\dot{\beta}}_{\dot{\alpha}} \,, \tag{C.3}$$

all of which can be proven through explicit spinor index manipulation.

The terms that need rewriting are the latter two of equation (3.36), i.e. the fermionic part of the transformation. Let's tackle each term separately, starting with the third. Using eq. (C.1), we get

$$\psi^\dagger \bar{\sigma}^\nu \sigma^\mu \varepsilon^\dagger \partial_\mu \partial_\nu \phi = \varepsilon^\dagger \bar{\sigma}^\mu \sigma^\nu \psi^\dagger \partial_\mu \partial_\nu \phi$$

Next, we exploit the fact that derivatives are commutative,

$$= \frac{1}{2} \varepsilon^\dagger \bar{\sigma}^\mu \sigma^\nu \psi^\dagger (\partial_\mu \partial_\nu + \partial_\nu \partial_\mu) \phi$$

Relabelling the contracted indices in the second term $\mu \longleftrightarrow \nu$ gives us

$$= \frac{1}{2} \varepsilon^\dagger (\bar{\sigma}^\mu \sigma^\nu + \bar{\sigma}^\nu \sigma^\mu) \psi^\dagger \partial_\mu \partial_\nu \phi$$

and, applying (C.3),

$$= \varepsilon^\dagger \psi^\dagger \partial_\mu \partial^\mu \phi \tag{C.4}$$

Finally, we apply the product rule in reverse to get

$$= -\epsilon^{\dagger}\partial_{\mu}\psi^{\dagger}\partial^{\mu}\phi + \partial_{\mu}(\epsilon^{\dagger}\psi^{\dagger}\partial^{\mu}\phi) \ .$$

Next, we look at the fourth term of equation (3.36), where we will essentially apply the same prescription in reverse. We start by adding a term that we know we'll need.

$$-\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} = -\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} - \varepsilon\psi\partial_{\mu}\partial^{\mu}\phi^{*} + \varepsilon\psi\partial_{\mu}\partial^{\mu}\phi^{*}$$

Then, applying (C.2) *in reverse* to the second term yields

$$= -\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} - \frac{1}{2}\varepsilon(\sigma^{\nu}\bar{\sigma}^{\mu} + \sigma^{\mu}\bar{\sigma}^{\nu})\psi\partial_{\mu}\partial_{\nu}\phi^{*} + \varepsilon\psi\partial_{\mu}\partial^{\mu}\phi^{*}$$

Relabelling the second term and then commuting the differential operators, we get

$$\begin{aligned} &= -\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} - \frac{1}{2}\varepsilon\sigma^{\nu}\bar{\sigma}^{\mu}\psi(\partial_{\mu}\partial_{\nu} + \partial_{\nu}\partial_{\mu})\phi^{*} + \varepsilon\psi\partial_{\mu}\partial^{\mu}\phi^{*} \\ &= -\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} - \varepsilon\sigma^{\nu}\bar{\sigma}^{\mu}\psi\partial_{\mu}\partial_{\nu}\phi^{*} + \varepsilon\psi\partial_{\mu}\partial^{\mu}\phi^{*} \end{aligned}$$

And finally, relabelling the indices of the first term as well, we can write this as

$$= -\varepsilon\partial_{\mu}\psi\partial^{\mu}\phi^{*} - \partial_{\mu}(\varepsilon\sigma^{\nu}\bar{\sigma}^{\mu}\psi\partial_{\nu}\phi^{*} - \epsilon\psi\partial^{\mu}\phi^{*}) \ .$$

Collecting our results then, we end up with

$$\begin{aligned} -\varepsilon\sigma^{\mu}\bar{\sigma}^{\nu}\partial_{\nu}\psi\partial_{\mu}\phi^{*} + \psi^{\dagger}\bar{\sigma}^{\nu}\sigma^{\mu}\varepsilon^{\dagger}\partial_{\mu}\partial_{\nu}\phi = & -\varepsilon\partial_{\mu}\psi\partial^{\mu}\phi^{*} - \varepsilon^{\dagger}\partial_{\mu}\phi\partial^{\mu}\psi^{\dagger} \\ & -\partial_{\mu}(\varepsilon\sigma^{\nu}\bar{\sigma}^{\mu}\psi\partial_{\nu}\phi^{*} - \varepsilon\psi\partial^{\mu}\phi^{*} - \varepsilon^{\dagger}\psi^{\dagger}\partial^{\mu}\phi) \ , \end{aligned}$$

which is exactly what we needed[39].

[39]This calculation is a lot simpler in the more elegant *superspace* formalism, in which supersymmetry is given a geometric interpretation in a space comprised of both ordinary and anti-commuting Grassman coordinates. Though it has become the standard way to express theoretical developments in supersymmetry, it is not necessary for phenomenology, and so it is not treated in the current work. For a review on superspace and superfields, see [5].

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