Optimal design of photovoltaic power plants

Master thesis in Applied and Computational Mathematics

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Abstract

The configuration of the photovoltaic panels is important to maximize the energy output of a photovoltaic power plant. In this thesis, I have developed an algorithm calculating the optimal solution for the design of a solar farm with fixed panels. The design of the photovoltaic power plant is decided by the configuration of the following parameters: tilt and azimuth of the panels, height and length of the rows, distance between the rows and number of rows. The parameters are constructed to maximize the net present value of the solar farm in a given field. The calculations will be based on local measurements and estimations. We will analyse and optimize the design of the solar farms for three different locations.

Contents

Chapter 1

Introduction

In 2015, United Nations spearheaded a set of goals: The Sustainable Development Goals. The goals are set to end poverty, protect the planet and ensure prosperity for all, as part of a new sustainable development agenda. One goal is to ensure access to affordable, reliable, sustainable and modern energy for all [35]. Solar energy is not dependent on a good infrastructure and is therefore easy and affordable to install in both central and rural areas. Solar energy is sustainable. I hope this thesis can contribute to the effort of making it affordable.

In the first chapter, I introduce some basic ideas and concepts needed to understand the production of photovoltaic electricity. In particular, what determines the optimal effect of a solar panel. We will also look at how the components of the solar farm can be configured for optimal design.

Solar energy

Solar energy is energy converted from the radiant energy from the Sun. Solar energy is renewable and once the construction is installed, it is emission free. The energy can be generated by different methods, such as solar thermal energy, concentrated solar power and photovoltaic cells (PV). In this thesis, we will focus on PV.

The price of PV panels is decreasing at a high rate, which makes it compelling.

Prices are expected to drop further, meaning PV could become even more compelling compared to other types of energy in the future. Some parts of the world are in need of sustainable, affordable and reliable electricity. Parts of this need could be covered by PV.

Solar irradiance

The radiant energy from the Sun is referred to as solar irradiance. It can be divided into three parts: beam, diffuse and reflected irradiance. Beam irradiance is irradiance coming directly from the Sun. Diffuse irradiance is the irradiance that has changed direction and been spread all over the sky. Reflected irradiance is irradiance that has hit another object before it hits the PV panels. The sum of the beam, diffuse and reflected irradiance is the total irradiance.

PV

A PV panel consists of several cells. A cell transforms radiant energy from the Sun into electricity. A photon from the Sun hits a cell of silicon in a panel. The silicon then releases an electron, producing direct current electricity. To transfer the electricity to the grid, the electricity needs to be converted to alternating current electricity.

To obtain as much electricity as possible, the panels should be facing directly towards the Sun. There are two ways of installing PV panels. The first is on a rotating mounting system tracking the Sun. This would result in more electricity from each panel but would require more space and a higher cost of both installing and maintenance. The other solution is on a fixed mounting system.

Solar PV power plant

A solar power plant, also known as a solar farm, is a collection of PV panels. The purpose of a PV farm is either to supply power into the electricity grid or produce power for an off-grid system. The amount of power produced depends on several parameters. Some of these parameters are decided when installing the PV farm and some of the parameters are fixed. The parameters to be decided include tilt and azimuth of the panels, height and length of the rows of panels, the distance between the rows and the number of rows. The purpose of this thesis is to find the optimal configuration of these parameters, so the PV farm will generate as much value as possible.

Figure 1.1: Pictures of a solar farm in Ghana. Photo: Daniel Engelsen

Chapter 2

The optimization problem

In this chapter, I will construct a profit function deciding the value of a solar farm. The solar farm will be designed to maximize the profit function. In this thesis, I assume we are given a limited area, on which we shall install a solar farm. Various techniques can be used to search for the optimal design, I will decide which algorithm I will use.

2.1 Value of the solar farm

I assume a situation where the income for the PV farm comes from selling electricity into the grid. This is a daily income, throughout the lifetime of the PV panels. In this thesis, we assume the PV farm is built without subsidies. The main expense is buying everything you need to the PV farm, and there will also be small running expenses after the construction is complete.

Income

The annual income (C_i) equals the amount of energy produced multiplied by the electricity price. The energy received by the panels per year (Q), is given in Watt-hours (Wh). The efficiency (η) tells the fraction of energy produced from the amount of energy received. To be able to convert the amount of produced energy into how much money we have earned, we need to know the electricity price (ep), given in NOK/KWh.

$$C_i = Q \cdot \eta \cdot 0.001 \cdot ep \tag{2.1}$$

Expenses

The expenses are divided into three categories: installation, materials and other. The sum of all the categories is estimated and divided by the area of the panels. This gives us a price per square meter of installed PV (pp). Multiplying the panel price with the area of installed PV gives the investment price of the solar farm (I_0).

$$I_0 = H \cdot L \cdot K \cdot pp \tag{2.2}$$

Where H and L are height and length of the rows of PV panels, and K is the number of rows.

Total energy hitting the panels

The total energy hitting the panels is converted from three different types of irradiance: beam, diffuse and reflected irradiance. In addition, we distinguish between the first and the succeeding rows, because the succeeding rows will produce less energy due to shade from the preceding row.

$$Q = H \cdot L \cdot (q_b + q_d + q_r + (K-1) \cdot (q_b^{sh} + q_d^{sh} + q_r^{sh})) \tag{2.3}$$

Where q_b, q_d and q_r are energy from beam, diffuse and reflected irradiance, on the first row, and q_b^{sh}, q_d^{sh} and q_r^{sh} are energy from beam, diffuse and reflected irradiance, on the succeeding rows.

2.1.1 Energy from irradiance

To calculate the amount of energy the solar farm produces, we need to know the amount of irradiance reaching the panels. The Energy received by a panel is found by integrating the amount of irradiance hitting the panel. We use the trapezoidal rule to integrate the discrete values.

$$
\begin{aligned}
q_\bullet &= \sum_{n=1}^{365} \int_0^{24} I_{\bullet,T} dt \approx \sum_{n=1}^{365} \sum_{k=1}^{\frac{24}{\Delta t}} I_{\bullet,T} \Delta t \\
q_\bullet^{sh} &= \sum_{n=1}^{365} \int_0^{24} I_{\bullet,T}^{sh} dt \approx \sum_{n=1}^{365} \sum_{k=1}^{\frac{24}{\Delta t}} I_{\bullet,T}^{sh} \Delta t
\end{aligned}
\tag{2.4}
$$

In these equations $q_\bullet$ represents q_b, q_d and q_r, the energy received by the first row of panels. $q_\bullet^{sh}$ represents q_b^{sh}, q_d^{sh} and q_r^{sh}, the energy received by the shaded rows of panels. $I_{\bullet,T}$ represents $I_{b,T}$, $I_{d,T}$ and $I_{r,T}$, the irradiance received by the first row of panels. $I_{\bullet,T}^{sh}$ represents $I_{b,T}^{sh}$, $I_{d,T}^{sh}$ and $I_{r,T}^{sh}$, the irradiance received by the shaded rows of panels. n represents day of the year, k is index of the irradiance measurements and Δt is the time between each irradiance measurement.

Sunrise and sunset

The solar panels are turned off during times with low irradiance, i.e. when the Sun is down. The sunrise and sunset can be calculated. In our case, we are given irradiance data from the specific locations. Therefore, we do not need to know when the Sun is up. This data are more site-specific. It will account for specific factors of the site, such as tall buildings or mountains surrounding the location. Therefore, we do not need to pay attention to whether the Sun is up or down, we can integrate over the whole day and night.

The profit function we want to optimize is simply:

$$
f = T \cdot C_i - I_0
$$

Where T is the number of years before we replace the panels. In the next section we will explain how $I_{\bullet,T}$ and $I_{\bullet,T}^{sh}$ from Eq. (2.4) are calculated, and describe the other parameters needed to calculate the profit function.

2.2 Irradiance on tilted panels

I want to find the irradiance on tilted panels. However, the standard way of measuring irradiance is to measure it on a horizontal surface. The irradiance is divided into the beam, diffuse and reflected irradiance.

$$I = I_b + I_d + I_r$$

The solar panels are often tilted, to increase the amount of beam irradiance.

$$I_T = I_{b,T} + I_{d,T} + I_{r,T}$$

The irradiance can be calculated using known formulas. We will look at some angles needed to construct these formulas.

2.2.1 Solar geometry

To derive the relation between the Sun and the panels, we need to specify the position of these objects in a reference frame. The reference frame is the framework for our coordinate system. We have four different reference frames.

Ecliptic reference frame
The ecliptic reference frame is constructed such that the Sun moves around the Earth in the xy-plane. This plane is tilted at an angle of 23.45° compared to the equator.

Fixed/Rotating equatorial reference frame
In the Fixed equatorial reference frame, the equator lies in the xy-plane, and the Earth is spinning around the z-axis. In the Rotating equatorial reference frame, the xy-axis is rotating along with the Earth's rotation.

Horizon reference frame
This reference frame is fixed on the surface of the Earth. The xy-plane is tangential to the surface of the Earth, from the observer's position. Where the x-axis is east-west, the y-axis is north-south and the z-axis is normal to the surface of the Earth. The horizon reference frame is the one we will use.

Angles in solar geometry

The angles needed to decide the Sun's position, are the Sun's elevation angle (α) and the Sun's azimuth (γ_s). Alternatively, the zenith angle (θ_z), which is the complementary angle of α, can be used. (See Figure 2.1)

$$\theta_z = 90^\circ - \alpha$$

These angles are continuously changing, due to the Earth's rotation around its own axis and the Earth's position in its orbit around the Sun. The azimuth of the Sun is measured from the south, with negative values towards east and positive towards west.

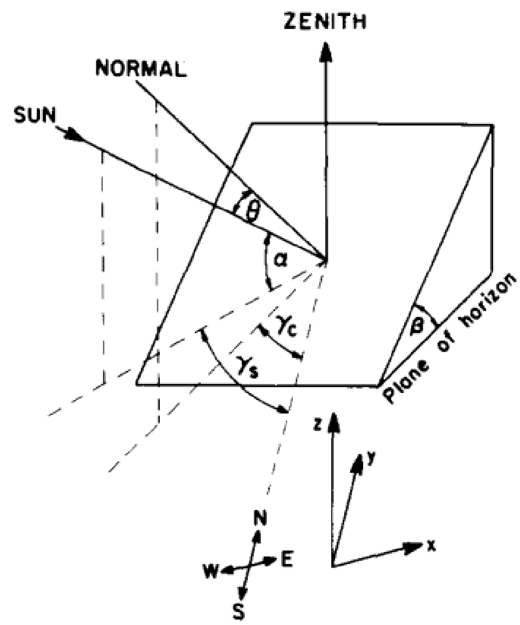


Figure 2.1: Angles involved in solar calculations. Figure from [2].

The angles we need to optimize, to obtain the optimal design of the solar farm, are the tilt of the panels (β) and the azimuth of the panels (γ_c).

The difference in azimuth between the panels and the Sun (γ) is given by:

$$\gamma = \gamma_c - \gamma_s$$

The angle of incidence (θ), is the angle between the normal of the panels and the Sun. If the panels would be tracking the Sun, then θ would be zero, and the panels would face directly towards the Sun. The latitude of the solar farm (ϕ), south negative, north positive. $-90^\circ \leq \phi \leq 90^\circ$. The solar declination angle (δ), vary throughout the year. An approximation of the declination angle is given by [3]:

$$\delta = 23.45 \sin\left(360 \frac{284+n}{365}\right)$$

Where n is day of the year, such that $n = 1$ on 1st January. Sun's azimuth, γ_s, is given by the following equation [7].

$$\gamma_s = \operatorname{sign}(\omega) \left| \cos^{-1}\left(\frac{\cos(\theta_z)\sin(\phi) - \sin(\delta)}{\sin(\theta_z)\cos(\phi)}\right) \right|$$

To calculate γ_s, we also need the hour angle (ω). Hour angle is negative in the morning, reaching zero at noon and positive values in the afternoon. The hour angle increases 15° per hour.

2.2.2 Beam irradiance

The irradiance reaching the surface of the Earth, is measured on a horizontal plane. Therefore, to obtain irradiance hitting the panels, we must calculate the relation (R_b) between irradiance on a horizontal plane and irradiance on a tilted plane.

$$I_{b,T} = I_b \cdot R_b$$

The factor R_b is given by [7]:

$$R_b = \frac{\cos(\theta)}{\cos(\theta_z)}$$

Where $\cos(\theta)$ and $\cos(\theta_z)$ could be calculated in two different ways. The most general way to calculate $\cos(\theta)$ and $\cos(\theta_z)$ is with the following equations [7]:

$$\begin{aligned}\cos(\theta) =& \sin(\delta)\sin(\phi)\cos(\beta) - \sin(\delta)\cos(\phi)\sin(\beta)\cos(\gamma) \\ &+ \cos(\delta)\cos(\phi)\cos(\beta)\cos(\omega) + \cos(\delta)\sin(\phi)\sin(\beta)\cos(\gamma)\cos(\omega) \\ &+ \cos(\delta)\sin(\beta)\sin(\gamma)\sin(\omega) \end{aligned} \tag{2.5}$$

$$\cos(\theta_z) = \cos(\phi)\cos(\delta)\cos(\omega) + \sin(\phi)\sin(\delta)$$

However, the optimal azimuth angle is usually 0° in the northern hemisphere and 180° in the southern hemisphere. Thus, it is a common situation that $\gamma_c = 0$ or $\gamma_c = 180$. In case $\gamma_c = 0°$ we could use the following equations [7]:

$$
\begin{aligned}
\cos(\theta) &= \cos(\phi - \beta)\cos(\delta)\cos(\omega) + \sin(\phi - \beta)\sin(\delta) \\
\cos(\theta_z) &= \cos(\phi)\cos(\delta)\cos(\omega) + \sin(\phi)\sin(\delta)
\end{aligned}
\tag{2.6}
$$

For $\gamma_c = 180°$ the minuses become pluses.

I will use Eq. (2.5) in my calculations, to not exclude the possibility for the azimuth to have any value between $-180°$ and $180°$.

Shading calculations

In a solar farm, we usually have multiple rows of panels. We have a limited available area, containing as many producing PV panels as possible. This requires the field to be compact, with a short distance between the rows. When two rows of panels are placed, one in front of the other, facing towards the equator, the row in front will cast a shadow covering parts of the row behind if the row is placed too tight behind. The part of a solar panel covered in shadow, will not receive beam irradiance. Therefore, it will produce less energy than the part receiving beam irradiance directly from the Sun. The size of the shaded area is dependent on interspacing between the rows, height of the rows, tilt of the rows and the Sun's elevation angle. The short interspacing will also result in less diffuse and reflected irradiance on the panels.

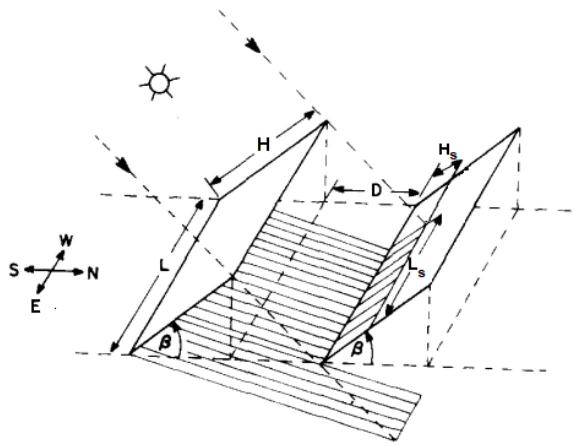


Figure 2.2: Shade from one panel to the next. Figure from [2].

From Figure 2.2 you can see the first row casting a shadow onto the next row. The relative area of the row covered by the shadow is given by the following equations. The normalised distance between panels and length of row are given by:

$$d = \frac{D}{H\sin(\beta)}$$

$$l = \frac{L}{H\sin(\beta)}$$

The relative shadow length is given by [2].

$$L_s = 1 - \frac{d\cdot\sin(\beta)+\cos(\beta)}{l}\cdot\frac{|\sin(\gamma)|}{\cos(\beta)\sin(\alpha)+\sin(\beta)\cos(\beta)}$$

The relative shadow height covering the panel is given by [2].

$$H_s = 1 - \frac{d\cdot\sin(\beta)+\cos(\beta)}{\cos(\beta)+[\sin(\beta)\cos(\gamma)/\tan(\alpha)]}$$

Where L_s, $H_s \in [0,1]$. The relative shaded area, the area that does not receive beam irradiance, becomes:

$$A_s = L_s \cdot H_s$$

Then the beam irradiance on the front row is:
$I_{b,T} = I_b \cdot R_b$
Beam irradiance on the shaded rows are:
$I_{b,T}^{sh} = I_b \cdot R_b \cdot (1 - A_s)$

2.2.3 Diffuse irradiance

To calculate the diffuse irradiance on a tilted surface $I_{d,T}$, we start with diffuse irradiance on a horizontal surface I_d. The diffuse irradiance hitting the first row is:

$$I_{d,T} = R_d \cdot I_d$$

Where R_d is the factor to be multiplied with. Representing the ratio between diffuse radiation on tilted and horizontal panels [2].

$$R_d = \cos^2\left(\frac{\beta}{2}\right)$$

Shading calculations

On the shaded rows of panels, diffuse radiation is also limited by the row in front. The diffuse irradiance on these rows is given by the following formula.

$$I_d^{sh} = F_d^{sh} \cdot I_d$$

Where F_d^{sh} represents the ratio between diffuse radiation on tilted and shaded panels and horizontal panels [2].

$$F_d^{sh} = \cos^2\left(\frac{\beta}{2}\right) - \frac{1}{2}(\sqrt{d^2+1} - d)\sin(\beta)$$

2.2.4 Reflected irradiance

Reflected irradiance can be measured by a downward-facing measure-instrument. If you are not given the reflected irradiance, it can be calculated provided you have the surface's albedo (ρ).

$$I_r = \rho \cdot I$$

The albedo is the measure of reflection from a surface, given by:

$$\rho = \frac{\text{The irradiance reflected by the surface}}{\text{The irradiance received by the surface}}$$

Reflected irradiance on a tilted surface is given by the equation [7]:

$$I_{r,T} = I_r \cdot \sin^2\left(\frac{\beta}{2}\right)$$

$I_{r,T}$ = Reflected irradiance on a tilted panel
I = Irradiance on a horizontal surface

Reflected irradiance on the shaded rows is calculated using the ratio between reflected irradiance on tilted and shaded panels and horizontal panels, given by [12].

$$F_r^{sh} = \frac{1}{2}\left[\frac{D}{H} + 1 - \sqrt{\left(\frac{D}{H}\right)^2 + \frac{2D}{H}\cos(\beta) + 1}\right]$$

Reflected irradiance on a tilted and shaded row ($I_{r,T}^{sh}$):

$$I_{r,T}^{sh} = \rho \cdot I \cdot F_r^{sh}$$

We have found the income and the expenses for a given solar farm, we are now ready to compare the two.

2.3 Net present value

As described above, the income is a more or less continuous stream, while most of the expenses are an upfront cost. To compare the two, we use net present value (NPV). NPV shows the present value of something you will receive in the future. The reason is that money available at the present time is worth more than the same amount in the future, due to its capability to increase. This gives us the opportunity to compare the value of different investments that gain its value at different times. I.e., invest in a solar farm or put the money in a bank account.

$$C \cdot \left(1 + \frac{p}{100}\right)^i = C_i \Rightarrow C = \frac{C_i}{\left(1 + \frac{p}{100}\right)^i}$$

C = The net present value
C_i = The value after i years
p = Percent discount rate
i = Number of years

The NPV of the solar farm equals the sum of NPV of the annual income, minus the investment of buying the solar farm.

$$NPV = -I_0 + \sum_{i=0}^{T-1} \frac{C_i}{\left(1 + \frac{p}{100}\right)^i}$$

Where I_0 is the initial investment and T is total number of years. This is a geometric series, and can therefore be written:

$$NPV = -I_0 + C_i \frac{1 - r^T}{1 - r}, \qquad \left(r = \frac{1}{1 + \frac{p}{100}}\right)$$

Inserting for I_0 using Eq. (2.2) and C_i using Eq. (2.1)

$$NPV = -H \cdot L \cdot K \cdot pp + Q \cdot \eta \cdot 0.001 \cdot ep \cdot \frac{1-r^T}{1-r} \tag{2.7}$$

A solar farm would need some maintenance e.g. changing panels when they stop working and cleaning the panels. We update the panel price per square meter (pp), to panel price per square meter included maintenance (ppm):

$$ppm = pp \cdot \left(1 + mc \cdot \frac{1-r^T}{1-r}\right)$$

mc = Annual maintenance cost
I insert this updated investment price into the objective function, Eq. (2.7).

$$NPV = -H \cdot L \cdot K \cdot ppm + (Q \cdot \eta \cdot 0.001 \cdot ep) \cdot \frac{1-r^T}{1-r} \tag{2.8}$$

This is the function deciding the NPV of the solar farm. If the NPV is greater than zero, we should invest in building the solar farm. We notice that most of the details are hidden within Q, given by Eq. (2.3). Among them, the dependence of the variables: tilt (β), interspacing between the rows (D), azimuth (γ_c), height of the rows (H), length of the rows (L) and number of rows (K).

2.4 Constraints

The design of the solar farm has some practical and theoretical constraints, e.g. in the process of installing and maintaining the solar farm. All the variables have a lower and upper bound on their possible values. The tilt of the panels is between 0° and 90°, and the azimuth is between −180° and 180°. In this thesis, we assume the piece of land, which the solar farm should be installed on, to be horizontal and rectangular. In our computational example, the size of the land is set to 20×40 m^2. Thus, the length of the rows has an upper bound at 40 m, and I set the lower bound to be 5 m.

I set the distance between each row to be no less than 0.2 m, in order to move in between the rows, i.e. when performing maintenance on the panels. I set the upper bound to be 2 m. The height of each row is set to be between 0.2 and 2 m.

The panels are installed 0.5 m above the ground, to avoid dust and other objects blocking the panels. For maintenance reasons and for the design to be a solid construction, an upper bound for the combined height is set to be 2 m above ground:

$$0.5 + H \cdot \sin(\beta) \leq 2 \tag{2.9}$$

The piece of land has a width of 20 m. For K rows to fit into the area, we need the following constraint:

$$K \cdot H \cdot \cos(\beta) + (K-1) \cdot D \leq 20 \tag{2.10}$$

2.5 Solving the optimization problem

Optimization is to find the best possible solution, under given constraints. The general mathematical model for an optimization problem is:

$$\begin{cases} Max & f(\mathbf{x}) \\ s.t. & \mathbf{g}(\mathbf{x}) \leq 0 \\ & \\ \forall \mathbf{x} \in & \mathbb{R}^m \times \mathbb{Z}^n \end{cases}$$

Where m and n are numbers of continuous and integer variables. If we have both continuous and integer variables, it is called a mixed integer problem. $f(\mathbf{x})$ is an objective function, $\mathbf{g}(\mathbf{x}) \leq 0$ is a vector of constraints and $\mathbf{x}$ is the variables. These variables are what needs to be optimized. In our case, the variables are the component configuration of the PV farm.

In our problem, the net present value of the PV farm Eq. (2.8) is the objective function. The constraints on the solar farm are the practical restrictions of the installation and the maintenance of the farm, discussed in sec 2.3. The configurations fulfilling all the inequalities are feasible solutions. The feasible solution with the highest objective function value is the optimal solution.

The objective function and the constraints are non-linear functions, and there are both continuous and integer variables. An algorithm that solves general mixed integer non-linear optimization problems in polynomial time, does not exist. However, there exist some iterative search methods, to search for an optimal solution to these problems.

The non-linear optimization problems may have local maximum points. Iterative methods may converge into one of these points, and not the global optimal solution. The iterative method needs to avoid these local maxima as often as possible. To be a good iterative method for solving a non-linear optimization problem, it needs to balance two abilities. A fast running time and the ability to avoid converging into the local maximum points.

For continuous variables, we may obtain gradient information which might be of great help when deciding on the search direction. However, no such information is available for integer variables. In the PV farm optimization problem, at least one of the variables is an integer, the number of rows in the PV farm. Hence all such gradient algorithms can't be selected.

Objective function

The optimization program contains a function to be maximized, called the objective function. In this thesis, the net present value of the solar farm over a 25 year period, will be the objective function. Most manufacturers offer a 25 years warranty on the solar panels.

The objective function is dependent on 6 variables. The variables are: tilt (β), interspacing between rows (D), azimuth (γ_c), height of row (H), length of row (L) and number of rows (K). At the end of the day, we are convinced that PV-generated electricity will never succeed unless it becomes profitable. Thus, it is crucial to configure your PV farm to maximize profit.

Constraints

There are some constraints on the objective function, e.g. we have a limited area. These constraints are given as inequalities using the variables in the optimization problem. These equations make up a set, called the feasible set of solutions. In other words, it will give us all the possible designs for a PV plant. In our case, the constraints are non-linear, which complicates solving the optimization problem.

2.5.1 Genetic algorithm

The genetic algorithm (GA) is a search algorithm inspired by natural selection and evolution. It is an iterative method, that can be used to solve complex problems. The genetic algorithm sample at random a set of size N, from the feasible set of solutions. Each sample, called an individual, represents a configuration of the solar farm. This sample of solutions is called the population. Then the whole population is put into the objective function, to get a fitness value corresponding to each individual. The individuals with the highest value are called the elite population.

There are two ways to create new individuals. The elite population is crossed, to make a new individual, a combination of individuals. Depending on the algorithm, some of the population not in the elite population can also be crossed to ensure genetic diversity. Another way to create a new individual is by mutation. By mutating the individuals, you change a small part of the genotype. When mutating an individual, you ensure genetic diversity.

The reason you want to keep the diversity is to ensure you converge to global maximum, and not end up in a local maximum point. Each iteration in the algorithm, construct a new generation. The algorithm runs until it reaches a stopping criterion.

Algorithm 1 Basic Genetic Algorithm

```
Initialisation:
Population ← Randomly created population
Iterations:
repeat
    Parental Generation ← Population
    Calculate fitness of each individual
    repeat
        Select individuals from parental generation
        Reproduce into new population
    until Population is full
until Stopping criteria met
```

A good algorithm is converging fast and converging towards the optimal solution. We could easily ensure either a fast running time or an algorithm with a high possibility of ending up in an optimal solution. The challenge is to balance the two abilities.

Chapter 3

Case Studies

In this chapter, we test our model for three specific locations. We will see characteristics of the different variables, and their influence on the NPV. The altitude of the Sun is an important parameter in the calculation of the design of the solar farm. The altitude is dependent on the latitude of the given location. We have been able to obtain data from Kumasi in Ghana, Bari in Italy and Bergen in Norway. The three chosen locations have latitudes 6° N, 41° N and 60° N respectively, spread evenly for a good coverage of different possible cases.

To calculate the optimal design of a PV farm, we need to know the amount of irradiance for the given location. The amount of irradiance can be measured with some measuring equipment. These measurements will be used to calculate the daily average of irradiance. The irradiance will be used in the algorithm to calculate power produced by the given configuration. We can then, by some estimated price parameters, find the NPV of the PV farm. In all cases, we compute the result for a fictive solar plant of size $40 \times 20\ m^2$.

3.1 Case Ghana

In this section, we will use Kumasi in Ghana as our location for the calculations. To calculate the optimal design of a PV farm, we need to know the amount of irradiance in Kumasi.

3.1.1 Irradiance measurements

We have obtained data from measurements of the irradiance in Kumasi [?]. The shortwave irradiance is measured by two pyranometers. One of them is facing upwards, for measuring the sum of the beam and the diffuse irradiance. The other pyranometer is facing downwards, for measuring reflected irradiance from the ground. The pyranometers measure the irradiance with a wavelength between 305 and 2800 nm.

Far infrared radiation is measured by two pyrgeometers. One for measuring the radiation from the sky, the other for measuring the radiation reflected from the ground. The range of the pyrgeometer is 5 000 to 50 000 nm. The data were sampled every 10 minutes over a period of two years.

The data file contained the following parameters, measured on a horizontal surface. Shortwave irradiance measured upwards (CM3_up), shortwave irradiance measured downwards (CM3_down), longwave irradiance measured upwards (CG3_up), longwave irradiance measured downwards (CG3_down) and temperature. Irradiance is given in (W/m^2). A quick look at the data file revealed some obvious errors, below we will explain them and discuss how to circumvent the problem.

Longwave irradiance

The intensity of longwave irradiance is measured in Kumasi. We have measurements from every 10th minute over two years.

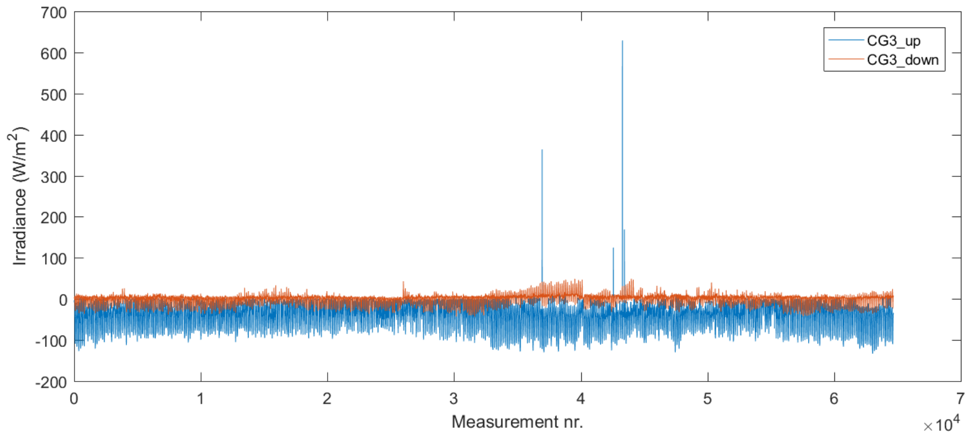


Figure 3.1: Longwave irradiance from the sky and from the ground.

The longwave irradiance coming from the sky is about zero W/m^2 during the night and reported to reach -100 W/m^2 during the day. While the reflected longwave irradiance, is measured to reach about -20 W/m^2 during the day and 10 W/m^2 during the night. This does not make sense, as the measurements should at least be positive, it is probably due to some data errors or mistakes in the installation.

The longwave irradiance has a wavelength between 5 000 and 50 000 nm. However, the amount of energy the irradiance contains w.r.t. the wavelength, and which wavelength that can be converted to electricity by silicon is shown below.

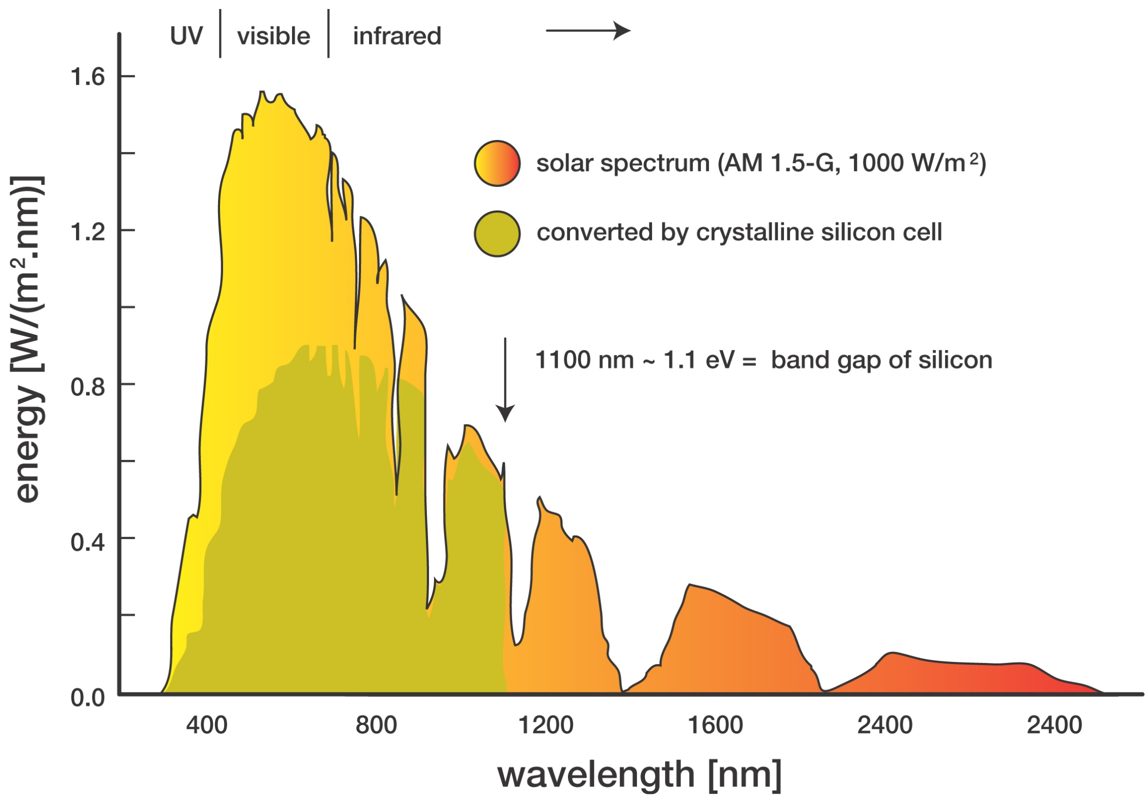


Figure 3.2: Energy in the solar spectrum [8].

The "Energy in the solar spectrum" chart, Figure 3.2, shows the amount of energy reaching the surface of the Earth from the different wavelengths of solar irradiance. This particular result is from a laboratory, using standard test conditions of 1 000 W/m^2 of irradiance and an air mass ratio of 1.5 [19]. Figure 3.2 also shows which wavelengths that theoretically can be converted into electricity by a crystalline silicon cell.

The longwave irradiance has wavelengths of 5 000 nm and longer, but a crystalline silicon cell can only convert irradiance with wavelengths shorter than about 1 100 nm into electricity. We can then conclude that the longwave irradiance doesn't have enough energy to produce electricity in a PV panel. We are only interested in the irradiance with the potential to produce energy. Therefore, we will focus only on the shortwave irradiance.

Shortwave irradiance

Shortwave irradiance will be referred to as irradiance. Irradiance measured upwards (CM3_up) is the sum of the beam and the diffuse irradiance ($I_b + I_d$), and irradiance measured downwards (CM3_down) is reflected irradiance(I_r). The dataset of measured irradiance from Kumasi had some shortage, some of the data were missing. The plot of all the available irradiance would look like this:

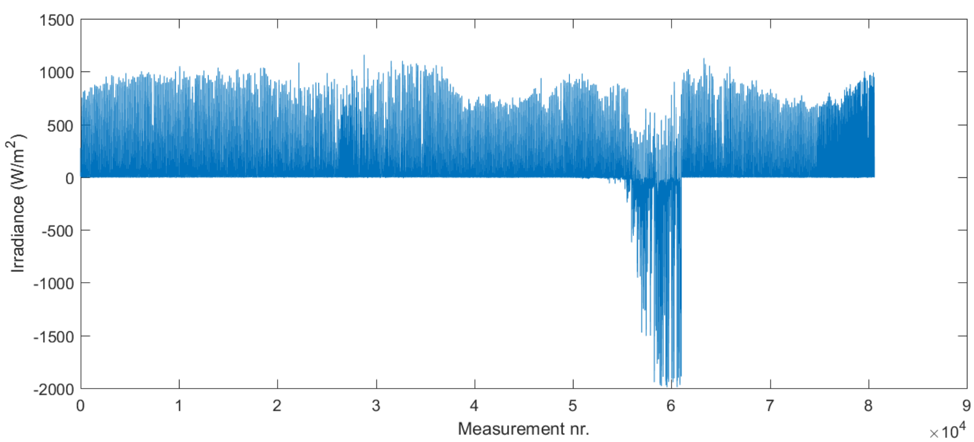


Figure 3.3: Plot of all irradiance data

As negative values are obviously non-physical, something wrong is happening around measurement number $6 \cdot 10^4$. This must be data errors. To avoid the corrupted data, we limit ourselves to the first year of measurements, in other words, the first 52 560 measurements.

Next, I remove all the days that are incomplete from the file. Then the plot of irradiance and reflected irradiance looks like this:

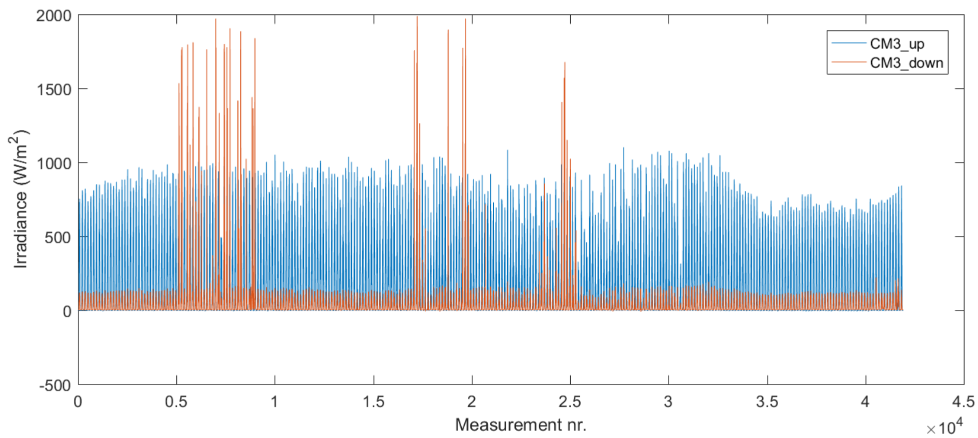


Figure 3.4: Shortwave irradiance and reflected irradiance.

Still, as we can see from Figure 3.4, there is something wrong with the measurements. The reflected irradiance can never exceed the irradiance. We must filter the values of reflected irradiance exceeding the irradiance.

We have processed the raw data the following ways, in order to find the irradiance through the day:
- Average of all the remaining days from the first year.
- Average for each month, the first year, and then the average of each month.
- Replace the removed or missing days, with days in the same month from the following year, then calculate the average.

Create a representative day

We will focus on the shortwave irradiance measured upwards. If we plot all the days in one plot and the average of all the days, it would look like this:

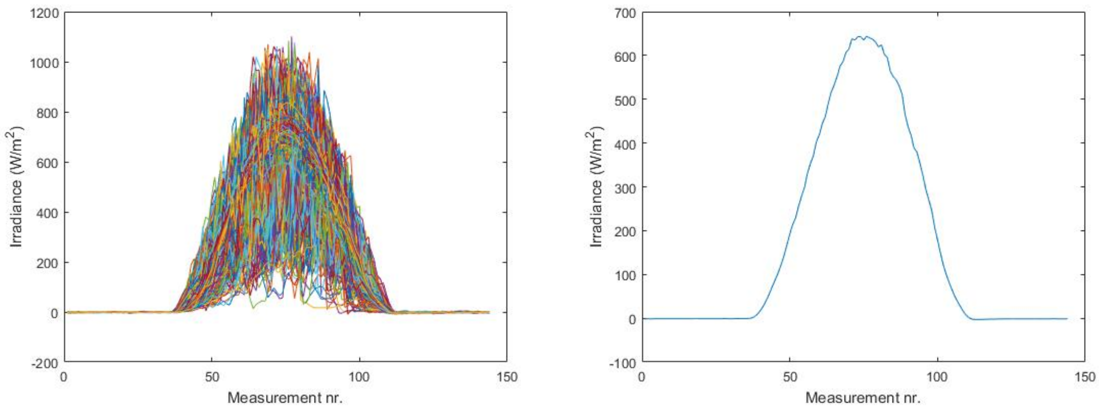
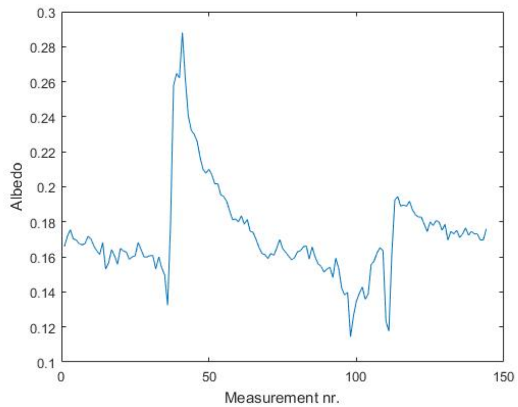


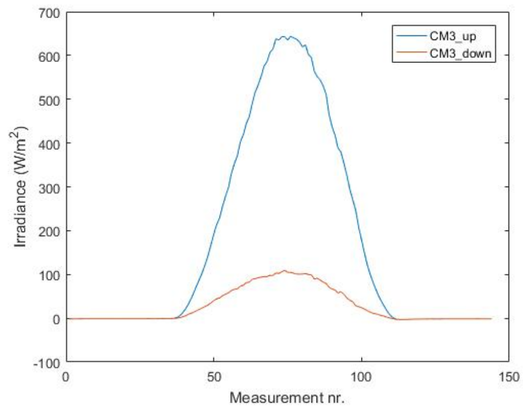
Figure 3.5: Plot of all days in left frame, and plot of the mean value in right frame.

The average of all the remaining measurements from the first year is 173.28 W/m^2 of shortwave irradiance. The graph in the right frame in Figure 3.5 looks plausible. No negative values and no values are higher than we would expect. However, due to lack of consistency in the data, there is only one complete day of measurements from November. Thus, the days sampled in the left frame of Figure 3.5, is not a random sample. If November has some deviation from the other months our result might contain a minor inaccuracy.

For the reflected irradiance, we filter out the data with values above 30% of the irradiance and values below zero. The reason for this is because some of the values are too high. The reflection cannot be higher than the irradiance. Hence, we replace it with irradiance multiplied by the albedo. The irradiance measurements are already filtrated and look like they are correct.



(a) The mean value of albedo



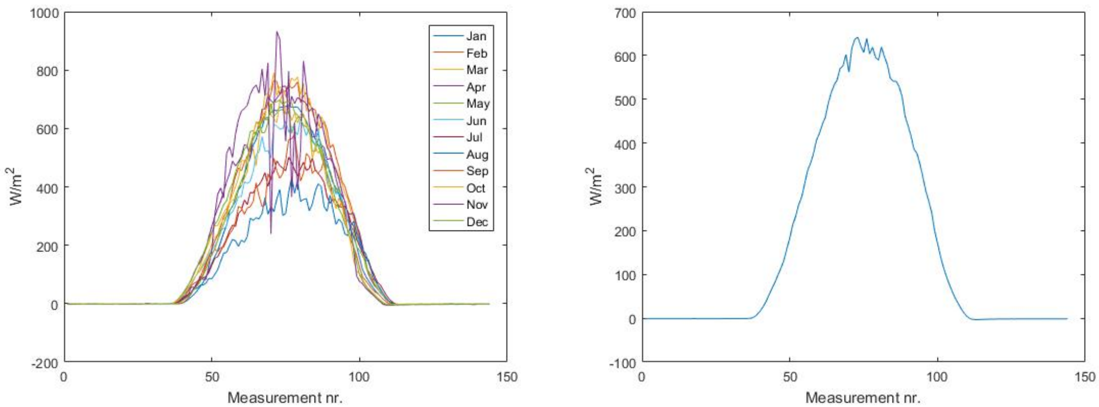
(b) Irradiance and Reflected irradiance

Figure 3.6: Results from the mean values of the first year of measurements

Figure 3.6b shows the result of average irradiance, found by removing incomplete days. And the result of average reflected irradiance, found by replacing measurements containing errors with approximations.

Create an average day from each month

If we calculate the average irradiance of each month, it doesn't matter how many days of data each month contains. Every month will count equally on the average, unlike if we calculate the average of all days, then the months with many days will make a greater influence on the average. However, if a month has few days, we will get low precision in the given months average.



(a) Plot of mean for all individual months (b) Plot of the mean of the different months

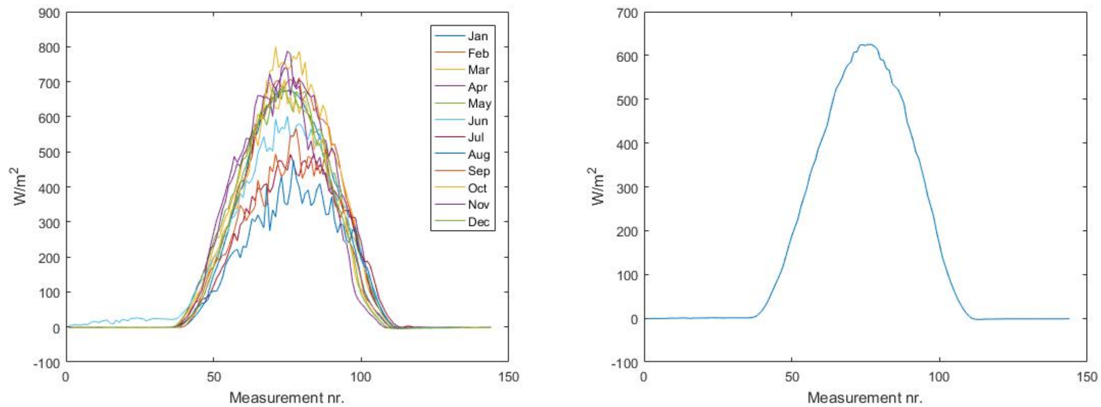
Figure 3.7: Plot of data divided in different months

Average irradiance is $169.89 W/m^2$. The Data from November is an average of only one day. This gives a significant impact on the result. Thus, this result is not representative for the given year.

Replace with data from the year after

We have measurements from two years. It is possible to combine data from both years. If we put data from the second year into the data from the first year, to fill some of the sparse months in the dataset. Then we will get a more thorough dataset for each month.

We sort the data by months, filter out the incomplete days, and filter out the days containing measurements under $-10 W/m^2$. We then take the average of the days in each month, resulting in a representative day for each month. We can also take the average of the months. Resulting in the following plots:



(a) Plot of mean for all individual months (b) Plot of the mean of the different months

Figure 3.8: Plots with data from two years

This gives an average of $167.47W/m^2$, that is lower than the two other calculations. The reason for this decrease in irradiance could be due to dust on the measuring equipment, which would cause an error in the measurements, or simply just less sunny weather the second year.

Satellite data

Due to the incomplete dataset from Kumasi, the calculations would give a result that is not accurate for the given area. However, there exists another method for measuring the irradiance. Based on satellite data, it is possible to calculate irradiance on the ground. The Photovoltaic Geographical Information System (PVGIS) [33] is an interactive map, showing estimates of solar irradiance for any location in Europe and Africa. The irradiance data is calculated by The Satellite Application Facility on Climate Monitoring (CM-SAF) [13]. The calculations are based on data from 12 years of measurements. After a validation of the data using 20 different locations, has shown that the overall mean bias error is about 2% [13]. The data is given in daily average per month. The chosen location is on the KNUST campus in Kumasi, Ghana with latitude 6.67 North and longitude 1.56 West. The dataset includes both beam and diffuse irradiance. The measurements are given every 15 minutes. The dataset does not include reflected irradiance. It will be calculated using the albedo. The albedo depends on the ground surface. In this case, I assume the albedo to be 0.2, i.e. 20% of the irradiance gets reflected from the ground.

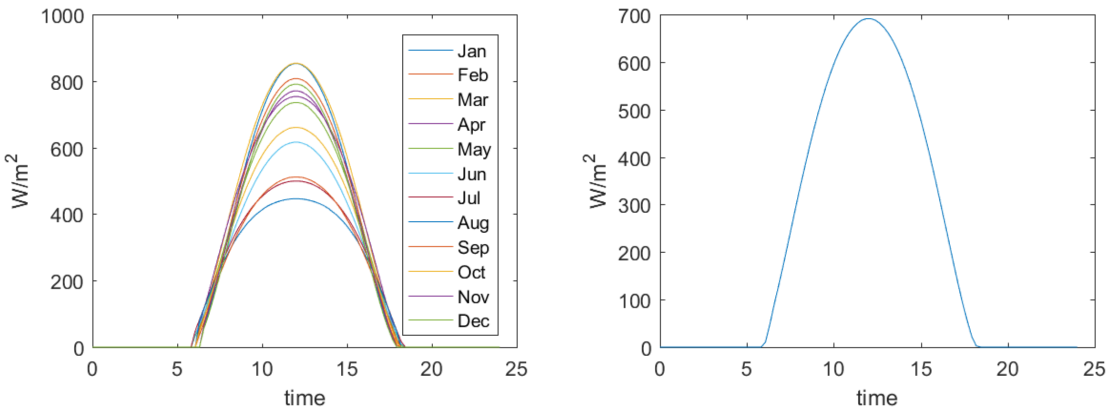


Figure 3.9: The daily average per month and the average of the months

These calculations give an average irradiance of 213.3 W/m^2, which equals 5.11 $KWh/m^2/day$. If we compare the average from the different months to the average of the ground measured data, we can see if there are any that stand out.

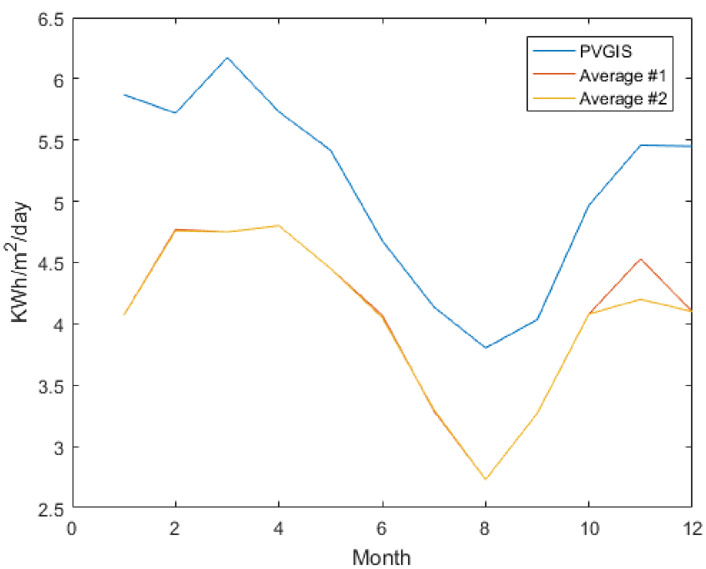


Figure 3.10: Comparison of the different monthly averages.

Average #1 is average of available data from 4.Feb.2012 to 3.Feb.2013. Average #2 is where we filled in the blanks with data from the following year.

There are no months that stand out and make a huge impact on the result. The satellite data is generally higher than the ground measurements. We will also compare our result by the time of the day.

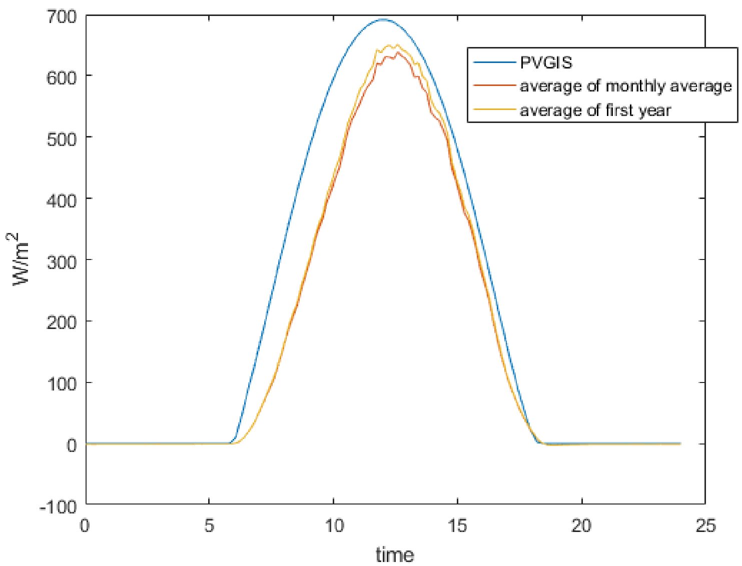


Figure 3.11: Comparison of the different daily averages.

This plot reveals that the measured irradiance is much lower than PVGIS before noon, while it is only slightly lower in the afternoon. This is most likely due to local conditions, such as shadow from a nearby object.

Conclusion on irradiance

The lesson learned from this exercise is that when theory meets reality things get muddled up. Instead of a simple readout of the data, substantial modifications were needed to filter out measurement errors. Various alternatives were tried to omit imperfect data and its influence on the final results. In short, the theory needs to be supplemented by common sense.

Average of the first year gives a nice and smooth graph, but it is only data from one year, and the data is incomplete. Average of each month gives an uneven graph, where one day gives too much impact on the average. Average of each month with data from both years is affected by the fact that data from the second year is significantly lower, possibly due to measurement errors. The satellite dataset is complete and covers 12 years of measurements, therefore it provides a representative representation of the irradiance in the given area. However, it does not include local conditions such as shadow from nearby objects. Thus, we assume in this thesis, that the location for our solar farm, does not receive any shadow from nearby objects.

3.1.2 Parameters for Kumasi

In addition to the variables, NPV is also dependent on some parameters. In this section, we will find estimations to the parameters needed to calculate the NPV.

Electricity price

In this case, we use Ghana as our location. We assume the electricity price is 0.2 USD per KWh, which is the feed-in tariff that has been publicly set for the Nzema solar power plant [28], a solar farm that was planned in Ghana. 0.2 USD $\approx$ 1.6 NOK.

Cost of initial investment

The initial investment for a solar farm includes:

Materials:
Modules and Inverters
Mechanical mounting equipment
Electrical mounting equipment
Operational monitoring equipment
Installation
Mechanical installation work
Electrical Installation, PV System (DC side)
Electrical installation work, network connection (AC side)
Other
Machine / Equipment / Tools
Piece of land

Multiconsult calculated these costs in the Enova report in 2013 [9]. The installed power of a solar farm is given in watt peak (Wp). Watt peak is the power output of the PV panels under standard test conditions. The standard test conditions are given by: 1 000 W/m^2 of irradiance, a temperature of 25° C and 1.5 air mass. For a 1 million Wp solar farm, Multiconsult estimated the total cost to be 12 million NOK, that equals 12 NOK/Wp. They used 250 Wp panels. Size $1.665m \cdot 0.991m = 1.65m^2$. Hence on this large scale, the total price per square meter was 1 818 NOK/m^2 in 2013.

The International Renewable Energy Agency (IRENA) has recorded the prices of some solar farms already built in Africa. They found the prices to vary between 1 and 2 USD/Wp [16]. In other words, the Enova estimate is acceptable (12 NOK $\approx$ 1.5 USD).

Cost of maintenance

Multiconsult estimates that the annual maintenance cost equals 2% of the initial investment [9]. We will use this estimate in our calculations.

Efficiency

The efficiency of the panel (η_{pv}) used in the Enova report is 15.15% [18]. We also have to account for energy loss in the inverter (η_{inv}), estimated to 98.2% [29], and other decreasing factors such as mismatch connections wiring, etc. (η_{other}) estimated to be 93% [26].

The efficiency is dependent of temperature, the panels are more efficient in cold temperatures. The standard test condition is $25°C$. The loss in efficiency, given by [6], is 0.45% per degree. In this case, the total efficiency (η) becomes.

$$\eta = \eta_{pv} \cdot \eta_{inv} \cdot \eta_{other} \cdot (1 - 0.0045(T - 25))$$

Where T is temperature in degree Celsius. The temperature in Ghana is given by [34].

Table 3.1: Monthly average temperatures (°C) in Ghana

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
27.03	29.46	29.97	29.55	28.58	27.13	26.09	25.69	26.22	27.09	27.57	26.89

Discount rate

The discount rate is set to 5%, like in the Enova report [9]. This is an expression of required return and risk of the project, where 5% shows a low risk of the investment.

Lifetime of PV panels

The efficiency of a solar panel decreases over the years. To obtain the maximum value from the solar farm, you should replace the panels after some years. Most manufacturers offer a 25 years warranty on the panels, that guarantees a minimum of 80% of the original efficiency after 25 years. Therefore, when we talk about the cost of a solar farm, we consider a 25 years period. After the 25 years, we expect to replace the panels. In this thesis, we assume the panels have 100% efficiency for 25 years.

Summary of cost estimates

Which variables should be optimized and which are predetermined? The variables: height (H), length (L), number of rows (K) and distance between the rows (D), decides the size of the farm. Tilt (β) and azimuth (γ_c) decides the direction of the panels. All these variables need to be optimized.

The parameters given are: price per square meter of PV farm (ppm), irradiance on a horizontal surface (I), efficiency (η), electricity price (ep), discount rate (p), lifetime of the panels (T) and latitude of the given location (ϕ).

We have now found an estimate of all the parameters we need and are ready to run the algorithm to find the optimal design of the PV farm.

The objective function Eq. (2.8), after inserting the estimated parameters:

$$NPV = -H \cdot L \cdot K \cdot 1818 \cdot (1 + 2\% \cdot \frac{1 - r^{25}}{1 - r}) + (Q \cdot \eta \cdot 0.001 \cdot 1.62) \cdot \frac{1 - r^{25}}{1 - r}$$

where Q is:

$$Q = H \cdot L \cdot (q_b + q_d + q_r + (K - 1) \cdot (q_b^{sh} + q_d^{sh} + q_r^{sh}))$$

3.1.3 Result in case Ghana

In this section, using estimated parameters in the algorithm, we find solutions and discuss the results. The piece of land used in this thesis is specified to 20×40 m^2.

Each variable has a lower and upper bound. These are set to be: $\beta = [0, 90]$, $\gamma_c = [-180, 180]$, $D = [0.2, 2]$, $H = [0.2, 2]$, $L = [5, 40]$, $K = [2, 20]$.

Running the genetic algorithm gives the solution 3 027 781 NOK. That is the net present value of the solar farm. In other words, since the value is positive, the investment would give a profit. This solution is obtained with the following variable values.

Tilt of the panels $\beta = 28.7°$ limited on the interval $[0, 90]$. It may look strange, that the optimal tilt is 28.7°. We are close to the equator, the latitude is 6.6°, which means the Sun's elevation angle is high. If we only had one row of panels, the optimal tilt would be a bit lower. When we have a small limited area, we also have a small number of rows of PV. When the tilt increases, each row occupies less space. That means you could eventually fit another row into the area, and that would increase the amount of power produced. It becomes clearer by plotting the NPV with the tilt as a variable, with values between 0 and 60, and the other variable values are fixed to the solution from the algorithm.

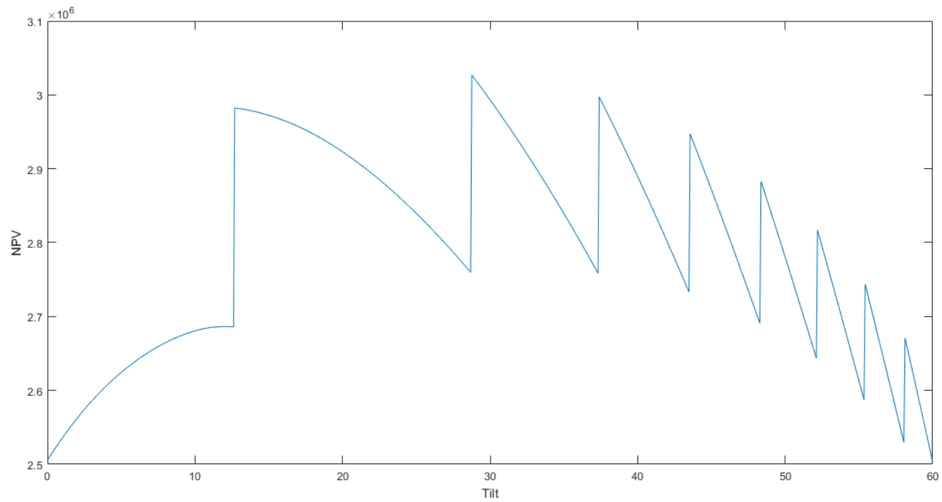


Figure 3.12: Plot of NPV as a function of tilt, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

From Figure 3.12 you see the NPV of the solar farm, where the tilt has values between 0° and 60° and the other variables are fixed to the values we got from the genetic algorithm, except K, the number of rows, which are maximized w.r.t. the constraint in Equation (2.4). Figure 3.12 shows a typical plot of a mixed integer/real function. It makes a jump every time the integer variable changes its value, making it piecewise continuous.

When the tilt is between 0° and 12°, you could fit 9 rows of PV panels. If you increase the tilt to 13°, you could fit another row of panels into the area, which would produce more electricity and increase the NPV. If you increase the tilt even higher, to 28.7°, you could fit 13 rows of panels. The tilt 28.7° gives a slightly higher NPV than when the tilt is 13°.

The distance between the rows of panels is 0.205 m. This variable is limited to the interval $[0.2, 2]$. In Ghana the Sun's elevation is high, that means the panels won't throw a big shadow onto the next row, so you don't need much space between the rows.

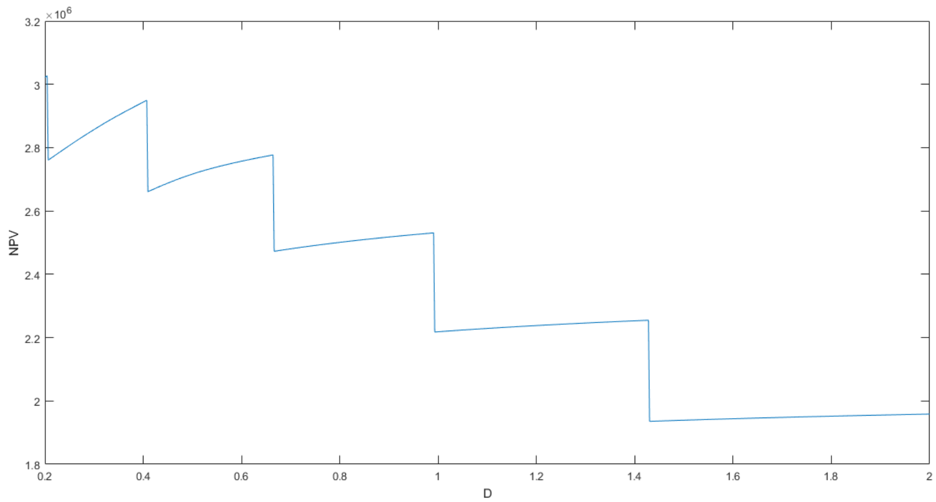


Figure 3.13: NPV as a function of D, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

It is clear from the plot that the lower values of D give the best solution, as expected. It also shows that the other variables are combined so that a new row of panels are introduced when the distance is just over 0.2 m.

The azimuth of the panels is $0.45°$ limited on the interval $[-180, 180]$. In this solution the azimuth is almost zero, that means the panels are facing south. The reason the panels face south is that Ghana is located north of the equator, such that the panels receive more irradiance from the south than from the north.

The height of the panels is 1.86 m limited on the interval [0.2, 2]. A higher value of H leads to a higher area of panels, but also more space occupied by each row. Each solar panel has a fixed size, but there is some variation from brand to brand, and some brands offer panels of different sizes. Therefore, the height is assumed to be a continuous variable.

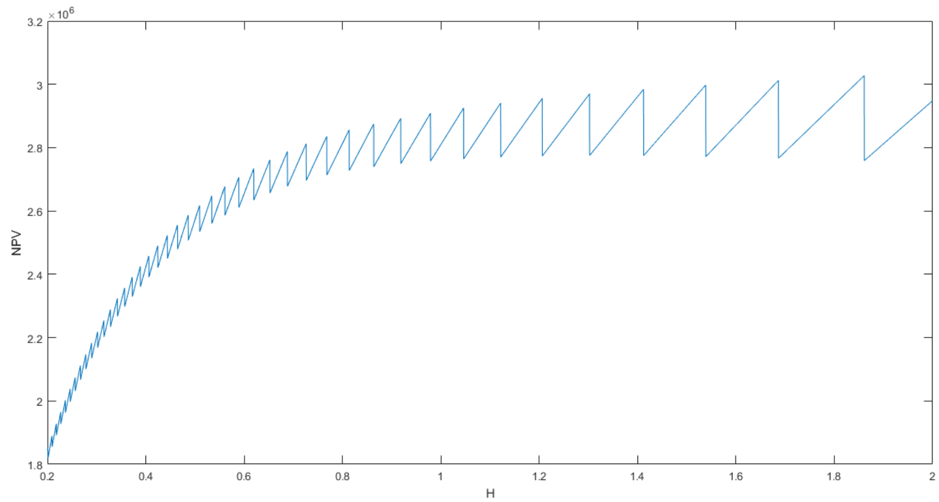


Figure 3.14: NPV as a function of H, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

Just like the tilt and distance between the rows, the value of H decides the number of rows that could fit into the area. If you increase the value of H, you will eventually run out of space and need to remove a row of panels, to fulfil the constraint. We can see from the plot that a new row is introduced when $H = 1.86$ m. If the last row was introduced closer to $H = 2$ m, we would find a better solution.

The number of rows is 11, limited on the interval [2, 20]. By increasing the number of rows, the other variables had to be changed to fulfil the constraints. By decreasing the number of rows, you would fulfil the constraints, but the solar farm would generate less electricity.

The length of the rows is 39.98 m limited on the interval [5, 40]. The variable should be maximized to the upper bound. This solution gave the value 39.98 m, 40 m would be a better solution. This shows that the genetic algorithm did not find the optimal solution. If we replace 39.98 with 40 into the objective function, the NPV is slightly increased.

By plotting the NPV with both β and H as variables, you can see some of the characteristics of the function.

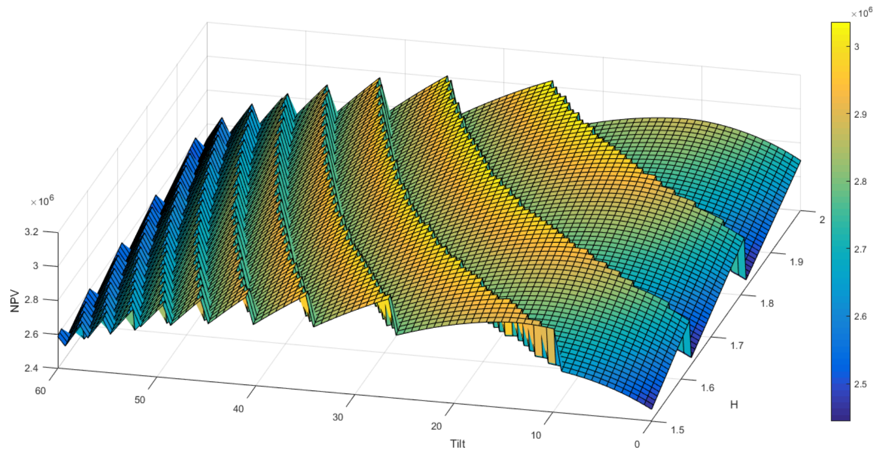


Figure 3.15: 3D plot of NPV, where H is between 1.5 and 2 and β is between 0 and 60, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

For an optimal NPV, each tweaking of H will change the optimal β.

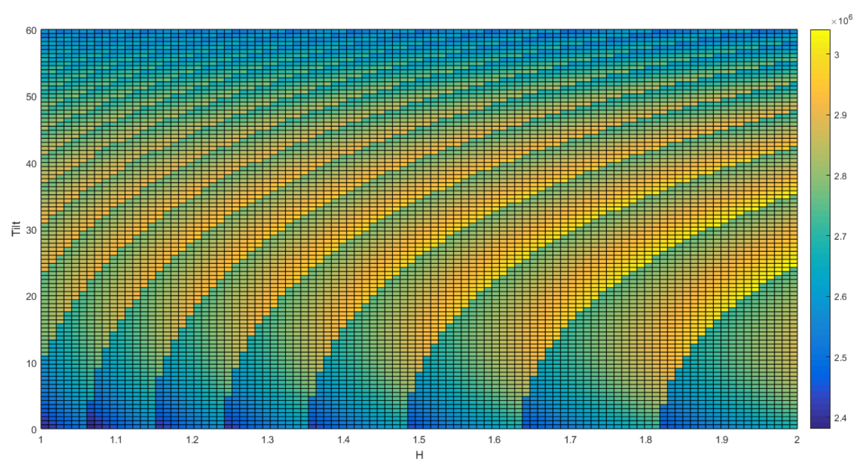


Figure 3.16: 2D plot of NPV, where H is between 1.5 and 2 and β is between 0 and 60, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

Tweaking H or β will also change the optimal D. Thus, H, β and D will have some relationship in the optimization of NPV, as one would expect.

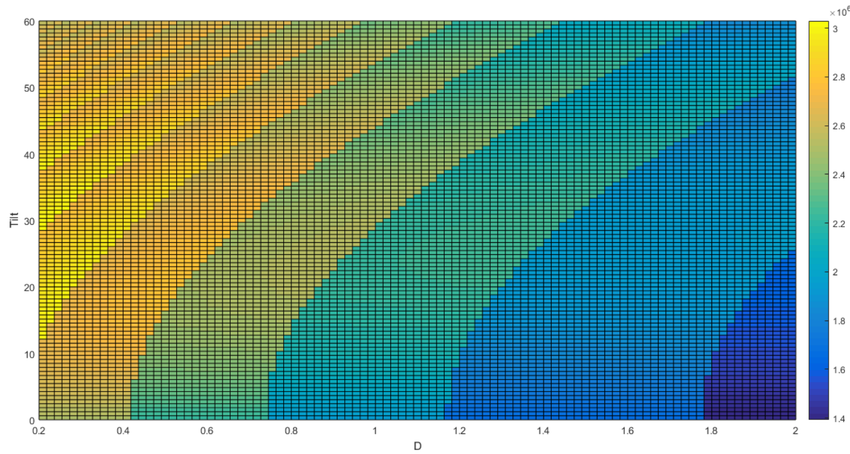


Figure 3.17: D between 0.2 and 2 and β between 0 and 60, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

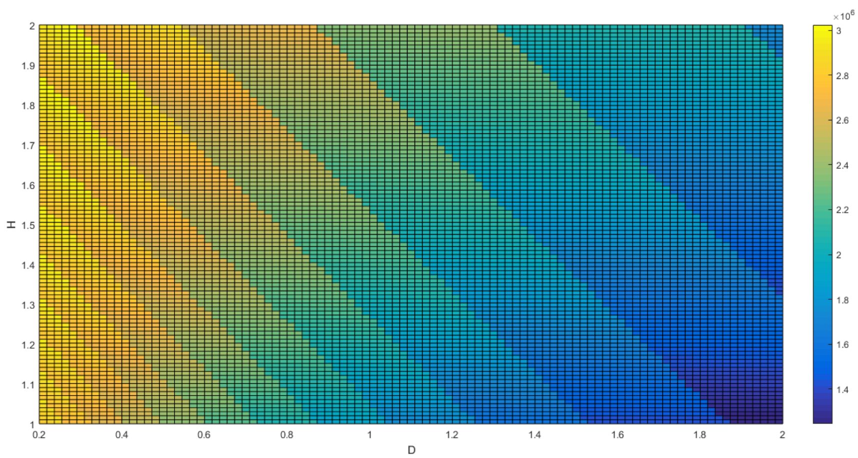


Figure 3.18: D between 0.2 and 2 and H between 1 and 2, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

It is clear from these plots that in this case, the highest NPV occurs when the distance between the rows are close to the lower bound. It is also clear that there are numerous local optima with NPV close to that of the global optimum, and it may be that small changes in our estimates of the parameters will make the global optimum jump to one of the nearby local optimum.

3.2 Case Italy

We will consider Bari in Italy as a possible location for a solar farm. The altitude of the Sun is dependent on the latitude of the location. Therefore, the Sun will most of the year have a lower altitude in Italy than Ghana. In Italy, some other parameters would also be different from what was used in Ghana. In this section, we will look at how this affects the design and NPV.

3.2.1 Irradiance

We could not get ahold of a complete dataset of irradiance measurements, measured from the ground in Italy. Just like in the case of Ghana, the dataset of irradiance obtained from PVGIS [33], will be used. This is not based on measurements from the ground, it is estimated measurements. The Measurements is obtained from calculations by CMSAF, on satellite images [13]. The location with latitude 41 and longitude 17, is chosen for this case. This location is located right outside Bari, in the southern part of Italy. The dataset is an average of 12 years of data, and it is a reliable and complete dataset. The dataset includes both beam and diffuse irradiance. The measurements are given every 15 minutes. The dataset does not include reflected irradiance. It will be calculated using the albedo. The albedo depends on the ground surface. In this case, I assume albedo to be 0.2, i.e. 20% of the irradiance gets reflected from the ground.

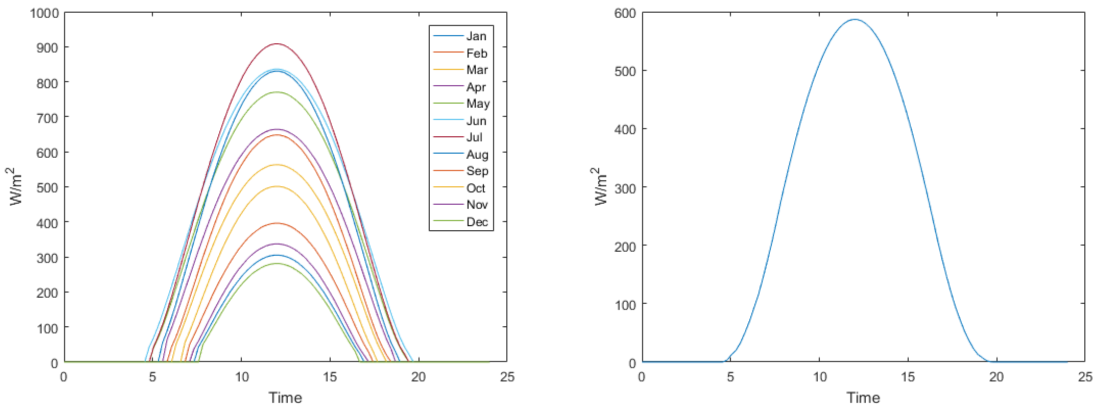


Figure 3.19: The daily average per month and the average of the months

Compared to the irradiance in Ghana, Italy experience a greater seasonal variation in both day length and intensity of the irradiance. On average Italy receives a weaker irradiance.

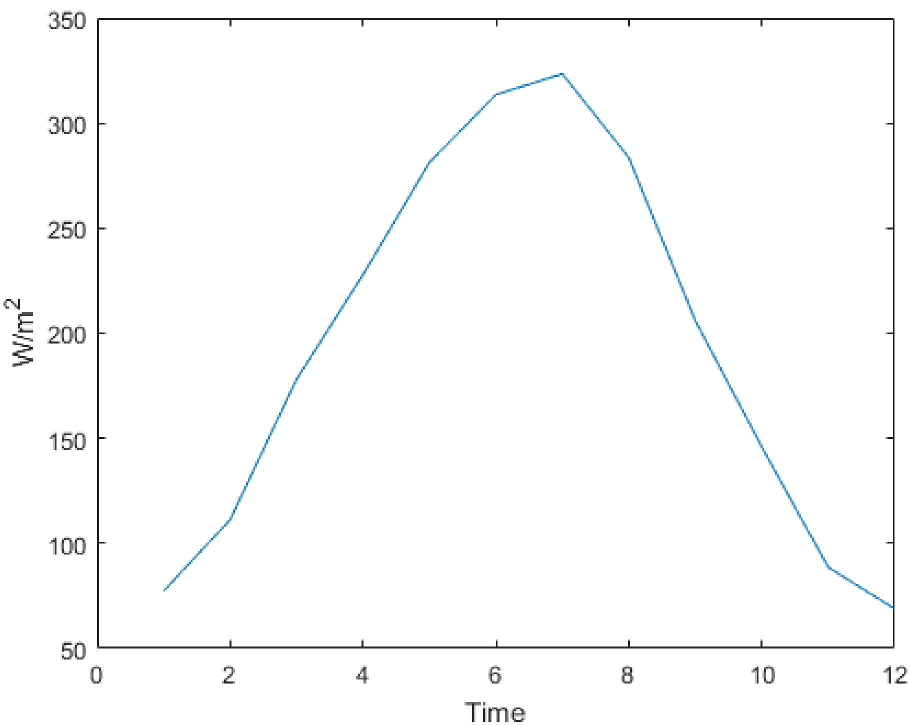


Figure 3.20: Average irradiance for the different months.

Italy receives the most irradiance during the summer months June and July and least irradiance in January and December.

3.2.2 Cost estimates

The installation investment cost in Italy is approximated to 1 818 NOK/m^2 [9], this is within ±10% of an estimation by IRENA in 2017 [17] and by World Energy Council in 2016 [36]. The annual maintenance cost is assumed to be 2% of the investment cost. The electricity price parameter is 0.148 EUR/KWh ≈ 1.4 NOK/KWh [10]. The discount rate is set to 5%, like in the case of Ghana.

The efficiency of the solar farm, is not the same in Italy as in Ghana, due to different temperatures. The PV panels have an efficiency of 15.15%, the efficiency of the inverter is 98.2% and the efficiency of other decreasing factors such as mismatch connections wiring, etc. is estimated to be 93%.

$$\eta = \eta_{pv} \cdot \eta_{inv} \cdot \eta_{other} \cdot (1 - 0.0045(T - 25))$$

The average temperature for the different months in Italy is given by [34].

Table 3.2: Monthly average temperatures (°C) in Italy

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
4.25	4.62	7.55	10.67	15.11	19.18	21.69	21.80	17.85	13.73	9.15	5.21

3.2.3 Result in case Italy

Running the algorithm in case Italy, the solution from GA is 2 842 736 NOK. A positive NPV, this means we should invest in the solar farm. The variable values found to get this result are discussed below.

The optimal tilt of the panels $\beta = 58.02$, limited on the interval [0, 90].

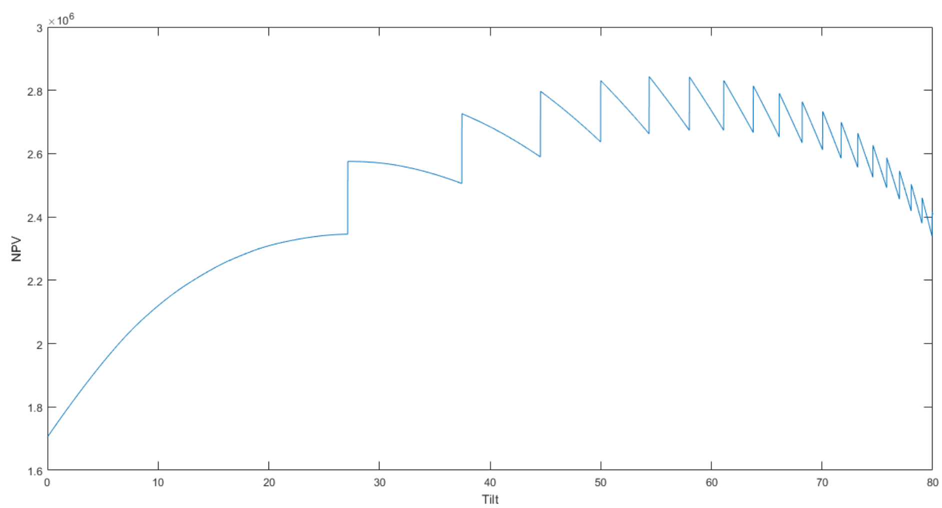


Figure 3.21: NPV with values of β between 0 and 80, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

From Figure 3.21 you can see the NPV from different values of tilt, with K set as the maximum value satisfying the constraints and the other variables fixed to the values given by the algorithm. On the interval where highest values occur, the NPV has a lot of local maximum points located close to each other.

The distance between the rows is 0.423 m. Limited on the interval [0.2, 2].

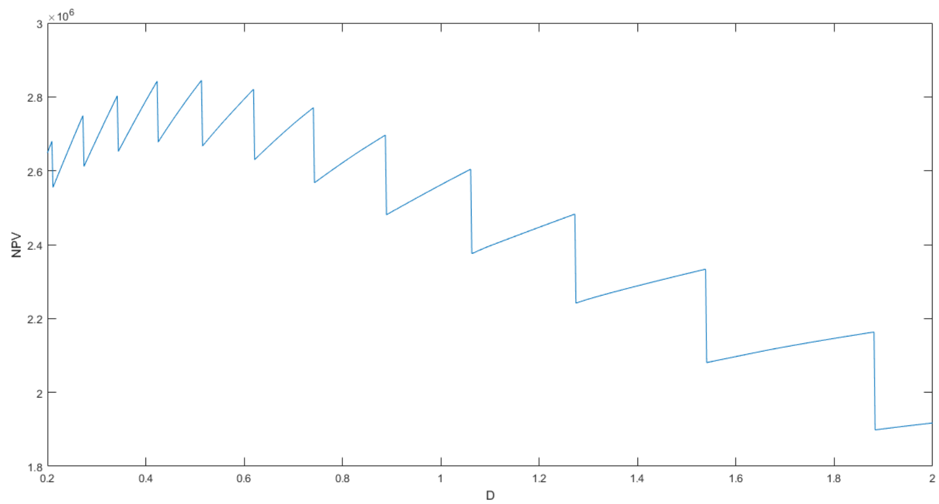


Figure 3.22: NPV with values of D between 0.2 and 2, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

In Ghana we wanted the interspacing to be as small as possible, but in Italy we want a bit more space between the rows. The reason for the increased space is because the elevation angle of the Sun is lower. Leading to a larger optimal tilt of the panels, which creates a larger shaded area. You can see from Figure 3.22 that the NPV is lower when $D \approx 0.2$m than when $D \approx 0.4$ m.

The azimuth of the panels is 0.32°, limited on the interval $[-180, 180]$. The panels are tilted towards the south, as expected. Hight of the panel 1.611 m limited on the interval $[0.2, 2]$.

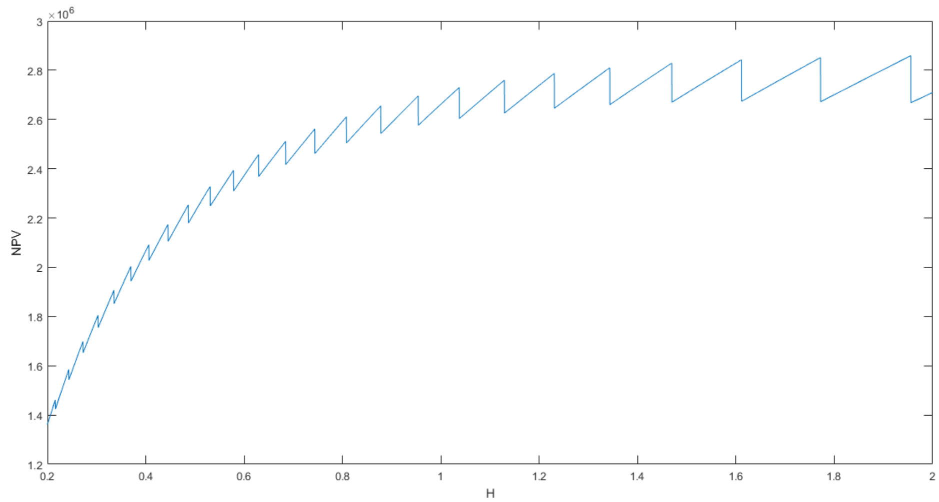


Figure 3.23: NPV with values of H between 0.2 and 2, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

We can see from Figure 3.23 that the highest NPV occurs when H is 1.955 m, not 1.611 as the algorithm found. From this observation, we can conclude that the genetic algorithm did not find the optimal solution to the problem, but a local maximum value close to the optimal solution.

The number of rows is 16 limited on the interval [2, 20]. This is the maximum value satisfying the maximum width constraint. Length of solar panel 39.999 limited on the interval [5, 40].

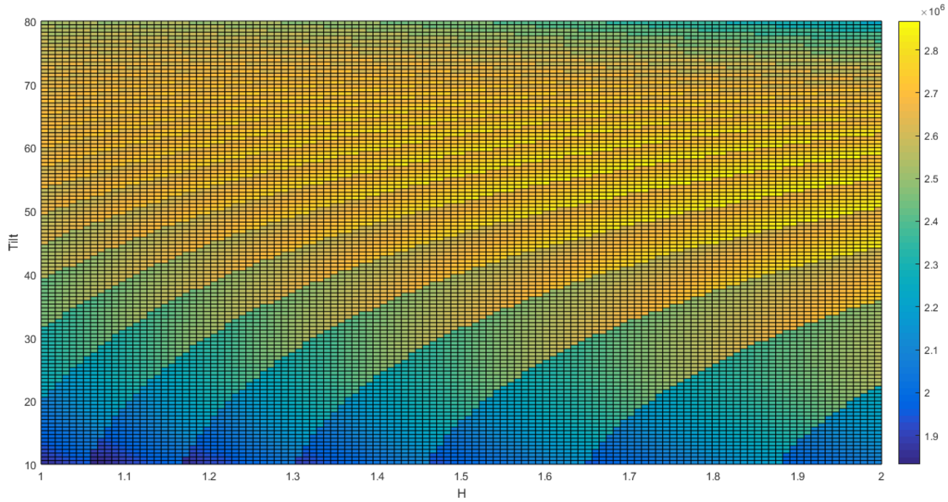


Figure 3.24: H from 1 to 2 and β between 10 and 80, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

3.3 Case Bergen

In this section, we will use Bergen as the location for the solar farm. Bergen is located at 66.3° north of the equator. The Sun has most of the year a lower altitude in Bergen than in Ghana and Italy. We will look at how this affects the results.

3.3.1 Irradiance

The dataset of irradiance is obtained from Geophysical Institute (UoB) [11]. The given data is measurements from 1.1.2004 to 1.1.2017, by observation site Florida in Bergen. The dataset is obtained from ground measurements, which are sampled every 10th minute. The dataset had some shortage, some measurements were missing and some were wrong. All the days lacking some measurements will be removed. The days containing measurements over 3 000 W/m^2 and the days where no measurements were under 200 W/m^2, will also be removed.

The given dataset includes the sum of the beam and the diffuse irradiance. The result from given dataset is represented in Figure 3.25.

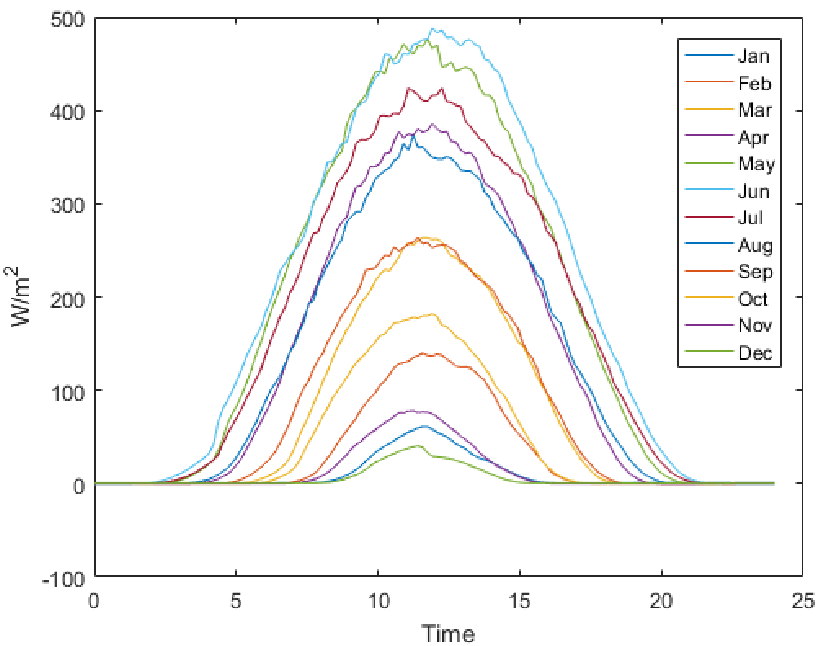


Figure 3.25: The irradiance in Bergen, from the different months.

From Figure 3.25 we can see the seasonal change of both the irradiance and the length of the day. In June the Sun is up for about 20 hours, while in December the Sun is barely over the horizon. The seasonal change in the irradiance becomes more clear when I plot the average irradiance from the different months.

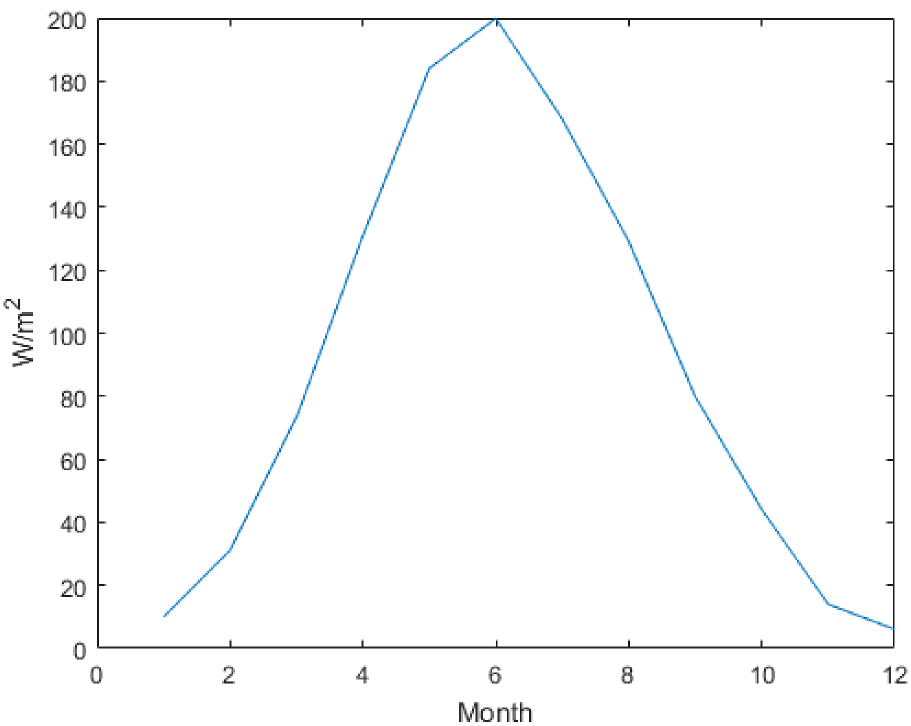


Figure 3.26: Average irradiance of the whole day, for the different months.

As expected, Bergen receives the most irradiance during May, June and July. In January, November and December, Bergen does not receive much irradiance due to the Sun's low elevation angle.

The irradiance data from Bergen does not include diffuse irradiance. It is only a measure of the beam and diffuse irradiance combined. We assume that 60% of the annual irradiance on a horizontal plane in Bergen is diffuse irradiance [31]. The dataset also does not include reflected irradiance. It will be calculated using the albedo. I assume albedo to be 0.2, i.e. 20% of the irradiance gets reflected from the ground.

3.3.2 Cost estimates

The investment price we found in case Ghana was based on calculations from Norway. In this case, the same estimate will be used, given by Enova to be 1 818 NOK/m^2 [9]. The annual maintenance cost is assumed to be 2% of the investment cost. The electricity price in Norway is 0.07 EUR/KWh ≈ 0.665 NOK/KWh, given by [10]. The discount rate is set to 5%, like in the other cases.

The efficiency of the solar farm, is not the same in all the locations, due to different temperatures. The PV panels have an efficiency of 15.15%, the efficiency of the inverter is 98.2% and the efficiency of the other decreasing factors such as mismatch connections wiring, etc. is estimated to be 93%.

$$\eta = \eta_{pv} \cdot \eta_{inv} \cdot \eta_{other} \cdot (1 - 0.0045(T - 25))$$

The temperature in Bergen is given by [37].

Table 3.3: Monthly average temperatures (°C) in Norway.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
−2.7	−3.1	−2.1	0.2	4.6	7.7	9.2	9.8	6.3	3.6	−0.3	−1.9

3.3.3 Result in case Bergen

The optimal value from GA is −1 301 NOK. The value is negative, that means we should not invest in a solar farm in Bergen. At least not for the profit of selling the electricity. We could build the solar farm if we got subsidies or the solar farm is built for other purposes than to make a profit of it.

If we change the electricity price to 0.952 NOK/KWh, which is what households would have paid for the electricity [32]. Including grid rent and taxes. The solution from the genetic algorithm, with the updated electricity price, is 5 057 NOK, a positive NPV. That means we might consider investing in a solar farm for private use. Given that you use all the energy you produce yourself, not to be dependent on getting the energy sold onto the grid.

We notice that it is not only less irradiance which gives us the negative NPV, the low price we get for selling the electricity is just as important.

In this case, the optimal tilt of the panels is 54.65°.

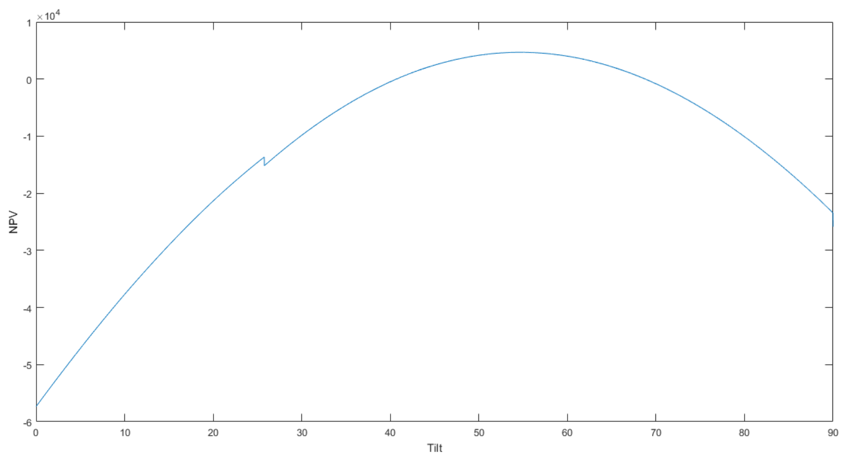


Figure 3.27: NPV for β between 0 and 90, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

When the tilt increases, each row will occupy less space. If the total space is reduced, such that it is possible to fit a new row, the NPV is changed and you can see a discontinuity in the graph. In case Ghana and Italy, the optimal solution and most of the local maximum points are found on the top of these discontinuous points. In Figure 3.27, this occurs only once, and it is not at the optimal solution.

In this case, the optimal distance between the rows $D = 2$ m. That is equal to the upper bound of this variable. Due to the low altitude of the Sun, every row will throw a longer shadow, resulting in longer optimal interspacing between the rows.

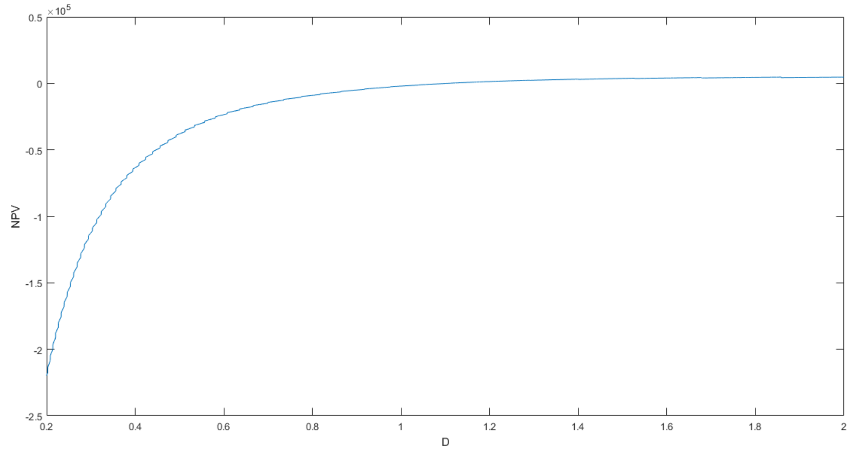


Figure 3.28: NPV for D between 0.2 and 2, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

This shows that the upper limit of D limits the value of the optimal solution. Therefore, in this case, the upper limit on D should be a higher value. By plotting the NPV where the upper bound on D is 4 m, we can see that the NPV could be increased.

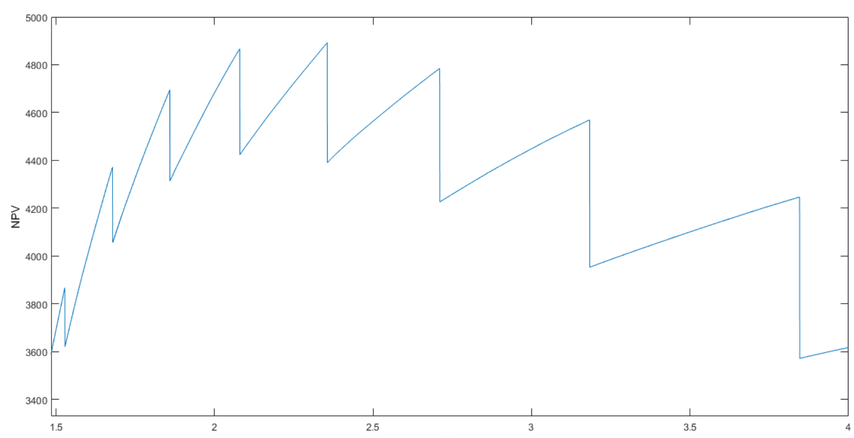


Figure 3.29: NPV for D between 1.5 and 4, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

The highest values occur when D is between 2 and 2.5 m. Thus, the interval of D should be changed for this case.

The height of the rows is 0.2221 m. This low value of H is, like the high value of D, due to the Sun's low elevation. A higher row would throw a longer shadow. With a height this low, the rows are able to have a high tilt which leads to high beam irradiance.

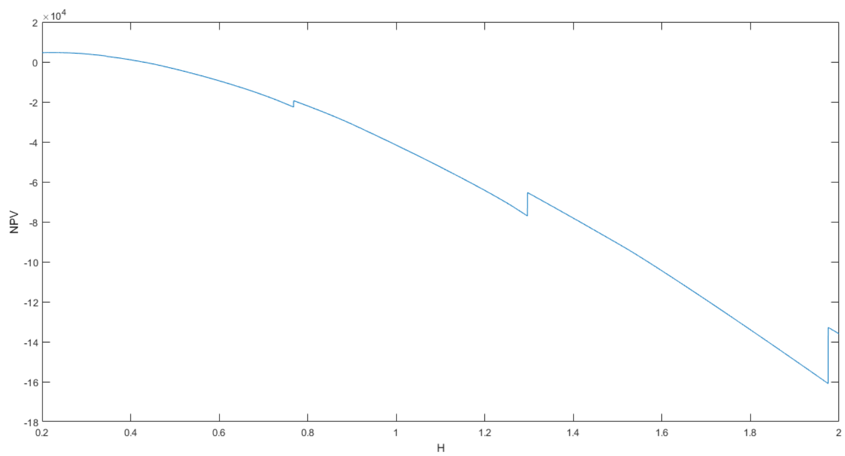


Figure 3.30: NPV for H between 0.2 and 2, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

The optimal value of H is close to the lower bound. A closer look at the values close to the lower bound, reveals that the NPV increases as H increases, under the lower bound.

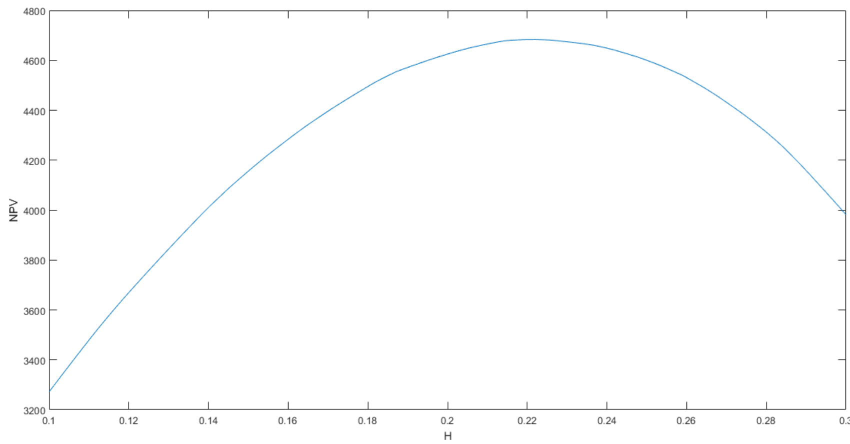


Figure 3.31: NPV for H between 0.1 and 0.3, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

Unlike D, which had its optimal value outside of the given bound, H has its optimal value between the bounds.

The optimal solution still gives the length of the rows equal 40 and azimuth almost at zero, in this case, azimuth is 1. With the low height of the rows, the number of rows still equals 10. That is the maximum value under the limited width constraint, given by Eq. (2.10).

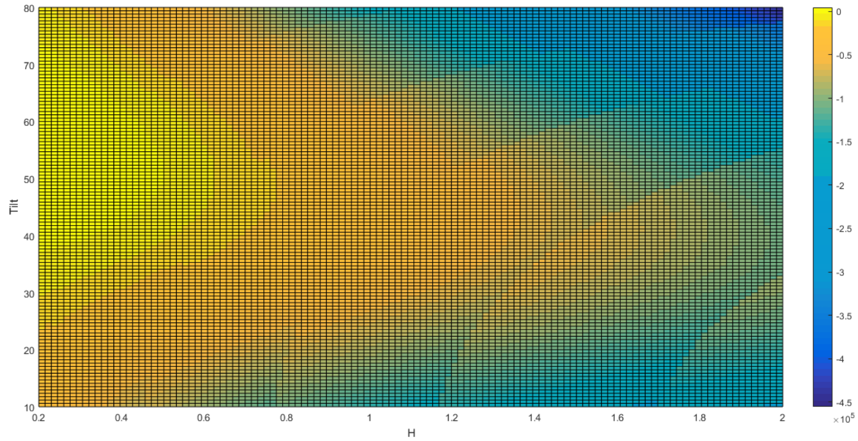


Figure 3.32: NPV for H from 0.2 to 2 and β between 20 and 80, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

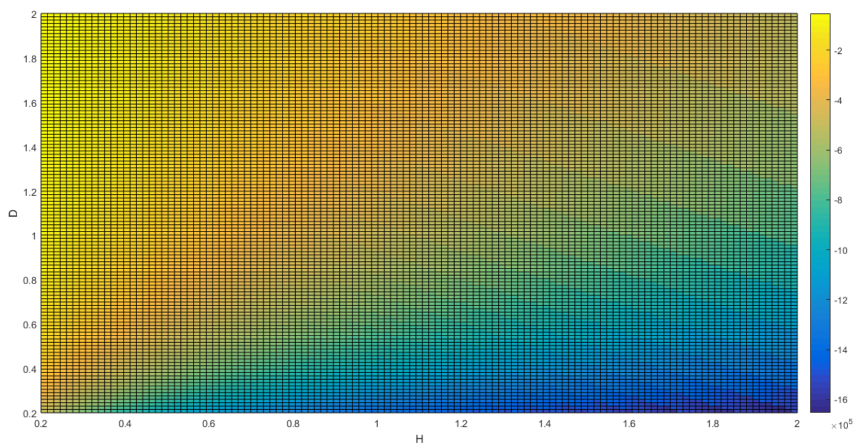


Figure 3.33: NPV for H from 0.2 to 2 and D between 0.2 and 2, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

In this chapter, we have seen the difficulty of optimizing the design of a solar farm. You can not find the best value for only a single variable, because when you alter a different variable, the value you find may no longer be optimal. Another solution could have appeared, that is better.

Table 3.4: The solutions found in this chapter:

	Kumasi, Ghana	Bari, Italy	Bergen, Norway
NPV	3 027 781 NOK	2 842 736 NOK	5 057 NOK
β	28.7°	58°	54.65°
D	0.205 m	0.432 m	2 m
γ_c	0.45°	0.32°	1°
H	1.86 m	1.611 m	0.222 m
K	11	16	10
L	39.98 m	39.99 m	40 m

Chapter 4

Analysis and discussion

In this chapter, we will analyse the algorithm, and try to improve its running time and accuracy. I use the built-in genetic algorithm function, in the Global Optimization Toolbox in MATLAB. We will also look at the fixed parameters we use, by analysing how a small change in the parameters will affect the solution.

4.1 Analysis of GA

In this section, we take a closer look at the genetic algorithm. We will look at how the algorithm handles crossover and mutation, how the selection process works and how to handle constraints and integer restrictions.

Parameters in genetic algorithm

The genetic algorithm is dependent on some parameters. These parameters decide the properties of the algorithm. MATLAB automatically chooses the values for these parameters. The default values from MATLAB are not necessarily the best for our problem. I will run some tests on these parameter values, to improve the running time of the algorithm and the ability to avoid converging to local maximum points. All the tests will be performed using the parameters from case Ghana.

One of the first things GA does is to create an initial population. To do that, it needs a creation function and it needs to know the population size. The creation function creates a random initial population with a uniform distribution. The default value of the population size is 60.

Testing different population sizes could lead to a better and more efficient algorithm. I will test 6 different population sizes, which are 50, 100, 200, 300, 400 and 500. The running time of the algorithm and maximum NPV will be measured. I will run the algorithm three times for each value, and then compare the results to find the best population size for this problem.

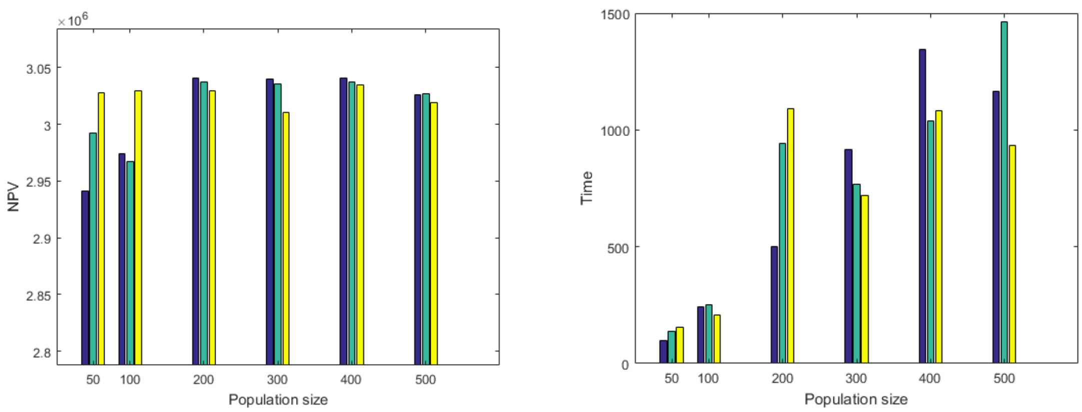


Figure 4.1: The solution (left) and running time (right) for the different population sizes.

The solution from the different population sizes shows some instability when the population size is 50 and 100, the algorithm terminates in some cases at suboptimal solutions approximately 3−4% below the global optimum. When the population size is 200 and higher, the given optimal value is stable and the different population sizes give equally good solutions. The running time of the different population sizes increases, as expected. The running time is almost linearly dependent on the population size. The preferred population size in the algorithm from now on will be 200, because it gives the best results.

After the population is created, it is sorted by fitness value. The best individuals, called the elite population, survive to the next generation. The algorithm then selects individuals to become parents for the next generation, based on their fitness value.

Each individual can be picked several times. There are different types of selection functions. The default selection function is the stochastic uniform selection function. The other possibility, when dealing with mixed integer problem, is the tournament selection function. I will run the algorithm three times for each selection function.

In the following tests, the different parameters will be given the same three initial populations. The blue, green and yellow bars will share the same initial population. By starting with the same initial populations, the result will be more dependent on the parameter value, and less dependent on the randomly selected initial population.

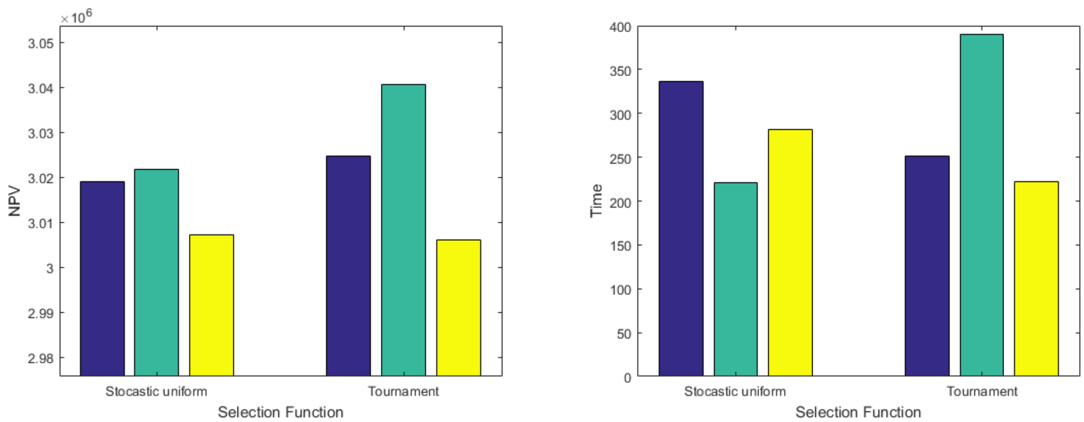


Figure 4.2: The solution (left) and running time (right) for the different selection functions.

Both functions score about equally good in both optimal values and running time. I will keep the tournament selection function, due to its best solution being better than the best solution from the stochastic uniform selection.

Next, we will look at some reproduction options, these options decide how the algorithm is searching for better solutions. The options are elite count and crossover fraction. Elite count decides how many individuals to be part of the elite population, which survives to the next generation without being changed. The default value of elite count is 5% of the population. In this case, with population size 200, the elite count is 10. I will test 5 different values of the elite count, which is 1, 5, 10, 20 and 50.

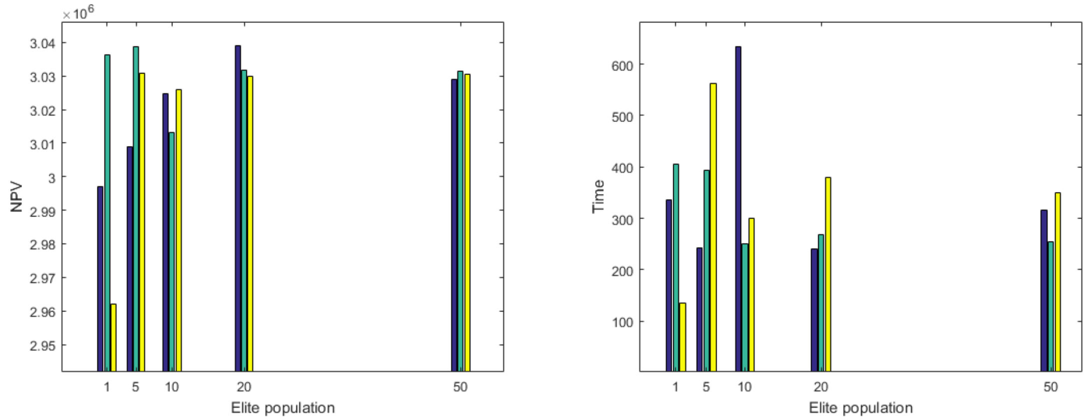


Figure 4.3: The solution (left) and running time (right) for different sizes of the elite population.

An elite population of 20 gives the best solutions and short running time. A higher value of elite population would dominate the total population, and make the search less efficient. A lower value of elite population would be unstable and often converge to a local maximum solution far from the optimal solution.

The rest of the next generation is created by crossover or mutation. Crossover fraction decides the fraction of the population, excluding elite population, to be created by crossover. The default value is 80%. I will try with 20%, 40%, 60%, 80% and 100% as crossover fraction.

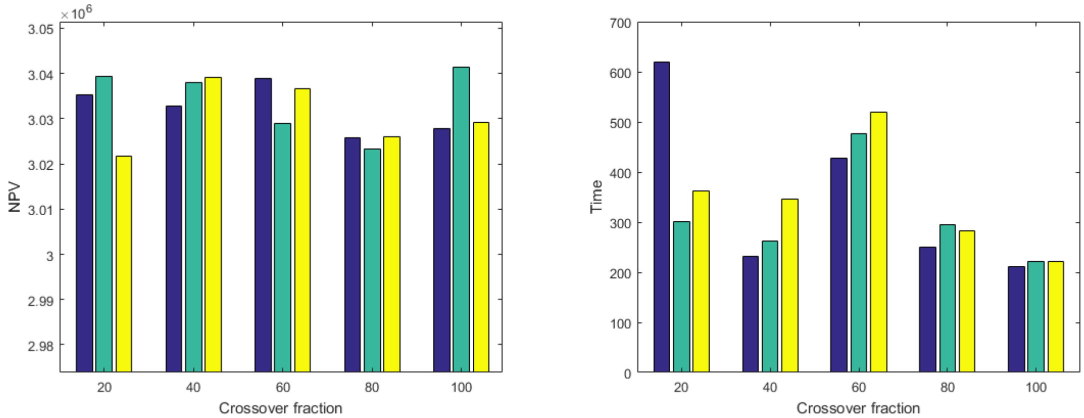


Figure 4.4: The solution (left) and running time (right) for the different crossover fractions.

With a crossover fraction of 100%, the new populations are created only by the elite population and crossover. When we exclude mutation, which is creating diversity in the population, we have a high probability of converging to a local maximum and being unable to escape. However, we should not let mutation be the dominating part of the population, that would lead to a more random search. The 40% crossover fraction finds good solutions at a short running time.

Because of the integer constraint on one of the variables, we have a limited selection of mutation and crossover functions. The only crossover function that can handle this problem is the Scattered function, and the only mutation function is the Gaussian mutation function.

After creating a new population, we need to make sure the individuals satisfy the constraints. The possible choices are Augmented Lagrangian and Penalty algorithm, both score equally in both optimal solution value and running time. I will keep Augmented Lagrangian, the default option.

Stopping criteria

Another way to improve the algorithm is by altering the stopping criteria. The stopping criteria decide when the algorithm is done, and the solution is found. However, the solution may not be optimal. We want the algorithm to not reach a stopping criterion before it reaches the optimal solution.

The algorithm stops if the average relative change in best solution value doesn't increase by more than a function tolerance over a given number of generations, called stall generations. The function tolerance is by default 10^{-6}. The default value of maximum stall generations is 50. I will try the following values of Function tolerance: 0, 10^{-9}, 10^{-6}, 10^{-3} and 1.

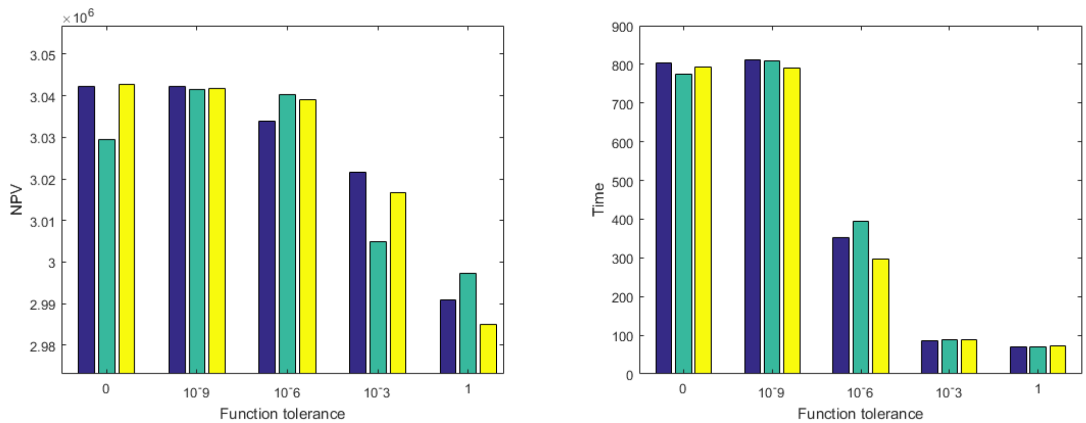


Figure 4.5: The solution (left) and running time (right) for the different values of function tolerance.

Notice in particular the poor performance when tolerance is 10^{-3} and 1, This needs to be avoided. The case where function tolerance is zero has the best ability to find good solutions. It will be preferred despite having a long running time.

With the function tolerance equalling zero, the algorithm will run until it reaches maximum generations or maximum stall generations. I will try different values of maximum generations. The values to be tested are 400, 600, 800, 1 000 and 1 200.

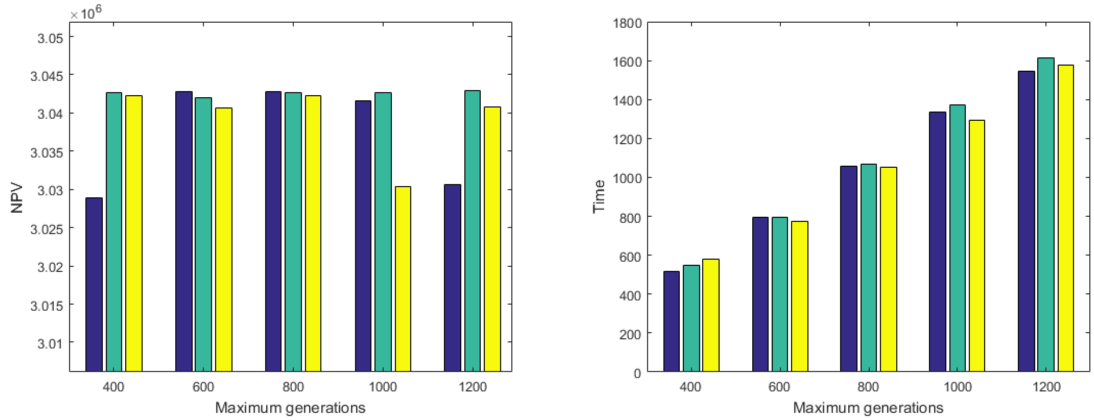


Figure 4.6: The solution (left) and running time (right) for the different values of maximum generations.

With function tolerance set to zero, the algorithm runs until it reaches the limit of maximum generations. Unless you reach maximum stall generations, which is rare. That leads to a nearly linear increase in run time when you increase maximum generations. Since we always keep the fittest individual, the solution will not decrease over the generations. Thus, we want a high limit on maximum generations. I choose 800 as the upper bound on the number of generations because it finds the best solutions.

The value of maximum stall generations determines how many generations before the best value must increase. If we choose the maximum stall generations to be a small number, the algorithm may stop even if the algorithm could have found a better solution later. I will try the following different values: 10, 20, 30, 40, 50 and 60.

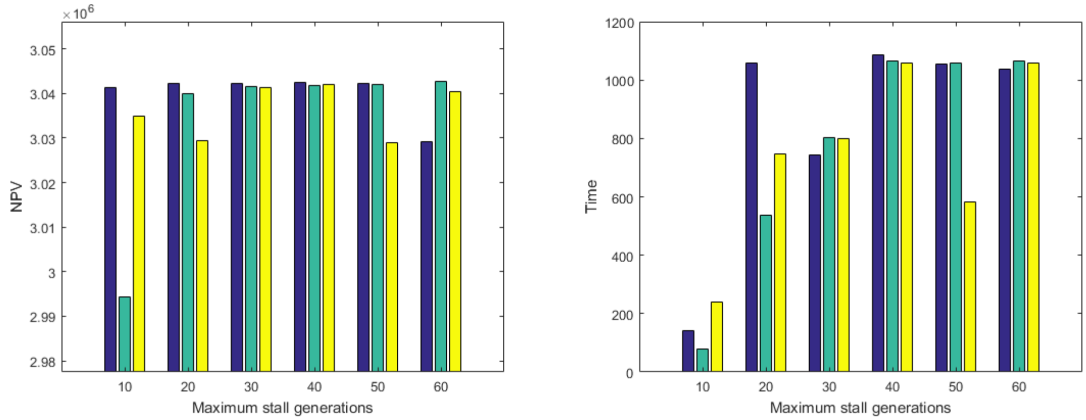


Figure 4.7: The solution (left) and running time (right) for the different values of maximum stall generations.

With a low value of maximum stall generations, the algorithm has a higher probability of converging to a local maximum. When maximum stall generations is set to 10 and 30, all three runs stops because it reaches maximum stall generations, and the solutions are a bit lower than the best solutions. When maximum stall generations is set to 20, two of the runs stops when it reaches maximum stall generations, and one run reaches maximum generations.

When maximum stall generations is set to 50, one of the runs stops when it reaches maximum stall generations, and two runs reach maximum generations. Generally, the algorithm finds the best solution when the algorithm reaches maximum generations. When maximum stall generations is 40 and 60, all three runs reach maximum generations. It finds the best solution when stall generations is 60. We will limit maximum stall generations to 60 in the further calculations.

With the new parameters, we will again test the algorithm with some different values of the population size. Some alterations in the other parameters may have changed what the best value of the population size is.

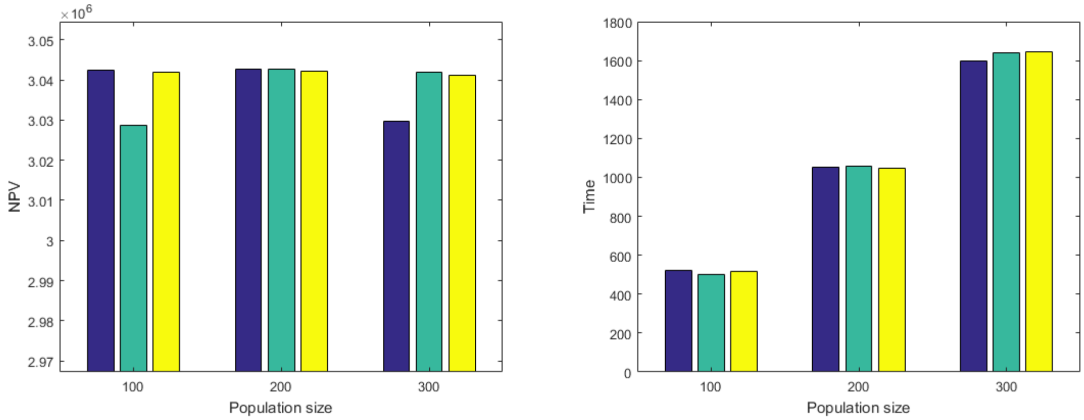


Figure 4.8: The solution (left) and running time (right) for the different population sizes.

The population size of 200 still gives the best solutions. Both of the other population sizes in this test converges to a local maximum in one of the runs. A population size of 200 is still the preferred value.

Through this testing, we have improved the algorithm. The parameters in the genetic algorithms are altered to increase that the probability of the algorithm converging to an optimal solution.

After running this algorithm a few times, with these new parameters, the best solution it found was 3 043 000 NOK in case Ghana. That is a minor increase from 3 027 781 NOK, which was the solution from GA before the tuning of the parameters. The improved solution was found with the following variable values, rounded to two decimal places. $\beta = 24.49$, $D = 0.2$, $\gamma_c = -0.05$, $H = 2$, $K = 10$ and $L = 40$.

4.2 Sensitivity analysis of the parameters

The optimization program contains a function to be maximized, called the objective function. The objective function is, in this case, the net present value of the solar farm. The objective function is dependent on some parameters. Some of the parameters are estimated and some are measured. In this analysis, we will use the estimated parameters from Case Ghana. The purpose of this analysis is to see how much a small adjustment in each parameter will change the end result, and if the adjustment will result in a different optimal design.

Irradiance

The irradiance measurements are not exact, as demonstrated in the discussion of the Ghana-case. What would the result be if we changed the irradiance with ±10%? If the irradiance values were 10 % higher, we would get a higher NPV, that case will be referred to as the favourable case. The case where we put the irradiance to be 10 % lower, will be referred to as the unfavourable case. We will use the variables from the best result in the previous section, referred to as the base case, to see how much the NPV changes when the irradiance is changed. We will also run the algorithm three times for each case, to see if the change in the parameter will lead to different variable values.

Increase irradiance by 10% gives an NPV of 3 535 720 NOK. That is an increase of 16.19%. Decrease irradiance by 10%, the NPV becomes 2 550 160 NOK. That is a decrease by 16.19%. The increase and decrease in the NPV is the same amount when the irradiance is increased and decreased. The optimal variable values are still the same as in the base case, an alteration in the irradiance values will not change the optimal design of the solar farm.

Efficiency

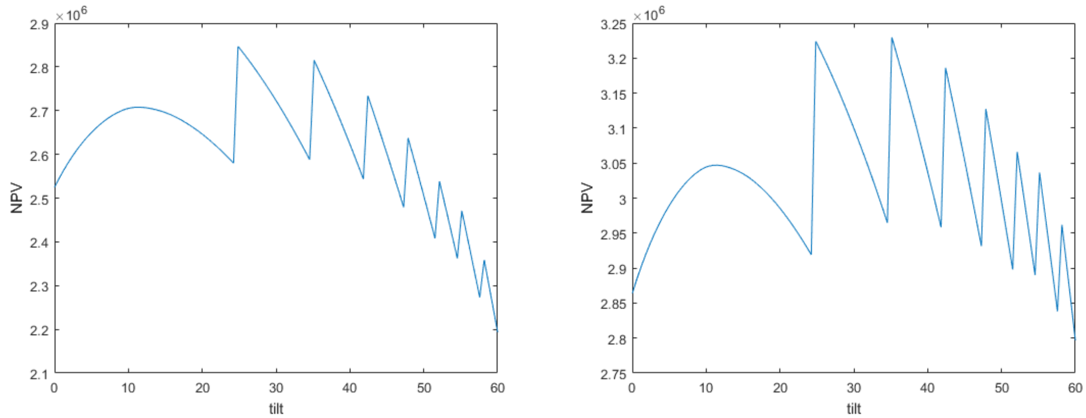
Different modules exist, with different efficiency and price. The total efficiency depends on the panels, inverter and other, it also depends on the temperature. What would the result be if we changed the total efficiency with ±10%?

We get the same result as when changing irradiance. The reason for them making the same impact on the result is that both parameters are factors in the equation that describes the income. Adjusting them by the same factor will give the same result. The optimal variable values are also still the same as in the base case, an alteration in the efficiency will not change the optimal design of the solar farm.

Investment price

The NPV is also dependent on the total investment price which is the sum of the initial investment and maintenance on the solar farm. What would the result be if we changed the total investment price with ±10%?

When we increase the investment price by 10% the NPV becomes 2 854 454 NOK, a decrease of 6.19%. The optimal variable values are still the same. When the investment is decreased by 10% the NPV becomes 3 231 695 NOK, a 6.20% increase. When the investment price is decreased, we get different optimal variable values. The decreased investment price leads to a different optimal design of the solar farm.



(a) Increased investment price.

(b) Decreased investment price.

Figure 4.9: NPV as a function of β, where the other variables are fixed to the result from the algorithm and K is maximized under the constraints.

From the two plots above, you can see NPV of the solar farm for different values of tilt. The plot on the left side shows the result of increased investment price. The maximum value on that plot is when tilt $\approx 25°$. The plot on the right side shows the result of decreased investment price. The maximum value on that plot is when tilt $\approx 35°$. And consequently, instead of 10 rows, the optimal number of rows is 11.

When we consider PV panels, we look at price and efficiency. You can get cheap panels with low efficiency, and you can get more efficient panels which are more expensive. Would the result be the same with different types of panels? We will look at the result of changing both price and efficiency by ±10%.

By increasing both price and efficiency, the NPV becomes 3 347 234 NOK. That is a better solution than the base case. By decreasing both price and efficiency the NPV becomes 2 738 646 NOK. Both cases have optimum for the same variable values as the base case.

Electricity price

In addition to the amount of irradiance and the efficiency, the revenue also depends on the electricity price. If we Increase the electricity price with 10% the NPV becomes 3 535 720 NOK, an increase of 16.19%. When the electricity price is decreased the NPV becomes 2 550 160 NOK a decrease of 16.19%. Both cases have the same variable values as the base case.

Discount rate

The discount rate is assumed to be 5%. If we increased the discount rate with 10% the NPV becomes 2 846 333 NOK, a decrease of 6.46 %. By decreasing the discount rate the NPV becomes 3 254 710 NOK an increase of 6.95 %. Both cases have the same optimal variable values as the base case.

Lifetime of panels

PV panels often come with a guaranteed lifetime of 25 years. More precisely, when the panels have produced electricity for 25 years, their efficiency should be at least 80%. In this thesis, we assume the panels to have a constant efficiency for 25 years. In real life, the efficiency will decrease over time. With this simplification, a longer lifetime would increase the NPV.

If the lifetime of the panels is increased by 10%, the NPV becomes 3 259 349 NOK, an increase of 7.11%. If the lifetime of the panels is decreased by 10%, the NPV becomes 2 798 457 NOK, a decrease of 8.03%. Both cases have the same optimal variable values as the base case.

4.3 Results from analysing GA and sensitivity of the parameters

We have observed that the two conditions for an algorithm to be improved, the running time and the ability to avoid converging in a local maximum, are conflicting. If we alter a parameter to improve one of the conditions, it's at the expense of the other. The optimization problem in this thesis has a lot of local maximum points, which makes the problem hard to solve. Therefore I modify the genetic algorithm focusing on being able to escape local maximum points.

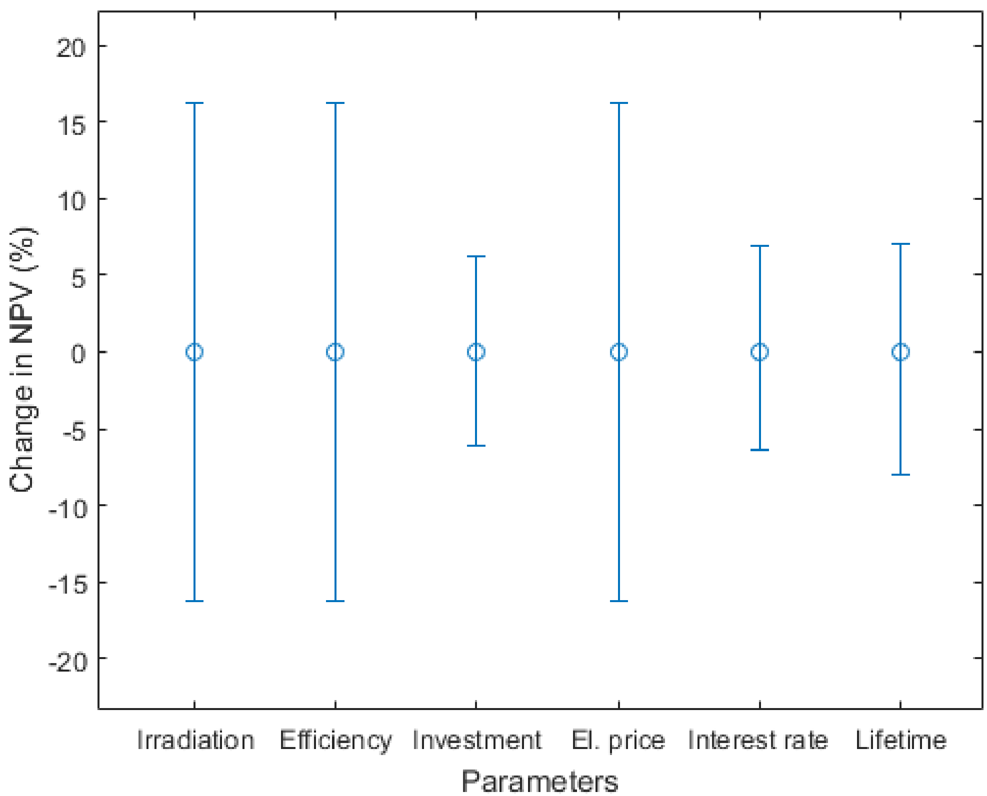


Figure 4.10: Change in NPV, when the parameters have been altered by ±10%.

Irradiance, efficiency and electricity price are the parameters which give the greatest change in the NPV when adjusted. By adjusting the investment price, the NPV does not change as much as some of the other parameters. Despite the fact that it does not make the biggest difference in the result, it is the only parameter that causes a different optimal design of the solar farm, when the parameters are altered by $\pm 10\%$.

Chapter 5

Optimal design of the solar farms

In this chapter, I will present the optimal design of the solar farms for the different locations. The solar farms are assumed to be installed on a horizontal $20 \cdot 40m^2$ area. I also assume the area receives no shade from any nearby objects.

The design of a solar farm in Ghana

The optimal design of a solar farm in Ghana was found with the following variables, rounded to four significant numbers.

The optimal tilt in case Ghana is 24.49°. If the farm only consisted of one row, the optimal tilt would be lower. But the fact that a higher tilted row requires less space, allows more rows into the area. Therefore, the optimal tilt is higher than expected.

The optimal distance between the rows of PV panels is 0.2 m. That is the lower bound we set on this variable. The optimal distance is equal to the lower bound, because of the high elevation of the Sun. That means the shade from the preceding row is short, and it is optimal to have the distance equal to the lower bound.

The optimal height is 2 m. That is the upper bound set on the height. This is because you want as much area of PV as possible. You don't want the shade from the panels to cover the next row, but with the given tilt, it won't shade the next row even when H takes the maximum value.

The optimal number of rows is 10. With the given values of height, distance and tilt, this is the maximum number of rows that fit into the area.

The optimal azimuth is $-0.05°$, the panels are facing south, as expected. The optimal length of the rows is 40 m. This is, like with H, to get as much area of PV as possible. γ_c and L do not have a relation with the other variables.

This results in an NPV of 3 043 000 NOK in case Ghana. That is a minor increase from 3 028 781 NOK, which was the solution from GA before the adjustment.

The design of a solar farm in Italy

The optimal design of a solar farm in Italy is given by the following variable values, rounded to four significant numbers.

The tilt of the panels is $50.79°$, the distance between the rows is 0.4837 m and the height of the rows is 1.935 m. Unlike in Ghana, where the optimal D and H equal the lower and upper bound. With the given variables, the maximum number of rows possible to fit into the area is 12.

The azimuth of the panels is $-0.1313°$ and the length of the rows is 40 m. With the given variables, the NPV is estimated to be 2 874 000 NOK.

The design of a solar farm in Bergen

The optimal design of a solar farm in Bergen, built to cover personal use, is given by the following variable values, rounded to four significant numbers.

$\beta = 54.89°$, $D = 1.873$ m and $H = 0.2007$ m. In the other cases, D were close to the lower bound and H were close to the upper bound, due to the high elevation angle of the Sun. In this case, D is close to the upper bound and H is close to lower bound, this will result in less shade on the panels. $K = 11$, $\gamma_c = 1.081°$ and $L = 40$ m. This design gives an NPV of 5 141 NOK.

Table 5.1: The optimal solutions

	Kumasi, Ghana	Bari, Italy	Bergen, Norway
NPV	3 043 000 NOK	2 874 000 NOK	5 141 NOK
β	24.49°	50.79°	54.89°
D	0.2 m	0.4837 m	1.873 m
γ_c	$-0.05°$	$-0.13°$	1.081°
H	2 m	1.935 m	0.2007 m
K	10	12	11
L	40 m	40 m	40 m

For all sites, $\gamma_c \approx 0°$ as expected for a solar farm in the northern hemisphere. The reason it is not exactly 0° is probably due to local effects making the graph of the irradiance not symmetric about noon, as seen in Figure 3.25. Likewise, it is no surprise that the optimal solutions in all cases are found when L is equal to the upper bound. We expect a dependency between β, H and D, but this is not straightforward, the interplay between these variables are quite subtle and unpredictable.

Chapter 6

Conclusion

I have constructed a mathematical model for computing the optimal design of a solar farm. This has been tested in three different locations. In addition to the design, we found the NPV of the solar farms. We also did a sensitivity analysis on the estimated parameters, to see if some error in the estimates could lead to a significant change in the result.

The solar farms in the different locations have different optimal designs and NPV. The cases with low latitude resulted in the best results. Investing in a solar farm in Ghana, would by our estimates, give a profit of 3 043 000 NOK.

The latitude in Bari is higher than in Kumasi. However, a solar farm in Bari would also give a significant profit. Investing in a solar farm in Italy, would by our estimates give a profit of 2 874 000 NOK.

The estimated parameters are important for the NPV. In Bari and Kumasi, the solar farm would be profitable even with an alteration on $\pm 10\%$ on the estimated parameters.

Investing in a solar farm in Bergen, would not give a profit. At least not with the design constraints from this thesis. However, if the solar farm is built for covering personal use, you should invest in a solar farm. Given the fact that all the power produced by the solar farm, replaces power bought from the electricity grid. Investing in a solar farm for personal use in Bergen, would by our estimates, result in an NPV of 5 141 NOK.

My model gives a good description of costs and income from a solar farm. It should be modified if the conditions are changed. With a good estimate of the parameters, it can tell if the investment is profitable. But what's likely most important, is that it provides guidance on how the farm should be configured.

6.1 Further work

6.1.1 Possible improvements

My results are based on various estimations. I have not had at my disposal any dataset containing the combination of production results from solar farms as well as irradiance measurements from the same location. This could help improve the calculations by making more accurate predictions of the NPV.

The genetic algorithm used in my thesis is chosen for convenience because it is the only algorithm that solves this type of problem in MATLAB, other algorithms should be tested. The genetic algorithm is not made specifically for this type of problem. It is also possible to try different options in the genetic algorithm, such as other crossover options. MATLAB does not offer a great variety of options in their built-in functions, on this specific problem.

6.1.2 Alternative models

There exist numerous types of ways to build a PV farm. Some design includes tracking, either one or two axes. Panels tracking the Sun would produce more electricity but at a higher cost of the installation and maintenance work.

Another design possibility is east-west, where the rows alternate between being tilted towards east and west. This design results in a more even production throughout the day. This design would be more compelling if the grid set a limit on the production, e.g. you were given a fixed electricity price until a given limit or that the el. price is decreasing with increased production, such that you want to spread the production more throughout the day.

There are numerous different solvers, design and parameter possibilities. All the possible cases could be considered for the individual location and customized to the specific use. The different cases could be modelled into an optimization problem and could be interesting continuations of this thesis.

Bibliography

[1] Badescu, V. (2002).
3d isotropic approximation for solar diffuse irradiance on tilted surfaces.
Renewable Energy, 26(2), 221–233.

[2] Bany, J. and Appelbaum, J. (1987).
The effect of shading on the design of a field of solar collectors.
Solar Cells, 20, 201-228

[3] Cooper P. I., (1969).
The Absorption of Solar Radiation in Solar Stills.
Solar Energy, 12, 3

[4] Deep K., Singh K. P., Kansal M.L., and Mohan C., (2009),
"A real coded genetic algorithm for solving integer and mixed integer optimization problems".
Applied Mathematics and Computation, 212, 505-518

[5] Dirk C.J., Kurtz S.R., (2012).
"Photovoltaic Degradation Rates — An Analytical Review". [online] `www.nrel.gov/docs/fy12osti/51664.pdf`
(Retrieved 16.01.2018).

[6] Dubey S., Sarvaiya J. N., Seshadri B., (2013)
Temperature Dependent Photovoltaic (PV) Efficiency and Its Effect on PV Production in the World – A Review. Energy Proc, 33, 311-321.

[7] Duffie J., Beckmann W., (2013),
Solar engineering of thermal processes. 4th edition, John Wiley, New York, USA

[8] Energy in the Solar spectrum.
[online] http://km2000.us/solar/readings.html (Retrieved 15.01.2018).

[9] Enova, *Kostnadsstudie, Solkraft i Norge 2013.*
[online] www.enova.no/upload_images/9EF9602A2B454C008F472DF2A98F6737.pdf
(Retrieved 21.11.2017).

[10] Eurostat, Electricity price statistics
0.148 eur in Italy and 0.07 eur per KWh in Norway. 1 EUR ≈ 9.5 NOK. [online] ec.europa.eu/eurostat/statistics-explained/index.php/Electricity_price_statistics
(Retrieved 11.01.18)

[11] Geophysical institute (UoB).
[online] veret.gfi.uib.no/?action=download (Retrieved 22.11.17).

[12] Goswami D. Y., (1986).
Effect of Row-to-Row Shading on the Output of Flat Plate South Facing Solar Arrays. DOE/JPL-957021-86/1, Prepared for JPL by North Carolina Agricultural State University, Greensboro, North Carolina.

[13] Huld T., Müller R., Gambardella A., (2012).
A new solar radiation database for estimating PV performance in Europe and Africa. Solar Energy, 86, 6, 1803-1815.

[14] Ibrahim A., Bourennani F., (2013)
Optimal Photovoltaic System Design with Multi-objective Optimization. Int J Appl Metaheuristic Comput (IJAMC), 4 (4).

[15] Instruction manual of the measuring device.
[online] https://s.campbellsci.com/documents/cn/manuals/cnr1.pdf (Retrieved 12.11.2017).

[16] IRENA (2016), Solar PV in Africa: Costs and Markets.
[online] http://www.irena.org/-/media/Files/IRENA/Agency/Publication/2016/IRENA_Solar_PV_Costs_Africa_2016.pdf (Retrieved 25.01.18).

[17] IRENA (2017), Renewable energy benefits: Leveraging local capacity for solar PV, International Renewable Energy Agency, Abu Dhabi. [online] www.irena.org/-/media/Files/IRENA/Agency/Publication/2017/

Jun/IRENA_Leveraging_for_Solar_PV_2017_summary.pdf?la=en&hash=9C90082E1722BD974B933FA565A2756586F9D33C (Retrieved 25.01.18).

[18] ITS Innotech Solar module GmbH.
[online] http://www.allecogroup.com/wp-content/uploads/2013/02/Innotech-Solar-productbrochure-EcoPlus-250Wp.pdf (Retrieved 21.11.17).

[19] Kumar B.S., Sudhakar K., (2015)
Performance evaluation of 10 MW grid connected solar photovoltaic power plant in India. Energy Rep., 1, 184-192.

[20] Liu B.Y.H., Jordan R.C., (1963).
The long-term average performance of flat-plate solar energy collectors. Sol Energy, 7.

[21] Marsland S., (2014),
Machine learning: An Algorithmic Perspective. Second edition. ISBN: 978-1466583283

[22] Mathworks. Documentation on GA.
[online] https://se.mathworks.com/help/gads/ga.html (Retrieved 18.01.2018).

[23] Mathworks. Options in GA.
[online] https://se.mathworks.com/help/gads/genetic-algorithm-options.html (Retrieved 18.01.2018).

[24] Mathworks. How GA works.
[online] https://se.mathworks.com/help/gads/how-the-genetic-algorithm-works.html (Retrieved 18.01.2018).

[25] Mitchell M., (1996),
An Introduction to Genetic Algorithms. Cambridge, MA: MIT Press. ISBN 9780585030944.

[26] Rhodes J.D., Upshaw C.R., Cole W.J., Holcomb C.L., Webber M.E., (2014).
A multi-objective assessment of the effect of solar PV array orientation and tilt on energy production and system economics.
Sol. Energy, 108, 28-40.

[27] Ringbeck S., Sutterlueti J., (2013).
BoS costs: status and optimization to reach industrial grid parity. Progress in Photovoltaics: Research and Applications, 21, 6, 1411-1428.

[28] Shoemake J., (2014).
Ghana Powers Up: The Largest Solar Plant In Africa Is Coming. *Ventures Africa.* [online] http://venturesafrica.com/ghana-powers-up-the-largest-solar-plant-in-africa-is-coming/ (Retrieved 30.10.2017).

[29] SMA, PV Inverter, SUNNY TRIPOWER,10000TL / 12000TL / 15000TL / 17000TL Installation Manual
[online] files.sma.de/dl/8552/STP10-17TL-IA-en-32.pdf (Retrieved 15.08.2017).

[30] Sproul, A. B. (2007).
Derivation of the solar geometric relationships using vector analysis.
Renewable Energy, 32(7):1187–1205.

[31] Starr, M. R., Palz, W., (1983).
Photovoltaic power for Europe : an assessment study. Page 76. D. Reidel Pub. Co. for the Commission of the European Communities; Hingham, MA: Sold and distributed in the U.S.A. and Canada by Kluwer Boston, Dordrecht, Holland; Boston, U.S.A.

[32] Statistics Norway.
[online] https://www.ssb.no/en/elkraftpris (Retrieved 25.01.2018).

[33] Šúri M., Huld T.A., Dunlop E.D. Ossenbrink H.A., (2007).
Potential of solar electricity generation in the European Union member states and candidate countries. Solar Energy, 81, 1295–1305.

[34] The World Bank, Climate Change Knowledge Portal.
[online] sdwebx.worldbank.org/climateportal/index.cfm?page=downscaled_data_download&menu=historical. (Retrieved 12.12.2017).

[35] UN, sustainable development goals. Goal 7: Affordable and clean energy.
[online] http://www.undp.org/content/undp/en/home/sustainable-development-goals/goal-7-affordable-and-clean-energy.html. (Retrieved 01.11.2017).

[36] WORLD ENERGY COUNCIL, WORLD ENERGY RESOURCES 2016. [online] www.worldenergy.org/wp-content/uploads/2017/03/WEResources_Solar_2016.pdf (Retrieved 25.01.18).

[37] Yr, Climate statistics. [online] http://www.yr.no/sted/Norge/Hordaland/Bergen/Bergen/klima.html (Retrieved 12.12.2017).

Appendix A

Nomenclature

A = maximum height above ground
A_s = Area of the row covered by shade
C_i = Annual income
D = Space between the rows
E = Clearance between the panels and the ground
ep = Electricity price
F_d^{sh} = Ratio between diffuse irradiance on an un-shaded plane and a shaded plane
F_r^{sh} = Ratio between reflected irradiance on an un-shaded plane and a shaded plane
GA = Genetic algorithm
H = Height of row
H_s = Height of shadow on the shaded rows
I = Irradiance
I_b = Beam irradiance
$I_{b,T}$ = Beam irradiance on tilted panel
$I_{b,T}^{sh}$ = Beam irradiance on tilted and shaded panel
I_d = Diffuse irradiance
$I_{d,T}$ = Diffuse irradiance on tilted panel
$I_{d,T}^{sh}$ = Diffuse irradiance on tilted and shaded panel
I_r = Reflected irradiance
$I_{r,T}$ = Reflected irradiance on tilted panel
$I_{r,T}^{sh}$ = Reflected irradiance on tilted and shaded panel
I_0 = Initial investment
K = Number of rows

L = Length of row
L_s = Length of shadow on the shaded rows
mc = Annual maintenance cost
n = Day of the year
NOK = Norwegian kroner
p = Percent discount rate
pp = Price of installed PV
ppm = Price of installed PV included maintenance
Q = Total irradiance on the panels (Wh)
q_b = Beam irradiance (Wh/m^2)
q_d = Diffuse irradiance (Wh/m^2)
q_r = Reflected irradiance (Wh/m^2)
q_b^{sh} = Beam irradiance on a shaded panel (Wh/m^2)
q_d^{sh} = Diffuse irradiance on a shaded panel (Wh/m^2)
q_r^{sh} = Reflected irradiance on a shaded panel (Wh/m^2)
R_b = Relation between beam irradiance on a horizontal plane and on a tilted plane
R_d = Relation between diffuse irradiance on a horizontal plane and on a tilted plane
T = Temperature
W = Width of field
Wp = Watt peak

$\mathbb{R}$ = Set of real numbers
$\mathbb{Z}$= Set of integers

α = Sun's altitude
β = Tilt of panels
γ = azimuth angle between the Sun and the panels
γ_c = Panels azimuth
γ_s = Sun's azimuth
δ = Declination
η = Efficiency
η_{pv} = Efficiency of panels
η_{inv} = Efficiency of inverter
η_{other} = Other decreasing factors
θ = Angle between the normal to the panels and the beam irradiance
θ_z = Angle between the horizontal plane and the beam irradiance
ρ= Albedo
ϕ = Latitude of location

ω = Hour angle
Δt = time interval