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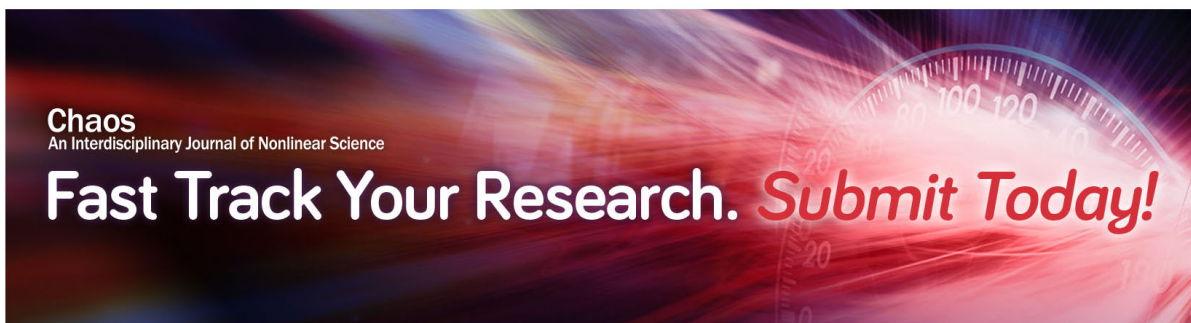
Introduction to focus issue: Synchronization in large networks and continuous media—data, models, and supermodels

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The synchronization of loosely coupled chaotic systems has increasingly found applications to large networks of differential equations and to models of continuous media. These applications are at the core of the present Focus Issue. Synchronization between a system and its model, based on limited observations, gives a new perspective on data assimilation. Synchronization among different models of the same system defines a supermodel that can achieve partial consensus among models that otherwise disagree in several respects. Finally, novel methods of time series analysis permit a better description of synchronization in a system that is only observed partially and for a relatively short time. This Focus Issue discusses synchronization in extended systems or in components thereof, with particular attention to data assimilation, supermodeling, and their applications to various areas, from climate modeling to macroeconomics. *Published by AIP Publishing.* https://doi.org/10.1063/1.5018728

Synchronization among regular oscillators such as a complex organism's circadian rhythms, pendulum clocks on a common wall, or blinking fireflies establishes a surprising order in natural systems. Theoretical and numerical results obtained over the past 25 years with coupled chaotic systems suggest that synchronistic relationships could possibly occur between systems whose internal behavior is not ostensibly regular, extending greatly the potential range of synchronism in nature. More recently, synchronization has been explored in naturally occurring, chaotic systems with very large numbers of variables and in models thereof; the latter are typically governed by sets of ordinary differential equations (ODEs) on large networks or by partial differential equations (PDEs) on continuous media. An important instance is the synchronization between a system and its real-time computational model that can be induced by a limited set of observations of the system. Such truth–model synchronization corresponds to the well-established practice of data assimilation, used extensively in meteorology, oceanography, and the climate sciences in general. An extension of this idea is to allow a set of alternative models of the same real system to synchronize with one another, as well as the real system, by exchanging data and thus forming a supermodel. Such a supermodel offers a potential solution to problems of divergent predictions by different expert models, and it has been shown to improve upon the common practice of merely averaging over model outputs. This Focus Issue sheds further light on the uses of synchronization for data assimilation and for supermodeling, as well as on current developments in synchronization within and between extended systems, natural and social, in general.

I. SETTING THE STAGE

Synchronization theory arose in the early 1990s in the context of secure communications methodology.[1] Over the last quarter-century, synchronization of chaotic systems that are coupled loosely, i.e., through only a few of their many dynamical variables, has begun to find real-world applications that are much broader and cover many areas of the sciences. In particular, the scientific community has been moving from synchronization in network models to synchronization of extended, classically continuous systems that arise in nature and society.

Synchronization is possible both within such a continuous system and between two or more such systems. In the former case, we have a new description of coherence. In the latter case, the principle remains the same as for a pair of coupled ODEs: exchange of a surprisingly small amount of energy or information will cause large systems to synchronize, despite spatio-temporal chaos within the individual systems and despite the complexity of the system as a whole.

Because of the great overall interest in synchronization, as well as because of its increasing importance within the field of chaos as a whole, there have been already a number of Focus Issues and an even larger number of individual articles in this journal on the topic. The present issue deals with current applications to extended real-world systems and detailed models thereof (G. S. Duane and M. Ghil conceived and planned this Focus Issue. All four Guest Editors carried out the editorial tasks.). These applications require a direct representation of the continuum or of a realistically large network.

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Of special interest is the relationship between such a system and its model, which can itself be characterized as one of synchronization, either in the context of control theory or of data assimilation. To the extent that chaos synchronization with very limited coupling describes relationships among real systems, and especially between a real system and its model, we have a validation[6] of the philosophical concept of synchronicity[2–5]—an organizing principle said at times to be on a par with causality and having captured the popular imagination.

A. A brief history of chaos synchronization

To recap the history of the subject, the earliest investigations in synchronized chaos were due to Fujisaka and Yamada[7] and Afraimovitch *et al.*[8] Truly widespread interest was spurred by the work of Pecora and Carroll,[1] who obtained synchronization of two Lorenz systems[9] by completely replacing one variable, e.g., the X variable, in the "slaved" system by the value of the corresponding variable in the "master" system.

Synchronization was later shown to occur for general types of loose coupling between chaotic systems, such as the addition of a simple relaxation term, labeled diffusive coupling. It was suggested that this phenomenon may be useful in cryptography, since the driving variable could be used as a carrier signal that would be difficult to distinguish from noise. However, as might be guessed from the Mañe-Takens theorem,[10] the resulting codes were not difficult to break, despite a series of proposals for more sophisticated synchronization-based encryption schemes.[11]

In applications to real chaotic systems, though, one does not expect to find perfect synchronization, especially when the systems to be synchronized are not identical. Complete synchronization typically degrades in two ways: (a) through intermittent bursting away from synchronized motion,[12] via on-off intermittency;[13] and (b) through generalized synchronization, in which case there is still a perfect correspondence between the states of two synchronized systems, but the correspondence function is not the identity.[14]

Intermittent synchronization has indeed been found in natural systems, including the form of spatially intermittent synchronization known as chimeras.[15] Generalized synchronization has remained largely a theoretical construct because the correspondence function is typically an intractable, often nowhere differentiable function,[16] whose existence can only be established indirectly. When this function is close to the identity in any sense, the observed behavior can be characterized simply as approximate synchronization. Such is the case in multi-scale systems, where synchronization is nearly perfect only on larger scales and the smaller scales act to compensate differences between the systems in a complex way, e.g., Ref. 17.

Chaos synchronization generalizes a phenomenon that had been studied already much earlier, namely, the synchronization of regular oscillators, presumably first discussed by Huygens,[18] who described anti-synchronization between pendulums suspended on a common wall. In chaotic systems, a form of regular synchronization still appears as *phase synchronization*, in situations in which it is possible to assign a phase to the state of the system, even when the trajectory is not cyclical.[19] Phase synchronization is relatively easy to detect in real systems, provided that the signal is suitably pre-filtered.[20]

Synchronization between PDE systems, as first described by Kocarev *et al.*,[21] opened the door to a theoretical description of synchronization between spatially extended systems. Early applications of synchronization to such physical systems were to ferromagnetic materials[22] and to lasers.[23]

Duane[24] applied these ideas to fluid dynamics, specifically to models of atmospheric flow in the Northern and Southern Hemispheres, demonstrating a weak synchronization effect. This application also pointed out a fundamental difficulty in applying the Pecora-Caroll[1] scheme to extended systems: The signals that connect the systems in both directions are transmitted by waves, with a resulting time delay in the connection. Substantial delays in the transmission of signals also occur in large socio-economic systems.[25]

Synchronization between different, widely separated parts of the same extended system is greatly weakened by such delays in the usual case, with nearly continuous bursting away from synchronized motion. Of course, synchronization between adjacent elements of a given system is not impeded. Implications for describing the resulting coherent structures are still a topic of investigation.

Synchronization has mainly been investigated as a phenomenon of classical physics. But the strongest evidence of synchronicity in the physical world is probably that provided by the Einstein-Podolsky-Rosen (EPR) correlations in quantum systems, lending such systems a fundamentally nonlocal character. Recent attempts to describe such behavior objectively[26,27] can be interpreted as implying synchronization[6] but remain highly speculative. A jump from classical to quantum systems is one future prospect for the extension of the developments discussed in this issue.

B. Synchronization between the system and the model

Synchronization between two extended systems of the same type, or where a correspondence function is obvious, has a straightforward application: We can imagine that one system is a model of the other. In a fairly common situation, the model may be an imperfect representation of the "real" system, e.g., have much lower dimension than the latter. An obvious application is the control of the real system by the model, thus extending the notion of controlled chaos.[28,29]

While unidirectional coupling of model to reality provides a useful view of control in general, and biological motor control in particular, the opposite direction of coupling, from reality to model, may be viewed as providing a description of perception. A computational model that receives a recurrent but limited stream of data from the system it represents carries out a form of machine perception[36]—and of machine learning, if the model adapts along the way—that is useful for predicting the future behavior of the system. That is, the model synchronizes with the real system, requiring only small intermittent adjustments to maintain the synchronization indefinitely.

The coupling between reality and model needed to maintain the synchronous state defines the computational process of data assimilation. Originating in weather prediction, cf. Bengtsson *et al.*[30] and references therein, where complex numerical models of a large fluid dynamical system are continually fed new data from observations, data assimilation has extended to oceanography[31] and other areas of the geosciences.

The synchronization view of data assimilation is a framework that conceptually encompasses several algorithmic approaches. The diffusive coupling form commonly used for synchronization is known as nudging in the data assimilation literature.[30–32]

If the coupling coefficients are allowed to vary in time, then one can show that—subject to linearity assumptions—their optimal time-dependent values have the same form in terms of the evolving error statistics as in the classical Kalman filter algorithm.[35] Variants and extensions of the latter subsume or are equivalent to many of the data assimilation methods used currently in operational practice.[31,33,90]

It is, in fact, difficult to devise new methods based on the synchronization view that has not already been considered in the data assimilation literature. An exception that remains to be fully investigated is the treatment of strong nonlinearities, as may occur at times of regime transition, where both sequential-estimation algorithms of the Kalman filter type and optimal-control–based algorithms of the variational type may have difficulties.[31,34] In this case, the synchronization view could possibly provide new algorithms that can improve the tracking of the transitions.[35,36]

A well-known extension of both operational data assimilation and the synchronization approach to it is to estimate model parameters at any given instant of time, as well as model states. In the augmented state-vector method of data assimilation, parameters to be estimated are treated as additional state variables and often assumed to be constant in time. The basic assumption of this method is not always consistent with the results of the estimation, since the estimated parameters may actually turn out to vary in time, thus compensating for the model's not being quite consistent with the observations.[37]

In synchronization, the dynamical equations can be readily extended so that model parameters are allowed to depend on time and have their own equations, which permit them to synchronize with the presumably time-independent parameters of the natural system being observed.[38] Duane and Hacker[39] have applied the latter dynamical approach to a mesoscale atmospheric model, but here too, the advantages of the synchronization view remain to be investigated further.

The goal of synchronizing a model with observed behavior in a natural system or with the simulations of a very detailed model may be much more general than having trajectories actually coincide, as in data assimilation. Ideally, one may wish for qualitative similarity, as might be achieved through the model's attractor approaching the geometry and properties of the natural system's or detailed model's attractor. Such is the case in predicting the future behavior of the climate system for instance, where the weather on a specific future date is of little interest, but one would like to have something deeper than merely statistical predictions of mean state and variance.

Actually, an early form of synchronization, from the study by Afraimovitch *et al.*,[8] achieved exactly this type of attractor matching, labeled non-isochronic synchronization, but this line of investigation was apparently abandoned. The phenomenon appeared later under the nomenclature of measure synchronization in investigations of Hamiltonian systems,[40] for which Liouvilles's theorem precludes collapse of the trajectories of two systems to a lower-dimensional, synchronized subspace, but their attractors still match in a suitably defined sense. The work of Afraimovitch *et al.*[8] suggests that measure synchronization is not limited to Hamiltonian systems, and hence, it remains a possible approach to attractor learning.

As in the case of data assimilation, a rich literature on model reduction exists: rigorous results on attractor similarity between full and reduced models have been proven,[41,42] and numerous examples of successful computations have been given.[43–45] Still, as in the case of data assimilation, the synchronization point of view might offer algorithmic improvements, as well as a new way of better understanding old results.

C. Supermodels

So far, we have progressed from synchronization between extended systems in nature (Sec. I A) to synchronization between systems and their models (Sec. I B). The next phase of development in the worldview under consideration is synchronization between models. We envision different models of the same reality, each imperfect but in a different way. Imagine, for instance, that each model is the result of a local optimization in the space of all possible models. Such is commonly the case when different groups of modelers make different *ad hoc* choices about the structure of the models in dimensions that are not well informed by theory or by empirical evidence.

The two dozen or so models of the Earth's climate that are used for future projections thereof[46] provide a good example. In a supermodel, one uses terms that are common to synchronization and data assimilation methods, such as diffusive coupling, to connect the models. As with natural systems that are sufficiently similar so that a simple correspondence function can be defined, the models may synchronize to a certain degree, and the differences become absorbed in the behavior of less important degrees of freedom, typically on smaller and faster scales.

The construction of such a supermodel effectively reduces the parameter-estimation task to a remarkable extent. Instead of having to learn a large number of model parameters or the model's qualitative structure, one only needs to estimate a much smaller set of model-to-model connection coefficients. There is one for each field in each ordered pair of numerically discretized PDE models, since it is natural to assume, to a first approximation, that the coefficients are spatially uniform.

As shown in Fig. 1, the models are all coupled to a single reality, as well as to each other, at least in the learning phase. Any algorithm that can be used for estimating parameters in a data assimilation or synchronization context can then be applied to estimate the connection coefficients. Several preliminary tests have resulted in approximate synchronization between models, as well as between the models and the system being observed.[48,49] The supermodel can then be used for predictive purposes in the same manner as any of the constituent models. If desired, the models can be disconnected, after training, from the natural system or the more detailed model thereof, so as to solve the sensitivity problem of how the synchronized attractor responds to changes in any ancillary parameters.

If data assimilation can be compared to perception, then the inter-model data assimilation in a supermodel might be compared to self-perception and thus—proceeding to a realm no longer regarded as metaphysical[47]—to consciousness.[36] More mundane applications are also envisioned in the near term.

The supermodeling concept has been validated using relatively simple models that are governed by systems of ODEs[48] or PDEs.[49] In the realm of practical modeling of extended systems, the scheme has been applied so far (i) to combine a pair of highly detailed climate models, but with very limited model-to-model connections;[50] and (ii) to simpler models of cancer tissue development, with untrained connection coefficients and again with limited coupling.[51] The stage has thus been set for further investigations in the
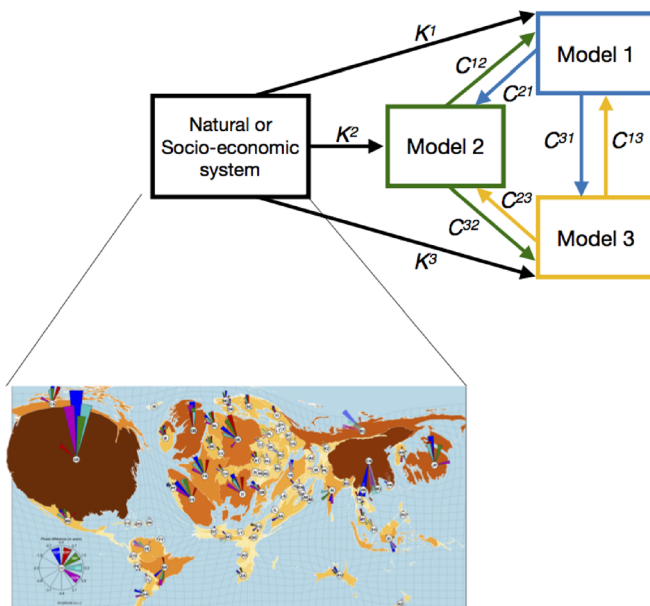

FIG. 1. Schematic diagram of a supermodel: A small set of alternative models are coupled to each other with connection coefficients C_l^{ij}, different for each model pair $\{(i, j): 1 \leq i, j \leq J\}$ and for each field variable $\{l: 1 \leq i, j \leq L\}$. The coefficients need to be optimized, while the models are coupled unidirectionally to the observations from a natural system or from a high-end simulation thereof. The latter coupling uses coefficients K^i, e.g., Kalman gain matrices as they arise in standard data assimilation schemes.[31] The trained connections induce partial synchronization between models as well as between the models and the data. (Adapted from Groth and Ghil,[87] and Duane[6]).

application of synchronization to combine alternative real-time computational models.

D. How synchronized are a set of time series?

An ancillary problem in the broad field of synchronization is that of determining from observational data alone to which extent a set of time series does reflect synchronization of the subsystems that have generated the available data. The issue arises in the study of real-world systems in a way that it did not for the synchronization of the highly idealized systems or circuits that were initially studied. Solving this problem entails the usual issues of pre-filtering the time series, to eliminate spurious noise, and then to analyze the pre-filtered time series in terms of their shared properties.

In its simplest form, the problem is to find out whether two time series $x_1(t)$ and $x_2(t)$ are, according to Osipov *et al.*,[52] frequency locked, phase locked, or completely synchronized. The classical way of doing this is to define, for each of the two time series, the associated analytic signal $\psi_k(t) = x_k(t) + iy_k(t)$, where $k = 1$ or 2. Here, i is the imaginary unit and $y(t)$, dropping henceforth the indices k, is the Hilbert transform $\mathcal{H}[x(t)]$ of $x(t)$, which is given by the Cauchy principal value of a singular integral. Analytically, then, one can define the polar representation of the complexified signal $\psi(t) = A(t) \exp[\phi(t)]$, where A is the amplitude and ϕ is the phase of the signal $x(t)$.

The two signals $x_1(t)$ and $x_2(t)$ are, for instance, phase locked[52] if the difference $\Delta_{1,2}\phi = |\phi_1(t) - \phi_2(t)|$ does not grow in time. The problem with this polar representation is that the Hilbert transform $y(t) = \mathcal{H}[x(t)]$ of a given $x(t)$ is numerically ill-posed and that, for the typically short and noisy time series found in climate records,[53,54] it is well-nigh impossible to test over just a very few full periods whether the phase difference $\Delta_{1,2}(t)$ between two nearly equal periodicities in the two time series increases with time or not.

These numerical issues are discussed in greater detail by Feliks *et al.*,[54] and successive steps toward a greatly improved pre-filtering and subsequent synchronization analysis have been proposed and tested in both the climatic[54–57] and the macroeconomic[58,59] context.

II. THIS FOCUS ISSUE

A. Synchronization in extended systems

We begin with a few papers on the general phenomenon of synchronization in extended chaotic systems.

Colon and Ghil[60] address synchronization in the context of Boolean delay equations (BDEs).[61] BDEs represent a mathematical framework for modeling networks that evolve continuously in time, unlike cellular automata, which are discrete in both the variables and the time. The use of continuous time allows one to use, in general, a distinct delay $\tau_{i,j}$ in the action of a given Boolean variable x_i on another Boolean variable x_j, with $\tau_{i,j} \neq \tau_{j,i} \neq \tau_{k,l}$ for $(i - k)(j - l) \neq 0$. Previous work had already shown that the presence of delays modifies substantially the dynamics on various networks, as one might suspect from the case of delay differential equations (DDEs) vs. ODEs: the solutions of the DDE $\dot{x} = -x(t - \pi/2)$ are

periodic, while those of the ODE $\dot{x} = -x(t)$ tend to the unique, stable fixed point $x = 0$.

The BDE framework thus allowed the authors to study damage propagation and synchronization in economic supply networks under the much more realistic assumption of heterogeneous time lags. The study proceeded from simple to complex network topologies, including both Erdös-Rényi and scale-free networks, as well as network structures based on the statistics of over 1 million Japanese firms.[62] Key results included criteria for the collapse, survival or partial survival of the network, given a local perturbation, as well as cases of cyclostationary waves propagating through the network. The heterogeneity of delays can have a crucial effect on synchronization of such waves or the loss thereof.

The paper by Yao *et al.*[63] is also a study of the effects of time lags in internally synchronized networks, with varying topologies. Here, the network elements are chaotic pendulums. The authors find that the time lags play a decisive role in the synchronization process, inasfar as the threshold value of the coupling strength for complete synchronization strongly depends on the time delay in the coupling, while the specific topologies are relatively unimportant in the synchronization behavior.

Moskalenko *et al.*[64] studied the relationship between chaotic synchronization and microwave signal amplification in unidirectionally coupled beam–plasma systems. This paper addresses a challenging issue in the synchronization of systems possessing multiple time scales that has not been sufficiently investigated. Can one detect synchronization between two or more systems on some time scales, while the trajectories of the systems on other scales are not correlated? The study's key tool is wavelet analysis, and the main result is that synchronization within a limited range of time scales—referred to as time scale–synchronization—leads to amplification of output power in numerical simulations, confirming the previously reported experimental results.

B. Synchronization between data and models

This section of the issue presents several extensions of and alternatives to the view of data assimilation as synchronization of a natural or socio-economic system and its model—as discussed in Sec. I B above.

Penny[65] presents a hybrid method for data assimilation that combines time-dependent coupling coefficients, as in Kalman filtering,[31] with time-independent coefficients, as in older methods. The method proves superior in situations where there is not enough information to get sufficiently complete and reliable error statistics, as required in the Kalman filter approaches.[66] The hybrid method is applied to a realistic ocean model.

Weiss and Grooms[67] introduce a coupling scheme for data-assimilation-as-synchronization that relies on coherent structures, an idea previously explored by Ide and Ghil,[68,69] among others. Here, it is ocean eddies that are represented as vortices, to reduce the number of observations required for given predictability. Various ways for doing so, including reliance on coherent structures or on preferred instability modes, had been reviewed by Ghil.[70] Specifically, since coherent structures manifest internal synchronization, the results of Weiss and Grooms[67] suggest that internal synchronization within a system facilitates synchronization with another system, as previously hypothesized by Duane[71] to be the case more generally.

Abarbanel *et al.*[72] eschew the notion of an objective nature state that synchronizes with a model, in keeping with the usual positivist worldview of contemporary physics. Instead, these authors maximize *a posteriori* probabilities for a given stream of input data. This maximization relies on an Euler-Lagrange path integral approach, in which entire trajectories are matched to the data, as suggested by Eyink and Restrepo.[73,74] The authors[72] find similarities between the path integral approach and synchronization.

This path integral approach results in a formalism for data assimilation that Abarbanel *et al.*[72] note is equivalent to the 4dVar scheme used in operational practice by several meteorological centers. Restrepo,[74] on the other hand, had focused on a comparison of the path integral method with ensemble Kalman filtering.

There is an interesting dichotomy between Abarbanel's (and Restrepo's[74]) path integral approach and resulting second-order equations, on the one hand, and the first-order synchronization by diffusive-coupling formulation, on the other hand. This dichotomy reflects the methodological dichotomy between 4dVar approaches and 3dVar–Kalman filter approaches that are both commonly used in operational practice. The latter dichotomy is but a modified form of the classical duality between optimal control and sequential estimation, respectively.[31]

The short paper by Duane[75] studies the recently proposed "FORCE" algorithm for learning in neural networks with fully general neuron-to-neuron connection patterns as a particular instance of data assimilation for parameter estimation. The parameters to be estimated in such recurrent neural networks are the synaptic weights. The FORCE algorithm, which ensures synchronization of network output with a training signal, is found to be equivalent to Kalman Filtering with a peculiar state-dependent form for the time-dependent couplings between training signal and model. Duane[75] sees therewith a promising role for applications of data-assimilation-as-synchronization in machine learning and, possibly, in biological learning.

C. Supermodeling: Synchronization among models

The papers in this section discuss general developments in the theory and practice of supermodeling.

Kirtman *et al.*[76] present results obtained with an interactive ensemble of climate models, a setting that is a forerunner of supermodeling. The climate models used have an atmospheric and an oceanic component, and the two components are coupled only at the ocean–atmosphere interface. In the common practice of ensemble forecasting or climate simulation,[46] one uses a unique model with distinct realizations that are started from an ensemble of initial states and might include also random changes in some parameter values. In an interactive ensemble, the realizations are coupled to each other during the climate simulation. The context of the study

is the predictability of atmospheric signals arising from ocean dynamics. In particular, the influence of oceanic meso-scale activity is analysed by the interactive ensemble approach.

Two sets of experiments are designed, which differ in the unique ocean model being used: one has higher horizontal resolution, which allows oceanic eddies to be resolved, and the other does not. In each one of the two sets, an ensemble of copies of the same atmospheric model, the Community Atmospheric Model version 4 (CAM4), are coupled by surface fluxes of heat, water mass, and momentum to a single, shared copy of the ocean model, namely, a high-resolution (HR) or a low-resolution (LR) version of the Parallel Ocean Program version 2 (POP).

Averaging the fluxes of the N atmospheric copies enhances the atmospheric signal that is induced by the ocean state since unsynchronized atmospheric fluctuations are averaged out. Presenting these averaged fluxes to the ocean leads to a higher degree of synchronization as measured by local correlations between sea surface temperature (SST) and convective precipitation.

The conclusion, as in the Weiss paper,[67] is that coherent, i.e., internally synchronized, ocean eddies enhance predictability. This enhancement is apparently due to an increased dependence of the atmospheric internal dynamics on the ocean state, so that the atmospheres increasingly synchronize with one another. The same generalized synchronization is likely to occur in a supermodel, in which the atmospheres differ from one another.

Selten *et al.*[77] present the first fully connected supermodel, composed of several atmospheric models coupled to each other, as well as to a common ocean–sea ice component and to a common land surface model. The models crudely represent all elements of the climate system but are simpler than those used in high-end climate projection experiments.[46] Synchronization-based parameter estimation is used to train the connection coefficients, as illustrated in Fig. 1 and discussed in Sec. I C. The paper demonstrates the robustness of the supermodel, with its trained connections, against variations in ancillary parameters that represent CO_2 concentration changes in both the atmosphere being observed and the constituent models. An important finding is that training of the supermodel on short time scales improves its long-term climate simulation.

Wiegerinck and Selten[78] address the problem of attractor learning in supermodels, in a situation where the models contain fewer degrees of freedom than the simulated ground truth, nature or control run. Two situations are investigated: (i) in the first one, the ground truth is given by a chaotically driven Lorenz[9] model and the imperfect models are two Lorenz models with constant forcing; (ii) in the second one, the ground truth is given by a highly simplified global atmospheric model with good climatological properties, the spectral three-level, quasi-geostrophic (QG3) model of Marshall and Molteni[79] and the imperfect models are lower-resolution QG models.

In both situations, the supermodel is defined in the limiting case where the connection coefficients become infinite but with fixed ratios between different connections. This setting yields essentially a single model defined by weighted combinations of the tendencies for each variable in the constituent models. It is referred to as weighted supermodeling as opposed to connected supermodeling. Minimizing the distance between attractors as a function of such weights is shown to give a performance superior to that of a supermodel with weights chosen by short-range optimization schemes for finite-horizon prediction skill.

D. Applications in the natural and social sciences

The remaining papers focus on applications of synchronization and supermodeling in various physical and socioeconomic systems.

Shen *et al.*[80] develop further the interactive ensemble construct of Kirtman *et al.*,[76] as reviewed in the previous subsection, by using two distinct atmospheric models coupled through surface fluxes to the same ocean model. In this case, both the atmospheric models and the ocean model are based on the Community Earth System Models (COSMOS) developed at the Max-Planck-Institut für Meteorologie, Hamburg. The atmospheric models are both 5th-generation European Centre–Hamburg (ECHAM5) general circulation models and differ only in their convection scheme, while the oceanic model is the Max Planck Institute Ocean Model (MPIOM).

The paper addresses the question of how two models exhibiting qualitatively similar erroneous behavior could combine in a supermodel to yield qualitatively correct behavior. The promising answer is worked out in focusing on the Tropical Pacific, where strong nonlinearities are at work in the coupled atmosphere–ocean system giving rise to the El Niño–Southern Oscillation[81,82] cycle. It is these nonlinearities that appear to be responsible for a better result than would arise from a simple linear combination of two different mechanisms, both of which, taken separately, give the same qualitative error in sea surface temperature patterns in the two models.

Read *et al.*[83] provide an instance of synchronization in experimental fluid dynamics. The authors study the classical apparatus of a rotating differentially heated annulus that mimicks large-scale atmospheric flows,[84] but they introduce periodic forcing that is meant to imitate seasonal changes in the pole-to-equator temperature difference. They investigate therewith phase synchronization of the fluid wave motion between the two concentric, rotating cylinders with the temperature oscillations imposed at the cylindrical boundaries.

The periodicity in the forcing is imposed by superposing on the otherwise constant temperature difference between the cylinders recurrent pulses of duration $0.1 < \delta < 1.0$, with 1.0 being the nondimensional length of the cycle, while the waves are amplitude-modulated at constant forcing with a period $P \neq 1.0$. Arnol'd tongues of complete synchronization were observed for sufficiently large δ, with some degree of synchronization occurring even for small δ. The authors imply that this result might point to a mechanism for so-called teleconnections[17,24,85] in the atmospheres of Earth and other planets on time scales that are both shorter and longer than a year.
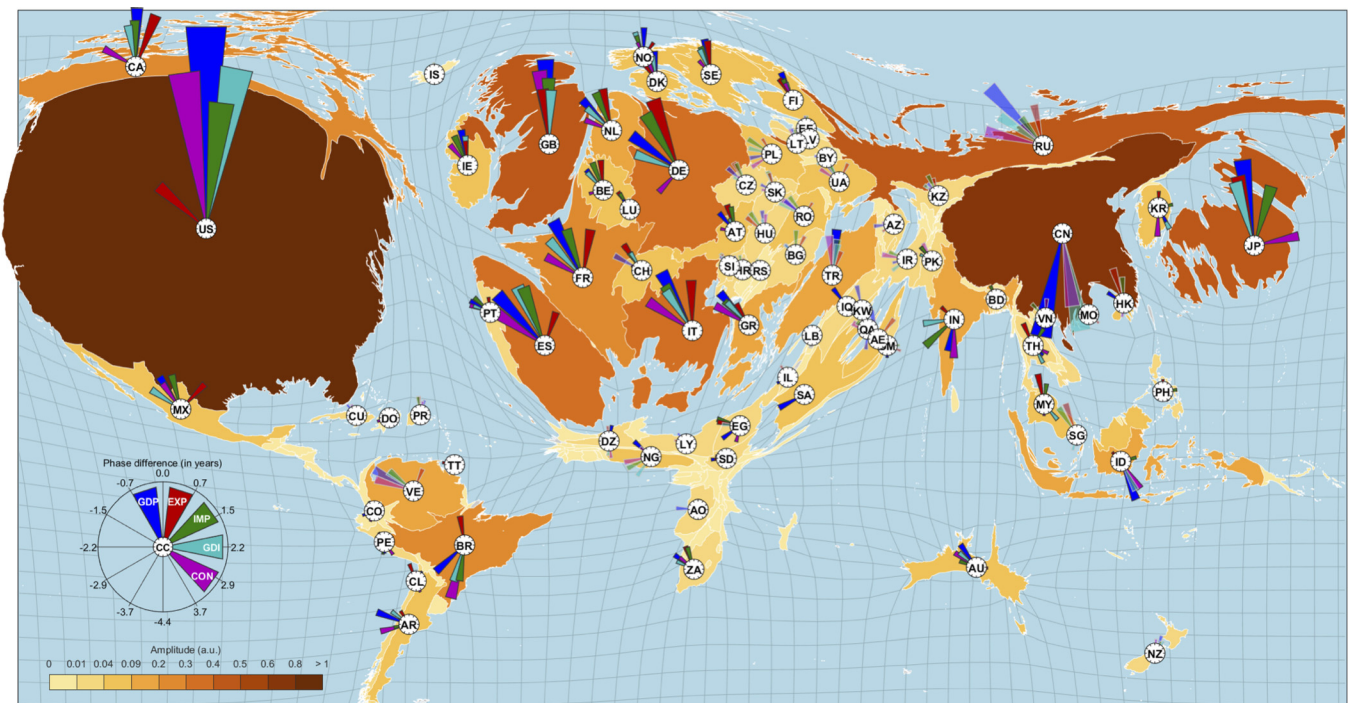
FIG. 2. Global map of phase and amplitude relations in the leading 7–11-year oscillatory mode of world economic activity. For each of the 104 countries analyzed, the relations among the variables' phase and amplitude are shown in a polar coordinate system, with the two-letter country code at the origin. The corresponding codes and colors of the pointers for the five macroeconomic indicators studied—GDP, GDI, CON, EXP, and IMP—are given in the small compass inset at the lower left. Estimates for variables with missing values are indicated by transparent pointers. Phase differences are given with respect to US GDP in a clockwise manner, i.e., positive and negative values indicate a phase that leads or lags the US GDP, respectively. The land area of each country is proportional to its maximum amplitude over all of its variables. Reproduced from Groth and Ghil,[87] with the permission of the American Institute of Physics.

Auer and Hellmann[86] study synchronization in large power grids involving distributed generation of power. Internal phase synchronization emerges among nodes of the grid, and disruptions to the grid are accompanied by loss of synchronization. A stability analysis then identifies the nodes most sensitive to such disruptions.

Groth and Ghil[87] present an application of advanced spectral methods to the study of synchronization in macroeconomic time series for over 100 countries. The authors are among the originators of the methodology of multivariate singular spectrum analysis,[56,88,89] and this methodology is applied here to detect internal phase synchronization without having to define a phase for each subsystem. The results include identifying synchronized clusters of activity, as well as sources of disruption. The key result is the characterization of an internally synchronized world business cycle, along with the relative phases of the five indicators—gross domestic product (GDP), gross fixed capital formation (GDI), consumption expenditure (CON), exports (EXP), and imports (IMP) of goods and services—used for each of the 104 countries in the sample (see Fig. 2).

ACKNOWLEDGMENTS

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