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**A NOTE ON THE WAVE FIELD  
IN THE SURFACE ZONE**

**BY  
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Abstract

The velocity potential is expressed as a surface integral by using a Green function approach, and from this integral solution a simple analytical wave solution can be obtained in the surface zone. This wave solution is found to satisfy the non-linear kinematic boundary condition on the free surface exactly. In this note the dynamic boundary condition at the free surface is satisfied to the second order in the expansion parameter (the wave steepness) on the surface itself and not at the mean surface level which is the case when the Stokes expansion procedure is applied to the wave problem. This wave solution in the surface zone may be considered as a hydrodynamic justification for the engineering wave model introduced by Rodenbusch and Forristall.

## 1. Introduction.

Linear theory (the Airy solution) is in principal incapable of predicting fluid velocities above the mean still water level and also fails to predict the intermittent nature of the velocities in the region between the still water level and the deepest wave trough. Therefore, in ocean engineering there has been suggested several kinds of wave models to improve the description of the wavefield, especially in the crest. Such engineering wave models have been suggested by Wheeler<sup>1</sup>, Chakrabarti<sup>2</sup>, Rodenbusch and Forristall<sup>3</sup>, Mo and Moan<sup>4</sup>, and Gudmestad and Connor<sup>5</sup>. Most of them are obtained by modifying the Airy solution. These engineering wave models have the advantage that they are simple analytical models which are easily applied, and the purpose of introducing them is to obtain theoretical results which are in better agreement with measurements made in laboratory investigations and in the offshore environment than are those obtained from the Airy solution. Common to these wave models is that they are not based on a strictly hydrodynamic approach, none of them satisfies the Laplace equation everywhere in the fluid.

The purpose of this note is to obtain a simple analytical solution which is valid in the surface zone, and which is easily applied. We call for a solution which satisfies the Laplace equation and in addition the free surface boundary conditions to a given order of approximation on the surface itself. In this note the second order problem is analysed, i.e. the boundary conditions are to be satisfied to the second order in the wave steepness on the free surface itself. This is obtained by using a Green function approach to the surface wave problem.

Of course there exist both analytical and numerical wave solutions which satisfy the Laplace equation (for a summary see Sarpkaya and Isacson<sup>6</sup>). In deep water for instance there are the Stokes solutions of orders up to and including the fifth order. However, in the Stokes expansion procedure the boundary conditions are applied at the mean surface level and not on the free surface itself. The method, which we use, allows for the boundary conditions to be applied on the surface itself, and it is the results of such an approach which are presented in this note.



## 2. Formulation and solution.

We consider waves on the free surface of a homogeneous, incompressible and inviscid fluid of finite depth. The wave motion is assumed to be two-dimensional and takes place in the  $(x, z)$ -plane, with the  $x$ -axis in the mean surface level and the  $z$ -axis directed vertically upwards. If the velocity potential is denoted by  $\Phi(x, z, t)$  and if the free surface is given by  $z = \eta(x, t)$ , then the equation and the boundary conditions which govern this wave problem are

$$\nabla^2 \Phi = 0 \quad -d \leq z \leq \eta(x, t) \quad (1)$$

$$\left. \begin{aligned} \frac{\partial \Phi}{\partial t} &= -g\eta - \frac{1}{2}(\nabla \Phi)^2 \\ \frac{\partial \Phi}{\partial n} &= \mathbf{n} \cdot \nabla \Phi = \frac{\partial \eta}{\partial t} \left\{ 1 + \left( \frac{\partial \eta}{\partial x} \right)^2 \right\}^{-\frac{1}{2}} \end{aligned} \right\} \text{ at } z = \eta(x, t) \quad (2)$$

$$\frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = -d, \quad (3)$$

where  $\mathbf{n} = \{\mathbf{k} - (\partial \eta / \partial x)\mathbf{i}\} \{1 + (\partial \eta / \partial x)^2\}^{-\frac{1}{2}}$  and  $\mathbf{i}$  and  $\mathbf{k}$  are the unit vectors in the  $x$ - and the  $z$ -direction respectively.

The boundary conditions (2) are the dynamic and the kinematic boundary conditions at the free surface, expressing that the pressure is continuous across the surface and that there is no mass flux through the surface respectively. The boundary condition (3) ensures that there is no mass flux through the bottom.

We assume that the surface elevation is a periodic function of  $u = kx - \omega t$ , where  $k$  is the wave number and  $\omega$  the frequency, and that it can be written as

$$\eta(x, t) = \eta_0(x, t)/k = \text{Re}\left\{ \frac{a_0}{k} e^{iu} + \frac{a_1}{k} e^{2iu} + \dots \right\} \quad (4)$$

where  $\text{Re}\{\dots\}$  means the real part of the complex function within the brackets.  $\eta_0, a_0$  and  $a_1$  are non-dimensional equations;  $a_0$  is taken to be real and  $a_1$  is to be found.

The Stokes expansion procedure is usually applied to solve this wave problem, i.e. the nonlinear free surface boundary conditions are expanded in series around  $z = 0$ , and the Laplace equation is then solved successively to satisfy the boundary conditions to the different orders of magnitude at  $z = 0$ .



In this note we will use a different approach, a Green function approach, in order to get the boundary conditions satisfied on the surface itself and not at  $z = 0$ . The first thing to do then is to find the expression for the velocity potential  $\Phi$  on the surface. From (2) and (4) we get (see Engevik<sup>7</sup> for the derivation)

$$\begin{aligned}\Phi_s &= \varphi_s + Ct, \quad \text{where} \\ \varphi_s &= -\text{Re}\left\{\frac{ig}{\omega k}(a_0 e^{iu} + \frac{a_1}{2} e^{2iu}) + \frac{ia_0^2 \omega}{8k^2} \left(1 + \frac{g^2 k^2}{\omega^4}\right) e^{2iu} + \dots\right\} \\ C &= \frac{a_0^2 \omega^2}{4k^2} \left(1 - \frac{g^2 k^2}{\omega^4}\right).\end{aligned}\tag{5}$$

where the index  $s$  means "at the free surface".

We write  $\Phi = \varphi + Ct$ , where  $\varphi$  is equal to  $\varphi_s$  at the surface, and introduce the new variables  $u$  (defined above) and  $v = kz$ .  $\varphi = \varphi(u, v)$  has to satisfy the Laplace equation, i.e.

$$\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}\right)\varphi = 0.\tag{6}$$

The kinematic boundary condition at the surface and at the bottom become

$$\begin{aligned}\frac{\partial \varphi_s}{\partial n} &= -\frac{\omega}{k^2} \eta'_0 \{1 + (\eta'_0)^2\}^{-\frac{1}{2}} \quad \text{at } v = \eta_0(u), \\ \frac{\partial \varphi}{\partial v} &= 0 \quad v = -d_0,\end{aligned}\tag{7}$$

where the prime denotes differentiation with respect to  $u$ , and  $d_0 = kd$ .

The solution can be written as (for details, see Engevik<sup>7</sup>)

$$\varphi(u_0, v_0) = \frac{1}{2\pi} \lim_{m \rightarrow \infty} \int_{S_m} \left(\varphi_s \frac{\partial G}{\partial n} - \frac{\partial \varphi_s}{\partial n} G\right) ds,\tag{8}$$

where the integration is along  $S_m$ , the part of the surface which is chosen to lie between  $u = -(2m+1)\pi$  and  $u = (2m+1)\pi$  where  $m$  is an integer.  $\varphi_s$  and  $\partial \varphi_s / \partial n$  are the velocity potential and the normal derivative of the velocity potential at the surface, and  $G(u, v; u_0, v_0)$  is the Green function, i.e.



$$G(u, v, u_o, v_o) = \ln\{(u - u_o)^2 + (v - v_o)^2\}^{\frac{1}{2}} + \ln\{(u - u_o)^2 + (v + v_o + 2d_o)^2\}^{\frac{1}{2}} \quad (9)$$

We introduce the expression for  $G$  and  $\partial\varphi_s/\partial n$  into the integral (8), and make an integration by parts of the integral into which  $\partial\varphi_s/\partial n$  enters. Furthermore we introduce the complex representations  $\varphi_s^c$  and  $\eta_o^c$  of the velocity potential and the surface elevation respectively, defined by

$$\varphi_s = Re\{\varphi_s^c\}, \quad \eta_o = Re\{\eta_o^c\}, \quad (10)$$

and which are given by (4) and (5) to the second order of approximation. Then the integral (8) can be expressed as

$$\begin{aligned} \varphi(u_o, v_o) = & \frac{1}{2\pi} Re \left[ \int_{-\infty}^{\infty} \varphi_s^c \left\{ \frac{(\eta_o(u) - v_o) - \eta_o'(u)(u - u_o)}{(u - u_o)^2 + (\eta_o(u) - v_o)^2} \right\} du \right. \\ & - \frac{\omega}{k^2} \int_{-\infty}^{\infty} \eta_o^c \left\{ \frac{(u - u_o) + \eta_o'(u)(\eta_o(u) - v_o)}{(u - u_o)^2 + (\eta_o(u) - v_o)^2} \right\} du + \int_{-\infty}^{\infty} \varphi_s^c \left\{ \frac{(\eta_o(u) + v_o + 2d_o) - \eta_o'(u)(u - u_o)}{(u - u_o)^2 + (\eta_o(u) + v_o + 2d_o)^2} \right\} du \\ & \left. - \frac{\omega}{k^2} \int_{-\infty}^{\infty} \eta_o^c \left\{ \frac{(u - u_o) + \eta_o'(u)(\eta_o(u) + v_o + 2d_o)}{(u - u_o)^2 + (\eta_o(u) + v_o + 2d_o)^2} \right\} du \right] \quad (11) \end{aligned}$$

The first two integrals in (11) give the solution of the infinite water depth problem. The last two integrals represent the effect from the bottom and must be included in order for the boundary condition at the bottom to be satisfied.

The integrals in (11) can be evaluated by using the residue theorem. They can be considered as integrals along the real axis in the complex  $w$ -plane, where  $w = u + is$ , and the  $u$ -axis is the real axis and the  $s$ -axis the imaginary axis. If we assume that  $\eta_o(w)$  is an analytic function of  $w$ , we get

$$\begin{aligned} \varphi(u_o, v_o) = & \frac{1}{2} Re \left[ \sum_j (\varphi_s^c - i \frac{\omega}{k^2} \eta_o^c)_{w_{1j}} - \sum_j (\varphi_s^c + i \frac{\omega}{k^2} \eta_o^c)_{w_{2j}} \right. \\ & \left. + \sum_j (\varphi_s^c - i \frac{\omega}{k^2} \eta_o^c)_{w_{3j}} - \sum_j (\varphi_s^c + i \frac{\omega}{k^2} \eta_o^c)_{w_{4j}} \right], \quad (12) \end{aligned}$$



where  $w_{1j}, w_{2j}, w_{3j}$  and  $w_{4j}$  are the zeros of  $F_1(w), F_2(w), F_3(w)$  and  $F_4(w)$  respectively in the upper half of the  $w$ -plane, and where

$$\begin{aligned}
 F_1(w) &= \eta_o(w) - v_o + i(w - u_o) \\
 F_2(w) &= \eta_o(w) - v_o - i(w - u_o) \\
 F_3(w) &= \eta_o(w) + v_o + 2d_o + i(w - u_o) \\
 F_4(w) &= \eta_o(w) + v_o + 2d_o - i(w - u_o)
 \end{aligned} \tag{13}$$

In obtaining (12) we have assumed that the zeros of  $F_1(w), F_2(w), F_3(w)$  and  $F_4(w)$  are simple zeros. However, if two zeros should coalesce to form a double zero for some values of  $(u_o, v_o)$ , then the residue at the pole corresponding to the double zero is equal to the sum of the residues at the two poles corresponding to the two coalescing zeros. This means that the expression (12) is valid even if two zeros should coalesce for some values of  $(u_o, v_o)$ . It is easily verified that the contributions from each of the poles satisfy separately the Laplace equation.

The expression (12) is the general solution of  $\varphi(u_o, v_o)$ , and it will be shown how it can give a simple approximation to the wave field in the surface zone.

### 3. Wave motion in the surface zone.

We assume that the depth  $d$  is so large that the bottom has negligible effect on the wave motion in the surface zone. (Numerical calculations have shown that when  $d_o = 4$ , the influence from the bottom on the wave motion in the surface zone is of order  $10^{-4}$  to  $10^{-5}$  when  $a_o = 0.1, 0.2$  and  $0.3$ , and largest when the amplitude is large.) In the following we therefore neglect the last two sums in (12).

The one of the zeros of  $F_1(w)$ , which we denote  $w_{10}$ , plays a more important role than both the other zeros of  $F_1(w)$  and the zeros of  $F_2(w)$  as far as the wave motion in the surface zone is concerned. Let  $(u_o, v_o)$  be a point in the fluid just below the free surface. We put  $u_o = \eta_o(u_o) - \epsilon$ , where  $0 < \epsilon \ll 1$ , into the equation  $F_1(w) = 0$ , and find that

$$\begin{aligned}
 w_{10} &= u_o + \alpha + i\beta, \quad \text{where} \\
 \alpha &= -\epsilon \eta'_o(u_o) / \{1 + (\eta'_o(u_o))^2\} + O(\epsilon^2) \\
 \beta &= \epsilon / \{1 + (\eta'_o(u_o))^2\} + O(\epsilon^2),
 \end{aligned} \tag{14}$$



which shows that  $w_{10}$  lies in the upper half of the  $w$ -plane near the point  $(u_o, 0)$ . Furthermore  $w_{10} \rightarrow u_o$  when  $\epsilon \rightarrow 0$ , i.e. when the free surface is approached from below.  $F_2(w)$  has a zero near the point  $(u_o, 0)$  as well, but this zero lies in the lower half plan and is therefore not among the zeros  $w_{2j}$ .

It is found that on and near the surface most of the contribution to the solution of  $\varphi(u_o, v_o)$  is due to the zero  $w_{10}$ , so that we can neglect the contributions from the other zeros here. However, into the fluid at some distance from the surface the contributions from the other zeros may become significant and must be taken into consideration. Consequently we put

$$\varphi(u_o, v_o) = \frac{1}{2} \text{Re}(\varphi_s^c - i \frac{\omega}{k^2} \eta_o^c) w_{10} \quad (15)$$

to be valid in the surface zone. As mentioned previously this solution satisfies the Laplace equation. It also satisfies the non-linear kinematic boundary condition at the free surface exactly. The dynamic boundary condition (or the condition (5)) is satisfied to order  $a_o^2$  if  $\omega^2 = gk$  and  $a_1 = a_o^2/2$ . Then

$$\left. \begin{aligned} \eta_o^c(u) &= a_o e^{iu} + \frac{a_o^2}{2} e^{2iu} + \dots \\ \varphi_s^c(u) &= -\frac{i\omega}{k^2} (a_o e^{iu} + \frac{a_o^2}{2} e^{2iu} + \dots) \end{aligned} \right\} \quad (16)$$

which can be put into (15) to give,

$$\begin{aligned} \varphi(u_o, v_o) &= -\frac{\omega}{k^2} \text{Re}[i(a_o e^{iw_{10}} + \frac{a_o^2}{2} e^{2iw_{10}}) + \dots] = \\ &= \frac{\omega}{k^2} [a_o \sin u_{10} e^{-s_{10}} + \frac{a_o^2}{2} \sin 2u_{10} e^{-2s_{10}} + \dots] \end{aligned} \quad (17)$$

where we have used that  $w_{10} = u_{10} + is_{10}$ . The velocity components become

$$\left. \begin{aligned} v_x &= \frac{\partial \varphi}{\partial x_o} = \frac{\omega}{k} \text{Re}[i(a_o e^{iw_{10}} + a_o^2 e^{2iw_{10}} + \dots)/(i + \eta'_o(w_{10}))] \\ v_z &= \frac{\partial \varphi}{\partial z_o} = \frac{\omega}{k} \text{Re}[(a_o e^{iw_{10}} + a_o^2 e^{2iw_{10}} + \dots)/(i + \eta'_o(w_{10}))]. \end{aligned} \right\} \quad (18)$$

$w_{10} = u_{10} + is_{10}$  satisfies the equation

$$\frac{a_o^2}{2} \cos 2w + a_o \cos w - v_o + i(w - u_o) = 0. \quad (19)$$

$w_{10}$  is the solution which is equal to  $u_o$  on the surface, i.e. when  $v_o = \eta_o(u_o)$ .

We notice that the surface elevation  $\eta_o = Re\{\eta_o^c\}$  is the same as the one obtained from the Stokes second order theory.

We have done some numerical calculations which are presented in Tables 1-6. The dimensionless velocity  $\bar{v}_x = v_x(\omega/k)^{-1}$  has been calculated both from the integral (11), the formulae (18) and the Stokes second order theory for different values of  $a_o$  (the wave steepness parameter) and with the depth  $d_o = 4$ . When  $d_o = 4$ , then the contribution from the bottom to the velocity field in the surface zone is of order  $10^{-4}$  to  $10^{-5}$ , and largest when the amplitude is large. Tables 1-6 show that the difference between the values of  $\bar{v}_x$  calculated from the integral (11) and the formulae (18) is less than 0.5 % when  $a_o = 0.1$ , about 2% when  $a_o = 0.2$ , and 4-5 % when  $a_o = 0.3$ . This means that the zero  $w_{10}$  contributes to the velocity field in the surface zone with more than 99.5 % when  $a_o = 0.1$ , with about 98 % when  $a_o = 0.2$ , and with 95-96 % when  $a_o = 0.3$ , so we can conclude that the simple expression for  $\varphi(u_o, v_o)$  given by (17) is a good approximation to the integral solution (11). We also notice that the value of  $\bar{v}_x$  calculated from the formulae (18) is closer to the integral solution (11) than is the value given by the Stokes second order theory. Moreover the horizontal velocities underneath the crest ( $u_o = 0$ ) calculated from the integral solution (11) and the formulae (18) are both less than those given by the Stokes second order theory, (which in deep water equals the Airy solution). It should be mentioned that it has long been recognized that the Airy solution leads to horizontal particle velocities in the crest which are too high when compared to velocities measured in laboratory investigations and in the offshore environment<sup>1,8,9</sup>.

Let us consider our solution (17) a little more, and also compare it with the wave model introduced by Rodenbusch and Forristall<sup>3</sup>. As most measurements indicate that the Airy solution predicts too high horizontal velocity in the crest, Rodenbusch and Forristall have suggested a linear extrapolation method to obtain the wave kinematics in the crest. In deep water their wave model in the crest reads



$$\left. \begin{aligned} \varphi(u_o, v_o) &= \frac{\omega}{k^2} a_o (1 + v_o) \sin u_o \\ v_x(u_o, v_o) &= \frac{\omega}{k} a_o (1 + v_o) \cos u_o \\ v_z(u_o, v_o) &= \frac{\omega}{k} a_o (1 + v_o) \sin u_o \end{aligned} \right\} \text{ for } 0 \leq v_o \leq \eta_o(u_o) \quad (20)$$

Beneath the mean water level ( $v_o = 0$ ) the Airy solution is used. We notice that the velocity potential in the crest does not satisfy the Laplace equation.

To apply our solution (17) we have to calculate  $w_{10} = u_{10} + i s_{10}$ , that solution of the complex equation (19) which is equal to  $u_o$  on the surface, i.e. when  $v_o = \eta_o(u_o)$ . This means that  $u_{10}$  and  $s_{10}$  are the solution of the following set of real equations

$$\left. \begin{aligned} \frac{a_o^2}{2} \cos 2u \cosh 2s + a_o \cos u \cosh s &= v_o + s \\ \frac{a_o^2}{2} \sin 2u \sinh 2s + a_o \sin u \sinh s &= u - u_o \end{aligned} \right\} \quad (21)$$

and with  $u_{10} = u_o$  and  $s_{10} = 0$  on the surface. If we put  $u_{10} = u_o$  and  $s_{10} = 0$  into the expression (17) for  $\varphi(u_o, v_o)$  we see immediately that, to order  $a_o^2$ ,  $\varphi(u_o, v_o)$  is equal to the velocity potential taken from Rodenbusch and Forristall's wave model, see (20). The same is found to be true for the velocity field. This shows that, although Rodenbusch and Forristall start from the linear wave solution, they obtain a wave model which satisfies the boundary conditions to order  $a_o^2$  on the free surface itself in the crest. However, the Laplace equation is not satisfied in the crest.

The set of equations (21) is easily solved numerically to obtain  $u_{10}$  and  $s_{10}$ . We have done some calculations which are shown in Tables 7-12. We write  $u_{10} = u_o + \Delta u$  and  $s_{10} = \eta_o(u_o) - v_o + \Delta s$ , where  $\eta_o(u_o) - v_o$  is the distance from the surface to the point  $(u_o, v_o)$  in the fluid. The calculations show that the phase  $u_{10}$ , which is equal to  $u_o$  on the surface, varies as we go down into the fluid, except when  $u_o = 0, \pi$ , in which cases  $\Delta u = 0$ , which follows directly from eqs. (21). For the other values of  $u_o$  the phase change deviates more and more from zero downwards. Also  $|\Delta s|$ , which is zero on the surface, is increasing downwards from the surface. Moreover, both  $|\Delta u|$  and  $|\Delta s|$  are increasing

with the amplitude  $a_o$ . However, although both  $\Delta u$  and  $\Delta s$  are small in the surface zone, they are vital in order for the velocity potential to satisfy the Laplace equation.

If now we introduce  $u_{10} = u_o + \Delta s$  and  $s_{10} = \eta_o(u_o) - v_o + \Delta s$  into our solution and compare it with Rodenbusch and Forristall's wave model, we find that the two solutions differ by terms of order  $a_o^3$ ,  $a_o \Delta u$  and  $a_o \Delta s$ , and of course also by the fact that our solution satisfies the Laplace equation but theirs does not. However, the terms of first and second order in  $a_o$  are equal in the two solutions.

#### 4. Discussion and conclusion.

The velocity potential has been expressed as a surface integral by using a Green function technique, and from this integral solution simple analytical expressions for the velocity potential and velocity field in the surface zone have been derived. Traditionally it is the Stokes expansion procedure which has been applied to the surface wave problem. If  $\varphi_s$  (the velocity potential on the surface),  $\partial\varphi_s/\partial n$  (the normal derivative of the potential on the surface) and  $\eta$  (the surface elevation) are approximated to some given order in the wave steepness parameter  $a_o$ , then these two approaches (the integral method and the Stokes method) will give solutions which are more or less good approximations to the exact solution of the wave problem. It is to be expected that these two approximations will not become exactly equal, and that the difference between them for a given order of  $\varphi_s$ ,  $\partial\varphi_s/\partial n$  and  $\eta$  will increase with  $a_o$ , which the numerical calculations also show. It should also be expected that this difference may become smaller if we go to a higher order of approximation of  $\varphi_s$ ,  $\partial\varphi_s/\partial n$  and  $\eta$ .

In this note we have evaluated  $\varphi_s$ ,  $\partial\varphi_s/\partial n$  and  $\eta$  to order  $a_o^2$ . It is found that on and near the surface most of the contribution to the integral solution comes from the zero  $w_{10}$  of  $F_1(w) = (a_o^2/2) \cos 2w + a_o \cos w - v_o + i(w - u_o)$ . This contribution represents a simple analytic wave solution valid in the surface zone. It satisfies the Laplace equation. It also satisfies the kinematic boundary condition on the free surface exactly, and the dynamic boundary condition to order  $a_o^2$  on the free surface itself. Therefore it should be a better approximation to the wave solution than is the Stokes second order solution which satisfies the boundary conditions at the mean surface level. (In deep water, which we consider here, the Stokes second order solution equals the Airy solution.)

Rodenbusch and Forristall<sup>3</sup> have introduced an engineering wave model which in the crest predicts better measured wave kinematics than does the Airy solution. Although



their wave model is not based on a strictly hydrodynamic approach and does not satisfy the Laplace equation in the crest, it yields a wave field in the crest which, to order  $a_0^2$ , is equal to the one given by our theory. Thus we may say that it has been given a hydrodynamic justification for their wave model.

There is no problem in going to higher orders of approximation in our theory, and it should perhaps be necessary in order to improve the results in the cases with large amplitude.

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Table 1. The dimensionless horizontal velocity when  $a_0 = 0.1$  and  $d_0 = 4$ .

$v_0 = kz_0$	$u_0 = 0.$			$u_0 = \pi/4.$		
	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd
0.105	0.1096	0.1100	0.1112	0.0765	0.0767	0.0759
0.095	0.1086	0.1089	0.1101	0.0758	0.0760	0.0752
0.075	0.1065	0.1068	0.1079	0.0742	0.0744	0.0737
0.055	0.1044	0.1048	0.1057	0.0727	0.0729	0.0722
0.035	0.1024	0.1028	0.1037	0.0713	0.0715	0.0708
0.005	0.0994	0.0998	0.1006			

Table 2. The dimensionless horizontal velocity when  $a_0 = 0.1$  and  $d_0 = 4$ .

$v_0 = kz_0$	$u_0 = 3\pi/4.$			$u_0 = \pi.$		
	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd
-0.0707	-0.0665	-0.0668	-0.0659	-0.0897	-0.0900	-0.0910
-0.0807	-0.0659	-0.0661	-0.0653	-0.0888	-0.0891	-0.0901
-0.1007	-0.0645	-0.0648	-0.0640	-0.0871	-0.0874	-0.0883
-0.1207	-0.0632	-0.0635	-0.0627	-0.0854	-0.0857	-0.0866
-0.1407	-0.0619	-0.0622	-0.0615	-0.0838	-0.0841	-0.0848



Table 3. The dimensionless horizontal velocity when  $a_0 = 0.2$  and  $d_0 = 4$ .

$v_0 = kz_0$	$u_0 = 0.$			$u_0 = \pi/4.$		
	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd
0.220	0.2365	0.2400	0.2495	0.1673	0.1689	0.1630
0.200	0.2322	0.2358	0.2445	0.1638	0.1653	0.1598
0.170	0.2258	0.2296	0.2373	0.1587	0.1603	0.1551
0.120	0.2156	0.2196	0.2257	0.1554	0.1570	0.1520
0.070	0.2057	0.2101	0.2147	0.1507	0.1522	0.1475
0.020	0.1963	0.2010	0.2043	0.1445	0.1461	0.1417
0.000	0.1926	0.1975	0.2002			

Table 4. The dimensionless horizontal velocity when  $a_0 = 0.2$  and  $d_0 = 4$ .

$v_0 = kz_0$	$u_0 = 3\pi/4.$			$u_0 = \pi.$		
	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd	$\bar{v}_x$ from integral (11)	$\bar{v}_x$ from formulae (18)	$\bar{v}_x$ from Stokes' 2nd
-0.1414	-0.1276	-0.1298	-0.1229	-0.1575	-0.1600	-0.1671
-0.1614	-0.1248	-0.1270	-0.1204	-0.1547	-0.1572	-0.1638
-0.1814	-0.1221	-0.1243	-0.1181	-0.1519	-0.1545	-0.1606
-0.2014	-0.1195	-0.1218	-0.1157	-0.1492	-0.1518	-0.1574
-0.2214	-0.1169	-0.1192	-0.1134	-0.1465	-0.1492	-0.1543
-0.2414	-0.1144	-0.1167	-0.1112	-0.1438	-0.1465	-0.1513

**Table 5.** The dimensionless horizontal velocity when  $a_0=0.3$  and  $d_0=4$ .

$v_0=kz_0$	$u_0 = 0.$			$u_0 = \pi/4.$		
	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from
	integral (11)	formulae (18)	Stokes' 2nd	integral (11)	formulae (18)	Stokes' 2nd
				$v_0=kz_0$		
0.345	0.3771	0.3900	0.4242	0.2121	0.2780	0.2624
0.325	0.3708	0.3842	0.4158	0.1921	0.2722	0.2572
0.295	0.3615	0.3757	0.4035	0.1621	0.2637	0.2496
0.245	0.3465	0.3620	0.3838	0.1121	0.2501	0.2374
0.195	0.3318	0.3489	0.3651	0.0621	0.2370	0.2259
0.145	0.3177	0.3366	0.3473	0.0121	0.2245	0.2149
0.095	0.3040	0.3251	0.3303			

**Table 6.** The dimensionless horizontal velocity when  $a_0=0.3$  and  $d_0=4$ .

$v_0=kz_0$	$u_0 = 3\pi/4.$			$u_0 = \pi.$		
	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from	$\bar{v}_x$ from
	integral (11)	formulae (18)	Stokes' 2nd	integral (11)	formulae (18)	Stokes' 2nd
				$v_0=kz_0$		
-0.2121	-0.1870	-0.1943	-0.1717	-0.255	-0.2100	-0.2326
-0.2321	-0.1825	-0.1898	-0.1683	-0.275	-0.2071	-0.2280
-0.2521	-0.1780	-0.1855	-0.1650	-0.295	-0.2042	-0.2235
-0.2721	-0.1738	-0.1814	-0.1617	-0.315	-0.2013	-0.2191
-0.2921	-0.1696	-0.1773	-0.1585	-0.335	-0.1984	-0.2147
-0.3121	-0.1656	-0.1734	-0.1554	-0.355	-0.1955	-0.2105



Table 7.  $u_{10}$  and  $s_{10}$  when  $a_0 = 0.1$

$\underline{u_0 = 0}$				$\underline{u_0 = \pi/4}$			
$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$	$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$
0.105	0	0	0	0.0707	$\pi/4$	0	0
0.085	0	0.02002	0.02	0.0507	0.7870	0.0199	0.02
0.065	0	0.0401	0.04	0.0307	0.7886	0.0398	0.04
0.045	0	0.0602	0.06	0.0107	0.7902	0.0597	0.06
0.025	0	0.0804	0.08	0.0007	0.7911	0.0697	0.07
0.005	0	0.1006	0.10				

Table 8.  $u_{10}$  and  $s_{10}$  when  $a_0 = 0.1$ .

$\underline{u_0 = 3\pi/4}$				$\underline{u_0 = \pi}$			
$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$	$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$
-0.0707	$3\pi/4$	0	0	-0.095	$\pi$	0	0
-0.0907	2.3574	0.0199	0.02	-0.115	$\pi$	0.01998	0.02
-0.1107	2.3586	0.0398	0.04	-0.135	$\pi$	0.0399	0.04
-0.1307	2.3598	0.0597	0.06	-0.155	$\pi$	0.0599	0.06
-0.1507	2.3610	0.0795	0.08	-0.175	$\pi$	0.0798	0.08
-0.1707	2.3622	0.0993	0.10	-0.195	$\pi$	0.0996	0.10

Table 9.  $u_{10}$  and  $s_{10}$  when  $a_0 = 0.2$

$u_0 = 0$				$u_0 = \pi/4$			
$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$	$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$
0.22	0	0	0	0.1414	$\pi/4$	0	0
0.20	0	0.02006	0.02	0.1214	0.7889	0.0194	0.02
0.16	0	0.0605	0.06	0.0814	0.7961	0.0583	0.06
0.12	0	0.1014	0.10	0.0414	0.8034	0.0974	0.10
0.08	0	0.1429	0.14	0.0014	0.8108	0.1366	0.14
0.04	0	0.1848	0.18				
0.00	0	0.2273	0.22				

Table 10.  $u_{10}$  and  $s_{10}$  when  $a_0 = 0.2$

$u_0 = 3\pi/4$				$u_0 = \pi$			
$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$	$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$
-0.1414	$3\pi/4$	0	0	-0.18	$\pi$	0	0
-0.1614	2.3582	0.0198	0.02	-0.20	$\pi$	0.01998	0.02
-0.1814	2.3602	0.0395	0.04	-0.22	$\pi$	0.0399	0.04
-0.2014	2.3621	0.0592	0.06	-0.24	$\pi$	0.0598	0.06
-0.2214	2.3641	0.0788	0.08	-0.26	$\pi$	0.0796	0.08
-0.2414	2.3660	0.0983	0.10	-0.28	$\pi$	0.0994	0.10



Table 11.  $u_{10}$  and  $s_{10}$  when  $a_0 = 0.3$

$u_0 = 0$				$u_0 = \pi/4$			
$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$	$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$
0.345	0	0	0	0.2121	$\pi/4$	0	0
0.305	0	0.0404	0.04	0.1721	0.7966	0.0367	0.04
0.225	0	0.1237	0.12	0.0921	0.8197	0.1107	0.12
0.145	0	0.2107	0.20	0.0121	0.8440	0.1850	0.20
0.065	0	0.3023	0.28	0.0021	0.8471	0.1943	0.21
0.005	0	0.3745	0.34				

Table 12.  $u_{10}$  and  $s_{10}$  when  $a_0 = 0.3$

$u_0 = 3\pi/4$				$u_0 = \pi$			
$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$	$v_0 = kz_0$	$u_{10}$	$s_{10}$	$\eta_0 - v_0$
-0.2121	$3\pi/4$	0	0	-0.255	$\pi$	0	0
-0.2321	2.3586	0.0197	0.02	-0.275	$\pi$	0.01998	0.02
-0.2521	2.3609	0.0393	0.04	-0.295	$\pi$	0.0399	0.04
-0.2721	2.3633	0.0588	0.06	-0.315	$\pi$	0.0598	0.06
-0.2921	2.3656	0.0722	0.08	-0.335	$\pi$	0.0796	0.08
-0.3121	2.3679	0.0976	0.10	-0.355	$\pi$	0.0994	0.10







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