

time grid label for this notebook: NBR_synMPT_fullMPTgrid_nB

Predictive asymmetry analysis of climate system dynamics during the MPT.

Written by Maria Salem, in supplement to master thesis.

In this notebook we compute the normalized predictive asymmetry (\mathcal{A}) between empirical climate proxy records over the entire time span of the MPT (**time interval 1250-700 ka BP**). The aim is to deduce causal coupling strength and directionality in the climate system between ice volume, northern hemisphere summer insolation, and Southern Ocean Fe fertilization prior to the Mid-Pleistocene Transition.

Preamble: We import all necessary packages, wrangled time series and predefined functions for analysis, by including notebook 3 MA_NB3_Toolbox.ipynb

In [9]:

```
using NBInclude
@nbinclude "../NBRs_ePalus_ns20/NB3_Toolbox_ns20.ipynb" ;
```

The LR04 d18O record spans from -5319.0 to -1.0 kyrs BP.
The La2004 insolation time series (Ins) spans from -5000.0 to -0.0 k
yrs BP.
Spratt Lisiecki GSL stack (SpratSL) spans from -797.0 to -1.0 kyrs B
P.
The Elderfield GSL record (EldSL) spans from -1574.0 to -8.0 kyrs B
P.
The Grant sea level record (GraSL) spans from -491.0 to -1.0 kyrs B
P.
The Rohling sea level record (RohSL) spans from -5329.0 to -1.0 kyrs
BP.
The Bereiter pCO₂ record (BerCO₂) spans from -802.0 to -3.0 kyrs BP.
The Chalk pCO₂ record (ChaCO₂) spans from -1240.0 to -1092.0 kyrs B
P.
The Lambert dust record (IceDust) spans from -799.0 to -13.0 kyrs B
P.
The Martinez-Garcia Fe flux record (MarFe) spans from -3999.0 to -2.
0 kyrs BP.
The higher resolution part of the MG record spans from -800.0 to -0.
5 kyrs BP. The record is now on the common time grid. The record is now
on the common time grid. Results are saved in the .jld2 file. 38.7532
63 seconds (164.14 M allocations: 11.093 GiB, 15.15% gc time)

The following records are used for analysis (references used as record labels, in bold):

- **Ins** / La2004: Numerical solution for northern hemisphere insolation (top of atmosphere solar flux mean for summer solstice at $65^{\circ} N$), by *Laskar et al.* (2004).
- **LR04**: $\delta^{18}O$ global reference stack - a principal component analysis of 57 $\delta^{18}O$ records, by *Lisiecki & Raymo* (2005).
- **EldSL** / E: Temperature deconvolution of $\delta^{18}O$ signal in marine sediment core from the Chatnam Rise (Pacific Ocean), spanning the last 1.5 Myrs, by *Elderfield et al.* (2012).
- **MedSL** / R: Relative sea level at Straight of Gibraltar, spanning the last 5.3 Myrs (Rohling et al., 2014).
- **MarFe** / MG: Marine sediment record of Fe accumulation in the Southern Ocean spanning the last 4 Ma, by *Martinez-García et al.* (2011).

Unfortunately, we do not have any proxy for pCO₂ with satisfactory resolution over this time interval.

Outline for this notebook:

Get the time series on a common time grid

1. Decide on a common time interval for analyses in this notebook.
2. Cut all records to the relevant time interval.

Run pairwise analyses for predictive asymmetry between the time series

1. Compute a family of transfer entropies and predictive asymmetries for each time series pair (function `computePredictiveAsymmetry` defined in NB3).
 2. Normalize the predictive asymmetries and produce a results plot showing the 95% confidence interval of predictive asymmetry between each time series pair. **Produce an overview plot of predictive asymmetry results over the notebook's time interval**
 3. Produce an overview plot of the ensemble of predictive asymmetry results for the time period here studied. The ensemble plot will be use in the results chapter of main text.
-

Get the time series on a common time grid

For our method to work, it is important we have the data on the same time grid - covering the exact same time interval and with a regular time step. The time step was set by binsize of the grid in the `BinnedResampling` in NB1, where we chose a regular grid with one value for every 1000 years. The common time interval, on the other hand, we set individually in each notebook.

1. Decide on common time grid

Determine the common time interval for analyses in this notebook. We decide on a time interval for analysis according to where the records overlap. Additionally, we can delimit the time interval to a period of interest - in our case, we want to see if there are any changes in causal dynamics before, during and after the Mid-Pleistocene Transition.

In [10]:

```
# Determine the common time interval

# the onset of the MPT defines the start of the common grid
gridstart = -1250

# the end of the MPT defines the end of the common grid
gridend = -700

print("Time interval for analyses in this notebook is from ", -gridstart, " ka B
P to ", -gridend, " ka BP")
# Note: opposite to NB1, tmin and tmax here indicate the binmidpoints of the com
mon grid.
```

Time interval for analyses in this notebook is from 1250 ka BP to 70
0 ka BP

Recall the time step for the common grid. This was given by the binsize in the grids for BinnedResampling in NB1. We also recall time steps for the additional analyses of the high resolution records.

In [11]:

```
# Recall the binsize
binsize_1 = 1           # 1 kyrs - is the default timestep on which all time seri
es are binned (NB1)

print("All records are on a regular grid with timestep of ", binsize_1, " kyr.")
```

All records are on a regular grid with timestep of 1 kyr.

We can now define the time grid for analyses in this notebook. Objects associated with this grid are labeled with the grid suffix `synMPT_fullMPTgrid_nB`.

In [12]:

```
# the common grids for the time series are then defined by

commongrid_1 = gridstart : 1 : gridend
Binmidpoints_commongrid_1 =[commongrid_1[i] for i in 1:length(commongrid_1)]

print(gridstart : 1 : gridend , " defines the main common grid for analyses in t
his notebook.")

#= *Note*:
the common grid for the time series in this notebook is defined by tmin and tmax
as the *bin midpoints*
(opposite to NB1, where tmin and tmax defined the *grid edges* for binned resamp
ling.) =#
```

-1250:1:-700 defines the main common grid for analyses in this notebook.

Let's see what binning/graining of state space is used for estimations of transfer entropy in this notebook. The transfer entropy is a probability distribution, which we estimate by the visitation frequency test. Our estimation of the transfer entropy is thus sensitive to the binning resolution of state space (the amount of bins we use to count visitations), which is given by ϵ . ϵ is chosen under the hood of the `computePredictiveAsymmetry` function (see NB3), as a function of time series length. We use Milan Palus' [] proposed rule of thumb to approximate the more optimal binning of state space, which we refer to as the Palus horizon. As a curiosity, let's see what gridding of state space is for estimations of transfer entropy in this notebook. For overview, let's check the ϵ used for the time series in this notebook.

In [13]:

```
# Palus horizon

# 1kyr time step
N_1 = length(Binmidpoints_commongrid_1)
eD = 3 # embedding dimension in the VisitationFrequencyTest - we have used the default 3
epsilon = Int(round( N_1^(1/(eD+1)) ))
print("For binning of state space to estimate transfer entropy,
the binning argument epsilon=", epsilon, " for the 1 kyr timestep grid (set by the Palus horizon).")
```

For binning of state space to estimate transfer entropy,
the binning argument $\epsilon=5$ for the 1 kyr timestep grid (set by the Palus horizon).

False positive rate. As shown by sensitivity tests by Haaga et al. (2020), the false positive rate f is a function of the time series length. Haaga et al. (2020) heuristically show \mathcal{A} to have a significant bettering of the false positive rate at a time series length of around 150 datapoints, for most types of systems, with a few weird-ass exceptions such as for Henon-systems). We therefore choose to set our f to 1 (%) for analyses with time series length above 200 datapoints (to be on the safer side). For time series of shorter length, we raise f to 1.5. A higher f means we will get fewer false positives (higher specificity), but also more false negatives (lower sensitivity). We make this choice to be sure we can draw robust conclusions.

In [14]:

```
# time series lengths in this notebook
print("Time series on the 1 kyr grid are of length ", length(Binmidpoints_commongrid_1), " - we will use f = 1 in interpretation of the results.")
```

Time series on the 1 kyr grid are of length 551 - we will use $f = 1$ in interpretation of the results.

In [15]:

```
f=1
```

Out[15]:

```
1
```

2. Cut the time series to the decided time interval.

We select the time series' common time array as following: all time values greater (younger) than, or equal to, the common grid's starting midpoint `tmin`, and all values smaller (older) than, or equal to, the common grid's ending midpoint `tmax`.

Insolation - La2004

La2004 is a numerical solution for insolation, with no associated uncertainty for the time interval here observed. Therefore no confidence. interval on this record

- Default version (`La2004_insol_cut`) is on a grid with 1 kyr timestep between each insolation value (as in the original dataset).

In []:

```
[ ]
```

In [16]:

```
# cut out the relevant time interval (tmin:tmax) from the time array
La2004_t_cut = La2004_t_fulllength[(La2004_t_fulllength .>= gridstart) .& (La2004_t_fulllength .<= (gridend))]
# cut out the relevant time interval from the insolation values array
La2004_insol_cut = La2004_insol65N_fulllength[(La2004_t_fulllength .>= gridstart) .& (La2004_t_fulllength .<= (gridend))]

# check that we have cut the correct time interval
print(La2004_t_cut[1] : La2004_t_cut[end] , " - the cut version of La2004 time series
", gridstart : gridend , " - is now the same as the common grid") # so all good

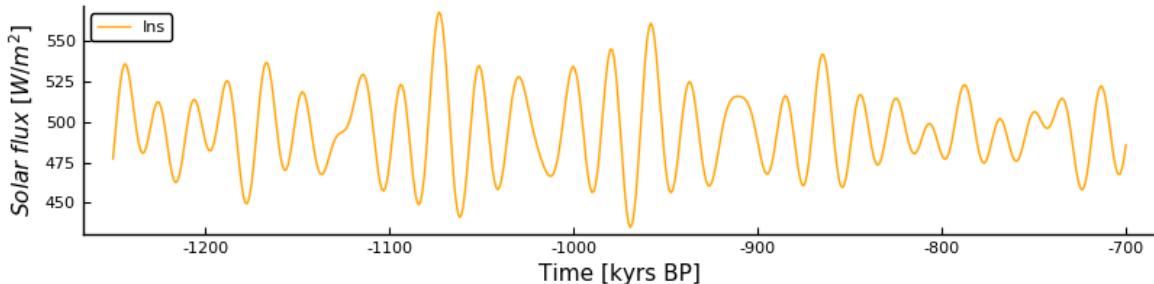
# define the time series plot

plot_La2004 =
plot(La2004_t_cut, La2004_insol_cut,
      color = :orange,
      label = "Ins",
      xlabel = "Time [kyrs BP]",
      ylabel = L"Solar \ flux \ [W/m^{2}]",
      legend = :topleft,
      grid = false,
      size = (800,200)
    )
```

-1250.0:1.0:-700.0 - the cut version of La2004 time series

-1250:-700 - is now the same as the common grid

Out[16]:



uvD cut troubleshooting

After several attempts, we see that neither the `>=` operator nor the `=` operator work for `UncertainIndexValue` format that our time series are given in. It therefore cuts one value too much at the beginning and end of our grid. We will therefore set the the grid edges `tmin` and `tmax` a littlebit before and after the start and end of the common grid, so that we don't lose the first and last datapoint of the record.

In [17]:

```
#= After several attempts, we see that the >= (greater than OR EQUAL TO) operator does not work for uivDs (our time series format, which we want to cut). We will therefore set the grid edges tmin and tmax a littlebit before and after the start and end of the common grid, so that we don't lose the datapoints on the edges =#
tmin = gridstart - 0.001
tmax = gridend + 0.001
;
#= select the time series' common time array as following:
all values greater (younger) than ``tmin``,
and smaller (older) than ``tmax`` =#
```

LR04

In [18]:

```
?cut_timeinterval_from_uivD
search: cut_timeinterval_from_uivD
```

Out[18]:

No documentation found.

```
cut_timeinterval_from_uivD is a Function.
# 1 method for generic function "cut_timeinterval_from_uivD":
[1] cut_timeinterval_from_uivD(ts::UncertainIndexValueDataset, tmin,
tmax, bmp_cg::StepRange{Int64,Int64}) in Main at /Users/maria/Jottacloud/MASTER_2.0/Koding/NBRs_ePalus_ns20/NB3_Toolbox_ns20.ipynb:In
[4]:18
```

In [19]:

```
LR04 = LR04_binned_fulllength_fullageunc

# Cut out the relevant time interval of the binned LR04 time series
LR04_cut = UncertainIndexValueDataset(
    LR04.indices[(LR04.indices .> tmin) .& (LR04.indices .< tmax)], # pick out all indices (ages) where ages are larger than tmin and smaller than tmax
    LR04.values[(LR04.indices .> tmin) .& (LR04.indices .< tmax)] # pick out all values where the corresponding ages are larger than tmin and smaller than tmax
)
```

Out[19]:

```
UncertainIndexValueDataset{UncertainIndexDataset,UncertainValueDataset} containing 551 uncertain values coupled with 551 uncertain indices
```

In [20]:

```
# check that the record was cut correctly and is now on the common time grid

record_timestep = LR04_cut.indices[2].value - LR04_cut.indices[1].value
record_timegrid = LR04_cut.indices[1].value : record_timestep : LR04_cut.indices
[end].value

if record_timegrid == Binmidpoints_commongrid_1
print("The record is now on the common time grid")
else
print("Something's wrong")
end

## or alternatively

binmidpoints_ts =[LR04_cut.indices[i].value for i in 1:length(LR04_cut.indices)]

if binmidpoints_ts == Binmidpoints_commongrid_1
print("The record is now on the common time grid")
else
print("Something's wrong")
end

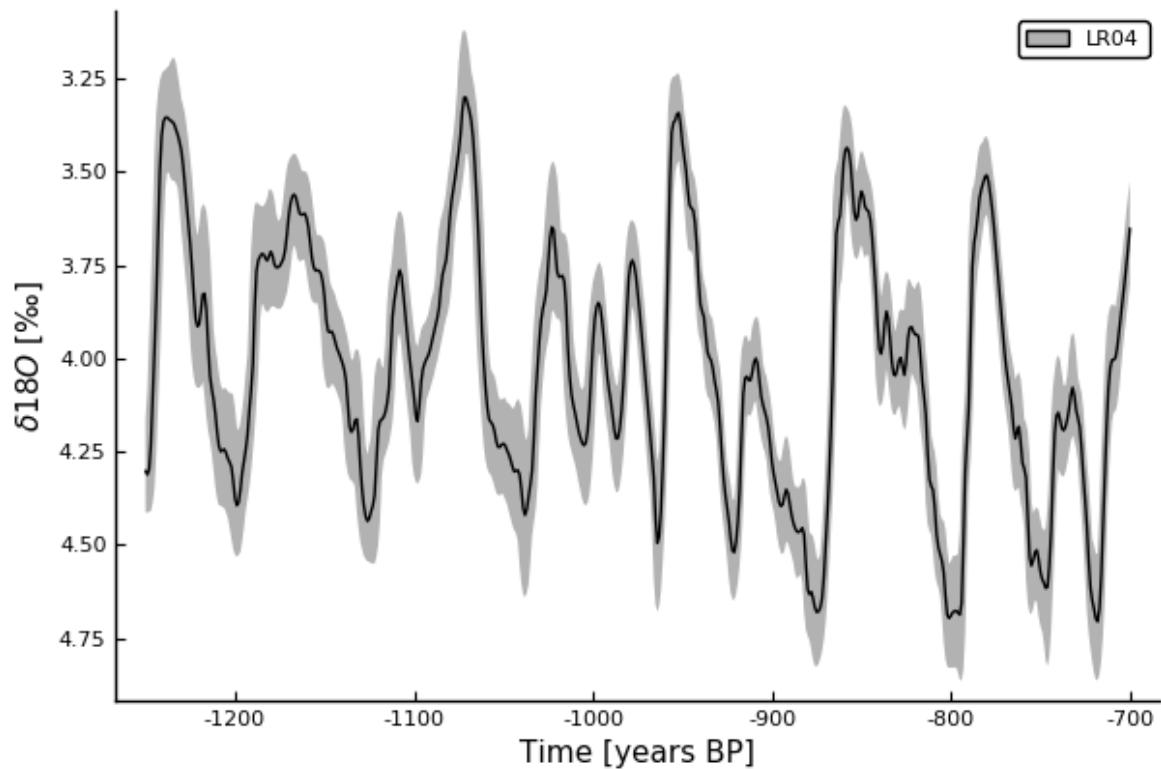
#### Plot time series with the 95% confidence interval

# computing the median in each bin (0.5 quantile), and the confidence interval we want to use (95%)
bin_median = quantile.(LR04_cut.values, 0.5)
bin_upperq = quantile.(LR04_cut.values, 0.975) .- bin_median
bin_lowerq = bin_median .- quantile.(LR04_cut.values, 0.025)
;

plot_LR04 =
plot(binmidpoints_ts, bin_median,
    ribbon = (bin_lowerq, bin_upperq),
    fillalpha = 0.3,
    color = :black,
    label = "LR04",
    xlabel = "Time [years BP]",
    ylabel = L"\delta^{18}\text{O} [\text{perthousand}]",
    grid = false, yflip = true
)
```

The record is now on the common time gridThe record is now on the co
mmon time grid

Out[20]:



In [21]:

```

LR04 = LR04_binned_fulllength_noageunc

# Cut out the relevant time interval of the binned LR04 time series
LR04_cut_noageunc = UncertainIndexValueDataset(
    LR04.indices[(LR04.indices .> tmin) .& (LR04.indices .< tmax)], # pick out all indices (ages) where ages are larger than tmin and smaller than tmax
    LR04.values[(LR04.indices .> tmin) .& (LR04.indices .< tmax)]# pick out all values where the corresponding ages are larger than tmin and smaller than tmax
)

# check that the record was cut correctly and is now on the common time grid
binmidpoints_ts = [LR04_cut_noageunc.indices[i].value for i in 1:length(LR04_cut_noageunc.indices)]
begin
    if binmidpoints_ts == Binmidpoints_commongrid_1
        print("The record is now on the common time grid")
    else
        print("Something's wrong")
    end
end

#### Plot time series with the 95% confidence interval

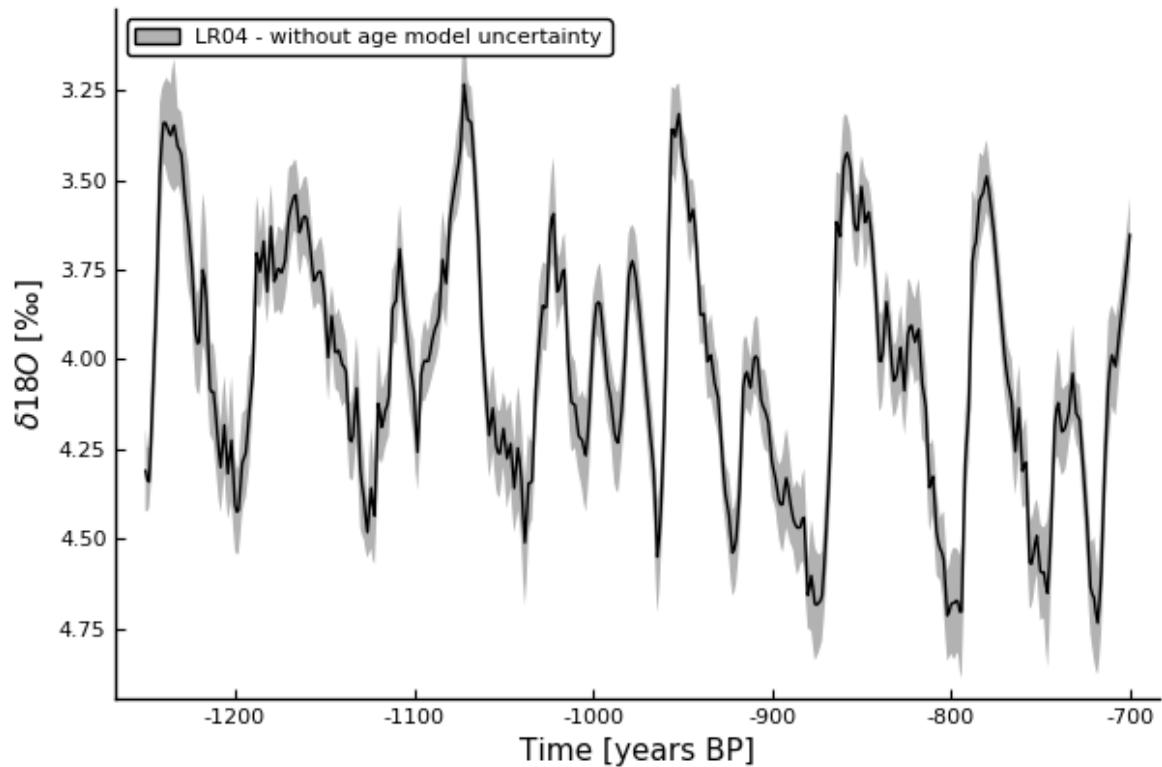
# computing the median in each bin (0.5 quantile), and the confidence interval we want to use (95%)
bin_median = quantile.(LR04_cut_noageunc.values, 0.5)
bin_upperq = quantile.(LR04_cut_noageunc.values, 0.975) .- bin_median
bin_lowerq = bin_median .- quantile.(LR04_cut_noageunc.values, 0.025)

plot_LR04_noageunc =
plot(binmidpoints_ts, bin_median,
    ribbon = (bin_lowerq, bin_upperq),
    fillalpha = 0.3, legend = :topleft,
    color = :black,
    label = "LR04 - without age model uncertainty",
    xlabel = "Time [years BP]",
    ylabel = L"\delta{18}O [\perthousand]",
    grid = false, yflip = true
)

```

The record is now on the common time grid

Out[21]:



Sea level - Elderfield record

In [22]:

```

ts = E_binned_fulllength_ageunc

# cut time series
ts_cut = UncertainIndexValueDataset(
    ts.indices[(ts.indices .> tmin) .& (ts.indices .< tmax)],
    ts.values[(ts.indices .> tmin) .& (ts.indices .< tmax)]) 

# check that the record was cut correctly and is now on the common time grid
binmidpoints_ts =[ts_cut.indices[i].value for i in 1:length(ts_cut.indices)]
begin
    if binmidpoints_ts == Binmidpoints_commongrid_1
        print("The record is now on the common time grid")
    else
        print("Something's wrong")
    end
end

# save cut time series in an unambiguous name
E_cut = ts_cut

### Plot time series with the 95% confidence interval

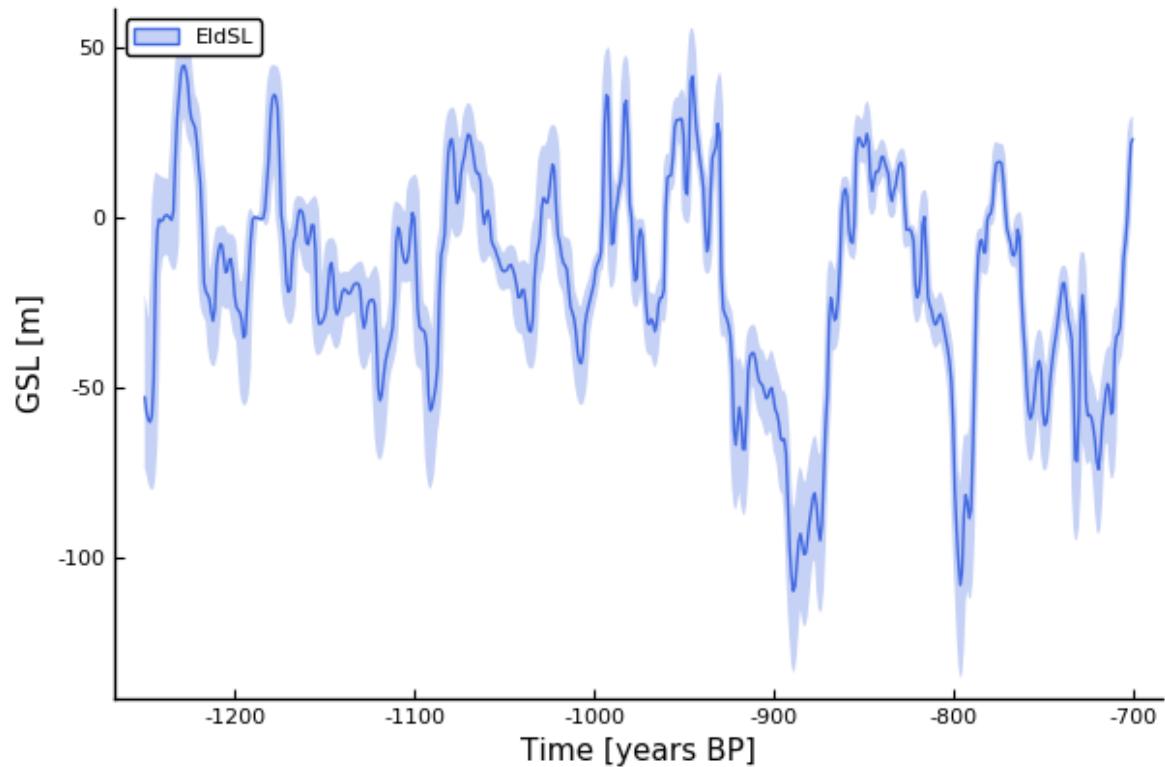
# computing the median in each bin (0.5 quantile), and the confidence interval we want to use (95%)
bin_median = quantile.(ts_cut.values, 0.5)
bin_upper = quantile.(ts_cut.values, 0.975) .- bin_median
bin_lower = bin_median .- quantile.(ts_cut.values, 0.025)
;

plot_E =
plot(binmidpoints_ts, bin_median,
    ribbon = (bin_lower, bin_upper),
    fillalpha = 0.3, legend = :topleft, color = :royalblue,
    label = "EldSL",
    xlabel = "Time [years BP]",
    ylabel = "GSL [m]",
    grid = false
)

```

The record is now on the common time grid

Out[22]:



Sea level - Rohling record

In [23]:

```

ts = R_binned_full

# cut time series
ts_cut = UncertainIndexValueDataset(
    ts.indices[(ts.indices .> tmin) .& (ts.indices .< tmax)],
    ts.values[(ts.indices .> tmin) .& (ts.indices .< tmax)])

# check that the record was cut correctly and is now on the common time grid
binmidpoints_ts =[ts_cut.indices[i].value for i in 1:length(ts_cut.indices)]
begin
    if binmidpoints_ts == Binmidpoints_commongrid_1
        print("The record is now on the common time grid")
    else
        print("Something's wrong")
    end
end

# save in an unambiguous name
R_cut = ts_cut

### Plot time series with the 95% confidence interval

# computing the median in each bin (0.5 quantile), and the confidence interval we want to use (95%)
bin_median = quantile.(ts_cut.values, 0.5)
bin_upper = quantile.(ts_cut.values, 0.975) .- bin_median
bin_lower = bin_median .- quantile.(ts_cut.values, 0.025)
;

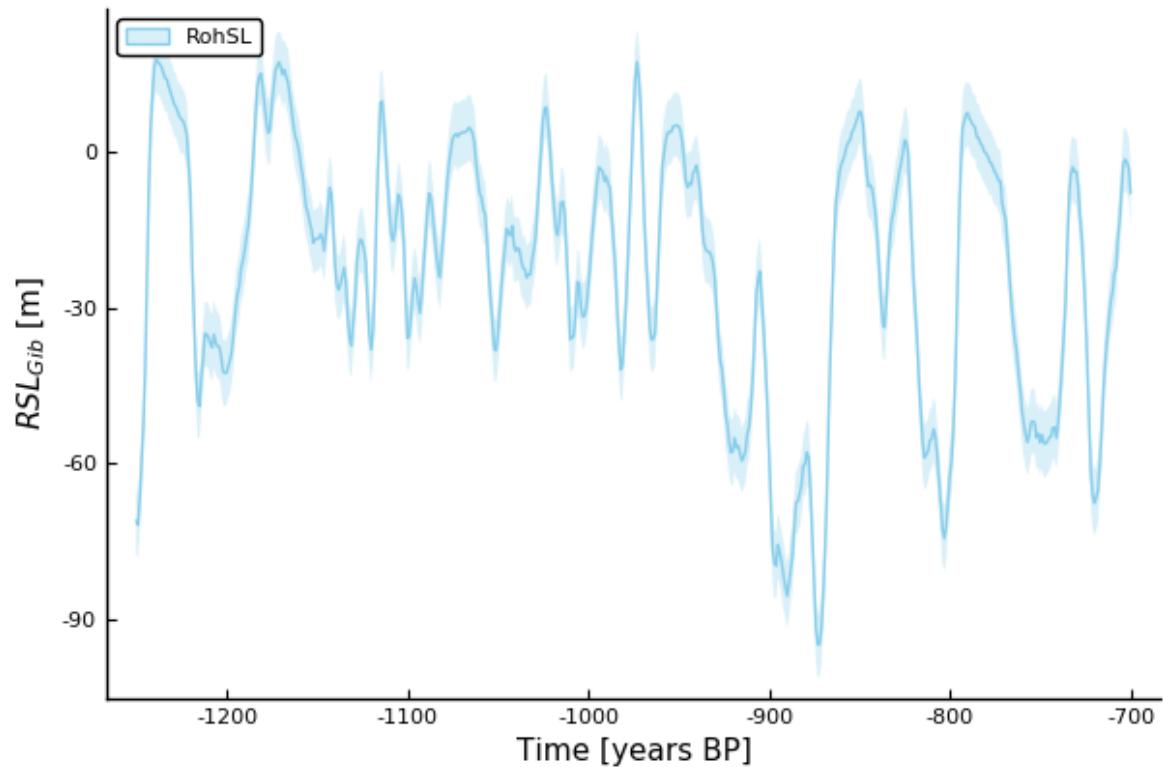
binmidpoints_commongrid =[ts_cut.indices[i].value for i in 1:length(ts_cut.indices)]

plot_R =
plot(binmidpoints_ts, bin_median,
      ribbon = (bin_lower, bin_upper),
      fillalpha = 0.3, legend = :topleft, color = :skyblue,
      label = "RohSL",
      xlabel = "Time [years BP]",
      ylabel = string(L"RSL_{Gib}", " [m]"),
      grid = false
)

```

The record is now on the common time grid

Out[23]:



dust - Martinez-García Fe MAR record

In [24]:

```

ts = MG_binned_fulllength

# cut time series
ts_cut = UncertainIndexValueDataset(
    ts.indices[(ts.indices .> tmin) .& (ts.indices .< tmax)],
    ts.values[(ts.indices .> tmin) .& (ts.indices .< tmax)])

# check that the record was cut correctly and is now on the common time grid
binmidpoints_ts =[ts_cut.indices[i].value for i in 1:length(ts_cut.indices)]
begin
    if binmidpoints_ts == Binmidpoints_commongrid_1
        print("The record is now on the common time grid")
    else
        print("Something's wrong")
    end
end

# save the cut time series uivD in an unambiguous name
MG_cut = ts_cut

### Plot time series with the 95% confidence interval

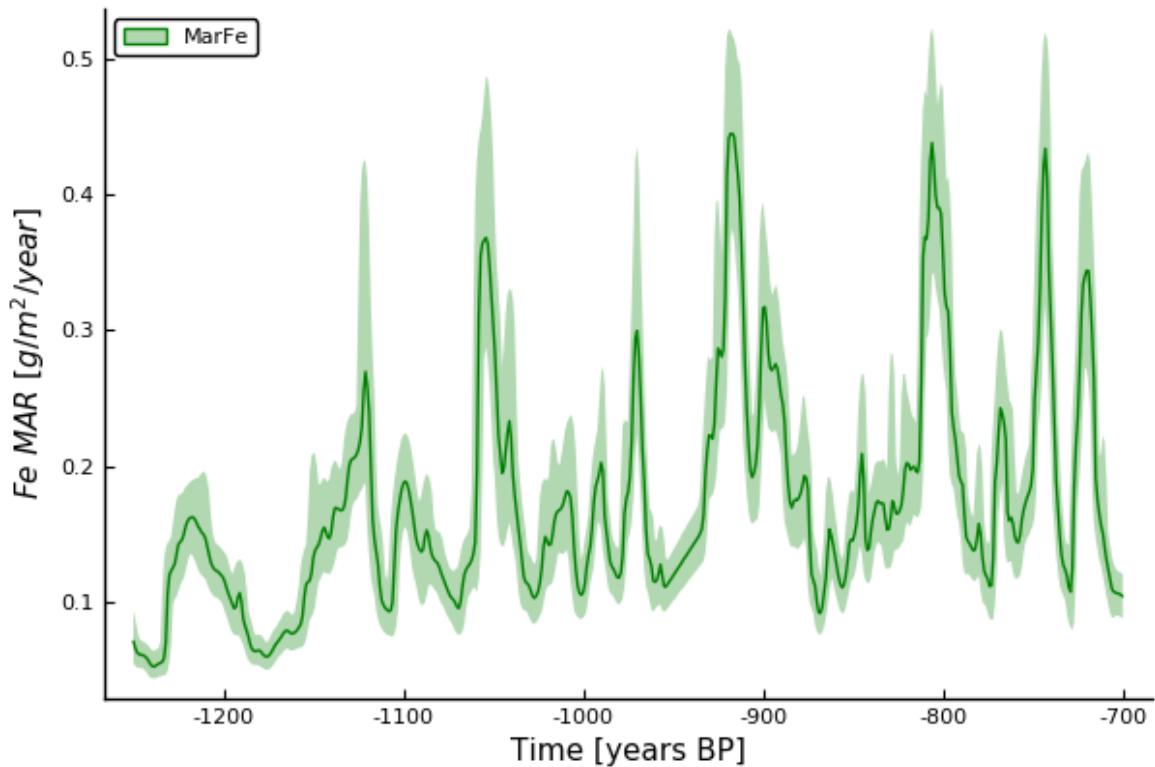
# computing the median in each bin (0.5 quantile), and the confidence interval we want to use (95%)
bin_median = quantile.(ts_cut.values, 0.5)
bin_upper = quantile.(ts_cut.values, 0.975) .- bin_median
bin_lower = bin_median .- quantile.(ts_cut.values, 0.025)
;

plot_MG =
plot(binmidpoints_ts, bin_median,
    ribbon = (bin_lower, bin_upper),
    fillalpha = 0.3, legend = :topleft, color = :green,
    label = "MarFe",
    xlabel = "Time [years BP]",
    ylabel = L"Fe \ MAR \ [g/m^{2}/year]",
    grid = false
)

```

The record is now on the common time grid

Out[24]:



Overview of time series MPT interval

In [50]:

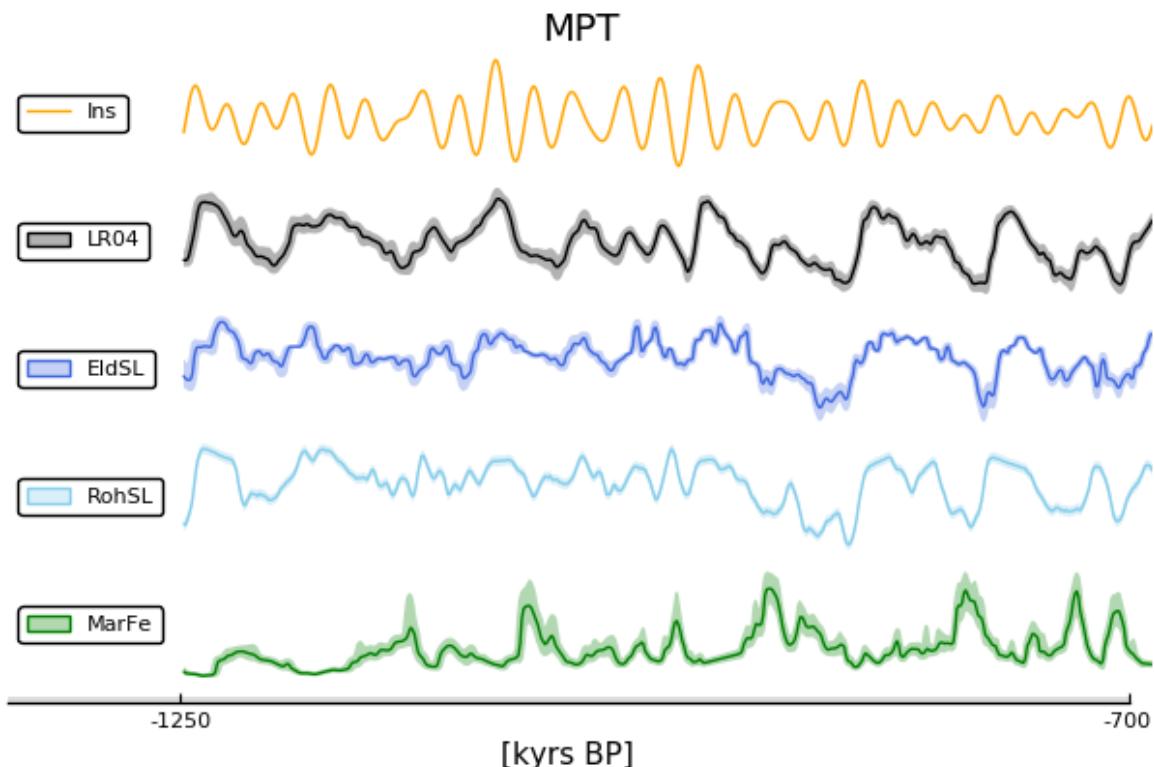
```
# overview time series

po0 = plot(plot_La2004, xlabel = "", xaxis = :off, title = "MPT")
#      (1240-1090 ka BP))
po1 = plot(plot_LR04, xlabel = "", xaxis = :off)
po3 = plot(plot_E, xlabel = "", xaxis = :off)
po4 = plot(plot_R, xlabel = "", xaxis = :off)
po6 = plot(plot_C, xlabel = "", xaxis = :off)
po8 = plot(plot_MG, xlabel = "", xaxis = :off)
p_xaxis = plot(xlabel = "[kyrs BP]", xlabelfont = 8, xaxis = :on, xmirror = false)

l = @layout [a;b;c;d;e;f{0.01h}]

po_alltimeseries = plot(po0,po1,po3,po4,po8, p_xaxis,
    layout = l, #grid(7,1),
    #size = (250, 600),
    xlims = (gridstart-100, gridend), xticks = (gridstart : gridend-gridstart : gridend),
    ylabel = "", ytickfont = false, yaxis = :off,
    legend = :left, bg_legend = :white)
```

Out[50]:



Compute the predictive asymmetry between the time series

- Compute the predictive asymmetries between each time series pair (method explained in NB2 and function defined in NB3)
 - Normalize the results (function defined in NB3), and make a plot showing the 95% confidence interval
-

Outline for analyses

1. $\delta^{18}O$ - insolation

- LR04 - La2004

1. GSL - insolation

- Elderfield - La2004
- Rohling - La2004

1. GSL / $\delta^{18}O$ - pCO₂ (syn-MPT)

2. GSL / $\delta^{18}O$ - dust

- Elderfield - Martinez-García
- Rohling - Martinez-García

1. pCO₂ - dust

2. pCO₂ - insolation

3. dust - insolation

- Martinez-García - La2004
-

1. \mathcal{A} between $\delta^{18}O$ and insolation

LR04 - La2004

Let's start with LR04 and insolation - because the LR04 d¹⁸O stack is tuned to the insolation signal, and this "bias" is carried on by tuning other gsl records to the LR04 stack, so it is interesting to see what to keep in mind from this.

(Both the Elderfield and the Spratt-Lisiecki sea-level records are tuned to the LR04, whose age model is based on a lag of $\delta^{18}O$ behind insolation. This is based on the understanding of the Milankovitch cycles as a key driver behind the ice age cycles, but is however, a caveat for our method, which seeks to infer this causal connection, rather than a priori assume it. We will therefore compute the predictive asymmetry between the LR04 $\delta^{18}O$ record and the La2004 northern hemisphere summer insolation record, to ... check for bias?)

What would we expect? Drive insolation -> response $\delta^{18}O$. But we would expect this either way if it was caused by real signal or age model assumptions... Not sure what to make out as an argument here..)

Compute the predictive asymmetry between the time series pair. We compute the normalized predictive asymmetry using the function defined in NB3.

In [23]:

```
# Recall the time series on the common grid as X and Y
X = LR04_cut
Y = La2004_insol_cut

# Compute the predictive asymmetry (function defined in NB3)
@time computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
    filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04_La2004.jld2")
```

Results are saved in the .jld2 file. 45.306102 seconds (242.31 M allocations: 17.507 GiB, 17.98% gc time)

In [25]:

```
# load the results to see the arrays saved in hte computation
@load "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04_La2004.jld2"
```

Out[25]:

```
7-element Array{Symbol,1}:
 :X
 :Y
 :ηmax
 :rsTE_XtoY
 :rsPA_XtoY
 :rsTE_YtoX
 :rsPA_YtoX
```

In [26]:

```
# normalize the results for comparability

@load "../../../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04_La200
4.jld2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f
= 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f
= 1);
```

Calculate the quantiles for the confidence interval we want to plot (2σ , aka 95%).

In [27]:

```
### Calculate the quantiles for the confidence interval we want to plot (2σ, aka
95%)

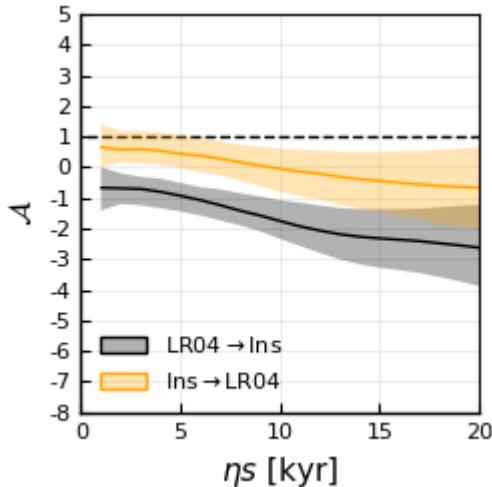
# ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .-
normPA_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .-
[quantile(normPA_XtoY[i,:], 0.025) for
i in 1:ηmax] # lower quantile
    # ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .-
normPA_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .-
[quantile(normPA_YtoX[i,:], 0.025) for
i in 1:ηmax] # lower quantile
;
```

Define the results plot

In [28]:

```
# define the predictive asymmetry plot
plot_normPA_LR04_La2004 =
plot(#title = string("Normalized # define the predictive asymmetry plot", L"\mathcal{A}", ") between ", L"\delta^{18}O", " (LR04) and northern hemisphere summer insolation (La2004")),
    xlims = (0, ηmax),
    xticks = (0 : 5 : ηmax),
    ylims = (-8,5), yticks = (-10:1:10),
    legend = :bottomleft, bg_legend = :transparent,
    xlabel = string(L"\eta s", " [kyr]"), # prediction lags
    ylabel = L"\mathcal{A}",
    size = (250,250),
    grid = true, border = true,
    hline([1], line = (:dash, :black)),
    label = "" # mean TE
)
plot!(normPA_XtoY_median, # ...from X to Y
    ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
    label = string("LR04", L"\rightarrow", "Ins"),
    fillalpha = 0.3, color = :black
)
plot!(normPA_YtoX_median, # ...from Y to X
    ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
    label = string("Ins", L"\rightarrow", "LR04"),
    fillalpha = 0.3, color = :orange
)
```

Out[28]:



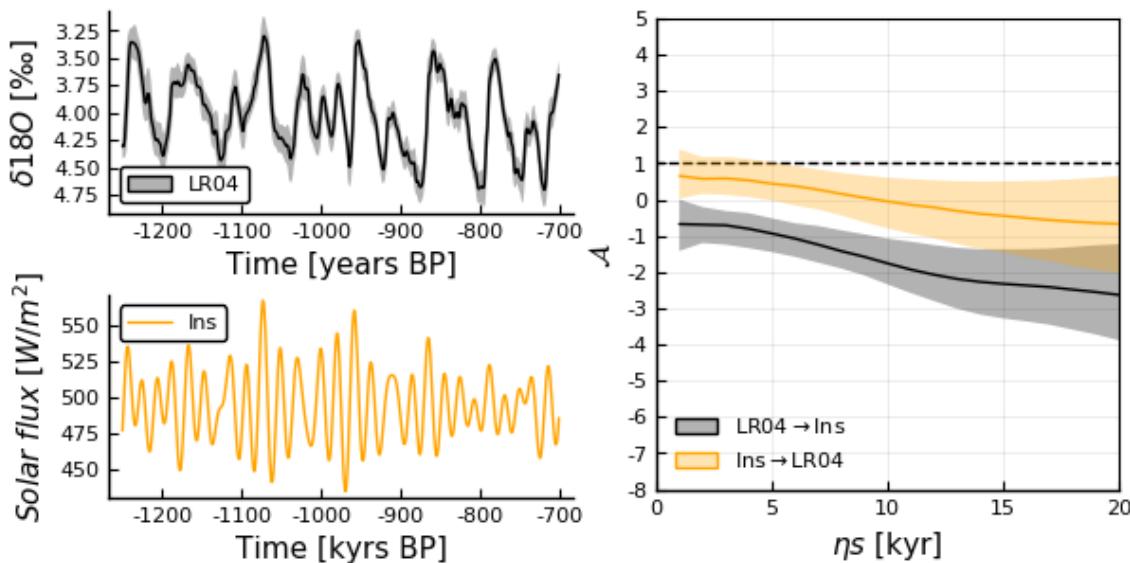
Plot the figure to include in thesis, including the time series and their \mathcal{A} results as subplots in one figure.

In [29]:

```
# Plot overview of time series pair to analyse
plot_overview_LR04_La2004 =
plot(layout = grid(2,1),
      size = (500, 400),
      plot_LR04,
      plot_La2004
    )

# join plots of time series and pa results to a results subplot
plot_results_LR04_La2004 =
plot(size = (600,300),
      layout = grid(1,2),
      plot_overview_LR04_La2004,
      plot_normPA_LR04_La2004)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/LR04_La2004.pdf") # This is the version with age model uncertainty
```



Caption: Normalized predictive asymmetry between $\delta^{18}\text{O}_b$ and northern hemisphere summer insolation. Insolation (orange) is the La2004 numerical solution of insolation intensity on June 31st at 65°N (Laskar et al., 2004). $\delta^{18}\text{O}_b$, shown in black, is the LR04 global stack by Lisiecki & Raymo (2005). Note that the age model of this record is built on the premiss that the $\delta^{18}\text{O}$ signal follows northern hemisphere insolation intensity with an assumed lag of 300 years. This poses a caveat in the results of our analysis.

- Without age model uncertainty

In [29]:

```
# Recall the time series on the common grid as X and Y
X = LR04_cut_noageunc # this is the version without age model uncertainty included
Y = La2004_insol_cut

# Compute the predictive asymmetry (function defined in NB3)
@time computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
    filepath = "../..../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04noageunc_La2004.jld2")
```

Results are saved in the .jld2 file. 44.625107 seconds (242.01 M allocations: 17.491 GiB, 17.06% gc time)

In [30]:

```
# Load and normalize the results

@load "../../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04noageunc_La2004.jld2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f = 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f = 1);

# calculate the quantiles for the 95% confidence interval

    # ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .- normPA_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .- [quantile(normPA_XtoY[i,:], 0.025) for i in 1:ηmax] # lower quantile
    # ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .- normPA_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .- [quantile(normPA_YtoX[i,:], 0.025) for i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_LR04_La2004_noageunc =
plot(xlims = (0, ηmax), xticks = (0 : 5 : ηmax),
      ylims = (-8,5), yticks = (-5:1:5),
      legend = :bottomleft, bg_legend = :transparent,
      xlabel = string(L"\eta s", " [kyr]", "# prediction lags"),
      ylabel = L"\mathcal{A}", "# normalized predictive asymmetry...",
      size = (250,250),
      grid = true, border = true,
      hline([1], line = (:dash, :black)),
      label = "" # mean TE
)
plot!(normPA_XtoY_median, # ...from X to Y
      ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
      label = string("LR04_noageunc ", L"\rightarrow", "Ins"),
      fillalpha = 0.3, color = :black
)
plot!(normPA_YtoX_median, # ...from Y to X
      ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
      label = string("Ins", L"\rightarrow", "LR04_noageunc "),
      fillalpha = 0.3, color = :orange
)

# defining the overview plot
plot_overview_LR04_La2004_noageunc =
plot(plot_LR04_noageunc, plot_La2004, layout = (2,1))

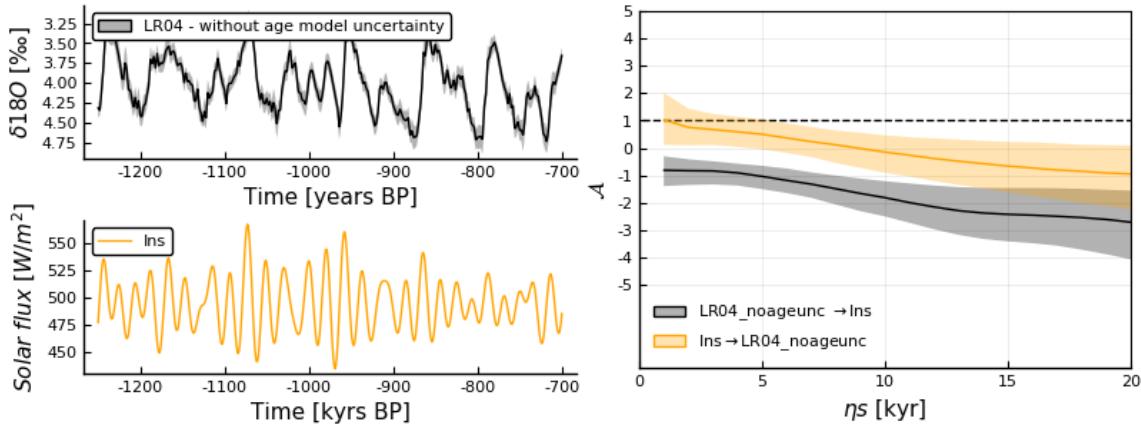
# join plots of time series and pa results to a results subplot
plot_results_LR04_La2004_noageunc =
plot(size = (800,300),
```

```

layout = grid(1,2),
plot_overview_LR04_La2004_noageunc,
plot_normPA_LR04_La2004_noageunc)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/LR04noag
eunc_La2004.pdf") # this is the version without age model uncertainty included

```



2. \mathcal{A} between sea level and insolation

Elderfield - La2004

In [31]:

```

# Compute the predictive asymmetry between the time series

# First, recall the time series on the common grid as X and Y
X = E_cut
Y = La2004_insol_cut

# Compute the predictive asymmetry (function defined in NB3)
@time computePredictiveAsymmetries(
    X, Y, timestep = 1, ηmax = 20,
    filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/E_
La2004.jld2"
)

```

Results are saved in the .jld2 file. 48.658684 seconds (241.68 M al
locations: 17.467 GiB, 16.98% gc time)

In [31]:

```
# Load and normalize the predictive asymmetry results

# Load and normalize the predictive asymmetry results
@load "../..../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/E_La2004.jl
d2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f
= 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f
= 1);

# calculate the quantiles to plot the 95% confidence interval
    # ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .-
normP
A_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .-
[quantile(normPA_XtoY[i,:], 0.025) for
i in 1:ηmax] # lower quantile
    # ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .-
normP
A_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .-
[quantile(normPA_YtoX[i,:], 0.025) for
i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_E_La2004 =
plot(xlims = (0, ηmax),
      xticks = (0 : 5 : ηmax),
      ylims = (-5,5),
      yticks = (-5:1:5),
      legend = :bottomleft,
      xlabel = string(L"\eta s", " [kyr]"),
      ylabel = L"\mathcal{A}",
      size = (250,250),
      grid = true, border = true,
      hline([1], line = (:dash, :black)),
      label = "" # mean TE
    )
plot!(normPA_XtoY_median,
      ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
      label = string("EldSL", L"\rightarrow", "Ins"),
      fillalpha = 0.3, color = :royalblue
    )
plot!(normPA_YtoX_median,
      ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
      label = string("Ins", L"\rightarrow", "EldSL"),
      fillalpha = 0.3, color = :orange
    )
;

# join plots of time series and pa results to a results subplot
plot_overview_E_La2004 =
plot(layout = grid(2,1),
      size = (1000, 400),
```

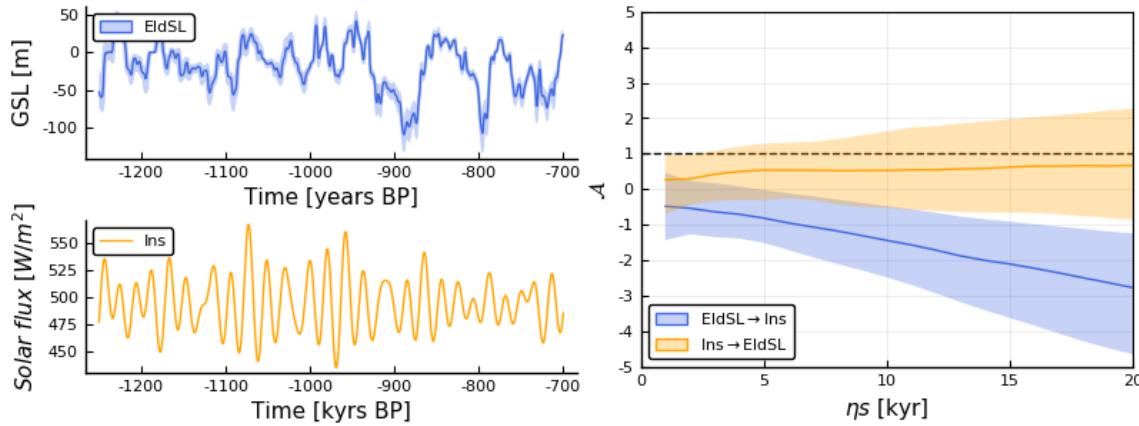
```

plot_E,
plot_La2004
)

plot_results_E_La2004 =
plot(size = (800,300),
      layout = grid(1,2),
      plot_overview_E_La2004,
      plot_normPA_E_La2004)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/E_La2004.pdf")

```



Rohling - La2004

In [33]:

```

# Recall the time series on the common grid as X and Y
X = R_cut
Y = La2004_insol_cut

# Compute the predictive asymmetry (function defined in NB3)
@time computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
    filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/R_La2004.jld2")

```

Results are saved in the .jld2 file. 40.787397 seconds (240.50 M allocations: 17.383 GiB, 19.17% gc time)

In [32]:

```
# Load and normalize the predictive asymmetry results

# Load and normalize the results
@load "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/R_La2004.jl
d2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f
= 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f
= 1)

# calculate the quantiles to plot the median and the 95% confidence interval
    # ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] -- normP
A_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .- [quantile(normPA_XtoY[i,:], 0.025) for
i in 1:ηmax] # lower quantile
    # ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] -- normP
A_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .- [quantile(normPA_YtoX[i,:], 0.025) for
i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_R_La2004 =
plot(#title =
    xlims = (0, ηmax),
    xticks = (0 : 5 : ηmax),
    ylims = (-5,5), yticks = (-5:1:5),
    legend = :bottomleft,
    xlabel = string(L"\eta s", " [kyr]"), # prediction lags
    ylabel = L"\mathcal{A}", # normalized predictive asymmetry...
    size = (250,250),
    grid = true, border = true,
    hline([1], line = (:dash, :black)),
    label = "" # mean TE
)
plot!(normPA_XtoY_median, # ...from X to Y
    ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
    label = string("RohSL", L"\rightarrow", "Ins"),
    fillalpha = 0.3, color = :skyblue
)
plot!(normPA_YtoX_median, # ...from Y to X
    ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
    label = string("Ins", L"\rightarrow", "RohSL"),
    fillalpha = 0.3, color = :orange)
;

# join plots of time series and pa results to a results subplot
plot_overview_R_La2004 =
plot(layout = grid(2,1),
    size = (800, 400),
```

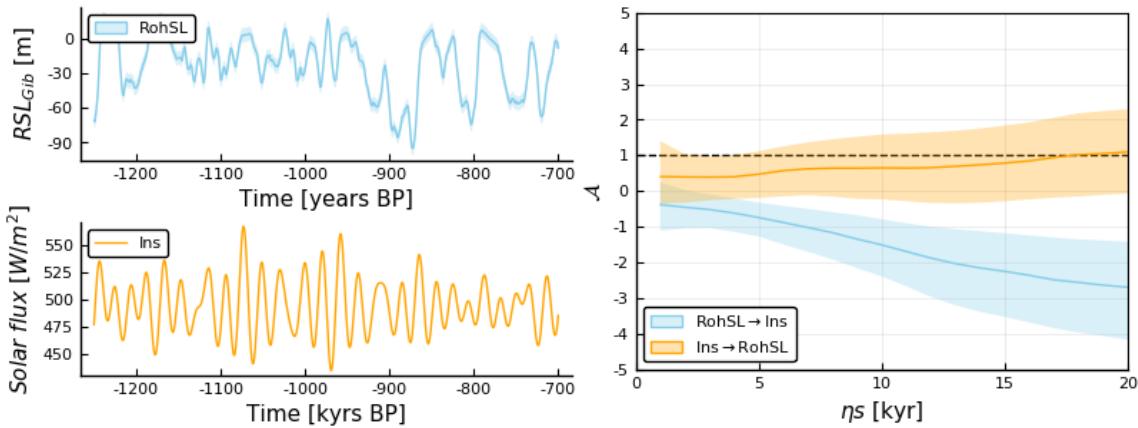
```

plot_R,
plot_La2004
)

plot_results_R_La2004 =
plot(size = (800,300),
      layout = grid(1,2),
      plot_overview_R_La2004,
      plot_normPA_R_La2004)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/R_La2004.pdf")

```



4. \mathcal{A} between sea level / $\delta^{18}\text{O}$ and dust

LR04 - Martinez-García

In [35]:

```

#Compute the predictive asymmetry between the two time series

# recall the time series on the common grid as X and Y
X = LR04_cut
Y = MG_cut

# Compute the predictive asymmetry (function defined in NB3)
@time computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
                                     filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04_MG.jld2")

```

Results are saved in the .jld2 file. 47.788809 seconds (246.53 M allocations: 17.595 GiB, 15.86% gc time)

In [33]:

```
# Load and normalize the predictive asymmetry results

@load "../../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/LR04_MG.jld2"
# unpack from structure
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f = 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f = 1)

# calculate the quantiles for the 95% confidence interval
    # ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .- normPA_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .- [quantile(normPA_XtoY[i,:], 0.025) for i in 1:ηmax] # lower quantile
    # ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .- normPA_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .- [quantile(normPA_YtoX[i,:], 0.025) for i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_LR04_MG =
plot(#title = L"$\mathcal{A}$ between #sea level and #insolation"
    xlims = (0, ηmax),
    xticks = (0 : 5 : ηmax),
    ylims = (-5,5), yticks = (-5:1:5),
    legend = :bottomleft,
    xlabel = string(L"\eta s", " [kyr]"),
    ylabel = L"\mathcal{A}",
    size = (250,250),
    grid = true, border = true,
    hline([1], line = (:dash, :black)),
    label = ""
)
plot!(normPA_XtoY_median,
    ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
    label = string("LR04", L"\rightarrow", "MarFe"),
    fillalpha = 0.3, color = :black)
plot!(normPA_YtoX_median,
    ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
    label = string("MarFe", L"\rightarrow", "LR04"),
    fillalpha = 0.3, color = :green)
;

# join plots of time series and pa results to a results subplot
plot_overview_LR04_MG =
plot(layout = grid(2,1),
    size = (1000, 400),
    plot_LR04,
```

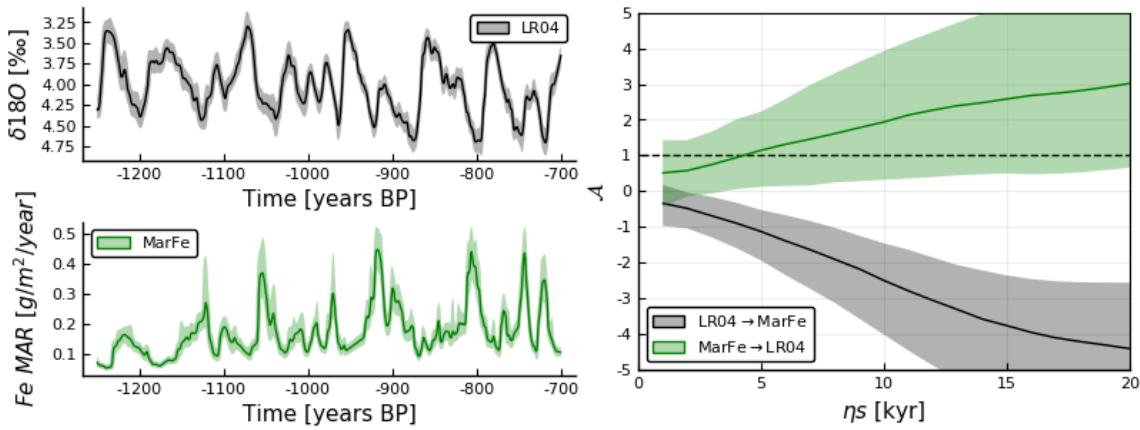
```

plot_MG
)

plot_results_LR04_MG =
plot(size = (800,300),
      layout = grid(1,2),
      plot_overview_LR04_MG,
      plot_normPA_LR04_MG)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/LR04_MG.pdf")

```



Elderfield - Martinez-García

In [37]:

```

# recall the time series on the common grid as X and Y
X = E_cut
Y = MG_cut

# Compute 150 families of transfer entropy and predictive asymmetry (function detailed in NB3)
computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
    filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/E_MG.jld2")

```

Results are saved in the .jld2 file.

In [34]:

```
# Plot the results, showing the 95% confidence interval

# Load and normalize the predictive asymmetry results
@load "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/E_MG.jld2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f = 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f = 1)

# calculate the quantiles for the 95% confidence interval
# ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .- normPA_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .- [quantile(normPA_XtoY[i,:], 0.025) for i in 1:ηmax] # lower quantile
# ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .- normPA_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .- [quantile(normPA_YtoX[i,:], 0.025) for i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_E_MG =
plot(#title = L"$\mathcal{A}$ between GSL and Southern Ocean Fe deposition",
      xlims = (0, ηmax),
      xticks = (0 : 5 : ηmax),
      ylims = (-5,5), yticks = (-5:1:5),
      legend = :bottomleft,
      xlabel = string(L"\eta s", " [kyr]"),
      ylabel = L"\mathcal{A}",
      size = (250,250),
      grid = true, border = true,
      hline([1], line = (:dash, :black)),
      label = ""
    )
plot!(normPA_XtoY_median,
      ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
      label = string("EldSL", L"\rightarrow", "MarFe"),
      fillalpha = 0.3, color = :royalblue
    )
plot!(normPA_YtoX_median,
      ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
      label = string("MarFe", L"\rightarrow", "EldSL"),
      fillalpha = 0.3, color = :green)
;

# join plots of time series and pa results to a results subplot
plot_overview_E_MG =
plot(layout = grid(2,1),
      size = (1000, 400),
      plot_E,
```

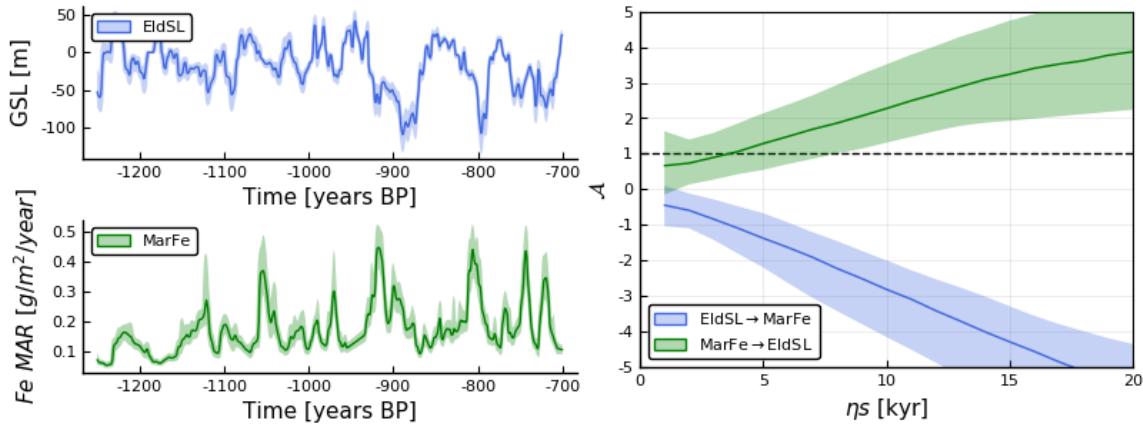
```

plot_MG
)

plot_results_E_MG =
plot(size = (800,300),
      layout = grid(1,2),
      plot_overview_E_MG,
      plot_normPA_E_MG)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/E_MG.pdf")

```



Rohling - Martinez-García

In [39]:

```

#Compute the predictive asymmetry between the two time series

# recall the time series on the common grid as X and Y
X = R_cut
Y = MG_cut

# Compute the predictive asymmetry (function defined in NB3)
computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
    filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/R_MG.jld2")

```

Results are saved in the .jld2 file.

In [35]:

```
# Plot the results, showing the 95% confidence interval

# Load and normalize the predictive asymmetry results
@load "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/R_MG.jld2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f = 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f = 1)

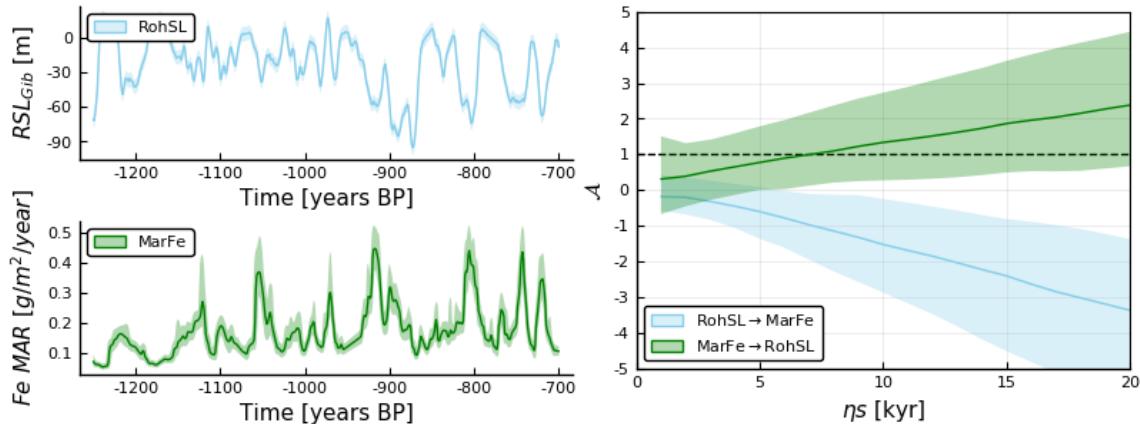
# calculate the quantiles for the 95% confidence interval
# ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .- normPA_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .- [quantile(normPA_XtoY[i,:], 0.025) for i in 1:ηmax] # lower quantile
# ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .- normPA_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .- [quantile(normPA_YtoX[i,:], 0.025) for i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_R_MG =
plot(xlims = (0, ηmax),
      xticks = (0 : 5 : ηmax),
      ylims = (-5,5), yticks = (-5:1:5),
      legend = :bottomleft,
      xlabel = string(L"\eta s", " [kyr]"),
      ylabel = L"\mathcal{A}",
      size = (250,250),
      grid = true, border = true,
      hline([1], line = (:dash, :black)),
      label = "")
plot!(normPA_XtoY_median,
      ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
      label = string("RohSL", L"\rightarrow", "MarFe"),
      fillalpha = 0.3, color = :skyblue)
plot!(normPA_YtoX_median,
      ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
      label = string("MarFe", L"\rightarrow", "RohSL"),
      fillalpha = 0.3, color = :green)
;

# join plots of time series and pa results to a results subplot
plot_overview_R_MG =
plot(layout = grid(2,1),
      size = (1000, 400),
      plot_R,
      plot_MG
```

```
)
plot_results_R_MG =
plot(size = (800,300),
      layout = grid(1,2),
      plot_overview_R_MG,
      plot_normPA_R_MG)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/R_MG.pdf")
f")
```



7. A between insolation and dust

La2004 - Martinez-García

In [41]:

```
# recall the time series on the common grid as X and Y
X = La2004_insol_cut
Y = MG_cut

# Compute 150 families of transfer entropy and predictive asymmetry (function detailed in NB3)
@time computePredictiveAsymmetries(X, Y, timestep = 1, ηmax = 20,
    filepath = "../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/La
2004_MG.jld2")
```

Results are saved in the .jld2 file. 43.877765 seconds (241.14 M al locations: 17.427 GiB, 17.92% gc time)

In [36]:

```
# Plot the results, showing the 95% confidence interval

# load and normalize the predictive asymmetry results computed above
@load "../..../results_ePalus_ns20/pa_jld2_files/synMPT_fullMPTgrid_nB/La2004_MG.jl2"
normPA_XtoY = normalizePredictiveAsymmetry(rsTE_XtoY, rsPA_XtoY, ηmax = ηmax, f = 1)
normPA_YtoX = normalizePredictiveAsymmetry(rsTE_YtoX, rsPA_YtoX, ηmax = ηmax, f = 1)

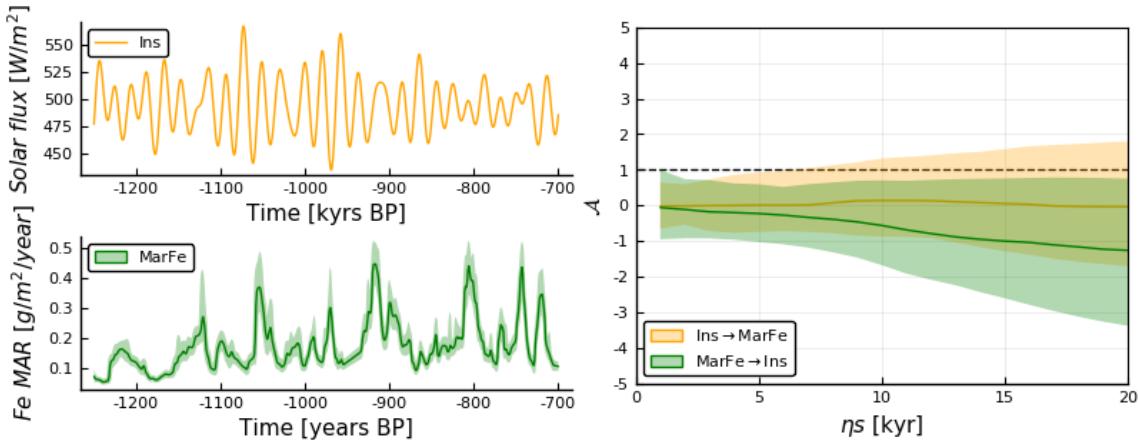
# calculate the quantiles for the 95% confidence interval
# ... for PA from X to Y
normPA_XtoY_median = [quantile(normPA_XtoY[i,:], 0.5) for i in 1:ηmax]
# median
normPA_XtoY_upper = [quantile(normPA_XtoY[i,:], 0.975) for i in 1:ηmax] .- normPA_XtoY_median # upper quantile
normPA_XtoY_lower = normPA_XtoY_median .- [quantile(normPA_XtoY[i,:], 0.025) for i in 1:ηmax] # lower quantile
# ... for PA from Y to X
normPA_YtoX_median = [quantile(normPA_YtoX[i,:], 0.5) for i in 1:ηmax]
# median
normPA_YtoX_upper = [quantile(normPA_YtoX[i,:], 0.975) for i in 1:ηmax] .- normPA_YtoX_median # upper quantile
normPA_YtoX_lower = normPA_YtoX_median .- [quantile(normPA_YtoX[i,:], 0.025) for i in 1:ηmax] # lower quantile
;

# defining the results plot
plot_normPA_La2004_MG =
plot(#title = L"$\mathcal{A}$ between insolation and Fe dust"
    xlims = (0, ηmax),
    xticks = (0 : 5 : ηmax),
    ylims = (-5,5), yticks = (-5:1:5),
    legend = :bottomleft,
    xlabel = string(L"\eta s", " [kyr]"),
    ylabel = L"\mathcal{A}",
    size = (250,250),
    grid = true, border = true,
    hline([1], line = (:dash, :black)),
    label = "")
plot!(normPA_XtoY_median,
    ribbon = (normPA_XtoY_lower, normPA_XtoY_upper),
    label = string("Ins", L"\rightarrow", "MarFe"),
    fillalpha = 0.3, color = :orange)
plot!(normPA_YtoX_median,
    ribbon = (normPA_YtoX_lower, normPA_YtoX_upper),
    label = string("MarFe", L"\rightarrow", "Ins"),
    fillalpha = 0.3, color = :green)

# join plots of time series and pa results to a results subplot
plot_overview_La2004_MG =
plot(layout = grid(2,1),
    size = (1000, 400),
    plot_La2004,
    plot_MG)
```

```
plot_results_La2004_MG =
plot(size = (800,300),
      layout = grid(1,2),
      plot_overview_La2004_MG,
      plot_normPA_La2004_MG)

savefig("../results_ePalus_ns20/pa_ResultPlots/synMPT_fullMPTgrid_nB/La2004_MG.pdf")
```



- high resolution analysis with time step of 500 years?

We decided not to interpolate the Martinez-Garcia record for this, as the 500 yr time step is higher resolution than the mean resolution across the time interval (900 years)

Ensemble plots of results

In [37]:

```
panel_labels = [ "a", "b", "c", "d", "e", "f", "g", "h", "i", "j", "k", "l", "m",
"n", "o", "p", "q", "r", "s", "t", "u", "v", "w", "x", "y", "z" ]
;
length(panel_labels)
```

Out[37]:

26

1. $\delta^{18}\text{O}$ - insolation

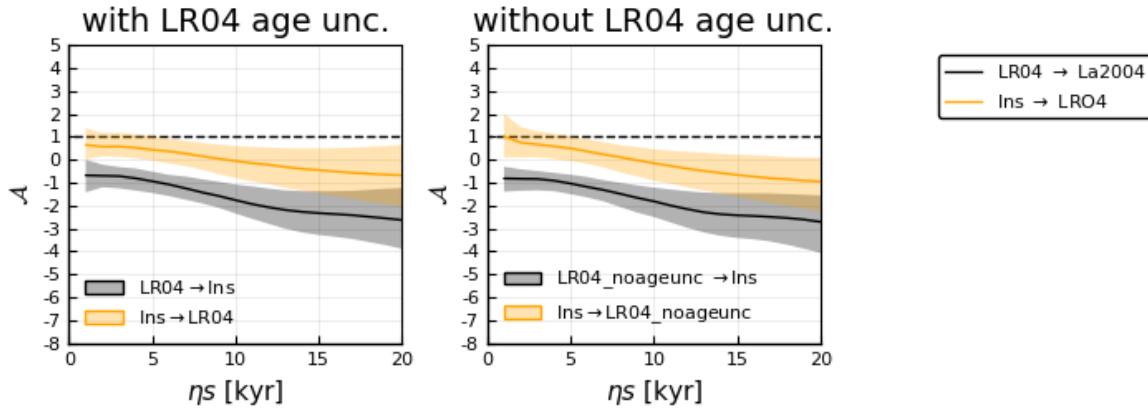
In [44]:

```
p01 = plot(plot_normPA_LR04_La2004, legend = :bottomleft, bg_legend = :transparent, title = "with LR04 age unc.")
p02 = plot(plot_normPA_LR04_La2004_noageunc, legend = :bottomleft, bg_legend = :transparent, title = "without LR04 age unc.")

legend = plot( # empty plot for legend
    [0 0],
    #ribbon = [(0,0) (0,0)], # How to make "fat" labels?
    showaxis = false, grid = false, legend = :outertopright,
    label = [string("LR04 ", L"\rightarrow", " La2004") string("Ins ", L"\rightarrow",
    " LRO4") ],
    color = [:black :orange])

plot(p01, p02, legend,
    layout = @layout([[A B ] E{0.1w}]), size = (700,250),
    ylims = (-8,5), yticks = (-10:1:10),
    border = :true
)
```

Out[44]:



Not a very big difference actually...

1. GSL - insolation

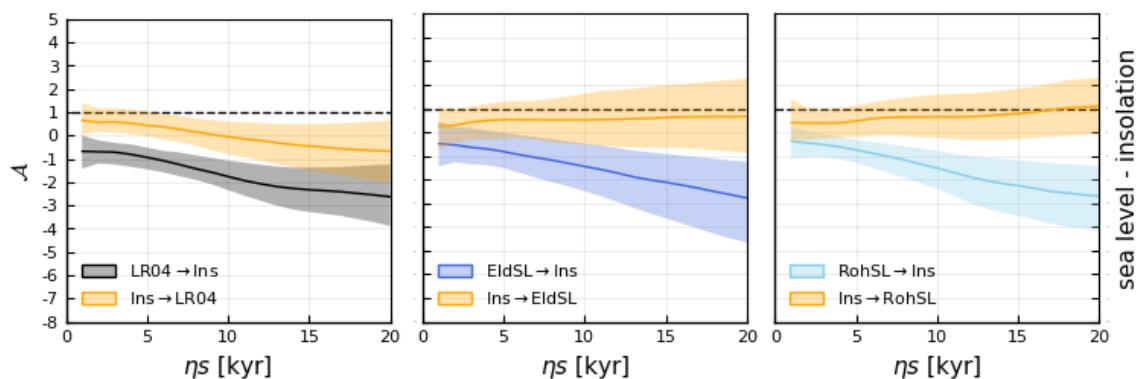
In [38]:

```
# remove the legends from the individual plots

p11 = plot(plot_normPA_LR04_La2004, legend = :bottomleft, bg_legend = :transparent, title = "", ylabel = L"\mathcal{A}")
p12 = plot(plot_normPA_E_La2004, legend = :bottomleft, bg_legend = :transparent, ylabel = "", ytickfont = false)
p13 = plot(plot_normPA_R_La2004, legend = :bottomleft, bg_legend = :transparent, ylabel = string("sea level - insolation"), ymirror = true, ytickfont = false
)

# plot the 6 results plots and the legend plot to the right
pe_gsl_insol = plot(
    p11, p12, p13,
    layout = grid(1,3), size = (750,250),
    border = true, xaxis = :on,
    ylims = (-8,5), yticks = (-10:1:10),
)

savefig("../results_ePalus_ns20/ensemble_normPA_plots/synMPT_fullMPTgrid_nB/gsl_insol.pdf")
```



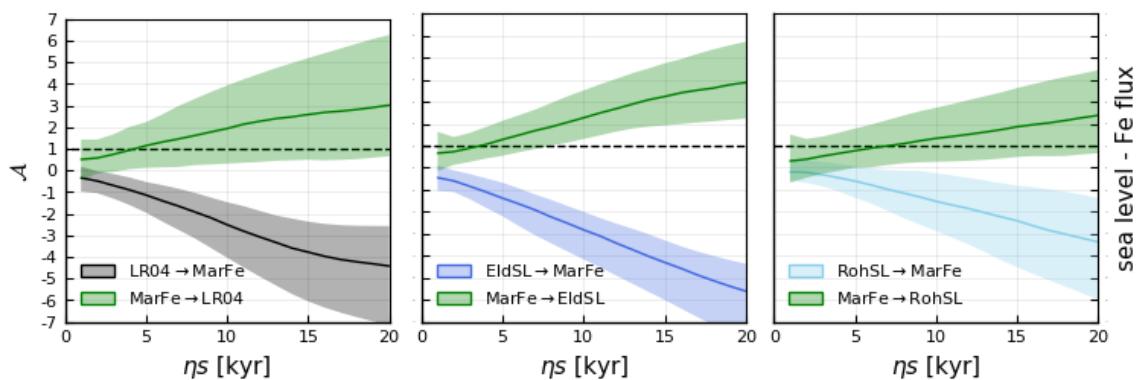
1. GSL - dust

In [39]:

```
p41 = plot(plot_normPA_LR04_MG, ylabel = L"\mathcal{A}")
p42 = plot(plot_normPA_E_MG, ylabel = "", ytickfont = false)
p43 = plot(plot_normPA_R_MG, ylabel = string("sea level - Fe flux"), ymirror = true, ytickfont = false)

pe_gsl_dust = plot(
    p41, p42, p43,
    layout = grid(1,3), size = (750,250),
    border = true, xaxis = :on,
    legend = :bottomleft, bg_legend = :transparent,
    ylims = (-7,7), yticks = (-10:1:10),
)

savefig("../results_ePalus_ns20/ensemble_normPA_plots/synMPT_fullMPTgrid_nB/gsl-dust.pdf")
```



1. insolation - dust

In [40]:

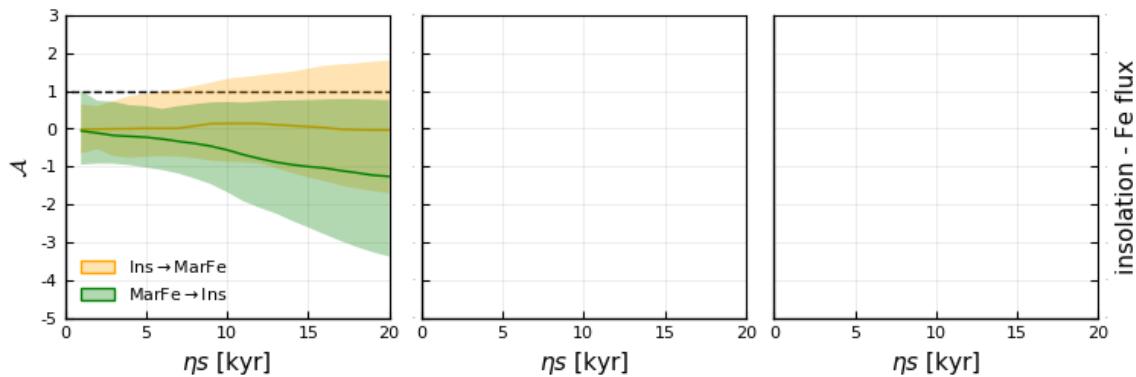
```

p74 = plot(plot_normPA_La2004_MG, ylabel = L"\mathcal{A}")
#p75 = plot(plot_normPA_La2004_MG_hr500, ylabel = "", ytickfont = false)
p_empty1 = plot(size = (250,250), border = false, xlabel = string(L"\eta s", " [ky
r]"), xticks = (0:5:20), lims = (0,20), yticks = :none, ytickfont = false)
p_empty2 = plot(size = (250,250), border = false, xlabel = string(L"\eta s", " [ky
r]"), xticks = (0:5:20), lims = (0,20), yticks = :none, ytickfont = false, ylabel
= string("insolation - Fe flux"), ymirror = true)

pe_insol_dust = plot(
    #plot_normPA_La2004_MG, size = (250,250),
    p74, p_empty1, p_empty2, layout = grid(1,3), size = (750,250),
    border = true,
    legend = :bottomleft, bg_legend = :transparent,
    ylims = (-5,3), yticks = (-10:1:10),
    )

savefig("../results_ePalus_ns20/ensemble_normPA_plots/synMPT_fullMPTgrid_nB/e
_insol_dust_synMPT_fullMPTgrid_nB.pdf")

```



Overview: predictive asymmetry for the synMPT_fullMPTgrid_nB grid

Make the overview ensemble plot of all the predictive asymmetry results gathered in this notebook

In [41]:

```

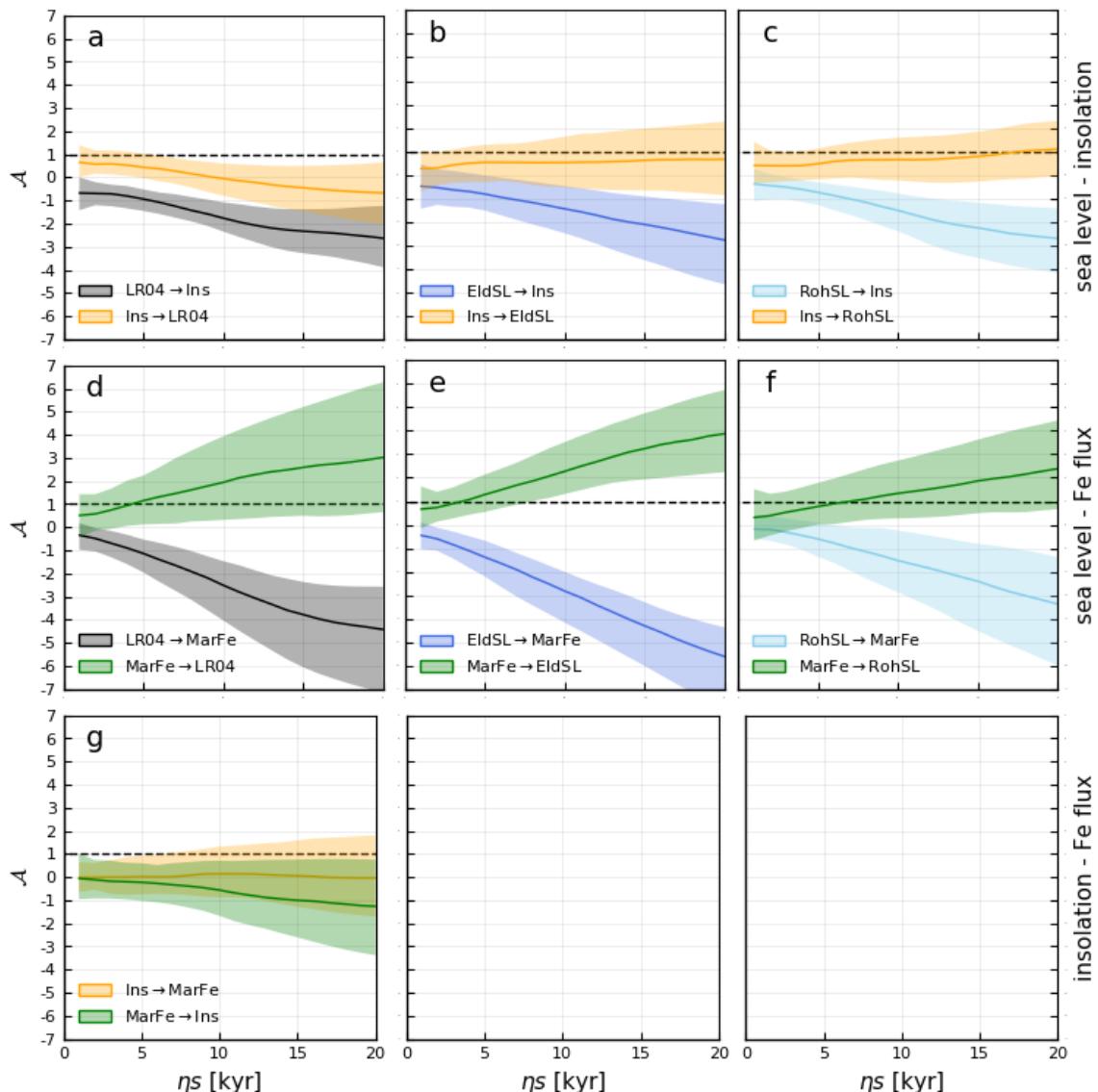
p101 = plot(pe_gsl_insol, xlabel = "", xtickfont = false)
#pe_gsl_co2,
p103 = plot(pe_gsl_dust, xlabel = "", xtickfont = false)
#pe_fit3,
#pe_co2_dust,
#pe_insol_co2,
p105 = plot(pe_insol_dust)

pe_allresults =
plot(p101,p103, p105,
      layout = grid(3,1), size = (750, 3*250),
      ylims = (-7,7)
    )

for i in 1:7
  annotate!(2,6, panel_labels[i], subplot = i)
end

# problem - annotate henger ved frå dei forrige plotta
pe_allresults
savefig("../..../results_ePalus_ns20/e_allresults/MA__synMPT_fullMPTgrid_nB_wPanel
labels.pdf")

```



Create master figure for appendix

In [42]:

```
# overview time series

po0 = plot(plot_La2004, xlabel = "", xaxis = :off, title = "MPT")
#      (1240-1090 ka BP))
po1 = plot(plot_LR04, xlabel = "", xaxis = :off)
po3 = plot(plot_E, xlabel = "", xaxis = :off)
po4 = plot(plot_R, xlabel = "", xaxis = :off)
po6 = plot(plot_C, xlabel = "", xaxis = :off)
po8 = plot(plot_MG, xlabel = "", xaxis = :off)
p_xaxis = plot(xlabel = "[kyrs BP]", xlabelfont = 8, xaxis = :on, xmirror = false)

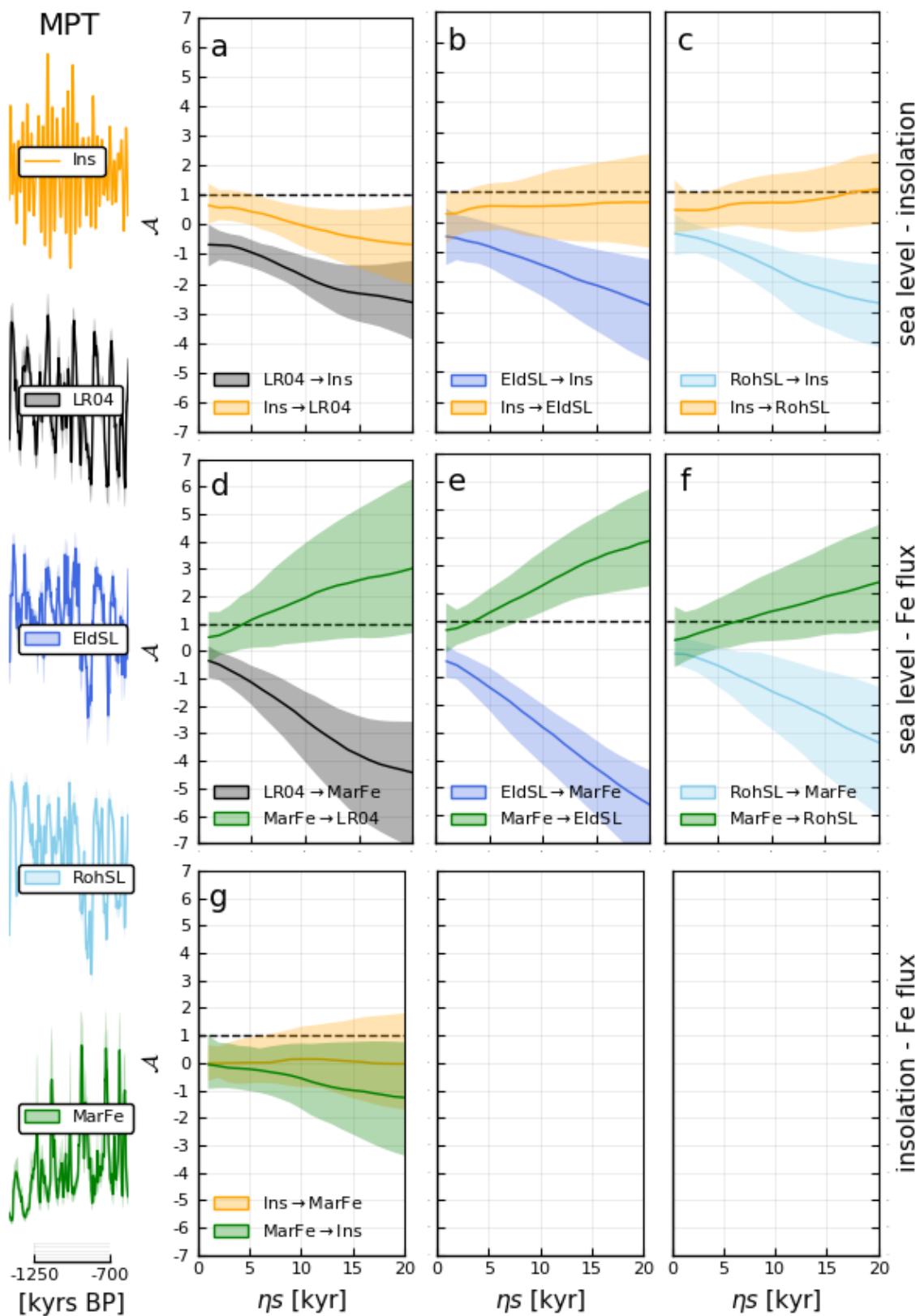
l = @layout [a;b;c;d;e;f{0.01h}]

po_alltimeseries = plot(po0,po1,po3,po4,po8, p_xaxis,
    layout = l, #grid(7,1),
    size = (250, 600),
    xlims = (gridstart, gridend), xticks = (gridstart : gridend-gridstart : gridend),
    ylabel = "", ytickfont = false, yaxis = :off,
    legend = :left, bg_legend = :white)

#savefig("../results_ePalus_ns20/timeseries/po_all_synMPT_Cgrid.pdf")

l = @layout [a{0.1w} b{0.9w}]

plot(po_alltimeseries, pe_allresults, layout = l, size = (72*8.27, 72*11.79)) # A4 size
savefig("../results_ePalus_ns20/e_allresults/MA__synMPT_fullMPTgrid_nB_Appendix_wPanellabels.pdf")
```



In []: