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Analyzing time series from eye tracking using Symbolic Aggregate Approximation

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Abstract

Being able to accurately, affordably and efficiently track the behavior of people has long been of interest to researchers. The eyes are no exception, as they can provide an insight directly into several behavioral aspects. Emerging technology has granted researcher the ability to explore how eye movement characterize particular issues for different purposes, from attention management, marketing, or investigating vision disturbances. Tracking the left and the right eyes simultaneously and at the same time knowing where a person needs to focus under a certain period can indicate how the eye coordination works. However, it is challenging to differentiate data pointing to normal eye coordination from data illustrating problems.

This thesis explores the viability of transforming the data produced when tracking the eyes into a discrete symbolic representation. For this transformation, we utilize Symbolic Aggregate Approximation to investigate a new possibility for effectively categorizing data collected via eye tracking technologies. This categorization illustrates tendencies for, e.g., tracking problems, problems with the set-up, normal vision, or vision disturbances. Accordingly, this will contribute to evaluating the eyes' performance and allow professionals to develop a diagnosis based on evidence from objective measurements.

The results are based on implementing a symbolic discretization method applied to experiments on a real-world dataset containing recordings of eye movements. In the future, the knowledge and transformation via the SAX method can be utilized to make sense of data and identify anomalies implemented in various domains and for multiple stakeholders.

Acknowledgements

I would like to express gratitude towards my sister Hanna Borgli and father Olav Borgli who helped me technically and mentally through this thesis, their knowledge has assisted me greatly. I thank my tolerant girlfriend Anna Louise Meland for her emotional support and motivation while working on the thesis and my friends for making me enjoy life during a difficult period. I would like to thank my main supervisor Carsten Helgesen for his excellent guidance, and for enabling me to complete my master under especially difficult circumstances. I would also like to thank the rest of the team Atle Geitung and Ilona Heldal for all the help they have given me and the countless hours in meetings they have sacrificed.

Finally, I want to pay my deepest respect to my mother Anneke Johanna Biesheuvel Borgli who passed away during the writing of this thesis. Her wisdom and strength took me on this path of higher education, and her support and advice throughout every struggle has guided me to reach this achievement.

Henrik Borgli
25 March, 2022

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Chapter 1

Introduction

Advances in eye tracking technology has opened a broad field in computer science where large scale and accurate recording of eye movement is achievable. The most advanced devices can record gaze, torsion, horizontal and vertical movement, blink rate and more. The resulting time series data can be analyzed in order to gain an understanding of the functional movement of the eyes, known as the oculomotor behaviour. This thesis is part of the research into analyzing eye tracking data in order to identify characteristics of unique eye movement which can be characterized and evaluated. In our case, analyzing recordings for use in the medical domain. In this introductory chapter, the background and motivation for the thesis is discussed, including the grand vision of the entire project. Finally, the problem domain is identified before presenting the research questions and goals.

1.1 Background and project motivation

Patterns in data have long been of interest to researchers. Every second of every day, inconceivable amounts of new data points are produced in every sector imaginable, all of which have a need to be structured, stored and analyzed in order to extract the knowledge and information they represent. One common way of representing a set of data points is on the axis of time, namely "time series", discussed in section 2.3. A time series can be represented as a graph with a set of data points over a sequential period of time (Wei, 2013).

Many methods have been proposed by researchers for extracting knowledge from this type of data, known as "data mining" (Keogh & Kasetty, 2002). This is further discussed in section 2.3.

The hardware needed has become commercialized, affordable and people with basic prior knowledge of eye tracking are now able to use the technology. Availability for vulnerable groups like poor people, children, elderly and, in the future, lesser developed countries includes the benefit of helping where little or no help is available.

During "Forsknings- og utviklingsarbeid-dagen" in November 2015, Gunvor Birkeland Wilhelmsen from the Department of Pedagogy in Teacher Education at Western Norway University of Applied Sciences asked for a solution for the problem of easily identifying functional vision problems using an inexpensive eye tracker. The problem was picked up by the Department of Computing, Mathematics and Physics at Western Norway University of Applied Sciences (Carsten Helgesen, Atle Geitung, Harald Soleim and Ilona Heldal) and a prototype called "C&Look", presented in section 2.4.1, was developed by two master students (Ruben Watanabe & Mads G. Eide) in 2017.

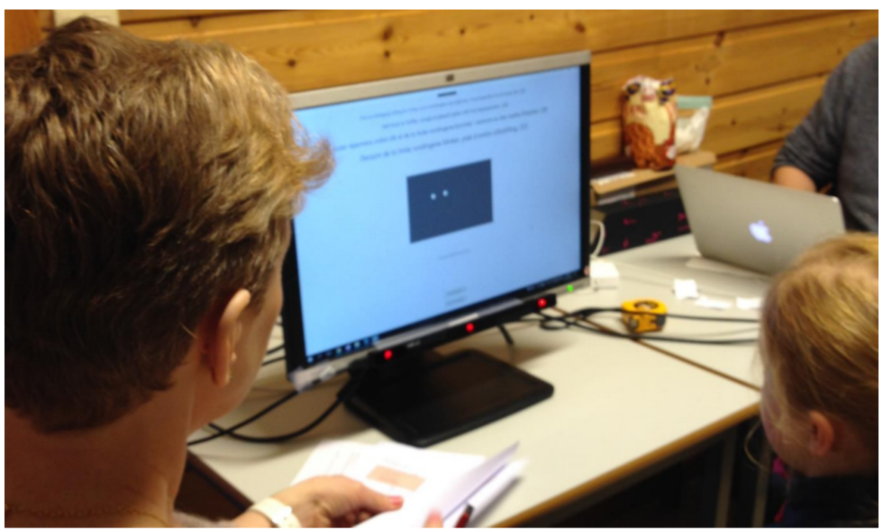

Figure 1.1: Image of video-oculography screening during calibration stage. (Photo taken by Ilona Heldal)

In essence the project was based on three motivating factors. First, the issue of preventing misdiagnosis of patients with functional vision problems, especially in children, as they are the largest vulnerable group and have the most future potential. Eye problems in children can be disguised as hyperactivity, ADHD, dyslexia or other conditions (Levantini et. al., 2020). A treatment of these conditions may negatively affect the behavior of a child, while the actual condition remains present. Secondly, the vision of the project is to further expand the possibilities of eye tracking technology in general and explore currently available, well documented methods on a new domain.

1.1.1 Thesis Motivation

Symbolic Aggregate ApproXimation (SAX), presented in chapter 4, is a method consisting of two parts which result in a discrete symbolic representation of a time series. The algorithm has shown great promise as a new representation tool for analytical research of time series data (Lin et. al., 2003). The use of time series to represent eye tracking data raises the question of the SAX method's viability in this domain. The method has been shown to tackle multiple known problems arising when analyzing time series (Lin et. al., 2007). Scalability, complexity and discretization have all been big issues when analyzing time series, which this method may handle. Being able to quickly and affordably test big groups within a short period of time may greatly improve detection of functional vision problems. This may prevent cases of misdiagnosis when a patient in reality has undiagnosed problems with their vision.

1.2 Problem Domain

In the domain of eye tracking, it is valuable to have methods able to characterize real-world behaviour in a set of numerical recording data. This behaviour is in general never identical, and usually consists of several unique events. Furthermore, the knowledge of eye movement from the medical domain has not yet been combined with the new eye tracking technology available for researchers. Therefore a problem arises of representing eye tracking data in a manner which can be combined with the knowledge from the medical domain. Today, the process of analysing the eyes requires a large amount of manual labor, and it is therefore simply not feasible to quickly test a large group. Being able to categorize performance of a screening as either good or bad, or even specific events, would be of great value, and by making the process automatic, would vastly improve the current capabilities of the technology.

1.2.1 Goals

The main goal of this thesis is to test the viability of the SAX algorithm on time series from eye tracking data. This will be done using conventional similarity measures as well as documented tools and methods, presented in section 4. If the algorithm successfully achieves accurate results, it can be further implemented into a complete system like the C&Look software, presented in section 2.4.1.

Furthermore, because the SAX algorithm is a generalized representation of any time series, the work done in this thesis may contribute knowledge to other domains such as finance, automation, bioinformatics, ect. These applications will be further discussed in chapter 7. Additionally we expect the technology to far exceed the requirements for us to solve the research questions presented. This means that we may not need to utilize the eye tracking device' full potential in order to achieve our goal, which may be further developed, researched and implemented in the future. The research and work done during this thesis will be a small leap towards a fully functional complete system. Some discoveries may unlock more potential than we initially thought and broaden the project goals.

1.2.2 Research questions

This thesis will explore well-documented approaches in combination with newer methods to develop an experimental system which will test and analyse the SAX algorithm on data gathered from eye tracking. This will be done with the intention of answering the following research questions:

1. How viable is SAX as a method for analyzing eye tracking data to recognize functional vision problems?

2. How accurate can we characterize a time series after the symbolic representation through SAX?

1.2.3 Limitations

There were several limitations to this thesis project: the time required to understand statistical methods to work with time series thoroughly, to understand data collected from eye tracking technologies to examine eye problems, and gain a better understanding of eye problems. While the vision behind this project was to demonstrate the capabilities of the SAX method for vision experts, this will be work for the near future. The background of the project considers knowledge and data validated by vision experts and, based on this, further developed the method using statistics to make eye movement understandable in extensive data sets collected from eye trackers. This step could not involve eye professionals with no knowledge of statistics. After implementing a more straightforward and accessible application for vision experts (often non-programmers), the next step will again involve experts as they need to validate the usability of this newly developed method. However, for data categorization, the help from vision experts who understand the possibilities of eye tracking could be valuable. Unfortunately, within the frame of this project, we do not have access to such knowledge; this also needs to be a next step before the validation of the method by practitioners.

Chapter 2

Background

The knowledge in this chapter is key in achieving the goals for this thesis and understanding the concepts behind the eye-tracking research we will conduct. Elements from eye movement and tracking research introduced here will be utilize throughout this thesis. First, the anatomy and movement of the eye is discussed. Following this is a section presenting the required background of eye tracking before presenting the datastructure time series. Finally, earlier work is presented where a system containing the tracking interface used to produce the dataset in this thesis is presented and explained.

2.1 The Eye

This section contains a brief description of the structure and anatomy of the human eye and the muscles involved in eye movement. In particular, describing the functions of the human visual system directly related to the way eye tracking is conducted.

2.1.1 Anatomy of the eye

The basic operation of seeing through the eyes consists of light going through the pupil, the light is then flipped through a lens and projected onto the retina which contains light-sensitive cells, called cones and rods, which transmits the signals to the brain through the optic nerve (Holmqvist & Andersson, 2017).

There are six parts of the eye which are of great importance in eye tracking research. These are (i) the *Retina*, (ii) the *Cornea*, (iii) the *Iris*, (iv) the *Pupil*, (v) the *Lens* and (vi) the *Fovea*. To be able to efficiently perform high-end eye tracking research, the researchers should have some knowledge of these parts of the eye, as they all relate to why we move our eyes the way we do.

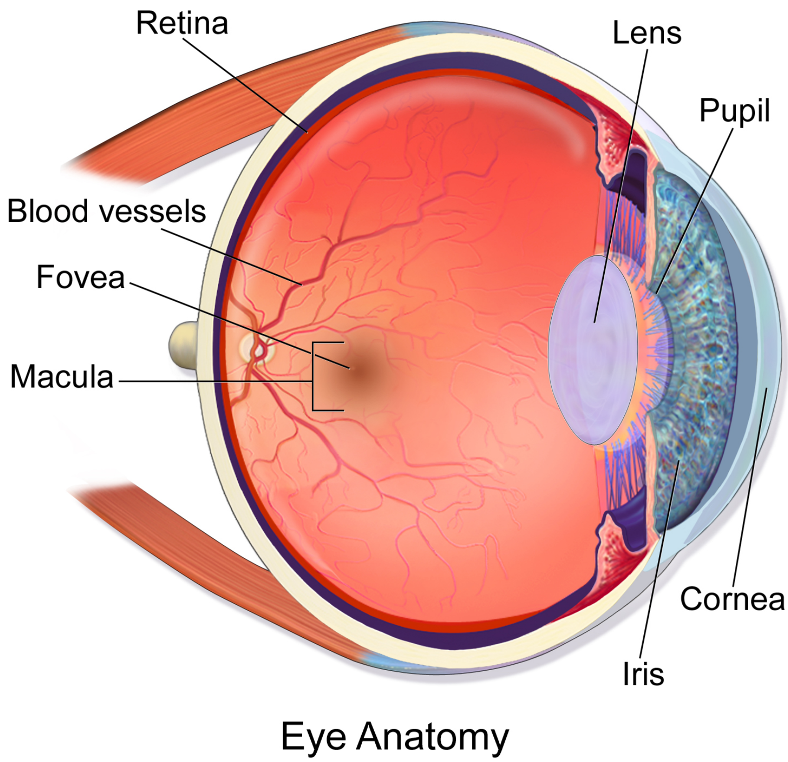

Figure 2.1: Illustration of the anatomy of the human eye. (Wikipedia Commons, 2022)

The *Cornea* is an transparent oval shape located above the Iris and Lens at the outermost part of the eye. It serves the purpose of refracting light as it passes through to the rest of the eye. This has the effect of focusing the light received by the eye and accounts for a large portion of the eyes total focal ability. Together with the Lens, it helps create a clear image (Bedinghaus, 2020).

The *Lens* is the second focal structure in the eye which refracts light in order to focus. In contrast to the cornea, the lens is suspended in elastic ligaments which can contract or relax to accommodate different focal lengths. This ability worsens by age as the lens hardens. Because of this elasticity, some inertial oscillations occur which can affect eye tracking.

The *Iris* is located between the Cornea and the Lens and consists of a pigmented muscular curtain with a hole in the middle. This hole is called the Pupil and combined they determine how much light reaches the sensory tissue of the retina. The amount of light passing through is controlled by two sheets of smooth muscles through dilation and contraction. (Gamm & Albert, 2020).

The *Pupil* is the opening where light passes into the eye. Research on the pupils have revealed several links between our cognitive state and the behavior of the pupil muscles. The pupil is especially important as it can be used as a measurement feature for gaze estimation in eye tracking. Notably, the stability of the pupil is affected by it's size and consequently may affect eye tracking.

The *Retina* is the innermost part of the eye and consists of a layer of tissue containing light-sensitive receptors. These receptors, primarily photoreceptor cells, translates the focused light into electrical neural impulses which are sent to the brain. These cells are divided in two groups: (i) rods which provide black-and-white vision and (ii) cones which provide color and perception of details.

The *Fovea* is a small depression on the retina with a greater amount of cones than the rest of the retina. Because of this, the fovea is the point where the most detail can be observed. This is especially important when doing tasks like reading. Issues with the fovea may result in blurry vision (Holmqvist & Andersson, 2017). Complicated issues when performing eye tracking arises when considering the geometrical direction of the eyes may be in one direction, while the actual focus and cognitive attention may lie in the direction of the fovea.

2.1.2 Oculomotor movement

The movement of the eyes are controlled each by six muscles which are connected in pairs to allow coordinated movement of both eyes. This enables eye movement in all directions. The direction of the eyes towards an object is referred to as the "gaze" and the point between the eyes in the direction of the gaze is called the "gaze point".

For the most basic analysis, there are three important movements which must be understood. When observing details, the eyes are moved to the object of interest and kept still.

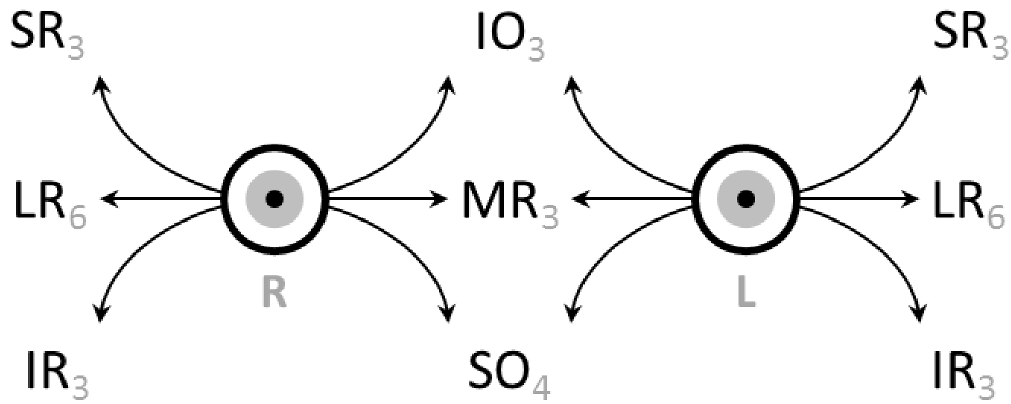

Figure 2.2: Illustration of eye movement. (Au.yousef, 2014)

This is called *Fixation*, and even though it is commonly believed that the eyes are locked onto a static point, it has been shown that the eyes are in constant movement when fixated. These involuntary movements are called micro saccades and are needed in order to maintain a visual image. A *Saccade* is the name of the action performed when rapidly shifting the gaze from one area to another. These movements are not always accurate, and are usually followed by smaller saccades to correct the gaze towards the point of fixation. These eye movements consist of a set of smaller fixations in contrast to *Smooth Pursuit* eye movements where the gaze is in constant motion. *Vestibulo-ocular movement* is connected to fixation and consists of the eye movement made to correct for change in perspective. This consists of the eyes moving closer or further apart from each other in order to maintain a fixation on a object which moves away or towards the observer (Holmqvist & Andersson, 2017).

A problem with the oculomotor function of the eyes is seen through these movements and is present due to issues with the six muscles controlling the movement. These issues are usually complex in nature, and may be part of other undiscovered issues. By understanding and being able to identify these individual movements, analysis can be performed on the collection of movements constituting a problem.

2.2 Eye Tracking

An explanation of the state of the art, methods and application of eye tracking will be given in this section. To get an understanding of the future possibilities and applications of this technology, we will also look at the history and a few important discoveries and milestones done on the way.

2.2.1 Introduction to Eye Tracking

Eye tracking has been defined by several researchers throughout the years as the process of measuring the position and direction of the eyes onto a real world stimulus. An eye tracker is the device in which we can perform these measurements with.

When performing eye tracking, several key factors must be considered. First, the type of hardware must be chosen. This is done based on the use case of the tracking. The trackers vary greatly in ability, price and accuracy. For example, cheaper devices are usually utilized for commercial use, while researcher usually require more accurate devices which have a higher price and complexity. Secondly, the environment in which the tracking is performed must be considered. This consists of the physical location of the eye tracker, the system on which it is performed and the tests which are completed. The physical environment requirements usually consist of evenly lit, quiet and neutral rooms. This also depends on the type of tracker used, discussed in section 2.2.3. The system on which the tracking is performed has not been standardized yet, and therefore most users create some form of their own system in combination with the framework of the eye tracker hardware. This thesis will use the data from such a system which is presented in section 2.4.1. The tests which are performed heavily affect the usability of the tracking data. During the writing of this thesis, the research has yet to reach the point of being able to efficiently analyze unstructured tests (Karamitopoulos & Evangelidis, 2007). A lot of information can still be derived using different techniques like heatmaps, but for research in domains of high complexity, like the medical domain, structured tests are usually required.

Figure 2.3: Modern adaption of a wearable eye tracking device. (Tobii Technology, 2022)

The final factor to consider is the subjects using the tracker devices which varies based on the goal of the tracking. In our case, the initial project began with the intent of analyzing children's vision. It would therefore be of little interest to have a dataset containing adults. In addition when testing, the subjects should somewhat reflect all possible outcomes such that all functional behaviour of interest, presented in section 2.1.2, can be analyzed.

2.2.2 Brief history of eye tracking

Some of the oldest valid research on the tracking of eye movement was made as early as the 1800's (Mohamed et. al. 2008). Significant observations and assumptions pushed the science forward, and by the 1900's, the first mechanical eye trackers were already built. Edmund Huey developed and built an early adaption of an eye tracker using a form of contact lens with a hole round the pupil where an aluminum stick could be placed. The physical movements of the stick were then recorded over a rotating smoked drum which could later be analysed (Huey, 1968). These first trackers paved the way for a new paradigm in the understanding of how the human eye behaves, but had the problem of being very intrusive, and highly inaccurate.

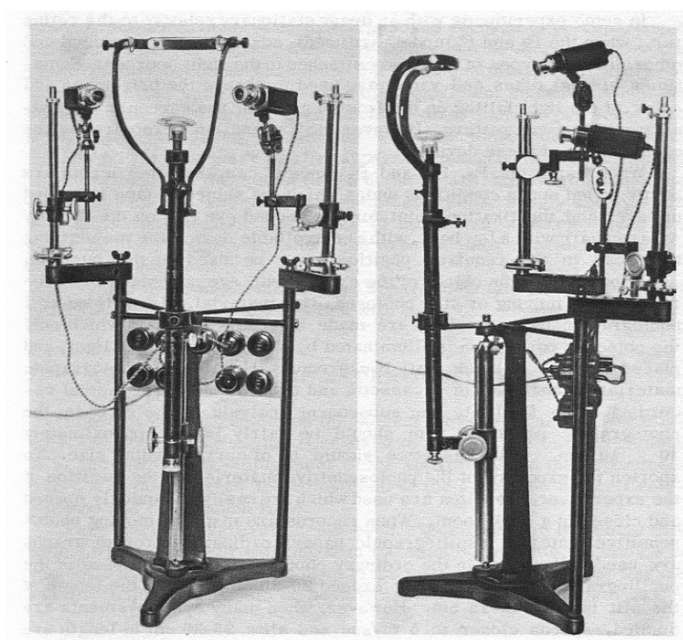

Figure 2.4: Early adaption of an eye tracking device. (Yarbus A. L, 1962)

The first non-intrusive eye tracker was developed by Charles H. Judd and Guy Thomas Buswell in the early 1900's. The device used beams of light that were reflected on the eye, then recorded on film (Marwah & Kharb, 2019). This was the beginning of Video-oculography which continued to become the most used type of eye tracking today, and the method used in this thesis. The studies performed during this research led to many leaps in the field of eye tracking research.

The following years, an explosion in research occurred and multiple new fields arose in the domain. The technology found it's place in finance, recreation, medicine, entertainment, gaming an many more domains and several complex theories were devised. Some theories defines our entire understanding of the eyes, like the "Strong eye-mind" hypothesis which shows a connection between the gaze and cognitive attention. Commercial use is also becoming of interest as the technology matures, and today, trackers and analysis tools are readily available for the general public.

2.2.3 Types of eye tracking

Special contact lenses

Special contact lenses were some of the first methods used to track the eyes. These lenses were crude and invasive, and lacked the accuracy of modern day trackers. Regardless, they helped push the boundaries of the new eye tracking domain and a lot of the knowledge gained during the research of these lenses is still used today.

There are two types of contact lenses used in eye tracking; lenses with mirrors which reflect light (Yarbus, 1967) or lenses containing magnetic search coils (Kenyon, 1985). There are multiple distinctive designs for both types of lenses, and they achieve different accuracy, comfort and results. The lenses are places directly on the eye and achieves high accuracy as it moves with the eyes of the participant. The most obvious negative aspect is the need for direct contact with the eye. Furthermore, the process of running the experiment may be time consuming. Compared to other types of eye tracking, the pre-experiment duration is extended, and the time used for actual recording of data is minimized.

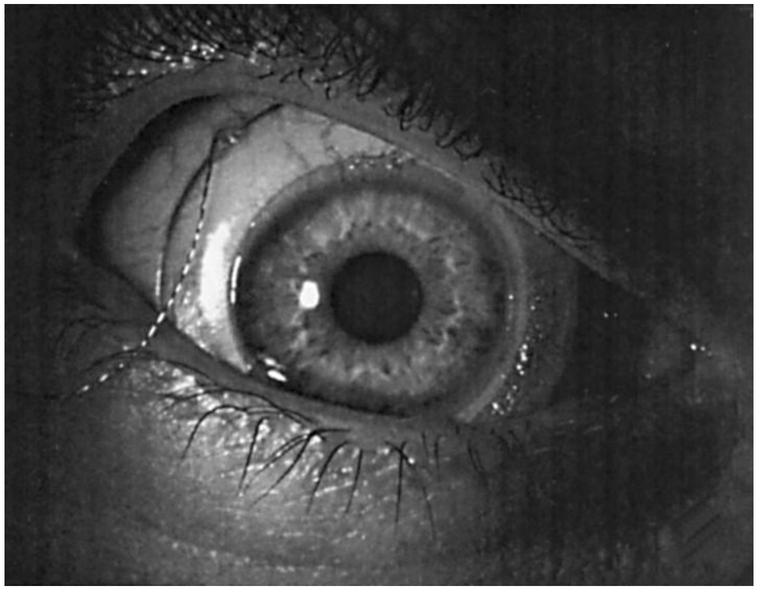

Figure 2.5: Scleral search coil lens on the eye. (Murphy et al., 2001)

Electrooculography

Electrooculography is a new technique which arose in the 2000's and consists of measuring the electrical potential between the front of the eye and the back of the eye (Brown et al, 2006). During movement, a potential difference occurs between electrodes which can be measured and mapped to specific movement. This is because the eye acts as a dipole separating positive and negative electromagnetic charge. While being highly accurate, this technique is usually complex and expensive, and is therefore mostly used by scientific researchers.

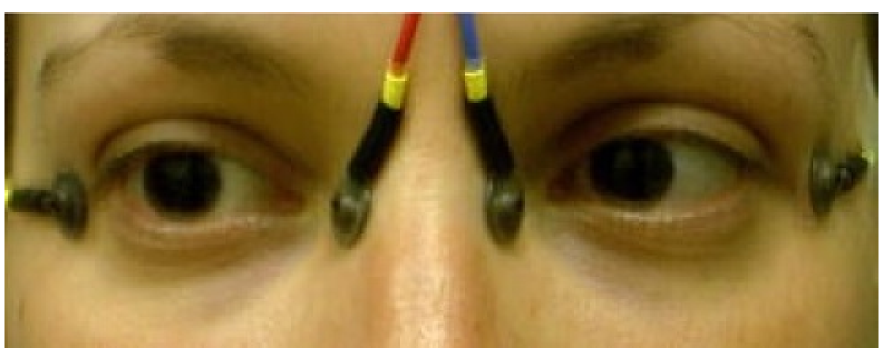

Figure 2.6: Placement of electrodes when performing Electrooculography. (Brown et al., 2006)

Video-oculography

Video-oculography is a technique for tracking the movement of the eyes using digital video. The video capture hardware faces a subject's eyes and picks up the reflection of the light on the eyes. There are two important distinction on how this can be done; using the visible light spectrum or the infrared light spectrum.

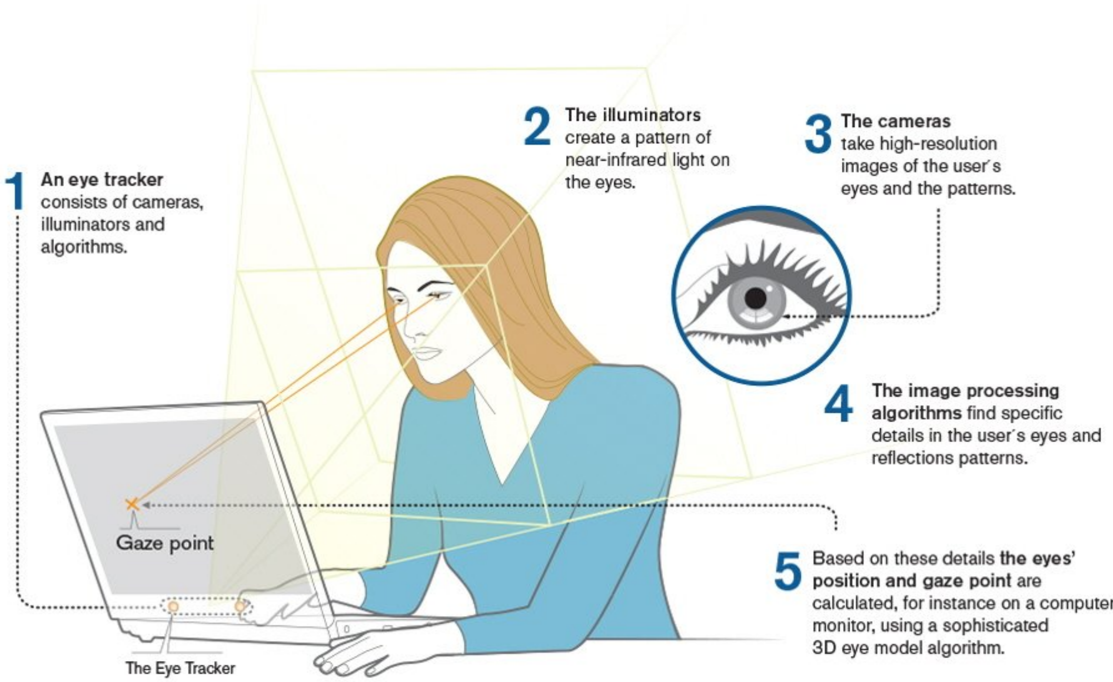

Figure 2.7: Overview of the components during video-oculography screening. (Tobii, 2022)

Visible light usually consists of relying on the ambient light in the room. The need for extremely well lit rooms, and consistent lighting is therefore a key factor of a successful tracking. Therefore, the environment the tracking is performed in has a great impact on consistency, accuracy and completeness. A way to overcome this issue is to directly shine a beam of light into the eyes of the subject (Larrazabal et. al., 2019). Using visible light for this task would be intrusive as the eyes wold react to the brightness, instead infrared light is utilized.

Video-oculography is a highly digital process compared to the other alternatives. The infrared lights are well understood, and video recording has been refined for a long time already. The brute of this process is in understanding the reflection of light on the eyes in order to deduce their position. There are several measuring features of the eyes, and eye tracking devices vary in how elaborately they use the different measurements. By using different planes of reflection, and combining multiple measurement features, higher accuracy can be achieved.

This method has seen the most popularity in the eye tracking domain due a combination of factors. The hardware components are affordable and easily available for research and commercial use. Using infrared light, the method can be used in a variety of environments. Being the simplest technique, the technology also has seen a rise in commercial use, which will further enable development of the technology in the future.

2.2.4 Application of eye tracking

Figure 2.8: Eye tracking combined with driving simulator. (University of Missouri, 2019)

Eye tracking has seen a rise in use by researchers and for commercial use in almost all domains, including gaming, education, transportation, advertisement, finance, security and many more. The technology is utilized in many ways, some more elaborate than others.

Most common and least complex is real-time interaction using eye tracking. This can be anything from moving the eyes over a simple button to press it, to interacting with objects in a full 3D world or through augmented reality.

Additionally, and of interest to us, this technology can support eye health professionals in a drastic fashion by decreasing diagnosis time, enabling earlier diagnosis and facilitating for more patients getting the correct treatment. It will especially help more vulnerable groups mentioned earlier as they may not have the financial possibility, knowledge or ability to seek medical help. Using the tracking and analysis systems proposed in this thesis, non-invasive and inexpensive tests can be performed in a known and comfortable environment. This may uncover symptoms not yet discovered, which may be brought to an eye health professional for further treatment.

2.2.5 Future of eye tracking

Research into the potential of eye tracking is still in it's early stage, and while promising results have already been produced, we are far from understanding the complexity of this domain. As mentioned earlier, eye tracking has already found it's place in many domains, and new implementations are created every day. The next generation of technology already promises functionality based on eye tracking, and games and other entertainment fully based on eye tracking is already in development.

In the coming years, eye tracking technology will be seen in several commercial applications, and will be actively used for recreation, advertisement, military engagement, education, safety infrastructure, transport and many more. While some uses of eye tracking are blatantly apparent, others remain unnoticed by the user. An example of this is the functionality in some phones which identifies the user's eyes in order to keep the screen active. Such types of tracking require ethical and moral discussions which may hinder the development. As the trackers become more portable and inexpensive, their use will likely be common in much of our technology and daily life.

2.3 Time series

A time series is a collection of numbers over an equally spaced sequential period of time. A number corresponds to one or more attributes and each number is paired with a corresponding point in the sequence. A time series is usually referred to as univariate (one-dimensional) or multivariate (multi-dimensional). A time series can be created from any variable that changes over time and occur naturally in many application areas like economics, finance and medicine (Wei, 2013).

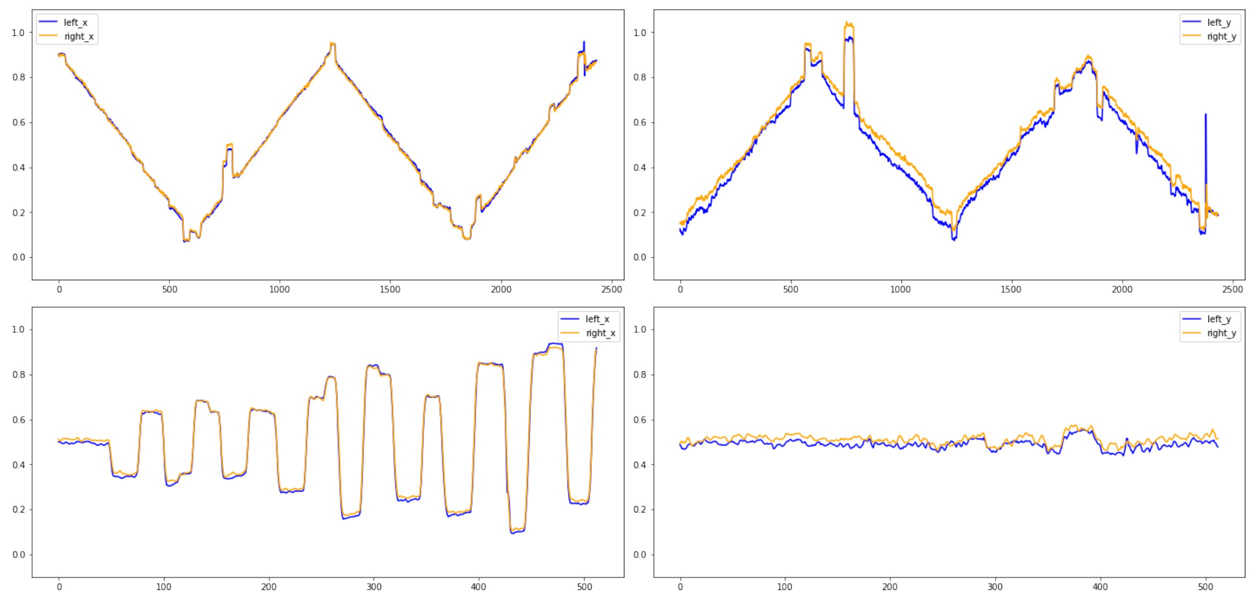

Figure 2.9: Examples of all time series from two recordings when tracking the eyes.

Generating data in the form of time series has exploded in the recent years. This comes as a consequence of the increasing computational power of our computers, the increased global usage of computers, and the current paradigm of mass data collection. When the dimensionality and complexity of the collected data increases, a need for better analysis tools and understanding arises. A surge of articles have emerged the recent years exploring new methods for researchers to adapt to the enormous amounts of data we currently amass (Leonidas & Evangelidis, 2007). The focus of these articles resides mostly in dimentionality/numerosity reduction, and feature extraction. More specifically: "the method of extracting the essential features of the original time series should guarantee that there would be no pattern missed and the number of patterns falsely identified as interesting will be minimized."(Karamitopoulos & Evangelidis, 2007).

2.3.1 Time Series data mining

The purpose of time series data mining is to extract all meaningful knowledge from the shape of the data. There are several ways to do this, all with vastly different results and applications. Following are some of the areas that are of value for this thesis, and has sparked the most interest of researchers in the recent years:

(i) *Clustering* is a method where the goal is discovering meaningful groupings in unlabeled data using some similarity/dissimilarity measure. There are a wide range of uses for this method, from statistics to astronomy, and also in medical research which we will discover in this thesis by trying to cluster sets of symbols with the final goal of classifying the clusters.

(ii) *Classification* is the act of arranging unlabeled data into groups or categories based on predefined criteria. After performing classification, information like behaviours, objects and anomalies can be understood. Classification is greatly used in machine learning.

(iii) *Anomaly detection* is the process of detecting unexpected section in the data which deviate significantly from the rest. It is also known as outlier detection or novelty detection. Such anomalies may reveal some knowledge of the data or mechanisms behind.

Not surprisingly, the choice of a representation of the data also greatly affects the output of the time series analysis. For most methods, alternate forms of data representation is required to overcome certain obstacles like noise, high dimensionality ect. Some well known representations include Discrete Fourier Transform and Haar Wavelet, but for the scope of this thesis we will only consider the Piecewise Aggregate Approximation and Symbolic Aggregate Approximation representations.

2.4 Frameworks and earlier work

This section discusses the choices made in previous research which may affect the results of this thesis. As mentioned in 1.1, this thesis is a continuation of the ideas and knowledge achieved during earlier work. The research resulted in the creation of a prototype system which created the data used for implementing and testing the SAX algorithm (Watanabe & Eide, 2017). This system is presented and explained, before the resulting dataset is discussed.

2.4.1 C&Look

"C&Look" is the chosen name of the prototype system created by Watanabe & Eide which handles the interaction with patients and data gathering when performing eye screenings. It was created with the intent of being a complete system capable of performing a full screening and characterization of a patients functional vision abilities. The program handles the hardware, tests and some analysis of the recorded data.

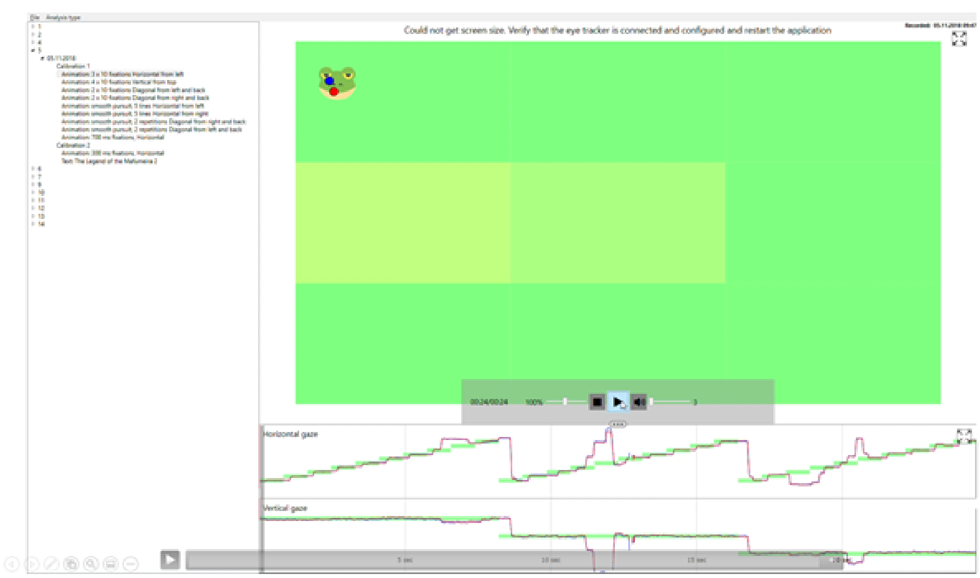

Figure 2.10: The user interface of C&Look during playback of a test. The stimulus object seen as a frog and recording data at the bottom. The position of the eyes can be seen as the two circles on the stimulus object. (Watanabe & Eide, 2017)

Eye Tracker Hardware

Today there are multiple eye trackers on the market which are suitable for a project like this but they vary greatly in price and functionality. A set of requirements seen in table 2.1 narrows the list of possible trackers down. The list was created using the goals for this thesis and the knowledge gained from earlier eye tracking projects.

Table 2.1: List of requirements used in choosing a suitable eye tracker.

Requirements of Eye Tracker
Must measure movements for each eye separately.
Must be easily portable between screenings.
Must allow users to wear glasses and contact lenses.
Must measure scanning and saccades on short distances.
Must provide number of fixations and fixation time.
Must be affordable to the general public.

The device which best fulfilled the criteria seen in table 2.1 was the "Tobii EyeX Pro" eye tracker seen in figure 2.11. This device achieved high accuracy, was cost effective, readily available and highly manageable. Additionally, software and frameworks for research, development, calibration and many more features are included with the Tobii devices.

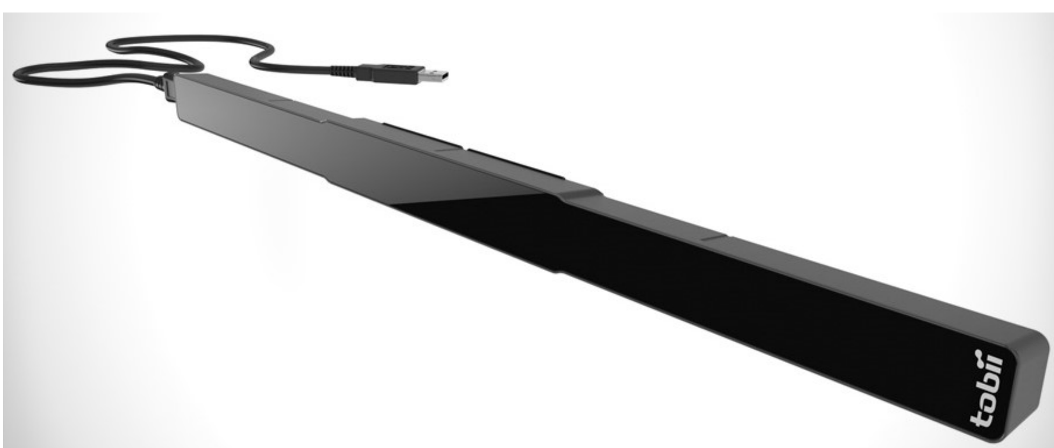

Figure 2.11: The Tobii EyeX recording device used with C&Look to gather the data used in this thesis. (Tobii Technology, 2022)

Calibration

An important aspect of the analysis of the data is considering the calibration of the eye trackers before the tests are performed. Calibration is performed because every eye is unique in some way, and therefore the reflective surfaces used to measure the position of the eye may vary from person to person. A general algorithm for calculating the position by measuring the reflections in the eyes is used, while the calibration process personalizes it for accuracy. Calibration can be performed in several ways, but most commonly performed by the subject looking at a set of focus points on the screen and adjusting until the points and the calculated gaze corresponds. In C&Look, the calibration is performed using this process, seen in figure 2.12.

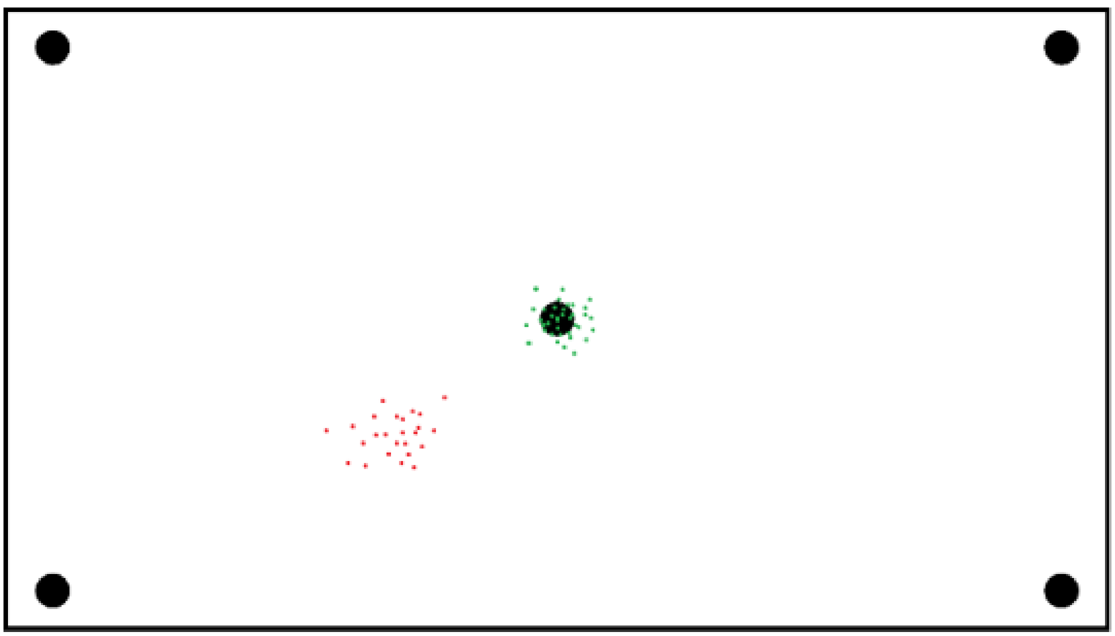

Figure 2.12: Calibration performed in C&Look. The red dots show the gaze points before calibration, blue dots show after calibration. (Watanabe & Eide, 2017)

When performing research on data where calibration is performed, it is important to consider how the calibration affects the data. As the goal for the calibration process is to compensate the calculations where needed, the effect may be that it "hides" or minimize some movements or problems that persists throughout the calibration. This is especially important in research like the one done in this thesis, where such movements may be used to detect the behavior of a subjects eyes.

Tests

C&Look contains 5 individual types of animation tests during the writing of this thesis: (i) diagonal, (ii) vertical, (iii) horizontal, (iv) speed and (v) reading, which can be seen in figure 2.13. The tests consist of either movement of a stimulus object or showing plaintext on the screen. Each movement test can be performed either with a set of fixations or as a smooth pursuit of an object. The number of fixations can vary. The complete set of available tests can be seen in table 2.2. The speed test is a variation of the horizontal test.

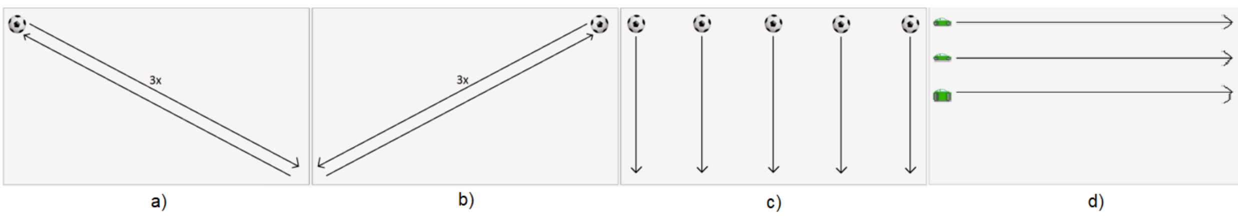

Figure 2.13: Visual of diagonal, vertical and horizontal stimulus shown on screen. a) DiagonalRightAndBack b) DiagonalLeftAndBack c) Vertical d) Horizontal (Watanabe & Eide, 2017)

While the tests are not exhaustive at all, they serve as a solid baseline to run our experiments. While more complex tests might render a more complete analysis, for the purpose of examining characterization capabilities, these tests suffice. Additionally, due to the generality of these tests, more advanced tests would in practice be a combinations of these.

Table 2.2: List of all available tests in the program C&Look.

	Horizontal	**Vertical**	**Diagonal**		**Other**
			Left	**Right**	
Fixations	h_10	v_10	dlb_10	drb_10	r
Smooth	h_s	v_s	dlb_s	drb_s	sst

A description of the names used in table 2.2 can be found in table 2.3. Horizontal, vertical and diagonal tests, with both fixations and smooth pursuit will be utilized in this thesis, as well as one saccadic speed test. The reading test is not used in the experiments due to their result being highly affected by the cognitive abilities of the user, especially when testing children.

Table 2.3: Description of the tests in the program C&Look.

ID	**Name**	**Description**
v	Vertical	Object moving on y axis
h	Horizontal	Object moing on x axis
dlb	DiagonalLeftAndBack	Diagonal movement from top right corner
drb	DiagonalRightAndBack	Diagonal movement from top left corner
r	Reading	Full page of text
sst	Saccade	Random movement of object

Data storage

The C&Look software utilizes the PostgresSQL database for storing all information. This includes anonymous patient data and identifiers, blueprints for test animations and the recorded eye tracking data. The latter, being of the highest space complexity, requires a scalable and efficient database, especially when hardware performance and test complexity increases.

PostgresSQL is an open-source object-relational database system. The system was originally developed in 1986 under the name postgres, but was later expanded to include support for SQL (Postgressql, 2022). It is widely used for computer science, and has a reputation for being versatile and expandable. With the addition of it being free of charge and including an admin UI for simpler management, it is among the best choices for the work done in this project.

Updates since release

Between the writing of Watanabe & Eide's master thesis and the collection of the data used in this thesis, some changes were made to the prototype system C&Look. This is of little importance to the goal of this thesis, but is mentioned for completeness. Some bugfixes have been made, mostly for the smooth functionality of the program. Additionally tests have been removed, and new ones have been created.

2.4.2 C&Look Dataset

In this thesis, the focus lies on the use case of medical recording data from eye tracking technology. Specifically, the real world tracking data data of young children in 3rd grade with a large variation of problems. This data was gathered during the writing of Watanabe & Eide's thesis with their prototype C&Look. The complete dataset consists of the recordings from 37 children from 3rd grade, acquired using the required documentation and consent. This thesis will only use parts of that dataset.

The reason for using this dataset consists of three parts. First, the dataset collected using C&Look is better validated than most available sources as it is gathered with the assistance of eye health specialists, which is highly desirable. Additionally, our need for specified data is hard to achieve from publicly available sources.

The second reason is that this thesis is a continuation of the work done by Watanabe & Eide. We wish to be able to use their prototype C&Look in collaboration with the methods presented in this thesis. Therefore implementing our solutions customized to handle their structural choices seem desirable in achieving this.

The third and final reason is that we want to initially focus on the group of patients where the most impact can be made. Children's eyes are difficult to diagnose as little information can be gained from the patient. Additionally, adults usually at some point discover their lack of visual ability through experience and reflection, while children usually lack the reasoning to be able to self diagnose vision problem. This may greatly affect their capabilities during their education which may have consequences the rest of their lives. The symptoms of vision problems are also similar to typical cognitive disturbances, which results is cases of misdiagnosis when the vision problem is not identified.

Chapter 3

Methodology

In this chapter, we discuss the methods used in this thesis. Initially, the research method is presented and the iterations performed are discussed. Finally, the evaluation process is discussed and the observations of interest are presented.

3.1 Research Method

In any research performed, a methodology should exist containing the step of works and decisions required to efficiently and correctly reach the goals. In natural science, e.g. mathematics or chemistry much of the processes may be evident. While this thesis is based on empirical motivations and using databases validated by experts from earlier research, the methodology to develop experiments to evaluate the data is coming from statistics, and does not require argumentation in a similar manner. Since this method use data from the practice, and after this work it is expected to be continued further to demonstrate practical usability, we define the approach behind this research in order to help understanding the completeness and rigor of the results. The grounding for the experimental process in this thesis lies in the scientific method, which can be seen in figure 3.1. This description is based on a rather generalized approach that consists of three parts: (i) *research on the background and formulating the hypotheses*, (ii) *defining the experimental study and performing the analysis* and (iii) *conclusions*.

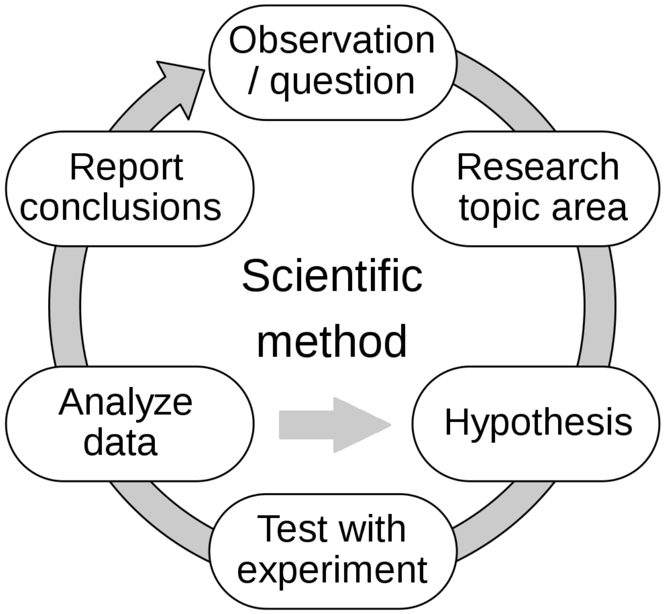

Figure 3.1: Illustration of the scientific method. (Wikipedia Commons, 2022)

The *research and hypothesis* section consists of finding a research topic, reading previous research, learning about the domain and creating a set of goals and assumptions. This section produces the contents found in the first four chapters of this thesis. The *experiment and analysis* consists of implementing a system for experimentation, running a dataset through the system and reporting observations found in the results. This is performed in chapter 5 in this thesis. The *conclusions* section combines earlier knowledge with the new findings. This is performed in chapter 6 & 7. In addition to moving through these sections during the research, there is a cycle between the experimental study section and the previous hypnotises section performed in iterations. This is crucial when performing exploratory research (Gauch H. Jr., 2003).

The processes in these sections were performed in three phases, illustrated in table 3.1. The initial phase was combined with the previous work done in this project by other students and the visions of the professors assisting in this thesis. This phase was much larger than anticipated, mostly due to the rigorous research of the methods in order to understand their use and the process of gathering and validating data. In the second phase a iteration cycle is introduced as the experimentation process begins and reveals new information which can provide further improvements. This cycle also contain adding complementary research and deriving conclusions. This was by far the most intense and time consuming phase as the

Table 3.1: Overview of the phases during the research process of this thesis.

Phase	**Content**	**Chapter**
Phase 1	- Topic of Eye Tracking is chosen and researched. - Method of SAX is chosen and researched. - Goals and hypothesis are created. - Relevant research and goals are reported.	Chapter 1 - 4.
Phase 2	- Data is gathered, inspected and organized. - Parameters values and configurations are determined. - Experiments are designed and implemented in a system over the span of multiple iterations. - Output is organized and presented. - The process is documented and reported.	Chapter 5.
Phase 3	- Experiment results are discussed. - Conclusions are drawn. - All findings are reported and the thesis completed.	Chapter 6 & 7.

entire implementation was created and all the experiments were designed and performed. Each iteration cycle focused on a part of the algorithm, followed by evaluation end testing. The third and final phase mainly consisted of the writing part of the research process, and revolved heavily on inspecting the results from the previous phases and reporting our findings. During this phase, final adjustments on the implementation and experiments were made.

3.2 Evaluation Method

In order to gain an understanding of the research performed in this thesis, we need a method for evaluating our results. In this thesis, the evaluation performed consists of a qualitative visual comparison of the raw data and resulting transformed data. To perform this comparison, a standardized set of identifiable behavior must be determined. In figure 3.2 we see the set of behavior chosen as the focus of this thesis.

Table 3.2: Categories of events to be identified using SAX.

Name	**Description**	**Characteristics**
Normal	Expected behavior from a subject with *no* functional vision problems.	Data from both eyes maintain a short consistent distance.
Problematic	Expected behavior from a subject *with* functional vision problems.	Data from both eyes move off the intended stimulus object and looses coordination.
Off-screen	The subject looks outside the screen.	A continuous line of 0 or 1 over a period of time.
Spike	Unknown recording errors, anomalies or blinking.	A large increase followed by a decrease in distance over a very short period.
Loss of attention	The subject loses focus of the stimulus.	The eyes move in parallel off the expected stimulus path.
Tiredness	The subject becomes tired at the end of the screening.	A greater average distance can be observed at the end of the recording.
Oscillation	The eyes temporarily or consistently oscillate.	Collection of spikes either on one eye or in coordination.

A distinction is made between characterizing specific behavior and characterizing the final conclusion of a screening. A unique problematic event does not necessarily constitute problems with the vision, and it is the presence of multiple problematic events which determines if a screening is problematic or not. In figure 3.2 we see two examples, one of a normal recording, and one containing enough problematic events to constitute problems in the vision.

The evaluation process is informal, which has a large impact on the quality of the research performed in this thesis. While a lot of knowledge can be derived from the results for use

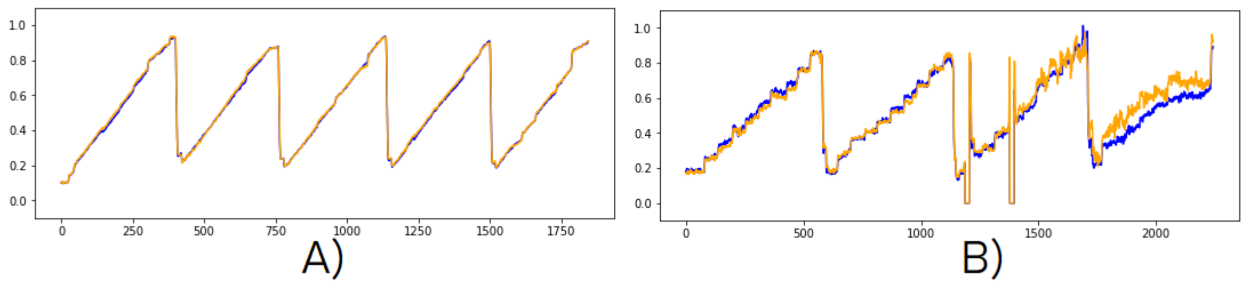

Figure 3.2: Example of normal and problematic behavior. A) Normal behavior B) misalignment, 2 spikes, oscillation and tiredness.

in further research, the findings are weighed with uncertainty. This uncertainty is reduced due to the nature of the experiments, specifically by analysing the behavior of changing the parameter values in addition to the transformed data. The evaluation is performed using a set of metrics. These are shown in table 3.3. The metrics posses the same informality of the evaluation process, but is valuable in the discussion and conclusions in this thesis.

Table 3.3: Metrics used in the evaluation of the results.

Name	**Description**
Readability	The ease of manually and visually inspecting the transformed data.
Discovering ability	The ability to identify and distinguish events.
Complexity	The amount of work and layers of analysis needed in order to produce the results.

Chapter 4

Algorithms and functions

In this chapter we will present and discuss the algorithms and functions used in our experiments. We begin by explaining the Piecewise Aggregate Approximation method, which is an intermediate step of Symbolic Aggregate Approximation, before explaining the final discretization process resulting in the symbolic representation of the time series. Finally, the implementation of these are presented and a system for running experiments is proposed.

4.1 Piecewise Aggregate Approximation

Piecewise Aggregate Approximation (PAA) is a supporting technique for mining time series, and results in a time series with reduced dimension. It was proposed in (Yi & Faloutsos, 2000) and is an intermediate step required to perform the Symbolic Aggregate Approximation which is presented later in the next section.

The PAA algorithm reduces a time series C of n length into a w-dimensional vector $\bar{C} = \bar{c}_i, ..., \bar{c}_w$. With the following equation we can calculate the ith element of $\bar{C}$:

$$\bar{c}_i = \frac{w}{n} \sum_{j=\frac{n}{w}(i-1)+1}^{\frac{n}{w}i} c_j \tag{4.1}$$

The time series is divided into w equal sized sections called "frames". If n is not divisible by w, there will be some points which are difficult to place. A solution presented in (Lin et. al., 2003) consist of splitting the affected points into parts and placing the parts in each their frame. This generalization is not used in our experiments as we can simply cut the original data into a length divisible by w.

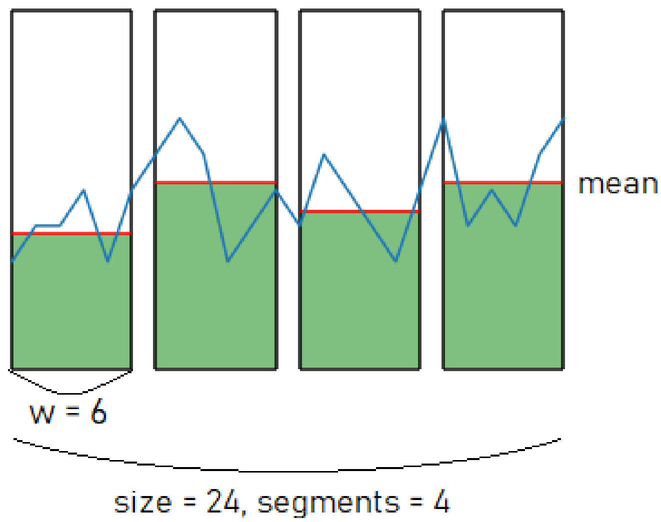

Figure 4.1: Example of segmented means when performing the PAA transformation.

Once the time series is split into frames, the mean of the values falling inside each frame is calculated. Figure 4.1 illustrates the complete process of the segmentation and mean calculations of a time series of length 24 using 4 segments. The vector of these means is the final data-reduced representation of PAA and is used in the discretization process of Symbolic Aggregate Approximation.

Before performing the PAA transformation, the time series is normalized. This step consist of normalizing the time series to have a mean of zero and a standard deviation of one. In our implementation, the mean and standard deviation of the entire dataset is used. This form of normalization is sometimes referred to as standardization and has two functions: (i) to compare time series which are created using different preconditions such as amplitude and offset (ii) determining a set of "breakpoints" which are used to divide the numerical ranges of the values corresponding to symbols in the discretization process. These breakpoints are further explained in the following section. The properties of PAA are carried on to the SAX representation through transitivity.

4.2 Symbolic Aggregate Approximation

Symbolic Aggregate ApproXimation (SAX) is proposed in (Lin et. al., 2003) and transforms a time series into a discrete string. The method is unique as a discretization procedure as it uses Piecewise Aggregate Approximation as an intermediate representation.

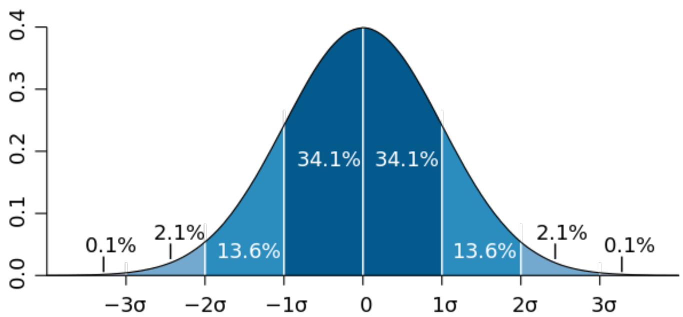

Figure 4.2: Standard Deviation under a normal probability distribution curve. (Wikipedia)

To achieve the discretization in SAX, a set of "breakpoints" must be determined. Breakpoints are a set of numerical values which dictates the assignment of symbols. They are a crucial part of the algorithm, as they produce the discrete areas of equal size, seen in figure 4.2, for use during the symbolic discretization process of SAX. From the theory, a system arises based on the values' odd or even property. The group a value has dictates weather a breakpoint lands on zero or if zero falls between two breakpoints. This respectively either creates areas which are over and under average or an area of average. All values of the same group has this property, the only difference when incrementing is how many parts the areas are divided into. This is property, and the division of symbols, is illustrated in figure 4.3.

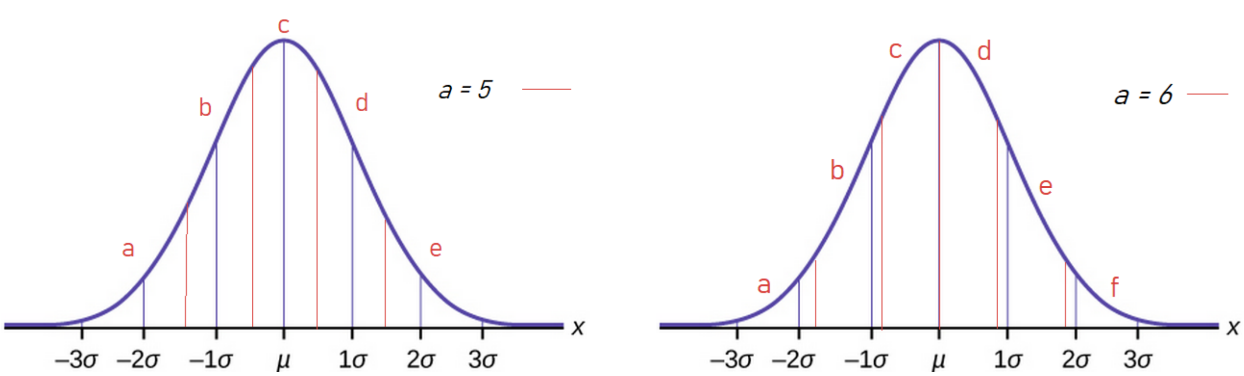

Figure 4.3: Odd and even effect on distribution of symbols on the probability curve.

Given that the time series are normalized in PAA, and that normalized time series have a highly normal probability distribution, the values of the breakpoints shown in table 4.1 can simply be looked up in a statistical table.

Table 4.1: Lookup table containing the breakpoints that divide a Gaussian distribution of equiprobable regions. (J. Lin et al. 2007)

β_i	5	6	7	8
β_1	-0.84	-0.97	-1.07	-1.15
β_2	-0.25	-0.43	-0.57	-0.67
β_3	0.25	0	-0.18	-0.32
β_4	0.84	0.43	0.18	0
β_5		0.97	0.57	0.32
β_6			1.07	0.67
β_7				1.15

Once the breakpoints are placed, the process of discretization is performed in the following manner. All coefficients below the smallest breakpoint are mapped to the symbol "**a**", coefficients greater than or equals the smallest breakpoint and less than the second smallest breakpoint are mapped to "**b**", ect. By running the loop through all the coefficient values, adding the corresponding symbol, we finally retrieve our dimensionally reduced discretization of the original time series. The process is visualized in figure 4.4 and the resulting string is referred to in the articles as a "SAX word" or "SAX string".

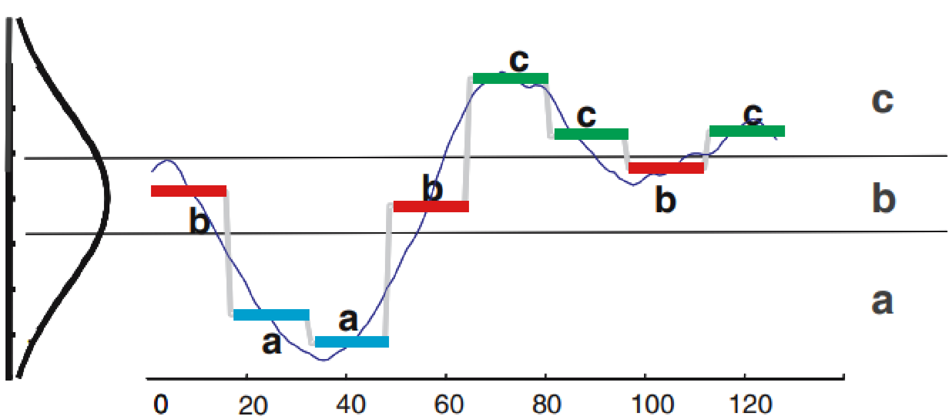

Figure 4.4: Example of SAX ran on a time series of length 128 with framesize of 8 and alphabet size of 3. (J. Lin et al. 2007)

4.2.1 Parameters

The choice of parameters is essential for the scope of this thesis. For SAX there are two parameters we can adjust. In this section, we will present these and discuss their impact on the results. We will also present earlier findings which limit the range of values we must investigate.

Frame Size (w) indirectly sets the dimensionality of the SAX representation by dictating how many datapoints from the original time series are used to calculate the new value in each frame. This is in contrast to the way it is done by the original authors where the w parameter directly sets the dimensionality of the resulting sax representation. This is because we found it of more interest to control the the size of a frame rather than the dimentionality.

After our change, the mathematical equation looks like the following:

$$\bar{c}_i = \frac{1}{w} \sum_{j=w(i-1)+1}^{wi} c_j \tag{4.2}$$

The lowest value of w can be chosen by assessing the use case of the method. There should be some form of reduction on the original dimention, therefore w must be over 1. The increment 1 to 5 is negligible when assessing the overall behaviour of the algorithm, therefore 5 can be used as a base value. Choosing lower values may be interesting, but not for the scope of this thesis. The highest value of w can also chosen based on the same criteria, with the addition of asserting the value from the frequency of the eye tracking device. In order to use the full potential of the eye tracker, the value of w should be set much lower than the frequency of the tracker.

Alphabet Size (a) controls the amount of symbols used in the dizcretization of the time series. The numerical value of a points to the length of the range of symbols in the alphabet starting from "**a**". In essence this means the value of a controls the size of the areas created by the breakpoints, in other words, determining which breakpoint values to use.

As explained earlier in section 4.2 and observed later in the results, the choice of values for a is narrowed down by a specific property controlled by the value being either odd or even. In short, this property dictates weather the middle breakpoint(s) lands on zero or if zero falls between two breakpoints. This respectively either creates areas which are over and under average or an area of average. All values in the same group has this property, and the only difference when incrementing is how many parts the areas are divided into. This results in only needing to test specific parameters to gain a full understanding of the rest.

The lowest value possible for a is 2. This is a special case of SAX which is called "clipping". Increasing the value further results in the effects from the odd and even property mentioned earlier, therefore increments similar to those of w may be used. The highest value of a is the length of the alphabet, but is in practice usually much less.

4.2.2 Euclidean distance

The Euclidean distance formula was proposed by the Greek mathematician Euclid using geometry. Several other distance measures have been proposed by others, but none are as versatile as the Euclidean distance. The formula can calculate the length of a line segment between two points on a one-dimensional plane. The formula can be further expanded to include more dimensions by applying the Pythagorean theorem to a right triangle on the plane which results in the following:

$$d(p,q) = \sqrt{(p_1 - q_1)^2 + (p_1 - q_1)^2} \tag{4.3}$$

This distance is the length of a line segment between two points on a Cartasian coordinate system. The Cartasian system is a coordinate system where each point is uniquely identified by a pair of numerical values instead of single value points. The euclidean distance between the pairs is calculated using the formula shown in 4.3. Initially this theory was suggested using geometry, but was later combined with Pythagoras to perform the calculation numerically in the 18th century. The geometrical explanation can be seen in figure 4.5.

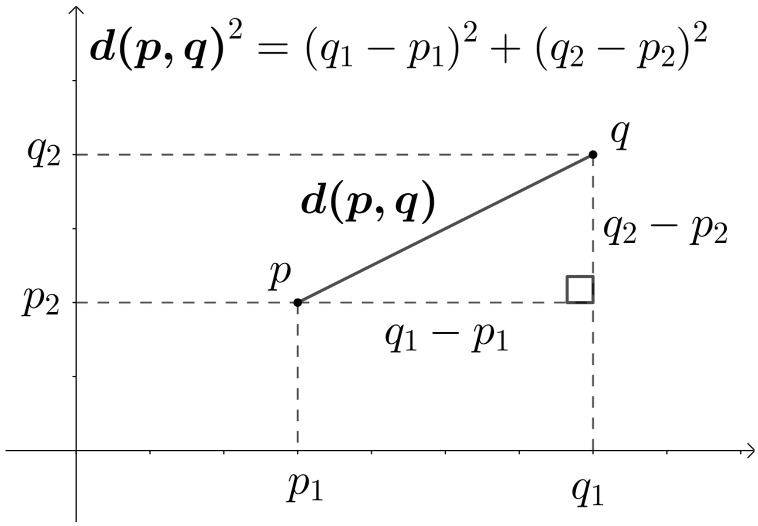

Figure 4.5: Using the Pythagorean theorem to compute two-dimentional Euclidean Distance. (Wikipedia Commons, 2022)

4.3 Implementation of SAX

To be able to test the effects of the symbolic representation in discovering anomalies in our data, we had to build a program running the algorithm on a set of parameters which are then visually presented for qualitative analysis. In the following section, the design and implementation of SAX is discussed.

4.3.1 Libraries, tools and IED

The system for running the experiments is written in Python using Jupyter Notebook IED. Both the IED and Python are versatile and easy to use, and for a time-sensitive project, this is imperative. The primary reason for utilizing Pyhton is the speed of development and testing. Other languages usually require larger frameworks and heavy programming to achieve the same results. Additionally, the Python community is large, and open-source code is readily available. This results in Python having a rich echo system of libraries for any need, but most notably data mining. We used libraries for reading data into datastructures, visualisation and mathematical functions to create our testing system. All libraries used are well-documented and regarded as trusted.

There are also libraries available for implementing SAX which we chose not to use. The reason for this is the nature of our experiment. First, we want to be able to evaluate all parts of the algorithm, which requires a different datastructure than what is used in the available libraries. Second, we can not rely on the choices made by the author(s) of a library unless they are provable and well-documented. Lastly, for us to create separate programs which we can freely use in our research we can not break the copyright.

Several libraries are implemented in the system to aid in research, but most of them are of little interest or redundant. The core logic of the system is ran by the following four libraries:

(i) **Pandas**
The Pandas library is a flexible open-source data analysis and manipulation tool. It works especially good with large amounts of data and provides various data structures and operations for manipulating numerical data and time series. For this system, the Dataframe structure is used to store and organize recording data.

(ii) **NumPy**
The NumPy library is an open-source library for scientific computing. The library is fundamental in most numerical computations performed in the Python language where basic arithmetic does not suffice. Combining the computational powers of NumPy with the datastructures of Pandas gives researches complete control over the data with well-defined and documented operations. The mean() and std() functions are used in the distance calculations and normalisation during our experiments. Furthermore NumPy includes it's own array structure which is utilized due to it's incompatibility with the Pandas datastructure.

(iii) **Math**
The Math library is included in the Pyhon Standard Library and contains important basic mathematical functions. It's sqrt() function is used to calculate the square root during calculations of distance measures.

(iv) **Matplotlib**
The Matplotlib library is an extention of the NumPy library and consists of methods for creating statistical visualisations in Python. The plots created by this library are used to present the results in this thesis.

4.3.2 System description

The system flow diagram, seen in figure 4.6, illustrates the flow of data in the system implementing SAX. The flow is divided into three sections: (i) pre-processing, (ii) experiments, containing the PAA and SAX algorithms and (iii) post-processing. The latter two are produced for each recording, while the initial processing can be done once for all experiments.

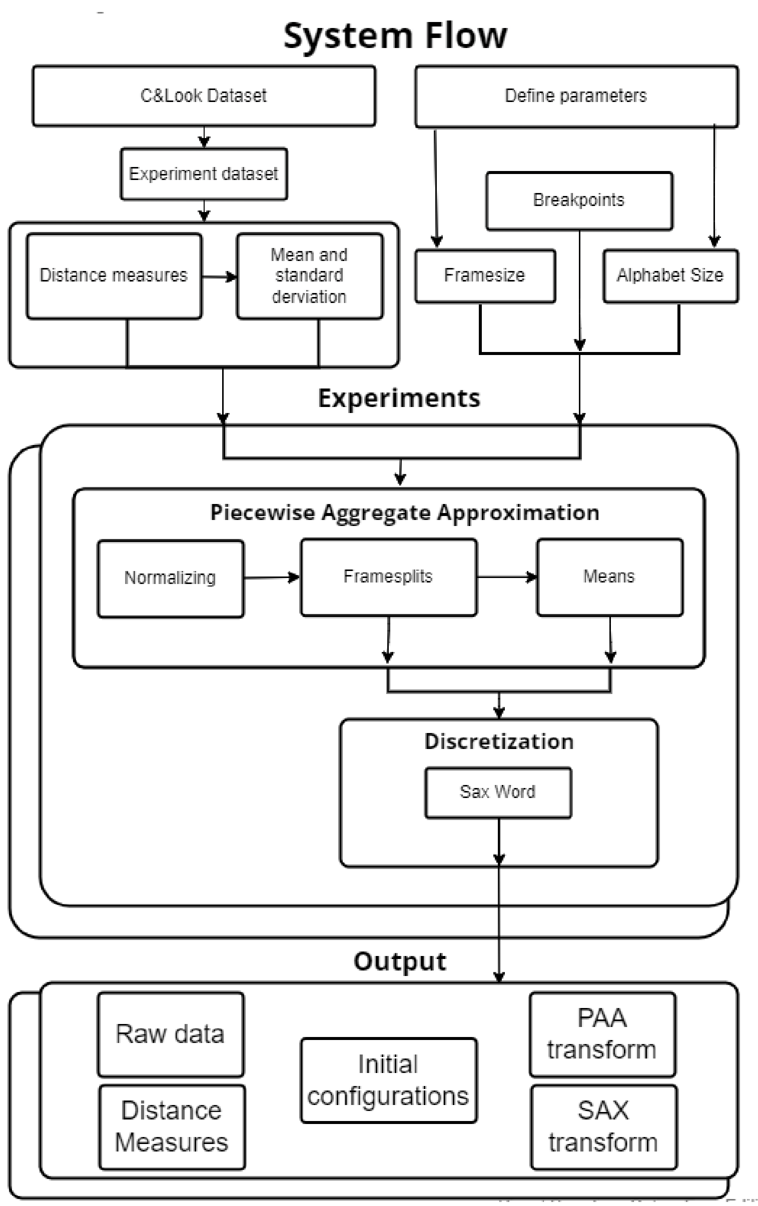

Figure 4.6: The flow of data in the proposed experiment system.

The system is designed with a step-by-step approach, where each functional step of the algorithm can be presented visually through graphs. This is done to be able to break down the SAX procedure and analyze each part separately.

- **Pre-processing (i):** The pre-processing needed before running the experiments relies heavily on manual inspection. This may be changed in future versions, but for the purpose of this thesis we decided to not automate this part. In this section of the system the dataset is fetched and organized into dataframes. A set of unique recordings are extracted, and the breakpoints and parameters are initialized. Finally, the required euclidean distance measures are calculated and the total mean and standard derivation is stored for later use.
- **PAA Algorithm (ii):** The entire algorithm is contained in a object class. For each of the distance measures from the previous step, several classes are created, one for each value of w chosen for the experiments. The class consists of the mathematical methods needed to normalize the data, split the data in frames and calculate the mean of each frame. The result of these calculations are performed upon creation of the object and is stored in the object for later use. The normalization is performed using the mean() and std() operations from the NumPy library. The mean and standard derivation stored during pre-processing is used to perform this transformation. The following code is used on a time series in the *normalization* process:

```
    def normalize_data(data):
        return (data - np.mean(all_euc_values, dtype=np.float64)) /
          ↪ (np.std(all_euc_values, dtype=np.float64))
```

Splitting the frames consists of creating an array containing the location of the slices between the frames, based on the value of w. In the following code, this array creation can be seen. Additionally, we also see the transformation of the w parameter, which is discussed in section 4.2.1.

```
    def calc_frame_splits(data, w):
        n_frame_split = int(len(data)/w)
        df_frame_splits = []

        i=0
        for frame in range(n_frame_split):
            df_frame_splits.append(w*i)
            i=i+1
        df_frame_splits.append(len(data))

        return df_frame_splits
```

In each frame, the mean is calculated using the NumPy library and the array of slices created earlier. The resulting array of means, produced by the following code, is the final output of the PAA transformation which is used to perform the disctretization in SAX.

```
    def calculate_frame_mean(data, frame_splits):
        all_mean = []

        i=0
        all_mean.append(
            np.mean(data[frame_splits[i]:frame_splits[i+1]],
               ↪ dtype=np.float64))

        i=i+1
        for val in range(len(frame_splits)-2):
            all_mean.append(
                np.mean(data[(frame_splits[i]+1):frame_splits[i+1]],
                   ↪ dtype=np.float64))
            i=i+1

        return np.asarray(all_mean)
```

- **SAX Algorithm (ii):** The rest of the steps required to perform the final discretization of SAX is contained in another object class. For each of the PAA objects, several classes are made, one for each combination of a and w chosen for the experiments. This class consists of the method to assign a symbol to each value in the array of means from the PAA object. The means are transformed to symbols by looping through each value and assigning the corresponding symbol based on the respective breakpoint. The resulting string is stored in the SAX object and is performed using the following code:

```
    def assign_symbols(paa_means, a):
        sax = ''

        for mean in paa_means:
            i = 0
            for breakpoint in breakpoints[a]:
                if mean < breakpoint: # Check from lowest breakpoint
                   ↪ to highest if mean is lower than breakpoint.
                   ↪ Increment alphabet(i) with breakpoint.
                    sax = sax + chr(97+i)
                    i=i+1
                    break
                if mean >= breakpoints[a][-1]: # If mean higher than
                   ↪ max breakpoint set to highest possible sax
                   ↪ char.
                    sax = sax + chr(97+a-1)
                    i=i+1
                    break
                i=i+1
        return sax
```

- **Post-processing (iii):** The post-processing needed after running the experiments rely on the intent of the research. In general, the output is contained within the object classes and can be called upon at will. In most of the experiments, only parts of the output are of interest, and those parts can be manipulated in different ways. These results are contained within the final section of the code and presented in the same order as the experiments in chapter 5.

This system does not reflect the entire vision of the underlying project. Namely, creating an *automated* system of which the analysis method is implemented. As our implementation is contained within the object classes, and automated execution of the analysis can easily be performed by tweaking the code. Additionally, in section 7.3 we present a solution of handling the w and a parameters using a "zoom" method which in collaboration with a GUI can automate the process of analysing using SAX.

Chapter 5

Experiments

In the previous chapters, we presented the background, methods and algorithms required to perform an analysis of eye tracking data. In this chapter, the details of how we utilized these methods in order to analyze a dataset are described. Parts of the dataset and configurations used in the experiments are presented before explaining in detail each experiment, their data and observations.

5.1 Experiment data

Table 5.1: List of requirements used in choosing suitable recordings for the experiments.

Required recordings for experiment dataset
Ideal behaviour
Problematic behaviour
Unusual behaviour

For our experiments, we do not require massive amounts of data. This is because we are performing a qualitative analysis of the results. This is discussed in section 3.2. Instead, a deliberately chosen set of time series are extracted from the dataset based on a set of requirements shown in table 5.1. The goal is to test most of the various types of behaviour during our experiments, even though this is not feasible in practice. These behaviors are discussed in section 2.1.2.

The experiment dataset consists of the time series seen in figure 5.1, 5.2 and 5.3 which were extracted from C&Look. In addition to reflecting the requirements from table 5.1, it also contains some redundant timeseries not used in the experiments. The redundancy is present for efficiency of research in case further testing is required. Additionally, both time series from one recording are always included for completeness.

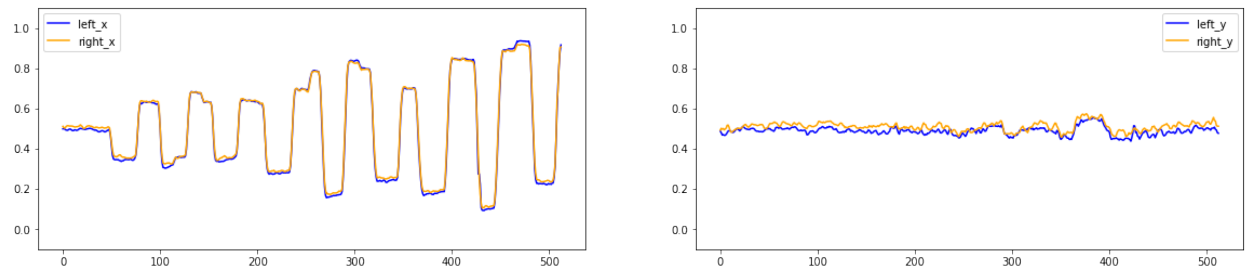

Figure 5.1: Pair of graphs from a recording chosen for the experiments. Left side shows left and right eye on the horizontal axis and right side shows the vertical axis. The stimulus used for this recording is increasingly oscillating on the horizontal axis.

The dataset contains the data from 10 unique recordings, each producing 4 time series; two for each axis of movement, horizontally or vertically, one for the left eye and respectively one for the right eye. In the result figures, the left graphs all show the horizontal axis of the left and the right eye, while the right graphs show the vertical direction of movement. In figure 5.1, we see the only recording using a random movement on the horizontal axis. In the other figures the stimulus is presented at the top, where figure 5.2 is split in the middle, while figure 5.3 is split on the last graph pair.

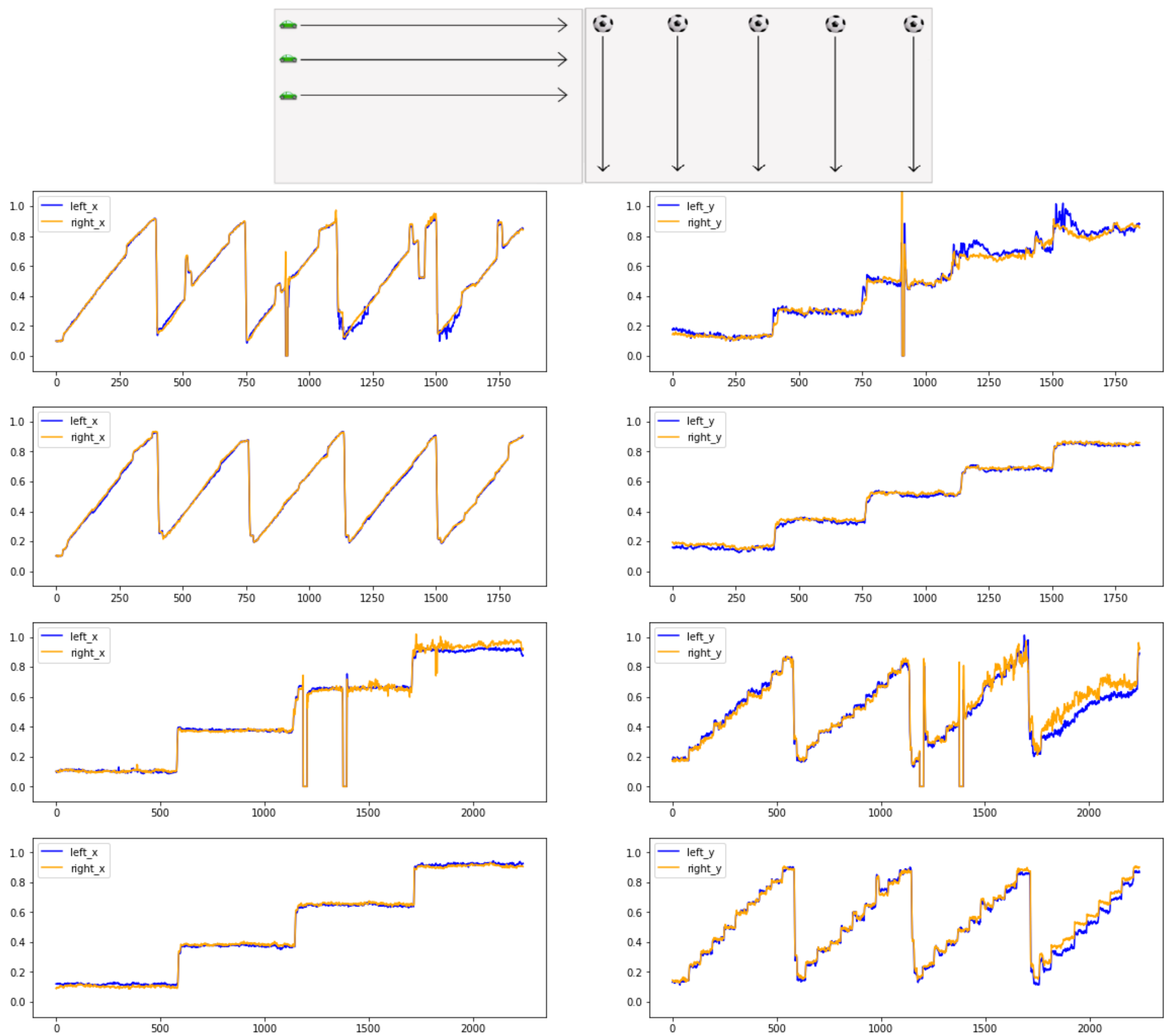

Figure 5.2: Second set of recordings chosen for the experiments. Left side shows the horizontal axis and right shows the vertical axis.

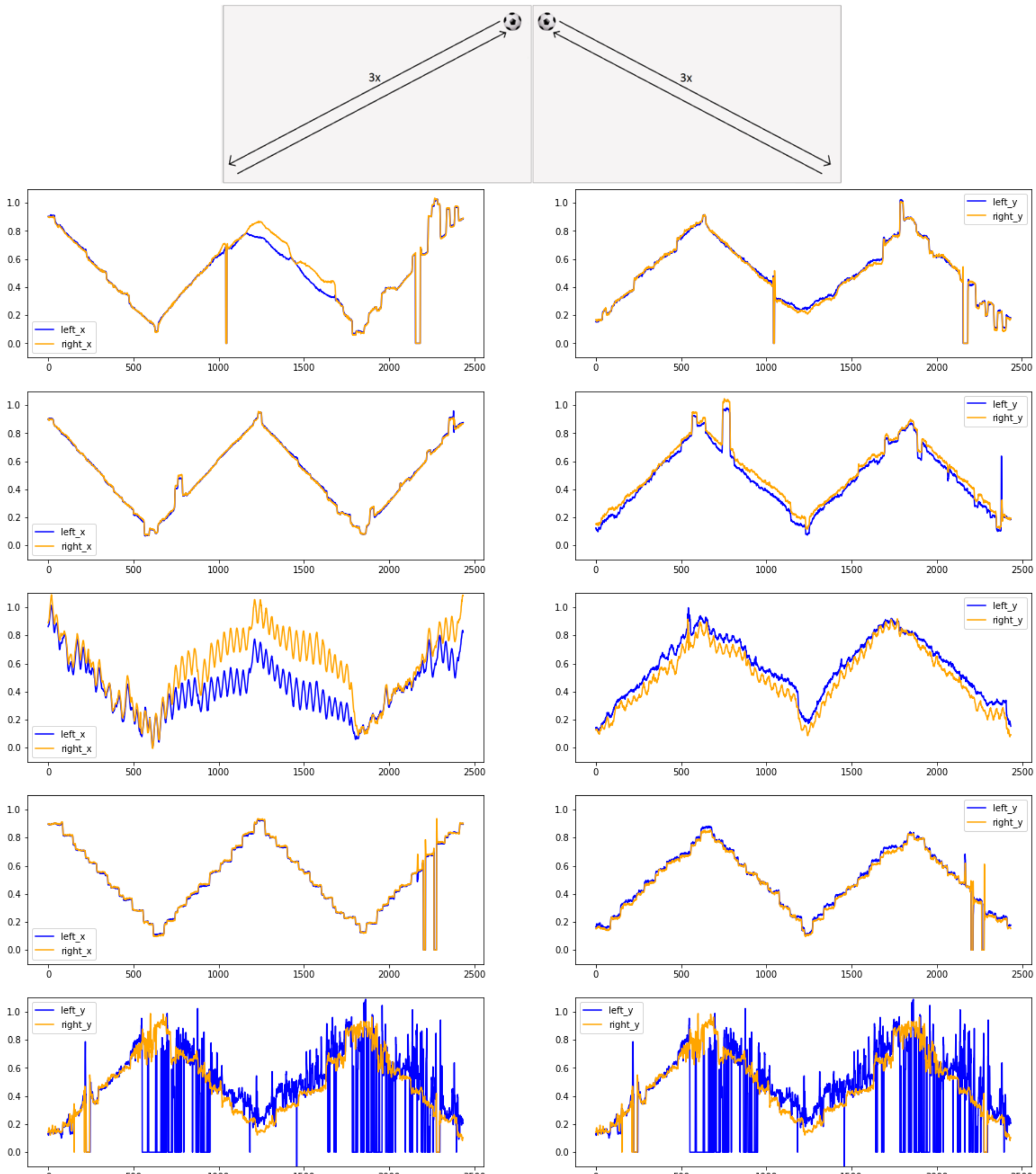

Figure 5.3: Second set of recordings chosen for the experiments. Left side shows the horizontal axis and right shows the vertical axis.

5.2 Design of experiments

Table 5.2: Overview of the experiment structure.

Sub-experiments	
Frame Size (w)	Experiment 1 - 3
Alphabet Size (a)	Experiment 4 - 6
Discovery of Functional Vision Problems (w x a)	Experiment 7 - 9

To achieve goals of this thesis, several experiments are completed in quick succession. These sub-experiments are created to highlight specific features we are interested in researching. They are divided into three categories shown in table 5.2. Each experiment produces a set of results presented later in this chapter. The choice of dividing in this manner was made so we could test the PAA and SAX separately. This is important because we don't have a full understanding of the complexity of changing the data representation on eye tracking data.

We ran our experiments through our proposed system presented in section 4.3 using the data from figure 5.1 and 5.2. The experiments are designed in the following manner:

1. **Euclidean distance measures** are calculated on all time series using the method from section 4.2.2. The results of the calculations are time series of distance values, one for each axis, which are later used as the input for the algorithm. Additionally, a distance measure between one of the eyes and the stimulus object is calculated for experiment 9. The result is three different distance measures:

   (i) Horizontal - Distance between Left x and Right x.

   (ii) Vertical - Distance between Left y and Right y.

   (iii) Eye and Object - Distance between Left OR Right x/y and Object x/y

2. **Piecewise Aggregate Approximation (PAA)** is initialized with the euclidean distance measures calculated in the first step. The transformation is ran using a set of parameter values discussed later in section 5.2.1.

3. **Symbolic Aggregate ApproXimation (SAX)** is initialized with the mean values calculated during the PAA transformation. These transformations are ran with the set of parameter values discussed later. At this point, each distance measure has now produced it's own set of results.

4. **Output** data is organized into sections shown in table 5.2. The output consist of the PAA and SAX objects, as well as the initial configurations for that particular experiment.

5.2.1 Experiment parameters

To achieve the goals of these experiments, a set of parameter values must be constructed such that they span over a large and precise enough range to identify all the different behaviours of the algorithm. This means that our largest value somewhat indicates how all larger values behave, and the smallest value somewhat indicates how all smaller values behave. Additionally, the increments between the values must be small enough so that no behaviour is lost between the larger or smaller value.

Table 5.3: List of parameter values used in the experiments.

Parameter	**Values**
Frame Size (w)	5
	15
	30
	45
	60
Alphabet Size (a)	5
	6
	7
	8

As mentioned earlier, the experiments are designed to produce results able to confirm the algorithms ability of detecting problems. This means that we wish to identify interesting combination of parameters, as well as identify the behavior of the algorithm when changing a particular variable. Consequently, we will highlight each parameters (i) effect on correctness of results, (ii) effect on readability and (iii) effect on discovering problems with vastly different characteristics. In table 5.3 the values chosen for the two adjustable parameters are shown. In addition to the range constraints mentioned before, values are also bound by several key pieces of knowledge extracted from earlier work. This is discussed in detail in section 4.2.1. Using all the bounds and the criteria discussed in this section we created the set of values seen table 5.3.

5.2.2 Explanation of output

In this thesis, the results are presented through graphs. In figure 5.4 we see 3 of the 4 types of output occurring in this thesis. The first graph shows the stimulus used when performing the recording. This is the computed simulation of the stimulus object on the screen used in experiment 9. The second graph shows the actual recorded movement of both eyes, in this case on the horizontal axis. In this graph, two separate time series are shown, each corresponding to the left or right eye. The final graph in the figure shows the euclidean distance values calculated between the two time series seen in the second graph. This graph is used as the input of the SAX algorithm.

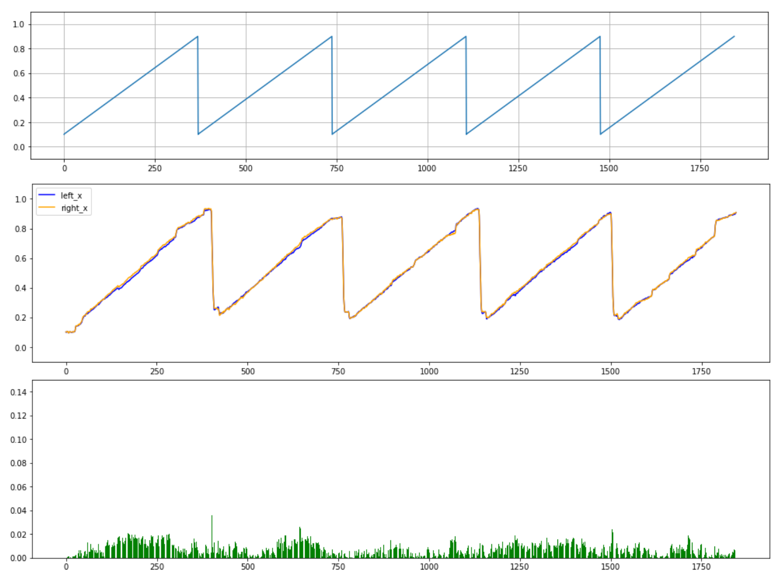

Figure 5.4: Example of experiment output. From the top: (i) the stimulus used in the test, (ii) raw data of the right and left eye and (iii) corresponding euclidean distance values between the eyes, in this case on the horizontal axis.

In figure 5.5, the final SAX output is seen. This graph consists of (i) the normalized mean vectors calculated in the PAA method seen as green pillars, (ii) the breakpoints seen as black dotted lines on the horizontal axis and (iii) the SAX symbol string seen at the bottom of the figure. On the left side we see how the symbols are divided. This is later only implied, as the reader simply needs to recite the alphabet from the bottom up. In the top right corner we first see the value of a as the dotted black line, and second the value of w as a green rectangle.

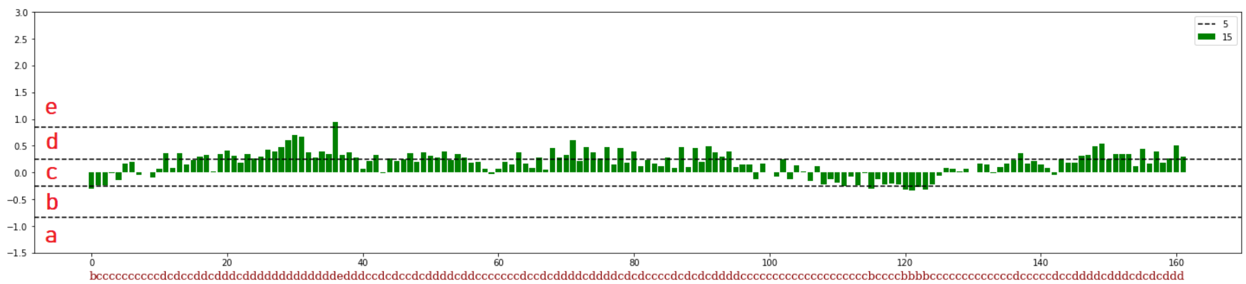

Figure 5.5: Example output produced when running the entire SAX algorithm.

Throughout our presentation and discussion of the experiments and results, we will use distinct language to point to parts of the figures of interest. When discussing the location of an observation, a value will be given for the reader to be able to locate the observation themselves. This value represents a point on the x axis and unless specified otherwise consists of the range of values surrounding that number.

5.3 Frame Size Experiments (w)

In this section we will discuss the details of experiment 1 - 3, what data and parameters were used, and finally present the results of the process. These results will be discussed in chapter 6.

5.3.1 Similarities of Experiments 1 - 3

Initially, we are interested in the effect of the w parameter value on readability of the resulting graphs. Consequently, we are not required to perform the full discretization as some understanding of the readability can already be achieved after the PAA transformation. Readability in this context means the ease of which a researcher can manually handle the resulting SAX string. In essence, there is a limit to the abilities of a human in analysing the resulting data. The reason is that we can intuitively deduce from the PAA transformation that large size vectors, produced by a low value of w, result in long sax words and vice versa. This has a great impact on the algorithms readability, and consequently usability, as small arrays may be utilized for manual analysis, while larger arrays can only be utilized by computers.

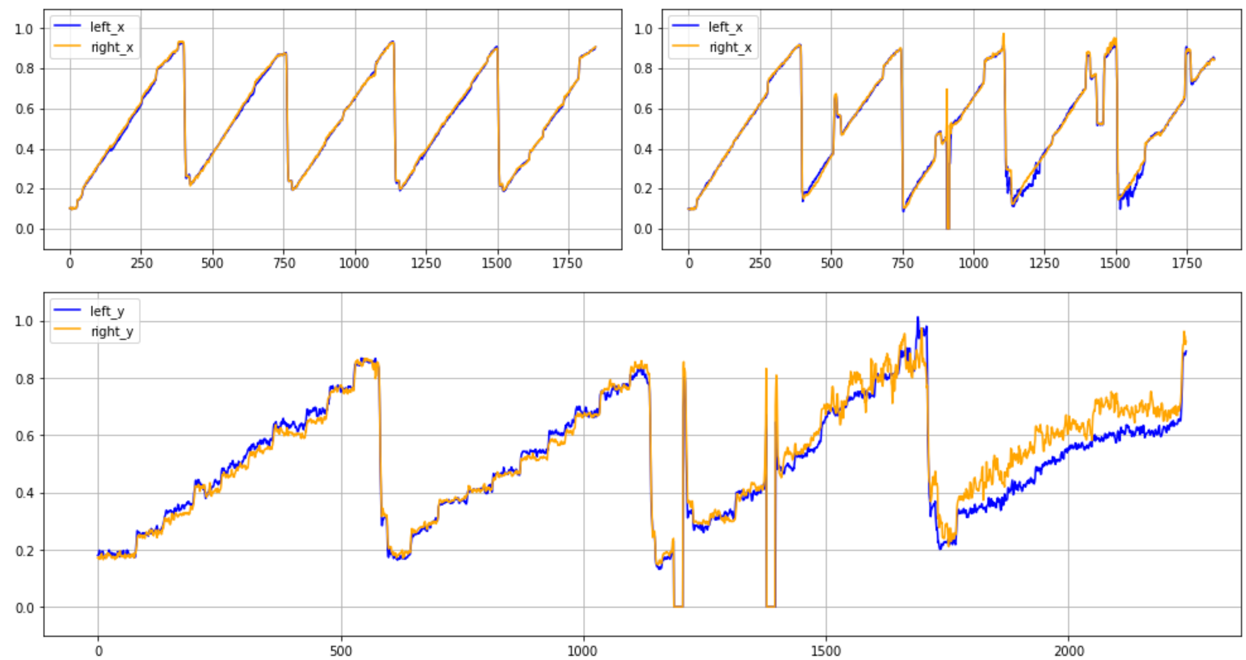

Figure 5.6: Dataset used for experiment 1 - 3.

The time series seen in figure 5.6 are divided into two groups (i) a pair of two time series using the same stimulus, one with identified functional problems and one ideal (ii) and a single time series containing problems with other characteristics. The first two are comparable with each other as they are created using the same stimulus. This will be important in being able to verify observations and behavior of the algorithm. The the last recording identifies new behaviour not shown in the initial two time series. This is done to identify undiscovered behavior or anomalies.

All the parameter values for the w parameter seen in table 5.4 are used in the experiments, namely (i) 5, (ii) 15, (iii) 30, (iv) 45 and (v) 60. This is initially done to narrow the set of values of w for use in later experiments. There are several reasons as to why this is done, but most importantly is to be able to perform the analysis without receiving redundant information. Redundancy in this case would be in testing different parameters which do not render different results. Testing these parameters more accurately might be interesting for further work, but not the scope of this thesis.

Table 5.4: List of parameter values used in experiments 1 - 3.

Experiment Parameter	**Values**
Frame Size (w)	5
	15
	30
	45
	60

5.3.2 Experiment 1

The goal for this experiment is to create a basis for further testing which can be used to compare with later results. This experiment also assists in verification of the implementation of the PAA algorithm as the results are expected to be highly uniform, which makes it easier to identify problematic behavior later.

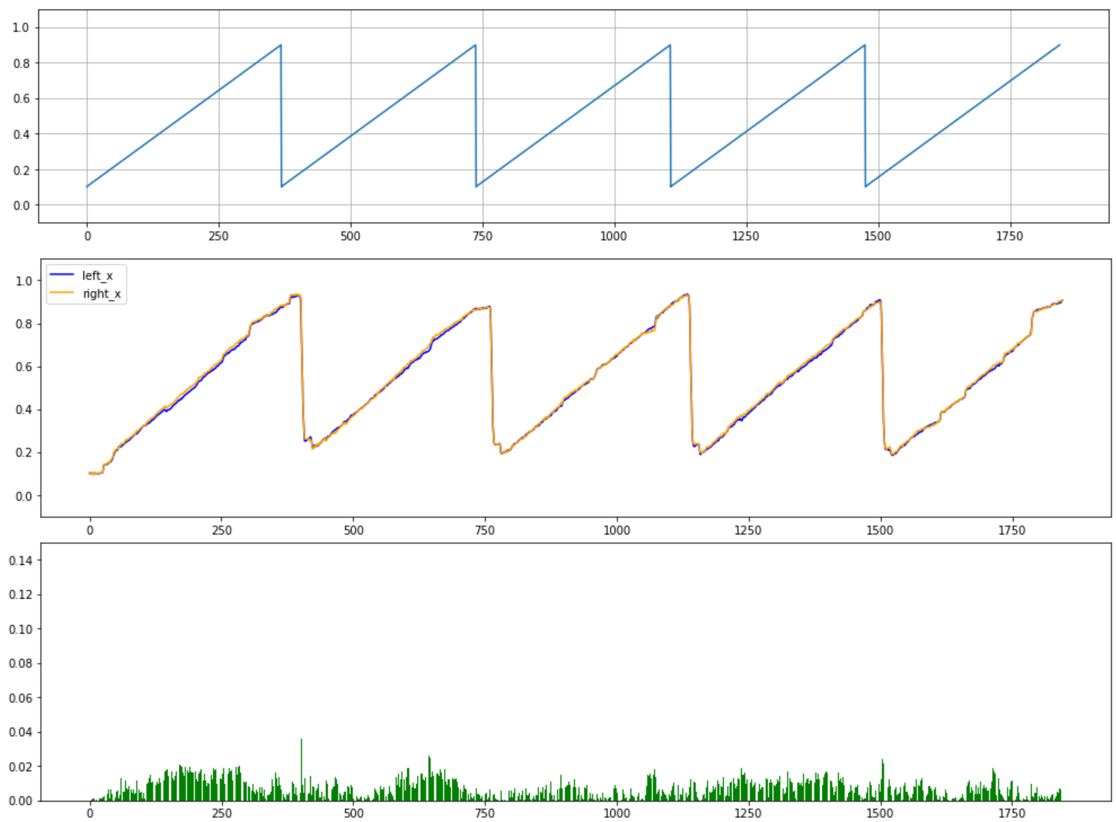

Figure 5.7: Stimulus, raw recording data and corresponding euclidean distance values between the left and right eye on the horizontal axis.

The graphs in figure 5.7 show the data which was used in experiment 1. The data contains zero identified anomalies and is what we call a normal recording. Some distance will always be observed because of the natural tendency of the gaze, discussed in 2.1.2. Additionally, other discrepancies may also arise from possible errors in hardware, software, calibration and more. Testing such recordings is also valuable as we can use the same stimulus and observe differences between other recordings containing identified behaviour.

Output and Observations

While the output from Experiment 1 in figure 5.8 initially seem unexciting, a lot can be observed on these graphs. For all values of w, we observe how every value falls under the average at 0, as is expected on a normal recording. We also observe how the accuracy of the transformed graphs decline when increasing w. At 400 a minor spike is observed where the gaze moves from one side of the screen to the other. These are not easily identified at the other similar events. The identified spike is only observed using the smallest value of $w = 5$. At 1750 the eyes move in parallel off the stimulus path, this is not observed in the transformed graphs. There is little variation in this recording which is reflected as a mostly linear horizontal division of values on the transformed graphs at all values of w.

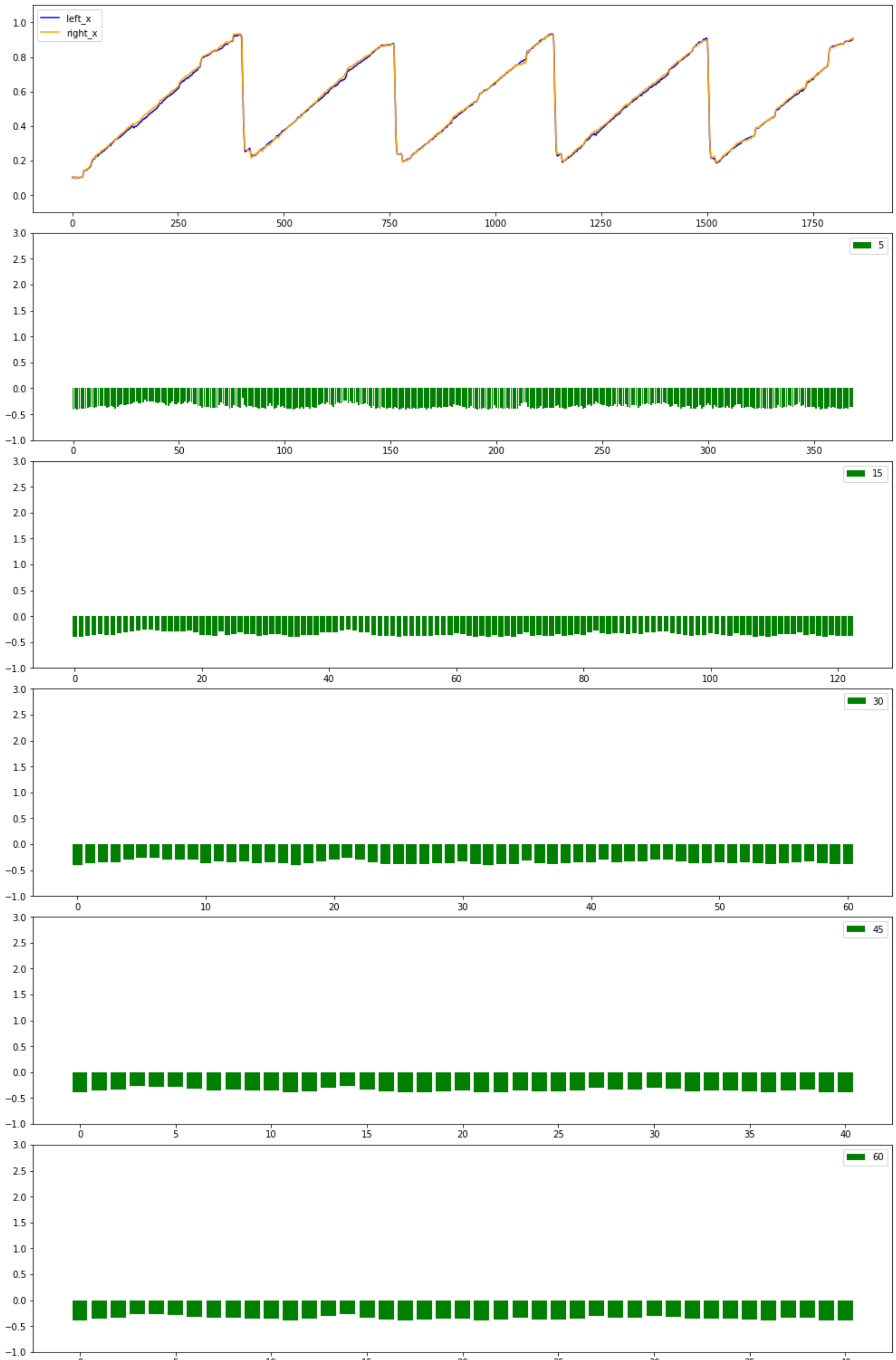

Figure 5.8: Output of experiment 1 using w = [5, 15, 30, 45, 60].

5.3.3 Experiment 2

This experiment is created to see if minor events are identified after the PAA transformation, and how the identification is affected by changing the value of w. All observations made in this experiment are comparable with the results from experiment 1. This experiment also further verifies the PAA algorithm.

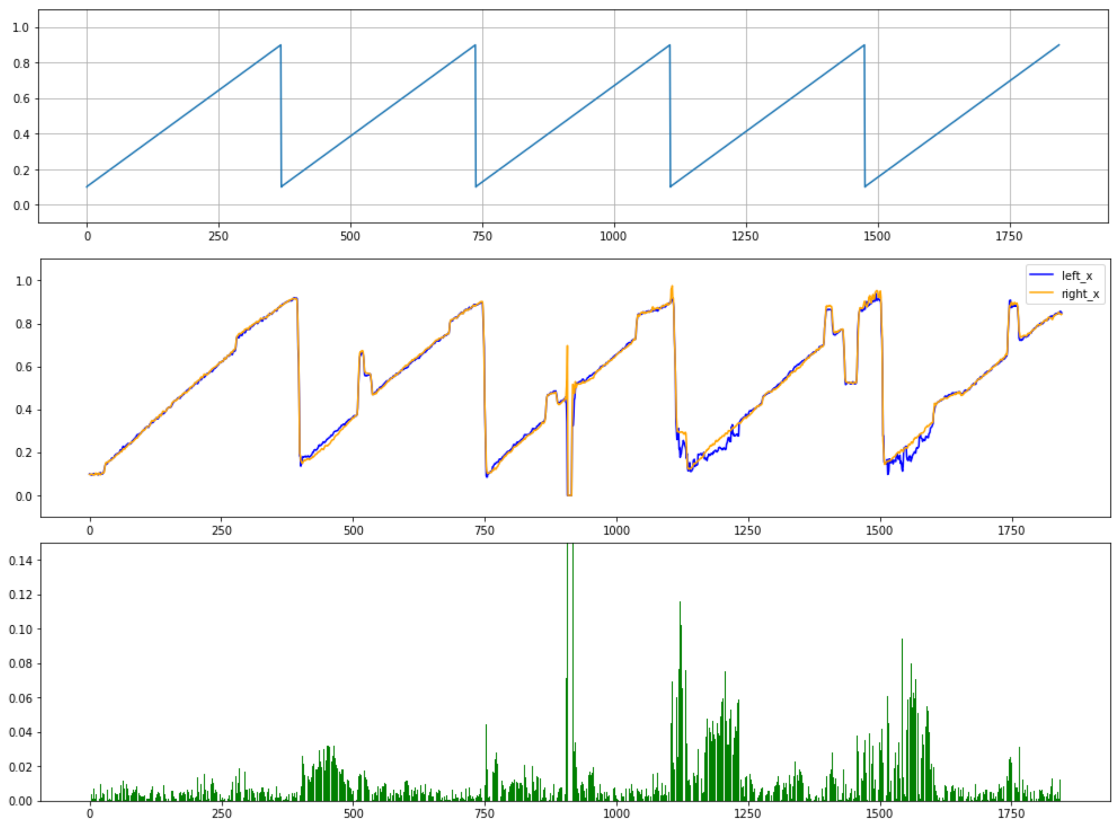

Figure 5.9: Stimulus, raw recording data and corresponding euclidean distance values between the left and right eye on the horizontal axis.

The graphs in figure 5.9 show the data which was used in experiment 2. This data contains three identified miss-alignment anomalies, four events where both eyes in coordination miss the path of the stimulus object and a large spike. The misalignment anomalies can be seen in both the original graph and the euclidean distance graph at 400, 1200 and 1550 in figure 5.10. The miss coordination events can be seen at 500, 1400 and 1750. Finally, the spike is seen at 800 which also occurs in several other recordings throughout the dataset.

Output and Observations

Experiment 2 gives us another perspective using identical stimulus with a different subject. Seen in figure 5.10, only 2 of the 3 misalignment events, seen at 400, 1150 and 1550, are identified at all values of w less than 30. The first event at 400 is still noticeable in the PAA transformation, but would fall under the normal distance. At 500, 850, 1400 and 1750 we can see 4 distinct events where both eyes move off the intended stimulus path in coordination. These are not identified at any w. The spike seen at 800 shows how the eyes are not always completely in coordination during these parallel events, especially when the event is very sudden.

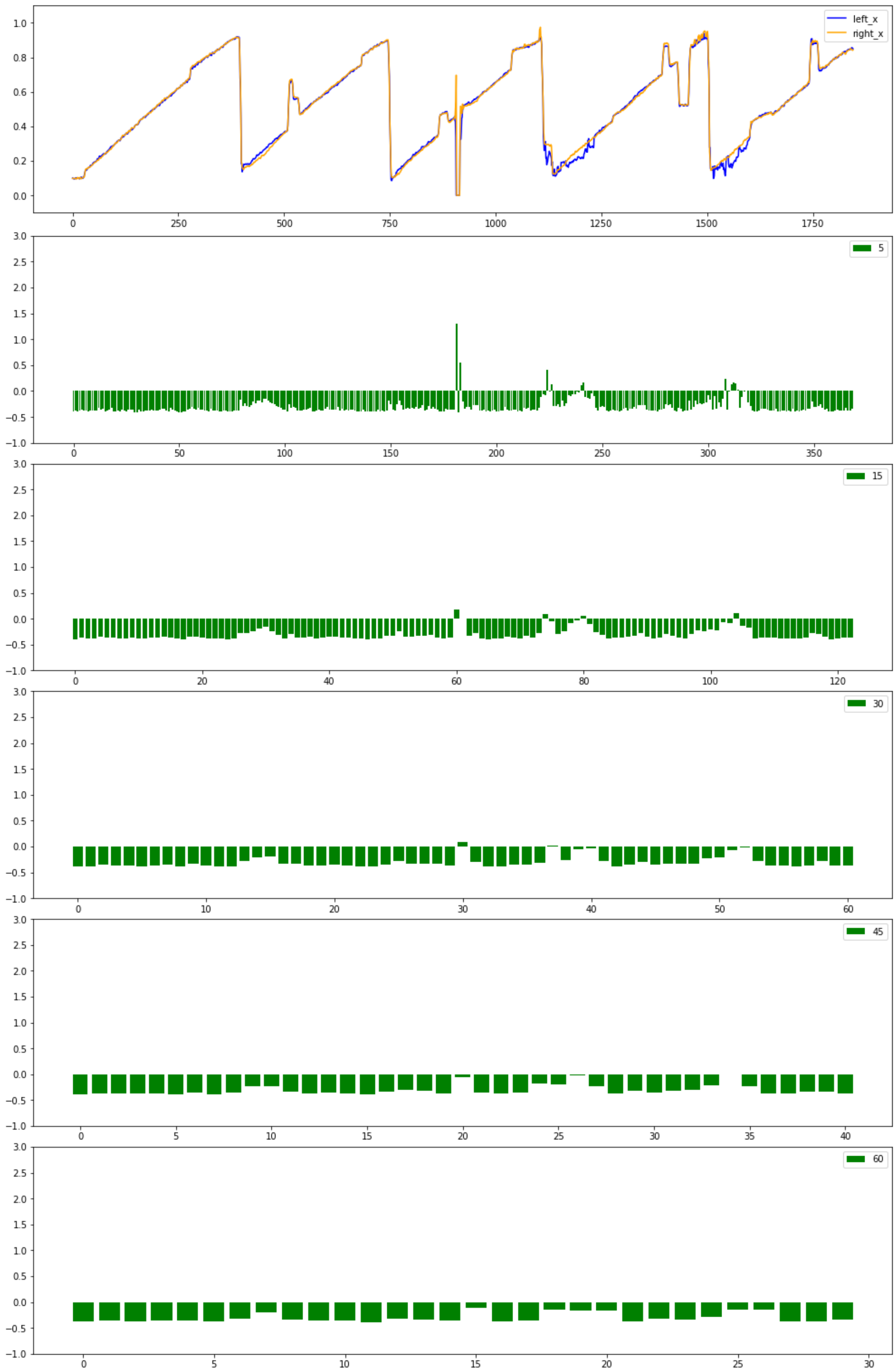

Figure 5.10: Output of experiment 2 using $w = [5, 15, 30, 45, 60]$.

5.3.4 Experiment 3

In experiment 3 the observations from experiment 1 and 2 are validated on a recording created using a completely different stimulus. The goal is to identify the same events seen in earlier experiments.

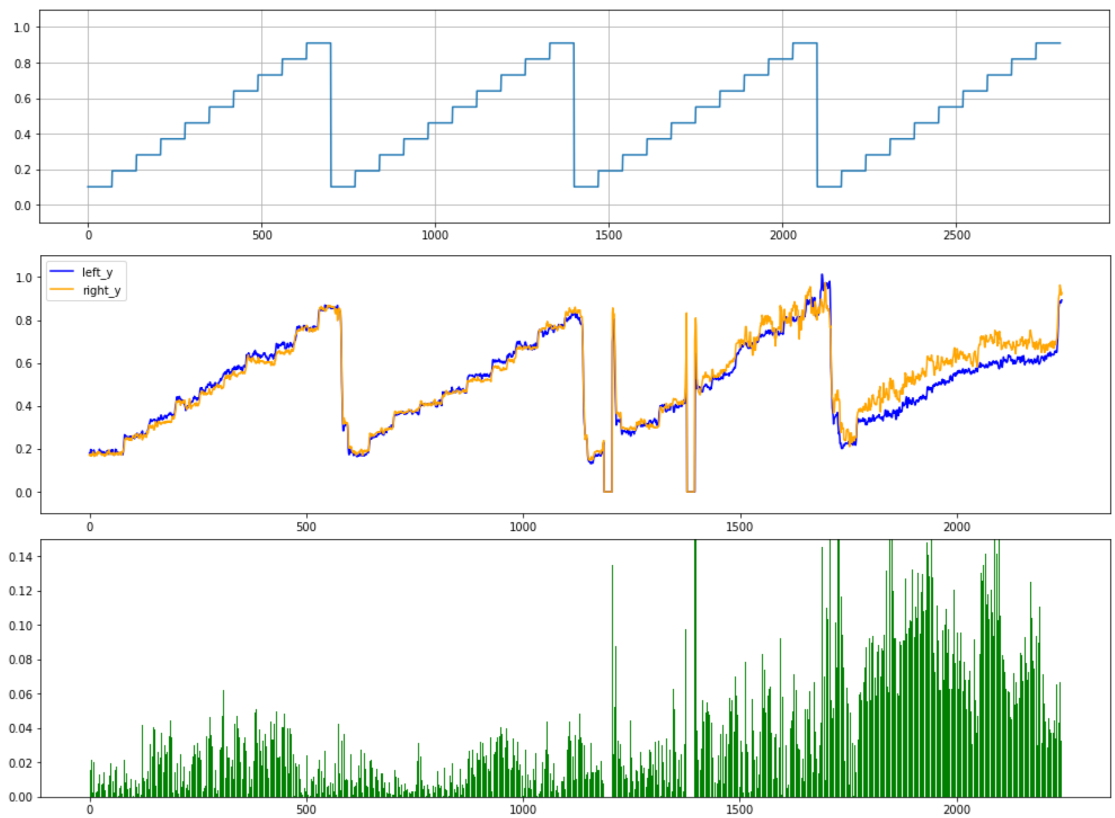

Figure 5.11: Stimulus, raw recording data and corresponding euclidean distance values between the left and right eye on the vertical axis.

The graphs in figure 5.11 show the data which was used in experiment 3. The recording is affected by some permanent oscillation, but still behaves mostly normal until the last cycle of the test. The event seen in the last cycle is identified as the loss of attention or the subject becoming tired. In the data we also see two examples of the same spike seen earlier, which has a large presence in our dataset. This event can be seen at the 1250 and 1400 on the graphs.

Output and Observations

Experiment 3 introduces a new stimulus of a more complicated nature. Seen in figure 5.12, the section of most interest is located after 1300 and is large and somewhat consistent. This section is identified in the results at all values of w. More specifically, the difference between the last section and the rest is interesting. Another observation is made at 1600 where we can see a clear oscillating behavior of the right eye. This is seen in our results when w is small. Increasing w hides this oscillating behavior, in our case when w is 30 or higher. Both spikes are observed on the PAA representation, and is highly exaggerated at lower values of w. While the spikes are of equal length, they result in very different outcome in the PAA representation.

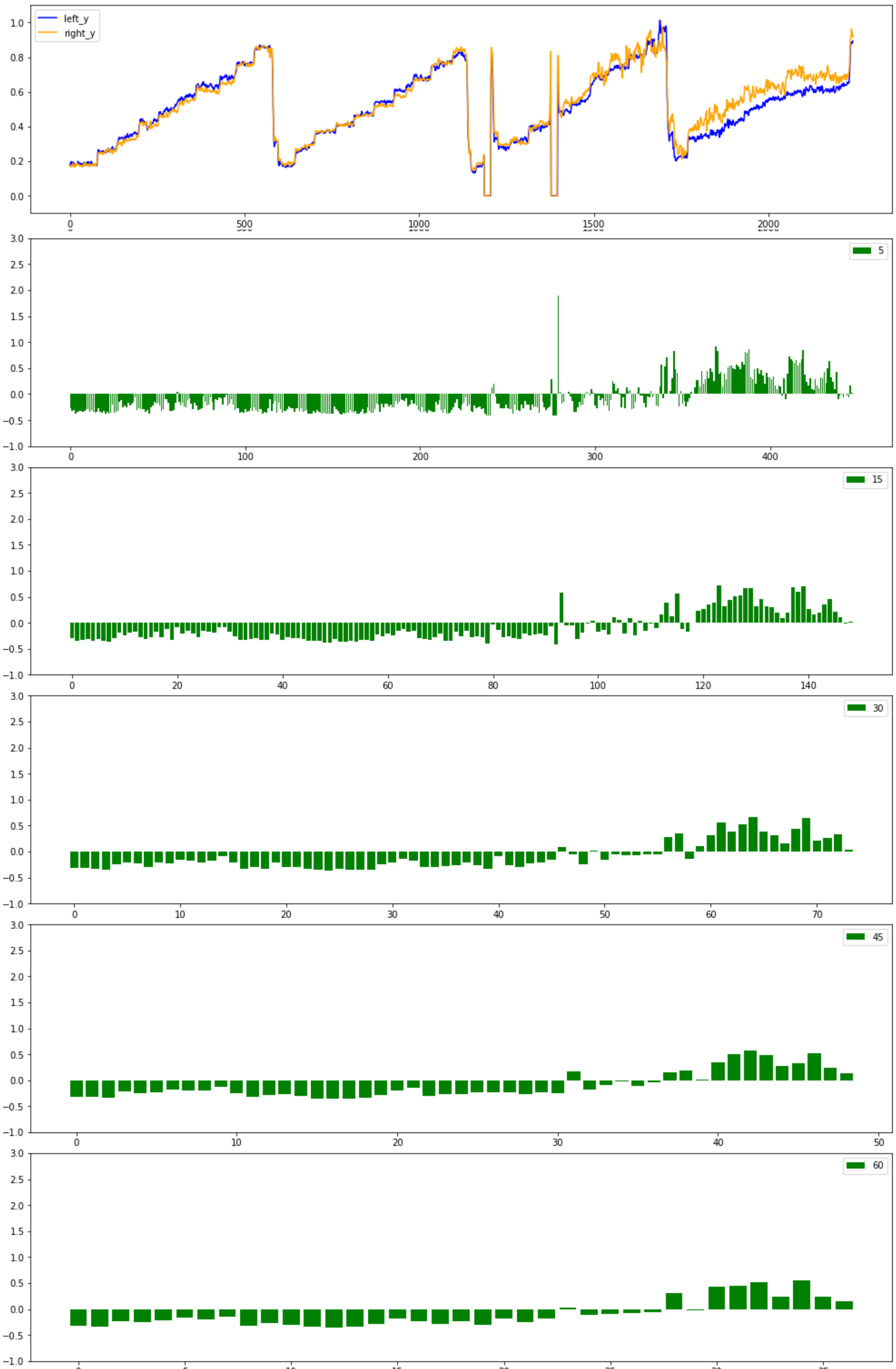

Figure 5.12: Output of experiment 3 using w = [5, 15, 30, 45, 60]

5.4 Alphabet Size Experiments (a)

In this section we will discuss the details of experiment 4 - 6, what data and parameters were used, and finally present the results of the process. These results will be discussed in chapter 6.

5.4.1 Similarities Experiments 4 - 6

After completing the first three experiments, we start testing the final part of the algorithm. The experiments are performed using the full SAX method from chapter 4. Here we directly focus on the a parameter value and it's effect on the discretization. The results from these experiments are combined with the knowledge from experiment 1 - 3 so that we can combine the two values to find specific combinations of parameters for later experiments.

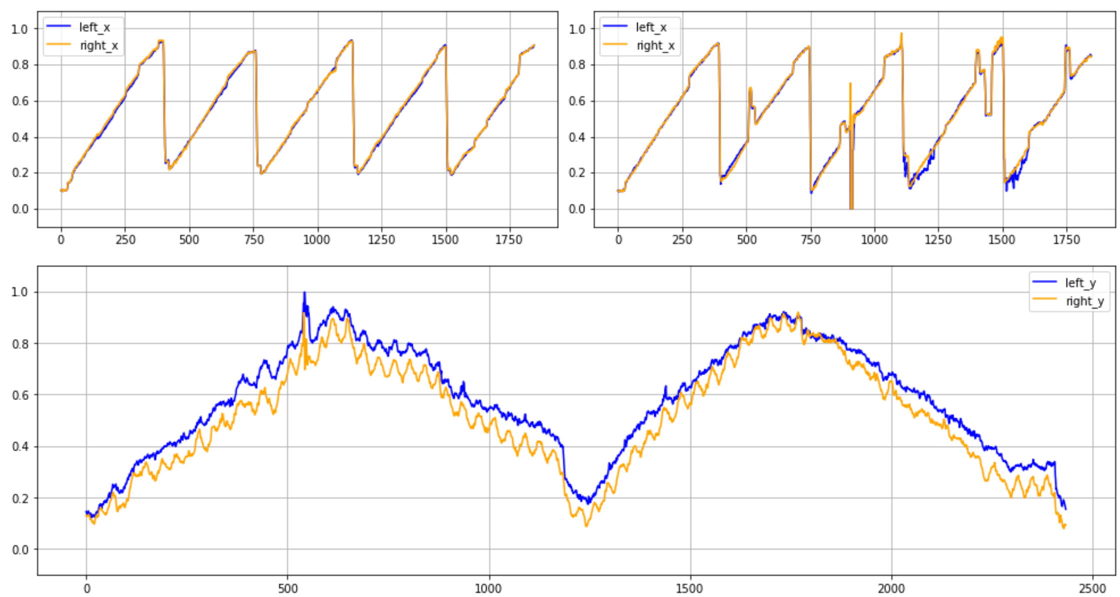

Figure 5.13: Dataset used in experiment 4 - 6.

The time series seen in figure 5.13 consists of the first two time series used in experiment 1 and 2, combined with a new time series containing problems with other characteristics. As mentioned, the first two are comparable with each other, in addition, we can also compare these with the results of experiment 1 - 2 for a clearer understanding.

Table 5.5: List of parameter values used in experiments 4 - 6.

Parameter Values	
Frame Size (w)	15 30
Alphabet Size (a)	5 6 7 8

Seen in table 5.5 are the parameter values used for these experiments. The two w values 15 and 30 are chosen based on the observation from experiments 1 - 3, discussed in the sections after each experiment. All the possible parameter values for a are used in the experiment, namely (i) 5, (ii) 6, (iii) 7 and (iv) 8. This is initially done to narrow the set of values of a for use in later experiments.

5.4.2 Experiment 4

The goal for experiment 4 is similar to experiment 1, but with the use of the complete SAX algorithm. In this manner we are able to see the differences between the recordings, while also being able to identify the specific parts of the method which produce the variations. As in experiment 1 and 2, this experiment also functions as a validation of the discretization part in the method used in SAX.

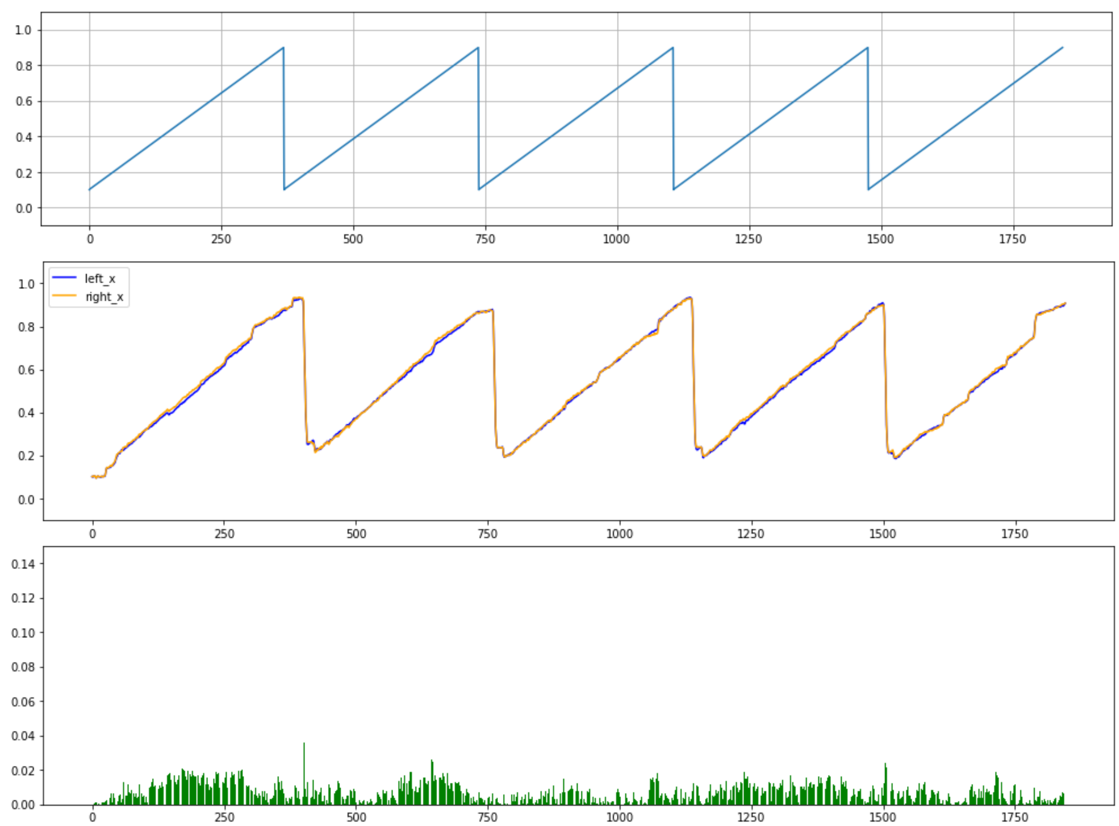

Figure 5.14: Stimulus, raw recording data and corresponding euclidean distance values between the left and right eye on the horizontal axis.

The graphs in figure 5.14 show the data which was used in experiment 4. This data is identical to the data from experiment 1 seen in figure 5.7. The data contains zero identified anomalies and is classified as a normal recording.

Output and Observations

In figure 5.15 and 5.16 from experiment 4, the effect of increasing the value of a can be observed. As expected, a string consisting of a single symbol can be seen at all values of a, except at our highest value when a is 8. From the set of output we see that we can adjust a in a way such that the meaning of a symbol is changed. In figure 5.15, we see this happening when the size of a is increased from 5 to 6. When changing w seen in figure 5.16, the meaning of the symbols seem unaffected during visual inspection.

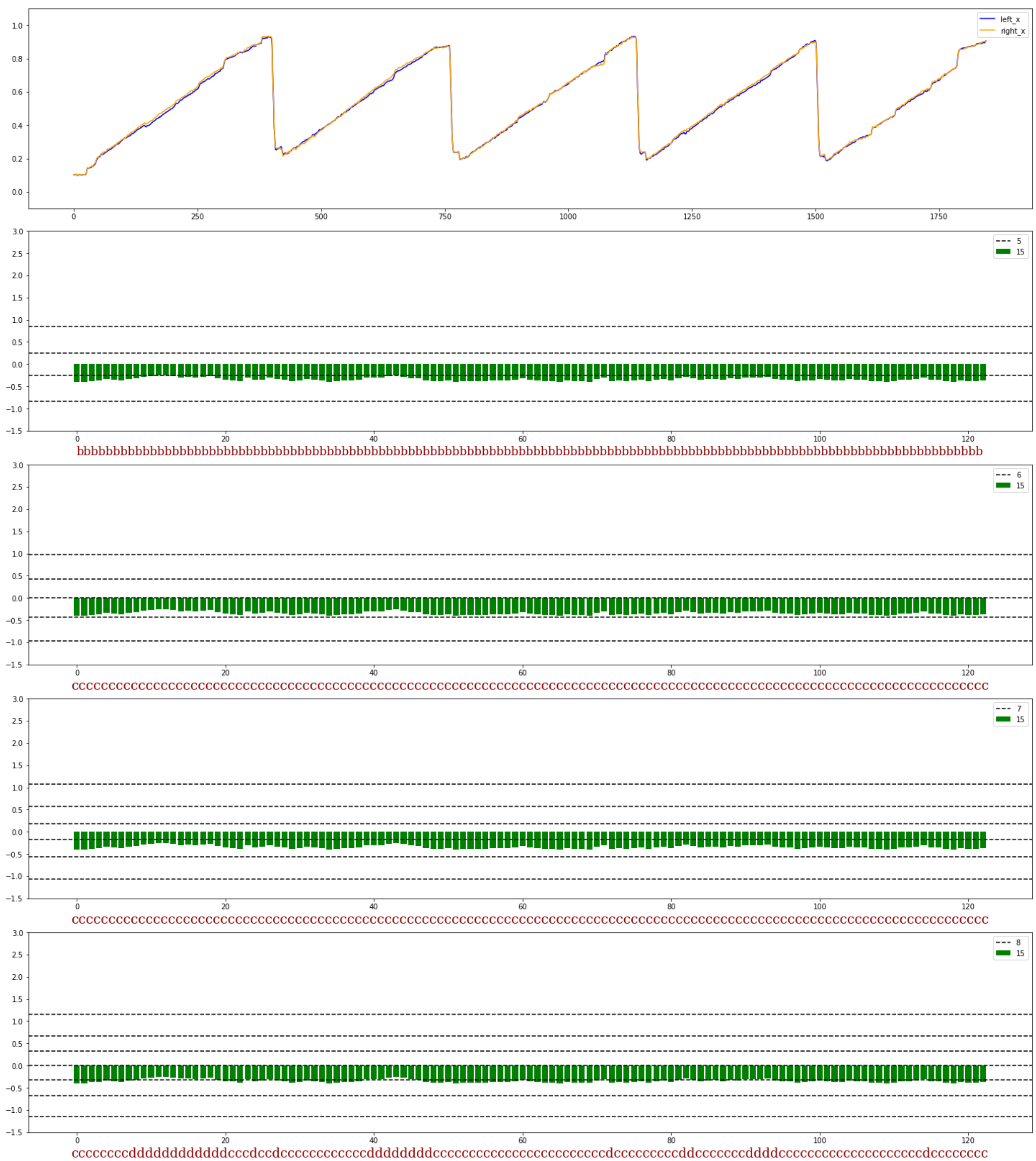

Figure 5.15: First output of experiment 4 using $w = 15$ and $a = [5, 6, 7, 8]$

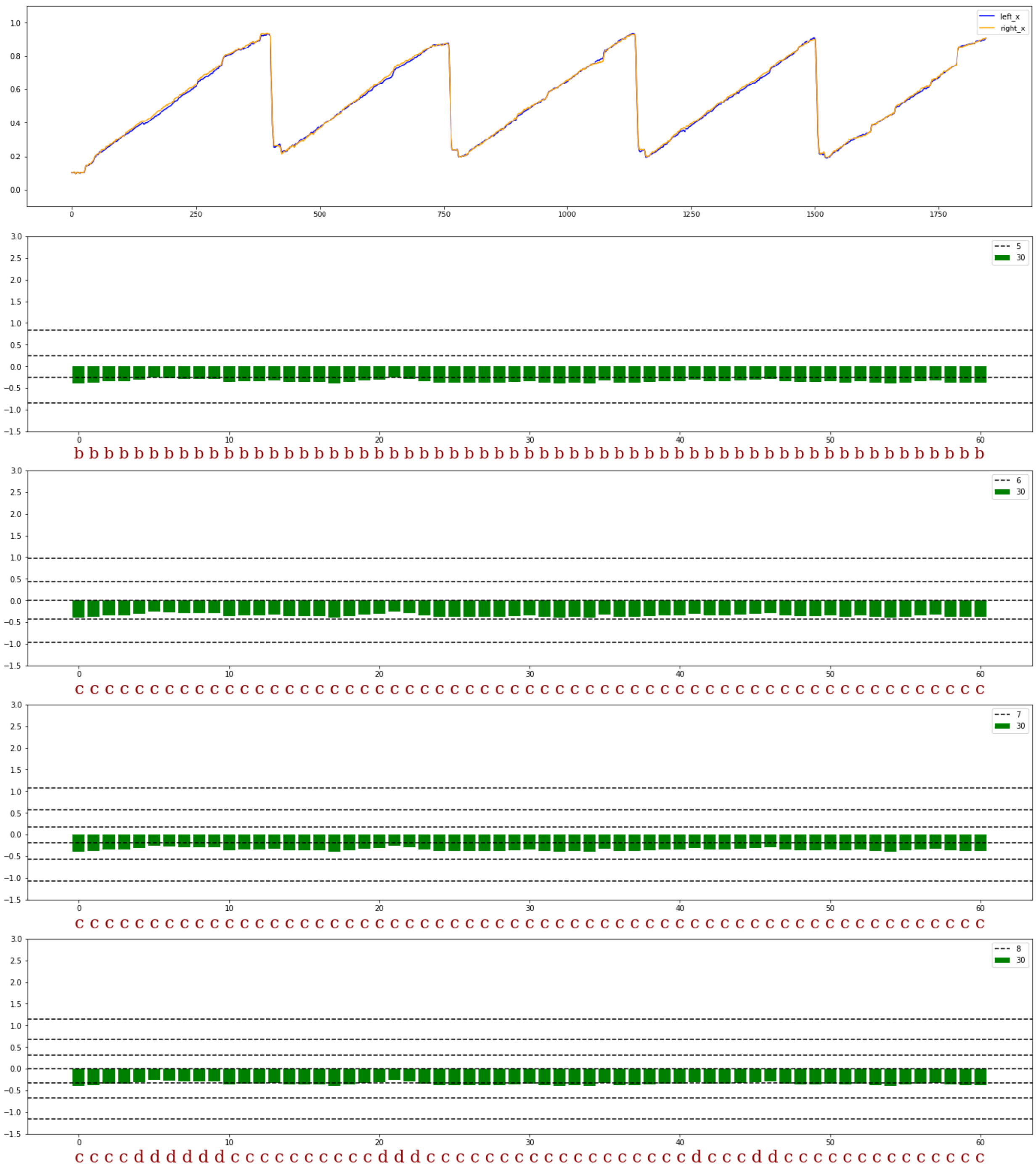

Figure 5.16: Second output of experiment 4 using w = 30 and a = [5, 6, 7, 8]

5.4.3 Experiment 5

The goal for this experiment is similar to the goal of experiment 2, but with the extension of using the full SAX method. After this experiment, a certain understanding of the methods' behavior starts becoming apparent. Also, connections between the effects of the two parameters can be made.

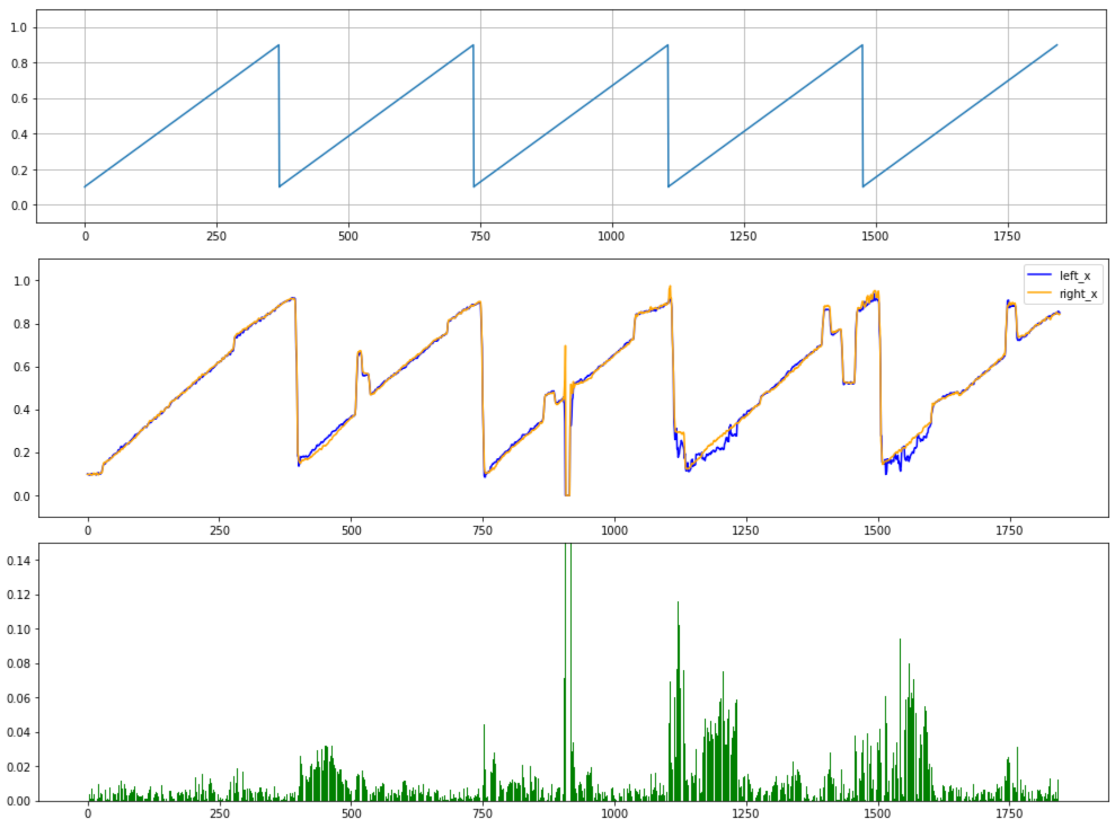

Figure 5.17: Stimulus, raw recording data and corresponding euclidean distance values between the left and right eye on the horizontal axis.

The graphs in figure 5.17 show the data which was used in experiment 5. This data is identical to the data from experiment 2, seen in figure 5.9. The recordings contain three identified misalignment anomalies, four events where both eyes in coordination miss the path of the stimulus object and a large spike. The misalignment anomalies can be seen at 400, 1200 and 1550 in figure 5.17. The miss coordination events can be seen at 500, 1400 and 1750. Finally, the spike can be seen at 800.

Output and Observations

Experiment 5, seen in figure 5.18 and 5.19, is again another perspective on the same stimulus with a different subject, but this time focusing on the SAX discretization. We observe that the area around 0, which indicates normal behavior, can be adjusted through the breakpoints when changing the value of a. Interestingly, the small event at 400 is transformed in a way that it is not visible when a is 6 and w is 15 but visible at all other values of a. Increasing w extends this observation to include when a is 7. An important observation is made at the spike at 800 where the anomaly event falls under the same area as the gaze movement events at 1200 and 1500.

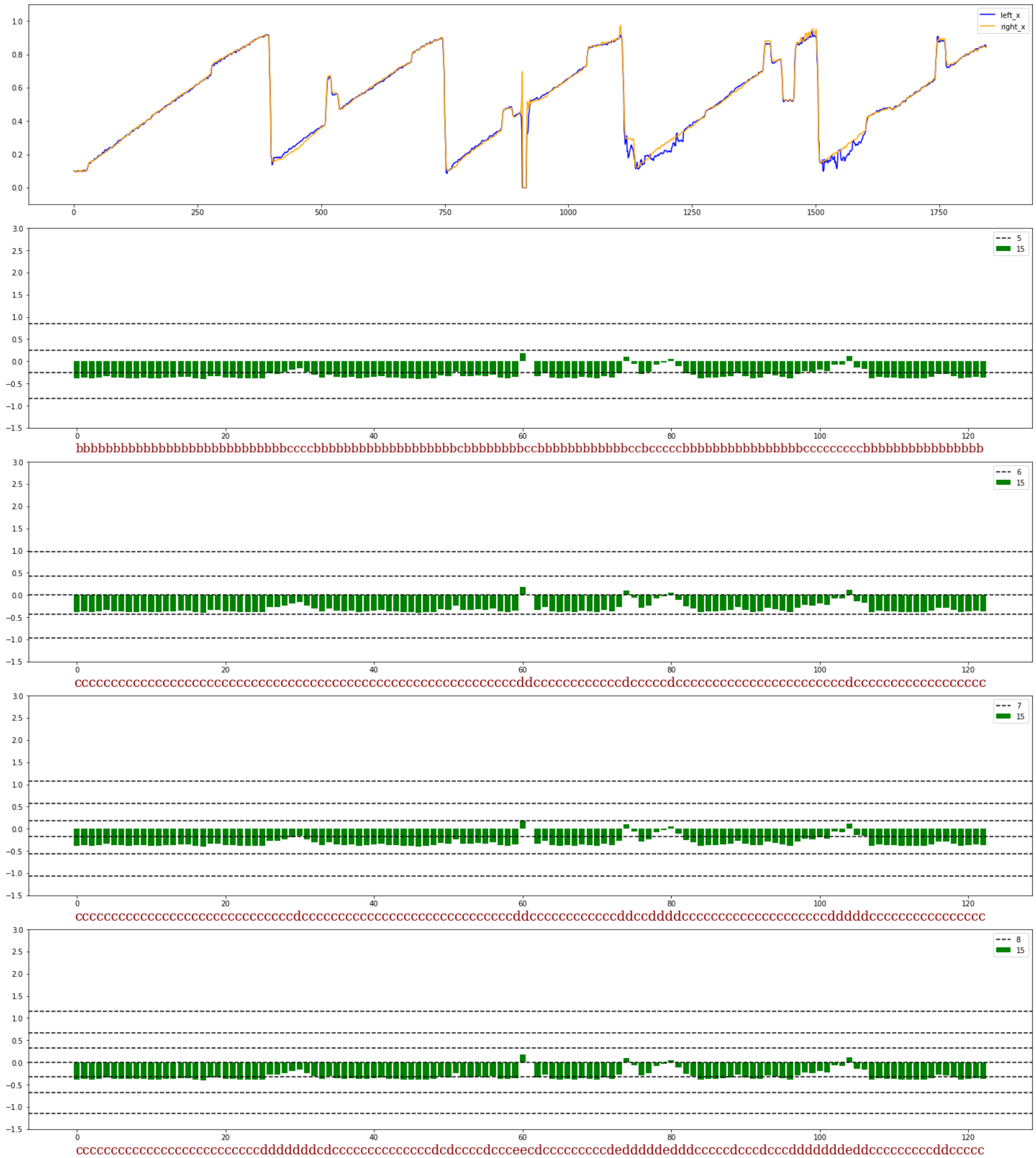

Figure 5.18: First output of experiment 5 using $w = 15$ and $a = [5, 6, 7, 8]$

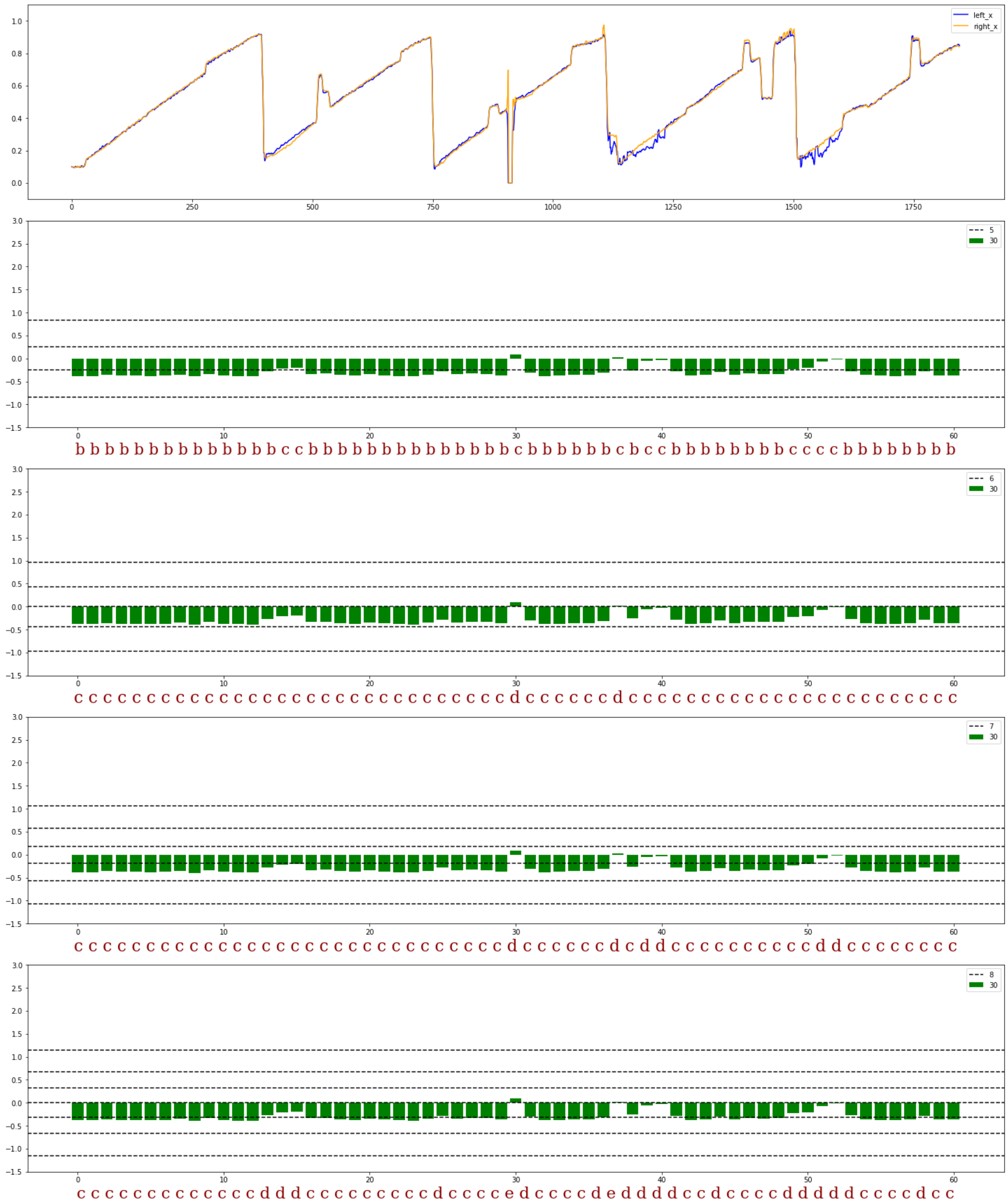

Figure 5.19: Second output of experiment 5 using $w = 30$ and $a = [5, 6, 7, 8]$

5.4.4 Experiment 6

The goal of experiment 6 is similar to the goals of experiment 3, with the addition of the SAX discretization. This final targeted test on the a parameter value is ran on data which vary greatly from our expected recording results. This is done to find the effects of changing the parameter value in extreme cases to reveal behavior we are not directly looking for.

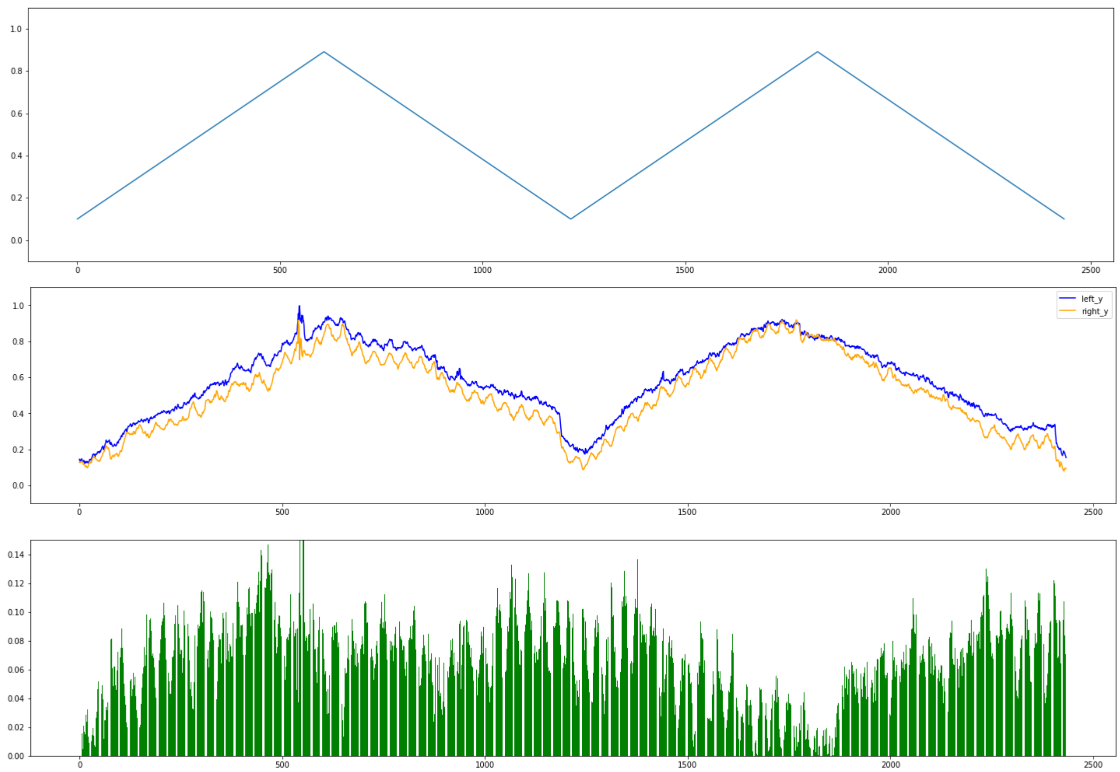

Figure 5.20: Stimulus, raw recording data and corresponding euclidean distance values between the left and right eye on the vertical axis.

The graphs in figure 5.20 show the data which was used in experiment 6. The data contains highly oscillating time series which usually stem from subjects with severe eye problems, for example albinism. In these time series the oscillation also varies over time which may stem from other factors. Some of the behaviour found in these recordings may be falsely identified due to the nature of our distance measures. Having data of this type may reveal behaviour which only occur in unique data. In the original data we identify at 1750 an area of less distance compared to the rest. Additionally, the oscillation between the eyes varies from being parallel, to one eye dominating. This difference can be observed when comparing the sections at 450 and 1500.

Output and Observations

Experiment 6 introduces a new stimulus and a case of very problematic ocomotular behaviour. As seen in figure 5.21, the symbolic patterns which emerges contain a lot of the oscillating behaviour seen in the original graph. In addition, a difference can be observed between the high frequency section at 450 and the low frequency section at 1700. The fluctuation in the symbols at high frequency is observed when a is 8 with the pattern "**fef**" at 500, while the low frequency fluctuation is observed when a is 7 with the pattern "**dcd**" at 1700. Notice that these are not observed in unison using one set of parameters. Not all information about the oscillation is hidden as we can still deuce the coordination of the eyes during oscillation. This is observed at 400 where both eyes oscillate in coordination resulting in a pattern of "**ddd**", while at 1500 one eye oscillates more, and we again see the "**dcd**" pattern emerging.

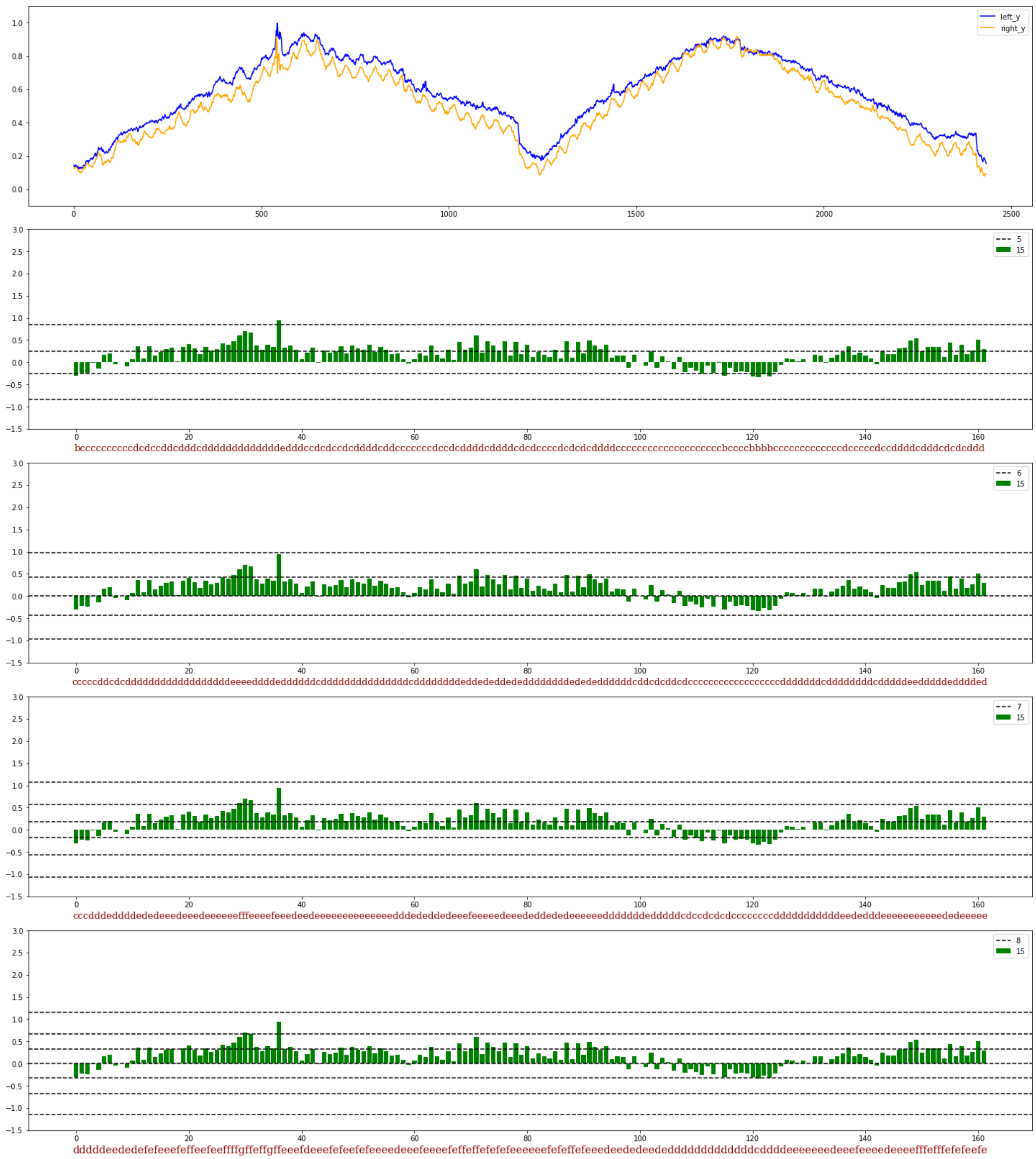

Figure 5.21: Output of experiment 6 using $w = 15$ and $a = [5, 6, 7, 8]$

5.5 Discovery Experiments (w and a)

In this section we will discuss the details of experiment 7 - 9, what data and parameters were used, and finally present the results. These results will be discussed in chapter 6.

5.5.1 Similarities Experiments 7 - 9

For our final experiments, we are interested in exploring the full extent of the method. These experiments are created to achieve an understanding of the algorithms ability to characterize behavior in the time series using the final SAX string produced. They are divided into three sections (i) discovery through horizontal OR vertical distance between the eyes, (ii) discovery by comparing horizontal AND vertical distance and (iii) discovery through horizontal OR vertical distance between eyes AND object.

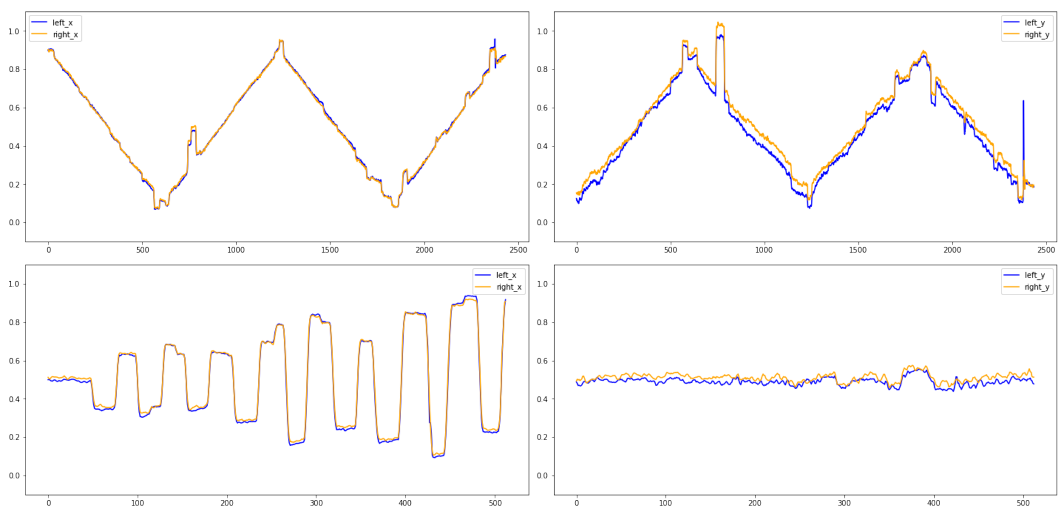

Figure 5.22: Dataset used for experiment 8.

For these experiments more data is needed than for the other experiments. This is because of the large set of possible behaviours the eyes may have, discussed in section 3.2. Our dataset must include most of these behaviours for our algorithm to be thoroughly tested. The dataset for experiments 7 - 9 can be seen in figure 5.24, 5.22 and 5.23. For these experiments, there are fewer similarities, therefore the detailss are discussed later when performing them.

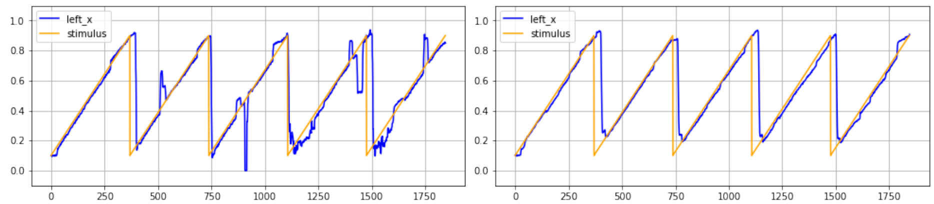

Figure 5.23: Dataset used for experiment 9.

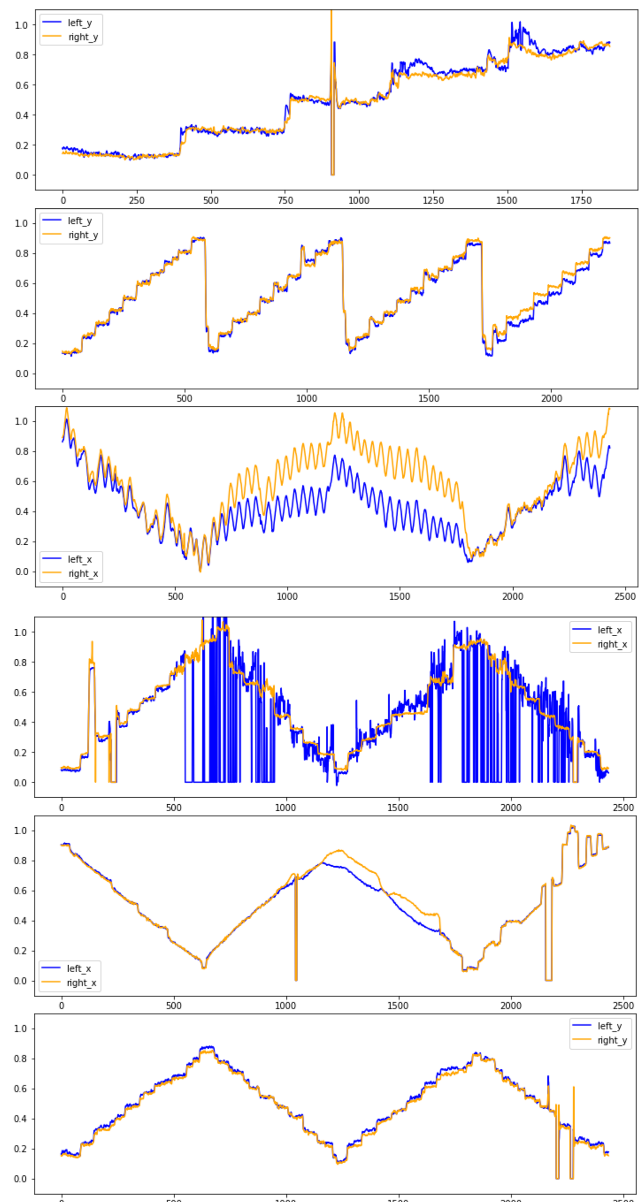

Figure 5.24: Dataset used for experiment 7.

5.5.2 Experiment 7

This experiment is the first experiment on the complete SAX algorithm where the focus lies in its ability to discover types of behavior. The goal of the experiment is to identify several different types of ocomotular movement using the same set of parameter values. The values used are chosen based on the knowledge gained from experiments 1 - 6. Additionally, because of constrains on time and complexity of this thesis, we chose to explore many different types of possible behavior on a set choice of parameter values instead of using a range of values like in the earlier experiments. This is because we know enough about the effect of changing the parameter values that we can, to a greater extent, focus on identifying different types of behavior instead of verifying the findings with many different parameter values. For this experiment, the value of w is set to 15 and the value of a is set to 6.

The graphs in figure 5.24 show the data which was used in experiment 7. The data contain multiple types of different recordings with a large variety of behavior. Both horizontal and vertical recordings are present in the dataset. Some repeating recordings from other experiments are also present as we already have a lot of knowledge of them and can use these to verify our final conclusions.

Output and Observations

In Experiment 7, seen in figure 5.25, we are able to identify patterns in the SAX string where gaze movement of interest is observed on the original data. In graph "A)" the initial misalignment at 0 is identified by the change from a row of "**ccc**" to a row of "**bbb**". Notably, random "**c**" symbols appear in the middle of a "**bbb**" row, even though this is not observed on the original data. The spike at 850 is identified as a minor misalignment, but can not be distinguished from the neighbouring data which are of a different nature. The sections of higher distance after 1100 is identified in it's whole by observing the frequency of the "**c**" symbols, but the shape of the event is lost using these parameter values. In graph "B)" we initially observe the overwhelming amount of spikes which are identified as the symbol "**e**". Notably is the large distance between the symbols "**c**" and "**e**" which indicates a very quick change in distance. The oscillation of one of the eyes seen at 1500 is not identified in our SAX string. In graph "C)" we initially observe the uniformity of the "**b**" symbols until 1650 when the symbols change to "**c**" as the distance increases. In graph "D)" we are not able to identify the spike observed at 1050 or the series of events with the gaze moving in parallel off the stimulus object path at 2300. Notably, the actual misalignment between 1250 and 1650 is somewhat identified with the symbol "**c**", but as in graph "C)" the difference between the misaligned section and the aligned section makes the misalignment more apparent. In graph "E)" the frequency of the "**cdc**" pairs implies the presence of oscillation, but the severity is highly obscured. In the final graph "F)" the uniformity of the symbols are observed throughout the recording, except at the two spikes seen at 2250 which are identified by the symbol "**c**".

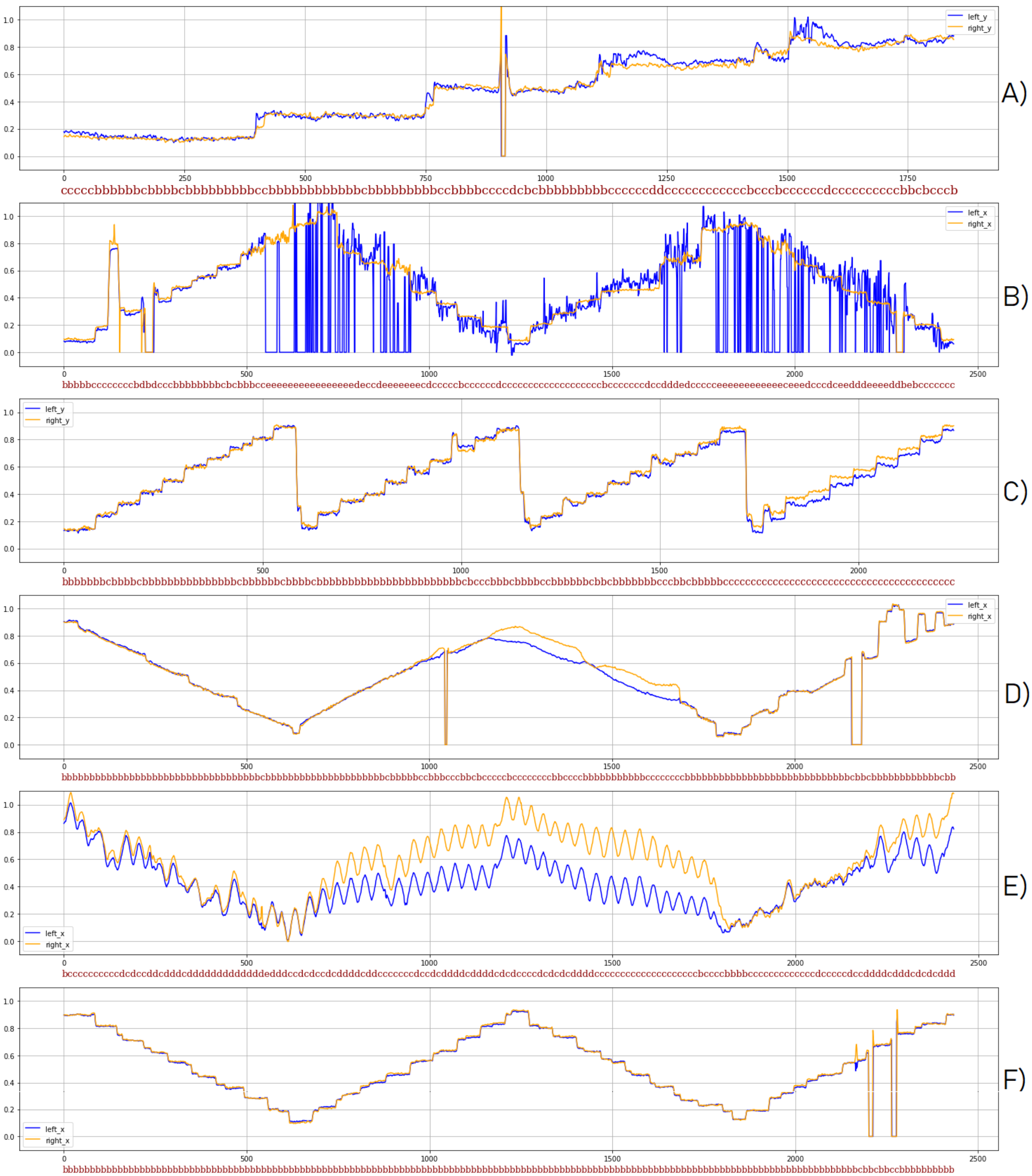

Figure 5.25: Raw recording data and corresponding SAX string using $w = 15$ and $a = 6$

5.5.3 Experiment 8

This experiment focuses on the correlation between the results on the horizontal axis and the results on the vertical axis. This is done to show what correlated behaviour looks like on each axis, using same parameter values. For this experiment, the value of w is set to 15 and the value of a is set to 6.

The graphs in figure 5.22 show the data which was used in experiment 8. The original data contains both the horizontal (left) and vertical (right) recording data of two unique recordings. In both recordings, the vertical data contains more variation in distance than the horizontal data. In the first pair, a misalignment can be seen on the vertical axis throughout the recording, while in the second pair, the vertical axis randomly shakes. For both pairs of recordings, the horizontal recording is mostly normal.

Output and Observations

Experiment 8 is interesting as we are now able to compare the transformations of the vertical and horizontal axis against each other. From visual inspection, the first two graphs in figure 5.26 indicate a consistent larger distance on the vertical axis, this is shown in our SAX string as an increment of the symbol with the highest frequency "**b**" to "**c**". An increased oscillation is observed in the second graph which is shown in our representation as the recurring patterns "**bcc**" and "**bcb**". This effect can also be seen in the last graph. Additionally, the dominant uniform symbol changes from "**b**" to "**c**" indicating misaligned gaze throughout the entire recording. This is present in both pairs of recordings.

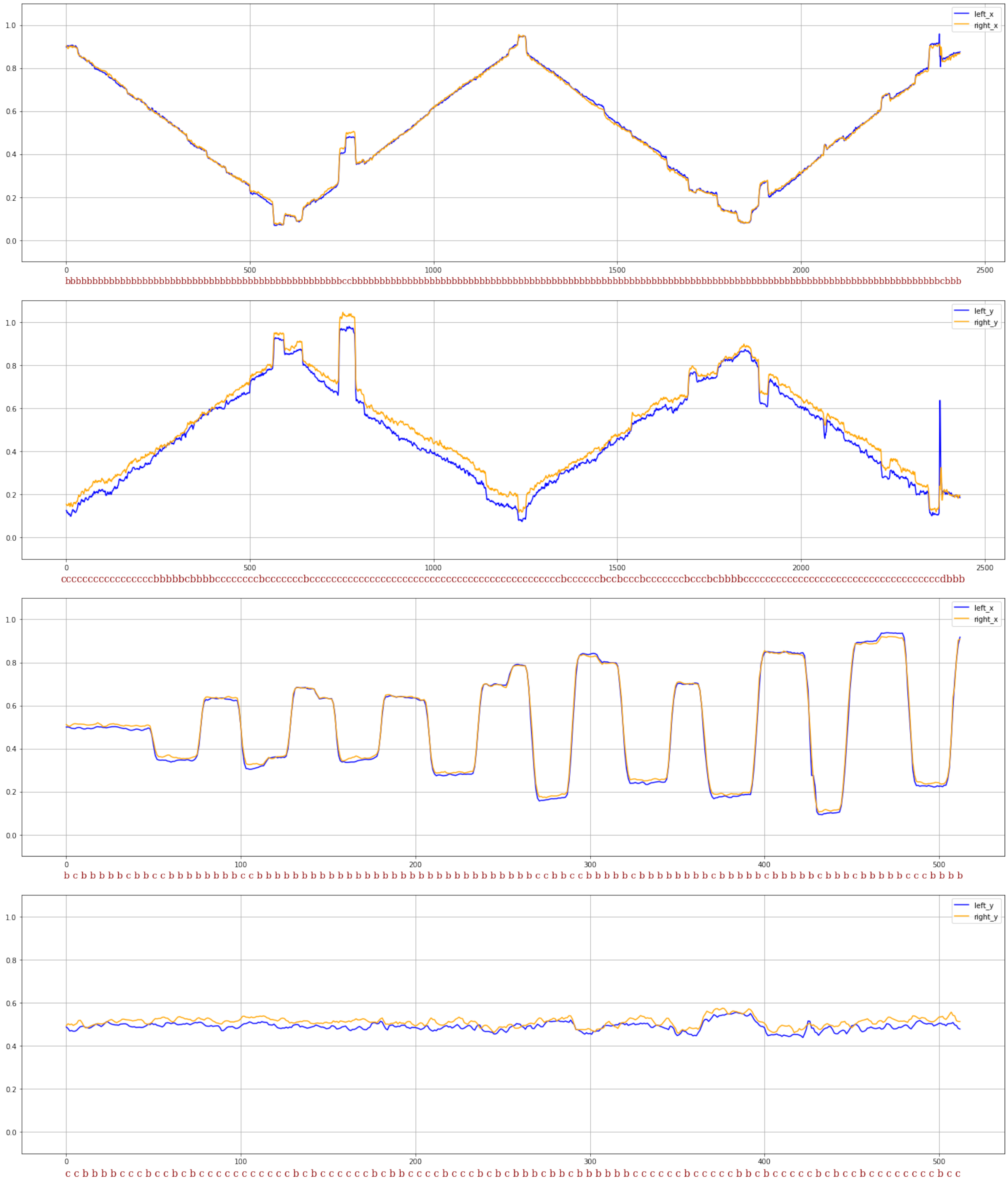

Figure 5.26: Raw recording data and corresponding SAX string using $w = 15$ and $a = 6$

5.5.4 Experiment 9

This final experiment is created to analyze the time series based on some earlier known knowledge. In this case it is the knowledge of where the original stimulus object is located. This can be of great value due to our use of a distance measure between the right and left eye. A distance between one of the eyes and the original stimulus would produce results which reflects the subjects ability to follow the stimulus, and not only the alignment of their own eyes. Additionally, identification of oscillations is highly affected by using the new distance measure.

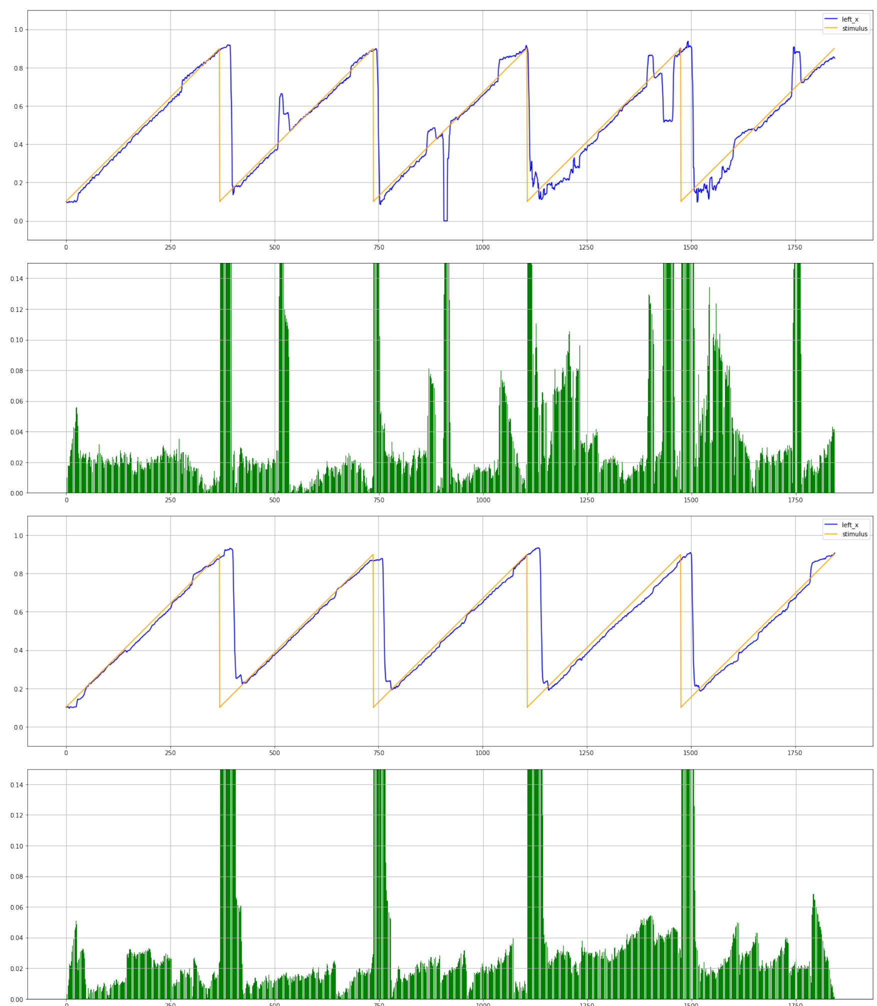

Figure 5.27: Raw recording data over stimulus followed by corresponding euclidean distance values between the left eye and stimulus on the horizontal axis.

The graphs seen in figure 5.27 shows the left eye of two unique recordings over the stimulus used during the recording. Beneath each recording is the euclidean distance vector between the eye and stimulus object. The recordings have been seen in earlier experiments, and are chosen to further build upon the knowledge gained from these.

Output and Observations

Most striking with the results from experiment 9, seen in figure 5.28, are the four large spikes seen at 450, 750, 1100 and 1500 in both graphs as the jump from the symbol "**c**" to "**f**" . Their similarity is more apparent on the normal recording. Notably, the events of the gaze moving in parallel off the stimulus target are now easily identified as the symbols "**f**" or "**e**", but are hard to distinguish from the other events. In the last graph, the short misalignments at 1400 and 1800 are correctly observed on the SAX word with the symbols "**dd**". Other insignificant distance variations are mostly hidden, even the somewhat significant variation at the start.

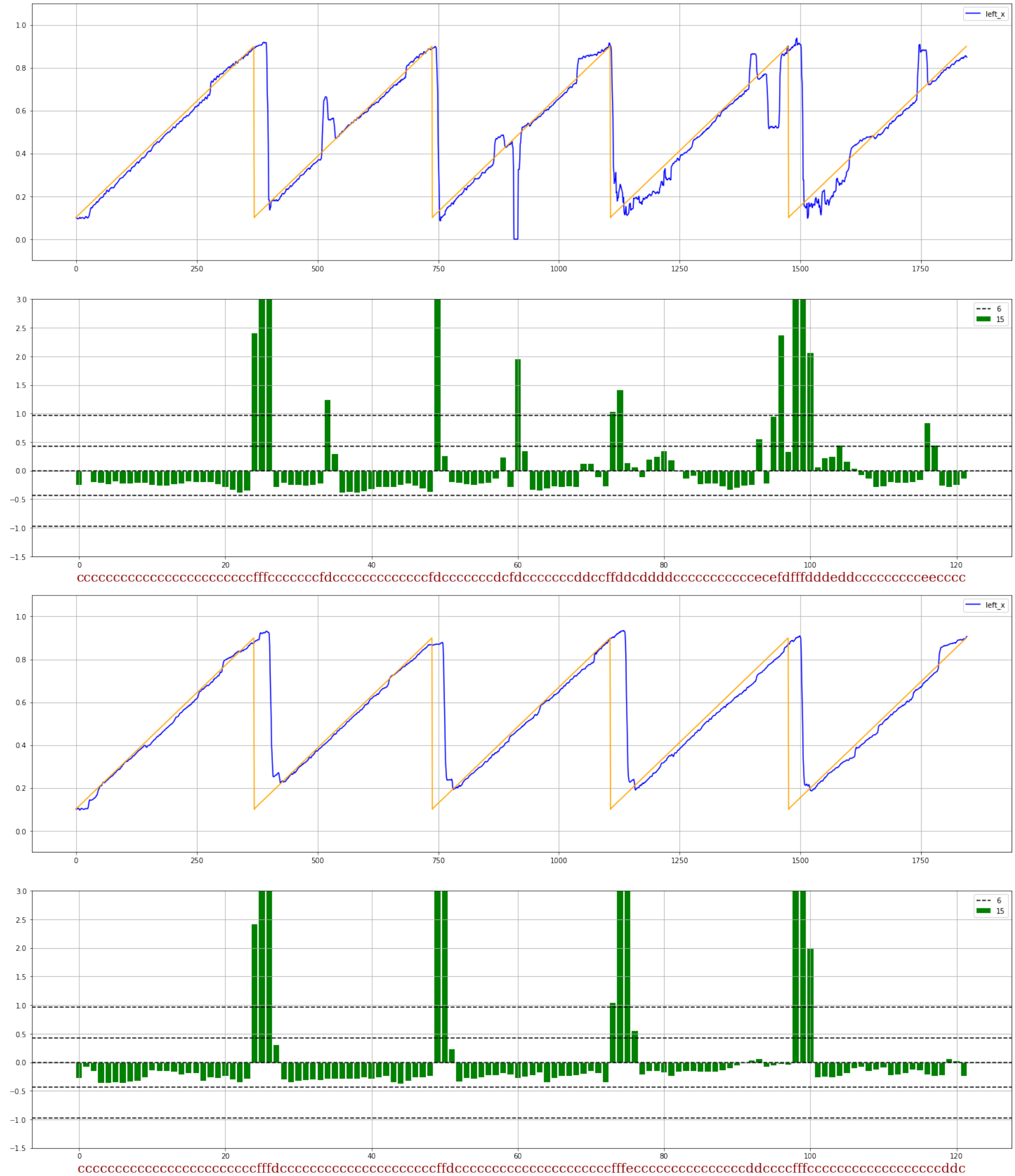

Figure 5.28: Raw recording data over the stimulus followed by corresponding PAA transformation and SAX string using $w = 15$ and $a = 6$

Chapter 6

Results and discussion

In this chapter, a summary of the findings is presented and the results are discussed. The chapter is divided using the same structure used in the experiments. First the observations during the targeted tests of the parameters are discussed and examples are shown, before ending the chapter discussing the final discovering ability of SAX using combinations of parameter values.

6.1 Assessing parameters

In the following section, we will discuss the results from adjusting the parameters of the algorithm. First we will focus on the parameters individually before we discuss combinations of values.

6.1.1 Frame size (w)

From the results in experiment 1, seen in figure 5.8, we gain a little knowledge of the effect when changing the value of w. We see that the higher the value of w is, the longer the behaviour must last in order to observe it after the PAA transformation. This means that using high values of w may be useful for detecting large differences in multiple time series, while lower values may be more efficient for analysing specific behavior on a single time series. We are most interested in seeing if the algorithm can observe specific behavior, but running the algorithm on a combination of two or more values may render more complete knowledge of the subject's eyes.

In experiment 2 we are able to see the difference of behavior when testing data created with the same stimulus as in experiment 1. Seen in figure 5.10, we are now able to observe several events on the graph. These events are short, and can be seen disappearing when w is increased. In order to differentiate between short and long events, a low value of w must be used. Using the knowledge of the location of the stimulus, we also observe the eyes moving off the intended path in parallel at several locations. This is not observed in our results because we only use the distance between the eyes. In later experiments, we run the algorithm on the distance between the gaze point and the stimulus path to show this.

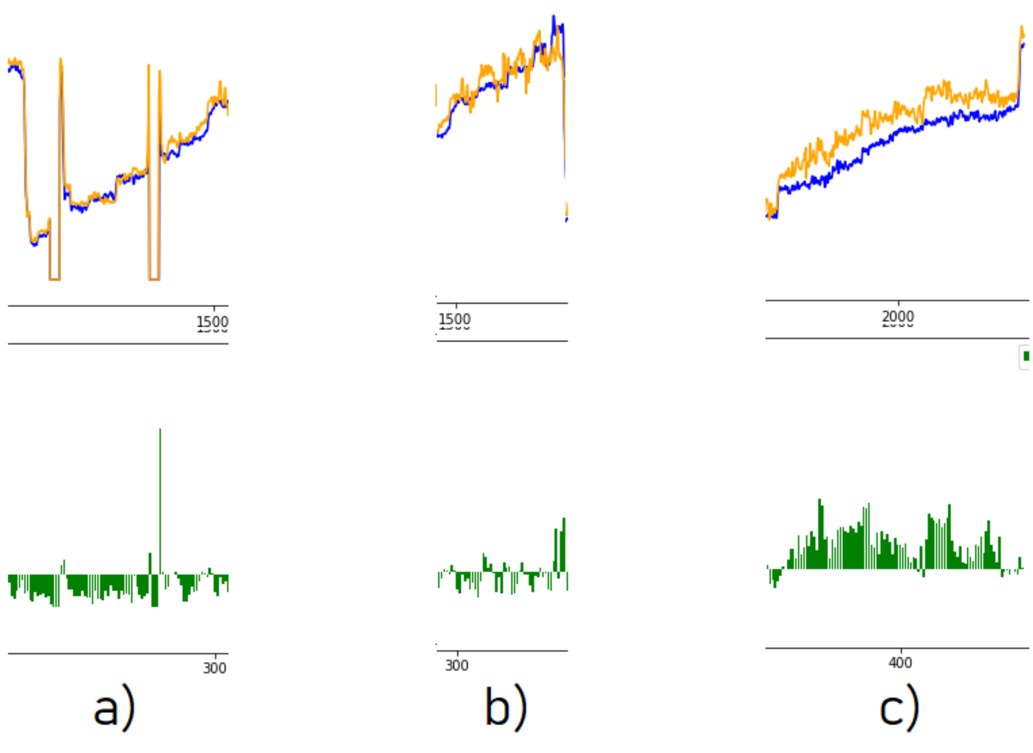

Figure 6.1: Observations from experiment 3. a) Two spikes b) Oscillation c) Tiredness/loss of attention

Experiment 3 shows us one of many recordings with data of high variation. High variation usually consist of high oscillation and many unique areas of unusual behavior. In this case the oscillation can be observed throughout the recording when using a low value of w. At higher values, these disappear. Interestingly, two identical events, seen in example "a)" in figure 6.1, are observed differently in the PAA transformation. Since these are so short, they are highly susceptible to change based on the value of w. In example "b)" the alignment between the eyes oscillate the most, which can be observed in the graph using low values of w. The event at the end of the recording seen in example "c)" is much easier to observe at all values of w. This is because of what was earlier discussed, that short events are hidden by large values of w. Oscillation would in this case be considered a set of short events, and are therefore hidden. A combination of initially using a high value of w may be used to first identify that something is happening, before decreasing the value of w for a more detailed analysis.

6.1.2 Alphabet size (a)

In experiment 4 we are able to observe the difference with the results of the w parameter experiments as well as continue exploring the full algorithm. The initial observation made is the uniformity of the symbols until the largest parameter value is used. As this is a normal recording, getting uniform symbols is expected. We can therefore argue that increasing a beyond 8 would be redundant for our goals on this particular recording. On the other hand, increasing a beyond 8 might still be of interest when analyzing subjects. This is because a combination of results using different values can be used to analyze a single subject. In addition, the set of experiment data affects how visible the events are after transformation, and if little average variation is seen in the experiment data, the individual variation within all graphs increase. We observe how the choice of a moves the breakpoints in a pattern of (i) areas over and under 0 and or (ii) an area around 0. This can be used to create standardized areas where each unique symbol has a discrete characteristic, which is discussed in section 4.2. Changing the value of w has little effect on the information gained from the symbols using this time series.

In experiment 5, the effect of changing the value for a can be observed, also at the higher values of a. All parameter values produce a non-uniform symbol string. Interestingly, most information is retained using the highest and lowest values of a. This is highly correlated to the pattern of the breakpoints, discussed earlier in section 4.2, where the characteristic meaning of the symbols change fundamentally. In these results, when a is 5 the observed symbols characterizes the events as either around average, while at a value of 8 they are characterized as either below or above average.

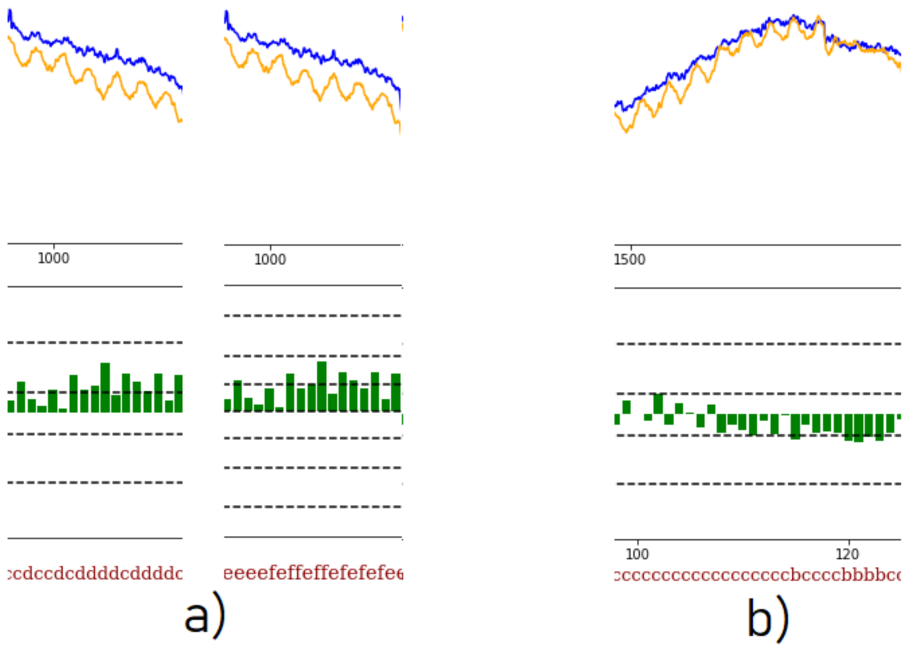

Figure 6.2: Observations from experiment 6. a) Misaligned oscillation of one eye b) Aligned oscillation

Experiment 6 introduces a highly oscillating dataset which also contain several misalignments. Oscillating data has been analyzed in earlier tests where we could see the problem of using the distance between the gaze points as our input to the algorithm. The effect of the oscillating can not be seen when the eyes are doing so in parallel. The results from experiment 6, seen in figure 5.21, further verifies this claim, while also identifying the bounds for this claim. We can see how the oscillation of only one eye, seen in example "a)" in figure 6.2, is correctly identified with a pair of symbols which changes back and forth. This effect can be seen at all values of a, but the pattern changes from a pair of interchangeable symbols "**dcdc**", to a row of interchangeable symbols "**feefee**" and other similar patterns. Notably in example "b)", we correctly observe an unique area where the eyes are aligned.

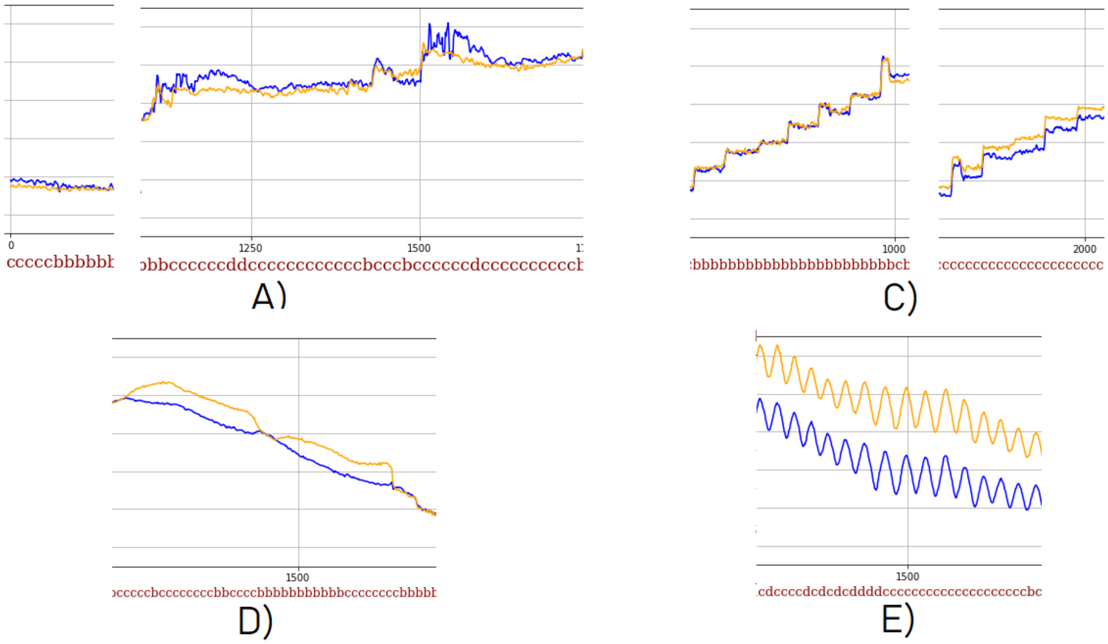

Figure 6.3: Observations from experiment 7. A) Short misalignment C) Tiredness D) Loss of attention/Long misalignment E) Coordinated oscillation

6.2 Assessing SAX symbol string

In Experiment 7, the observations from experiment 1 - 6 can be identified on a set of recordings of high variation using only the SAX symbol string. In graph "A)" in figure 5.25, we are not able to see the spike at 800 using these parameters. This may be possible using smaller values of w, but might not be of interest. Two minor misalignments happen at 1200 and 1550 which are observed on the SAX string as a series of "**ccc**". Interestingly, the section after the misalignments is also identified as a series of "**ccc**", even though they have little resemblance. This can indicate that the value of a used on this graph is too small as the difference between these observations should be big enough to discterizise the behavior to another symbol in order to characterize the behaviors. This is more apparent when observing the negligible difference at 0 and 150. Graph "B)" is interesting to analyze because of the apparent loss of data on one of the eyes. This should be observed in some way, which our SAX method somewhat achieves by identifying most of the occurrences with a large distance between two neighbouring symbols "**ceee**" and "**eeed**". In figure "C)", we see a shaky, but normal recording containing a minor misalignment at the end. With these parameters, we observe little variation of the symbols, and using the knowledge from the graphs as well, we can see how the symbol "**b**" indicates normal behavior while "**c**" indicates minor misalignment. There possibly exists more accurate parameter values we can chose where the random "**c**" symbols throughout the graphs are removed. This behavior can also be seen in graph "D)". Noticeably, we see at 1500 how the breakpoints are incorrectly placed in order to characterize the event using a single symbol. That means the area between the breakpoints of the symbol "**c**" is not large enough, and/or is placed exactly around the average value of this section resulting in the interchangeable "**c**" and "**b**". In figure "E)" we analyze another case of a highly oscillating graph. We see at 1400 and 1500 how a small change in the oscillation can drastically affect the uniformity of the symbols. Finally, figure "F)" in 5.25 shows a normal graph with two large spikes at the end. As mentioned earlier, the identification of such spikes through the symbol string is only possible if the value of w is small enough for the data to fall inside of a frame.

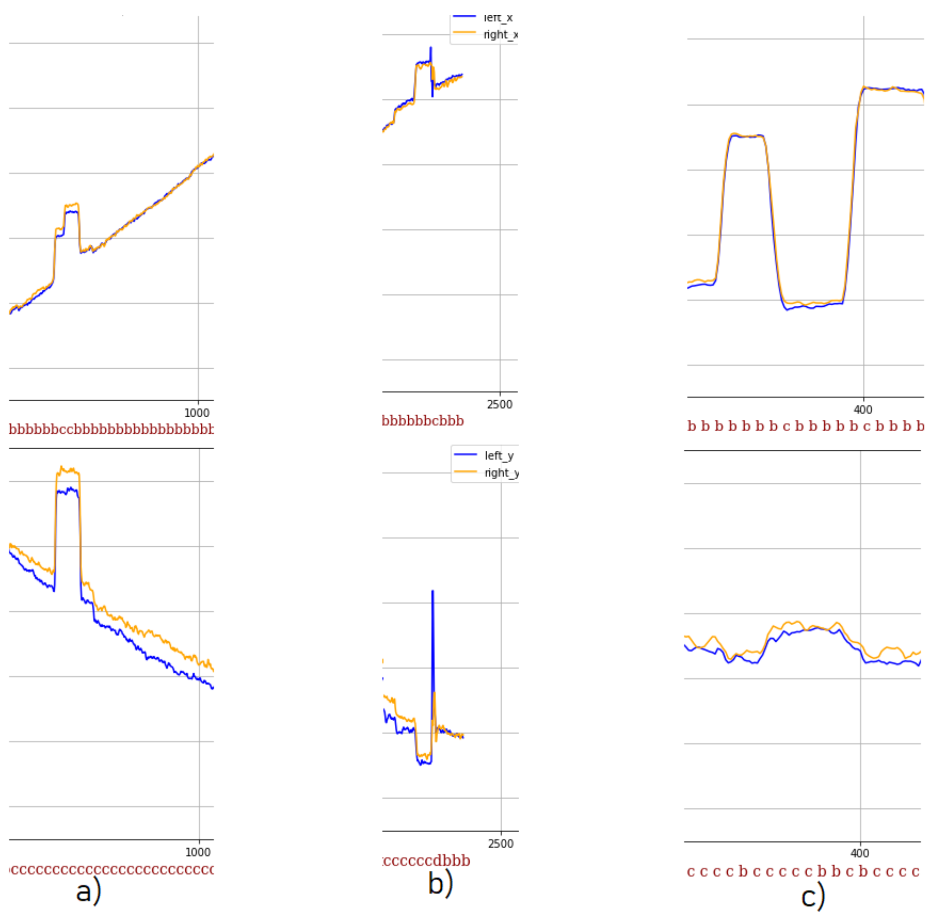

Figure 6.4: Observations from experiment 8. a) Coordinated off the stimulus path b) Spike c) Coordinated off the stimulus path

In Experiment 8, seen in figure 5.26, we are able to compare the results of transformations performed on both axis of the eyes. Another way of doing this comparison is using the euclidean distance formula on all four points, but this is restricted as the axis must be identical. Using the same parameters and initial configurations, we are able to characterize both graphs using symbols with the same meaning. For the initial two graphs, the symbol "**b**" indicates normal behavior, while "**c**" indicates a minor misalignment. Comparing the two strings, we can immediately identify that the first graph has less variation in distance due to the frequency of the "**b**" symbol. This can only be done as long as we are able to give meaning to the symbols. At 750 in the first two graphs, seen in figure 6.4, we can see how the misalignment is identified when everything else is normal, but not when everything else is misaligned. In the last two graphs, we observe some shaking on the vertical axis, which is not identified on the horizontal axis. Additionally, at 400 the eyes move off the intended path on the vertical axis which can not be detected on the horizontal axis. Finally, a spike similar to what has been observed earlier is seen in the first two graphs at 2400, which is observed to a greater extent in the latter problematic recording.

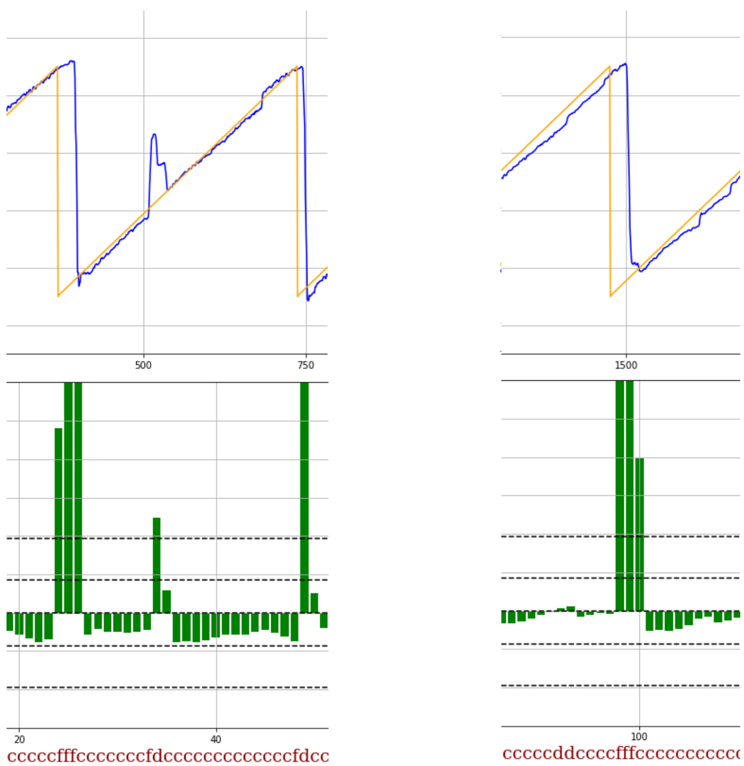

Figure 6.5: Observations from experiment 9. First graph shows delay and spike, second shows delay and misalignment.

Finally, in experiment 9 in figure 5.28, we are able to observe the eyes misalignment to the stimulus object. This is one of the extra layers of analysing one can perform to gain a better understanding of the behavior seen on a recording. Initially, we observe the delay in vision where the stimulus moves from one side of the screen to the other. This can be seen 4 times, but with varying identification through the SAX string. A clear distinction cannot be made between the movement of the stimulus and the other spikes seen, using only the SAX string without having knowledge of the position of the stimulus. This distinction can be made using the methods from earlier experiments. Additionally, we see how a normal recording also contains these delays. For these tests we are also able to separate the left and right eye in order to more specifically identify behaviour. Comparing the results we can negate the similar stimulus delay for a clearer idea of the behaviour.

Chapter 7

Conclusions

In this final chapter, the conclusion of the work done in this thesis is presented. This chapter is structured in the same manner as the experiments and result chapter. Once our conclusions are presented, answers to the research questions are given before completing the thesis with discovered improvements and a discussion of what requires further work.

7.1 Summary and Contributions

In this thesis, we presented our experiences during the research on Symbolic Aggregate Approximation for use in detection of functional vision problems in eye tracking data. The eye tracking domain has exploded in popularity recent years as the technology becomes cheaper, better and more available, therefore new methods for analysing and utilizing this data is much needed. Tools for analysing time series on data which is comparable with the data from from eye trackers already far exceed the current implementations in the domain, therefore the challenge lies in combining the knowledge we already have from earlier research, with the new emerging technology.

The Symbolic Aggregate Approximation method is chosen as it shows the potential of serving as a tool in characterizing differences in the alignment of the eyes. These assumptions are made due to its already documented abilities on similar data. This thesis' main contribution lies in the use of SAX on the euclidean distance of two time series in order to identify and characterize eye movement. Once the observations made in this thesis are connected to knowledge from the medical domain, this method may be used in future versions to perform a medical assessment of a subjects eyes. This is also discussed in section 7.3.

Initially, we performed three experiments in order to discover the basic behaviour of the first part of the algorithm, and learn the effects of changing the value of w. The experiments showed the PAA algorithms ability to differentiate large areas of some distance using all the chosen parameter values. While visual inspection became more difficult when the distances became shorter, distinctions could still be made on values falling under 0. The experiments revealed the direct link between the frequency of the recording and the value of w, which was early hypothesized. We observed that changing w affected our ability to identify "off-screen" events, specifically, how higher values of w render less accurate SAX results. In practice, this affects the method's characterizing ability by requiring events of interest to be occurring over a longer period of time in order to be detected. From our early experiments, we have also been able to identify oscillation events, but can only do so as long as the eyes do not oscillate in coordination. This also becomes hidden at larger values of w. It can be concluded that the value of w determine the length of events we are able to observe.

Following the isolated experiments on w, the same is done for the values of a. Additionally, these experiments test some of the same data as in the first experiments in order to isolate the behaviour affected by the value of a. Already before testing, in section 4.2, we presented the idea of the natural groupings which appear in (i) the areas under the normal distribution, consisting of sections over and under average at 0, (ii) or sections around average. This grouping is shown in our results when changing the value of a and observing the corresponding placement of the breakpoints. Doing this fundamentally changes the "meaning" of a symbol and can be used to characterize general performance by assigning a weight metric to each symbol. This metric can be used to decide if the recording is overall problematic or not. It can be concluded that the value of a determines the accuracy of which we are able to observe events.

Finally, after the isolated experiments on w and a, the focus shifts from the behavior of the parameters to combinations of parameter values and the transformed SAX symbol string. From the results, we are now able to evaluate the methods ability to discover the various behaviours presented in section 3.2. These experiments consist of three separate sections: (i) the discovering abilities (ii) extensions of method (iii) characterization of events.

The *discovering abilities* are analyzed by running a set of recordings through our method containing multiple different anomalous events. Our experiments showed promising results as we were able to identify most of the behaviour to some extent, even without optimizing the parameter values. The main issue lies in distinguishing between these, which was apparent during the discussion of the results. Possible solutions for this have been presented, which consist of extending the SAX algorithm, utilizing all the available data and performing multiple layers of SAX. We have identified multiple ways of *extending the method* during our experiments. In conclusion, our results have shown positive outcome using distance measures between more than just the eyes, additionally using these in combination has shown to render further understanding of the results. This was only shown as a proof of concept in our experiments, therefore, the method struggled to identify oscillating behavior of both eyes in parallel. No conclusive parameter combinations are suggested for optimally *characterizing events*. This is because we have found that the parameters must be empirically determined for optimal results or rendered obsolete, which is suggested in section 7.3. Additionally, as we have shown the effect of changing the parameter values, we can deduce that the performance will greatly improve by using more optimal values. Only partial characterization is possible using the qualitative approach of this thesis, but by using well-known methods for pattern recognition and anomaly detection this process can be both greatly improved and automated.

7.2 Answer to research questions

In conclusion, we have answered the research questions in section 1.2.2:

1. *How viable is SAX as a method for analyzing eye tracking data to recognize functional vision problems?*
   We have presented results showing the use of Symbolic Aggregate Approximation on a small set of eye tracking data using a range of parameter values. Our results show that different behaviour observed on raw data from screenings can be identified by analyzing the corresponding SAX string, using the methods in this thesis. Our results have revealed the limitations of our thesis, presented in section 1.2.3, some of which are further relaxed in section 7.3. Additionally, our results implies a greater potential when running the algorithm multiple times on the same data, using different parameter values.

2. *How accurate can we characterize a time series after the symbolic representation through SAX?*
   Our results have shown great limitations in distinguishing unique observations, but by analyzing the effect of changing the parameter values, assumptions can be made of the methods ability to better characterize the observations using other values than those in our experiments. The results have shown a great difference in the SAX string when analyzing recordings identified as normal, compared to recordings identified as containing anomalies. Additionally, the results have shown that when using additional distance measures, stimulus data and a set of parameter values that we are able to characterize the events to a greater extent.

We conclude that the method of using SAX to analyze time series created using eye tracking is viable in characterizing functional behavior of the eyes. Through the SAX string, we have been able to identify observations made on the raw time series, and we have shown that by using several layers of SAX we are able to identify and distinguish patterns to a greater extent.

7.3 Further work

Further work is devised in two parts. In part one, we discuss improvements on the existing algorithm, the data, the process and other parts of the method used in this thesis. In part two, we discuss the areas of particular interest for further research. Finally, we present a few optional uses for SAX other than what was presented in this thesis.

Improvements

The results of this thesis have revealed specific improvements required to optimize our method. These have been discussed in the previous chapter, and possible solutions and discussion for these are presented in the following summary:

(i) *Improvements on the dataset.*

The pre-processing needed before running experiments relies heavily on manual inspection. Therefore, the dataset used in this thesis, and the datasets available elsewhere, lack the required authentic labeling and characterizations as well as verified data. Because of this, observations made when experimenting on this data must be repeatedly verified. In this thesis, there was an apparent need for labeled data which was not satisfied, this affected the quality of our observations and assumptions throughout the testing.

(ii) *Parameter value range and empirical estimation.*

Even though we were able to identify a lot of the behaviour using the parameter values presented in this thesis, better observations were implied using values not tested in this thesis. A general solution for this using a "zoom" method is discussed later in further research. Additionally, we have identified possible methods for empirically choosing the SAX parameter values. These consist of using the frequency of the oscillations to calculate the value of w needed for the oscillation to be fully contained in a frame and, in collaboration with eye experts, create a set of breakpoints where the areas determine the label of the data contained in it.

(iii) *Breakpoints created using other methods.*

In section 4.2, we discuss how the breakpoints used in the discretization process are determined. In addition to this, other methods for assigning breakpoints can be used, and an example was given earlier, by using the assistance of professionals to create values which collaborate with real-world bounds actually used to diagnose functional vision problems.

(iv) *Other distance functions.*

In section 4.2.2, we discuss how the distance between the eyes are calculated using the Euclidean distance measure. While this has shown to be a valid choice, other distances like Hamming distance can also be used.

Further research

(i) *Analysing reading patterns using SAX.*

Testing the SAX method on screenings created using texts as stimulus is of great interest, as similar analysis on reading patterns have gained much attention due to its connection to the cognitive mind. However, at this point, further research using basic movement is still required.

(ii) *Additional tools for characterization.*
In section 3.2, we discuss the use of visual inspection as the evaluation method used in this thesis. While valuable in exploring new approaches, the method achieves reduced scientific significance due to its lack of numerical grounding. In order to properly create a model of behavioural patterns on the SAX string, more advanced evaluation methods are required. There are several well-known methods of pattern recognition available at the writing of this thesis, as well as newer methods using machine learning. Additionally, quantitative approaches should be explored to further validate the research.

(iii) *Eye Specialist Collaboration.*
While this research heavily relies on computer science, there is already a need to involve researchers from other domains, particularly professionals in ocomotular movement. Understanding the complexity of functional eye movement requires education and is key in characterizing the patterns occurring when using SAX.

(iv) *SAX on raw time series.*
During the experimentation process, assumptions were made of the use of SAX on the raw screening data instead of the distance vectors. This should be further explored and may reveal uses which can either be an extension of SAX or used to identify other characteristics.

(v) *Multiple layers of SAX.*
During the result discussion, this idea has been mentioned several times and consists of running multiple combinations of parameter values on the same data in order to identify different types of behavior. This assumption was made due to our ability to identify different observations in the results using different types of parameter values. By implementing this in a GUI, a screening can be thoroughly inspected by sliding the values of choice. Additionally, distance vectors can be created from multiple distances in the data, which is shown in experiment 9, and can be used in combination with each other, shown in experiment 8.

(vi) *Using stimulus data.*

While this has been discussed earlier, an important distinction lies in how the data is used. While it is shown in the experiment that the distance of an eye and the stimulus can be used to identify observations on the recording data, more elaborate uses of the knowledge of the stimulus can be used to both explain behaviour and improve accuracy.

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