The perception of quantity ain't number: Missing the primacy of symbolic reference

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60-word abstract:
Clarke and Beck's defense of the theoretical construct "approximate number system" (ANS) is flawed in serious ways —from biological misconceptions to mathematical naïveté. The authors misunderstand behavioral/psychological technical concepts, such as numerosity and quantical

cognition, which they disdain as “exotic”. Additionally, their characterization of rational numbers is blind to the essential role of symbolic reference in the emergence of number.

Main text:
The target article by Clarke and Beck —written with an unnecessary condescending tone— is flawed at many levels, from biological misconceptions to mathematical naïveté, and exhibits serious inconsistencies. Here we only address those most crucial.

First, the article lacks clarity regarding the central concept of "number". The authors open by assuming that humans and other animals possess an “*approximate number system* (ANS) that represents number”, but never provide a working definition of *number*. They simply take number as pre-given and unproblematic. For a highly polysemous term like “number” this presents major problems, especially when the goal is to defend the ANS hypothesis and claim that it (also) represents rational numbers. Little clarity can emerge from arguments that blur perceptual, linguistic, and conceptual dimensions of quantity treatment, all falling under the umbrella of “number”. The authors’ use of mathematical concepts such as “natural numbers” and “rational numbers” doesn’t help either. These are technical concepts that refer to infinite sets governed by specific axiomatic systems which, among others, determine their elements via a categorical membership relation. One entailment is that, for a given set, no element is more familiar or typical than another one —mathematically, 38,980,254,332,198 is “as natural” as 2, and 1/2 “as rational” as 577843/97816. While the authors mention that the ANS does not represent *every* natural (or rational) number “or even *most*” rational numbers (whatever “most” may mean in the case of this infinite set, dense in the real numbers), they provide no theory of *which* rational numbers —and by means of what criteria— are supposed to be represented by the ANS (other than saying that they are “of a familiar sort”). A more appropriate title for their article would thus be “The number sense represents *some* rational numbers (but it is unclear which)”. The authors’ confusing use of mathematical concepts and terminology (e.g., stating that “real numbers are continuous”) just makes things worse.

Second, the authors erroneously criticize behavioral/psychological technical concepts, such as “numerosity”, which they disdain as “exotic”. While they are right in that the term “numerosity” has been misused in the numerical cognition literature (Davis & Pérusse, 1988; Núñez, 2017a), they ignore that this term was coined by the psychophysicists of the 1940s who were seeking for conceptual clarity when investigating the problem of scales of measurement of psychological magnitudes (Stevens, 1939/2006, 1951). Renowned experimentalist S.S. Stevens referred to numerosity as “a property defined by certain operations performed upon groups of objects” (1939/2006, p. 23), with the goal of evaluating their numerousness by means of which an experimenter ultimately establishes the cardinal attribute of physical collections of objects. Contrary to the authors’ claim, numerosity was not coined as an “exotic substitute for number”, but as a careful attempt to disentangle the abstract conceptual content of “number” from the degree to which an experimenter could reliably evaluate the attribute of numerousness of stimuli. Thus, the sound and well-defined statement “five is a prime *number*” was never meant to (and cannot) be substituted by “five is a prime *numerosity*”.

The authors also brush off the term *quantical* (Núñez, 2017a) as "exotic", misconstruing its meaning and its theoretical entailments. They erroneously characterize it as a *noun* ("quanticals") serving "as a substitute for number" (with "mysterious properties") whereas, in fact, "quantical" was proposed as an *adjective*—in contrast to "numerical"— meant to characterize some biologically endowed forms of non-symbolic quantity-related cognition and capacities. The authors also misrepresent the quantical-numerical distinction as about "imprecision", conceived to critique the ANS hypothesis on this ground. But the essence of the distinction is about the capacity of *symbolic reference* (Deacon, 2011)—rich in humans and largely absent in non-human animals— which the authors fail to appreciate. Subitizing, for example, is a form of quantical (non-symbolic) cognition, yet still precise. The quantical-numerical distinction is not in the business of making claims about the ANS representing anything (let alone the authors' imagined "quanticals"). Rather, by pointing to the symbolic reference property inherent in number (but not in purely perceived quantities of items) it leads to the critique that the construct "ANS" teleologically puts number (hence the "N") directly in the category of what is biologically endowed, without symbolic (and therefore cultural) mediation. Attacking the "quanticals" strawman to defend the ANS hypothesis is therefore fallacious.

Third, the authors' arguably only novel claim is that the ANS represents rational numbers because it "represents *ratios* among positive integers". Numerically, however, ratios presuppose a binary *arithmetic operation* (division) which, beyond numbers proper, would have to be biologically endowed and implemented *qua arithmetic operation*, a biological no-go. Moreover, statements such as "while the ANS probably represents 2.5 and 2.75, there is no evidence that the ANS can represent 2.7452294861" are theoretically untenable. There is no evidence, or reason to believe, that the hypothesized ANS (or any biological system) "represents" numbers in base 10, which would render 2.75 "more representable" and familiar than 2.7452294861 (presumably due to its shorter decimal expansion). Indeed, 2.75 expressed in, say, base 7 yields $2.51515151\ldots_{(7)}$, with an infinite decimal expansion. The taken-for-granted *expression* of rational (or any) numbers reveals the crucial miss of symbolic reference in the argument. It prevents the authors from seeing that (i) psychophysical perception of quantities of items and (ii) the numbers obtained by the measurement of stimuli's attributes (loudness in decibels, relative quantity in numerical ratios, etc.) are fundamentally different phenomena. The former— shared by many animal species— evolved largely via natural selection, the latter requires symbolic reference implicated in language and specific cultural practices on the part of the schooled experimenter or philosopher, and has evolved via cultural evolution (Beller & Bender, 2008; d'Errico et al., 2018; Gray & Watts, 2017; Núñez, 2017ab). The evolution of such bio-cultural underpinnings of quantification and number is the subject matter of exciting new areas of multidisciplinary research such as those implemented in QUANTA, an endeavor supported by the European Research Council (Barras, 2021). Essential in this enterprise is the recognition of the primacy of symbolic reference in the evolution of cognitive tools for quantification.

Conflict of interest: None

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