“Freak” waves and large-scale simulations of surface gravity waves

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Abstract. Can extreme waves on the open ocean, away from bottom influence and current refraction, be explained by state-of-the-art wave statistics, or are there exceptions deserving the names “freak” or “rogue” waves? We believe that some findings from the large scale numerical simulations of Socquet-Juglard *et al.* (2004) are relevant for answering these questions. The 3D simulations use a computational domain containing typically 10^4 waves, starting from a truncated JONSWAP spectrum with various angular distributions giving both long-crested and short-crested waves. For short-crested waves, the probability distributions of surface elevation and crest height are seen to fit the theoretical distributions found by Tayfun (1980)very well up to 5 standard deviations. After giving a short review of the simulation results, we try to relate them to storm wave data from the Central North Sea.

Introduction

The occurrences of dangerous wave conditions in coastal waters are well known. These “freak” or “rogue” waves may often be explained by focussing (or caustics) due to refraction by bottom topography or current gradients, or even reflections from land. Well-documented in that respect are the giant waves sometimes found in the Aguhlas current on the eastern coast of South Africa (it Lavrenov, 1998).

On the open ocean, away from bottom influence and current refraction, there does not seem to be any special geophysical reasons for extreme waves other than the wind forcing. Extreme waves that are just rare events of a Gaussian or nearly Gaussian population of waves on the sea surface, do not count as “freak’ or “ rogue” waves (see *Haver and Andersen*, 2000)[1]. *Are there waves out there that cannot plausibly be explained by the state-of-the-art wave statistics*?

Any indications that they exist?

First of all, there is indirect evidence in the form of damage done to ships and off-shore installations. Then, there also exists direct and reliable measurements showing exceptional waves. We could mention those described by *Sand et al.* (1990) and *Skourup et al.* (1996) of extreme events that are not plausibly explained by the current state-of-the-art wave statistics. *Skourup et al.* (1996) analyzed 12 years of wave records from the Gorm field in the central North Sea (depth about 40 m) and found several events where the wave crest height, a, exceeds $2H_s$ (H_s is the significant wave height). These are crest heights larger than 8 standard deviations of the sea surface! One of these wave records is shown in Figure 1. The probability of occurrence by Gaussian linear wave theory, $\sim 10^{-14}$, and by second order wave theory, $\sim 10^{-11}$, are clearly much too small for several such events to be recorded within a time span of 12 years.

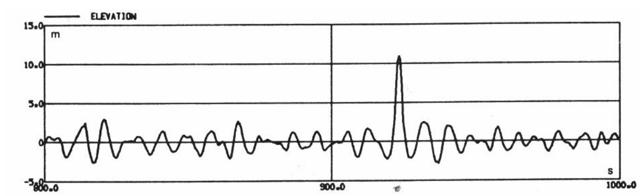

Figure 1. Wave record from the Gorm platform in the central North Sea (17th of November 1984). Sand *et al.* (1990).

Sand et al. (1990) also mention an episode from the Ekofisk field where considerable wave induced damage was reported on a deck more than 20 meters above the still water level.

The much referred to “Draupner wave” that was recorded in January 1995 had a crest height of 18.5 meters, while the significant wave height was estimated to be 11–12 meters.

[1]In fact, these authors suggest as a definition of a freak wave event that it is not plausibly explained by so-called second order models.

Thus, the crest height was a little more than 6 standard deviations. The probability of occurrence by Gaussian and second order theory are $\sim 10^{-8}$ and 10^{-6}, respectively. The wave conditions lasted for approximately 6 hours so that roughly 2000 waves passed. Thus, the a priori chances (by second order statistics) for the event to happen in that storm would be roughly 1 in 500 (or 1 in 100 if the highest estimate of H_s is chosen). Unless a number of such waves should occur within a few years, the "Draupner wave" will probably not be counted as a freak in the meaning of *Haver and Andersen* (2000).

Warren et al. (1998) analyzed some other North Sea data. Comparisons were made with the modified Rayleigh distribution of *Tayfun* (1980), which take into account second order nonlinear effects. For the case of deep water waves (see their figure 11) some of their data may not easily be reconciled with the theoretical distribution.

Let us then consider what kind of physics that may be invoked to produce them.

The physics of "rogue" waves

Clearly extreme waves represent a very high concentration of wave energy compared to the average[2]. A number of mechanisms are known that produce large waves from moderately small ones by focusing the energy:

Focusing by current refraction

Far offshore on the open ocean with only very small current velocities (less than 20 cm/s, say) it would seem that these effects are negligible. *White and Fornberg* (1998) have shown by refraction studies, however, that even small random current fluctuations with RMS values of the order 10 cm/s and typical scales of the order of 10 km can give formation of caustics provided the incoming wave field is unidirectional and narrow banded.

From experience with similar refraction calculations (*Trulsen et al.*, 1990; *Dysthe*, 2000) we suggest that even a small directional distribution of the incoming wave field will "smear out" the caustics and thus reduce the effect of weak refraction to minor fluctuations in energy density. An analogous phenomenon can be observed on the bottom of a swimming pool: With direct sunshine surface waves refract the sun rays giving patterns of moving caustics on the bottom. When a cloud diffuses the sun radiation, the caustic pattern disappears.

Focusing by inverse dispersion

The effect is used in a well-known technique for producing a large wave at a given position in wave tanks. The wave maker is programmed to make a long wave group with steadily decreasing frequency. With proper design of this frequency-chirped "signal" , dispersion contracts the group to a few wavelengths at a given position in the tank. This type of focussing has been suggested as a possible mechanism for freak waves. It has been shown (see the review article by *Kahrif et al.*, 2004) that if a given chirped "signal" produces strong focusing in the absence of other waves, it will still do so (although somewhat weaker) when a random wave field is added. If the amplitude of the deterministic "signal" is below the RMS value of the random waves it will remain "invisible" until it focuses. The problem with this idea in a geophysical setting is to find a source for the spontaneous occurrence of suitable "signals" .

Nonlinear focusing

The so-called Benjamin–Feir (B-F) instability of regular wave trains is well-known. *Henderson et al.* (1999) have investigated what they call "steepwave events" by simulating the evolution of a periodically perturbed regular wave train. Due to the B-F instability the wave train breaks up into periodic groups. Within each group a further focusing takes place producing a very large wave having a crest height roughly 3 times the initial amplitude of the wave train. They point out that these steep wave events seems to have a strong similarity to the so-called breather type solutions of the NLS equation (see also *Dysthe and Trulsen*, 1999). The problem with this kind of B-F instability in a geophysical setting is the initial condition: a slightly perturbed *regular* wave train.

It seems that all the above mechanisms for producing large waves need some special preparation or coherence to work. A spontaneous occurrence of such favorable conditions seems rather unlikely.

Does this leave us with the old idea that the extreme waves are simply very unlikely constructive interference phenomena that can be explained by linear or slightly nonlinear (second order) theory? Not necessarily.

Spectral instability

A different idea is the conjecture (based on 2D theory and fairly small scale simulations) that freak waves are associated with a B-F type spectral instability (*Onorato et al.*, 2000; *Mori and Yasuda*, 2000; *Janssen*, 2003). In the 2D case, a recent experiment in a waveflume (*Onorato et al.*, 2004) has given results in support of this idea. Although this effect may be feasible from a physical point of view the geophysical implication is still problematic. The instability evolves on the so-called Benjamin-Feir timescale which is much shorter than the synoptic timescale (on which the wave spectrum changes due to input from the wind). So how does a spectrum slowly develop (by wind) into an unstable form which is known to change rapidly towards a stable form as a result of the instability?

It was also suggested by *Dysthe* (2000) that 4-wave interactions, even in the absence of spectral instability, might influence the probability of constructive interference due to the dependence that is introduced between the interacting waves.

[2]For a wave with crest height $a = 1.6H_s$ the concentration is roughly a factor 20, if the energy density is estimated by $\rho g a^2/2$.

To investigate these ideas, and also to look for the spontaneous occurrence of extreme waves, *Socquet-Juglard et al.* (2004) carried out large scale 3D simulations with a computational domain containing roughly 10^4 waves. In the following we discuss some of these results and try to relate them to the question of "freak" or "rogue" waves.

The Simulation Model

The simulations referred to in the following are published elsewhere in more detail (*Socquet-Juglard et al.*, 2004).The higher order NLS-type equations used in the numerical model have been discussed in *Dysthe* (1979), *Trulsen and Dysthe* (1996), and *Trulsen et al.* (2000), and are not reproduced here. The wave field is assumed to have a moderately narrow spectral width, so that the surface elevation, η, is represented as

$$\eta = \bar{\eta} + \frac{1}{2}(Be^{i\theta} + B_2 e^{2i\theta} + B_3 e^{3i\theta} + \cdots + \text{c.c.}), \quad (1)$$

where $\theta = \mathbf{k}_p \cdot \mathbf{x} - \omega_p t$. Here $\mathbf{k}_p = (k_p, 0)$, with k_p corresponding to the peak of the initial wave spectrum, and $\omega_p = \sqrt{gk_p}$. The coefficients B, B_2, and B_3, of first, second and third order in wave steepness, respectively, have small rates of variation in time and space. The higher order coefficients can be expressed by B and its derivatives, so that we end up with an evolution equation for the first harmonic coefficient B. When the complex variable, B, has been computed, the surface can be constructed to third order in the wave steepness.

Length and time are scaled by k_p^{-1} and ω_p^{-1}, respectively. Also, $k_p\eta \to \eta$ and $\mathbf{k}/k_p \to \mathbf{k}$. The initial wave spectrum has the form $F(\mathbf{k}) = F(k, \phi) = S(k)D(\phi)$, where k, ϕ are polar coordinates in the $\mathbf{k}$-plane. For $S(k)$ we use a truncated form of the JONSWAP spectrum,

$$S(k) = \frac{\alpha}{k^4} \exp\left[-\frac{5}{4}k^{-2}\right] \gamma^{\exp\, -\frac{(\sqrt{k}-1)^2}{2\sigma_A^2}} \quad (2)$$

Here γ is the so-called peak enhancement coefficient and the parameter σ_A has the standard values; 0.07 for $k < 1$ and 0.09 for $k > 1$. The dimensionless parameter α in (2) is chosen such that the steepness, s, gets a desired value. In the scaled variables, the steepness s is defined as $\sqrt{2}\sigma$, where

$$\sigma = \left(\int_{\mathbf{k}} F(\mathbf{k})\, k \mathrm{d}k \mathrm{d}\phi \right)^{1/2} \quad (3)$$

is the scaled standard deviation of the surface.

The angular distribution $D(\phi)$ is taken to be of the form

$$D(\phi) = \begin{cases} \frac{1}{\beta}\cos^2(\frac{\pi\phi}{2\beta}), & |\phi| \le \beta, \\ 0, & \text{elsewhere}, \end{cases} \quad (4)$$

where β is a measure of the directional spreading.

Table 1. Initial directional and JONSWAP parameters for three simulation cases used to demonstrate the temporal evolution of the spectra. All spectra are normalised to an initial steepness, $s = 0.1$.

Case	β	γ
A	0.7	3.3
B	0.35	5
C	0.14	5

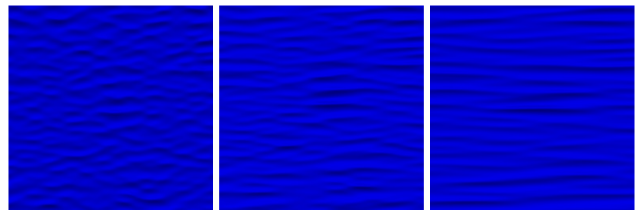

Figure 2. The surfaces of small sections of the computational domain for the cases A, B and C

The JONSWAP spectrum (2) is truncated[3] such that for $\gamma > 3$, less than 15% of the total energy is left out.

To solve the modified NLS equation for B we use the numerical method described by *Lo and Mei* (1985, 1987) with periodic boundary conditions. Moreover, the model is based on a perturbation expansion (e.g., *Trulsen and Dysthe*, 1996) that only allows for a limited time horizon, τ, for the calculations of the order of $(\omega_p s^3)^{-1}$. Quantitative comparisons with wave tank experiments and other numerical wave models (*Lo and Mei*, 1985; *Trulsen and Stansberg*, 2001; *Shemer et al.*, 2001, 2002; *Stocker and Peregrine*, 1999; *Clamond and Grue*, 2002) have indicated that a reasonable time horizon is indeed $\tau \approx (\omega_p s^3)^{-1}$, which for the present simulations (with $s = 0.1$) gives $\tau \approx 150T_p$.

Within these limitations, our model appears to be in very good agreement with the physical- and numerical experiments mentioned. Due to its very high efficiency it is well suited for large computational domains and repeated simulations.

In the simulations referred to here the steepness s is taken to be 0.1. This seems to be nearly an extreme value as demonstrated by *Socquet-Juglard et al.* (2004) using a (T_p, H_s) scatter diagram of data from the Northern North Sea (Pooled data 1973–2001, from the Brent, Statfjord, Gullfaks and Troll platforms).

Three different simulation cases, ranging from a broad to a very narrow directional spread, are listed in Table 1. See also Figure 2, where 2% of the computational area is displayed for each of the three cases.

[3] All the included Fourier modes satisfy the condition $|\mathbf{k} - (0, 1)| < 1$

Spectral development

The stability of nearly Gaussian random wave fields of narrow bandwidth around a peak frequency ω_p was considered by *Alber and Saffman* (1978), *Alber* (1978), and *Crawford et al.*, (1980) Their results were based on the NLS equation and suggested that narrow band spectra may evolve on the B-F timescale $(s^2\omega_p)^{-1}$, provided that the relative spectral width $\Delta\omega/\omega_p$ does not exceed the steepness, s. If that threshold is exceeded, change should only occur on the much longer timescale $(s^4\omega_p)^{-1}$, i.e., the Hasselmann timescale (*Hasselmann*, 1962; *Crawford et al.*, 1980).

Dysthe et al. (2003) investigated the stability of moderately narrow wave spectra by large scale 3D direct simulations. Starting from a bell-shaped initial spectrum, it is found that regardless of whether the initial spectral width satisfies the above mentioned condition for instability, the spectra evolve on the B-F timescale from an initial symmetrical form into a skewed shape with a downshift of the peak frequency. The angularly integrated spectrum shows an evolution towards a power law behavior $k^{-2.5}$(or ω^{-4}) above the peak frequency. It has recently been shown (*Socquet-Juglard et al.*, 2004) that a similar development takes place when starting from a truncated JONSWAP spectrum. The development becomes more pronounced as the initial spectral width decreases. Examples of the temporal evolution for the directional spectra of the cases A, B and C are shown in Figure 3.

In 2D, similar results have been obtained by *Mori and Yasuda* (2000) using the full Euler equations, and by *Janssen* (2003) using the Zakharov integral equation. In 3D, *Onorato et al.* (2003) using the full Euler equations demonstrated a spectral evolution and the power law behavior ω^{-4} to occur for a wide spectral range.

As can be seen from the figures most of the action is taking place during the first 70 wave periods. The peak region is broadened along with a downshift of the peak. As shown by *Soquet-Juglard et al.* (2004) there is still a tendency towards a power law $k^{-2.5}$ above the peak.

The probability distribution of the surface

In the following, it is convenient to scale the surface elevation, η, and the crest height, a, by the standard deviation, σ, defined in equation (3). *Soquet-Juglard et al.* (2004) investigate the probability distributions of the surface elevation and the crest height for the simulation model and parameters mentioned above. For the bulk of the waves, say $\eta < 3$, the simulated data are found to be in very good agreement with the second order theoretical distributions due to *Tayfun* (1980). Tayfun's results assume that the first harmonic in a narrow band development (Eqn. 1) is Gaussian. This assumption is found to be in good agreement with the simulations.

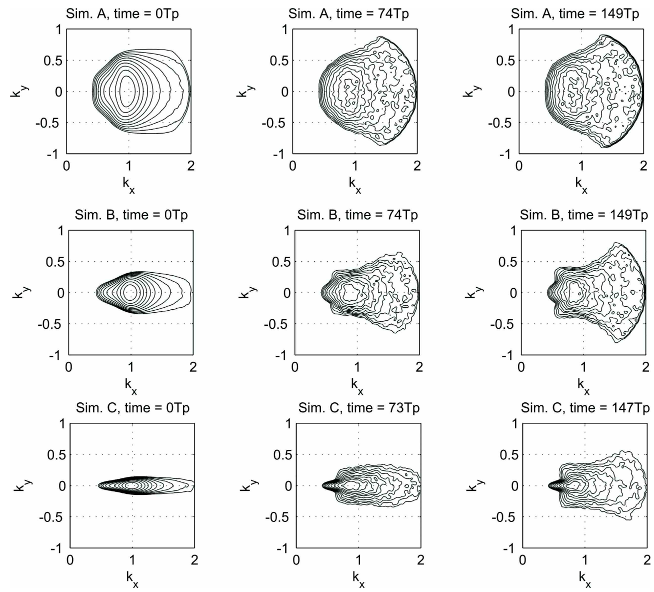

Figure 3. The spectral evolution of the truncated JONSWAP spectra A, B and C (see table).

Using the fact that $\sigma^2 \ll 1$, Tayfun's distribution (see *Socquet-Juglard et al.*, 2004) of the surface elevation can be expanded asymptotically to give

$$p_\eta(z) \sim \frac{1 - 7\sigma^2/8}{\sqrt{2\pi(1 + 3G + 2G^2)}} \exp(-\frac{G^2}{2\sigma^2}), \qquad (5)$$

where

$$G = \sqrt{1 + 2\sigma z} - 1.$$

Note that the asymptotic form (5) has the mild restriction $\eta > -3/(8\sigma)$.

For the crest height, a, Tayfun found the distribution

$$p_a(z) = \frac{1}{\sigma}\left(1 - \frac{1}{\sqrt{2\sigma z + 1}}\right) \exp\left[-\frac{1}{\sigma^2}(\sigma z + 1 - \sqrt{2\sigma z + 1})\right]. \qquad (6)$$

It is readily verified that in the limit $\sigma \to 0$, the expression (5) tends to the Gaussian distribution, and the expression (6) tends to the Rayleigh distribution. For comparisons of (6) with data from storm waves, see *Warren et al.* (1998).

As an example Figure 4 shows a comparison of (5) with typical data from simulation case A. It is seen that the correspondence is remarkable at least up to 4 standard deviations. For the cases B and C the agreement was equally good up to 3 standard deviations.

Influence of spectral evolution

In 2D simulations it was also found by *Onorato et al.* (2000) and by *Mori and Yasuda* (2000) that large waves appeared to be associated with rapid spectral development due to the modulational instability. In 3D, *Onorato et al.* (2002)

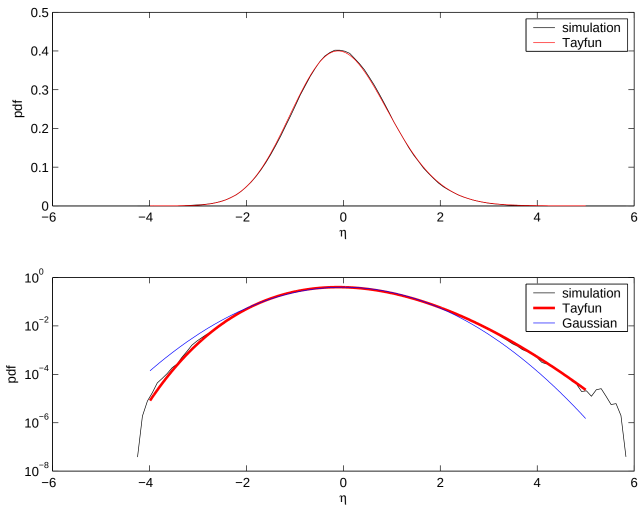

Figure 4. Typical simulated distribution (black) of the surface elevation for the case A compared to the Tayfun distribution (red). The Gaussian distribution is also shown in the log-plot.

made small-scale simulations using the NLS equation. They found that with increase of the angular spread, the occurrence of extreme events decreased.

Recently *Onorato et al.* (2004) have performed experiments in a long wave flume. A wave maker generated a modulationally unstable JONSWAP spectrum that developed along the flume. They demonstrated an increase in the abundance of extreme waves especially during the early development of the instability.

To test these indications that enhanced occurrence of large waves is associated with rapid spectral development (or instability) the more extreme part of the distributions must be inspected. For this purpose it is convenient to exhibit the simulated probability of exceedance for the crest height, a, at different times. In Figure 6 this is compared with the Rayleigh and Tayfun probabilities of exceedance given by

$$P_R(a > x) = \exp(-x^2/2), \tag{7}$$

$$P_T(a > x) = \exp\left[-\frac{1}{\sigma^2}(\sigma x + 1 - \sqrt{2\sigma x + 1})\right], \tag{8}$$

respectively.

The simulated data used in Figure 4 are from one “snapshot” (in time) of our computational domain, covering roughly 10^4 waves. Figure 5 shows such “snapshots” (henceforth referred to as a “scene”) are taken at the height of the evolution process of the wave spectra (at $t = 25T_p$) for the cases A, B and C. A clear deviation from the Tayfun distribution (8) is seen only for the case C. Early in the spectral development there is an increased probability of crests higher than 3 standard deviations for this case. The same trend can be seen in

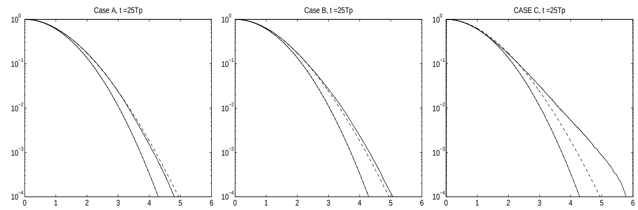

Figure 5. Probability of exceedance of the scaled crest height. Simulations (solid) at time $25T_p$ compared to the Rayleigh (solid with dots) and Tayfun (dashed) probabilities.

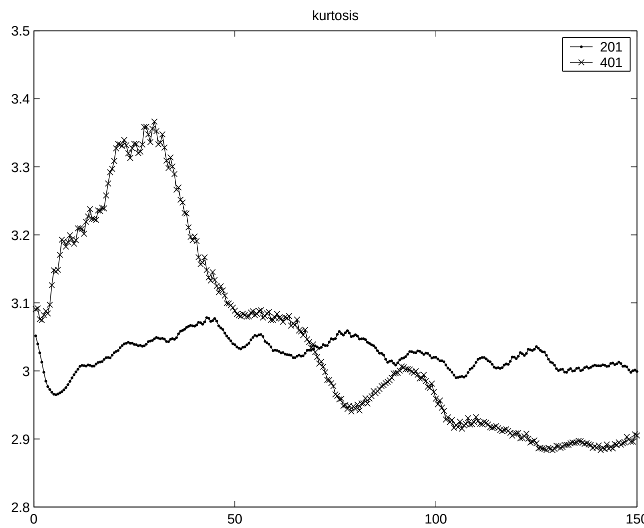

Figure 6. Development of the kurtosis for the cases A and C.

the development of the kurtosis as shown in Figure 6.

The distribution of extremes

For the short-crested case, the simulated data fit the Tayfun distributions very well both for the surface elevation (5) and the crest height (6) up to four times the standard deviation of the surface (*Socquet-Juglard et al.*, 2004). This does not seem to change with time despite the fact that the spectrum is changing.

The simulated data of more extreme waves (say for crest heights larger than 4 standard deviations) should be compared with theoretical predictions of asymptotic extremal distributions. Our starting point is a theorem due to *Piterbarg* (1996) for asymptotic extremal distributions of homogeneous Gaussian fields. For applications to the sea surface, the stochastic field is the surface elevation $\eta(x, y)$.

Let S be a part of the computational domain. The surface elevation (at any given instant of time) attains its maximum η_m at some point in S. The theorem gives the distribution of η_m over independent realizations of the surface, assuming the surface elevation to be Gaussian. Obviously, the distribution of η_m depends on the size of S. This size is conveniently measured in terms of the "number of waves", N, that it contains. The size (or area) of one "wave" is taken to be $\lambda_0\lambda_c/\sqrt{2\pi}$, where λ_0 is the mean wavelength and λ_c the mean crest length defined in terms of the wave spectrum $F(k_x, k_y)$ (see, e.g., *Krogstad et al.*, 2004). If the domain, S, is rectangular with sides $N_x\lambda_p$ and $N_y\lambda_p$, the number of "waves" is

$$N = \sqrt{2\pi}\frac{N_x N_y \lambda_p^2}{\lambda_0\lambda_c}.$$

In the present simulations the computational domain has $N_x = N_y = 128$, giving $N \backsim 10^4$ for the full domain (varying somewhat with the angular distribution of the spectrum).

Piterbarg's theorem states that the asymptotic cumulative probability of η_m for realizations of such a Gaussian ocean containing N waves is

$$P(\eta_m \leqslant x, N) \sim \exp\left[-Nx\exp(-\frac{1}{2}x^2)\right]. \qquad (9)$$

By computer simulations of Gaussian surfaces with ocean wave-like spectra, (9) has turned out to be very accurate indeed (see, e.g., *Krogstad et al.*, 2004). *Socquet-Juglard et al.* (2004) have shown how Piterbarg's result can be modified to Tayfun distributed surfaces simply by applying the transformation

$$x \to \frac{1}{\sigma}(\sqrt{1+2\sigma x} - 1) \qquad (10)$$

in (9). We shall refer to this as the Piterbarg-Tayfun distribution. The asymptotic Gumbel limit of the Piterbarg-Tayfun distribution is easily found to be

$$\exp\left[-\exp\left(-\frac{h_N - 1/h_N}{1+\sigma h_N}\left[x - (h_N + \frac{\sigma}{2}h_N^2)\right]\right)\right],$$

where h_N is a solution of the equation $h\exp(-h^2/2) = 1/N$, that is

$$h_N = \sqrt{2\ln N + \ln(2\ln N + \ln(2\ln N + \cdots))}.$$

The corresponding expectation value of η_m is then

$$\mathsf{E}(\eta_m) \simeq h_N + \frac{\sigma}{2}h_N^2 + \frac{\gamma(1+\sigma h_N)}{h_N - 1/h_N}, \qquad (11)$$

where $\gamma \simeq 0.5772$ is the Euler-Macheroni constant.

In Figure 7, data from the simulations (case A) are compared to the theoretical prediction of equation 9 (11). Each data point in the figure is the average value of η_m from roughly 100 simulation scenes of the same size. The sizes of the scenes in terms of the "number of waves" ranges from 40 to 10.000.[4] As can be seen from the figure there is a very good correspondence up to five standard deviations.

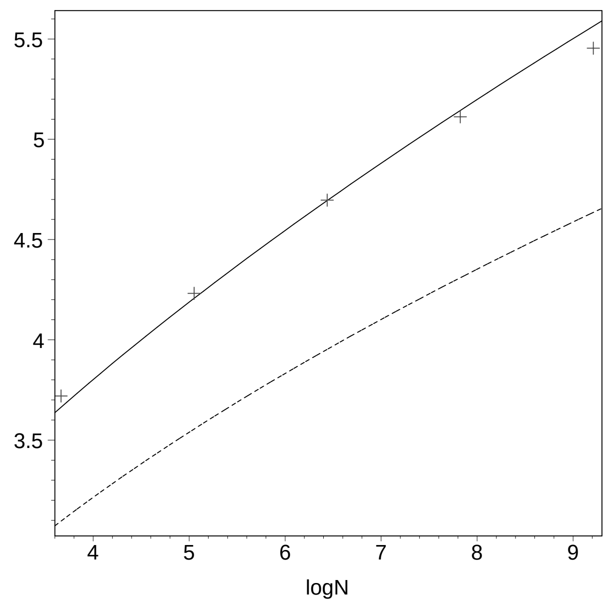

Figure 7. The average largest surface elevation of scenes containing N waves. Simulations, case A, (crosses) are compared with the expected value given by equation (11) for $\sigma = 0.071$ used in the simulations (solid), and $\sigma = 0$ corresponding to the Gaussian case (dashed).

On some observational data from the North Sea

Sand et al. (1989), *Skourup et al.* (1996)[5], and *Warren et al.* (1998) have analyzed storm wave data from the Central North Sea. In the following we shall try to compare some of their data with the Tayfun distributions that seems to work so well with our simulated data. To do so, we have to make simplifying assumptions. We shall assume that the average steepness for the storm conditions they are considering is generally high (in fact we take s to be 0.1 as in our simulations)[6].

We want to investigate how extreme value estimates based on the Tayfun distributions compare to the measured data. Since these are in the form of time series we cannot use the Piterbarg distribution equation (9) which works for 2D. Instead, we use the 1D version which reduces to the well-known Poisson/Rice asymptotic formula. With the transfor-

[4]As a comparison, a 20 min. wave record of storm waves (T_p in the range 8-12 seconds) contains 100-150 waves.

[5]Remark that the extreme cases considered by *Sand et al.* (*e.g.* the example shown in figure 1) are from the same collection of data as those of *Skourup et al.*

[6]*Warren et al.* use a steepness corresponding to $s \approx 0.09$ in their figures 9-11.

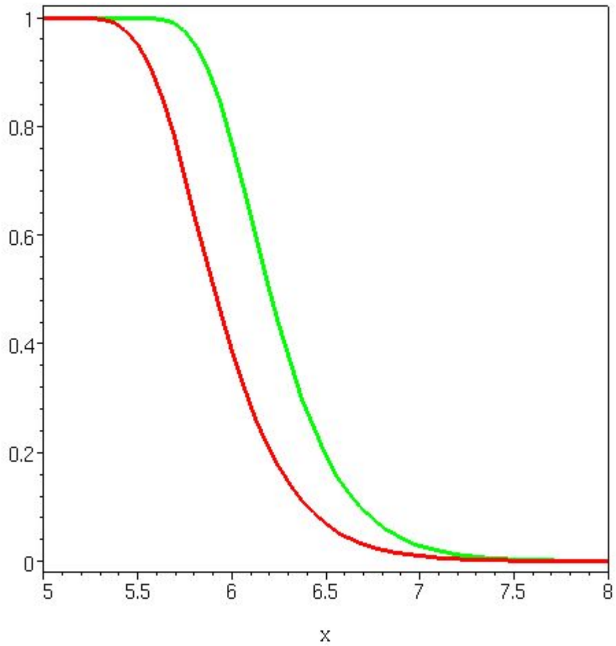

Figure 8. Exceedance probabilities (equation 12) for the Gorm case (green) and for the North Everest case (red).

mation (10) we find the probability that the maximum surface elevation, η_m, (having N waves) exceeds x, is given by

$$P(\eta_m \geqslant x, N) \sim 1 - \exp\left[-N\exp(-\frac{1+\sigma x-\sqrt{1+2\sigma x}}{\sigma^2})\right]. \tag{12}$$

Warren et al. used 2 years (1993–1995) of data from the British sector (platforms Lomond and North Everest at depths 90 meters). From these records they use roughly $2 \cdot 10^5$ waves. The maximum surface elevation, η_m, is found to be ≈ 6.4, and there are three cases exceeding 6. Using equation (12) we find that $\mathsf{E}(\eta_m) \simeq 6.0$.

Skourup et al. had 12 years (1989–1991) of data from the Danish sector (the Gorm platform at depth 40 meters). From these they extracted more than 1600 hours of storm wave records corresponding to roughly $6 \cdot 10^5$ waves. The maximum surface elevation in their data, η_m, is found to be ~ 8.8, and there are nine cases exceeding 8. Using (12) we find that $\mathsf{E}(\eta_m) \simeq 6.3$. In Figure 8 the exceedance probabilities (12) are shown for the two cases.

While the North Everest/Lomond data are not far off what could be expected, the Gorm data are obviously way off! In the following we consider some of the other findings from the Gorm field.

The waves satisfying at least one of the following two criteria

$$a > 1.1 H_s \text{or} H > 2H_s \tag{13}$$

were collected as possible "freaks". Here H is the wave height and $H_s = 4\sigma$ is the significant wave height. The number of waves found to satisfy the first criterion was 446 while only 51 satisfied the second one. The ratio $446/51 \simeq 8.7$ is then a rough estimate of the probability ratio between the two events in equation (13). To make a comparison we assume that H is distributed by

$$P(h > H) = \exp(-H^2/\overline{H^2}). \tag{14}$$

For an extremely narrow spectrum, $\overline{H^2} = 8m_0$. When compared to real data, this is known to give too high estimates. *Longuet-Higgins* (1980), however, fitted the distribution (14) to observational data compiled by *Forristall* (1978) from storms in the Gulf of Mexico, and demonstrated a good agreement if the variance was chosen as $\overline{H^2} \simeq 6.85m_0$. Later, *Næss* (1985) generalized this result, relating the correction factor to the first minimum of the correlation function.

Now, using (14) for H (with $\overline{H^2} \simeq 6.85m_0$) and the two distributions (7) and (8) for a, we get for the Rayleigh distribution $P_R(a > 1.1H_s)/P(H > 2H_s) \simeq 1.4$, while for the Tayfun distribution with $\sigma = 0.071$ (corresponding to $s = 0.1$), the ratio is $P_T(a > 1.1H_s)/P(H > 2H_s) \simeq 6.5$.

Both empirical data and simulations indicate that the wave-group in which an extreme wave occurs is rather short, containing on the average only one big wave. In a sense this group is a more important object than the large wave it contains because it has a longer lifetime than the individual large wave. Roughly one period T_p after the realization of an extreme crest height, the characteristic feature of the same group will be a deep trough (a "hole in the ocean").

For a Gaussian surface, the average waveform in the neighborhood of an extreme wave maximum is given by the scaled auto-covariance function, see, e.g., *Lindgren* (1972). In Figure 9, cuts through averaged wave shapes over the maximum in the wave- and crest directions are compared to the auto-covariance function. This indicates, in accordance with the above, that on the average, the extreme wave belongs to a very short group, with "room" only for one big wave. Also, two other differences are obvious: The simulated large crest, a, is more narrow and the depth of the following trough, a_t, is more shallow than indicated by the covariance function. For their collection of "freaks", *Skourup et al.* found that the average ratio a/a_t was approximately $\simeq 2.2$. For the cases shown in Figure 9 the covariance function prediction is $\simeq 1.5$ and the simulations give $\simeq 2.3$. Although this seems to fit the findings of *Skourup et al.* (1996) for this average value, the fact still remains that their data have a number of extreme cases that cannot be explained as rare events from the Tayfun distribution. Remark that the extreme case shown in Figure 1 we have $a/a_t \sim 5$! A possible reason for the occurrence of "freak" waves at the Gorm field, is the modest depth (40 meters) and its proximity to the Dogger Bank, so perhaps refraction effects are

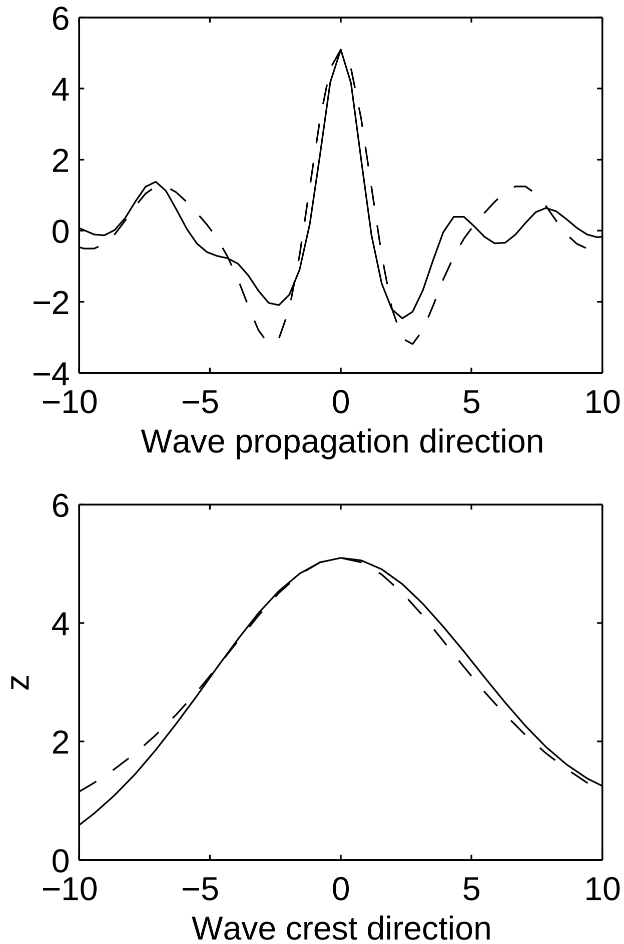

Figure 9. Cuts through the maximum of the average wave profile in the main wave and crest directions (full curves), compared with the the spatial covariance function (dashed curve). Simulation A.

important even that far from coasts.

Conclusions

In our simulations (*Socquet-Juglard et al.*, 2004) we tested the idea that spectral instability increases the occurrence of extreme waves. A significant correspondence between the two was only seen for the case of very long crested waves. For this case we found the same development as in the experiment by *Onorato et al.* (2004).

For short-crested waves no such correspondence between spectral change and the occurrence of extreme waves was seen. Here we found that the distribution of extreme waves are very well approximated by the Tayfun distributions at least up to five standard deviations (*i.e.* $a = 1.25H_s$).

We have compared the prediction of the Tayfun distribution to some fairly long series of data from the Central North Sea. For the northernmost fields (North Everest/Lomond at 90 meters) the maximum crest height is only slightly underpredicted. For the southernmost field (Gorm), however, the prediction is way too low. A reason for the occurrence of "freak" waves at the Gorm field, could perhaps be related to the modest depth (40 meters) and the proximity to the Dogger Bank. Thus refraction effects might be important even that far from the coasts.

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