# Some aspects of the work of V.W. Ekman

# **Alastair D. Jenkins**

Bjerknes Centre for Climate Research, Geophysical Institute, University of Bergen, Allégaten 70, 5007 Bergen, Norway (alastair.jenkins@bjerknes.uib.no)

# John A.T. Bye

School of Earth Sciences, University of Melbourne, Victoria 3010, Australia (jbye@unimelb.edu.au)

Received April 2005

ABSTRACT. The climatic conditions in the Arctic, and observations of the wind-induced drift of ice made in connexion with Fridtjof Nansen's *Fram* expedition, led to the discovery 100 years ago by Vilhelm Bjerknes' pupil, V.W. Ekman, of the importance of the Earth's rotation in limiting the depth of the wind-induced shear current in the ocean. This essay commemorates the centenary of Ekman's seminal paper, which was published in 1905. The paper presents a concise summary of Ekman's contributions to physical oceanography and a brief review of their continuing impact. The two key concepts are the Ekman spiral (the helical rotation of the velocity vector, in atmospheric and oceanic boundary layers) and the resultant Ekman pumping (vertical motions in the water column associated with the divergence of the flow in the surface layer in the presence of a rotational wind-stress field). Later work has revealed how the magnitude and direction of the turbulent flow in the near-surface layer of the ocean. However, the structure of Ekman's original viscous coupling model remains a permanent legacy to physical oceanographers.

### Contents

Introduction	15
Background	15
Theoretical ideas	16
Experimental achievements	17
Other works	18
Ekman theory in a modern context	18
Concluding remarks	21
Acknowledgements	21
References	21

### Introduction

The importance of V.W. Ekman to physical oceanography cannot be overestimated. This article summarises some of his main contributions in a contemporary context. Excellent brief biographies of this Swedish oceanographer, who was strongly influenced by his Norwegian teachers and friends, can be found in Welander (1971) and Svansson (1996).

The life of Vagn Walfrid Ekman began in Stockholm on 3 May 1874. During his studies at the University of Uppsala, he became interested in fluid dynamics, which culminated in the publication of his thesis in 1902 on the theoretical topic for which he is famous. During the years 1902–08, he worked in the Central Laboratory of the International Council for the Exploration of the Sea in Christiania (Oslo), where the significance of his work became known, and he also embarked on other aspects of oceanography, including instrumentation and laboratory experimentation.

This period was followed by appointment to the University of Lund in 1910, where he worked until 1939 in mechanics and theoretical physics as professor of mechanics and mathematics. After a period of interest in other aspects of physics, his focus returned in 1923 to theoretical oceanography, in particular the horizontal variation in ocean currents (Svansson 1996). The years that followed were devoted to a ceaseless endeavour to make long-term measurements at sea, without which theoretical models cannot be verified. Ekman was awarded the Agassiz Medal in 1928 and the Vega Medal in 1939. The results of his oceanographic cruises in the Atlantic Ocean in 1930, partly due to loss of documents during the Second World War, were not, however, published until 1953. He died in Gostad, Stockaryd, on 9 March 1954.

# Background

The relative importance of the wind and of temperature and density differences in driving the ocean circulation was a topic of scientific discussion in the latter part of the nineteenth century, and at that time there were two main theoretical approaches (Krümmel 1907, 1911).

The first theory was that of Zöppritz (1878), who maintained that the current was primarily forced by the wind, and had an average direction the same as that of the wind. His theory did not take into account the deflecting effect of the Coriolis acceleration due to the rotation of the Earth, and, in fact, required extremely long ('geological') lengths of time for the effect of the wind to reach into the depths of the ocean. Mohn (1887), who applied the Zöppritz theory in his analysis of observations made during the Norwegian Northern Seas Expedition of 1876–78, concluded that the effect of the Earth's rotation on the currents was small.

The second theory, promoted by Vilhelm Bjerknes (1904), was that the driving force for the ocean circulation was the effect of density and temperature differences. A brief historical overview of the two theories is given in Pettersson (1909):

In the 1870s it was usually thought that physical conditions, above all the differences in specific weight and in temperature of the ocean water in tropical and arctic regions, supply the primary driving forces for the ocean circulation. The German geophysicist K. Zöppritz (1838-85), however, came forward with the view that wind forcing is the primary cause of the circulation. He was of the view that a wind blowing in a particular direction over a given sea area, which is stronger and more frequent than those blowing in other directions, will gradually produce a surface current in the same direction, which will in turn after an indefinitely long time induce the same motion even in the ocean's deepest layers... This hypothesis has been maintained until the present day, and is accepted by most foreign hydrographers. The experience of Swedish [and presumably also Norwegian] hydrographers, among others O. Pettersson and G. Ekman, is at variance with Zöppritz's theory. In our waters ... the difference between the different water layers is so marked, as regards their temperature, salinity, and density, that it is a simple matter to determine the boundaries of the surface and deep water masses, as well as their respective movements. Investigations have revealed that the deep water does not derive its kinetic energy from the effect of the wind on the surface layer . . .

The author went on to explain that density differences, along with tidal effects, appeared to give rise to the dominant driving forces, and that 'W. Bjerknes' had derived, by means of the hydrodynamic circulation theory of Helmholtz and Kelvin, some mathematical expressions 'from which it is possible in a straightforward manner to represent graphically the internal forces within the ocean and compute their effects.'

Observations of the drift of the vessel *Fram* as it was frozen into the ice during Nansen's polar expedition of 1893–96 (Nansen 1900–06) provided significant new insight into the response of the surface layer of the ocean to wind forcing, which was to unite these two apparently divergent theories. Nansen found that the drift of the ice, and thus the current immediately underneath it, was on average directed between 20 and  $40^{\circ}$  to the right of the wind direction. He explained this fact as a consequence of the Earth's rotation, and made the qualitative prediction that the current vector would spiral clockwise with increasing depth.

#### **Theoretical ideas**

These polar observations were the mainspring from which a key dynamic concept in physical oceanography would be launched. According to H.U. Sverdrup, this came about in the following way: 'Nansen empirically recognised the possibility of rotation of the current vector as a function of depth and suggested to Bjerknes that it should be examined more formally. Bjerknes assigned the problem to a young mathematical physicist, V. Walfrid Ekman, who solved it and thereby his name was given to the phenomenon known as the Ekman Spiral' (Nierenberg 1996). Hence, following Nansen's request, Ekman investigated the problem mathematically, and published the results firstly in Swedish in a Norwegian journal (Ekman 1902), and subsequently in English (Ekman 1905) and German (Ekman 1906). He showed how the rotation of the Earth influenced the way that the ocean currents responded to the force of the wind, and pointed out how purely viscous effects were unable to transfer horizontal momentum down into the water column, so that it was necessary to take the turbulent downward transport of momentum into effect, by, for example, introducing a turbulent 'eddy viscosity' coefficient  $v_E$ . In the case where  $v_E$  has a constant value, he showed that in the steady-state limit, the following formulae give the current at depth z:

$$u = V_0 e^{-\alpha z} \cos[(\pi/4) - \alpha z],$$
  

$$v = (f/|f|)V_0 e^{-\alpha z} \sin[(\pi/4) - \alpha z],$$
  

$$\alpha = [|f|/(2\nu_E)]^{1/2}, \quad f = 2\omega \sin \varphi$$
  

$$V_0 = \tau_2/(\rho \nu_E \alpha \sqrt{2}),$$
  
[1]

where *u* and *v* are the horizontal Cartesian components of the current, along 0*x* and 0*y* respectively,  $V_0$  is the magnitude of the surface current,  $\omega$  is the angular speed of rotation of the Earth, *f* is the Coriolis parameter,  $\varphi$  is the latitude,  $\rho$  is the water density, and  $\tau = (0, \tau_2)$  is the constant shear stress vector at the sea surface (the wind stress), which for convenience is assumed to be directed along the *y*-axis. The surface current is directed at 45° to the right of the wind stress in the Northern Hemisphere, and 45° to the left of the wind stress in the Southern Hemisphere. A straightforward derivation of [1] is given in Proudman (1953).

If the system is initially at rest, and the wind is 'switched on' at time t = 0, the system will approach the above steady state, with the tip of the current vector executing a spiral curve with slowly decreasing oscillations. Ekman (1905) attributed the corresponding mathematical formula [2], represented here in the notation of complex algebra, where  $i = \sqrt{-1}$  and  $\zeta$  is a 'dummy' variable, to I. Fredholm.

$$u + i\nu = \frac{\tau}{\rho\sqrt{\pi\nu_E}} \int_0^t \frac{i}{\sqrt{\zeta}} \exp\left(-if\zeta - \frac{z^2}{4\nu_E\zeta}\right) d\zeta \quad [2]$$

The period of the oscillations is equal to the inertial period  $\pi/(\omega \sin \varphi)$ , and their sense is anticyclonic, that is, clockwise in the Northern Hemisphere, and anticlockwise in the Southern Hemisphere. The steady-state transport (Ekman transport) in the surface layer, when integrated over the depth, is equal to  $\tau/(\rho |f|)$  in the direction 90° to the right of the wind-stress vector in the Northern Hemisphere, and 90° to the left of the wind stress in the Southern Hemisphere.

The angle of the surface current to the wind-stress vector depends on the vertical profile of the eddy viscosity, and will be reduced if  $v_E$  decreases as moving toward

the surface. The direction and magnitude of the Ekman transport, however, do not depend on the eddy viscosity profile.

The originality of Ekman's solution was disputed by Nierenberg (1996), who pointed out that the result was the same as that of the Coq effect (skin effect) when electromagnetic waves impinge on a conducting body. However, the first known reference to this type of solution is not in the field of electromagnetism, but in Fourier's treatment of heat conduction in the Earth, when it is subjected to a periodic temperature fluctuation at the surface (Fourier 1826). The solution of Fourier's problem, in which, in a medium of uniform conductivity, the surface temperature lags the surface heat flux by a phase angle of  $45^{\circ}$ , controls the thermal lag over the land, whereas convection in the mixed layer modifies the thermal lag over the ocean. These two thermal regimes give rise to the geographical variation of the lag of the seasons (for example, Byers 1974). Thus the progenitor of the Ekman solution is also highly significant for the global climate, although it appears not to have been studied in this context. The originality of Ekman's result is physical rather than mathematical: the rotation of the frame of reference leads, as in the theory of the gyroscope and in the theory of the atmospheric and oceanic circulation of Bjerknes (1904), to steady motions at an angle to the applied forcing.

The length scale  $L = (2v_E/|f|)^{1/2}$  is often termed the Ekman depth: at vertical distances of order *L* from horizontal boundaries, the combined effects of friction and rotation have a direct influence on the flow: at distances much greater than *L*, the effects are indirect. Ekman (1902, 1905) discussed in great detail how to determine appropriate and physically realistic values for  $v_E$  and *L*, which he assumed depended on surface wind speed divided by the square root of  $\sin \varphi$ . Had he instead stipulated  $L \sim u_*/(\sin \varphi)$ , so that  $v_E \sim u_*^2/f$ , in which  $u_* = (\tau/\rho)^{1/2}$  is the friction velocity, he would have provided the basis for Rossby number similarity theory about three decades before Rossby (1936).

Ekman also derived corresponding equations and results for the boundary layer at the ocean bottom, which, of course, is directly applicable to the flow of the atmosphere over the land or over water bodies. He considered additionally what would happen in the case of oceans that are bounded by coastlines. In the case of a single, straight coastline, if the wind blows parallel to the coast, the Ekman transport in the surface will cause the surface to slope in the direction perpendicular to the coast, and the sea-surface gradient will in turn be balanced by a resulting flow throughout the entire water column, parallel to the coast, in the same direction as the wind.

#### Technical terms named after Ekman

The importance of the work of Ekman is obvious from the host of technical terms in common usage amongst physical oceanographers.

Ekman dynamics: The process, described above, where the rotation of the frame of reference results in

frictional boundary layers of limited thickness in which the velocity of the fluid is at an angle with respect to the direction of the applied shear stress. Alternatively, it is the aspects of the fluid dynamics controlled by a finite Ekman number.

Ekman transport: The flux of fluid in the Ekman layer. In the steady state, when integrated over depth, it is equal to  $\tau/(\rho | f |)$  in the direction 90° to the right of the wind-stress vector  $\tau$  in the Northern Hemisphere and 90° to the left in the Southern Hemisphere. Experimental verification is discussed below.

Ekman spiral: The path traced out by the tip of the velocity vector in an Ekman layer, as the vertical coordinate changes. Experimental verification is discussed below.

Ekman veering: The tendency of a water mass to move in an anticyclonic direction, for example, a gravity current flowing along the sea bottom will tend to turn to the right in the Northern Hemisphere.

Ekman layer: The boundary layer where the flow is directly influenced, via friction, by the interfacial shear stress.

Ekman depth: A length scale characterising the vertical thickness of the Ekman layer. If the eddy viscosity is assumed constant, a suitable value is  $L = (2v_E/|f|)^{1/2}$ .

Ekman pumping: A horizontal gradient of the wind stress will cause a corresponding horizontal gradient in the Ekman transport. Since the Ekman transport is directed at right angles to the wind-stress vector, a diverging wind stress will cause a rotating Ekman transport, and a rotating wind stress (wind-stress curl) will cause a divergence in the Ekman transport. By continuity, this will induce a vertical velocity:

$$w = (\partial \tau_2 / \partial x - \partial \tau_1 / \partial y) / (\rho f)$$
[3]

Ekman number: The ratio of the (eddy) viscous force to the Coriolis force. For motions with a characteristic horizontal length scale *l*, this will be  $2v_{\rm E}/(|f| l^2)$ 

Ekman balance: Refers to the steady state, given in [1], where the frictional and rotational forces balance each other.

#### **Experimental achievements**

The instrumentation designed by Ekman includes an insulated water sampling bottle, used in Nansen's expedition in *Fram* (Nansen 1900–06), and a number of current meters, which are described below.

#### **Ekman current meters**

The self-recording current meter devised by Ekman, which could be turned on and off remotely, recorded the mean speed of the current using a shielded propeller and a gearing mechanism driving a recording dial, and dropped metal balls into boxes to indicate the current's directional distribution within the measurement period. The mechanism of the repeating current meter (Ekman 1926, 1932) had a more complex 'balls into boxes' means of recording up to 47 successive measurements. The current meter was set into operation using 'messengers'



Fig. 1. Ekman operating a repeating current meter. Photograph copyright University of Bergen.

consisting of a pair of weights locked together with a spring, which, after hitting a buffer weight attached to the current meter, were collected in a bucket suspended underneath it (Fig. 1). Measurements with the recording current meter were made down to 1200 m depth during the 1930 cruise of *Armauer Hansen* in the eastern Atlantic between Lisbon and the Canary Islands (Ekman 1953). In a preliminary paper, Ekman (1939) stated (translated into English by A. Svansson [personal communication]):

We made during two and three-quarter days and nights current measurements at five different horizons down to 100 m. The currents varied in a rather confused and complicated manner, also when some periodic components had been eliminated. But by computing average velocities and directions for the whole observation period as well as applying some further interpolation and smoothing, a distribution was found, which has some similarity with the theoretical one.

Ekman's work as a pioneer in making current measurements at great depth in the ocean provided valuable direct measurements that complement the use of waterdensity variations to determine currents, and are an essential component of present-day understanding of ocean circulation and climatic variations.

## The equation of state of sea water

In the laboratory, Ekman carried out a series of pioneering experiments in which he determined the pressure dependence of the density of sea water (Ekman 1910). These measurements were used in developing tables for the equation of state for sea water (Sverdrup and others 1942), which were used for about 70 years, before being replaced by an algorithm based on more recent higherprecision measurements (UNESCO 1982).

#### Other works

Although Ekman is remembered primarily for his work on the effect of the Earth's rotation on ocean currents near horizontal boundaries, he also performed significant work in other fields. His first publication (Ekman 1899), which concerned the propagation of salt wedges upstream as rivers enter the sea, used the concept of eddy viscosity, which played an important role in his famous study. Later, he authored a very comprehensive experimental and theoretical study of the 'dead water' phenomenon, in which sailing vessels in particular have their progress seriously impeded by the generation of internal waves in places where the water column is stratified (Ekman 1904), and also observed internal waves in the open ocean (Ekman and Helland-Hansen 1931).

As professor of mechanics and mathematics at the University of Lund, he published a textbook on mechanics (Ekman 1919), with a second edition in 1942, and a third edition in 1949. The book contains thorough discussions of the philosophical basis of the subject, illustrated by short biographies of Galileo, Newton, and Huygens. Some of the exercises in the book refer to his activities in geophysics and polar exploration: in the first exercise, the reader is asked to calculate the mean velocity of an icebreaking vessel. Ekman was also an active participant in discussions on the philosophy of religion, publishing a pamphlet on how belief in a deity can be reconciled with past and current scientific knowledge (Ekman 1936), and also several other popular articles (Anders Persson, personal communication).

### Ekman theory in a modern context

The theory of viscous boundary layers under the influence of rotation has wide application within meteorology, oceanography, and other fields of geophysics and planetary physics. There are also engineering applications in the design of rotating machinery.

## **Ekman dynamics**

Section III of Ekman's 1905 paper (Ekman 1905) considers 'Wind-currents influenced by the continents etc.' In the case of wind blowing over a deep ocean bounded by a straight coast, Ekman determined that the stationary state would be composed of three parts: two shallow vertically varying current layers at the surface and bottom (the Ekman layers), and a uniform current in

- the interior, directed parallel to the coast. He remarked:
  - The most striking result of the coast's influence is that a wind is able indirectly to produce a current more or less in its own direction from the surface down to the bottom, while in the absence of coasts the wind's effect would be limited to a comparatively thin surface-layer. The bulk of this current — the 'midwater-current' — is directed along the coast and its velocity is proportional simply to the wind [stress] component parallel to the coast.

The uniformity of the current in the interior is a manifestation of the 'Taylor column' effect, whereby the fluid velocity tends to be uniform along the rotation axis in a rotating frame of reference under conditions where the Ekman number  $2v_{\rm E}/(f l^2)$  and Rossby number u/(f l)are small, *u* being the characteristic velocity scale and l the length scale of the motion. In other words, the required condition is that the motion is on such a large space and time scale that the Coriolis effect dominates over both viscosity and advection. This phenomenon is the subject of the Taylor-Proudman theorem, derived by Proudman (1916), 11 years after Ekman's paper, and verified experimentally by G.I. Taylor (1917) in the following year. Ekman also made estimates of the time necessary for the current to attain its stationary value using [2]: the time necessary is related to how long it would take for water to be transported in the Ekman layers in order for the surface slope and the current to come into geostrophic balance.

The resolution by Ekman of the current system in the water column into three parts in which, in the interior, the effects of friction are unimportant and the flow is geostrophic (at low Rossby numbers), is an essential constituent of the theory of the general circulation in ocean basins. It forms the basis for the theorem of conservation of potential vorticity (Rossby 1940; Ertel 1942), which, for a homogeneous ocean, implies that, in the absence of external forcing, the mean interior flow is largely along contours of f/h, where h is the ocean depth. In general, Sverdrup balance applies: divergences set up by the windstress curl, see [3], are balanced by a transport across the f/h contours (Sverdrup 1947), with a return flow along the western boundary of the basin (Stommel 1948). This picture breaks down, however, in basins where the f/hcontours form closed curves, for example, in the Southern Ocean, and in the basin comprising the Nordic seas and Arctic Ocean.

#### The influence of waves on the sea surface

At first sight, it might appear that surface waves have little to do with the ideas developed by Ekman. However, it is important to note that Ekman's work was first prompted by observations of surface wind-induced drift, which, it was noted, was directed at a greater angle to the wind direction if the ocean was covered with ice, than in open water. Although this effect can be explained by employing a turbulent eddy viscosity that increases with depth (for example, Madsen 1977), it is fundamentally the result of dynamical and kinematic effects of the surface waves. The oscillating wave motions have fluid particle paths that are not closed, and cause floating objects such as surface drifting buoys, drift cards, etc, to move in the direction of wave propagation, even in the absence of wind forcing. A consistent understanding of the effect of surface waves is therefore necessary in the interpretation of ocean current measurements, and may also be vital for understanding of climate variability in order to analyse historical data, from drift card experiments, or from reports of locations of floating derelict vessels.

Surface waves have periods that are very much less than that of the Earth's rotation, so that they are to a first approximation unaffected by the Coriolis force. In the presence of friction, if waves maintain a steady amplitude, they must be subject to some external forcing, otherwise they will be damped and their amplitude will decrease as they propagate. The mean momentum associated with the wave motion will then diffuse downward, indefinitely far in the absence of rotation, in agreement with the theory of Zöppritz (1878), but if rotation is taken into account, the wave-induced current will be restricted to the Ekman layer. Calculations in Lagrangian coordinates, with an applied wind stress and a monochromatic wave field, by Weber (1983) for the steady state, and by Jenkins (1986) for space- and time-dependent cases, indicate that if the eddy viscosity  $v_E$  is constant, the steady surface current is directed at an angle to the wind stress significantly less than 45°. Jenkins (1987) found values of wind-drift current (2.2-2.8% of wind speed) and its angular deviation from the wind-stress direction  $(12^{\circ}-17^{\circ})$  to the right in the Northern Hemisphere) that are in general agreement with observed values. Similar results were obtained (Jenkins 1989) when the monochromatic wave field was replaced by results from a numerical wave model (Komen and others 1994).

The magnitude of the influence of the surface wave field on the air-sea momentum flux and the sea-surface drag coefficient has in the past decade been a subject of some controversy: some, for example, Taylor and Yelland (2001), have found the influence to be small, but recent eddy-correlation measurements over the Baltic Sea by Smedman and others (1999) and Sjöblom and Smedman (2003), as well as earlier observations of 'waveinduced wind' by Donelan (1990), now leave little doubt that waves have a major effect. Two other factors that influence the momentum flux from the atmosphere to the ocean should be mentioned: one is the enhancement of the momentum flux when the waves are breaking (Banner 1990), and the other is the wave-damping effect of surface films, for example, van den Tempel and van de Riet (1965), Hühnerfuss and others (1987), and Jenkins and Jacobs (1997), which reduces the surface drag.

All these processes may seem far removed from the original concept of eddy viscosity, which was used by Ekman to develop his theories. The important question is whether they affect the validity of the model of ocean dynamics presented above. The contemporary consensus



Fig. 2. Observed and theoretical Ekman spirals adapted from Figure 9 of Chereskin (1995). Note that the shallowest measurement depth is 8 m, and also that for a mean wind speed of 7 m s<sup>-1</sup>, the authors estimate that the thickness of the wave boundary layer is 5 m. The eddy viscosity and the Ekman depth for the fitted theoretical profile are, respectively,  $v_E = 2.7-10^{-2}$  m<sup>2</sup> s<sup>-1</sup> and L = 25 m. The theoretical spiral has been offset from the origin for clarity. Scales are cm s<sup>-1</sup> for currents, and m s<sup>-1</sup> for wind.

is that they do not. The dominance of the surface frictional layer is unchallenged, even in a wave environment, which as every swimmer knows, is much more inertial than viscous.

This has recently been verified theoretically in the inertial coupling theory (Bye 1995, 2002), in which the atmosphere and the ocean are coupled through a wave boundary layer that takes account of the wave field, rather than through a viscous boundary layer. In this formulation, the Stokes drift due to the particle motion of the waves in the water, and the phase velocity of the waves in the air are explicitly represented. A key finding is that in Ekman balance the non-wave component of the velocity at the sea surface is the surface geostrophic velocity in the interior of the ocean. Thus, quite generally, the Ekman analysis can be applied in this reference frame.

The analysis also shows that, relative to this reference frame, the surface drift current, which connects the two fluids, lies at an angle to the surface shear stress (to the right in the Northern Hemisphere) that is determined by the friction at the sea surface. In a purely inertial system, which occurs during the active generation of the wave field, this angle is small, but as the wave field develops and spray production (which is essentially the waste product of the wave generation mechanism) increases, this angle increases due to the formation of a slip surface at the sea surface, typically reaching the values found in Jenkins (1987). The Ekman limit of  $45^{\circ}$  would only be applicable at the sea surface in the absence of waves. Under the 'wave boundary layer,' in which the particle motion due to wave motion is significant, however, it is predicted that the classical Ekman spiral should occur (Bye 2002).

This has recently been demonstrated in an excellent series of observations obtained from a surface mooring using a downward-looking acoustic Doppler current profiler (ADCP) in the California Current at  $37^{\circ}$ N,  $128^{\circ}$ W, approximately 400 km off the coast of California in a water depth of 4800 m (Chereskin 1995). It was found that, on daily time scales over a period of several months, the frictional circulation was in Ekman balance with a mean observed transport to the right-hand side of the wind stress within 3% and  $4^{\circ}$  of the predicted Ekman

transport. The mean velocity profile also showed a smooth spiral which the author indicates 'was qualitatively similar (although flattened) to the theoretical Ekman spiral' (Fig. 2). The (small) difference between the observed and theoeretical velocity profiles is due to the effects of transient stratification processes in the upper water column, which were not included in Ekman's original analysis.

## **Concluding remarks**

The work of Vagn Walfrid Ekman, a distinguished member of the school of Vilhelm Bjerknes, has, over the past century, had a great influence on the development of geophysical fluid dynamics, and other branches of science and engineering in which fluid boundary layers occur in rotating reference frames. Although the mathematical basis of his wind-induced boundary layer concept (Ekman 1902, 1905) is straightforward, having a natural relation to Fourier's work on heat conduction in the Earth (Fourier 1826), the conceptual advance that it represents is profound. Ekman's subsequent career as an applied mathematician, oceanographer, and educator, shows his great inventiveness, talent for communicating fundamental concepts, thorough attention to detail, and concern for historical, spiritual, and philosophical issues.

Although the interior structure of Ekman boundary layers may be different in detail to the 'constant eddyviscosity' complex exponential profile he first introduced, their influence on adjacent fluid regions has some common properties (Ekman transport, Ekman pumping) that are independent of such detailed structure. In the early development of basin- and global-scale ocean models, before the development of the massive computational resources of today, it was absolutely essential to use these properties in order to account for dissipative processes and for meridional and cross-isobath flow. Even today, it is extremely valuable to use the integral properties of Ekman boundary layers, in addition to theorems by other leading figures, for example, J. Proudman, G.I. Taylor, C-G. Rossby, and H. Ertel, in order to obtain insight from the still all-too-sparse measurements available from the world oceans.

The authors conclude by quoting Ekman's final remarks in his 1902 paper, which are very much in line with the approach taken by Bjerknes with regard to meteorological research and its application:

Undoubtedly it will also be advantageous... to conduct experimental investigations and make direct observations of currents in the ocean. These should give the theoretical analysis a firmer foundation. Conversely, application of the theoretical results in the interpretation of experiments and field observations will lead us to conclusions which are of greater applicability than would otherwise have been possible.

The subsequent history of oceanographic and meteorological research has indeed shown that great progress can be made when there is a close and fruitful interaction between investigators united by common experimental and theoretical goals. This is especially true today with respect to climate studies.

## Acknowledgements

Alastair D. Jenkins was funded by the Research Council of Norway under contract 155923/700 (Polar Ocean Climate Processes). An earlier version of this paper was presented as a poster at the Conference on Climate Change in High Latitudes in Bergen, Norway, September 1–3, 2004. The authors especially express their thanks to Dr Anders Persson, Dr Artur Svansson, and two anonymous referees, for informative and helpful comments. This is publication no. A0021 of the Bjerknes Centre for Climate Research.

### References

- Banner, M.L. 1990. The influence of wave breaking on the surface pressure distribution in wind-wave interactions. *Journal of Fluid Mechanics* 211: 463–495.
- Bjerknes, V. 1904. Das Problem der Wettervorhersage, betrachtet vom Standpunkte der Mechanik und der Physik. *Meteorogische Zeitschrift* 21: 1–7.
- Bye, J.A.T. 1995. Inertial coupling of fluids with large density contrast. *Physics Letters* A202: 222–224.
- Bye, J.A.T. 2002. Inertially coupled Ekman layers. *Dynamics of Atmospheres and Oceans* 35: 27–39.
- Byers, J.R. 1974. *General meteorology*. 4th edition. New York: McGraw-Hill.
- Chereskin, T.K. 1995. Direct evidence of an Ekman balance in the California Current. *Journal of Geophysical Research* 100 (C9): 18, 261–18,269.
- Donelan, M.A. 1990. Air-sea interaction. In: LeMehaute, B., and D. Hines (editors). *The sea: ocean engineering science*. Volume 9. New York: Wiley: 239–292.
- Ekman, V.W. 1899. Ein Beitrag zur Erkärung und Berechnung des Stromverlaufs an Flussmündungen. *Öfversigt af Kungliga Vetenskaps-Akademiens Förhandlingar* 56 (5): 479–507.
- Ekman, V.W. 1902. Om jordrotationens inverkan pa vindstrommar i hafvet. Nyt Magazin for Naturvidenskab 40 (1): 1–27.
- Ekman, V.W. 1904. On dead water. In: Nansen, F. (editor). *The Norwegian North Polar Expedition 1893–1896: scientific results.* Vol. V: 15. Christiania: Dybwad.
- Ekman, V.W. 1905. On the influence of the Earth's rotation on ocean-currents. Arkiv for Matematik, Astronomi och Fysik 2 (11): 1–52.
- Ekman, V.W. 1906. Beiträge zur Theorie der Meeresströmungen. Annalen der Hydrographie und maritimen Meteorologie 9: 423–430; 10: 472–484; 11: 527–540; 12: 566–583.
- Ekman, V.W. 1910. Tables for sea water under pressure. Copenhagen: Conseil Permanent International pour l'Exploration de la Mer (Publication de Circonstance 49).
- Ekman, V.W. 1919. Mekanik. Stockholm: Norstedt.
- Ekman, V.W. 1926. *On a new repeating current meter*. Copenhagen: Conseil Permanent International pour l'Exploration de la Mer (Publication de Circonstance 91).
- Ekman, V.W. 1932. An improved type of current-meter. *Journal du Conseil* 7: 3–10.

- Ekman, V.W. 1936. Gudstron och den Fysiska Vetenskapen. Stockholm: Sveriges Kristliga Studentrörelses Bokforlag.
- Ekman, V.W. 1939. Några särdrag hos havsströmmarna enlight teori och iakttagelse. *Ymer* 3: 177–187.
- Ekman, V.W. 1953. Studies on ocean currents: results of a cruise on board the *Armauer Hansen*. Geofysiske publikasjoner/Det norske videnskaps-akademi 19 (1): Part I: Text, Part 2: Tables and Plates.
- Ekman, V.W., and B. Helland-Hansen. 1931. Measurements of ocean currents (experiments in the North Atlantic). *Kunglinga fysiografiska sällskapet i Lund, Förhandlingar* 1 (1).
- Ertel, H. 1942. Ein neuer hydrodynamischer Wirbelsatz. Meteorologische Zeitschrift 59: 22.
- Fourier, J.B.J. 1826. Théorie du mouvement de la chaleaur dans les corps solides. *Mémoires de l'Académie Royale des Sciences de l'Institut de France, années 1821 et 1822* 5: 153–246. [Available for download from http://gallica.bnf.fr]
- Hühnerfuss, H., W. Walter, P. Lange, and W. Alpers. 1987. Attenuation of wind waves by monomolecular sea slicks and the Marangoni effect. *Journal of Geophysical Research* 92: 3961–3963.
- Jenkins, A.D. 1986. A theory for steady and variable wind and wave induced currents. *Journal of Physical Oceanography* 16: 1370–1377.
- Jenkins, A. 1987. Wind and wave induced currents in a rotating sea with depth-varying eddy viscosity. *Journal of Physical Oceanography* 17: 938– 951.
- Jenkins, A.D. 1989. The use of a wave prediction model for driving a near-surface current model. *Deutsche Hydrographische Zeitschrift* 42: 133–149.
- Jenkins, A.D., and S.J. Jacobs. 1997. Wave damping by a thin layer of viscous fluid. *Physics of Fluids* 9: 1256– 1264.
- Komen, G.J., L. Cavaleri, M.A. Donelan, K. Hasselmann, S. Hasselmann, and P.A.E.M. Janssen. 1994. *Dynamics and modelling of ocean waves*. Cambridge: Cambridge University Press.
- Krümmel, O. 1907. *Handbuch der Ozeanographie(I)* 2e Auflage. Stuttgart: J. Engelhorn.
- Krümmel, O. 1911. Handbuch der Ozeanographie(II) 2e Auflage. Stuttgart: J. Engelhorn.
- Madsen, O.S. 1977. A realistic model of the windinduced Ekman boundary layer. *Journal of Physical Oceanography* 7: 248–255.
- Mohn, H. 1887. Nordhavets Dybder, Temperaturer og Strömninger. Den Norske Nordhavs-Expedition 1876– 1878, volume 2, Christiania: Grøndahl.
- Nansen, F. (editor). 1900–06. *The Norwegian North Polar expedition 1893–1896: scientific results.* 6 vols. Christiania: Dybwad.
- Nierenberg, W.A. 1996. Harald Ulrik Sverdrup, November 15, 1888–August 21, 1957. *Biographical Memoirs, National Academy of Sciences* 69: 338–375.

- Pettersson. S.O. 1909. Hydrografi. *Nordisk Familjebok, Konversationslexikon och Realencyklopedi*, volume 11 Stockholm: Harrisburg-Hypereides 1457–1458.
- Proudman, J. 1916. On the motion of solids in a liquid possessing vorticity. *Proceedings of the Royal Society* of London 92: 408–424.
- Proudman, J. 1953. *Dynamical oceanography*. London: Methuen; New York: John Wiley.
- Rossby, C-G. 1936. On the momentum transfer at the sea surface: part I. Papers in Physical Oceanography and Meteorology 3 (3).
- Rossby, C-G. 1940. Planetary flow patterns in the atmosphere. *Quarterly Journal of the Royal Meteorological Society of Canada* 66: 68–87.
- Sjöblom, A., and A. Smedman. 2003. Vertical structure in marine atmospheric boundary layer and its implication to the inertial dissipation method. *Boundary-Layer Meteorology* 190: 1–25.
- Smedman, A., U. Högström, H. Bergström, A. Rutgersson, K. Kahm, and H. Pett. 1999. A case-study of airsea interaction during swell conditions. *Journal of Geophysical Research* 104: 25,833–25,851.
- Stommel, H.M. 1948.. The westward intensification of wind-driven ocean currents. *Transactions of the American Geophysical Union* 29: 202–206.
- Svansson, A. 1996. Ekman, Vagn Walfrid. In: Encyclopedia of Earth Sciences. London: Macmillan: I, 256–257.
- Sverdrup, H.U. 1947. Wind-driven currents in a baroclinic ocean; with application to the equator. *Proceedings of the National Academy of Sciences Washington* 33: 318–327.
- Sverdrup, H.U., M.W. Johnson, and R.H. Fleming, 1942. The oceans: their physics, chemistry and general biology. Englewood Cliffs, NJ: Prentice-Hall.
- Taylor, G.I. 1917. Motion of solids in fluids when the flow is not rotational. *Proceedings of the Royal Society* A 104: 213–218.
- Taylor, P.K., and M.J. Yelland. 2001. The dependence of the sea surface roughness on the height and steepness of the waves. *Journal of Physical Oceanography* 31: 572–590.
- UNESCO. 1982. International oceanographic tables. Vol 4. Paris: UNESCO (Technical Paper Marine Sciences 40).
- van den Tempel, M., and R.P. van de Riet. 1965. Damping of waves by surface-active materials. *Journal of Chemical Physics* 42: 2769–2777.
- Weber, J.E.1983. Steady wind- and wave-induced currents in the open ocean. *Journal of Physical Oceanography* 13: 524–530.
- Welander, P. 1971. Ekman, Vagn Walfrid. In: Gillispie, C. (Editor). *Dictionary of scientific biography*. New York: Charles Scribner's Sons: IV: 344–345.
- Zöppritz, K. 1878. Hydrodynamische Probleme in Beziehung zur Theorie der Meeresstromungen. *Annalen der Physik und Chemie* 3: 582–607.