New Phenomenological Constraints on the Gaugino Mediation Model

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Abstract

In this thesis, phenomenological consequences of the Gaugino Mediation model for SUSY breaking are explored. First, the very important trilinear coupling is proven to exist in Gaugino Mediation. Then, the parameter space is investigated in the cMSSM framework using the SUSY spectrum generator and running parameter calculator `SPheno`. The dependence of the lightest Higgs mass and the LSP on the cMSSM input parameters is explored, and the emerging patterns are discussed in light of analytical equations.

Finally, the the parameter space is constrained using recent experimental results. The first constraint is the lightest Higgs mass, required to be ~ 126 GeV. Points fulfilling this constraint are then checked for flavor violating observables by hand, and checked against the run 1 8 TeV LHC collider results using `SmodelS`.

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A Word on Collaboration

This thesis project was part of a collaboration between my advisor, Jörn Kersten, his current PhD student Inga Stümke, his former PhD student Dr. Jan Hesig (currently holds a postdoctoral position at the University of Aachen in Germany), and myself. Inga and I worked closely together on the Higgs-mass constraints, the LSP investigations and on the derivation of the trilinear couplings. However, all the scans and the plots visualizing the results thereof used in this thesis were coded and produced by myself, and the work on `SModelS` and flavor observables, along with the analytical analysis of the parameter space patterns, is my own. We hope the results of the collaboration will come together into a published article later this year.

0.1 Notation and Conventions

- The Einstein summation convention is implied throughout the text. Greek indices run from 0-4, while roman letters usually run from 1-3 unless otherwise explicitly stated, as in the discussion of Lie groups in the section on gauge theories.
- $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$
- The Minkowski metric $g^{\mu\nu}$ is defined in the mostly minus convention $(1, -1, -1, -1)$.
- Natural units are used throughout, i.e $c = \hbar = 1$.
- The Pauli spin matrices are denoted by σ^i, where

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

- The four component object σ^μ has components $(1,\ \vec{\sigma})$ and $\bar{\sigma}^\mu$ has components $(1,\ -\vec{\sigma})$,where $\vec{\sigma}$ has components σ^i.
- The Dirac gamma matrices are defined in the chiral representation.
-

$$\gamma^\mu = \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix} \tag{2}$$

- $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$, where $\mathbf{1} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- The Dirac conjugate, or bar, of a fermion field is defined by $\overline{\psi} \equiv \psi^\dagger\gamma^0$
- The Feynman slash notation implies contraction with the Dirac gamma matrices, i.e $\not{A} = A_\mu\gamma^\mu$

Contents

Chapter 1

Introduction

The Standard Model (SM) theory of particle physics is a beautiful theory that has been extremely well-verified experimentally. With the recent discovery of what is almost certainly the Higgs Boson, it is safe to say that this theory is a good description of nature, at least in some approximation relevant to the energy regions we have tested thus far. However, with no Dark Matter candidate and no consistent way to incorporate quantum gravity into the theory as of yet, we believe the model is still incomplete. The mass of the Higgs is also highly unstable in the theory, meaning that it seems theoretically more natural for the Higgs mass to be much heavier than its observed value of $\sim$126 GeV. Although this observation is not necessarily a direct inconsistency, hiding it away involves some slight of hand that makes a mockery of common sense. Supersymmetry is an extension of the Standard Model that not only remedies the theoretical instability of the Higgs mass, but also has new Dark Matter particle candidates and possible gravitational relevance. The theory postulates that our universe's Lagrangian is invariant under a new type of symmetry known as a supersymmetry, hence the name. This new symmetry demands the introduction of new superpartners, called sparticles. In the simplest case, there is one such partner for each Standard Model particle. Unfortunately, the theory has some issues of its own, the most significant of which demands that SUSY must be a broken symmetry.

SUSY breaking must take place in a very special hidden sector in order for it to be consistent with current experimental observations. There exist many models for this symmetry breaking, each of which have different predictions regarding the mass spectrum of the SUSY particles. This thesis explores the implications of a model known as *Gaugino Mediation*, where SUSY is broken on a 4-dimensional "plane" separated by *extra dimensions* from the 4-dimensional world we live in. The result of the breaking is communicated to our 4d world via specific supersymmetric particles permitted to transverse the extra dimensions.

Gaugino Mediation, as with all of the SUSY breaking models, has free parameters that cannot be theoretically predicted or calculated, and therefore must be analyzed using other methods. Fixing these free parameters in the model fixes the mass spectra and other relevant observables, so the free parameters can be constrained by forcing the model to be consistent with all the most modern experimental results. First of all, the model must predict a neutral Higgs mass within 3 GeV of the experimentally observed value of $\approx$ 126 GeV, allowing for theoretical uncertainty. It is also necessary that the model predicts mass spectra that do not violate any of the lower bounds determined by recent LHC runs restricting the masses of possible new, fundamental particles. In addition, there are constraints on the size of flavor violating processes that can test validity of the model. A computer code called `SPheno` can be used to generate accurate mass spectra and calculate the size of flavor violating observables for the SUSY particles in the Gaugino Mediation framework, and the output of `SPheno` can be constrained using another code called `SModelS` specifically designed to test particle spectra against LHC results. This thesis uses these tools to find regions in parameter space where Gaugino Mediation could still be a consistent model for SUSY breaking.

This thesis begins with an introduction to the essential concepts of the Standard Model in chapter two. Chapter three develops the SUSY formalism necessary for our discussion of Gaugino Mediation and our research therein. Chapter 4 outlines the most important features of the Gaugino Mediation model. This is followed by a discussion of the results of the parameter space scans in chapter 5. Chapter 6 summarizes the results and sets forth possible avenues for future work.

Chapter 2

The Standard Model

2.1 Quantum Field Theories

2.1.1 Beginnings

Quantum field theory was born out of the need for a relativistic theory of quantum phenomena. The relativistic formalism developed alongside non-relativistic quantum mechanics in the 1920's, the earliest progressions credited to the likes of Dirac, Heisenberg, and Schrödinger, among others[1]. The first attempts aimed to extend Schrödinger's one-particle wave equation to a relativistic version, which resulted in the Klein-Gordon Equation. This theory, however, ran into difficulties because it did not predict positive-definite probabilities for all observables. Dirac remedied this with his famous Dirac Equation, which very accurately described the behavior of spin 1/2 particles with relativistic velocities. But with the discovery of other fundamental particles with spin 0 and 1 like the W± bosons that the Dirac equation did not apply to, it became clear that this theory was incomplete. One-particle theories like the Dirac equation were also incapable of describing processes where the number of particles change, like the absorption and emission of photons in an atom. These deficiencies lead to the introduction of the field concept and the birth of Quantum Field Theory.

2.1.2 Fundamentals of a Quantum Field Theory

A Quantum Field Theory, like any physical theory, must first of all obey the basic observed symmetries in our universe. The two most important of these symmetries are spacetime translation invariance (spacetime is homogeneous) and Lorentz invariance. These two transformations together form elements of the *Poincaré group*, denoted **ISO**(3,1) or *isometries of Minkowski space*. So all observables calculated in the theory must be invariant under Poincaré transformations.

[1] For a nice history of the development of Quantum Field Theory, see chapter 1 of [1]

Quantum mechanics is a tremendously successful formalism for describing the dynamics of one-particle fermions, but as we discussed in the previous section, it has limitations. It is also a non-relativistic theory, so a good place to start in building a quantum field theory is to reformulate quantum mechanics in a Poincaré invariant way. From standard QM, we know that all observables correspond to matrix elements describing the overlap of particle states, i.e

$$O = \langle \psi_1 | \psi_2 \rangle. \tag{2.1}$$

If we wish the the theory to be Poincaré invariant, we must have

$$\langle \psi_1 | \psi_2 \rangle = \langle \psi_1 | \mathcal{P}^\dagger \mathcal{P} | \psi_2 \rangle \Rightarrow \mathcal{P}^\dagger \mathcal{P} = 1, \tag{2.2}$$

In order for 2.2 to hold, the particle states must transform in a definite way under these Poincaré transformations. Now, particles have many physical characteristics like mass, intrinsic spin, momentum and spin projected along an axis. Poincaré transformations change characteristics like momentum and spin projection, but not invariants like mass and spin. With this is mind, we define particle to be *objects that mix only amongst themselves under unitary Poincaré transformations*, meaning that they are characterized by their Poincaré invariant attributes, mass and spin.

With a rough picture of the particle states, we now need to include the dynamics. Again from standard QM, we know that in the interaction picture[2], the quantum states obey the equation of motion

$$i\frac{d}{dt}|\Phi(t)\rangle = H_I(t)|\Phi(t)\rangle, \tag{2.3}$$

where H_I is the Hamiltonian of the system in the interaction picture. Now, from classical field theory, $H_I = \int d^3x \mathscr{H}_I(x)$, where $\mathscr{H}_I$ is the interaction Hamiltonian density composed on the fields in the theory. If H is to be interpreted as an operator on the states, then so must $\mathscr{H}_I$, implying the fields themselves must be operators. Just like in QM, these fields must then obey commutation relations, except these relations are now between fields instead of generalized coordinates. What fields are found in the system's Hamiltonian? Just the fields corresponding to the particles involved in theory. So we must "embed" the particles into appropriate fields, so that there is a one-to-one correspondence between particles and fields. Doing this involves assigning fields with the same number of degrees of freedom as the particle. For example, a spin 0 boson is associated with a scalar field with one degree of freedom. If we define the particle states to be states operated on by the field operator assigned to them, then the states will automatically adopt the well-defined Poincaré transformation properties of the classical field as well, which fits our requirement that our particles should transform in a well-defined way under the Poincaré group. Then, using the commutation relations to define how the interaction Hamiltonian acts on the particle states, we can construct an interacting field theory for quantum particles that will be Poincaré invariant as long as the interaction Hamiltonian obeys these symmetries.

[2]see [2], chapters 1 and 6 for more on the interaction picture.

There is one more complication. How do we determine what types of interactions are allowed in our theory? Our principle of Poincaré invariance tells us only that the interaction Hamiltonian must be Lorentz and translation invariant, but that is insufficient. For this, we need the principle of gauge invariance, the topic of the next section.

2.2 Gauge Theories

2.2.1 An Example: QED

A *gauge theory* is a Quantum Field Theory defined to be invariant under so-called *gauge transformations*. A formal understanding of this concept is best obtained through the example of Quantum Electrodynamics (QED). To construct the theory, one begins with the Lagrangian densities (referred to from now on as simply the Lagrangian) describing the free fields involved in the theory, which in the case of QED are the free photon and fermion fields.

The Lagrangian density of the free photon field is

$$\mathscr{L}_{\text{free photon}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{2.4}$$

where

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{2.5}$$

and A^μ is the photon field. The Lagrangian density 2.4 is defined such that when applying the Euler-Lagrange equations

$$\frac{\partial \mathscr{L}}{\partial A^\mu} - \partial^\alpha \left(\frac{\partial \mathscr{L}}{\partial(\partial^\alpha A^\mu)} \right) = 0, \tag{2.6}$$

the photon field obeys Maxwell's equations for the free electromagnetic field

$$\Box A^\mu(x) - \partial^\mu(\partial_\nu A^\nu(x)) = 0. \tag{2.7}$$

It is easy to see that this Lagrangian is both Lorentz invariant and hermitian, as it involves only real fields and all Lorentz indices are contracted. The free fermion field, otherwise known as the Dirac field, has free Lagrangian

$$\mathscr{L}_{\text{Free Dirac}} = \overline{\psi}(i\partial\!\!\!/ - m)\psi, \tag{2.8}$$

where ψ is a four-component spinor. Applying the Euler-Lagrange equations to this Lagrangian gives the Dirac equation

$$(i\partial\!\!\!/ - m)\psi \equiv (\hat{P\!\!\!/} - m)\psi = 0. \tag{2.9}$$

This Lagrangian is also Lorentz invariant and hermitian[3]. From equations 2.4 and 2.8, we can now construct a Lagrangian for the two free fields by simply adding the two together

$$\mathscr{L}_0 = \mathscr{L}_{\text{Free Photon}} + \mathscr{L}_{\text{Free Dirac}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\partial\!\!\!/ - m)\psi. \tag{2.10}$$

This, however, is not a very interesting theory as there are no interactions between the fields. This is where the principle of gauge invariance comes in. We postulate that the total Lagrangian must be invariant under the following coupled field transformations

$$\begin{aligned} \psi(x) &\rightarrow \psi'(x) = \psi(x)e^{-iqf(x)} \\ \overline{\psi}(x) &\rightarrow \overline{\psi'}(x) = \overline{\psi}(x)e^{iqf(x)} \end{aligned} \tag{2.11a}$$

$$A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) + \partial^\mu f(x), \tag{2.11b}$$

where equations 2.11 taken together define a *gauge transformation*. The function $f(x)$ can be any differentiable function of the spacetime coordinates. $F_{\mu\nu}$ is already invariant under 2.11b, and since it does not contain any fermion field, we conclude that the free photon part of the Lagrangian is already gauge invariant. Under the transformations 2.11a, the free Lagrangian density transforms as

$$\mathscr{L}_{\text{Free Dirac}} \rightarrow \mathscr{L}_{\text{Transformed Dirac}} = \mathscr{L}_{\text{Free Dirac}} + qe\overline{\psi}(x)\gamma^\mu\psi(x)\partial_\mu f(x), \tag{2.12}$$

i.e it is not gauge invariant. If however, we define a new derivative operator D^μ, known as a *covariant derivative*, by

$$D^\mu\psi(x) \equiv [\partial^\mu + iqA^\mu(x)]\psi(x), \tag{2.13}$$

it is not hard to show that the this transforms as

$$D^\mu\psi(x) \rightarrow e^{-iqf(x)}D^\mu\psi. \tag{2.14}$$

Defining a new Dirac Lagrangian

$$\mathscr{L}_{\text{Dirac}} \equiv \overline{\psi}(iD\!\!\!\!/ - m)\psi, \tag{2.15}$$

this new Lagrangian clearly gauge invariant under the gauge transformation (1.8). This new Lagrangian also has the form

$$\mathscr{L}_{\text{Dirac}} = \mathscr{L}_{\text{Free Dirac}} + \mathscr{L}_{\text{I}}, \tag{2.16}$$

where $\mathscr{L}_{\text{I}}$ stands for *interaction Lagrangian*, and

$$\mathscr{L}_{\text{I}} = -q\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu(x). \tag{2.17}$$

[3]Lorentz invariance is not as obvious to see in this example, as one must define how a Dirac spinor transforms under representations of the Poincaré group. See chapter 2 of [3] for more.

The final, fully interacting, gauge invariant Lagrangian of QED is then

$$\mathscr{L}_{\text{QED}} = \overline{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.18}$$

Here we see the power of gauge theories. Purely from the postulate of gauge invariance, we have derived the Lagrangian for the relativistic quantum field theory QED, the predictions of which have been verified with astounding experimental precision[4].

2.2.2 Historical Motivation for Gauge Theories

We now see what a gauge transformation is and how the concept is used to generate interaction terms in quantum field theories. However, the procedure may seem a bit ad hoc at this point, and so we now give a little more motivation for where the idea originated.

The concept of gauge invariance is rooted in classical electrodynamics, and is a statement about the invariance of Maxwell's equations under a change in the electromagnetic potential

$$A_\mu(x) \to A'_\mu = A_\mu(x) + \partial_\mu f(x). \tag{2.19}$$

This idea was extended to non-relativistic quantum mechanics when theorists were attempting to describe the dynamics of a charged particle is an electromagnetic field quantum mechanically. Their guess was to use the classical Hamiltonian this system [4]

$$H = \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}\right)^2 + e\phi \tag{2.20}$$

as the quantum Hamiltonian by promoting the momentum coordinate to an operator through the standard procedure

$$p_\mu \to -i\partial_\mu. \tag{2.21}$$

This corresponds to making the substitution

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu(x). \tag{2.22}$$

in the quantum Hamiltonian for a free particle, where $q \equiv -e$. This procedure is known as the *minimal substitution*. This procedure turned out to agree well with experimental observations, so when extending to the relativistic case, the same substitution was used, this time in the Dirac equation. Since it was known that classical observable electromagnetic fields were invariant under 2.19, it was desired that the Dirac Lagrangian with the minimal substitution also be invariant under 2.19. However, the only way to achieve this is to transform the spinor fields as in equation 2.11a, thus motivating the form of the gauge transformation given in 2.11. Invariance under 2.11 was then elevated to the fundamental symmetry to be obeyed by more complicated quantum field theories.

[4]The anonmalous magnetic moment of the electron predicted by QED, for example, agrees perfectly to 8 digits with the experimentally observed value [2].

2.2.3 Lie Groups

In order to generalize the idea of a gauge transformation, we must first introduce the concept of a *matrix Lie Group*. First of all, we briefly introduce the concept of a group. A group is a set G with elements g, together with an abstract operator, denoted $\cdot$, that obeys the following defintions:

1. (Closure) $g,\, h \in G$ implies $g \cdot h \in G$.
2. (Associativity) For $g,\, h,\, k \in G,\; g \cdot (h \cdot k) = (g \cdot h) \cdot k$.
3. (Existence of Identity) $\exists\, e \Rightarrow\; g \cdot e = e \cdot g \quad \forall g \in G$.
4. (Existence of Inverse) $\forall g \in G,\; \exists\, h \in G \Rightarrow g \cdot h = h \cdot g = e$.

A matrix Lie group is a group whose elements are matrices, where the group operator $\cdot$ is defined to be standard matrix multiplication. Lie groups belong to a special class of groups known as *continuous groups*. What makes these groups special is that each element in the group can be written as a unique function of a continuous parameter. It can be shown[5] that this property of matrix Lie groups implies that any group element can written in the form

$$e^{itX} \equiv e^{it^i X_i}. \tag{2.23}$$

The $\{X_i\}$[6] are a set of matrices, each of the same dimension as the elements of the Lie group, the t_i are a set of constants, varying which produces new group elements, and the exponential is defined by its power series expansion. The matrices $\{X_i\}$ are known as the generators of the group, and form what is known as the *associated Lie Algebra* of the Lie group. This Lie algebra forms a real vector space, and the matrices $\{X_i\}$ form a basis for this space. These Lie Algebras are *closed under commutators*, meaning that if X and Y are both elements of the algebra, then so is $[X, Y]$. For any elements X, Y, and Z in the algebra, the Jacobi identity also holds:

$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0. \tag{2.24}$$

All these properties of the Lie algebra can be shown to follow directly from the properties of a Lie group.

A *representation* $(\Pi,\, V)$ is defined by a map Π from elements g of a group G to linear operators $\Pi(g)$ on a vector space V. This map must be an isomorphism, which means that is must be one-to-one and onto, and preserve the group multiplication structure in the sense that

$$\text{for } g\ h \in G,\; \Pi(g \cdot h) = \Pi(g)\Pi(h). \tag{2.25}$$

[5]see [5]

[6]Note here that in this case, i does not run from 1-3, but runs over the number of elements in the Lie algebra.

The most basic representation for complex (real) matrix Lie groups is the *fundmental* representation, where the map Π is just the identity, and the matrices of the group are defined to act on the vector space $\mathbb{C}^n(\mathbb{R}^n)$ of complex (real) n-dimensional vectors, where the dimension n depends on the dimension of the matrices in the Lie Group.

2.2.4 General Gauge Transformations

Now we use these ideas to generalize the idea of a gauge transformation. If we make the following definitions

$$U \equiv e^{-igf^i(x)X_i} \tag{2.26a}$$

$$A_\mu \equiv A^i_\mu X_i, \tag{2.26b}$$

then a general gauge transformation has the form

$$\psi(x) \to \psi'(x) = U\psi \tag{2.27a}$$

$$A_\mu(x) \to A'_\mu(x) = UA_\mu(x)U^\dagger + \frac{i}{g}(\partial_\mu U)U^\dagger \tag{2.27b}$$

Note the form of U. This is just the form of a Lie group element! So a general gauge transformation is induced by Lie group elements acting on the field operators. If the theory is to be invariant under such transformations, these operators U must be unitary, by arguments similar to those given in 2.1.2. This then implies that the generators of the group must be hermitian. Also note the inclusion of the constant factor g. This is analogous to electric charge e in QED. It is known as the *coupling* of the force described by the gauge group, and is a measure of the "strength" of the force.

The relevant Lie groups for formulating the standard model are elements of $SU(n)$, which stands for the *special unitary group in n dimensions.* The elements of these groups are complex, $n \times n$ dimensional unitary matrices with determinant $= +1$. The Lie algebras corresponding to the groups $SU(n)$ are vector spaces of $n \times n$ dimensional *hermitian* matrices, and the algebra has $n^2 - 1$ basis vectors. So, by the discussion above, our gauge transformations U are just n dimensional matrices, which implies that for equations 2.26, ψ is now an n-dimensional vector[7] and A_μ is an n-dimensional matrix. These equations reduce to equations 2.11 in the case of QED because U is one-dimensional. QED is an *abelian* field theory, meaning the gauge transformation corresponding to QED is generated by commuting Lie algebra elements. Theories where the generators do not commute are referred to as *non-abelian*, which is the case for all $SU(n)$ gauge theories with $n > 1$.

Every gauge symmetry is associated with gauge fields and describes a force mediated by these fields. In the case of QED the gauge field is the photon field, mediated by the photon. There is one gauge field for each basis element of the corresponding Lie Algebra, so in the case of one-dimensional QED, there is only one gauge field.

[7]Note that this dimension of ψ is separate from the Dirac spinor dimension! The gauge transformation does not mix Dirac indices!

With these ideas in mind, we can now build a Lagrangian that is invariant under an $SU(n)$ gauge transformation. If we define our gauge field strength tensor as

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \tag{2.28}$$

using (1.20b) we can show $G_{\mu\nu}$ transforms as

$$G_{\mu\nu} \to G'_{\mu\nu} = UG_{\mu\nu}U^\dagger. \tag{2.29}$$

We can use this to build a gauge invariant Lagrangian term

$$-\frac{1}{2}\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) = -\frac{1}{4}G^i_{\mu\nu}G^{\mu\nu}_i, \tag{2.30}$$

where Tr is the Trace operator, the factor of 1/2 is chosen such that each individual gauge field has normalization similar to that of the pure photon field case, and the implied sum over index i is again over the number of gauge fields. If we define a covariant derivative by

$$D_\mu\psi \equiv (\partial_\mu + igA^i_\mu X_i)\psi, \tag{2.31}$$

this transforms exactly like 2.14 under transformations of the form 2.27. Therefore, we are finally able to construct a Lagrangian invariant under a general $SU(n)$ gauge transformation which describes the interactions between gauge fields A^μ and matter fields ψ charged under this particular gauge force:

$$\mathscr{L}_{gauge} = -\frac{1}{2}\mathrm{Tr}(G^{\mu\nu}G_{\mu\nu}) + \overline{\psi}(i\not{D} - m)\psi. \tag{2.32}$$

Terms of the form of the first term in 2.32 are known as *gauge kinetic terms*, as they correspond to the kinetic energy of the gauge fields. Terms of the form $\overline{\psi}\not{D}\psi$ are known as *kinetic fermion terms*, and any term corresponding to the mass of a field is referred to as a *mass term.*

2.3 The Standard Model

2.3.1 Overview

The Standard Model is a gauge theory aiming to describe the behavior of the so-far experimentally observed particles in our universe and the forces between them. These particles involve the force carriers, or gauge bosons, and matter particles. The matter particles in the theory are the quarks and leptons, along with the scalar Higgs. The gauge bosons are the $W\pm$, Z_0 and photon, which mediate the *Electroweak force*, and the gluons, which mediate the *Strong force*. The Electroweak force is a unification of the Weak force and the Electromagnetic force, which were believed in the early stages of particle physics to be two distinct forces. It was later discovered that they were in fact one force, and the Weak force was "weaker"[8] simply because it was mediated by *massive* gauge bosons, the $W\pm$ and Z_0, while the Electromagnetic force was mediated by the massless photon. The gluons are also massless force carriers.

Like the case of QED, each gauge boson is associated with invariance under a gauge transformation. The gluons are associated with an $SU(3)$ transformation, which has eight generators and therefore there are 8 gluon fields. The gauge theory associated with the the strong force and the $SU(3)$ group is known as *Quantum Chromodynamics*, or QCD. The $W\pm$, Z_0 and photon are associated with an $SU(2) \otimes U(1)$ symmetry, which has $3+1$ generators.

The standard model is postulated to be invariant under the tensor product gauge transformation $SU(3)_C \otimes SU(2)_I \otimes U(1)_Y$, where C, I and Y stand for color charge, weak isocharge and weak hypercharge, respectively. Color charge is the charge of the color force. Any field with color charge has either color red, green or blue. Weak isocharge is a charge related to the weak force, and can have value $\pm\frac{1}{2}$. Hypercharge is a combination of weak isocharge charge and electromagnetic charge defined by $Y \equiv Q - I$, where Q is the electromagnetic charge in units of e, the electric charge of the electron. The statement of invariance means that all observables calculated from the theory must be gauge invariant under the gauge symmetry transformations.

Fields corresponding to particles charged under certain forces must have gauge structure that allows gauge transformations to act on them. For example, fields charged under $SU(2)$ must live in a doublet that can be acted on by a 2×2 matrix. This doublet structure is related to the Electroweak force, and therefore each field in this doublet must have a well-defined weak isocharge. This is defined such that fields living in the top of the doublets have isocharge $+1/2$, and those living in the bottom have weak isocharge $-1/2$. A doublet is an example of a *mutlipliet*, which is defined to be group of fields that are rotated into one another under a gauge transformation.

2.3.2 The Multiplets

To build the multiplets of the standard model, we must specify which particles and fields are charged under which forces. We start with the leptons, which are defined to be the muon, electron and tau fermions and their corresponding neutrinos. First we note that the weak force is a *chiral* force. This means that fields with different chirality are effected differently by the Weak force. *Left-chiral* fields are charged under the Weak force, whereas *right-chiral* fields are not. The chirality of a field refers to the eigenvalue of the corresponding Dirac spinor when acted on by the γ^5 matrix (only fermions have chirality). A lepton doublet in the Standard Model is built from a left chiral electron field and a left chiral neutrino field. The right chiral versions of the electron, muon and tau do not have weak

[8] The strength of the force refers to the distances over which the force is significant, for example, the Weak force is weak because it only has meaningful consequences for very short distances.

Leptons	$\begin{pmatrix}\nu_{e_L}\\ e_L\end{pmatrix}$ $\begin{pmatrix}\nu_{\mu_L}\\ \mu_L\end{pmatrix}$ $\begin{pmatrix}\nu_{\tau_L}\\ \tau_L\end{pmatrix}$, e_R, μ_R, τ_R
Quarks	$\begin{pmatrix}u_L\\ d_L\end{pmatrix}$ $\begin{pmatrix}c_L\\ s_L\end{pmatrix}$ $\begin{pmatrix}t_L\\ b_L\end{pmatrix}$, u_R, c_R, t_R d_R, s_R, b_R each of which can be red, green or blue
Gauge Bosons	$W\pm$, Z_0, photon, gluons
Higgs	$\begin{pmatrix}h^+\\ h_0\end{pmatrix}$

Table 2.1: The Standard Model Particles

isocharge, and therefore do not live in doublets, while there are no right chiral neutrinos[9]. There are three *families* of leptons, also known as lepton *generations*, corresponding to the three multiplets built by the process described above[10]. These generations are sometimes referred to as the *flavor* of the lepton. However the use of the word "flavor" is not always consistent in the lepton sector. Sometimes, it is said that there are six flavors of leptons, where the neutrinos and charged leptons all define their own flavors, whereas other times it is said that there are three flavors of leptons corresponding to the generations. We will use the six-flavor convention in this thesis to parallel the quark sector. The electron, muon and tau have electric charge -1, and their corresponding neutrinos are electrically neutral.

There are six flavors of quarks in the standard model, the up, down, charm, strange, top and bottom quarks. Quarks are defined to be charged under both the electroweak

[9]When the standard model was formed, it was unknown whether or not neutrinos were massive or not. However, upper limits on the masses were so small that massless neutrinos were assumed. As a result, right chiral neutrinos cannot exist in the theory. If they did, they would obtain mass through electroweak symmetry breaking. We know today that neutrinos are in fact massive, but too much is still not understood about the neutrino sector to definitively include massive neutrinos in the Standard model in a unique way.

[10]It is still a mystery why there are three families of quarks and leptons.

and strong forces, meaning they must have both doublet and triplet structure, as well as hypercharge. Each quark, therefore, may have one of the three colors, and the left chiral components also live in doublets of two quarks. Left chiral up-type quarks (those living in the top part of the Weak doublets) have electric charge $+2/3$, and the down-type quarks have electric charge $-1/3$, and their corresponding right chiral components have the same electric charge. Like the leptons, there are also three families of quarks.

All that is left is the Higgs field. It is responsible for the breaking of the $SU(2)_I \otimes U(1)_Y$ symmetry, the process which gives matter particles mass (to be explained in greater detail later). In order to break this symmetry, it must be charged under the corresponding electroweak force, so the Higgs also lives in a doublet. Also, in order to make the Higgs doublet term in the Lagrangian invariant under the $U(1)_Y$ transformation, the top member in the doublet must have electric charge +1 and the bottom member must be electrically neutral. Therefore, because only *complex* scalar fields can have electric charge, the Higgs field must be complex.

All charged fields also have corresponding fields that operate on antiparticles, which are particles that are identical to their particle counterparts except they have opposite electric charge. These are denoted by a bar over the particle, for example $\bar{e}$ denotes an anti-electron (not to be confused with the Dirac conjugate $\overline{\psi}$!). A complete table of the field multiplets in the standard model is given in table 2.1, where it is to be understood that every particle in this table has an antiparticle version.

It is also a slightly subtle matter what one calls a particle in the Standard Model. For example, an "electron" refers to spinor with both right and left chiral components. If an electron is produced thought a purely electromagnetic interaction, then it has no definite chirality. The same goes for the quarks. So according to the electromagnetic force or the Strong force, the electron is in fact a fundamental particle, but according to the Weak force, it is not. This subtlety really comes to down to which energy scale one is probing. If one is at relatively low energies (large distances), then the Weak force is irrelevant and the electron can be considered a fundamental particle. However, at higher energies when the Weak force becomes a significant contributor, then the left and right chiral electrons are the fundamental particles.

2.3.3 The Standard Model Lagrangian

Now that we have built the field multiplets, we are prepared to give a short overview of the form of the Lagrangian. There are gauge fields corresponding to each of the gauge symmetries. The gauge kinetic terms of the Lagrangian look like

$$\mathscr{L}_{\text{gauge}} = -\frac{1}{2}\text{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2}\text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}(F_{\mu\nu}F^{\mu\nu}), \tag{2.33}$$

$$\begin{aligned} G_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \\ W_{\mu\nu} &\equiv \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu], \\ F_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{2.34}$$

$$\begin{aligned} A_\mu &\equiv A^i_\mu \frac{\lambda_i}{2}, \\ W_\mu &\equiv W^i_\mu \frac{\tau_i}{2}, \end{aligned} \tag{2.35}$$

where the λ_i are the *Gell-Mann matrices*, which are the generators of the $SU(3)$ Lie group, and the τ_i are the Pauli spin matrices[11]. The reason for the new notation τ is to specify that these matrices act on field doublets and not spinors. Here the B and W fields do not correspond to the observable gauge bosons discussed in 2.5. This has to do with *electroweak unification*, which was a discovery that the observed weak force and the electromagnetic force were actually closely related. QED was known to be a very accurate description of electromagnetic interactions, and a Lagrangian describing weak interactions for *charged* gauge bosons had also been worked out where the form of the interaction terms was known. Working backwards from the Lagrangian terms given in equation 2.41 aiming to arrive at the interaction terms of QED and the charged weak interactions, it can be shown[12] that once the electroweak symmetry is broken, suitable redefinitions of the B and W fields lead to Lagrangian terms corresponding to the observable $Z_0, W\pm$ and photon fields with their respective mass terms and correct interaction terms. These field redefinitions take the form

$$\begin{aligned} W_\mu(x) &= \frac{1}{\sqrt{2}}[W_{1\mu}(x) - iW_{2\mu}(x)] \\ W^\dagger_\mu(x) &= \frac{1}{\sqrt{2}}[W_{1\mu}(x) + iW_{2\mu}(x)] \end{aligned} \tag{2.36}$$

$$\left.\begin{aligned} W_{3\mu}(x) &= \cos\theta_W Z_\mu + \sin\theta_W A_\mu(x) \\ B_\mu(x) &= -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x), \end{aligned}\right\} \tag{2.37}$$

where θ_W is known as the weak mixing angle. A remarkable consequence of electroweak unification was the prediction of an electrically *neutral* gauge boson, the Z_0 boson.

We introduce the matter fields using the method of section (1.2), where the covariant derivatives are defined by which gauge groups the fields transform under. We have leptons, quarks and the Higgs fields. The left chiral fermions and Higgs have respective covariant derivatives

$$\begin{aligned} D^\mu_{\text{quark}_L} &\equiv (\partial^\mu + i\frac{g_s}{2}\lambda_i A^{\mu i} + i\frac{g}{2}\tau_i W^{\mu i} + i\frac{Y}{2}g' B^\mu), \\ D^\mu_{\text{lepton}_L,\text{higgs}} &\equiv (\partial^\mu + i\frac{g}{2}\tau_i W^{\mu i} + i\frac{Y}{2}g' B^\mu), \end{aligned} \tag{2.38}$$

[11] i here runs over the number of basis vectors of the algebra again, so from 1-8 for the gluon fields and from 1-3 for the W fields.

[12] see [2]

where Y is the hypercharge and equals 1/6 for left chiral quarks, and 1/2 for left chiral leptons and the Higgs field. The right chiral fermions[13] have the following covariant derivatives:

$$D^{\mu}_{\text{lepton}_R} \equiv (\partial^{\mu} + ig'B^{\mu}), \tag{2.39}$$

With these operators in hand, we can form gauge invariant kinetic Lagrangian terms for the fermions and the complex scalar Higgs field of the form

$$\overline{\Psi}\not{D}\Psi \text{ (fermion terms)} \qquad [D^{\mu}H]^{\dagger}[D_{\mu}H] \text{ (Higgs term)} \tag{2.40}$$

where the capital Ψ indicates that these fields live in mutiplets, H is the Higgs doublet, and the covariant derivatives are defined according to equation 2.38 depending which type of field the operator is acting on. Now, we may think we have finished our gauge theory, as we now have a gauge invariant Lagrangian and interaction terms. However, we still have no masses, and we haven't considered a few other points. First of all, with the introduction of the Higgs Field, there are now other gauge invariant combinations of the fields that may be constructed, and we have no reason to exclude those. These are known as *Yukawa terms.* An example of such a term is given by

$$y^{ij}\overline{\Psi}_{Li}H\psi_{e_R j}. \tag{2.41}$$

Here,

$$\overline{\Psi}_{Li} \equiv \begin{pmatrix} (\overline{\psi}_L)_{e_i} \\ (\overline{\psi}_L)_{\nu_{e_i}} \end{pmatrix}$$
$$H \equiv \begin{pmatrix} H^{+} \\ H^{0} \end{pmatrix} \tag{2.42}$$

where $(\psi_L)_{e_i}$ stands for the left chiral charged lepton field of flavor i and similar for the neutrino field, ψ_{e_R} stands for the right chiral charged lepton field of flavor i, and i runs from 1-3 over the three lepton families. The y^{ij} are elements of what is known as a *Yukawa coupling matrix*, which are 3 x 3 complex matrices of coupling constants describing the strength of the various Yukawa interaction terms. There are Yukawa terms for quarks and leptons, and it is through these terms that the fermions obtain their mass. For reasons having to do with the definition of the Higgs field minimum, terms of the form found in 2.41 give mass only to up-type particles, i.e those particles residing in the top component of any $SU(2)$ doublet. For the leptons, this is fine, because only the charged leptons have mass, and they are found in the top component. For the quarks, however, a Yukawa term like

[13] No neutrinos here, remember.

2.41 is not sufficient because it gives only up-type quarks mass. In order to give down-type quarks mass, a term of the form

$$y^{mn}\overline{Q}_{Lm} i\tau_2 H^{\dagger T}\psi_{d_{Rn}} \equiv y^{mn}\overline{Q}_{Lm} \circ H^{\dagger T}\psi_{d_{Rn}}, \tag{2.43}$$

where Q is a doublet of left chiral quarks, and d_r is a right chiral down-type quark of generation n. The Pauli spin matrix is necessary in order to ensure gauge-invariance. The Yukawas are still not the only gauge invariant terms one may include in the Lagrangian. In fact, there an infinite number of such terms one could create. However, it turns out that inclusion of new, gauge-invariant terms in the Lagrangian ends with the Yukawas because of a different constraining principle known as *renormalizablility*, to be discussed in 2.5. There is also one other part of the Lagrangian, referred to as the *Higgs potential*, which is necessary for spontaneous symmetry breaking, and will be elaborated upon in next section.

2.4 Spontaneous Symmetry Breaking

As we have mentioned several times before, the Standard model with unbroken electroweak symmetry is a massless theory. This is due to a clash between the principle of gauge invariance and massive gauge bosons. Mass terms for the gauge bosons are not gauge invariant if one simply puts them into the Lagrangian explicitly. When the likes of Peter Higgs and competitors were working on this problem in the 1960's, they were not ready to simply abandon the powerful principle of gauge invariance, so they searched for a way to introduce these masses into the theory while still retaining a manifestly gauge-invariant Lagrangian. The method these great scientists developed is known as *spontaneous* symmetry breaking[14].

The method exploits the consequences of postulating that the vacuum, or state of lowest energy of the quantum system, is not unique. If this is true, then there must be some way to differentiate the degenerate vacuum states from one another. If we can find some quantity in the vacuum is gauge dependent, and then choose one of the vacuum states related to the others by the gauge transformation to be *the* ground state, then a gauge transformation will take us from our chosen vacuum state to a different state that is not the vacuum. So the vacuum will no longer be gauge invariant, and we will have broken the gauge symmetry! The ingenious choice of this "vacuum identifier" is the vacuum expectation value (VEV from now on) of a quantum field. Remember we want this VEV to break the gauge symmetry but *not* our Poincaré invariance. The only possibility is then to give a non-zero VEV to a Poincaré invariant scalar field,

$$\langle 0|\phi(x)|0\rangle = \text{some constant}. \tag{2.44}$$

[14]As opposed to *explicit* symmetry breaking, where one simply puts terms directly into the Lagrangian that are not invariant under the symmetry transformation

To see how this VEV allows us to identity a particular vacuum state that relates to the gauge symmetry, we consider the Higgs model for a simplified $U(1)$ gauge theory with only one gauge boson. The Lagrangian for this model is

$$\mathscr{L}(x)_{\text{HM}} = [D^\mu \phi(x)][D_\mu \phi(x)]^* - (\mu^2|\phi(x)|^2 + \lambda|\phi(x)|^4) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x). \qquad (2.45)$$

This Lagrangian is invariant under the $U(1)$ gauge transformation defined in equation 2.11. Note that we have introduced the new term $\mu^2|\phi(x)|^2 - \lambda|\phi(x)|^4 \equiv \mathscr{V}(\phi)$ into the Lagrangian. This is the *Higgs potential* mentioned at the end of the previous section. The Hamiltonian corresponding to the total energy of the classical field in just $\mathscr{L}(x)_{\text{HM}}$ with the sign of the Higgs potential reversed. Now the vacuum is defined to be the state of minimum energy, so taking a vacuum expectation value of our Hamiltonian should minimize it. The scalar kinetic terms vanish because taking the derivative of a constant of the form 2.41 is zero. The gauge kinetic terms play no role, so we must only minimize $V(\phi)$.

The coefficient λ must be positive so that the energy is bounded from below. The μ^2 term must also be negative, or else $\mathscr{V}$ is minimized for the trivial and more importantly *unique* value $\phi = 0$, which is exactly the scenario we wish to avoid. With this form of the potential we find that a whole circle of minima exist with values

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} e^{i\theta}, \qquad 0 \leq \theta \leq 2\pi. \qquad (2.46)$$

Now, we chose a particular one of these vacuum states and define it to be *the* vacuum, for convenience, the value with $\theta = 0$

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2} \equiv \frac{1}{\sqrt{2}}v \qquad (2.47)$$

We see that under a U(1) gauge transformation, this vacuum state is not invariant, as it takes the minimum to another one of the minima on the ring of minima, which is by definition now a *different* vacuum!

The next step is to redefine the complex scalar field in terms of two real fields and its VEV,

$$\phi(x) \equiv \frac{1}{\sqrt{2}}(v + \sigma(x) + i\eta(x)). \qquad (2.48)$$

We could proceed to plug this into the Lagrangian and rewrite the Lagrangian in terms of these new fields. However, it turns out that the η field is unphysical[15], and the generation of boson masses is easiest to understand if we rotate away this field by a special choice of gauge. This special gauge is known as the *unitary gauge*, and in this gauge ϕ reduces to the form

$$\phi(x)_{\text{unitary}} \equiv \frac{1}{\sqrt{2}}(v + \sigma(x)). \qquad (2.49)$$

[15] see [2]

The Lagrangian then can be rewritten in the form

$$\mathscr{L}_{\mathrm{HM}} = \mathscr{L}_0 + \mathscr{L}_I, \tag{2.50}$$

where the kinetic part looks like

$$\mathscr{L}_0 = \frac{1}{2}[\partial_\mu \sigma(x)][\partial^\mu \sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x) + \frac{1}{2}(qv)^2 A^\mu(x)A_\mu(x), \tag{2.51}$$

where q is the charge of the $U(1)$ gauge transformation and $A^\mu(x)$ is the gauge field, and the interaction part looks like

$$\mathscr{L}_I = -\lambda v \sigma^3(x) - \frac{1}{4}\lambda\sigma^4(x) + \frac{1}{2}q^2 A^\mu(x)A_\mu(x)[2v\sigma(x) + \sigma^2(x)]. \tag{2.52}$$

To see the significance of what has been achieved, we note that a massive scalar field obeys the free *Klein-Gordon* equation

$$(\Box + m^2)\varphi(x) = 0, \tag{2.53}$$

which comes from a Lagrangian of the form

$$\partial_\mu \varphi(x)\partial^\mu \varphi(x) - m^2\varphi^2(x), \tag{2.54}$$

and a massive vector fields obeys the *Proca* equation

$$(\Box + m^2)A^\mu(x) = 0, \tag{2.55}$$

which comes from a Lagrangian of the form

$$-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}m^2 A^\mu(x)A_\mu(x). \tag{2.56}$$

Now we see that the form of $\mathscr{L}_0$ corresponds exactly to a massive scalar field with mass $\sqrt{(2\lambda v^2)}$ and a massive vector boson of mass $|qv|$, while the $\mathscr{L}_I$ term corresponds to new interactions between our old gauge and fermion fields and the new scalar field. So we have accomplished our goal. We have given masses to the troublesome gauge bosons while still managing to maintain gauge invariance of the overall Lagrangian. The two degrees of freedom in the complex scalar field we started with have been converted into one degree of freedom for a real, massive scalar field and one extra degree of freedom for a gauge boson that is now massive.

Extending this idea to the Standard Model is not particularly difficult. It simply involves a Higgs doublet instead of just a single complex scalar field ϕ because we wish to break the $SU(2) \otimes U(1)$ symmetry instead of just a simple $U(1)$ symmetry. The subtlety arises in making sure that only the $W\pm$ and Z_0 gauge bosons obtain masses, while keeping the photon and the gluon massless. More formally, this means that the $SU(3)_c \otimes SU(2)_I \otimes$

$U(1)_Y$ is broken down into a $SU(3)_c \otimes U(1)_{em}$ symmetry. The Higgs doublet in the SM has the form

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \tag{2.57}$$

where the top component is positively charged and the lower component is neutral. By giving a vev only to the neutral, colorless Higgs field, we ensure that only the electroweak symmetry is broken, which results in mass terms for only the $W\pm$ and Z^0 bosons. [2]

2.5 Renormalization

The next topic in our Standard Model introduction is known as *renormalization*. The Standard model is plagued by divergent quantities that must be removed in order to use the theory to compute finite-valued observables. *Renormalization* refers to the process of removing these divergences in a consistent fashion. However, to understand where these divergences come from and what they mean, it is first necessary to introduce the idea of *perturbation theory* and its relation to the Standard Model.

2.5.1 Perturbation Theory

In quantum field theory, we want to be able to compute *transition probabilities*, probabilities for changing from an initial state to a different final state using matrix elements. These matrix elements should look like[16]

$$|\langle \text{final state}|S|\text{initial state}\rangle|^2 \equiv |\langle f|\Phi(\infty)\rangle|^2, \tag{2.58}$$

where S encodes how the interaction Hamiltonian changes the initial state over time. This gives us the probability that a physical process described by S takes the initial state into a certain final state. What does S look like? Recall that the dynamics of the theory are described by equation 2.3. We can rewrite this equation as an integral equation in the form

$$|\Phi(t)\rangle = |i\rangle + (-i)\int_{-\infty}^{t} dt_1 H_I(t_1)|\Phi(t_1)\rangle. \tag{2.59}$$

One can solve this equation *iteratively*, where an infinite number of iterations brings us to

$$|\Phi(\infty)\rangle = S|i\rangle, \tag{2.60}$$

where S takes the form [2]

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int \cdots \int d^4x_1 d^4x_2 \ldots d^4x_n \mathrm{T}\{\mathscr{H}_I(x_1)\mathscr{H}_I(x_2)\ldots\mathscr{H}_I(x_n)\}. \tag{2.61}$$

[16]We wish to evolve the initial state to $t = \infty$ since we want this state to describe the new state long after the physical process (usually a scattering process) is over and the initial state particles are far away and non-interacting before overlapping with the final state.

Here,

$$\mathscr{H}_I(x_n) \equiv -\mathscr{L}_I \tag{2.62}$$

is the interaction Hamiltonian density and T is the *time-ordered product*, which we can ignore for our purposes.

Now, we cannot evaluate this infinite sum exactly. However, if higher-order terms (terms corresponding to large n in the sum) quickly become smaller than the lower order terms, then we can arrive at a fairly accurate approximation to the exact answer by evaluating only the first few terms in the sum. If this is the case, then the theory is said to be a *perturbative theory*. In QFT, the interaction Lagrangians (and therefore Hamiltonians, by 2.62) are coupling dependent, as can be seen, for example, in equation 2.17. If these couplings are small enough, then higher order terms proportional to higher and higher powers of this small coupling become much less significant than the lower order terms, and the theory can be considered perturbative.

In practice, this is how calculations are done in QFT. The whole process is dependent on small coupling constants. QED and the theory of weak interactions are perturbative theories. QCD, on the other hand, is only perturbative at high energy scales[17] which makes QCD calculations at the low scale very difficult.

2.5.2 The Divergences

Now we are prepared to understand the origins of the divergences in the Standard Model. To low orders in the coupling, calculations using 2.61 are finite and well-behaved. However, once we begin moving to higher orders ($\sim O(4)$) these integrals begin to diverge, meaning that they are unbounded and infinitely large. If we consider that relevant probabilities in Quantum Field Theory are calculated using 2.58, these divergences lead to infinite probabilities that cannot be physically interpreted, and therefore must be removed.

The technique involves separating out the divergent portion of the integrals and redefining our constant parameters in the Lagrangian like the mass and couplings in such away that our divergences are "hidden". As an example, we again consider QED. If δm and δe are the lowest-order divergent integrals arising in 2.61, and one defines the physical, observable mass and charge of the electron to be

$$\begin{aligned} m_{\text{observable}} &= m_0 + \delta m, \\ e^2 &= e_0^2[1 - e_0^2 \delta e] \end{aligned} \tag{2.63}$$

where m_0 and e_0 are the Lagrangian parameters occurring in 2.18,[18] then one can show that all observable calculations depend only on the physical mass and charge and are finite

[17] This phenomenon is known as *asymptotic freedom*. All the coupling constants are functions of the energy scale at which they are measured at, as explained in section 2.5.4. The couplings of QED and of weak interactions grow with the energy scale, while QCD exhibits the opposite behavior.

[18] These are referred to as the *bare mass* and *bare charge*, respectively.

up to this order in perturbation theory. The divergent quantities only relate unphysical, bare parameters to physical ones, a relationship which itself is unobservable and therefore no longer of concern.

2.5.3 Renormalizable Theories

A theory is renormalizable if all the divergences occurring in the theory can be hidden away into constants in the Lagrangian as in equation 2.63. So now the question arises, how do we know that a theory is renormalizable? A derivation of the necessary and sufficient condition utilizes the idea of *Feynman diagrams*, which is an ingenious method invented by Richard Feynman to simplify the calculation of the perturbation theory integrals from 2.61 using a diagrammatic method. However, a discussion of this topic would take us too far astray, so we simply quote the result.[19]

Renormalizability Theorem: *A Quantum Field Theory is non-renormalizable if and only if it contains any interaction whose coupling constant g has negative dimension*

To put the theorem to use, we need to consider the dimension of the fields in a Quantum Field Theory, so that we may determine the dimension of the coupling constants. First we note that the action $S \equiv \int dt L(t)$ (here L is the Lagrangian, not the Lagrangian density) is a dimensionless quantity. The integration measure d^4x is of natural unit dimension -4, which from $S = \int d^4x \mathscr{L}(x)$ implies that every term in $\mathscr{L}$ has dimension 4. Using the fact that all mass parameters have dimension 1, we can deduce from the dimension of our Standard model fields from the Lagrangian. Scalar and gauge fields have dimension 1, while fermion fields have dimension 3/2. Now we are prepared to understand why the Standard model does not have any further terms beyond those described in section 2.3.3 (plus Higgs potential). Any new gauge-invariant term we might construct from the given fields would inevitably have dimension greater than 4, implying that its coupling must have negative dimension to assure that the term still has overall dimension 4. Therefore, by our renormalizability theorem, any such term would make our theory non-renormalizable!

2.5.4 Renormalization Group Equations

The method of renormalization discussed in section 2.5.2 is actually only one of several methods of handling divergences in Quantum Field Theories. The method we have demonstrated is known as the *on-shell* scheme. It hides divergences into the physically observed parameters like the mass and charge of the particles in the theory. There exist other schemes, such as the $\overline{MS}$ scheme, which introduce an unphysical mass scale μ into the theory. All physical predictions of the theory must be independent of this scale, a fact which leads to equations like $\frac{dX}{d\mu} = 0$ for certain observables X. The set of differential

[19] See [2], chapters 6 and 11.

equations derived from this scale invariance is known as the renormalization group. These equations can be solved perturbatively, and as a result all parameters in the theory become functions of energy. In this way, higher order corrections to observables take the form of *running couplings* instead of correction terms added to the lower order calculations. The remarkable thing about the renormalization group equations is that even if they are solved to a certain order in the couplings, the results also give information about higher order calculations without needing to calculate higher order perturbations.

2.6 Beyond the Standard Model

Now that we have more of less developed all the elements of the Standard Model, it is necessary to ask what is next. First we note that both the force of gravity and any possible Dark Matter candidate are explicitly absent. So we know that the Standard model cannot be a complete description of nature. Then the question arises of how to proceed to find a more complete theory. One approach is to postulate that the Standard model is not a fundamental theory of nature, but is in fact only valid up to a certain energy scale. This is the modern viewpoint, which has ushered in the age of *effective field theories*.

2.6.1 Effective Field Theories

An effective field theory is one that is assumed to be valid only up to a certain energy scale. Above this scale, it is expected that "new, unknown physics" sets in. Effective field theories can in fact be derived from the more fundamental theory that is valid up to higher energy scales. If one starts from the fundamental, renormalizable theory, one can show using the *path integral* approach that when one introduces the scale Λ, the Lagrangian can be rewritten in terms of a set of renormalizable terms plus a infinite sum of non-renormalizable terms that are proportional to[20]

$$\left(\frac{p}{\Lambda}\right)^n \tag{2.64}$$

for increasing powers of n, and where p is the external momentum of particles involved in the quantum process for which we are calculating observables. For large enough Λ and small enough momenta, these contributions are negligible, and we are left with a renormalizable effective theory valid up to the scale Λ, which is devoid of infinities. The infinities are still present in the more fundamental theory, and take the form of an infinite sum of non-renormalizable interaction terms suppressed at the low scale.

The Standard Model could be such an effective theory, where the renormalizable SM Lagrangian is just the low-energy approximation to more fundamental theory. This modern approach to field theory introduces some new philosophical issues, among which the

[20] See [6], chapter 12.

hierarchy problem has been especially influential in forming new ideas of what the Standard Model might be telling us about deeper physical theories.

2.6.2 The Hierarchy Problem and Supersymmetry

The hierarchy problem is based on a concept known as *naturalness*. To get a feel for what naturalness refers to, we again consider the first of the two equations in 2.63, this time in light of the effective field theory concept. The divergent part can be shown to take the form [2]

$$\delta m = \frac{3m_e\alpha}{4\pi}\ln\left(\frac{\Lambda}{m}\right) + O(\alpha^2), \tag{2.65}$$

where $\alpha = \frac{e}{4\pi}$. If the SM is considered an effective theory, then Λ here would correspond to the scale at which the SM breaks down and new physics sets in. A *natural* theory is one in which quantum corrections of this form are on the order of the observed mass. In the case of the electron mass, the correction is only logarithmically dependent on the scale, so even for $\Lambda \sim 1$ TeV, the correction to the electron mass is only $\approx 2.5\%$ of the observed electron mass of 511 keV. Therefore, QED is considered a natural theory. The same can be said for all the SM fermions including the colored quarks, as loops from strong QCD interactions also produce only logarithmic corrections.

Things are not so pretty for the Higgs mass. The corrections to the Higgs mass are proportional to $-\Lambda^2$ instead of $\ln\Lambda$. So somehow the bare Higgs mass and the correction term have to be *fine-tuned* extremely precisely so that they cancel in just the right way to arrive at the observed Higgs mass of $\approx$ 126 GeV. This instability of the Higgs mass to quantum corrections is known as the *hierarchy problem*. Now, although the hierarchy problem is concerning from a theoretical standpoint, it is not an observable prediction that could be used to rule out the SM (as we have noted before, the difference between the observed mass and the bare mass is not observable). Nonetheless, it seems very peculiar that all the other masses in the Standard model are somewhat stable up to very large energy scales, while the Higgs mass is very unstable. It could be the case that the new physics sets in at a relatively low scale, or there could exist some new symmetry that manages to stabilize the Higgs mass against these Λ^2 corrections.

This is where Supersymmetry comes in. The $\sim -\Lambda^2$ correction to the Higgs mass in the Standard Model comes from the Yukawa interactions in the Lagrangian, i.e interactions of the form $\overline{\psi}H\psi$, where the Higgs field couples to two fermions. If there existed some other scalar in the theory that coupled to the Higgs field in the form $\lambda|H|^2|S|^2$, where λ is the coupling, it turns out that this would contribute an order $+\Lambda^2$ correction to the Higgs mass. Notice that this is of opposite sign! So if the proportionality constants agree correctly, these contributions could cancel the quadratic divergences and eliminate the hierarchy problem. Supersymmetry (abbreviated SUSY) exploits this by introducing new symmetry transformations that transform bosons into fermions and vice versa. If such transformations are to be a symmetry of the Lagrangian, then there must exist new particles such that the transformation acting on the Standard Model particles can be cancelled by transformations acting on these new particles. These new super particles are known as *sparticles*, and in SUSY every Standard Model particle has one. We have now laid the framework for SUSY and proceed to develop the theory in more detail in the next chapter.

Chapter 3

Supersymmetry

This chapter is devoted to motivating the MSSM, or minimal supersymmetric model, which is the supersymmetric model made compatible with the Standard Model by adding the least amount of new sparticles. It is the simplest supersymmetric theory, and it is the model explored in this thesis.

3.0.1 SUSY Notation and Conventions

To begin with, we start with a brief summary of some SUSY conventions and new notations that are convenient for writing down supersymmetric theories. First of all, SUSY is usually written in terms of two-component *Weyl* spinors instead of the four-component Dirac spinors of the Standard Model. This is due to the fact that each particle with a unique set of quantum numbers is given a unique superpartner. We know that left and right chiral particles have different Weak quantum numbers, and so they must have different superpartners. Now we consider in more detail the notion of chirality. Recall that chirality depended on the eigenvalue of the spinor when operated on by the γ^5 matrix. Using the form of γ^5 given in section 0.1, we clearly see that the eigenvectors of this matrix have the form

$$\begin{pmatrix} \eta \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 \\ \chi \end{pmatrix}, \tag{3.1}$$

where both η and χ are two-component spinors. The first has γ^5 eigenvalue +1, and the second eigenvalue -1. The first is a *right chiral* spinor, and the second is a *left chiral* spinor. What really *defines* left and right chiral spinors, however, is that they transform differently under representations of the Lorentz Lie group, the group of Lorentz transformations, and that they have different weak isocharge. All the important information, i.e the Standard Model quantum numbers and Lorentz transformation properties, therefore, is summarized entirely in the two-component spinor, so we can just drop the zeros and continue to write things in terms of Weyl spinors. *From now on, we will use χ to always represent left chiral particles and η to represent right chiral particles.*

Since we wish to build Lorentz invariant Lagrangian terms from Weyl spinors, it is useful to note how these spinors transform under under the Lorentz group and what Lorentz invariants can be constructed from them. The left and right chiral spinors transform as

$$\begin{aligned} \eta \to \eta' &= (1 + \frac{i}{2}\vec{\epsilon} \cdot \vec{\sigma} - \frac{1}{2}\vec{\beta} \cdot \vec{\sigma})\eta \\ \chi \to \chi' &= (1 + \frac{i}{2}\vec{\epsilon} \cdot \vec{\sigma} + \frac{1}{2}\vec{\beta} \cdot \vec{\sigma})\eta, \end{aligned} \tag{3.2}$$

where $\vec{\epsilon}$ and $\vec{\sigma}$ are arbitrary, infinitesimal parameters. It can be shown[1] that under these transformations, some important Lorentz invariants are

$$\eta^\dagger \chi, \ \chi^\dagger \eta \tag{3.3a}$$

$$\chi^T(-i\sigma^2)\chi \equiv \chi \cdot \chi, \qquad \chi^\dagger i\sigma^2 \chi^{\dagger T} \equiv \overline{\chi} \cdot \overline{\chi}, \tag{3.3b}$$

$$\eta^T i\sigma^2 \eta \equiv \bar{\eta} \cdot \bar{\eta}, \qquad -\eta^\dagger i\sigma^2 \eta^{\dagger T} \equiv \eta \cdot \eta \tag{3.3c}$$

$$\chi^\dagger \bar{\sigma}^\mu i\partial_\mu \chi \tag{3.3d}$$

where we introduced the definition of a new spinor dot product for convenience.

3.0.2 The SUSY Multiplets

The first step to building supersymmetric theories is to consider the particle multiplets in our theory. Just like the gauge group multiplets, these supersymmetry multiplets are defined to be groups of particles or fields that mix under SUSY transformations. These transformations are generated by the SUSY generators or charges, in parallel to the generators of gauge transformations. There are two charges, Q and $Q^\dagger$, corresponding to what is known as N=1 supersymmetry.[2]. We want invariance under SUSY transformations to require adding new particles to the theory. Therefore, these charges should change bosons into fermions and vice versa, so that new sparticles are required if the theory is to be SUSY invariant. So

$$\mathrm{Q}|\text{boson}\rangle \to |\text{fermion}\rangle, \qquad \mathrm{Q}|\text{fermion}\rangle \to |\text{boson}\rangle, \tag{3.4}$$

and each multiplet therefore has both bosons and fermions. SUSY transformations must also commute with gauge transformations, since two symmetry transformations of the Lagrangian in any order should also be a symmetry transformation. This implies that all particles in a multiplet must have the same gauge quantum numbers, i.e transform in the same representation of the gauge groups. It can also be shown that each SUSY multiplet must have the same number of boson and fermion degrees of freedom.

[1] see [3], chapter 2

[2] There exist what are called *extended* supersymmetries where there are more charges, but these are necessarily non-chiral, and therefore ruled out phenomenologically because of the observed chirality of the weak force.

Using these criteria, we can build the basic multiplets we need to construct the MSSM. *Chiral* multiplets are composed of a single spin 1/2 Weyl fermion and its spin 0 complex scalar superpartner. The scalar sparticle is specified by taking the name of the fermion and adding an 's' in front, (i.e sfermion, squark, or more specifically, sup, sdown, selectron, etc...). *Vector* multiplets contain a spin 1 gauge boson and a spin 1/2 Weyl fermion. The fermionic superpartner is named by adding the suffic "ino" to the end of the SM gauge boson name (for example, the superpartner of the gluon in the *gluino*). The gauge superpartners in general are known as *gauginos*. With just these criteria, it is still ambiguous which multiplets quarks and leptons should be put into. To remove this ambiguity, it can be shown that a chiral theory for the gaugino, i.e one in which left and right-chiral components transform differently under the gauge groups, is forbidden. Therefore, the quarks and leptons must live in chiral multiplets, and our SM gauge bosons must live in vector multiplets.

However, we are not quite finished. The charges Q and $Q^\dagger$ obey a SUSY algebra, much like the generators of gauge transformations obey commutation rules (in SUSY, this algebra involves anticommutators as well as commutators). The transformations of the fields are determined entirely by the algebra these charges obey. It can be shown that *on-shell*, i.e when the Weyl fermions in the chiral multiplets obey the massless Weyl equations

$$i\bar{\sigma}_\mu \partial^\mu \chi = 0 \qquad i\sigma_\mu \partial^\mu \eta = 0, \tag{3.5}$$

and the vector fields in the gauge multiplets also obey their equations of motion, everything works nicely and no additional fields are needed in the multiplets. The algebra is said to *close on-shell*. But the algebra does not close *off-shell*. This is problematic in a Quantum Field Theory, where the fields are operators that are not necessarily constrained to obey the*classical* equations of motion. This complicates matters when defining how fields and Lagrangians transform under SUSY, because it turns out that the members of a multiplet only transform in the same way under SUSY $on-shell$, a result of the fact that the algebra determines how the fields transform. In order to fix this problem, additional *auxiliary* fields are needed to ensure that the algebra closes both on and off shell. It is also necessary that these fields propagate no on-shell degrees of freedom, so that we do not introduce any new, undesired dynamics. The way to make this true is to simply add a term to the Lagrangian in the form $F^\dagger F$ for the chiral multiplet Lagrangian and a term $\frac{1}{2}D^2$ to the vector multiplet Lagrangian so that the classical equations of motion for the fields are $F = D = 0$.

The auxiliary field for the chiral multiplets is conventionally called an F field, This field is defined to be a complex scalar, and its Lagrangian term forces it to have dimension 2. The auxiliary field textitD for the vector multiplet is a real scalar field, also with dimension 2[3].

[3] The complex/real nature of F and D has to do with ensuring that the multiplets have matching degrees of freedom for bosons and fermions.

3.0.3 The SUSY Transformations

Now we are ready to define how the fields in our theory transform under SUSY. We begin with the two simplest cases, where we develop the appropriate SUSY transformations that leave invariant the Lagrangians corresponding to a free chiral multiplet and a free vector multiplet. For the free chiral multiplet, the transformations have the form

$$\begin{aligned}\delta\phi &= \zeta\cdot\chi \\ \delta\chi &= -i(\partial_\mu\phi)\sigma^\mu i\sigma^2\zeta^* + F\zeta \\ \delta F &= -i\zeta^\dagger\bar{\sigma}^\mu\partial_\mu\chi.\end{aligned} \tag{3.6}$$

Here, the infinitesimal SUSY parameter ζ corresponding to a global, spacetime independent SUSY transformation[4] is seen to be a two-component spinor, defined by convention to be left chiral.

To motivate this form for the transformations, we note that we want the transformations first of all to turn scalars into something involving fermions and vice versa. This we see see from equation 3.6 is the case. We also want the transformations to have the same Lorentz transformation properties as the transforming field. A scalar is Lorentz invariant, and from 3.3, so is $\delta\phi$. It can also be shown that $\delta\chi$ transforms like a left chiral field. In addition, the transformation must have the same dimension as the transforming field. For a scalar field, this dimension is 1, and since a fermionic field is of dimension 3/2, ζ has dimension -1/2. $\partial_\mu\phi$ has dimension 0, so $\delta\chi$ has the same dimension as the ζ, which is fermionic, so this analysis holds up here as well. The auxiliary field is a scalar, so like the scalar field its transformation must be Lorentz invariant, which can be seen to be true using 3.3d with $\chi^\dagger$ replaced with $\zeta^\dagger$. It also must contain a fermion in order to preserve dimensions, since it is of dimension 2 and the SUSY parameter is of dimension -1/2. With these transformations, the simple Lagrangian

$$\mathscr{L} = \partial_\mu\phi\,\partial^\mu\phi^\dagger + \chi^\dagger i\bar{\sigma}^\mu\partial_\mu\chi + FF^\dagger \tag{3.7}$$

is invariant, and the SUSY algebra closes off-shell.

The simplest free gauge multiplet is described by the Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda^\dagger\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}D^2 + \xi D, \tag{3.8}$$

It is assumed that this an abelian theory, and in this case, the field D is gauge invariant, so we may add the last term, known as a *Fayet-Illiopoulos* term. This term is not present in non-abelian gauge theories because D is this case is not gauge invariant. We use λ to denote gauginos instead of χ because there is no distinguishing left chiral gauginos from right chiral gauginos. The SUSY transformations for these fields are slightly more complex because now the transformed field must obey all the same criteria as with the

[4]We only consider global SUSY transformations, as compared to gauge transformations in the Standard Model which are local. Local SUSY transformations lead to supergravity theories.

chiral multiplet, plus the additional constraint that it must transform in the same way under gauge transformations so that SUSY transformations commute with gauge transformations. Then, field transformations of that leave the Lagrangian invariant, close off shell, and obey the necessary constraints imposed by Lorentz invariance and gauge transformations have the form

$$\delta A^\mu = \zeta^\dagger \bar{\sigma}^\mu \lambda + \lambda^\dagger \bar{\sigma}^\mu \zeta \tag{3.9a}$$

$$\delta\lambda = \frac{i}{2} F_{\mu\nu}\sigma^\mu \bar{\sigma}^\nu \zeta + D\zeta \tag{3.9b}$$

$$\delta D = -i\zeta^\dagger \bar{\sigma}^\mu \partial_\mu \lambda + i(\partial_\mu \lambda)^\dagger \bar{\sigma}^\mu \zeta. \tag{3.9c}$$

The transformation for D is the same as the F field plus the hermitian conjugate because D is a real field.

Now of course, we would like to build a supersymmetric gauge theory with interactions between particles in chiral multiplets and those in vector multiplets. However, guessing the form of these Lagrangians and the possible interactions becomes quite complicated, even for the simplest case of combining an abelian multiplet with a single chiral multiplet. This is where the concept of *superspace* comes in. This new space is a mathematical extension of spacetime, which allows one to derive supersymmetric Lagrangians in easy and formulaic fashion.

3.1 Superspace

In the previous section, we proposed that the SUSY transformations took the form given in equations 3.6 and 3.9 and that these transformations left some basic Lagrangians invariant. However, this wasn't the most rigorous discussion. More rigorously, one can *derive* the SUSY transformations using the *algebra* obeyed by the charges. The process involves defining the charges as generators of coordinate translations in some space, and then using the algebra obeyed by the charges to find how exactly the coordinates transform. From there, one can write the charges as differential operators of fields depending on the coordinates in this space. The SUSY algebra complicates matters by mixing spacetime with fermionic components, since the SUSY charges themselves are fermionic. This can be seen by the key relation in the SUSY algebra,

$$\{Q_a, Q_b^\dagger\} = (\sigma^\mu)_{ab} P\mu. \tag{3.10}$$

So, in order to define the SUSY charges as generators of translations, one must extend spacetime to include fermionic components as well. Once the charges are written in differential operator form, one can easily determine how these more general fields transform under SUSY, and can find SUSY invariant components of these fields.

3.1.1 Coordinate Translations and the Algebra of the Charges

To get an idea of what we wish to accomplish, we consider a simple example showing how one may use the Poincaré algebra to write the generators of the Poincaré group as differential operators[5]. We start with the fact that the momentum operator is the generator of translations. A quantum field operator transforms under unitary transformations as

$$\phi(x) \to \phi(x') = U(a)\phi(x)U^{\dagger}(a), \tag{3.11}$$

where

$$U(a) \equiv e^{(ia^{\mu}P_{\mu})} \tag{3.12}$$

where P is the generator of translations in the Poincaré group. Writing $\phi(x)$ as

$$U(x)\phi(0)U^{\dagger}(x), \tag{3.13}$$

we can then write

$$\phi(x') = U(a)U(x)\phi(0)U^{\dagger}(x)U^{\dagger}(a), \tag{3.14}$$

or

$$U(x') = U(a)U(x). \tag{3.15}$$

Then, using the Baker-Campell-Hausdorff formula

$$e^{A}e^{B} = e^{\left(A+B+\frac{1}{2}[A,B]+\cdots\right)} \tag{3.16}$$

and setting the arguments of the exponentials on both sides of 3.31, we arrive at

$$i(a^{\mu}+x^{\mu})P_{\mu} - \frac{1}{2}a^{\nu}x^{\mu}[P_{\nu},P_{\mu}] + \cdots = ix'^{\mu}P_{\mu}. \tag{3.17}$$

Now, using that the fact that $[P_{\nu}, P_{\nu}] = 0$ in the Poincaré algebra, we see that $x'^{\mu} = a^{\mu}+x^{\mu}$. So, using only the algebra, we have seen how the coordinates transform under Poincaré spatial translations. Now we can write the momentum operator as a differential operator by treating ϕ as a continuous field now instead of a quantum operator. In this case, ϕ transforms as

$$\phi(x') = \phi(x+a) \approx \phi(x) + a^{\mu}\partial_{\mu}\phi(x) = e^{i\hat{P}_{\mu}a^{\mu}}\phi(x) \tag{3.18}$$

and expanding the exponential gives to first order

$$\phi(x) + a^{\mu}\partial_{\mu} = \phi(x) + ia^{\mu}\hat{P}_{\mu}\phi(x), \tag{3.19}$$

or

$$\hat{P}_{\mu} = -i\partial_{\mu}. \tag{3.20}$$

So we have achieved our goal of writing the generator as a differential operator using purely the algebra.

[5]This argument closely follows an exposition given in chapter 11 of [3]

3.2 Superspace Extension of Spacetime

Now we have an idea of how the algebra can be used to show how the coordinates transform and how this may be used to write the charges as differential operators. To do the same for SUSY, we need to extend spacetime to include fermionic coordinates so that the SUSY charges can act on fields that are functions of spacetime and these new coordinates to generate translations in this space. So we introduce two new coordinates, which we define to be two-component complex vectors where each component is a *Grassman* variable[6]. These new variables take the form[7]

$$\begin{aligned} \theta &= \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \\ \bar{\theta} &= \begin{pmatrix} \bar{\theta}^{\dot{1}} \\ \bar{\theta}^{\dot{2}} \end{pmatrix} \end{aligned} \tag{3.21}$$

These Grassman quantities are simply quantities that anticommute with one another. We also define these new variables to be left-chiral. With these new coordinate, we can extend our fields to be functions of these coordinates as well as spacetime. Such fields are called *superfields*. A superfield can be expanded in the Grassman coordinates in a very general fashion because of the anitcommuting property of the Grassman variables. For example, a scalar superfield depending only on θ and x can be expanded as

$$\check{F}(x,\theta) = \phi(x) + \theta \cdot \chi(x) + \frac{1}{2}\theta \cdot \theta F(x), \tag{3.22}$$

where χ is a left-chiral spinor and F and ϕ are scalars. We will place a check above all superfields to separate these fields from standard fields. Each term in $\check{F}$ is referred to as a *component* of the superfield. A general SUSY transformation in the differential operator representation acting on one of these superfields can be written as

$$e^{ia^\mu P_\mu + i\bar{\zeta}\cdot\bar{Q} + i\zeta\cdot Q}\check{S}(x,\theta,\bar{\theta}). \tag{3.23}$$

Using the method described in the previous section, one can use the algebra to read off the following coordinate transformations of superfields in superspace:

$$\begin{aligned} x' &= x + a + \frac{i}{2}\zeta\sigma^\mu\bar{\theta} - \frac{i}{2}\theta\sigma^\mu\bar{\zeta} \\ \theta' &= \theta + \zeta \\ \bar{\theta}' &= \bar{\theta} + \bar{\zeta}. \end{aligned} \tag{3.24}$$

[6]See appendix B for details involving the manipulation of Grassman coordinates.

[7]This is the first time we have used the so-called *van der Waerden* notation. See appendix A for the details of this new notation.

Then, one finds that the charges can be written as differential operators in a fairly complicated form that we do not write here, but which can be found in [3]. What we have accomplished by doing this is that now we can easily find how any superfield transforms under SUSY. Now the next step is to find special superfields that have components that are invariant under SUSY.

3.3 SUSY Invariants

The first special type of superfield is known as a *left chiral* superfield. These superfields are defined to obey the condition

$$\bar{D}_{\dot{a}}\Phi = 0, \tag{3.25}$$

where

$$\begin{aligned} D_{\dot{a}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{a}}} - \frac{i}{2}\theta^{c}(\sigma^{\mu})_{c\dot{a}}\partial_{\mu} \\ D_{a} &= \frac{\partial}{\partial\theta^{a}} - \frac{i}{2}(\sigma^{\mu})_{a\dot{b}}\bar{\theta}^{\dot{b}}\partial_{\mu}, \end{aligned} \tag{3.26}$$

where the second equation we list for completeness and later use. It can be shown that this constraint is SUSY invariant[8], meaning that if a superfield satisfies this constraint, a SUSY transformed version of the field also satisfies the constraint. These fields have the most general form

$$\check{\Phi} = \phi(x) - \frac{i}{2}\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\phi(x) - \frac{1}{16}\theta\cdot\theta\bar{\theta}\cdot\bar{\theta}\Box\phi(x) + \theta\cdot\chi(x) - \frac{i}{2}\theta\sigma^{\mu}\bar{\theta}\theta\cdot\partial_{\mu}\chi(x) + \frac{1}{2}F(x)\theta\cdot\theta, \tag{3.27}$$

where $\phi(x)$ is a scalar field and $\chi(x)$ *must be* a left chiral spinor field. Hence the name left chiral superfield. Here, $\theta\cdot\partial_{\mu}\chi(x) = \theta^{a}\partial_{\mu}\chi_{a}$. These types of fields have a few very important characteristics that make them very useful. First of all, a product of left chiral superfields is also left chiral. These fields also commute with one another. Thirdly, the term proportional to $\theta\cdot\theta$, known as the F term[9], can be shown to transform with a total derivative under SUSY, which means that these terms are invariant as far as the Lagrangian is concerned. Also note that these superfields have all the fields necessary to describe a chiral multiplet!

It can also be shown that the term proportional to $\theta\cdot\theta\bar{\theta}\cdot\bar{\theta}$, known as the D term, of *any, more general* superfield also transforms as a total derivative under SUSY, and so therefore is also an invariant Lagrangian density. In the case of the left chiral superfield, it turns out to *be* a total derivative, and therefore contributes nothing to the action.

[8]See [3]

[9]Warning! The F term is not the same as the auxiliary field F. The name comes from the fact that the F term of a left-chiral superfield is the auxiliary field F. However, this is not true for more general superfields. However, there shouldn't be any confusion, as in each circumstance F will be identified as an "F term" or as an auxiliary field. The same goes for D terms and D fields.

This exposition suggests that we may now construct SUSY invariant Lagrangians by simply taking the F and D terms of the appropriate superfields!

3.4 General SUSY Gauge Theories

We only cover the abelian case here, but generalizing to the non-abelian case is not too much trouble or different. To build supersymmetric gauge theories using the superspace method, one must define a supergauge transformation of a superfield. Following the impetus of gauge theories in the standard model, we define a gauge transformation of a charged left chiral superfield to be

$$\check{\Phi} \rightarrow e^{2iq\check{\Lambda}(x)}\check{\Phi}, \tag{3.28}$$

where q is the gauge charge and $\check{\Lambda}$ itself is also a chiral superfield so that it preserves the left chiral nature of the field it is acting on. However, now terms like

$$\check{\Phi}^\dagger\check{\Phi} \rightarrow \check{\Phi}^\dagger e^{-2iq(\check{\Lambda}^\dagger - \check{\Lambda})}\check{\Phi} \tag{3.29}$$

is not invariant, because we cannot take a left-chiral superfield to be real. So we define a new superfield known as a *vector superfield* that transforms as $V \rightarrow V - i(\Lambda - \Lambda^\dagger)$. Then the new term

$$\check{\Phi}^\dagger e^{2q\check{V}}\check{\Phi} \tag{3.30}$$

is supergauge invariant. Taking the D term of this combination of fields gives us all the gauge invariant interactions between chiral and vector multiplets. In a certain gauge, known as the Wess-Zumino gauge, the field V takes the form

$$\check{V} = \frac{1}{2}\theta\sigma^\mu\bar{\theta}A_\mu + \frac{1}{2\sqrt{2}}\theta\cdot\theta\bar{\theta}\cdot\bar{\lambda} - \frac{1}{8}\theta\cdot\theta\bar{\theta}\cdot\bar{\theta}D. \tag{3.31}$$

This field has all the fields needed to build a vector multiplet, hence it is called a vector superfield even though it is a Lorentz scalar. Under a supergauge transformation, both λ and D are gauge invariant, while A_μ transforms just like it transforms in a standard gauge theory from chapter 1. We also need the free kinetic terms of gauge fields, i.e a field strength tensor. We want this to be gauge invariant, and we see that if we could differentiate out the term proportional to A_μ in equation 3.31, this term would be gauge invariant. So with this motivation, we define

$$\check{F}_a \equiv \bar{D}\cdot\bar{D}D_a\check{V}, \tag{3.32}$$

where the three D operators get rid of the term proportional to A_μ as desired. Here, a is a spinor index. F is also left chiral. Now we want to use this to add a gauge invariant, SUSY invariant term to the Lagrangian, so this term must also be Lorentz invariant and therefore we must contract the a index. Because this term is left chiral, we also want the F term of the final superfield expression to obtain the SUSY invariant part.

So now we have kinetic terms for the gauge fields and the interactions of these fields with the chiral fields. However, we can also have interactions between the chiral fields themselves. To get gauge invariant interactions of this type, we must multiply left chiral fields together and keep only those terms where the gauge numbers of these combinations exactly cancel to ensure gauge invariance. We also cannot multiply together more than three such superfields, because any term with more than three superfields will result only in terms with dimension greater than 4, and are therefore nonrenormalizeable interactions. Interaction terms generated in this way are members of what is known as the *superpotential*, denoted by $W(\Phi)$.

We are now ready to construct a gauge invariant SUSY Lagrangian for an abelian gauge theory. The final Lagrangian looks like

$$\mathscr{L} = W(\check{\Phi})\big|_F + \frac{1}{4}(\check{F}^a \check{F}_a)\bigg|_F + \check{\Phi}^\dagger e^{2q\check{V}} \check{\Phi}\Big|_D , \tag{3.33}$$

where the notation indicates taking the F and D terms respectively. To construct the MSSM using this formalism, all one needs to do is to extend the formalism to include non-abelian gauge theories and then define relevant vector and left chiral superfields that contain all the SM particles and antiparticles.

3.5 The MSSM fields

For completeness, we include a table (table 3.1) of the SUSY fields in the MSSM and the notation used to describe them. There are some differences from table 2.1 that are important to explain. Firstly, there are no left and right chiral components specified, and instead we see bars above those fields that were right chiral in the SM. In table 3.1, all the fields in this table are understood to be left chiral. The barred fields denote fields corresponding to *antiparticles*. The reason is because, as we saw in section 3.3, we can easily construct SUSY invariant theories using only left chiral fields. Therefore, in order to replace the right chiral fields necessary in the theory, one must include the left chiral component of the field corresponding to the antiparticle. It can be shown[10] that a right chiral field can be replaced by a left chiral field using the relation

$$\eta_p = i\sigma^2 \chi_{\bar{p}}^{\dagger T}, \tag{3.34}$$

where the p subscript indicates a particle field, and the barred p indicates an antiparticle field. Using this relation, one may replace any right chiral field with a left chiral field to create theories written only in terms of left chiral fields, as is customary in the MSSM.

We also note that the gauge sector is written in terms of the B and W fields instead of the Z_0, $W\pm$ and photon fields. The latter are convenient in the SM because they are

[10] See [3], chapter 4.

the gauge bosons of definite mass after electroweak symmetry breaking. However, in the MSSM, the winos and binos can mix with the higgsinos as well and the other gauginos to form mass eigenstates, so one must define the superpartners to be those of the unrotated gauge fields.

Lastly, we see that there are *two* Higgs doublets, the reason for this will be elaborated on in section 3.7.

It is to be understood that each field in the figure has a superpartner, denoted by placing a tilde above the field. For example, the superpartners of the left chiral doublet of the first generation leptons is denoted $\tilde{L}_1$, and a gluino is denoted $\tilde{g}$. There are also auxiliary fields for each field in the table, but since these do not correspond to physically observable particles, we omit them.

Leptons	$L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix},$ $\bar{e}_1 = \bar{e},\ \bar{e}_2 = \bar{\mu},\ \bar{e}_3 = \bar{\tau}$
Quarks	$Q_1 = \begin{pmatrix} u \\ d \end{pmatrix} \quad Q_2 = \begin{pmatrix} c \\ s \end{pmatrix} \quad Q_3 = \begin{pmatrix} t \\ b \end{pmatrix},$ $\bar{u}_1 = \bar{u},\ \bar{u}_2 = \bar{c},\ \bar{u}_3 = \bar{t}$ $\bar{d}_1 = \bar{d},\ \bar{d}_2 = \bar{s},\ \bar{d}_3 = \bar{b}$ each of which can be red, green or blue
Gauge Sector	W_i, B, gluons
Higgs Sector	$H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \qquad H_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$

Table 3.1: SUSY Particles

3.6 SUSY Breaking

While SUSY offers a very nice way out of the hierarchy problem, it comes with new problems of its own. The most glaring issue with SUSY is phenomenological in nature. The masses of the superpartners in a SUSY multiplet have the same masses as their SM model partners, which is obviously ruled out phenomenologically because we would have observed them in experiments otherwise. This is a consequence of another relation in the SUSY algebra, namely

$$[Q_a, P_\mu] = 0, \tag{3.35}$$

i.e the SUSY charges commute with the momentum operator. This means that since Q operating on a state generates its superpartner, both a particle and its superpartner have the same eigenvalue of the operator $P^\mu P_\mu$, which by definition is m^2, the mass squared. So, SUSY must be broken in order to generate a phenomenologically viable theory.

This is fine, as we know that also our SM gauge symmetry needed to be broken in order to introduce masses for the gauge bosons. However, breaking SUSY introduces two new problems. The first is that in supersymmetry, the beautiful method of spontaneous symmetry breaking, i.e finding a quantity in the vacuum that is not invariant under the symmetry transformations, does not work. It can be shown that[11] no method of spontaneous SUSY breaking leads to a viable spectrum if one only considers the fields in the MSSM.

The second is that perfect supersymmetry is exactly what allows us to solve the hierarchy problem. So there is no guarantee that if we break SUSY, our quadratic divergences will still cancel. It turns out that certain SUSY-violating terms can in fact be added that still preserve this cancelling effect. These types of terms are known as *soft supersymmetry breaking terms*. The most general soft SUSY-breaking terms that may be added to supersymmetric Lagrangians take the form

$$\mathscr{L}_{\text{soft}} = -\left(\frac{1}{2}M_i\lambda^i \cdot \lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j\right) + h.c - (m^2)^i_j\phi^{j*}\phi_i, \tag{3.36}$$

where the index i attached to the M matrices runs from 1-3, ensuring there is one such gaugino soft mass term for the gauge group superpartners the bino, wino and gluino. Every term in the above soft Lagrangian must be gauge invariant, and also must respect something called *R-Parity* symmetry. R-parity is an extra symmetry that must be added to the MSSM to guarantee lepton number conservation, a well-observed experimental fact. The consequence of such a theory is that all interactions involving sparticles must contain at least two such sparticles. This fact has very important consequences for Dark Matter phenomenology, as we will explain later on. We see that gaugino and scalar mass terms are permitted, which is just what we need to make sure all our superpartners do not have degenerate masses with the Standard Model particles. The a terms that describe

[11] See [7] for details.

interactions between three scalars are known as *trilinear couplings*, which will see play an important role in the research of this thesis.

The soft Lagrangian for the MSSM looks like

$$\begin{aligned}\mathscr{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2}(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + \text{c.c}) \\ - (\tilde{\bar{u}}\mathbf{a_u}\tilde{Q}H_u - \tilde{\bar{d}}\mathbf{a_d}\tilde{Q}H_d - \tilde{\bar{e}}\mathbf{a_e}\tilde{L}H_d + \text{c.c}) \\ - \tilde{Q}^\dagger\mathbf{m_Q^2}\tilde{Q} - \tilde{L}^\dagger\mathbf{m_L^2}\tilde{L} - \tilde{\bar{u}}\mathbf{m_{\bar{u}}^2}\tilde{\bar{u}}^\dagger - \tilde{\bar{d}}\mathbf{m_{\bar{d}}^2}\tilde{\bar{d}}^\dagger - \tilde{\bar{e}}\mathbf{m_{\bar{e}}^2}\tilde{\bar{e}}^\dagger \\ - m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + (bH_u^T i\tau^2 H_d + \text{h.c}),\end{aligned} \quad (3.37)$$

where quark and lepton family indices have been suppressed. The matrices $\mathbf{a}$ and $\mathbf{m}$ are 3×3 complex matrices in family space, with the restriction that the $\mathbf{m}$ matrices must be hermitian so that the Lagrangian is real.

The most promising way to remedy the two problems outlined above is to introduce new fields that are only resolvable at very high energy scales. These fields are said to be part of a *hidden sector*. SUSY is spontaneously broken in this sector, and the result of this breaking is communicated in some way to the *visible sector*. The hidden-sector SUSY breaking must generate soft mass terms in the visible-sector Lagrangian, these terms being elements of the most general form given in equation 3.36. There are various models for the hidden sector and the method of communication to the visible sector. This thesis investigates the implications of a SUSY breaking model known as *gaugino mediation*, or g̃MSB.

3.7 Higgs Sector in the MSSM

There is one other aspect of the MSSM relevant to the research of this thesis that must be elaborated on before we continue on to Gaugino Mediation. In the Standard model, Yukawa-type interactions generate masses for all the massive particles in the theory. As discussed in section 2.3.3, we need both the Higgs doublet H and its complex conjugate $H^\dagger$ to ensure that all the relevant particles obtain masses. However, in SUSY, we generate interactions between chiral multiplets, which themselves correspond to Yukawa-like interactions, by taking the F term of the superpotential. This superpotential must be a product of left chiral superfields only so that the F term is SUSY invariant, and the complex conjugate of a left chiral superfield is *not* left chiral. So we cannot generate all the necessary Yukawa's using the hermitian conjugate of a single Higgs doublet in SUSY. To step around this, we must introduce a second Higgs doublet with opposite hypercharge to obtain all the necessary masses. So SUSY has two Higgs doublets, denoted H_u and H_d for up-type and down-type Higgs doublets, plus the superpartner *higgsinos* and auxiliary fields.

This means that we have eight scalar degrees of freedom. After spontaneous electroweak symmetry breaking, three of these degrees of freedom must become the masses of the $W\pm$ and Z_0 bosons, leaving 5 degrees of freedom, which become 5 Higgs particles. Of the five mass eigenstates in the Higgs sector, 3 become neutral Higgs particles of different mass, denoted h_0, H_0 and A_0. The other two Higgs particles, denoted $H^{\pm}$, carry opposite electric charge and have degenerate masses. It is theoretically predicted that the lightest of the five is a neutral Higgs particle, denoted h_0, which fits the criteria for the already discovered Higgs boson in the Standard Model.

Chapter 4

Gaugino Mediated SUSY Breaking

4.1 The Framework

Gaugino Meditation[1] is an extra-dimensional model for SUSY breaking. The model postulates that there exist more than four dimensions, with the space consisting of all D dimensions being referred to from now on as the *bulk*. Certain fields are allowed to propagate in the bulk, while others are constrained to 4d planes known as "branes" in the higher dimensional manifold. A Lagrangian describing such a theory with D spacetime dimensions would take the form

$$\mathscr{L}_{\text{higher-dimensional}} = \mathscr{L}_{\text{bulk}}(\Phi(x,y)) + \sum_j \delta^{D-4}(y-y_j)\mathscr{L}_j(\Phi(x,y_j),\phi_j(x)), \tag{4.1}$$

where x gives the coordinates for the 4d branes, y are coordinates for the $4-D$ dimensions, Φ is any field allowed to propagate in all dimensions, j runs over the branes and ϕ_j is a field localized to the j^{th} brane. In the version of $\tilde{g}$MSB explored in this thesis[2], the bulk fields are the fields in the Higgs and gauge supermultiplets. All other MSSM fields are constrained to the "visible sector" brane, spatially separated by a distance r from the "hidden sector" branes to which the SUSY breaking fields are constrained. This is visualized in figure 4.1 below. The distance r determines the size of the extra dimensions and the energy scale needed to resolve them. This scale is known as the *compactification scale*, denoted μ_c.

It turns out that the only way to couple the gauge fields to the hidden-sector fields in $\tilde{g}$MSB is through non-renormalizable interactions. Therefore, we must assume that the theory is an effective theory valid up to some more fundamental scale. We assume this to be the Plank scale ($\sim 1.22 \times 10^{19}$ GeV), as this is the scale where gravitational effects

[1] The name Gaugino meditation stems from an earlier, slightly different version of the model where only gauge fields were allowed to propagate in the bulk, which meant that all soft mass terms were derived from loops involving gauginos. In our model, Higgs fields are also allowed to propagate in the bulk, so the name is a bit of a misnomer.

[2] This model was developed in [8].

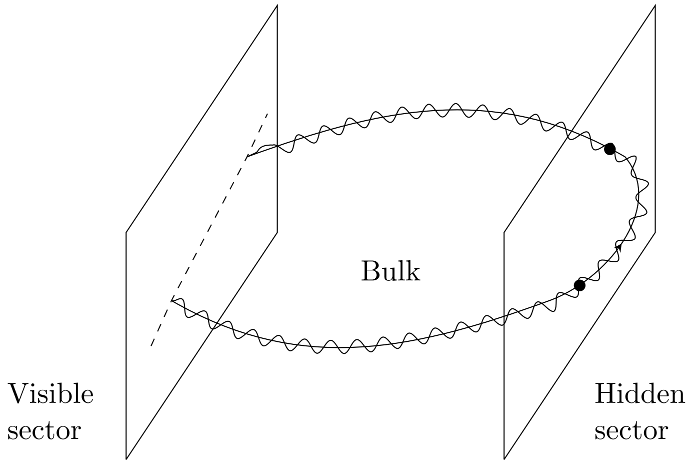

Figure 4.1: Gauginos propagating between the hidden and visible sectors. The solid black dots in the hidden sector are "mass insertions", and correspond to diagrams like the one given in figure 4.3.

become of similar strength to those of the SM forces. Between the fundamental scale and μ_c the theory is a D-dimensional theory. Below μ_c, the theory must match onto a 4d theory effective theory valid up the scale μ_c, in our case the MSSM plus soft mass terms.

It can be shown that direct contact interactions between the visible and hidden-sector fields are suppressed by a *Yukawa factor* e^{-Mr}, where r is the spatial separation between the branes and M is the mass of the state involved. If we assume very roughly that the masses in the theory are on the order of the fundamental scale, i.e $M \sim$ Plank scale, and we choose the separation to be large in units of the mass scale, these tree-level contact terms will be strongly suppressed. This is convenient because it automatically suppresses possible *flavor changing neutral currents* (FCNC's), which are essentially tree-level interaction terms in the Lagrangian that allow for the flavor of a fermion to change without its charge changing. As we will discuss in the next chapter, the Standard Model predicts that no such currents exist to tree level and should therefore be strongly suppressed. This has been strongly experimentally verified for Kaons[3], which are mesons with one strange-flavored quark and one down-flavored quark. However, the soft terms given in equation 3.36 introduce new interactions that contribute to flavor violating currents, and are therefore strictly constrained by experiment to be small. An example of such a new type of contribution in the Kaon system is given in 4.2 below. (This is an example of a Feynman diagram, where each vertex corresponds to an interaction term in the Lagrangian.)

To see how these types of vertices are suppressed in Gaugino mediation, we first note that the cross in diagram 4.2 corresponds to a term of the form $(m^2)^i_j \phi^{j*}\phi_i$ in the soft

[3] See [7], chapter 6.

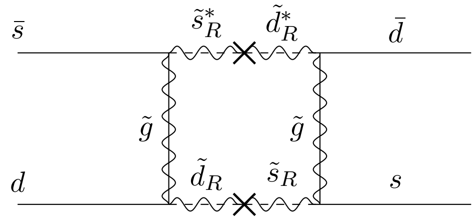

Figure 4.2: An example of a dangerous flavor changing neutral current in Kaons introduced with the addition of new SUSY particles and vertices [7].

Lagrangian. Such a term could arise from a SUSY invariant Lagrangian term like

$$\frac{1}{M^2}\int d^2\theta d^2\bar{\theta}\check{\Phi}^\dagger_{\text{hidden}}\check{\Phi}_{\text{hidden}}\check{\Phi}^\dagger_{\text{visible}}\check{\Phi}_{\text{visible}}, \tag{4.2}$$

where all four fields in the equation are superfields. The reason this is integrated over the given metric is because when SUSY is spontaneously broken, the scalars that obtain VEVs must necessarily result in a vev for the auxiliary field F in the chiral multiplet as well. Therefore, if we consider equation B.10 and the form of a chiral superfield 3.27, and we take the scalar field component from each of the visible superfields, we see that integrating over $d^2\theta d^2\bar{\theta}$ will give a term (among others)

$$\sim \frac{|\langle F\rangle|^2}{M^2}\phi^\dagger\phi \tag{4.3}$$

Here, $\langle F\rangle$ denotes the vev of the F field. The $1/M^2$ term can be understood in terms of dimensions. The F field has dimension 2, so F^2 has dimension 4, whereas a scalar mass term must have dimension 2. If the two scalars are not of the same particle type, such a mass term cannot be interpreted as a mass, and therefore is an interaction like the one denoted by the cross. However, this is a direct contact term between hidden and visible sector fields, so it is suppressed by the Yukawa exponential term. All such scalar soft mass terms are suppressed in this version of Gaugino Mediation.

Now, although these scalar soft mass terms are zero at the high scale, this is at tree level. Scalar soft mass terms can still arise from loops of the form given in figure 4.3. This means soft scalar mass terms are not necessarily small when they are run using the renormalization group equations down to lower energy scales. The question is then if the running of the couplings could reintroduce dangerous flavor violating interactions in the soft Lagrangian. The answer is no, because all scalar soft masses in $\tilde{g}$MSB are generated through interactions with Gauginos. Gauge interactions do not mix families, so no off-diagonal terms can be generated in matrix $(m^2)^i_j\phi^{j*}\phi_i$, keeping in mind that these indices run over family space.

It also be shown that the couplings of the gauge and Higgs multiplet fields are suppressed by a factor of $1/(Mr)^{D-4}$. This means that if one wants to generate tree level soft mass

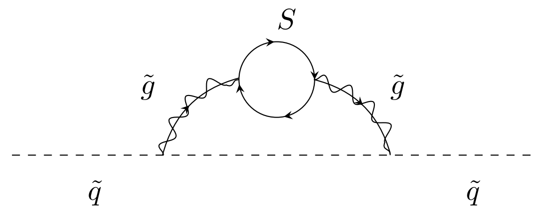

Figure 4.3: Loop diagram, MSSM sector

terms for the gauginos and the higgsinos, then r cannot be too large in units of the mass scale M, or else these terms will also be very suppressed and no soft terms will be generated. However, because contact interactions are *exponentially* suppressed, there exist regions where the contact terms between hidden and visible sector fields are sufficiently suppressed while soft-mass terms generated by Higgs and gauge supermultiplet fields are still relatively large.

It was also originally believe that no trilinear soft terms could occur in Gaugino Meditation. From equation 3.37, we see that the relevant allowed terms from the MSSM soft Lagrangian are

$$\tilde{\bar{u}}\mathbf{a_u}\tilde{Q}H_u - \tilde{\bar{d}}\mathbf{a_d}\tilde{Q}H_d - \tilde{\bar{e}}\mathbf{a_e}\tilde{L}H_d + \text{c.c.} \tag{4.4}$$

In order to generate such terms, squarks and sleptons would have to come into direct contact with the SUSY breaking fields, which is forbidden in Gaugino mediation. However, there is a trick to recover such terms, which we discuss in the following section.

4.2 Trilinear Soft Terms

It turns out that although no tree-level trilinear terms can be generated by directly coupling MSSM sfermions to the SUSY breaking fields, these can be generated by solving for the F term from the Higgs up-type supermultiplet. What is meant by "solve" is to apply the Euler-Lagrange equation to the Lagrangian, differentiating with respect to the auxiliary field in the Higgs up type multiplet. Because the auxiliary fields have no kinetic components in the Lagrangian, this process will give us the "unphysical" F fields in terms of the physical fields in our Lagrangian.

We begin by noting that to properly break SUSY, there must exist a field configuration such that the SUSY Lagrangian scalar potential is minimized when the auxiliary fields in the theory obtain non-zero VEVs[4]. Now, we want to break SUSY in the hidden sector, so we must give VEVs to some fields living in the hidden sector. The exact form of the fields in the hidden sector needed to properly break SUSY is generally very complicated, but for

[4]See [3], chapter 14.

our purposes giving a vev to the F term[5] of a hidden gauge singlet[6] chiral multiplet serves the purpose.

So the goal is to find all terms in the Lagrangian depending on the Higgs up-type auxiliary F field, denoted F_{H_u}, apply the Euler-Lagrange equation to these terms, eliminate the F field and see if this produces trilinear couplings like those in equation 4.4. First we find all the terms in the Lagrangian containing F_{H_u}. Since $\check{S}$ is a gauge singlet, it can couple to the Higgs up-type superfield like $\check{S}\check{H}_u^\dagger\check{H}_u$ + c.c. Now, this is a product general superfields, so to find contributions to the Lagrangian, we need the D term, i.e any term proportional to $\theta\cdot\theta\bar{\theta}\cdot\bar{\theta}$. Using equation 3.27, we find the following terms:

$$\text{Relevant contributions from } \check{S}\check{H}_u^\dagger\check{H}_u + \text{h.c} \Rightarrow$$
$$\phi_s F_{H_u}^\dagger F_{H_u} + F_s F_{H_u}^\dagger H_u - F_{H_u}^\dagger \chi_s\cdot\tilde{\chi}_{H_u} + \text{c.c}, \quad (4.5)$$

where the subscripts tell you which superfield the components came from, and the tilde indicates that this field is a superpartner of a SM field. There are also contributions from the MSSM superpotential, which has two relevant terms of the form

$$y_u^{ij}\check{u}_i\check{Q}_j^T i\tau_2\check{H}_u + \mu\check{H}_u^T i\tau_2\check{H}_d + \text{c.c}, \quad (4.6)$$

where the y_u matrix is just the up-type Yukawa matrix from the SM. Here, Q is the superfield containing the left handed quark doublets and their superpartners, and u is the superfield containing the left-handed antiquarks of up-type and their superpartners. This is a superpotential term, so we are interested in taking the F term. The relevant contributions are

$$\text{Superpotential contributions} \Rightarrow y_u^{ij}(\tilde{\phi}_u)_i\tilde{Q}_j^T i\tau_2 F_{H_u} + \mu F_{H_u}^T i\tau_2 H_d + \text{c.c.} \quad (4.7)$$

There is also one more relevant contribution from the term of the form found in equation 3.30, which describes the interactions between the gauge supermultiplets and the Higgs multiplets. So, we want

$$\check{H}_u^\dagger e^{\left(Yg'\check{B}+g\check{W}^i\tau_i\right)}\check{H}_u\Big|_D, \quad (4.8)$$

where Y is the hypercharge of the up-type Higgs field and $\check{B}$ and $\check{W}^i$ are the vector superfields containing the B and W bosons. Although this looks very complicated, it turns out to be very simple if we consider the form of a general vector superfield in the Wess-Zumino gauge given in 3.31. We want only terms which have F_{H_u}, which is already proportional $\theta\cdot\theta$, and no term in a vector superfield is proportional to $\bar{\theta}\cdot\bar{\theta}$, so the only contribution we get is from the zeroth order term in the expansion of the exponential, i.e from the D term of $\check{H}_u^\dagger\check{H}_u$. This gives only one relevant contribution, the obvious $F_{H_u}^\dagger F_{H_u}$ term.

[5]See [9] for a use of this approach.
[6]A singlet is a field that does not transform in any gauge group.

Now we have all the relevant terms in the Lagrangian, which we summarize below:

$$\mathscr{L}_{\text{relevant}} = (\phi_s F^\dagger_{H_u} F_{H_u} + F_s F^\dagger_{H_u} H_u - F^\dagger_{H_u} \chi_s \cdot \tilde{\chi}_{H_u} + \text{c.c}) \\ + (y_u^{ij} (\tilde{\phi}_u)_i \tilde{Q}_j^T i\tau_2 F_{H_u} + \mu F^T_{H_u} i\tau_2 H_d + \text{c.c}) + F^\dagger_{H_u} F_{H_u}. \quad (4.9)$$

Differentiating with repect to F_{H_u} gives

$$\frac{\partial \mathscr{L}}{\partial F_{H_u}} = \phi_s F^\dagger_{H_u} + y_u^{ij} (\tilde{\phi}_u)_i \tilde{Q}_j^T i\tau_2 + \mu i\tau_2 H_d + F^\dagger_{H_u} + \phi_s^* F^\dagger_{H_u} + F_s^* H_u^\dagger - \bar{\chi}_s \cdot \bar{\tilde{\chi}}_{H_u} = 0, \quad (4.10)$$

which we can easily solve for $F^\dagger_{H_u}$ to give

$$F^\dagger_{H_u} = \frac{-(\mu H_d^T i\tau_2 + y_{ij} \tilde{u}_i \tilde{Q}_j^T i\tau_2 + F_s^* H_u^\dagger - \bar{\chi}_s \cdot \bar{\tilde{\chi}}_{H_u})}{(1 + 2Re(\phi_s))}. \quad (4.11)$$

Taking the complex conjugate gives

$$F_{H_u} = \frac{(\mu^* i\tau_2 H_d - y^\dagger_{ij} \tilde{u}^\dagger_i i\tau_2 \tilde{Q}_j^{\dagger T} + F_s H_u + \chi_s \cdot \tilde{\chi}_{H_u})}{(1 + 2Re(\phi_s))}. \quad (4.12)$$

The denominator can be expanded, noting that ϕ_s is not of the correct mass dimension and must be divided by the mass scale, making the term small enough to justify expansion. Inserting this expression for $F^\dagger_{H_u}$ into equation 4.11 gives rise to nice trilinear term in $\mathcal{L}$ of the form

$$\sim \frac{\langle F_s \rangle}{M} y_{ij} \tilde{u}_i \tilde{Q}_j H_u \quad (4.13)$$

when we give the auxiliary field F_s a vev. So now we see that tree level trilinear couplings are obtainable in Gaugino Mediation.

These trilinears could pose a problem in the MSSM, however. Off-diagonal trilinear terms also mix families, which gives dangerous contributions to FCNC diagrams. However, if the trilinear couplings are proportional to the Yukawa matrices, as we see they are from equation 4.11, then these mixing coefficients vanish. This can be seen in light of the Standard Model. It turns out that in order to do perturbation theory in the Feynman diagram approach, one needs to use fields with definite mass . We know from section 2.3.3 that masses arise from Yukawa terms. However, as we mentioned, these are general, 3×3 complex matrices. In order to find the mass eigenstates, one must rotate the Yukawas into *diagonal* form. This rotates the fields, and these new rotated fields are the mass eigenstates and the fields needed for perturbation theory. This leads to field redefinitions in the entire Lagrangian. In the MSSM, the same Yukawas appear in the superpotential, and must be rotated in exactly the same way, the only difference being that this rotation also introduces a redefinition of the superpartner sfermion states as well. These field redefinitions carry over into the soft Lagrangian, and because our trilinears couple the same types of terms as the superpotential and are proportional to the Yukawas, these trilinears are automatically diagonalized as well!

Chapter 5

Parameter Space Investigation

We now have all the theoretical background needed to begin to explore the Gaugino Meditation model. We wish to identify relevant free parameters in the model and determine regions in the parameter space that are realistic considering modern phenomenological constraints. We begin by determining the relevant free parameters we wish to vary, and then explain the constraints we wish to implement on the parameter space. We then elaborate a little bit on the public computer codes `SPheno` [10], used to produce SUSY particle mass spectra, and `SmodelS` [11] used to constrain these spectra using LHC limits. We then start implementing the constraints to try and arrive at regions in parameter space that are allowed as of modern experiment, where the model could still be a valid description of nature.

5.1 The cMSSM in Gaugino Mediation

The soft Lagrangian in its most general form introduces 105 new parameters into the theory, making it very difficult to investigate or constrain the MSSM parameter space. The cMSSM, or *constrained Minimal Supersymmetric Model*, is a framework of assumptions that greatly reduces the number of parameters in the theory. The guiding principle is flavor violation. As discussed before, off-diagonal terms in the scalar mass-squared soft matrices and soft trilinear coupling matrices are constrained to be very small by experiments relating to FCNC's. As we have also shown, these problems do not exist if the scalar mass-squared matrices are diagonal and the trilinears are proportional to the Yukawa matrices. One can go one step further and assume that the scalar mass squared matrices are proportional to the unit matrix. The final step is not entirely well motivated by physical principles, but is based on the fact that at a certain energy scale, known as the *GUT* (Grand Unified Theory) scale, all the gauge couplings in the MSSM run to the same value. This scale is on the order of $\sim 2 \times 10^{16}$ GeV. Based on this idea, the cMSSM assumes that all the proportionality factors are equal at some high energy scale, i.e the matrices in 3.37 obey the following relations

$$\mathbf{a_u} = A_0\mathbf{y_u}, \quad \mathbf{a_d} = A_0\mathbf{y_d}, \quad \mathbf{a_e} = A_0\mathbf{y_e}$$
$$\mathbf{m^2_{\bar{u}}} = \mathbf{m^2_{\bar{d}}} = \mathbf{m^2_{\bar{e}}} = \mathbf{m^2_Q} = \mathbf{m^2_L} \equiv m_0\mathbf{1}. \tag{5.1}$$

It is also assumed that all three gaugino soft masses obey

$$M_1 = M_2 = M_3 \equiv m_{1/2}, \tag{5.2}$$

and that the b parameter is proportional to μ,

$$b = B_0\mu. \tag{5.3}$$

However, the b parameter in equation 3.37 can be replaced by $\tan\beta \equiv \frac{v_u}{v_d}$, defined to be the ratio of the VEVs acquired by the two neutral Higgs fields when the electroweak symmetry is broken. This is due to constraints from EWSB. If the MSSM scalar potential is to have a minimum where the neutral Higgs fields obtain VEVs, it can be shown [7] that the following relations must hold:

$$m^2_{H_u} + |\mu|^2 - b\cot\beta - (m^2_Z/2)\cos 2\beta = 0$$
$$m^2_{H_d} + |\mu|^2 - b\tan\beta - +(m^2_Z/2)\cos 2\beta = 0. \tag{5.4}$$

These allow us to exchange both μ and b for $\tan\beta$. The sign of μ is still undetermined by these formulas, so it must be chosen in order to completely define the Lagrangian. We choose positive μ in all our parameter scans. These five parameters technically fully define the cMSSM, as in the basic cMSSM, the Higgs soft masses are also assumed to be equal to m_0 at the high scale. However, our model of Gaugino mediation introduces a new twist.

The method of parameter space investigation in this thesis involves inputing values for the free parameters at the high scale, which we take to be the GUT scale. This scale is assumed to be on the order of the compactification scale, so there exist only Higgs soft masses, trilinear couplings and gaugino soft masses. This means that as an input parameter, m_0 is always set to zero. However, it is certainly not justifiable to set the Higgs soft masses equal to zero as well, so $m^2_{H_u}$ and $m^2_{H_d}$ are regarded as free parameters. This leaves finally 6 free parameters we must investigate. In summary, these are $\tan\beta$, A_0, $m_{1/2}$, $m^2_{H_u}$, $m^2_{H_d}$ and the sign of b.

5.2 The Constraints

We implement several experimental constraints on the mass spectra produced by `SPheno`. The primary constraint in this thesis is the experimentally observed Higgs boson mass, experimentally determined to be 125.7 ± 0.4 GeV [12] by the LHC. The Higgs mass calculations in `SPheno` are done up to two-loop, and the theoretical uncertainty due to the remaining higher loop corrections is estimated to be on the order of $\sim$ 3 GeV in the cMSSM [13]. Therefore, we consider all points with a lightest Higgs mass within the limits $122.7 < m_{h_0} < 128.7$ GeV to be allowed points.

`SModelS` implements *upper limit constraints* (ULC's) determined from the CMS and ATLAS data from run 1 at the LHC. These constraints place experimental upper limits on the theoretically calculated probability that certain processes will occur. If a model predicts that a process will occur more frequently than the ULC's, these parameter points can be considered to be ruled out experimentally, up to 95% confidence level. More specifically, `SModelS` constrains what is known as the *cross-section times branching ratio*, denoted $\sigma \times Br$. The cross-section determines how likely it is for specific particles to be produced in a reaction given the initial particles. In the cases tested by `SModelS`, the cross-sections describe decay processes where SUSY particles are initially produced by a proton proton collision, and the products decay into various observable lighter particles. All the SUSY particles can decay by means of various decay channels, and the branching ratio weights these channels relative to one another according to how likely it is for a particle to decay through each of the possible channels.

We also analyze constraints on the FCNC's discussed in section 4.1. Here is a useful place to elaborate on these types of interactions and to understand why they are forbidden in the Standard Model. The rule is a consequence of electroweak unification. In section 2.3.3, we discussed rotating the W and B fields into the observable Z_0, A and $W\pm$ fields. The electroweak interaction Lagrangian resulting from theses field redefinitions is [2]

$$\begin{aligned} \mathscr{L}_{I_{\text{weak}}} = -s^\mu(x)A_\mu - \frac{g}{2\sqrt{2}}[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)] \\ - \frac{g}{\cos\theta_W}[J_3^\mu(x) - \sin^2\theta_W s^\mu(x)/e]Z_\mu, \end{aligned} \tag{5.5}$$

where

$$\left.\begin{aligned} J^\mu &= \sum_{\substack{\text{all fermion}\\ \text{families}}} \bar{u}_i(x)\gamma^\mu(1-\gamma^5)d_i(x) \\ J_3^\mu &= \sum_{\substack{\text{all fermion}\\ \text{families}}} \frac{1}{4}[\bar{d}_i(x)\gamma^\mu(1-\gamma^5)d_i(x) - \bar{u}_i(x)\gamma^\mu(1-\gamma^5)u_i(x) \end{aligned}\right\} \tag{5.6}$$

and

$$s^\mu = \sum_{\substack{\text{lepton}\\ \text{generations}}} -e\bar{\psi}_l(x)\gamma^\mu\psi_l. \tag{5.7}$$

The vector boson fields were defined in equations 2.36 and 2.37, and s^μ is the QED current. The coupling g is the coupling to the W_i fields. The u and d notation stands for any fermion with up-type or down-type electroweak quantum numbers, respectively. From this Lagrangian, we see that only the J^μ and $J^{\mu\dagger}$ currents mix fermion flavors, and they couple only to the charged vector bosons. The Z_μ field couples the s^μ and J_3^μ currents, which *do not* mix flavor, hence flavor changing neutral currents are forbidden in the Standard Model. Today, experiments give us very small upper limits on these kinds of processes. However, we know that the MSSM soft Lagrangian introduces new terms that could make flavor changing neutral currents possible, so it is important to make sure that our parameter points do not produce FCNC's above the observed upper limits.

There have been extensive searches looking for flavor-violating interactions. These experiments are particularly relevant to SUSY searches because such FCNC effects could be detected at energy scales far below those of the sparticle masses themselves. Some of the most accurate measurements come from B meson decays. B mesons come in a few different varieties. Each has one bottom antiquark paired with a different flavor quark. The relevant ones for this thesis are the B_d^0, which has a down quark, and the B_s^0, which has a strange quark. The first "forbidden" decay we check is $b \to s\gamma$, where the bottom quark in the B_s^0 decays into a strange quark and a photon that carries away the excess mass energy coming from the difference in their masses. SUSY has terms that not only mix quark flavors, but also could allow quarks to transition into leptons and vice versa. The next two decays consider this possibility, considering the process $B_d^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-$.

The limits for these processes are taken from [14], given at 2σ confidence level with the systematic and statistical errors added in quadrature. For $b \to s\gamma$ and $B_s^0 \to \mu^+\mu^-$, we use both the upper and lower limits, given below at 95% CL:

$$\begin{gathered} 2.99 \times 10^{-4} < Br(b \to s\gamma) < 3.87 \times 10^{-4} \\ 1.7 \times 10^{-9} < Br(B_s^0 \to \mu^+\mu^-) < 4.5 \times 10^{-9} \end{gathered} \tag{5.8}$$

For $B_d^0 \to \mu^+\mu^-$, the experimentally observed lower limit is in fact lower than the SM prediction, so we discard this constraint, since interpreting it as a strict constraint would technically rule out the Standard Model. Instead, we use the experimentally observed upper limit, given at 90% CL:

$$Br(B_d^0 \to \mu^+\mu^-) < 6.3 \times 10^{-10}. \tag{5.9}$$

It was also the intention of this project to include *vacuum stability* constraints. In the MSSM, the addition of many new scalars into the theory introduces the possibility that these scalars could also obtain VEVs in the same way the Higgs field does. However, all of these fields are charged under the various forces, and if they obtained VEVs, they would break the $SU(3)$ color symmetry or the $U(1)$ electromagnetic symmetry leading to massive gluons and photons, which is of course experimentally ruled out. In the MSSM, it is known that the electroweak vacuum, i.e the vacuum in which both Higgs doublets obtain VEVs in the MSSM, is a *local* minimum of the scalar potential. However, it is not known whether this is also a *global* minimum. Therefore, it is possible that lower minima of the scalar potential exist. `Vevacious` [15] searches for minima lower than the EWSB minimum, and if it finds one, it calculates the tunneling time from the EWSB vacuum to the global minimum. If this tunneling time is less than a user-defined fraction of the age of the universe, then the parameter point is ruled out[1]. Unfortunately, the program has been out of use for many years, and some of the necessary dependencies have not been updated properly. We encountered problems with the program that the designers were unable to resolve before the deadline for this thesis, although they are still working with the issues and we hope to include the constraints in later work.

5.3 The Computational Tools

5.3.1 `SPheno`

`SPheno` is a program used to calculate mass spectra in supersymmetric theories. It does this by taking a set of input parameters and using them as boundary conditions to solve the MSSM renormalization group equations. It solves the RG equations to "two-loop" order, which means all Feynman diagrams with up to two loops are included. This is equivalent to solving to sixth order in the coupling constant. The RG equations take care of the infinities in the theory, but there are still finite quantum corrections to the mass spectra at higher orders in perturbation theory that must be considered. These corrections are dependent on the couplings. So, one can calculate the corrections to fourth order, and then insert the two-loop corrected couplings to obtain an even more accurate result. This is the procedure adopted by `SPheno`, except in the case of the lightest neutral Higgs mass. Here, the mass corrections are calculated to two-loop accuracy as well.

`SPheno` also calculates other important quantities, the most important of which for our purposes are the decay widths and branching ratios of all the supersymmetric particles, and the branching ratios of flavor violating decays of B mesons. The decay widths and relevant branching ratios of the SUSY particles turn out to be necessary as input to `SModelS`, and we will also use the B meson flavor observables to test against experimental constraints "by hand".

`SPheno` uses the SM masses and gauge couplings as input parameters, but these have all already been experimentally determined to high accuracy. For completeness, we include below a list of these parameters and the values for them used in our scans:

$$G_F = 1.166379 \cdot 10^{-5}\,\text{GeV} \quad m_Z = 91.18760\,\text{GeV (Pole Mass)} \quad \alpha_s(M_z) = 1.184 \cdot 10^{-1}\ (\text{SM}\,\overline{\text{MS}})$$
$$m_b(m_b) = 4.18\,\text{GeV}\ (\text{SM}\,\overline{\text{MS}}) \quad m_\tau = 1.77682\,\text{GeV (Pole Mass)} \quad m_t = 1.731 \cdot 10^2\,\text{GeV (Pole Mass)}.$$

[1]Even if the tunneling time is calculated to be less than the age of the universe, this is still a quantum tunneling and therefore probabilistic effect. Even if the tunneling time is only a fraction of the age of the universe, the probability for tunneling, which is a function of the lifetime, might still be very small.

Here, $\alpha_s \equiv \frac{g_s^2}{4\pi}$, where g_s is the strong QCD coupling. `SPheno` uses the third-generation quark and lepton masses because they are the heaviest and give the most significant contributions to the RG equations. G_F is the Fermi constant, and m_z is the mass of the Z boson, which `SPheno` takes as the default energy scale to run the couplings to down from the GUT scale. This is also the scale at which `SPheno` outputs the values for all the running parameters in the theory. For the two parameters input at a certain energy scale, α_s and m_b, it is necessary to define which renormalization scheme was used to calculate them, in this case the $\overline{\text{MS}}$ scheme is used.

5.3.2 `SModelS`

In order to implement ULC's for SUSY spectrum files, it is necessary to connect the spectra to models that have been tested by experiment. The MSSM is a very complicated model with many new particles and parameters. So complicated, in fact, that most experiments do not search for particles in the entire SUSY framework. This is because experimentalists rely on computer simulations of theoretical models to develop an idea of where to look for things in the massive amount of data produced in LHC experiments. The MSSM is simply too large to simulate accurately and in a reasonable amount of time. This is the case with many other *Beyond the Standard Model*, or BSM models. Instead, experimentalists work with what are known as *Simplified Model Spectra*, or SMS[2]. These models are effective Lagrangians with only a few number of new particles that are much easier to simulate. `SModelS` has a database of simplified models and all the *topologies* tested within each model. These topologies are simply Feynman diagrams of allowed decay processes in the simplified model. One can think of these as "process skeletons" that `SModelS` fills with the masses and the $\sigma \times Br$ for each of the intermediate processes taken from the input file. `SModelS` then calculates an overall $\sigma \times Br$ for each total decay and compares to the experimental limits. `SmodelS` returns an *r-value*, which is the ratio of the theoretically calculated $\sigma \times Br$ to the experimental UL. If this r-value is greater than one, the point is excluded up to 95% CL.

`SModels` is given the masses and the widths for the most significant decay channels for the particles in the model. It must use this input first to generate cross-sections for the decay processes. It calls another program called `Pythia` for this. `Pythia` calculates cross-sections through event simulation rather than direct computation using perturbation theory. It uses the spectrum file with decay widths to simulate a user-defined number of LHC production and detection events, and then gives a cross-section estimate. In this thesis, we use 10,000 events for each spectrum file.

One limitation of `SModelS` is that it cannot check points with charged LSP's. This is because all the experiments in the database are *Missing Energy Transfer*, or MET, experiments. This means that the experiments search indirectly for the particles by searching for

[2]This acronym is where `SModelS` gets it name

energy present before the collision that is not present in the final state, which by energy conservation implies some undetected particle has carried away the missing energy. The amount of missing energy can be used to determine the mass of the undetected particle. When it comes to detecting SUSY particles, it is assumed that all the particles will quickly decay into the LSP and SM particles before exiting the detection chamber. Therefore, in order to use the MET method, the LSP must be neutral so that it is not observed in the detector.

5.4 Explorations

Even having reduced the parameter space down to six free variables, this is still a difficult space to analyze and constrain given finite computing power, and it also very difficult to visualize 6-dimensional spaces in an easily understandable manner. To simplify matters, we consider that we wish to use the recent discovery of the Standard Model Higgs boson as a first constraint. So it makes sense to try and find which of these six parameters have the largest impact on the lightest supersymmetric Higgs mass so that we may scan only over those variables and simply set the other less significant values to suitable constant values.

5.4.1 Free Parameters and the Lightest Higgs Mass

The first parameter we look at is $\tan\beta$. We scan over $m_{1/2}$ first for a fixed value of A_0 and various values of $\tan\beta$ and plot the effect on the lightest Higgs, and repeat the process reversing the roles of $m_{1/2}$ and A_0. Both Higgs soft masses are set to zero in each plot. The results are visualized in the two plots below.

The reason the lines corresponding to different $\tan\beta$ do not extend the full length of the plot for larger $\tan\beta$ is because `SPheno` does not find sensible mass spectra for the input values corresponding to the missing sections. More specifically, SPheno finds that some of the sfermions have negative soft masses, which implies that the scalar potential is minimized when colored or electrically charged scalars obtain VEVs, violating the $SU(3)_c$ and $U(1)_{em}$ symmetries observed in the SM and in nature. It is clear from the plots that changing $\tan\beta$ does not have a large effect on the Higgs mass, and these differences in fact become smaller and smaller for larger $\tan\beta$.

Next we look into how the two Higgs soft masses $m^2_{H_d}$ and $m^2_{H_d}$, often denoted $m^2_{H_1}$ and $m^2_{H_2}$ respectively, affect the lightest Higgs mass m_{h_0}. We plot the behavior of m_{h_0} against both $m_{1/2}$ and A_0, with one of the two set fixed while the other is scanned over. For each of these scans, we set one of the two Higgs soft masses to zero, and then set the other to either 0 or 1×10^6 $(\mathrm{GeV})^2$ and note the changes in the Higgs mass. The results are given in figures 5.3 through 5.6. As we can see, neither of the Higgs soft masses has a large impact on the lightest Higgs mass in either planes of constant $m_{1/2}$ or constant A_0. The axes for each graph were chosen to highlight the areas where the differences in the

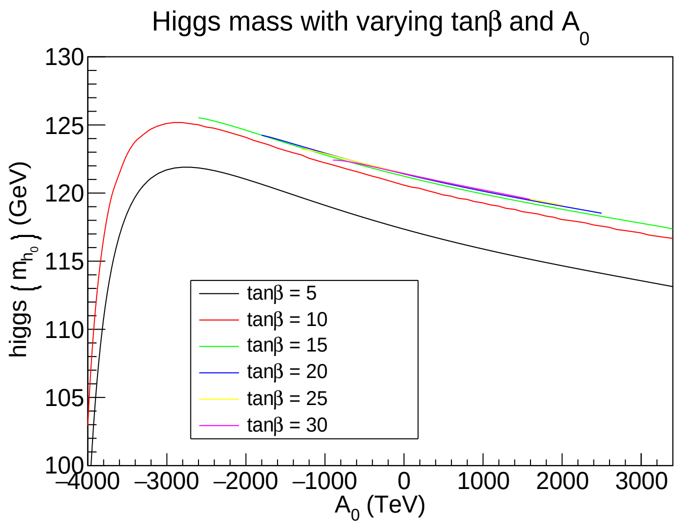

Figure 5.1: $m_{1/2} = 1$ TeV, $\tan\beta = 10$, $m_{H_d}^2 = m_{H_u}^2 = 0$

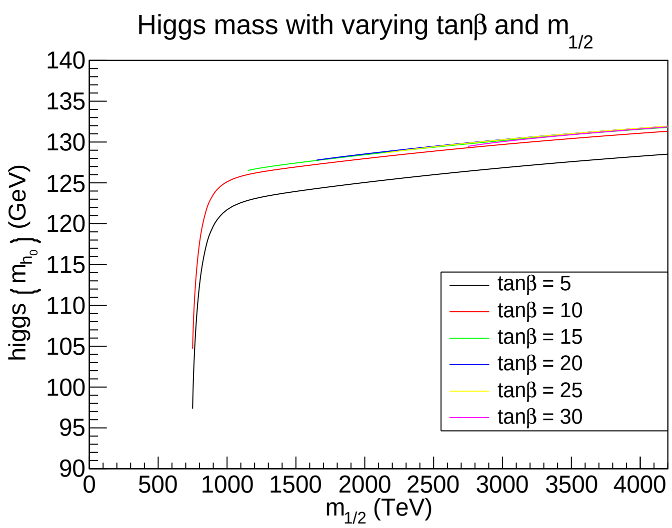

Figure 5.2: $A_0 = -3$ TeV, $\tan\beta = 10$, $m_{H_d}^2 = m_{H_u}^2 = 0$

lines corresponding to different Higgs soft mass values were greatest. Outside of the axis ranges chosen for the plots, the two lines essentially overlap perfectly in each plot.

Finally, we isolate the Higgs mass dependence on $m_{1/2}$ and A_0 in figures 5.7 and 5.8. The $m_{1/2}$ dependence is simpler. The Higgs mass m_{h_0} is increasing with $m_{1/2}$ regardless of the value of A_0, and shifting A_0 to more negative values only pushes the sharp decline in

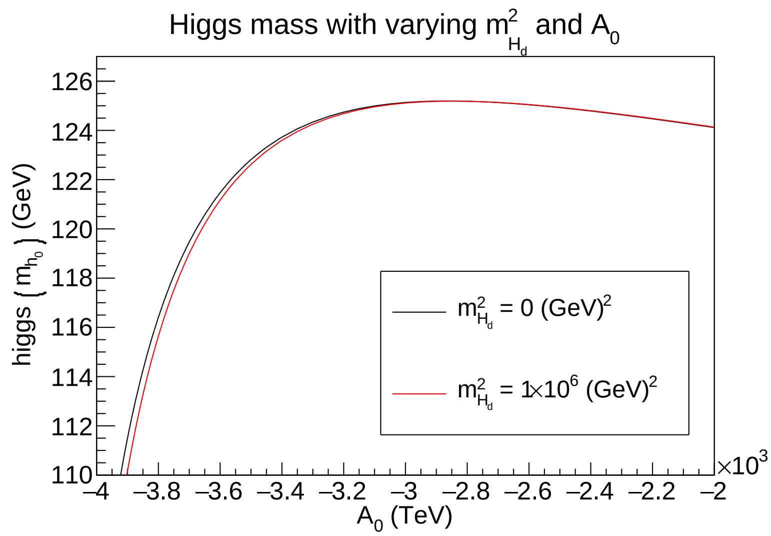

Figure 5.3: $m_{1/2} = 1$ TeV, $\tan\beta = 10$, $m^2_{H_u} = 0$

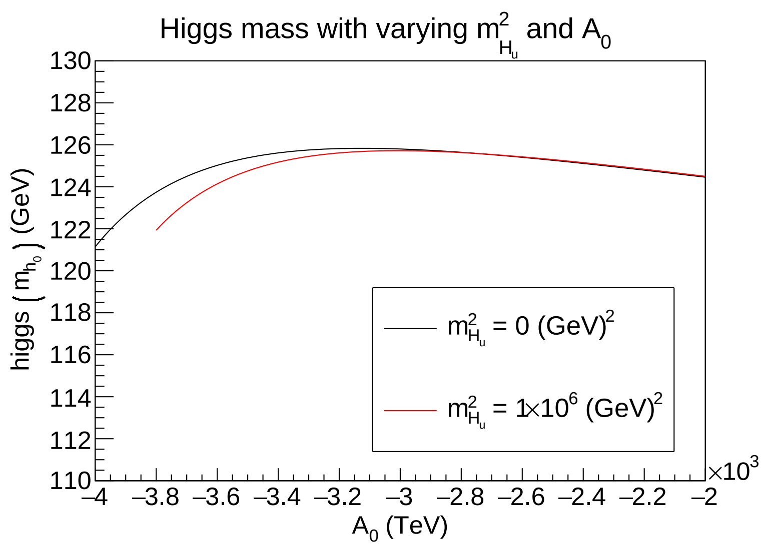

Figure 5.4: $m_{1/2} = 1.1$ TeV, $\tan\beta = 10$, $m^2_{H_d} = 0$

the plots further left. The A_0 dependence is more complicated but consistent. For larger negative values of A_0, Increasing A_0 causes a sharp increase in the Higgs mass. However, there is a turning point in all the graphs where this behavior reverses and the Higgs mass begins to decline for increasing A_0. Increasing $m_{1/2}$ stabilizes this decreasing behavior

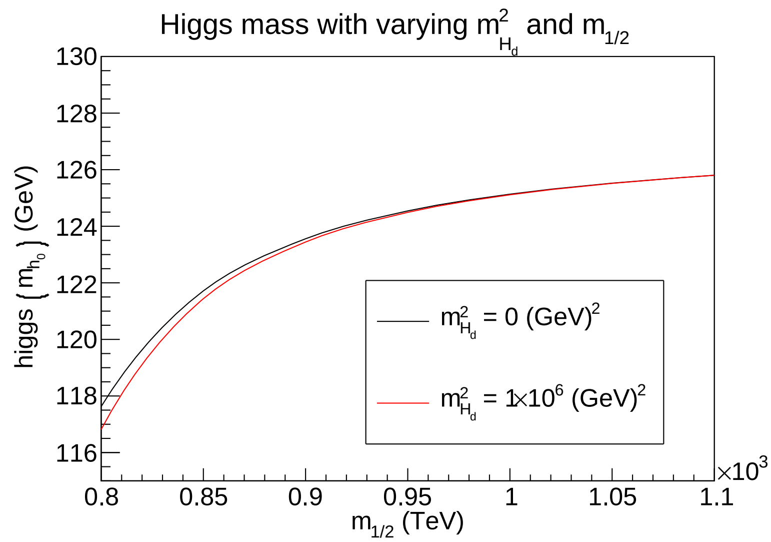

Figure 5.5: $A_0 = -3$ TeV, $\tan\beta = 10$, $m^2_{H_u} = 0$

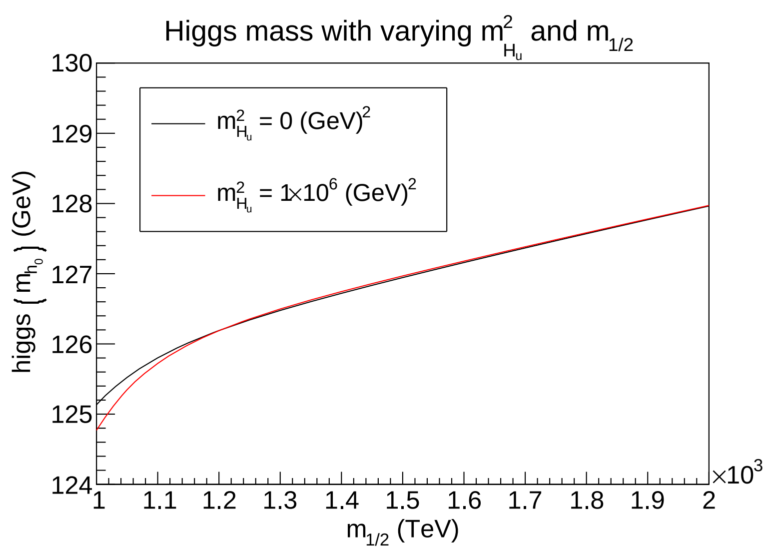

Figure 5.6: $A_0 = -3$ TeV, $\tan\beta = 10$, $m^2_{H_d} = 0$

to more negative values of A_0. It is interesting to note that for $M_{h_0}(A_0)$, the shift from sharply increasing to slowly decreasing occurs near $h_0 \sim 126$ GeV.

In summary, it can be safely said that both A_0 and $m_{1/2}$ have a relatively large effect on this Higgs mass compared to $\tan\beta$ and the two Higgs soft masses. So, if we are considering

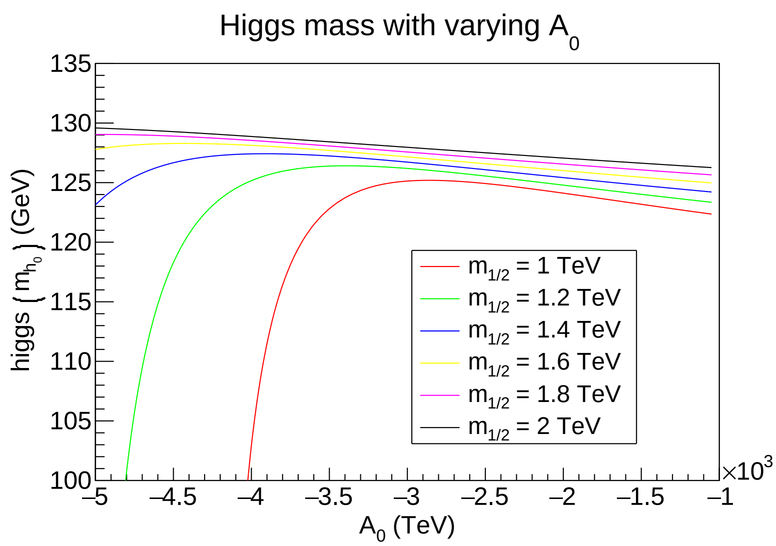

Figure 5.7: $m^2_{H_d} = m^2_{H_u} = 0,\ \tan\beta = 10$

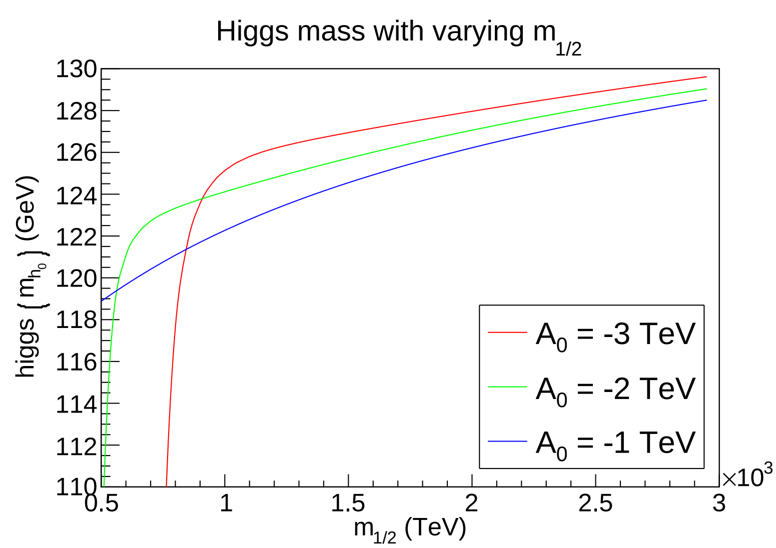

Figure 5.8: $m^2_{H_d} = m^2_{H_u} = 0,\ \tan\beta = 10$

only the dependence of the lightest Higgs mass on the model parameters, we can scan over only $m_{1/2}$ and A_0 while choosing fixed values for $\tan\beta$ and the two Higgs soft masses, and can be reasonably sure that our results will also roughly describe the behavior of the model for other values of these fixed parameters.

5.4.2 The LSP

Although we showed in the previous section that the Higgs soft masses do not have a significant effect on the mass of the lightest Higgs particle, they play a very important role in deciding which particle is the *lightest supersymmetric particle*, or LSP, in the model. The LSP is very significant because of the R-parity symmetry we discussed in section 3.6. Since every interaction term in the MSSM Lagrangian containing sparticles must contain at least two of these super partners, the LSP must be completely stable. It cannot decay into two SM particles, neither can it decay into a heavier SUSY particle and a lighter SM particle because of energy conservation. This means that since all SUSY particle decays eventually produce the LSP some place in the decay chain, this particle should be extremely abundant in our Universe, and is an excellent candidate for the as of yet undetected Dark Matter particle. However, we know that Dark Matter cannot be charged, so this model for Dark Matter is only allowed if the LSP is *neutral*. If the MSSM LSP is charged, then it must actually be the nLSP, or *next-to-lightest supersymmetric particle*. If the LSP were charged and there existed no lighter sparticle it could decay into, R-parity would require that this charged LSP be completely stable, meaning there would be heavy, charged matter everywhere in our universe, which is clearly not observed. This requires the introduction of a lighter, neutral sparticle into the theory. In this thesis, we exploit the fact that no model for quantum gravity is yet present in the MSSM. One can assume that there exists a quantum theory of gravity, mediated by a graviton, with a *gravitino* superpartner. If the MSSM LSP is charged, we can simply assume that the real LSP is the gravitino, and that this is the Dark Matter candidate. It turns out that in $\tilde{g}$MSB, a large majority of the points in parameter space that give a lightest Higgs mass that agrees with experiment (up to the theoretical uncertainty, of course) have a charged stau as the nLSP, and so we assume a gravitino LSP.

The LSP also has relevance for `SModelS` scans, for reasons explained in section 5.3.2. We need to find regions in parameter space with a Higgs mass in the given mass range and that also have neutral LSP's, so it is imperative that we understand how the LSP changes with the free parameters so that we may systematically search for these regions.

With these motivations in mind, we explore the LSP's dependence on the free parameters. Figure 5.9 below shows the dependence of the LSP on the two Higgs soft masses. Our choices for $m_{1/2}$ and A_0 are based on examining regions already determined to correspond to a lightest supersymmetric Higgs mass that agrees with experimental and theoretical limits. Every point in all the figures in this section obeys $122.7 < h_0 < 128.7$ GeV, and it is this criteria which governs our choices for $m_{1/2}$ and A_0. We will justify this claim in the next section.

The axes are chosen with a minimum value of 1×10^5 for each of the soft masses to highlight the interesting behavior. For the "good" regions in parameter space, all values for either the of the Higgs soft masses smaller than the axis limits lead invariably to a stau LSP. As we proceed from left to right to higher values of $m^2_{H_d}$, the LSP changes from the stau $\tilde{\tau}_1$ to the neutralino $\tilde{\chi}_1^0$ to the sneutrino $\tilde{\nu}_{\tau_L}$. The neutralino is a mass eigenstate that is a mixture of Binos and Higgsinos. The empty region in the upper right hand corner of the plot is a region where `SPheno` failed to produce a spectrum file.

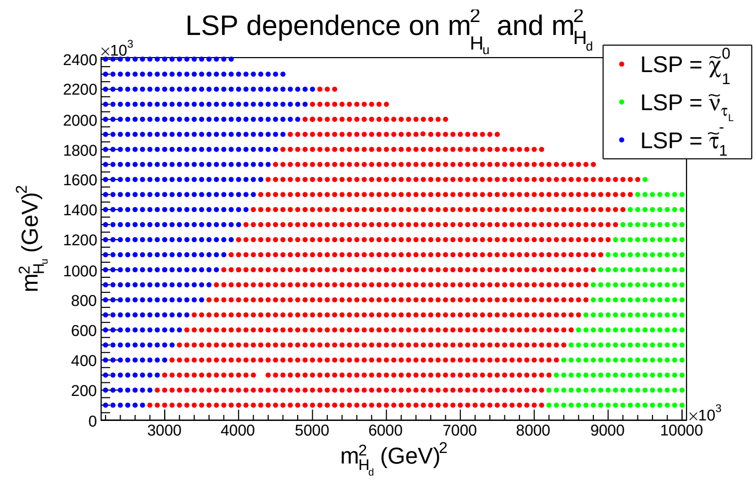

Figure 5.9: $m_{1/2} = 1$ TeV, $A_0 = -3$ TeV, $\tan\beta = 10$

It would be nice to achieve a semi-analytic understanding of the LSP dependence on the Higgs soft masses. To begin, we note that the $\tilde{\tau}_1$ is a mixture of the $\tilde{\tau}_{\mathrm{L}}$ and $\tilde{\tau}_{\mathrm{R}}$, and is defined to be the lighter of the two staus. However, we will assume that the two mass eigenstates are predominantly $\tilde{\tau}_L$, $\tilde{\tau}_R$, i.e we assume small stau-sector mixing.[3] This means that we must look at the the renormalization group equations for the soft masses of the $\tilde{\tau}_{\mathrm{L}}$ and $\tilde{\tau}_{\mathrm{R}}$. Using the notation of equation 3.37, these take the form [7]

$$16\pi^2 \frac{d}{dt} m^2_{L_3} = X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2 S \tag{5.10a}$$

$$16\pi^2 \frac{d}{dt} m^2_{\bar{e}_3} = 2X_\tau - \frac{24}{5}g_1^2|M_1|^2 + \frac{6}{5}g_1^2 S, \tag{5.10b}$$

where

$$\begin{aligned} X_\tau &= 2|y_\tau|^2(m^2_{H_d} + m^2_{L_3} + m^2_{\bar{e}_3}) + 2|a_\tau|^2 \\ S &= m^2_{H_u} - m^2_{H_d} + \mathrm{Tr}[\mathbf{m^2_Q} - \mathbf{m^2_L} - 2\mathbf{m^2_{\bar{u}}} + \mathbf{m^2_{\bar{d}}} + \mathbf{m^2_{\bar{e}}}], \end{aligned} \tag{5.11}$$

and g_1 and g_2 are the SM couplings to the B and W fields, respectively. The stau dependence on the Higgs soft masses is found in the terms X_τ and S. However, the Higgs mass

[3]We will justify this assumption at the end of section 5.5. The analysis in this section turns out to be strongly dependent on the no-mixing assumption, which in turn is dependent on the size of $\tan\beta$.

dependence in X_τ is proportional to y_τ, which is always a few factors smaller than g_1. So the Higgs soft mass dependence arises predominantly from the S term. From the form of S, we see that it is in fact the difference between $m^2_{H_u}$ and $m^2_{H_d}$ that is most significant.

Now, because the two terms have *opposite* signs for S, we must determine which of these two is most relevant. The $\tilde{\tau}_1$ is defined to be the lighter of the two staus, so it must be predominantly composed of the lighter of the two soft mass terms. In our regime of large $m_{1/2}$, equation 5.10a is dominated by the two negative gaugino mass terms that will tend to push the mass up as we run down from the electroweak scale. However, $m^2_{\tilde{e}_3}$ has only one gaugino soft term, and it is M_1, which is always smaller than M_2 in our regime (we will explain this later), so as long as S is not too large, it is safe to assume that $m^2_{\tilde{e}_3}$ will be the smaller of the two and our $\tilde{\tau}_1$ mass is predominantly governed by 5.10b.

With this is mind, we analyze the effect of the Higgs soft masses in the S term. If the difference is large and negative (i.e $m^2_{H_d}$ is larger), then this difference term will dominate S and S will have a tendency to push the $m_{\tilde{\tau}_1}$ to higher values as we run down from the high energy scale. This is certainly true in the case of Gaugino Mediation, where m_0 and therefore the soft scalar mass terms are zero at the high scale, and therefore S is dominated by the Higgs soft mass terms. When $m^2_{H_d} < m^2_{H_u}$ or when the difference is relatively small, then either S is positive or the S term is actually smaller the X_t term, both affects will instead push the stau mass *down*. This is why we find a stau LSP for all points with relatively small $m^2_{H_d}$.

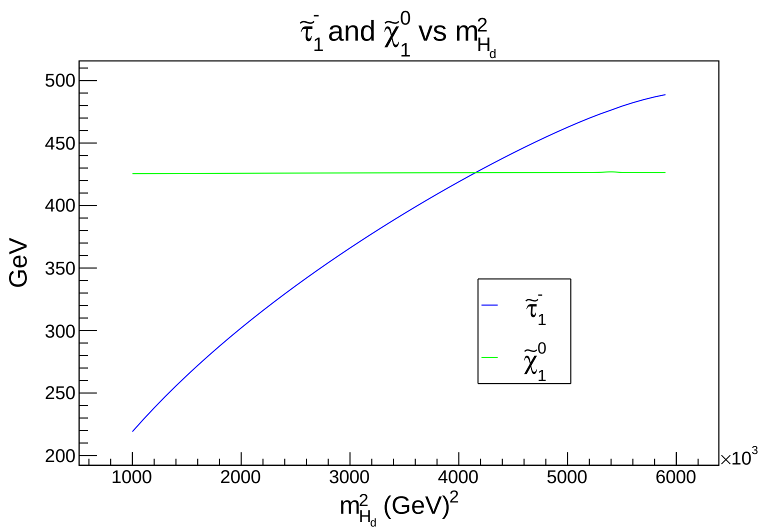

Figure 5.10: $m_{1/2} = 1$ TeV, $A_0 = -3$ TeV, $\tan\beta = 10$, $m^2_{H_u} = 1.4 \times 10^6$ (GeV)^2

The neutralino mass on the other hand, is steered primarily by the gaugino masses and the μ parameter because these particles are neutral and therefore couple to the neutral

Higgs particles. In the limit that $m_z << |\mu \pm M_1|, |\mu \pm M_2$ and $M_1 < M_2 << \mu$, satisfied for the parameter regions of interest in this thesis because of the relatively large values of $m_{1/2}$, the lightest neutralino mass takes the form [7]

$$m_{\tilde{N}_1} = M_1 - \frac{m_Z^2 \sin^2\theta_W (M_1 + \mu \sin 2\beta)}{\mu^2 - M_1^2} + \cdots . \tag{5.12}$$

To see how the neutralino mass changes, we therefore look at the RG equations for the gaugino masses and the μ parameter,

$$\frac{d}{dt} M_a = \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad (b_a = 33/5, 1, -3) \tag{5.13a}$$

$$\frac{d}{dt}\mu = \frac{\mu}{16\pi^2}[3y_t^* y_t + 3y_b^* y_b + y_\tau^* y_\tau - 3g_2^2 - \frac{3}{5}g_1^2], \tag{5.13b}$$

where a in the first of the two equations runs from 1-3, g_1 is the hypercharge coupling, g_2 is the $SU(2)_L$ coupling, and g_3 is the strong coupling g_s. The first of these two equations also explains why we should find $M_1 < M_2 < M_3$. The negative sign of b_3 means that M_3 will increase as we run down to the electroweak scale, opposite to the other gaugino soft masses, meaning M_3 will be much greater than the other two at the low scale. Because $g_1 \sim g_2$, the large b_1 factor will push M_1 below M_2. From these two equations, we also see that neither of the Higgs soft masses play a role, so we expect that the neutralino mass should be independent of changes in these soft masses.

To verify these observations we plot the $\tilde{\tau}_1$ and χ_1^0 mass versus $m_{H_d}^2$ in figure 5.10, where we see exactly this behavior. Note that for the values chosen in this plot, increasing $m_{H_d}^2$ pushes the difference in the Higgs soft masses to larger negative values, pushing the stau mass up above the neutralino mass. This is also what we see in figure 5.9, where for the fixed value $m_{H_u}^2 = 1.4 \times 10^6$ (GeV)^2, the LSP changes from the stau to the neutralino right around the intersection point seen in 5.10.

The sneutrino case occurs for very large values in the difference between $m_{H_d}^2$ and $m_{H_u}^2$, which in turn implies large values of S. There are no right chiral neutrinos or sneutrinos in the MSSM, so the sneutrino mass comes entirely from equation 5.10a. When S is large enough, the behavior of $m_{L_3}^2$ and $m_{\bar{e}_3}^2$ reverses. In this region, $m_{L_3}^2$ will end up being the lighter of the two third generation lepton soft mass terms, meaning that the lighter stau is governed by equation 5.10b, and both $m_{\tilde{\tau}_1}$ and $m_{\nu_{\tau_L}}$ will begin to decrease with increasing $m_{H_d}^2$. However, because the sneutrino receives no right chiral contribution, it will at some point be lighter than the stau, and and with large enough values of S will also become the LSP. We can see some of this behavior in figure 5.11.

We would also like to investigate the dependence of the LSP on $m_{1/2}$ and A_0. Figures 5.12 and 5.13 show the behavior on a larger scale than the more focused region of figure 5.9, and both use a logarithmic scale on the axis for $m_{H_d}^2$. Again we see that increasing $m_{H_d}^2$ allows us to change the LSP, and that the stau is the LSP for the vast majority of

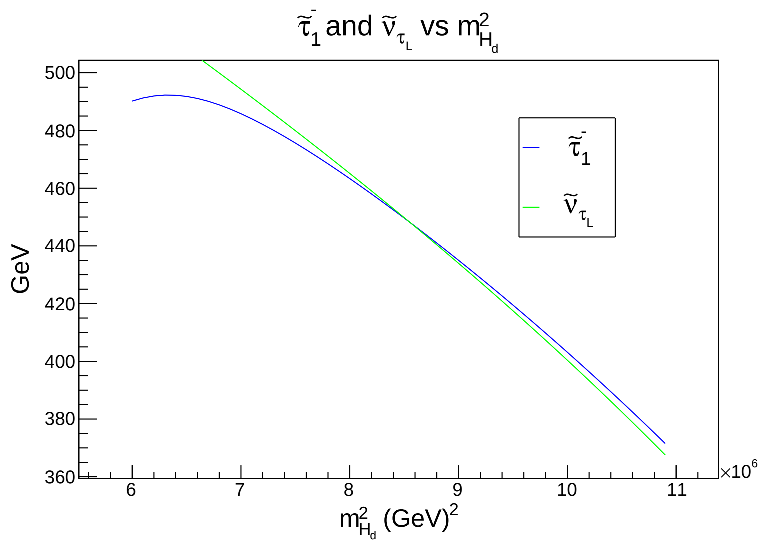

Figure 5.11: $m_{1/2} = 1$ TeV, $A_0 = -3$ TeV, $\tan\beta = 10$, $m^2_{H_u} = 1.4 \times 10^6$ (GeV)^2

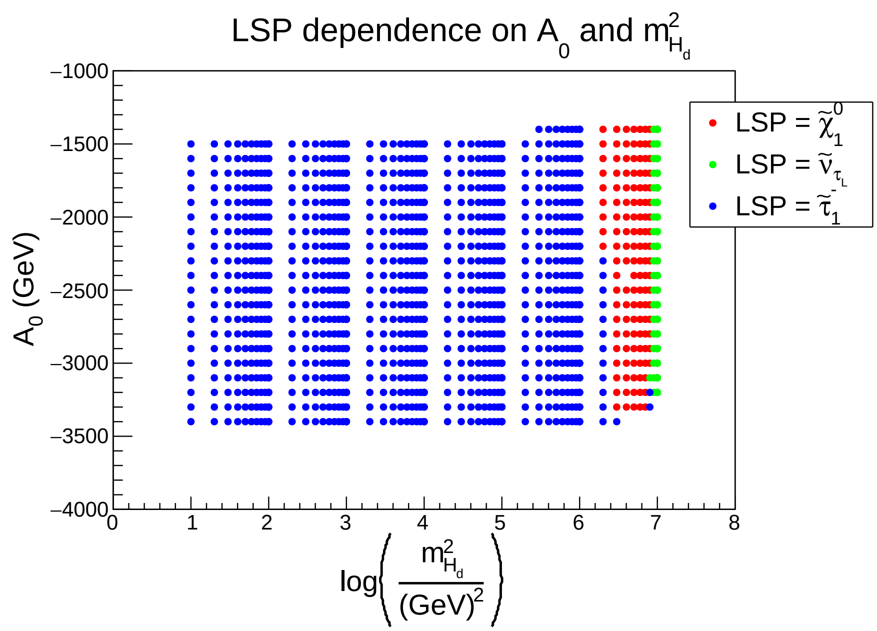

Figure 5.12: $m_{1/2} = 1$ TeV, $m^2_{H_u} = 0$, $\tan\beta = 10$

our points. The plot cuts off if we proceed any further to the right towards higher values of $m^2_{H_d}$, as again SPheno produces an error and no mass spectrum for these regions.

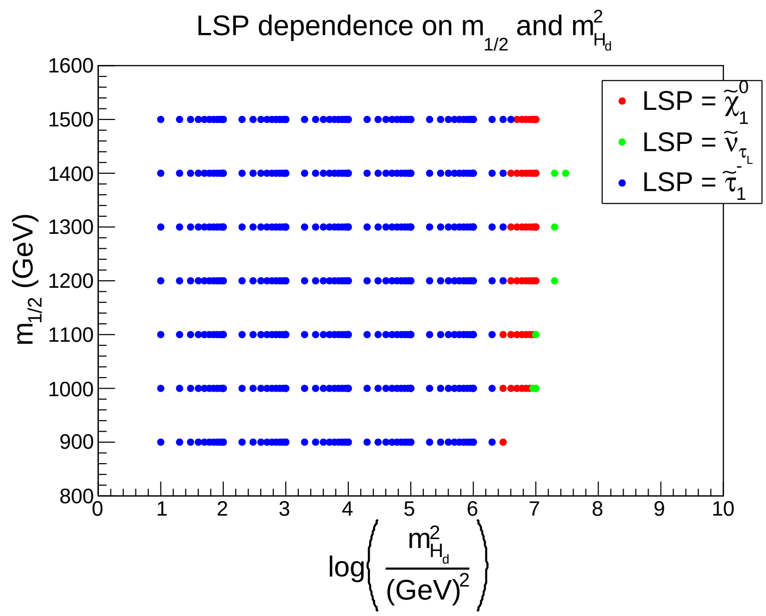

Figure 5.13: $A_0 = -1$ TeV, $m^2_{H_u} = 0$, $\tan\beta = 10$

If we zoom in on the interesting regions in the previous two figures where the LSP begins to change, we see the behavior displayed in the next two plots. Figure 5.14 shows that in the A_0 dependence is not as strong as the $m^2_{H_d}$ dependence, while figure 5.15 shows that for high enough values of $m^2_{H_d}$, the LSP has strong dependence on $m_{1/2}$.

This is a desirable feature, for now we can be fairly sure that for scans showing the dependence of the lightest Higgs mass on $m_{1/2}$ and A_0, we will be able to generate LSP's of all three types by simply setting a value for $m^2_{H_d}$ on the order of $\sim 5 \times 10^6$ (GeV)^2.

We can also use equations 5.10 and 5.13 to analyze this behavior analytically. We see from equation 5.15 that X_τ has an a_τ dependence. Large absolute values of A_0 will push up $|a_\tau|^2$ and therefore X_τ.

Larger values of X_τ will tend to push the stau mass down to lower values under RG running, so we expect that the stau mass should decrease with larger negative values of A_0. Again, the neutralino has little A_0 dependence, so we expect that for large negative values of A_0, the stau mass should be pushed below the neutralino mass and become the LSP. This is the behavior we see in figure 5.14, and is also verified by figure 5.16. However, this X_τ dependence is more or less irrelevant for small values of $m^2_{H_d}$, and the stau is almost always the LSP because of the small y_τ Yukawa coupling in X_τ. The S dependence is much stronger, and for small values of $m^2_{H_d}$, the positive portion of S dominates and pushes the stau mass down. The X_τ dependence only becomes relevant for large values of $m^2_{H_d}$ when the S influence reverses and we begin to see neutralino LSP's.

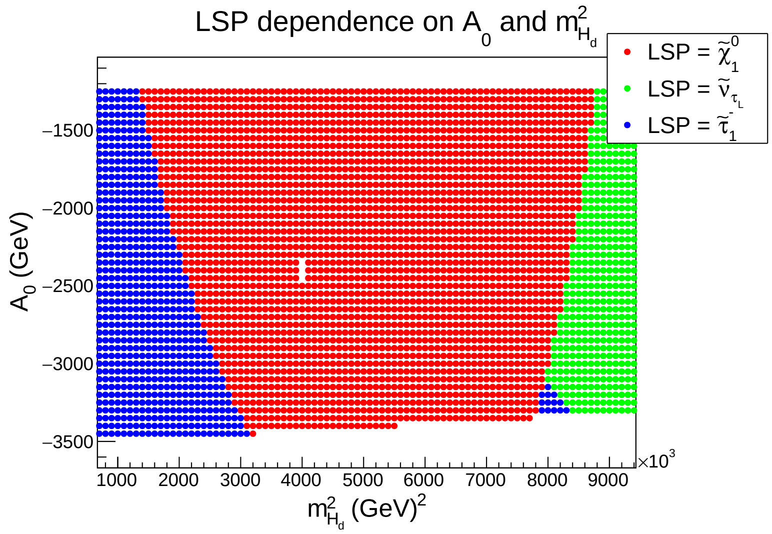

Figure 5.14: $m_{1/2} = 1$ TeV, $\tan\beta = 10$, $m^2_{H_u} = 0$

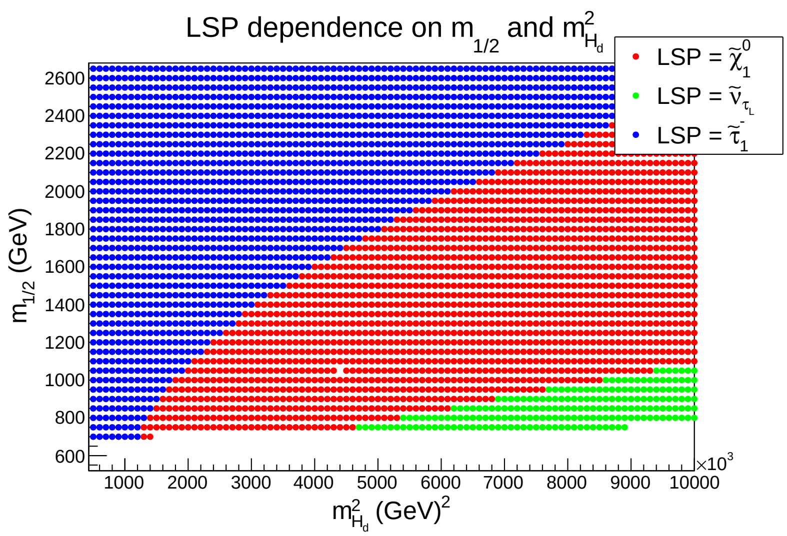

Figure 5.15: $A_0 = -2$ TeV, $\tan\beta = 10$, $m^2_{H_u} = 0$

The LSP dependence on $m_{1/2}$ is tricky. Larger values of $m_{1/2}$ increase the slope in equation 5.13a, but the small values of g_1 and g_2 still keep this slope relatively flat, so although larger $m_{1/2}$ has a tendency to push M_1 and M_2 down at the electroweak scale, M_1 and M_2 are still increasing functions of $m_{1/2}$. This means that larger $m_{1/2}$ pushes

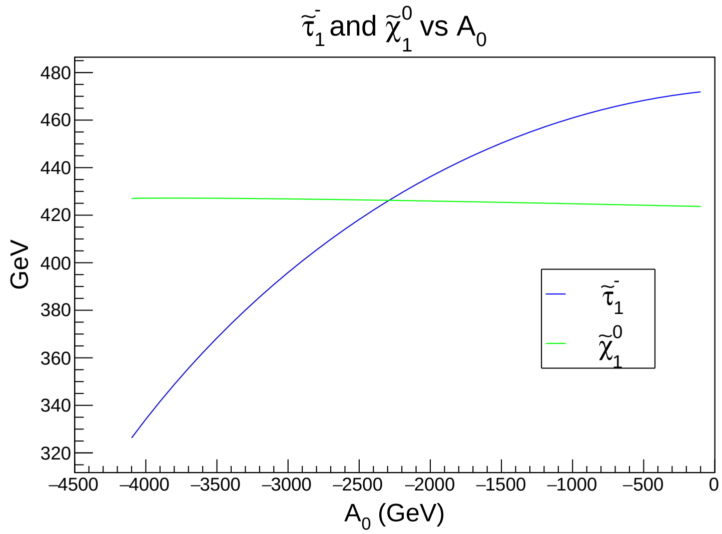

Figure 5.16: $m_{1/2} = 1$ TeV, $m^2_{H_u} = 0$, $m^2_{H_d} = 2 \times 10^6$ (GeV)^2

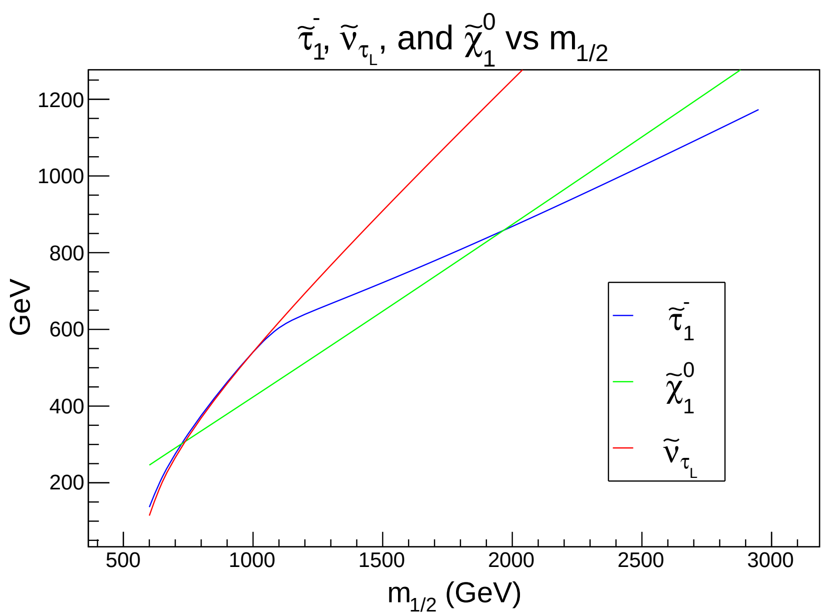

Figure 5.17: $m^2_{H_u} = 0$, $m^2_{H_d} = 5 \times 10^6$, $A_0 = 0$, $\tan\beta = 10$

up $m_{\tilde{N}_1}$ from equation 5.12. However, equations 5.10 tells us that both stau components should also be increasing functions of $m_{1/2}$, so it becomes a bit of battle between which mass increases fastest. It turns out the neutralino increases fastest, but there are regions where it begins below the lightest stau for the right value of $\tan\beta$, a fact hinted at by the $\sin\beta$ term in equation 5.12 and that will be clarified further in the next section. For small enough values of $m_{1/2}$, the behavior of the third generation lepton soft masses reverses again, and the sneutrino is pushed below both the stau and the neutralino. This behavior is summarized in figure 5.17. Again, however, this behavior is only relevant for large, negative S, which has the tendency to push up the stau mass. For small values of S, the mass-reducing M_1 dependence of the stau is too large, and keeps the stau below the neutralino, which is the large-scale behavior we see in figure 5.13

5.5 The Lightest Higgs Mass

The plots below shows regions in parameter space compatible with our mass constraint on the lightest Higgs mass for two different values of $\tan\beta$. We have cut off the scan for

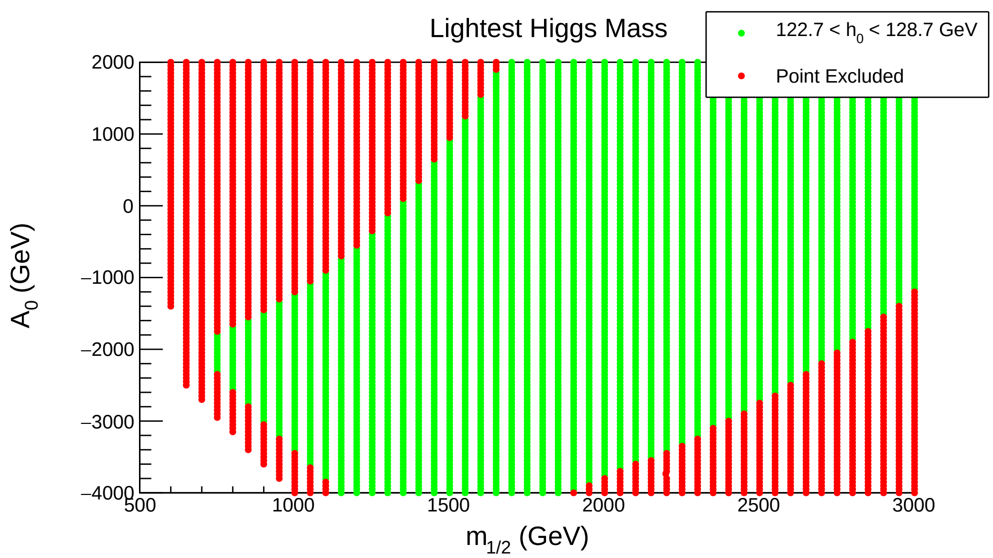

Figure 5.18: $m^2_{H_d} = 5 \times 10^6$, $m^2_{H_u} = 0$, $(\text{GeV})^2$, $\tan\beta = 10$

values of $m_{1/2}$ above 3 TeV to keep the spectrum light enough to be relevant to current LHC searches. One can extend the plot to much larger values of $m_{1/2}$ and A_0 and still find Higgs masses within the limits, however.

The excluded regions in the upper left of the two plots correspond to a Higgs mass that is too light, and the excluded regions in the lower right have Higgs masses that are too heavy. For the plot with $\tan\beta = 10$, 1631 points has a neutralino LSP, 2296 points has a stau LSP, and 11 had a sneutrino LSP. For $\tan\beta = 30$, all the points have stau LSP's. The

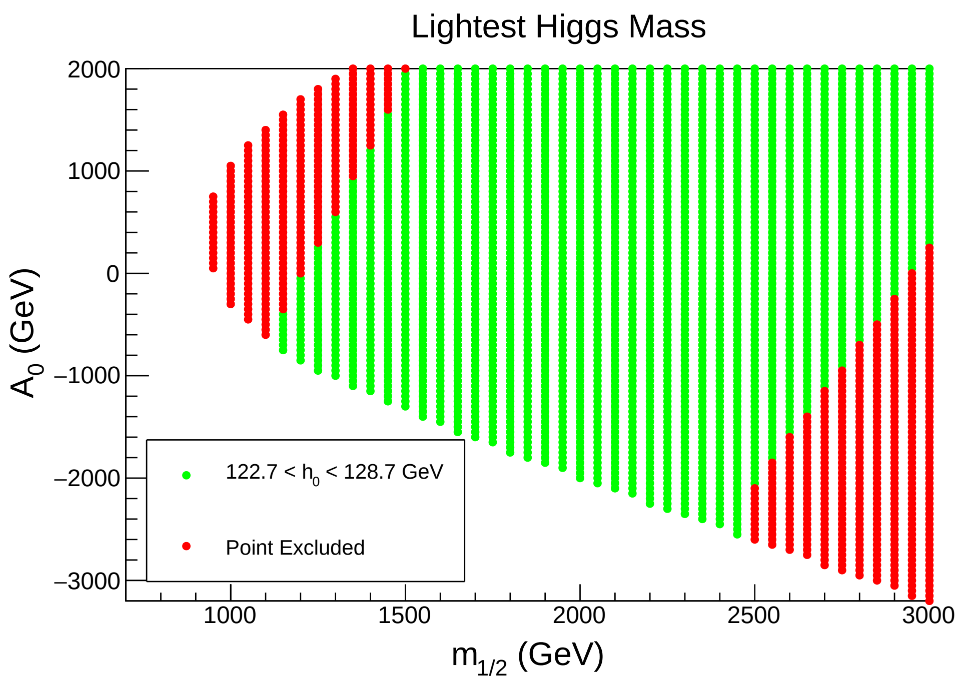

Figure 5.19: $m^2_{H_d} = 5 \times 10^6$, $m^2_{H_u} = 0$, $(\text{GeV})^2$, $\tan\beta = 30$

large blank region in figure 5.19 is where `SPheno` does not produce output, usually due to negative masses in the final result. Nonetheless, we do see that the green region is shifted slightly to the left and up. This behavior makes sense if we consider the effect of $\tan\beta$ and $m_{1/2}$ on the lightest Higgs mass explored in section 5.4.1. Increasing the values of either pushes up the Higgs mass, which should therefore shift our plot to lower values of $m_{1/2}$ for higher values of $\tan\beta$.

We would like to understand the result semi-quantitatively. To do so, we analyze the formula for the 1-loop corrected Higgs mass[4] in a particular limit known as the *decoupling limit*, defined by $A_0 >> m_Z$. Here, A_0 is *not* the GUT scale universal trilinear coupling, but the mass of another neutral Higgs particle present in the MSSM. This limit is obeyed by our points, with A_0 usually 2 orders of magnitude larger than $m_Z \approx 91$ GeV. This equation also only considers loop corrections from the top quark sector, which is a good approximation since the top mass is so much heavier than the other SM quarks. This

[4]`SPheno` actually includes two-loop corrections to m_h as well, but these are of course small compared to the 1-loop corrections, and for a rough understanding of what is going on, the 1-loop corrections suffice.

equation takes the form [16]

$$m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\left(\log\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right), \tag{5.14}$$

where

$$X_t \equiv A_t - \mu \cot\beta, \tag{5.15}$$

A_t is the top trilinear coupling, m_t is the top mass, m_Z is the Z boson mass, and v is the square root of the two Higgs VEVs, $v \equiv \sqrt{v_u^2 + v_d^2} = 174$ GeV. M_S is known as the supersymmetry breaking scale, defined as the square root of the stop masses, $M_S \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. All running parameters in this equation are defined at the scale M_S. Now we see the relevance of the universal trilinear coupling A_0 to the Higgs mass. The size of A_0 governs the size of A_t, although in a non-trivial manner described by the renormalization group equations, which in turn effects the Higgs mass through 5.15 and 5.14. This $m_h(A_0)$ dependence is not straight forward from the formula, so we explore it graphically.

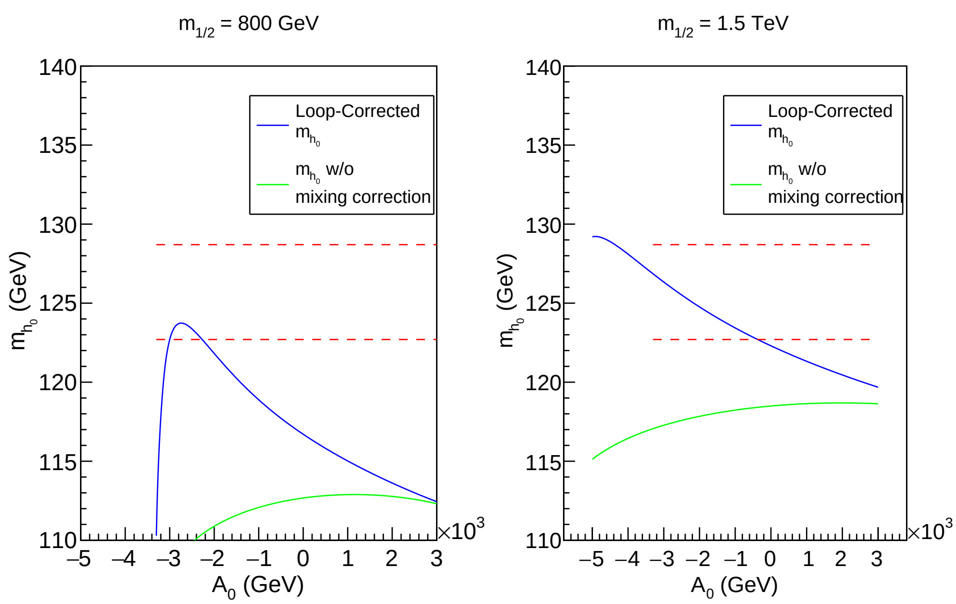

Figure 5.20: $\tan\beta = 10$, $m_{H_u}^2 = m_{H_d}^2 = 0$

In figures 5.20 and 5.21, we plot the universal trilinear coupling A_0 dependence of both the full 1-loop corrected Higgs mass m_h and m_h in the case of zero top mixing, $X_t = 0$. Figure 5.20 shows the two cases $m_{1/2} = 800$ GeV and $m_{1/2} = 1.5$ TeV. The region between the dotted red lines indicates points for which $122.7 < h_0 < 128.7$ GeV. We see that for these smaller values of $m_{1/2}$, the loop-corrected Higgs mass without the mixing contribution is below our desired range. In the $m_{1/2} = 800$ GeV case, the mixing contribution is only capable of raising the Higgs mass sufficiently around $A_0 \sim 2500$ GeV, while for $m_{1/2} = 1.5$ TeV, the logarithmic contribution is much larger and the mixing contribution is capable of bringing up the Higgs mass into the desired region for a much wider range in A_0. This agrees nicely with the result shown in figure 5.18, and allows us to explain why the exclued points in the upper left portion of the plot have a Higgs mass that is too light. We also can conclude that raising $m_{1/2}$ raises the logarithmic contribution to the Higgs mass and therefore the stop masses, explaining the increasing logarithmic-like dependence of h_0 on $m_{1/2}$ demonstrated in figure 5.8. It is also clear from these two plots that the mixing correction is largest for large negative values of A_0, an interesting point to keep in mind.

Figure 5.21 shows the two cases $m_{1/2} = 2$ TeV and $m_{1/2} = 2.5$ TeV. Here the much larger values of $m_{1/2}$ have pushed the base Higgs mass with $X_t = 0$ nearly into the desired range already, and so the mixing corrections easily raise the mass the needed amount. We see that proceeding to higher values of $m_{1/2}$ will eventually lead to regions where the Higgs mass is already in the good region and the mixing correction will push the Higgs mass up above the region when the mixing correction is largest, i.e for large negative values of A_0. This is exactly what we find in figure 5.18, and explains why the Higgs is too heavy in the excluded region in the lower right corner of the plot.

Equation 5.14 also helps explain the Higgs mass dependence on $\tan\beta$ observed in figures 5.1 and 5.2. For $\tan\beta > 1$, $\cos^2(2\beta) = \cos^2(2\tan^{-1}(\tan\beta))$ is an increasing function of $\tan\beta$ that asymptotically approaches one. For $\tan\beta < 10$, increases in $\tan\beta$ induce large changes in $\cos^2(2\tan^{-1}(\tan\beta))$. But for larger values of $\tan\beta$, $\cos^2(2\beta)$ begins to aysmptotically approach one, and increases in $\tan\beta$ have little effect. The X_t dependence is not as significant, but also serves to increase the Higgs mass slightly for higher $\tan\beta$. This is because for the regions we are considering, X_t is a positive function of A_0, and from 5.15, we see that increasing $\tan\beta$ decreases $\cot\beta$, and therefore pushes X_t up. The effects are less significant for larger $\tan\beta$ because of the asymptotic behavior of $1/x$ type functions. Both these effects influence m_{h_0} in the same way, and are just what we see in plots 5.1 and 5.2.

In order to check points with `SModels`, it is also necessary to isolate parameter points with neutral LSP's. Figures 5.22 and 5.23 show the LSP regions for the $\tan\beta = 10$ and $\tan\beta = 30$ scans in the same A_0 versus $m_{1/2}$ plane. We would like to understand why $\tan\beta$ has such a strong effect on the LSP. To see this, we must consider the tree level masses for the third generation sleptons after electroweak symmetry breaking. The tree-level mass-squared matrix takes the form [7]

$$m_{\tilde{\tau}}^2 = \begin{pmatrix} m_{L_3}^2 + \Delta_{\tilde{e}_L} & v(a_\tau^* \cos\beta - \mu y_\tau \sin\beta) \\ v(a_\tau \cos\beta - \mu^* y_\tau \sin\beta) & m_{\bar{e}_3}^2 + \Delta_{\tilde{e}_R} \end{pmatrix}, \tag{5.16}$$

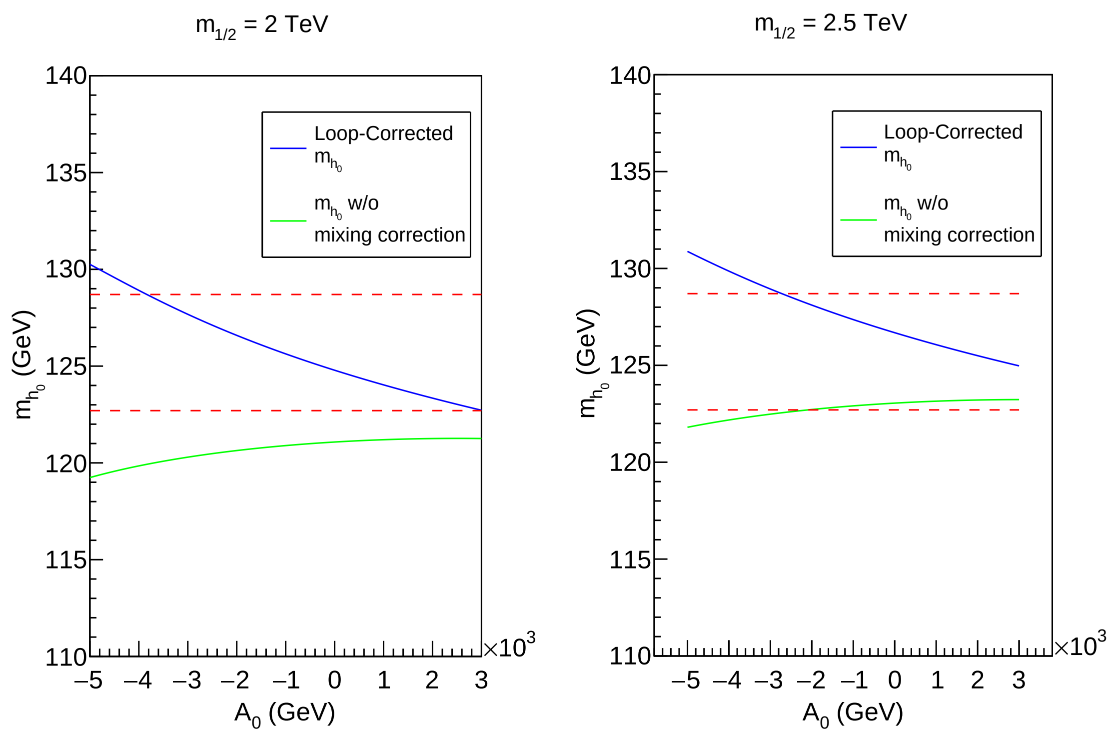

Figure 5.21: $\tan\beta = 10$, $m^2_{H_u} = m^2_{H_d} = 0$

where

$$\begin{aligned} \Delta_{\tilde{e}_L} &\equiv \left(-\frac{1}{2} + \sin^2\theta_W\right)\cos(2\beta)m_Z^2 \\ \Delta_{\tilde{e}_r} &\equiv \sin^2\theta_W \cos(2\beta) m_Z^2. \end{aligned} \tag{5.17}$$

For small $\tan\beta <\approx 10$, the off diagonal terms are small and can be neglected. This is not so obvious, and requires the introduction of a few more important relationships regarding the Yukawa couplings. The running masses obey the following [7]:

$$m_t = y_t v \sin\beta, \qquad m_b = y_b v \cos\beta, \qquad m_\tau = y_\tau v \cos\beta, \tag{5.18}$$

which together imply

$$\begin{aligned} \frac{y_b}{y_t} &= \left(\frac{m_b}{m_t}\right)\tan\beta \\ \frac{y_\tau}{y_t} &= \left(\frac{m_\tau}{m_t}\right)\tan\beta. \end{aligned} \tag{5.19}$$

This means that for relatively small $\tan\beta$, $y_b, y_\tau << y_t < 1$, so assuming $a_\tau \propto y_\tau$, we can assume small mixing in the stau sector. Then, we are in the regime analyzed in section

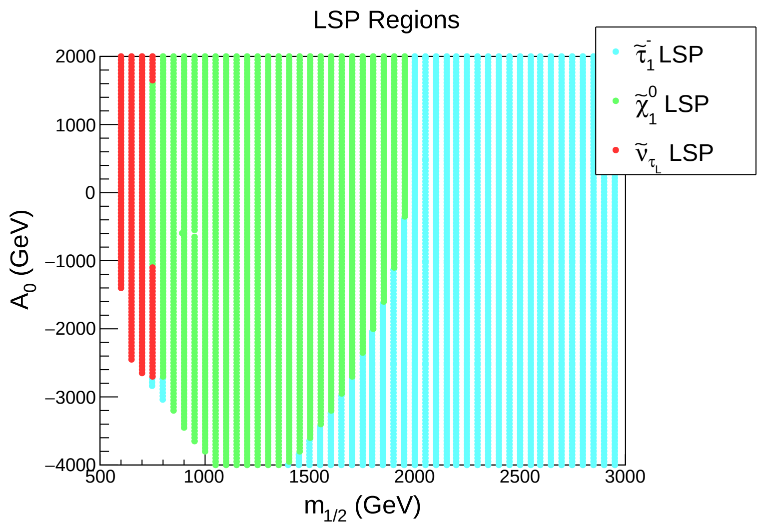

Figure 5.22: $m_{H_d}^2 = 5 \times 10^6$ (GeV)^2, $\tan\beta = 10$

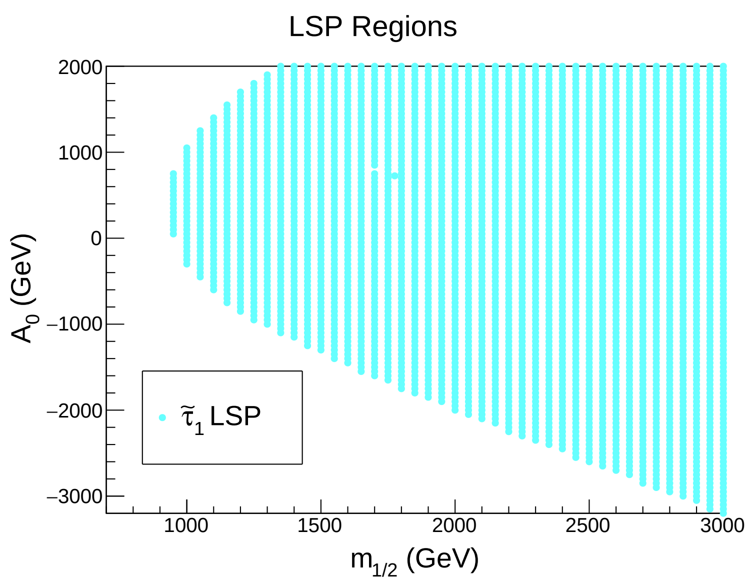

Figure 5.23: $m_{H_d}^2 = 5 \times 10^6$ (GeV)^2, $\tan\beta = 30$

5.4.2, explaining why we move through the different LSP regions as $m_{1/2}$ decreases, as seen in figure 5.22. However, for larger $\tan\beta$, we cannot neglect the mixing. The off diagonal

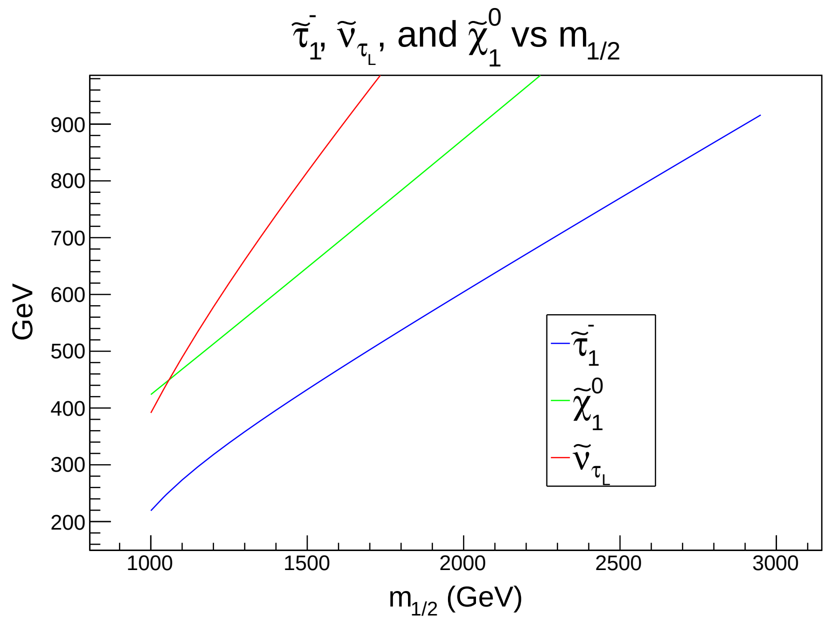

Figure 5.24: $m^2_{H_u} = 0$, $m^2_{H_d} = 5 \times 10^6$, $A_0 = 0$, $\tan\beta = 30$

terms invariably push the lower mass eigenvalue down, which is true of any 2×2 matrix with real eigenvalues. Large $\tan\beta = 30$ also pushes the $\sin 2\beta$ term in equation 5.12 down, which raises the neutralino mass. This means that the stau now starts below the neutralino, opposite to the $\tan\beta = 10$ case. The LSP dependence on $m_{1/2}$ for $\tan\beta = 30$ is plotted in figure 5.24.

5.6 `SModelS` and Flavor Observables Results

`SmodelS` was run only for the Higgs scan with $\tan\beta = 10$, as the $\tan\beta = 30$ scan produced only stau LSP's. Of the 1643 points with a good Higgs mass and neutral LSP's from the $\tan\beta = 10$ scan, `SModelS` does not produce an r-value for the majority of them. This is due to two reasons. `SModelS` uses the input mass spectra and cross-sections to decompose the spectra into a set of possible decay chains and the theoretical $\sigma \times Br$ associated with this decay topology. There is a user-defined limit, known as a *minimum decomposition weight* or *sigma cut*, which sets a limit on how small $\sigma \times Br$. Below the sigma cut, `SModelS` will not test the point because it is very unlikely that very tiny cross-sections are going to violate any ULC's. In this project, we use .003 femtobarns as the sigma cut.

The cross-sections for producing heavier particles are smaller as a general rule. The tested points all are relatively heavy, and for a small number of the tested points, so heavy that no decomposed topologies have a cross-section above the sigma cut, so these points are not even tested. For the majority of the tested points, only one or two or the decomposed topologies are above the sigma cut, but the spectra are so heavy that the experiments corresponding to these topologies have not been extended out to these mass ranges and therefore `SModelS` places no limits on these points. Only points with $m_{1/2}$ below 1200 GeV give r values, and among those there is sensitive dependence on A_0, where only small ranges of A_0 for values of $m_{1/2} \sim 1200$ GeV give r-values. This makes sense because in general, larger values of $m_{1/2}$ push the mass spectra to heavier values, while we saw for example in case of the lightest Higgs mass that the A_0 dependence is more complicated.

For the points that do give an r-value, the largest r-value is .177, well within the allowed limits. So we may conclude that all our neutral LSP points from the $\tan\beta = 10$ scan are allowed by the LHC experiments contained in `SmodelS` database.

The flavor constraints given in equation 5.8 turned out to be somewhat trivial. None of the good Higgs mass points from either the $\tan\beta = 10$ or $\tan\beta = 30$ scans violated any of the constraints.

Chapter 6

Conclusions and Outlook

6.0.1 Overview of Results

In this thesis, we have explored some phenomenological elements of an extra-dimensional model for supersymmetry breaking known as Gaugino Mediation. In particular, we have investigated the model within the framework of the cMSSM to make parameter space more manageable. We began by deriving soft trilinear coupling terms present at the GUT scale by solving for the auxiliary F-terms from the two Higgs superfields. These were overlooked during first explorations of Gaugino Mediation, and are crucial in obtaining a lightest neutral Higgs mass consistent with experiment results.

Once we had established the presence of the trilinear terms, we explored the dependence of the lightest neutral Higgs mass on the free cMSSM parameters in regions with $m_{h_0} \sim 126$ GeV. We found that both the Higgs soft masses had almost no influence on this Higgs mass, while $\tan\beta$ also had a relatively small impact, with larger values shifting the Higgs mass up slightly, and this effect becoming virtually unnoticeable for $\tan\beta > 20$. We determined that it was $m_{1/2}$ and A_0 that were the most relevant parameters to explore in this regard, with h_0 showing a fairly simple positive derivative behavior with respect to $m_{1/2}$, while for A_0 this behavior shifted from positive to negative derivative as A_0 increased from large negative values towards zero.

We also explored the influence of the free parameters on the LSP. We found that the difference in the Higgs soft masses $(m^2_{H_u} - m^2_{H_d})$ plays a very significant role in deciding which particle ends up as the LSP. Assigning non-zero and independent values to $m^2_{H_d}$ and $m^2_{H_u}$ is an extension of the cMSSM particular to $\tilde{g}$MSB resulting from allowing the Higgs field to propagate in the bulk of the extra-dimensions to directly obtain a mass term through interaction with the SUSY breaking fields. We found that pushing this difference to larger and larger negative values changes the LSP from the $\tilde{\tau}_1^-$ to the $\tilde{\chi}_1^0$ to the $\tilde{\nu}_{\tau_L}$. We found that the A_0 dependence was not very strong, but the $m_{1/2}$ dependence was quite significant, with smaller values of $m_{1/2}$ corresponding to a sneutrino LSP, which flowed through neutralino LSP regions and ended up in stau LSP regions as $m_{1/2}$ increased up to the 2 TeV range. Later we also found that the LSP had important $\tan\beta$ dependence, related to the influence of $\tan\beta$ on mixing in the stau sector and on the size of the lightest neutralino.

We proceeded to plot regions in the A_0 vs $m_{1/2}$ plane corresponding to a lightest neutral Higgs mass on the order of $\sim$ 125.7 GeV using the computer code `SPheno`, including all points within the theoretical uncertainty estimated to be on the order of 3 GeV. "Good" Higgs regions require $m_{1/2}$ to be relatively large, on the order of at least a TeV, while A_0 is large in absolute value and negative for small $m_{1/2}$, but is pushed up to zero and even positive as $m_{1/2}$ reaches $\sim$ 3 GeV.

Finally, we attempted to further constrain the points with a good Higgs mass using B-meson flavor constraints and the results of simplified model BSM searches at the LHC and the computer code `SModelS`. Only points with neutral LSP's could be analyzed by the missing energy transfer experiments in the `SModelS` database, so we needed to use our LSP analysis to find points fitting this criterion. Our SUSY mass spectra for the good points turned out to be quite heavy, with most of our points lying outside the range of the experimental results that `SModelS` checks against, and are therefore totally unconstrained. For many of the lighter points, `SModelS` does give an r-value, the largest of which being .177, well within the allowed range. So as far as `SModelS` can tell us, all of our good points are allowed. All our points with a good Higgs mass were also found to agree with the most recent constraints on three of the most sensitive flavor-violating decay processes.

6.0.2 Possibilities for Further Investigation

As mentioned in chapter 5, we hoped to use `Vevacious` to eliminate points that violate the color and electromagnetic charge gauge symmetries present in the Standard Model. Unfortunately, we were unable to resolve technical difficulties that arose from old software that was no longer maintained and is now incompatible with many of the modern versions of necessary dependencies like the `python` library `scipy`. A new, rewritten version, to be titled `Vevacious++` and coded entirely in `C++`, is currently being developed and is near a beta release. Unfortunately, the final version was not ready before the deadline of this thesis.

There exists another LHC-limit calculator released around the same time as `SModelS` called `CheckMate`. `CheckMate` does not work by decomposing into simplified topologies, but instead works by fast simulation. It uses theoretically calculated cross-sections and so-called event files as input. These input files are generated externally by an event generator such as `pythia` and statistically mimic a real LHC proton-proton collision. `Checkmate` uses this input, then simulates the detection process and then attempts to constrain the point by following the procedure of a specific experimental analyses chosen by the user. This is a much slower process, but avoids the complications of choosing which BSM experiments constrain which topologies. Another benefit of `CheckMate` is that the user has greater control of which experiments to compare with. `SModelS` only tests for strong production, meaning all the decays of SUSY particles are assumed to decay through the heavy gluino. This is because g_s is much larger than the electroweak couplings, so the colliding protons are much more likely to produce colored particles, which then decay into lighter sparticles. However, as we mentioned in section 5.6, large masses in the final state of a process tend to reduce the cross-section. If the gluino and squarks are extremely heavy (or the electroweak spectrum is very light), the cross-section-diminishing effect of the large mass of the colored particles outweighs the strength of the strong coupling, and direct electroweak production (i.e $pp \to \text{charginos}, \text{neutralinos}$) becomes the dominant decay mode. In this case it is important to check against bounds for electroweak decay channels. `CheckMate` can do these checks, and might reveal some interesting behavior for lighter spectra.

There also exists an extended version of `SModelS` specifically designed to check parameter points with charged LSP's, developed by Dr. Jan Heisig and co-authors [17]. This computer code excludes points based on direct detection of Heavy Stable Charged Particles (HSCP's) instead of MET signatures. The analysis will be included in the paper mentioned in the introduction.

Appendix A

van der Waerden Notation

This notation was invented to simplify the notation of Lorentz invariant quantities using two component spinors. The motivation comes from the convention of Lorentz invariants in the Einstein notation taking the form $a^\mu b_\mu$. The most basic Lorentz invariant quantity built from two-component Weyl spinors is $\eta^\dagger \chi$. If we define the components of χ and η as

$$\begin{aligned} \chi_a &\equiv \text{components of } \chi \\ \eta^a &\equiv \text{components of hermitian conjugate of } \eta, \end{aligned} \tag{A.1}$$

we can write $\eta^\dagger \chi$ as $\eta^a \chi_a$. If we define the components

$$\begin{aligned} \bar{\chi}_{\dot{a}} &\equiv \text{components of hermitian conjugate of } \chi \\ \bar{\eta}^{\dot{a}} &\equiv \text{components of } \eta, \end{aligned} \tag{A.2}$$

then we can write $\chi^\dagger \eta$ as $\bar{\chi}_{\dot{a}} \bar{\eta}^{\dot{a}}$. The rule now becomes that only indices of the same kind, i.e dotted and un-dotted can be summed over, and all quantities with bars are defined to have dotted indices. We use equations 3.3b and 3.3c to motivate the definitions of the remaining components.

$$\begin{aligned} \bar{\chi}^{\dot{a}} &\equiv (i\sigma^2)^{\dot{a}\dot{b}} \bar{\chi}_{\dot{b}} \\ \chi^a &\equiv (\bar{\chi}^{\dot{a}})^\dagger \\ \eta_a &\equiv (-i\sigma^2)_{ab} (\eta^{\dot{b}})^\dagger \\ \bar{\eta}_{\dot{a}} &\equiv (\eta_a)^\dagger. \end{aligned} \tag{A.3}$$

The indices of the $i\sigma^2$ matrices have no special meaning and always indicate the components of the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. They are simply defined such that the rule of summing only over matching index type is obeyed. Using these, we can construct the other relevant Lorentz invariants in equations 3.3. For example,

$$\chi^a \chi_a = \chi_b (i\sigma^2)^{ab} \chi_a = \chi_b (-i\sigma^2)_{ba} \chi_a = \chi^T (-i\sigma^2) \chi. \tag{A.4}$$

These equations imply relationships among the upper and lower coordinates. The most relevant for us are just those relating the components of the left chiral spinor, which we give below:

$$\begin{aligned} \bar{\chi}^{\dot{1}} &= \bar{\chi}_{\dot{2}}, \qquad \bar{\chi}^{\dot{2}} = -\bar{\chi}_{\dot{1}} \\ \chi^1 &= \chi_2, \qquad \chi^2 = -\chi_1 \end{aligned} \tag{A.5}$$

This also suggests that we can raise and lower indices with the matrix $(i\sigma^2)$. So we define a new metric-like quantity by

$$\begin{aligned} (i\sigma^2)^{ab} = (i\sigma^2)^{\dot{a}\dot{b}} &\equiv \epsilon^{ab} \equiv \epsilon^{\dot{a}\dot{b}} \\ (-i\sigma^2)_{ab} = (-i\sigma^2)_{\dot{a}\dot{b}} &\equiv \epsilon_{ab} \equiv \epsilon_{\dot{a}\dot{b}}, \end{aligned} \tag{A.6}$$

which allows for things like $\chi_a = \epsilon_{ab}\chi^b$, etc. Note that ϵ is *antisymmetric*, unlike the metric in relativity, so the order is important.

This new notation can be a source of ambiguity in the case of index-free quantities, like $\chi \cdot \chi$ or $\bar{\eta} \cdot \bar{\eta}$. So, we define these quantities such that unbarred spinor dot products have indices that go diagonally downward, and the opposite for barred quantities:

$$\begin{aligned} \eta \cdot \chi &\equiv \eta^a \chi_a \\ \bar{\chi} \cdot \bar{\eta} &\equiv \bar{\chi}_{\dot{a}} \bar{\eta}^{\dot{a}}. \end{aligned} \tag{A.7}$$

An ambiguity also arises in terms of the form 3.3d because of the σ^μ and $\bar{\sigma}^\mu$ matrices. We define these matrices as follows:

$$\begin{aligned} \text{Indices of } \sigma^\mu &\equiv (\sigma^\mu)_{a\dot{b}} \\ \text{Indices of } \bar{\sigma}^\mu &\equiv (\bar{\sigma}^\mu)^{\dot{a}b}, \end{aligned} \tag{A.8}$$

so that we may write, for example, 3.3d as $\bar{\chi}_{\dot{a}}(\bar{\sigma}^\mu)^{\dot{a}b} i\partial_\mu \chi_b$.

Appendix B

Grassman Coordinates

This appendix serves as a short introduction to Grassman variables, in particular, the Grassman variables defined in section 3.2 with help of the van der Waerden notation. Grassman quantities are defined to obey the basic algebra

$$\{\theta_1, \theta_2\} = 0. \tag{B.1}$$

This implies the important relation $\theta_1^2 = 0$. We define the derivative in the most obvious way possible

$$\frac{\partial \theta_1}{\partial \theta_1} = 1. \tag{B.2}$$

Using this van der Waerden notation, we also define the four variables $\theta_1,\ \theta_2\ \bar{\theta}_{\dot{1}}\ \bar{\theta}_{\dot{2}}$ to be independent in the sense that the derivative of any with respect to any other is 0. However, we see from equation A.5 that quantities with upper and lower components depend on one another, and that these relationships imply things like

$$\frac{\partial \theta^{\dot{1}}}{\partial \theta_{\dot{2}}} = 1. \tag{B.3}$$

The derivative is also a Grassman quantity, so the product rule looks like

$$\frac{\partial}{\partial \theta_1}(\theta_1\theta_2) = \left(\frac{\partial \theta_1}{\partial \theta_1}\right)\theta_2 - \left(\frac{\partial \theta_2}{\partial \theta_1}\right)\theta_1 = \theta_2, \tag{B.4}$$

because the derivative itself is an anticommuting quantity. As for integration, we define

$$\int d\theta_1 = 0, \qquad \int d\theta_1\theta_1 = 1, \tag{B.5}$$

where the measures themselves are also considered to be anticommuting. So, for example, we have

$$\int d\theta_1 \int d\theta_2\theta_1\theta_2 = -\int d\theta_1\theta_1 \int d\theta_2\theta_2 = -1. \tag{B.6}$$

Using equation A.7, we see that

$$\theta \cdot \theta = \theta^a \theta_a = (i\sigma^2)^{ab} \theta_b \theta_a = -\theta_a (i\sigma^2)^{ab} \theta_b = -\theta_1\theta_2 + \theta_2\theta_1 = 2\theta_2\theta_1. \tag{B.7}$$

If we define

$$d^2\theta \equiv d\theta_1 d\theta_2, \qquad d^2\bar{\theta} \equiv -d\theta_{\dot{1}} d\theta_{\dot{2}}, \tag{B.8}$$

we then have

$$\frac{1}{2}\int d^2\theta \theta \cdot \theta = 1, \qquad \frac{1}{2}\int d^2\bar{\theta}\bar{\theta} \cdot \bar{\theta} = 1, \tag{B.9}$$

as well as

$$\frac{1}{4}\int d^2\theta d^2\bar{\theta}\theta \cdot \theta\bar{\theta} \cdot \bar{\theta} = 1. \tag{B.10}$$

We will see that these relationships are very important when constructing SUSY invariant quantities from superfields.

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