A Simulation Model of the Interaction between Nitrogen and Oxygen Levels Influenced by the Hydrodynamics in a Fjord Using a System Dynamic Approach

by

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Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Candidata/Candidatus Rerum Politicarum

Department of Information Science

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Abstract

Title:	A Simulation Model of the Interaction between Nitrogen and Oxygen Levels Influenced by the Hydrodynamics in a Fjord Using a System Dynamic Approach
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Degree and year:	Submitted in Partial Degree of the Requirements for the Degree of Candidata/Candidatus Rerum Politicarum, Department of Information Science, University of Bergen, Norway, 2001
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A simulation model of the interaction between nitrogen and oxygen levels influenced by the hydrodynamics in a fjord is developed, using a system dynamic approach. The purpose of the project is to identify the main variables and structures that influence this interaction, and generate the general behavior of the nitrogen and oxygen levels at different locations over time.

The Sørfjord in Western Norway is surrounded by a small, industrialized community, and for one century waste from the local factories has been discharged into the fjord. It was not until the 1970's that severe contamination of the fjord was discovered. Since then, major improvements have been made, and a wide range of cleaning methods has been implemented. However, the oxygen levels are at times so low that fish and other mobile organisms may move to more oxygen rich areas. The main theory states that the low oxygen levels stem from nitrogen pollution from one of the local factories, in combination with a low water exchange rate. The nitrogen pollution causes a large chemical oxygen consumption.

Two models have been developed. First, the microbiological processes concerning oxygen production and consumption were modeled in order to describe the major elements influencing oxygen consumption and production in a liter of water. The model contains nitrification, which is the chemical process of transforming the nitrogen compound

ammonium into nitrate. This process consumes large amounts of oxygen. The biology model also describes oxygen production through photosynthesis by phytoplankton. Phytoplankton utilize ammonium as a nutrient for growth.

The biology model is merged with a simple hydrodynamic model in order to see the interaction between the various stock variables at different locations and depths of the fjord, and how the hydrodynamics of the fjord may influence the stock variables, and thereby the processes that occur.

Data from the Sørfjord in Western Norway is used as a basis for the model. However, being a simple model, it could, with a few parameter changes represent the interaction between nitrogen and oxygen in any fjord without a socket, and with a river in the innermost part of the fjord.

Acknowledgements

We would like to thank our advisor, Associate Professor Sigmund Nævdal for his outstanding commitment and belief in our project and us. His knowledge and experience was essential for the development of the model. His engagement passed by far what can be expected from an advisor, and we hope he did not let too many dinners turn cold in order to help us.

Special thanks to Professor Pål Davidsen, who was our advisor during the pre-project, and who has also encouraged us later in the process.

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We would like to thank environmental executive Rolf Bøen, who has shown great interest in the project, and the City of Odda for financial support.

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Professor Harald Svendsen, Department of Geophysics, University of Bergen
Per Strømsnes, Norzink

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1. Introduction

1.1 Problem Definition

Over the last 30 years extremely low levels of oxygen have been measured in the Sørfjord in Hardanger, Western Norway (Skei, 1973, 1998; Molvær, 1997, 1998, 1999). If these conditions continue they may have a large impact on the biological life in the fjord. Fish and mobile organisms may periodically tend to move to more oxygen rich waters, while those not capable of moving may die (Molvær, 1999).

The main assumption is that the recurring oxygen problems primarily stem from nitrogen pollution in combination with a low water exchange rate (Molvær, 1999; Skei, 1998). There are several factories located along the fjord, and one of them, Odda Smelteverk is responsible for 55.8% of the total nitrogen input to the fjord (Molvær, 1997). The nitrogen pollution causes a large chemical oxygen consumption. Other variables, such as sewage and natural nitrogen inflow from the rivers and farmlands surrounding the fjord may also influence the oxygen consumption. The oxygen levels are assumed to be fairly dependent on the water exchange rate, which is the rate at which fresh seawater, possibly containing higher oxygen levels, flow into the fjord. Oxygen also flow into the fjord through the river in the innermost part of the fjord. Oxygen is produced by phytoplankton through photosynthesis, which represents a contribution to the oxygen levels.

The goal of the project is to identify the main variables and structures that influence the interaction between the oxygen and nitrogen levels in a fjord through the development of a system dynamic model that generates the general behavior of the oxygen levels at different locations over time.

Data from the Sørfjord is used as a basis for the model, but being a simple model it does not exhibit the flow patterns and topography that is unique for the fjord. Thus, with a few parameter changes, the model could represent the behavior of the oxygen levels in any fjord without a socket, and with a river outlet at the innermost part of the fjord.

Two models have been developed:

1. A biology model

The biology model incorporates microbiological variables related to oxygen production and consumption in a liter of water in the fjord.

2. A combined biology and hydrodynamic model

The biology model is merged with a hydrodynamic model of the water movements in the fjord, in order to see how oxygen consumption and production vary at different locations in the fjord, and how the outflow and inflow of fresh seawater influence this. The hydrodynamic model is developed by our advisor, Sigmund Nævdal (Nævdal, 2001). The model divides the fjord into fjord cells, and the biology model is applied to each fjord cell. Hence, there are two aspects concerning the flows of biological state variables in the hydrodynamic model:

- The flows caused by the microbiological processes
- The flows caused by the hydrodynamics, i.e. the water movements between the fjord cells

1.2 Division of Work Between Students and Advisor

Figure 1-1 illustrates the division of the work between Andreas Hervig, Magnhild Viste, and advisor, Associate Professor Sigmund Nævdal, who has developed the hydrodynamic model of the water movements in the fjord (Nævdal, 2001). Andreas Hervig and Magnhild Viste have developed a model of the microbiological flows within a fjord cell, and integrated it with the hydrodynamic model. The integration process involved applying the variables of the biological model to all fjord cells, and making the levels of the biological model flow with the water between the various fjord cells.

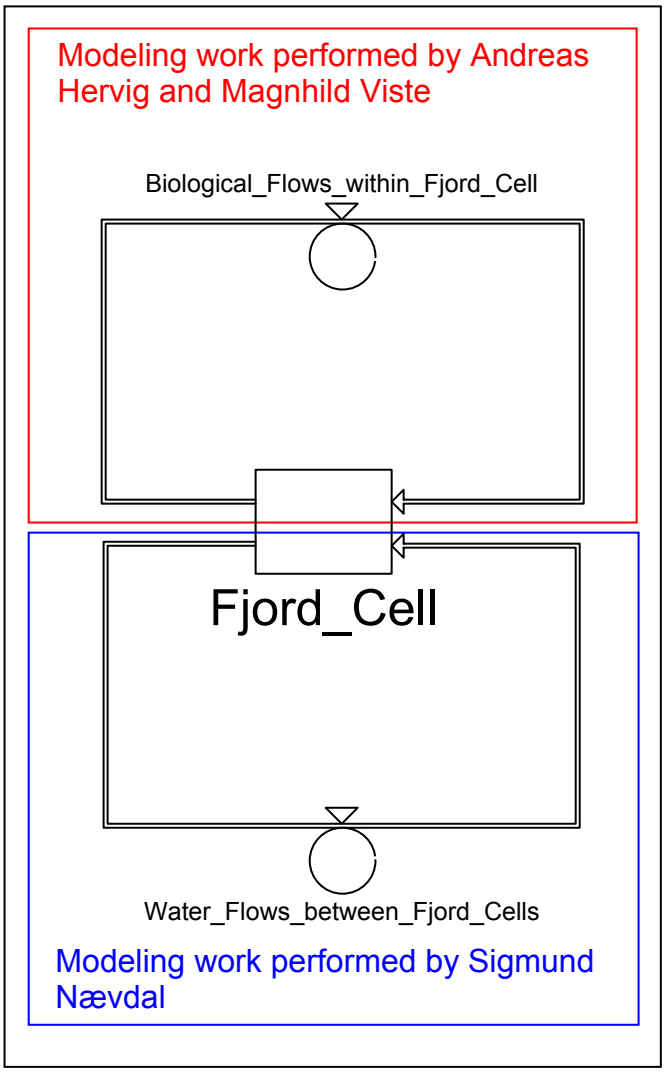


Figure 1-1: The division of modeling work between students and advisor.

1.3 Intended Contribution

As stated previously the goal of the project is to develop a simple system dynamic model that generates the main behavior of the oxygen levels over time. Such a model may be advantageous for several reasons, however the use and usefulness of the model is not studied here.

One of the major problems in dealing with environmental issues is the clear division between lawmakers, decision-makers, and experts, and that the knowledge of the system often involves people with different backgrounds. The rules for how much and what kind of waste a factory may discharge is set by the State Pollution Authority (SFT). This is a governmental agency that surveys and informs about environmental development, and uses its authority in order to sustain and improve the environmental quality through regulations and controls (Norwegian State Pollution Authority, 2000). The company that discharges the nitrogen must decide how to handle the waste, based on the regulations placed by authorities, reports from researchers, and the financial situation of the company, but must make their own decisions of how to meet the criteria. However, they may not have sufficient expert knowledge to address the problem, and must therefore hire experts from various environmental institutions that gather data and generate reports that may help them make the decisions. These services are expensive, and decisions may be made without the decision-maker knowing how the system will react, and why certain reactions are generated.

The local authorities must work to enhance the environmental conditions for the community through clean air and a clean fjord. However, the community will not have an economic basis without the companies. The sustainability of both the company and the fjord is crucial for the sustainability of the community surrounding the fjord. The balance is difficult to obtain, and the environmental officers of the community surrounding the fjord constantly face this problem.

It is in the interest of both the local authorities and the companies both to obtain a clean fjord and the production at the factories. Therefore, a close cooperation has been developed involving the local authorities, the decision-makers at the companies, researchers at the Norwegian Institute of Water Research (NIVA), Alex Stewart Environmental Services, and the Norwegian State Pollution Authority (SFT). The fjord is part of a national surveillance program, where the pollution levels of the fjord periodically are measured. However, a common model where they can assemble their knowledge, and that identifies which data is important to gather, is lacking.

One of the main intentions for building a system dynamic model of the oxygen problem in the Sørfjord, is for decision-makers to obtain a better understanding of the system they are

making decisions about, and create more awareness by enlightening the problem. These results are however not studied. Further, the Sørfjord model is only an attempt to model the main structure and generate the main behavior of the oxygen levels over time. For decisions to be made based on the model, further development and adjustments are considered necessary.

1.4 Pre-project

System dynamics has not typically been used to model this kind of problem. A short pre-project was therefore carried out in December 1999 in order to find out whether system dynamics is a suitable method (Hervig & Viste, 1999, 2000). The chemical reactions causing oxygen consumption resulting from nitrogen waste from Odda Smelteverk were modeled. Throughout the process it was made clear that the problem was of interest to local authorities, decision-makers, and scientists, but that they lacked a common model in which they could organize their knowledge. There was a lack of models that could transform data into relevant and coherent information, and help identify needs for additional data. Decision-makers and researchers revealed that there were uncertainties of how a change in pollution strategies would affect the system.

The pre-project showed that system dynamics is well suited for modeling ecological processes. They are extremely complex, and it is difficult to understand how the variables relate to each other and cause certain behavior patterns. The water in the fjord consists of accumulations such as oxygen, various forms of nitrogen, bacteria, and plankton. These accumulations influence each other through positive and negative feedback loops. The accumulations create delays in response to changes in the system and this cause the behavior to be relatively unpredictable. By gathering the information that exists about the problem into a system dynamics model it may be easier to analyze how the accumulations affect each other and the behavior of the system.

1.5 Information Gathering

In January 2000 it was decided that the pre-project would be followed by a larger model of the oxygen production and consumption in the fjord. The initial collection of data started through informal conversations with environmental officers at the three factories in Odda; Odda Smelteverk, Norzink, and Tinfoss Titan & Iron. At Odda Smelteverk the process of producing DCD, and how the DCD in the filtercake was discharged into the fjord was explained. Environmental officers at the town of Odda helped with information about the environmental situation in Odda and the Sørfjord in general, and about sewage discharged into the fjord by the community. Alex Stewart Environmental Services in Odda had previously participated in collecting data about various pollution levels in the fjord, and information about this was provided.

Meetings were held at the Norwegian Institute of Water Research in Oslo. Scientists who had long experience in doing research on the Sørfjord gave information about pollution in the Sørfjord. An overview of the nitrification process was also given. Researchers at the Institute of Microbiology at the University of Bergen helped in giving an overview of photosynthesis, phytoplankton and nitrification, while a researcher at the Institute of Oceanography gave an overview of the main water movements in the fjord.

After obtaining an overview of the domain through meetings with researchers and decision-makers, a literary study was performed in order to develop a more detailed picture of the processes in the fjord.

1.6 Decisions of Development of the Model

It was decided that the model should comprise of two different parts. First, a biology model, including the main variables concerning oxygen production and consumption in an average liter of water would be developed. This model would later be merged with a simple hydrological model developed by Sigmund Nævdal (Nævdal, 2001).

The biology model would include oxygen production by phytoplankton and oxygen consumption through nitrification. Reports on a model of eutrophication in the Oslofjord developed by Bjerkeng at NIVA (Bjerkeng, 1994, 1995) were used as a basis for oxygen production, however only the main variables from this model would be extracted.

The reports used were:

- NIVA Rapport Lnr. 3113/94 Eutrofimodell for Indre Oslofjord, Rapport 2: Faglig Beskrivelse av Innholdet i Modellen
  (NIVA Report Lnr. 3113/94 Eutrophication model for the Inner Oslofjord, Report 2: Technical Description of the Content in the Model).

- NIVA Rapport Lnr. 3116/94 Eutrofimodell for Indre Oslofjord, Rapport 5: Fytoplankton-prosesser – Et Litteraturstudium
  (NIVA Report Lnr. 3116/94 Eutrophication model for the Inner Oslofjord, Report 2: Phytoplankton Processes – a Literary Study).

The time perspective used in the Oslofjord model is different from that of the Sørfjord model. The Oslofjord model focuses on the short-term development of the variables in the fjord, such as the phytoplankton adaption to light during the day (Bjerkeng, 1994). Hours are therefore used as time units. In the Sørfjord model it is not necessary to know the changes that occur every hour or day. The decision-makers are mainly interested in the oxygen development in the fjord over years. The overall structure and behavior is considered most important, in order to be able to focus on seasonal changes and the effect of changes in nitrogen pollution over months. A time unit of days is therefore chosen.

Different time scales may also require different model boundaries (Nihoul, 1975). The Oslofjord model is a more detailed model of the biological environment in the fjord than is needed for the Sørfjord model. Many of the variables are therefore left out for the Sørfjord model since the goal of this model is to give a representation of the major behaviors of the fjord, not a detailed description. In addition, variables necessary for modeling the particular situation in the Sørfjord are added.

1.7 Overview of the Thesis

Richardson and Pugh (1981) present the following seven stages in the development of a system dynamic model:

1. Problem identification and definition
2. System conceptualization
3. Model formulation
4. Analysis of model behavior
5. Model evaluation
6. Policy analysis
7. Model use or implementation

These steps are used as a basis for the written presentation of the thesis, however the model development has been an iterative process, and it is not possible to follow one step at a time. Step '6. Policy analysis' and step '7. Model use or implementation' are omitted. Policy development is not considered relevant because decisions are not intended to be made based on the model, and model use and implementation is not documented because it is not studied.

The thesis has the following structure:

Chapter 1: Introduction

This chapter presents the major goals of the thesis and the pre-project that the initial work on the thesis is based on. The problem is briefly defined. The structure of the thesis and the division between the candidates is also indicated.

Chapter 2: Context

A definition of the problem is given in chapter 2. Here, the background for the oxygen problem and the major theories about the main variables concerning oxygen production and

consumption, are described. There is also an explanation of why system dynamics is applicable to the problem at hand.

Chapter 3: Conceptual model

In this chapter there is an explanation of why the particular model boundary was chosen. The theoretical backgrounds for the selected variables are described. The theories behind the equations that will later be used in the model are also explained. Causal loop diagrams for the overall model, and later for the different parts are included in order to give a better conceptual picture of the problem at hand.

Chapter 4: Implementation

Chapter 4 gives an explanation of the implementation of the model in Powersim Constructor™ 2.51. First, the implementation of the biology model and later the merging between the biology model and the hydrodynamic model is documented.

Chapter 5: Validation

Various tests are run on the model in order to increase confidence in the model. The model is evaluated regarding how well it is in coherence with existing data and theories about the system. The model is tested with respect to both structure and behavior.

Chapter 6: Analysis

An analysis of the model behavior based on the domination of different feedback loops is presented in chapter 6. The model is run with different input variables in order to find out how it reacts.

Chapter 7: Conclusion

Conclusions and results from the work on the project are presented in chapter 7. Suggestions to further research and further development of the model are also presented.

1.7.1 Division of Written Work Between the Candidates

The development of the biology model and the merging of the biology model with the hydrodynamic model were done through close cooperation between the candidates.

The written thesis however, is divided between the candidates as follows:

Magnhild Viste:

1. Introduction

2. Problem Description

3. Conceptual model

3.1 Model Boundary
3.2 Simple Causal Loop Diagram – The Biology Model
3.3 Conceptual Model – the Hydrodynamic Model
3.4 Simple Causal Loop Diagram – The Biology and Hydrodynamic Model
3.5 Conceptual Model – Nitrification
3.6 Theoretical Background for Nitrification
3.9 Conceptual Model for Predator-Prey
3.10 Theoretical Background for Predator-Prey

4. Implementation

4.1 Implementation of Nitrification in the Biology Model
4.3 Implementation of Predator-Prey in the Biology Model
4.4 Implementation of the Hydrodynamic and Biology Model
4.5 Implementation of Nitrification in the Hydrodynamic Model
4.7 Implementation of Predator-Prey in the Hydrodynamic Model

5. Validation

Andreas Hervig:

3. Conceptual Model

3.7 Conceptual Model for Phytoplankton and Photosynthesis
3.8 Theoretical Background for Phytoplankton

4. Implementation

4.2 Implementation of Phytoplankton in the Biology Model
4.6 Implementation of Phytoplankton in the Hydrodynamic Model

6. Analysis

Written by both candidates:

7. Conclusion

Each page is marked with the name of the candidate it was written by.

2. Problem Description

2.1 Context

The Sørfjord is a branch of the Hardangerfjord in Western Norway. Figure 2-1 is a map of the innermost part of the fjord. At the end of the 19th century, Odda, a small town by the Sørfjord was viewed as an attractive destination for wealthy European tourists (Storaas & Skei, 1996). People came from all over the continent to view the beautiful scenery, and tourism was an important income source for the community. During the industrial revolution in Norway, many types of heavy industry factories were built throughout the country. The need for electricity was great, and at the time transmission losses over power lines were substantial. Therefore, factories were often built in places where development of hydroelectric power was advantageous. Steep mountains and large waterfalls made some villages in Western Norway well suited for this purpose. The Tysso Waterfall in Tyssedal by the Sørfjord was ideal for power exploitation, and in 1906 the construction of a power station began. Two years later a carbide factory opened in Odda, and hydroelectric power was transferred from the new power

Figure 2-1: Odda and the Innermost Part of the Sørfjord.

plant in Tyssedal. This was the end of Odda's era as a tourist destination. The pollution from the factories became unbearable, and because of a lack of knowledge of the effects, no environmental precautions were taken. Later, two additional factories were built in the area around the Sørfjord; DNN Aluminium (1916, later Tinfos Titan & Iron, 1986), and Norzink (1929) (Skei, 1998).

For years waste from the factories was discharged directly into the fjord or into the air. Concerns about the pollution were largely ignored (Skei, 1998). When pollution became a political issue in the 1970's, tests were taken, and the Sørfjord was referred to as the most polluted fjord in the world (Akselsen, 1999). Since that time the factories have been forced to put a wide range of cleaning methods in use and some of the discharges have been terminated completely. The pollution is now reduced by 98%, and the factories are viewed as exemplary in international context as they have made major improvements, and invented new products of part of the waste.

In spite of these improvements the oxygen level in the Sørfjord has periodically for the last 30 years shown critical values (Skei, 1973, 1998; Molvær, 1997, 1998, 1999). Oxygen levels are classified in five categories illustrated in figure 2-2. The values are measured in microgram O_2 per liter of water. In the innermost part of the Sørfjord the level has at times reached class V, which is considered very critical. Figure 2-3 shows data of oxygen levels gathered in the Sørfjord (Molvær, 1999). These low values may have a negative influence on the biological organisms. Fish and other mobile organisms may leave the fjord and move to more oxygen rich locations (Molvær, 1999).

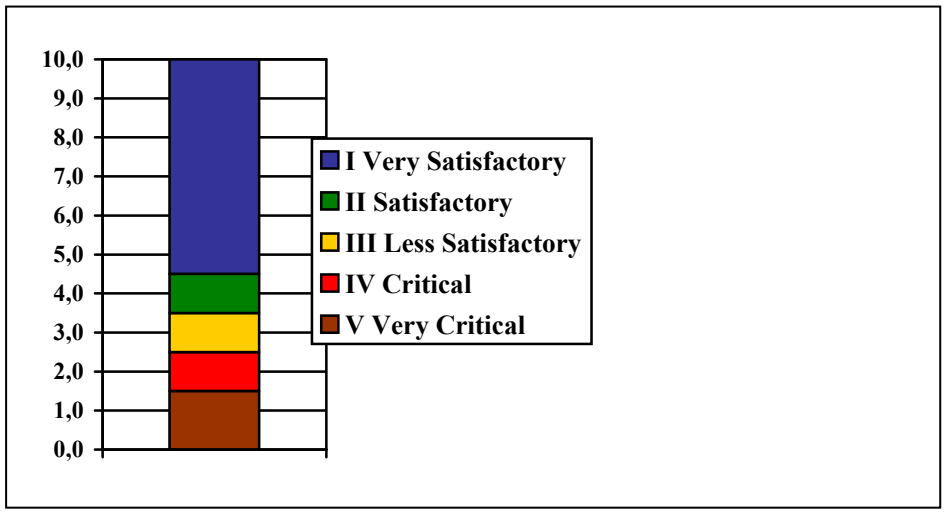


Figure 2-2: Classification of oxygen levels measured in O_2 per liter water (Molvær, 1999).

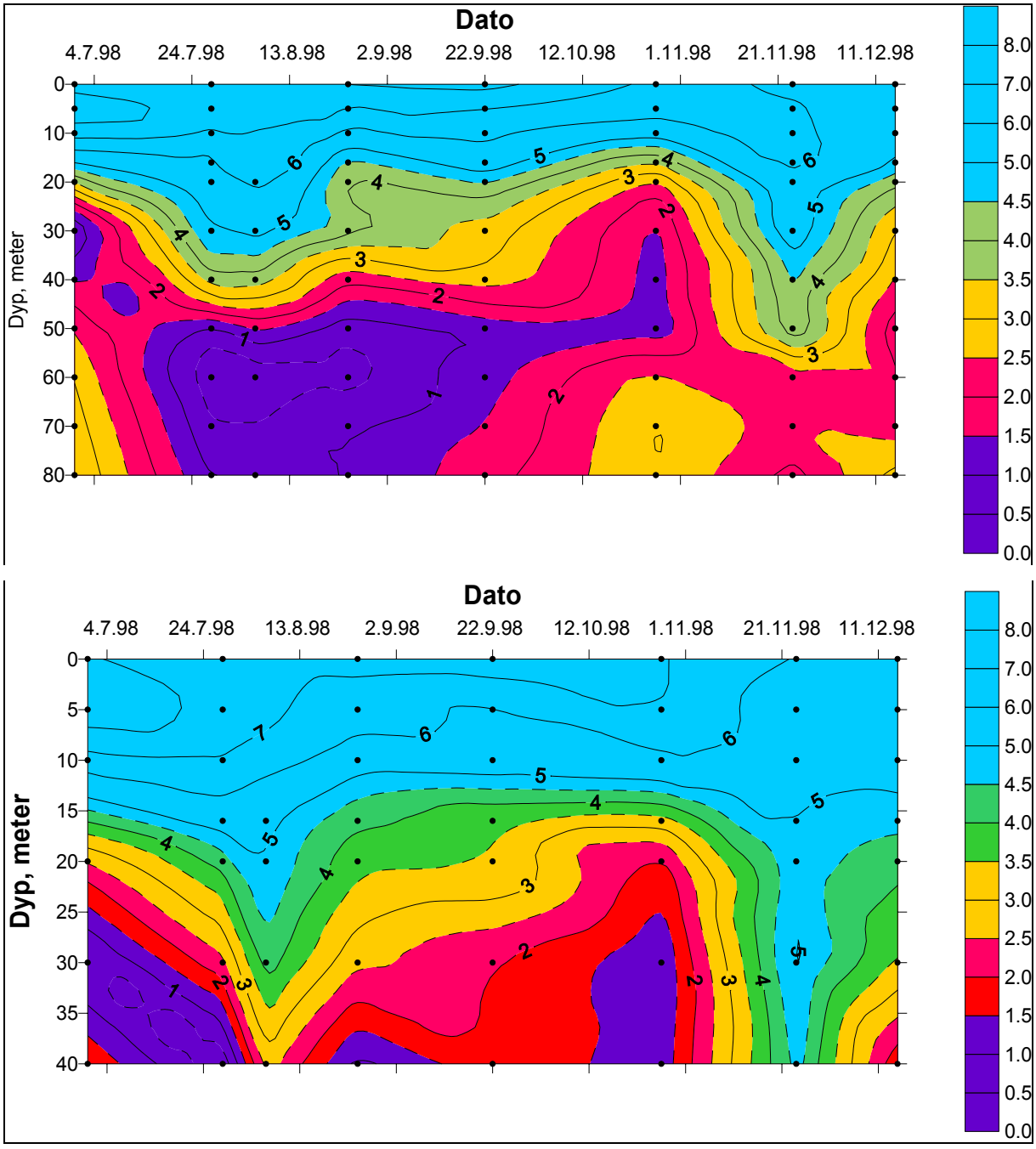


Figure 2-3: Oxygen levels in the innermost part of the Sørfjord (top) and at Lindenes (bottom), June-December 1998 (Source: Molvær, 1999, NIVA 4105-99, p. 10).

2.2 Oxygen Consumption

The low oxygen levels may arise from a number of causes. Oxygen levels can be a problem in fjords in general because the water exchange rate from the ocean is sometimes not sufficient to sustain the oxygen levels necessary for a blooming aquatic life. Many fjords have sockets at the outlet that hinder fresh seawater from flowing into the bottom layer. The Sørfjord does however not have a large socket, and has a relatively good estuarine circulation (Holtan, Molvær, Rygg, & Pleym, 1989).

The most accepted assumption of the low oxygen level in the Sørfjord is that it mainly stems from nitrogen waste from Odda Smelteverk, and that this in combination with a low water exchange rate creates the problem (Skei, 1998; Schaanning, 1999; Molvær, 1999). Nitrogen causes a large chemical oxygen consumption. The nitrogen is part of waste from a filter that is used in the melting process at Odda Smelteverk. Since 1954, 40 000 tons of filter-cake have been discarded into the fjord annually, and it is estimated that 1.2 percent of this is DCD. DCD is an abbreviation of dicydiamide, and has the chemical formula $(CN)_2(NH_2)_2$ (Schaanning, 1999). It is used for inflammable impregnation, pharmaceutical products, epoxy, explosives, paint, and fertilizers.

The filter cake is mixed with water, and flows into the fjord through a pipe located 20 meters below sea level. The area below the pipe is gradually filled up, and when the pile reaches some height, the pipe is moved. In the dock area in the innermost part of the fjord there are piles up to 10 meters containing filter cake. However, only part of the filter cake ends up in the sediment. About 60% is dissolved relatively immediately into the water (Schaanning, 1999).

Both DCD that is dissolved directly into the water and DCD in the sediment will within a time span of a couple of weeks be transformed into ammonium (Schaanning, 1999). The ammonium will be transformed into nitrite (NO_2^-) and later nitrate (NO_3^-). This process is called nitrification and consumes large amounts of oxygen. The process is performed by several types of nitrifying bacteria, also called nitrifiers. The most common species are nitrosomonas and nitrobacts, responsible for each nitrification step, respectively (Henriksen and Kemp, 1988).
There are other factors that may influence the low oxygen levels as well. Sewage from Odda has for a long time been discarded directly into the innermost part of the fjord through several

different pipes, causing a higher supply of nitrogen in shallow waters. This may also cause oxygen consumption through nitrification. Most of the pipes have been moved further out in the fjord, where the outflow to the ocean is expected to be better (Garmann & Co, 1999). The last pipes will be moved within a couple of years. By 2003 a purification plant that removes 5-10% of the nitrogen in the sewage will be ready, and all the small sewage systems will be gathered into one outlet. However, there are disagreements of the effect on oxygen consumption, since sewage is mixed with fresh water. Fresh water has a lower density than salt water, and the sewage will therefore float to the top where there is a higher oxygen production. Also, the nitrogen from sewage is only estimated to be about 3.6% of the total nitrogen input (Molvær, 1997).

The natural inflow of nitrogen to fjords is usually fairly high, and it is estimated that about 40.6% of the nitrogen that flows into the Sørfjord comes from the river Opo in the innermost part of the fjord and other rivers along the sides of the fjord (Molvær, 1997). This also includes nitrogen from the farmland around the Sørfjord. However, there are uncertainties of the contribution of nitrogen from the river to the oxygen problem. The fresh water from the river mainly mixes with the top layer of the fjord water since it does not contain any salt. The lower the salinity of the water, the lighter it is, causing water from the river with no salinity to remain at the top and float towards the outer end of the fjord. The oxygen level is usually high in the top layer of the fjord, and mainly a problem only in deeper layers (Molvær, 1999).

2.3 Oxygen Production

The oxygen levels are influenced by the oxygen production from photosynthesis in the fjord. The photosynthesis process is performed by microorganisms called phytoplankton (Paasche, 1988). The phytoplankton contain chlorophyll and utilize the light in order to produce oxygen. Temperature also influences this process. The phytoplankton assimilate nutrients such as ammonium, phosphorus and carbon through the photosynthesis in order to grow. A lack of nutrients hinders the process, and there will be less oxygen and phytoplankton production. Phytoplankton are predated by zooplankton which are above them in the food chain (Chapra, 1997).

2.4 The Water Exchange Rate

The Sørfjord is a fjord arm of the Hardangerfjord (see figure 2-4). The time it takes for all the water in the fjord to be exchanged with water from the Hardangerfjord is called the water exchange rate. According to researchers at NIVA, this rate is assumed to be between 2 and 12 weeks, however, limited data to base the assumptions on is gathered. Data gathering of the water exchange rate in the deeper layers is a difficult task to accomplish because the water from the Hardangerfjord flows in quite frequently and in varying amounts (Svendsen, 1973). The outflows in the middle layers are equally difficult to determine because they are dependent on the inflowing water.

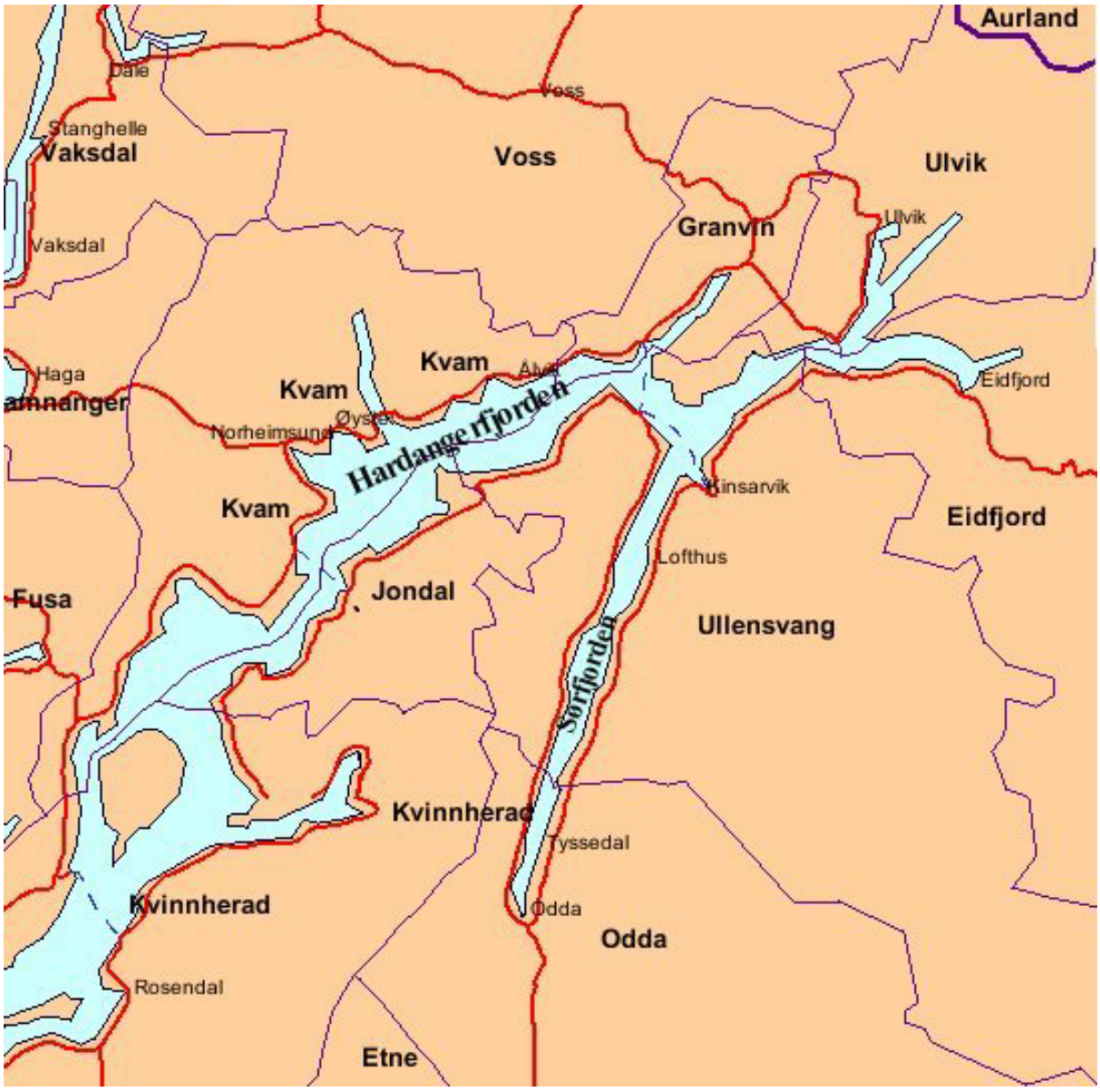


Figure 2-4: The Hardangerfjord and the Sørfjord.

The water exchange may be an important factor that influences the oxygen levels in the Sørfjord (Molvær, 1999: Skei, 1998). It may represent a net oxygen inflow or outflow, depending on the difference in the oxygen levels in the Sørfjord and the Hardangerfjord. If the water in the Hardangerfjord contains higher oxygen levels than the Sørfjord, there will be a net inflow of oxygen to the Sørfjord, if it is lower there will be a net outflow. Oxygen levels may often be a problem in fjords, and it is therefore likely that most often, the water from the Hardangerfjord, which is closer to the ocean, represents an inflow of oxygen to the Sørfjord. Further, all the other chemical and microbiological elements also flow with the water from the Hardangerfjord into the Sørfjord. They also represent net inflows and outflows depending on the difference between levels in the Sørfjord and the Hardangerfjord.

2.5 The Opo River

The water from the Opo River in the innermost part of the fjord represents a contribution both to the oxygen and nitrogen input to the fjord. Nitrogen from the river and other natural factors are estimated to represent 40.6 % of the total nitrogen input to the fjord.

The water from the river contains no salt, and therefore has a higher density than the brackish water in the fjord. The water therefore mainly remains at the top of the fjord, but gradually mixes with some of the fjord water as it flows towards the outer end of the fjord (Svendsen, 1973). The water that flow out of the fjord at the top is compensated by a deeper ingoing flow. Hence, the water from the river contributes to the hydrodynamics of the fjord.

The amount of water from the Opo River that flows into the fjord varies greatly, and is at its peak during the summer months when snow is melting in the mountains (Svendsen, 1973). Data from 1972, suggest that the water in the Opo River is at its lowest from December to April (Svendsen, 1973). Data from 1995 and 1996 indicate that there is a peak in the water inflow from the river during the summer months, and also in November-December (Molvær, 1998). The depth of the top layer of the fjord also varies with the amount of water from the river.

2.6 Overview of the Problem

Figure 2-5 presents an overview of the relations between the main variables concerning the problem. Polarities are not shown here since the causal relationships will be further elaborated in chapter 3.

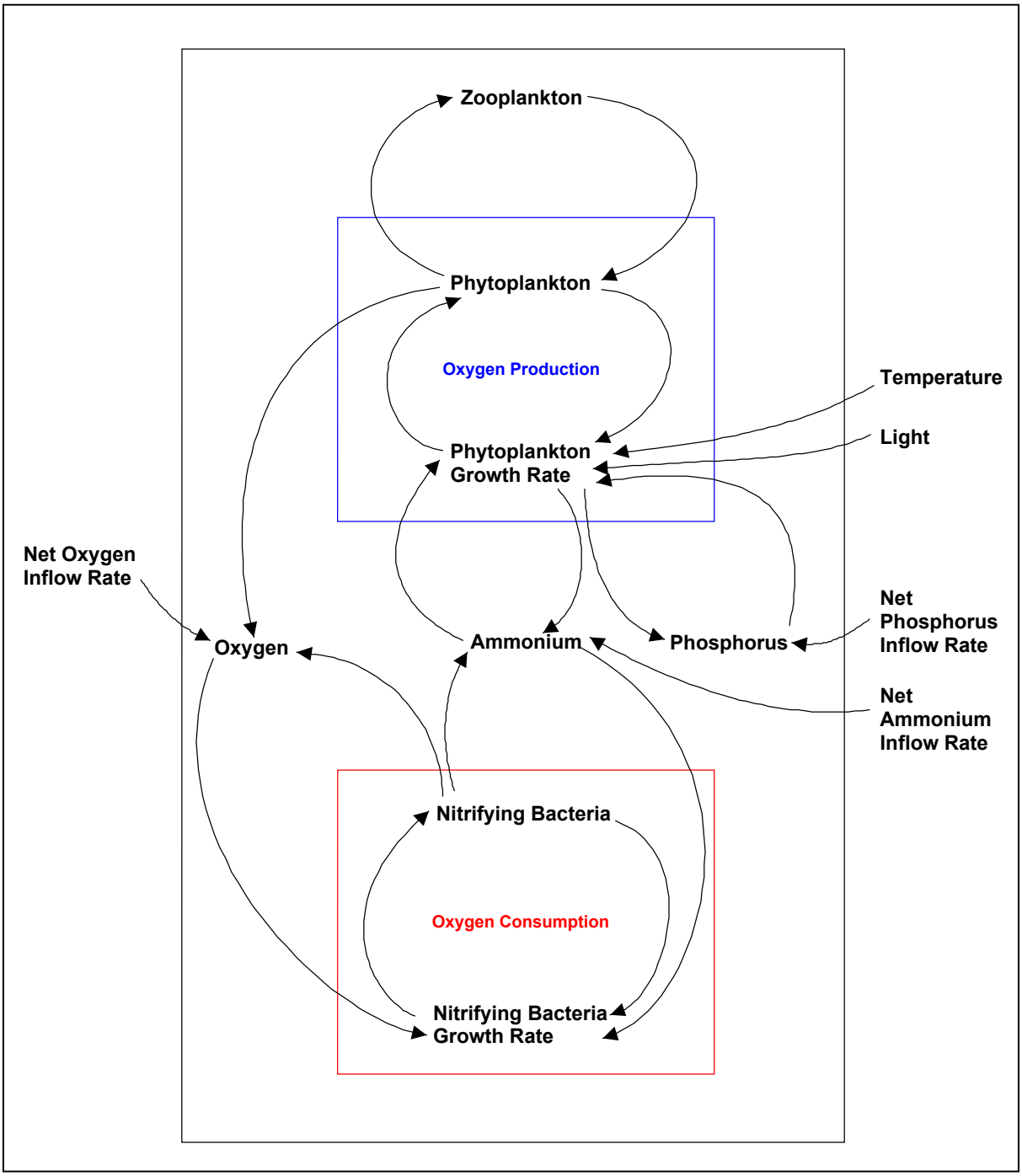


Figure 2-5: An overview of the major components of the problem.

Oxygen is produced through the phytoplankton growth rate. This is influenced by the temperature in the water, the light intensity, and the ammonium, phosphorus, and phytoplankton levels. The phytoplankton influences the zooplankton level and vice versa. The oxygen inflow rate also has a positive effect on the oxygen level.

Oxygen consumption is determined by the nitrification rate. The nitrification is performed by the nitrifying bacteria through their growth. The process is influenced by the ammonium and oxygen level in the water. Both oxygen and ammonium is consumed through the process.

The levels are also influenced by the water exchange rate. Oxygen, phosphorus, ammonium, zooplankton, phytoplankton, and nitrifying bacteria flow in and out of the system with the water exchange at the outermost part of the fjord. For simplicity it is not shown for phytoplankton, zooplankton and nitrifying bacteria in figure 2-5.

2.7 Why System Dynamics is Applicable to the Problem

One of the aims, but also difficulties of system dynamic modeling is to discover and represent dynamics caused by feedback, non-linearity, and delays (Sterman, 2000). A system or problem must contain these characteristics to be suitable for system dynamic modeling. In the following, examples are given of how the oxygen problem in the Sørfjord contains these characteristics, which makes it suitable for system dynamic modeling.

2.7.1 Dynamic System

A dynamic system contains levels or quantities that change over time (Richardson and Pugh, 1981). The problem in the Sørfjord is the changing oxygen level, and one would like to develop a better understanding of the underlying feedback structure that causes these changes. Figure 2-6 shows data of the oxygen levels from 05.01.95-11.06.97 collected by NIVA (Skei, 1998). From a system dynamic perspective the varying oxygen levels change due to the system structure, that is, due to changes in state variables that effect the oxygen level.

Through development of a system dynamic model it may be possible to identify the major variables and their interaction that cause the changes in the oxygen level.

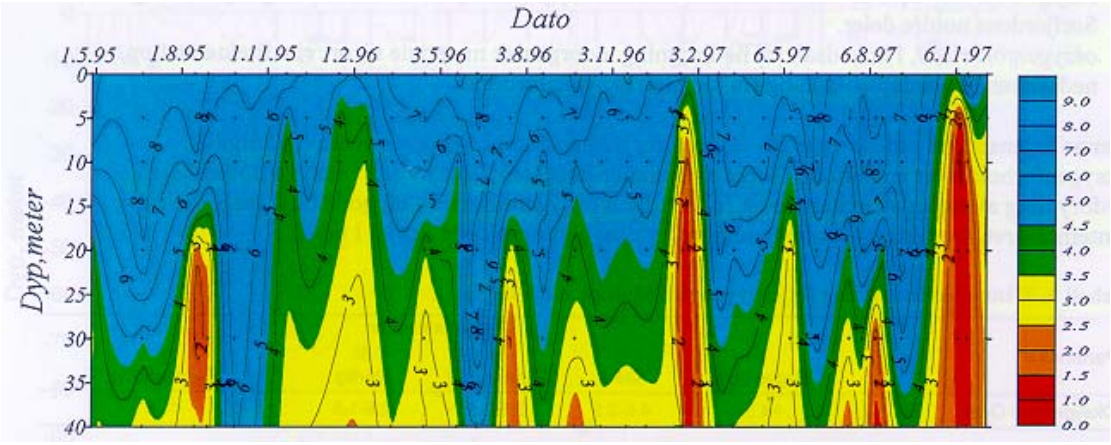


Figure 2-6: The Oxygen Levels in the dock area in the innermost part of the Sørfjord, 05.01.95-11.06.97 (Source: Skei, 1998, NIVA 742/98, p. 24).

2.7.2 Feedback System

An essential element of the system dynamic method is the importance of causes, effects, and system feedback. The dynamics of a system arise from the interaction of the feedback structure in the system (Sterman, 2000). Feedback means that if a change in the value of variable A effects variable B, the change in variable B may inflict a change in variable A. According to Richardson and Pugh (1981) 'a feedback loop is a closed sequence of causes and effects, a closed path of action and information'. A set of interconnected feedback loops is called a feedback system. Figure 2-7 shows an example of three feedback loops in the Sørfjord. The nitrifying bacteria level influences the oxygen

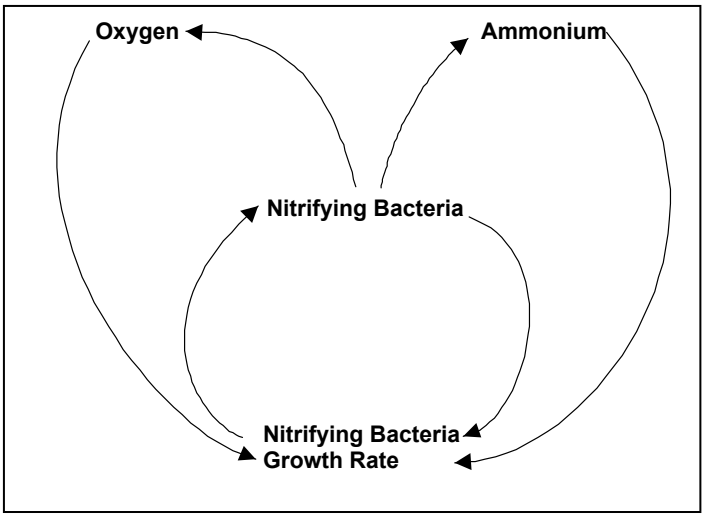


Figure 2-7: An example of feedback loops in the Sørfjord.

level, which influence the nitrifying bacteria growth rate, which again influence the nitrifying bacteria level. A similar loop exists for nitrifying bacteria, ammonium and the nitrifying bacteria growth rate. There is also a feedback loop between the nitrifying bacteria level and the nitrifying bacteria growth rate.

2.7.3 Nonlinear Relations

A system of oxygen production and consumption in water contains several nonlinear relationships. Equation 2.6-1 describes the rate at which nitrogen is nitrified. As the ammonium and oxygen levels in the water approaches 0, the nitrification rate levels off. This is further explained in chapter 3.6.

$$R = a \cdot N \cdot K_{\max} \cdot \frac{\left[NH_4^+\right]}{\left(K_m + \left[NH_4^+\right]\right)} \cdot \frac{\left[O_2\right]}{\left(K_m O_2 + \left[O_2\right]\right)}$$

Equation 2.7-1

2.7.4 Delays

Delays occur when it takes time for one variable to react to the change in the value of another variable in the system. A delay occurs when the output from a process lags behind the input of that process (Sterman, 2000). Material or information must be accumulated within the process to cause the delay. A system with delays must therefore always include stocks. In the Sørfjord the phytoplankton do not increase immediately when the ammonium supply to the water increases. The phytoplankton must first react to the increasing ammonium level in the water. The ammonium will then be consumed and later used for actual phytoplankton cell growth. This is illustrated in figure 2-8.

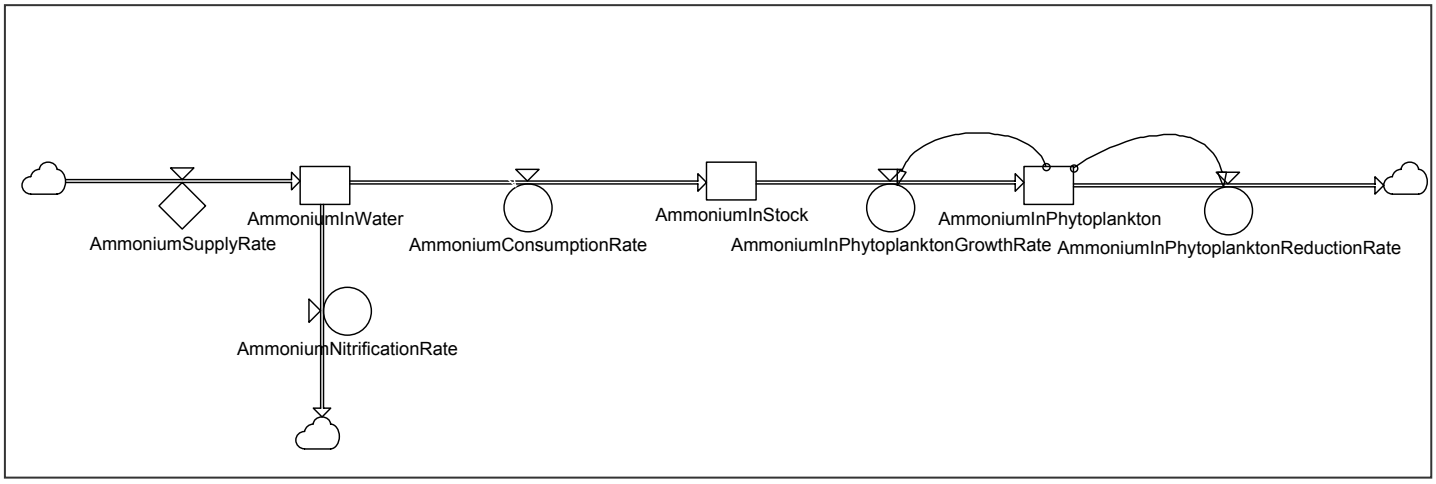


Figure 2-8: An illustration of delays in the Sørfjord system.

3. Conceptual Model

3.1 Model Boundary

Based on conversations with researchers at the Norwegian Institute of Water Research (NIVA) and the Department of Microbiology at the University of Bergen, it was decided that the overall model boundary should comprise of a microbiological model including two main components. One component would consist of phytoplankton growth and the related variables causing the growth. The main function of this component would be oxygen production through photosynthesis. The other component would represent the nitrification process and the variables influencing the process. This part of the model would be oxygen consuming. The first version of the model would show the components of an average liter of water, and their state and behavior under various conditions. The microbiological model would later be integrated into a simple hydrodynamic fjord model of the water exchange and movements, in order to give a better picture of the oxygen consumption and production in different parts of the fjord. These decisions were used as fundaments for elaborating a more specific model boundary.

The definition of the model boundary is a difficult process when building any kind of system dynamic model. When making a model of a segment of an ecological system it becomes particularly problematic. A clear distinction is made between modeling a problem versus a whole system, because focusing on the problem helps define the model boundaries (Richardson and Pugh, 1981). Nevertheless, it is difficult to decide which level of detail to include, and where to set the model boundaries. An ecosystem is defined as 'the complex of a community of organisms and its environment functioning as an ecological unit' (Webster, 1996). The boundaries of the ecological unit is however not always clear (Hessen, 1989). Further, the complexity of an ecosystem is enormous, with countless reciprocal interactions between the various components of the system. The close connection between the numerous variables in the real system may cause a change in the entire behavior of the system when adding or removing one of the variables. For example, if all the predators of a specie are killed or removed, the specie may obtain extremely advantageous growth conditions, causing it to grow until all nutrients are depleted and the system collapses. A similar real incident happened in Australia where feral rabbits were introduced in the second half of the 19th

century (Biodiversity Group Environment Australia, 1999). This is an example of a real ecosystem, where a specie was introduced which was not part of the original system of reciprocal interactions. Thus, the rabbit lacked its main European predators, which had hunted rabbits in the original ecosystem the rabbit was part of. The rabbit also received extremely favorable growth conditions due to the Australian vegetation, and its ability to adapt to the climate. The result was a rapid increase in the rabbit population, which contributed to the extinction of other species, and had a profound effect on the flora and fauna.

This may explain the reason it is also difficult to decide which variables to include and omit in a model of an ecosystem. A model of a fjord may exhibit a similar behavior if for instance zooplankton, which are the predators of phytoplankton, are omitted. Then there may be no limitations to the net phytoplankton growth other than the nutrient content of the water. The phytoplankton will consume all the nutrients in the water until they are depleted, and the phytoplankton may die because of lacking nutrients. In the real system, part of the phytoplankton would be consumed by zooplankton, limiting the net growth of the phytoplankton population, and thereby the phytoplankton level, and further the phytoplankton nutrient consumption.

Considering that a model is intended to illustrate a problem, not a system, most variables must be left out. High leverage variables are variables that have a great effect on the behavior of the system. The main problem in defining the system boundary is to identify these variables, and make sure that they are included in the model. Finding the high leverage variables with respect to the Sørfjord model was an especially difficult process, having limited experience in microbiology, oceanography and other research areas that the problem comprises. Little previous knowledge of the scientific domains made it difficult to judge conflicting, and sometimes opposing advice received from various experts and findings in the literature.

3.1.1 Endogenous Variables

Table 3-1 shows an overview of the main endogenous variables that are included in the model. They are divided into two columns, representing levels and flows respectively. The levels are the substances that exist in the water and flow with the water movements in the fjord. The four main inorganic substances in the water are carbon (C), ammonium (NH_4^+),

Endogenous Variables

Levels:

- Carbon in water
- Ammonium in water
- Phosphorus in water
- Oxygen in water
- Carbon in phytoplankton
- Ammonium in phytoplankton
- Ammonium in phytoplankton stock
- Phosphorus in phytoplankton
- Phosphorus in phytoplankton stock
- Chlorophyll in phytoplankton
- Zooplankton
- Nitrifying bacteria

Flows:

Oxygen

- Oxygen production rate
- Oxygen consumption rate

Phytoplankton

- Carbon in phytoplankton growth rate
- Carbon in phytoplankton reduction rate
- Ammonium consumption rate by phytoplankton
- Ammonium in phytoplankton growth rate
- Ammonium in phytoplankton reduction rate
- Phosphorus consumption rate by phytoplankton
- Phosphorus in phytoplankton growth rate
- Phosphorus in phytoplankton reduction rate
- Chlorophyll in phytoplankton production rate
- Chlorophyll in phytoplankton reduction rate

Zooplankton

- Zooplankton growth rate
- Zooplankton death rate

Nitrifying bacteria

- Bacteria production rate
- Bacteria death rate
- Ammonium nitrification rate

- Inflow and outflow rates to all levels through the hydrodynamic of the fjord

Table 3-1: The main endogenous variables in the Sørfjord model.

phosphorus (P), and oxygen (O_2). Phytoplankton is an endogenous variable that produces oxygen. In the model it is divided into six variables: carbon, ammonium, phosphorus and chlorophyll in phytoplankton cells, and ammonium and phosphorus in phytoplankton stock. Carbon in phytoplankton cells is included because in much of the literature phytoplankton are measured in the amount of carbon they contain (Bjerkjeng, 1995). The phytoplankton growth rate is also measured in the amount of carbon that is assimilated into phytoplankton cells per time unit. However, carbon in water will never be a constraint on the growth, and does not have the same limiting function as ammonium and phosphorus in water. Ammonium and phosphorus in phytoplankton are included because the ammonium and phosphorus levels in the water are constraints to the phytoplankton growth. It is critical to monitor which substance is lacking in the phytoplankton cells, what they require in order to grow, and the levels of the substances in the water.

The reason for the distinction between ammonium and phosphorus in phytoplankton cells and ammonium and phosphorus in phytoplankton stock, is that studies show that there is not a constant relationship between ammonium and phosphorus consumption from the water and the growth of phytoplankton cells (Goldman & Glibert, 1983; Bjerkeng, 1995). This suggests that part of the ammonium and phosphorus can be stored in the phytoplankton before it is used for actual cell growth. Ammonium and phosphorus in phytoplankton stock represent ammonium and phosphorus that is consumed by phytoplankton from the water, but not yet transformed into phytoplankton cells. It is possible for phytoplankton to take up more phosphorus than is needed to store for when there is a shortage in the water (Bjerkeng, 1995). The stock will be used for growth at a later stage.

Chlorophyll in phytoplankton is important in the model, as it is utilized by phytoplankton in order to produce oxygen. Zooplankton are above phytoplankton in the food chain, and are included as phytoplankton predators, contributing to the reduction of the phytoplankton level. Nitrifying bacteria initiate the nitrification process, and are therefore included as an endogenous part of the system.

The flows in the second column of table 3-1 represent endogenous inputs and outputs to and from the endogenous levels in the model. The oxygen level is controlled internally by the phytoplankton oxygen production rate and the oxygen consumption rate through nitrification.

The levels representing phytoplankton cells are controlled by a carbon, ammonium, and phosphorus growth rate. This is the actual phytoplankton growth where the substance is accumulated into phytoplankton cells. Ammonium and phosphorus is consumed from the water and stored in the phytoplankton cells before it is actually turned into cells through the ammonium and phosphorus consumption rates. Chlorophyll is produced through the chlorophyll production rate, which is closely linked to the phytoplankton growth rate. The phytoplankton die through the carbon, ammonium, phosphorus, and chlorophyll reduction rates.

The zooplankton level is controlled through the zooplankton growth and death rates. The growth rate is based on the amount of phytoplankton that is predated by the zooplankton. The nitrifying bacteria level is controlled by the bacteria production and death rates. The production rate is influenced by the nitrification rate.

In the hydrodynamic model there are inputs and outputs to and from all levels, through diffusion and the flow of the water between the various fjord cells. These are also considered endogenous flows of the model.

3.1.2 Exogenous Variables

Table 3-2 presents the exogenous variables of the model. Exogenous variables are outside the model boundary, and represent the inputs to endogenous variables in the model. They are divided into two sections: controllable and uncontrollable. The controllable exogenous variables are variables that it would be possible to alter in the real system. Changing the input of the controllable exogenous variables are generally the only way of controlling the behavior of the real system. In this case, few variables can be changed directly by human impact because they are controlled by nature. Inevitably, it is difficult for humans to alter the light and temperature in the fjord, although there may be conceivable ways to interfere with them. More phytoplankton may reduce the light intensity, but phytoplankton is still not regarded as

a controllable exogenous variable. Also, it is not considered realistic to change the input from the seawater and the river, as these are natural factors flowing into the system. Theoretically, oxygen may be lead down into the water in order to increase the exogenous oxygen input to the system. However, this is not viewed as a likely solution to the problem. The only controllable exogenous variable that is considered realistic, is the ammonium supply to the water from Odda Smelteverk and other artificial ammonium sources, such as sewage.

There are several uncontrollable exogenous variables in the system that are included as inputs to the model. There are inputs and outputs to all levels through the water exchange rate. This is only the case in the hydrodynamic model, not in the biology model. Oxygen and ammonium are also supplied to the water from the Opo River. Light and temperature are exogenous variables, which have a positive effect on the phytoplankton growth rate.

Exogenous Variables	
Controllable	**Uncontrollable**
- Ammonium supply through industrial waste	- Supply to all levels through the water exchange rate
- Artificial oxygen supply?	- Oxygen and ammonium supply in water from the Opo River
	- Light
	- Temperature

Table 3-2: Exogenous variables.

3.1.3 Omitted Factors

Several factors and related variables that are part of the real system are purposely not included in the model. These are presented in table 3-3. Sewage from the Odda community is omitted because the amount of nitrogen associated with this contamination is considered too small to have a large effect on the system (Molvær, 1999). The pipes have been moved further out in the fjord where the water exchange is better, and it only represents a small fraction of the total nitrogen input. Further, blue mussels are also omitted in order to simplify the model. Nitrate is not included, as phytoplankton mainly prefer ammonium, provided that ammonium is not in scarcity (Bjerkeng, 1995). In the Sørfjord there is an ammonium surplus, and it is therefore likely that the phytoplankton mainly consume ammonium rather than nitrate.

The effect of the salinity in the water on phytoplankton growth is excluded. It is estimated that its effect is minor, compared to the effect of temperature and light (Bjerkeng, 1994). Two factors are omitted related to the death of phytoplankton. First, the oxygen consumption caused by the decomposition of dead phytoplankton is not included. When the phytoplankton die, they sink to the bottom of the fjord, and the destruction of the dead phytoplankton material consumes oxygen. It is not known to what extent this process effects the oxygen consumption in the Sørfjord. Second, the release of ammonium, carbon, and phosphorus substance to the water when the phytoplankton and zooplankton die is also not included. These factors would represent both oxygen consumption and an endogenous ammonium supply, both substances, which are an important part of the oxygen production and consumption. However, it is not known to what extent they would influence the behavior of the model. In a later version of the model the inclusion of these variables should be considered.

The influence of the phytoplankton level on the light intensity is also not included. High levels of phytoplankton make less light shine through the water, thereby limiting oxygen production by phytoplankton. Differences in light intensities at different depths are also not included, however, the model can be adjusted to represent these variations. This does not have a great effect on the behavior of the model, as temperature is differentiated in the various layers, limiting phytoplankton growth in the deeper layers.

The flow of thermal energy between the fjord cells is also omitted. The model could have been modeled in a manner where if for instance the temperature of one fjord cell was 6°C, and the neighboring cell was 8°C, the temperature of the first cell would influence the temperature in the second cell, and thus reduce the temperature. The higher temperature in the second cell would influence the temperature in the fist cell and contribute to an increase in its temperature, depending on the movement of the water.

Omitted factors
- Sewage
- Blue mussels
- Nitrate
- The effect of salinity on phytoplankton growth
- Oxygen consumption caused by decomposition of dead phytoplankton and zooplankton
- Difference in light intensities at different depths
- Release of substance to the water when phytoplankton and zooplankton die
- The influence of the phytoplankton level on the light intensity
- The flow of temperature in the water between the fjord cells
- Zooplankton predators

Table 3-3: Factors that are purposely omitted from the Sørfjord model.

Zooplankton are part of the food chain, and are also being predated by those above them in the chain. A superior predation variable is left out in order to make the model less complex.

3.2 Simple Causal Loop Diagram – The Biology Model

Figure 3-1 shows a causal loop diagram of the overall structure of the biology model, while figure 3-2 presents the names of the main feedback loops. The diagram consists of two reinforcing growth loops, 'R1: Phytoplankton growth' and 'R2: Nitrifying bacteria growth'. Both reinforcing loops are controlled by several balancing loops that limit the growth. Further, there are inputs to the system that cause the variables to change their values and

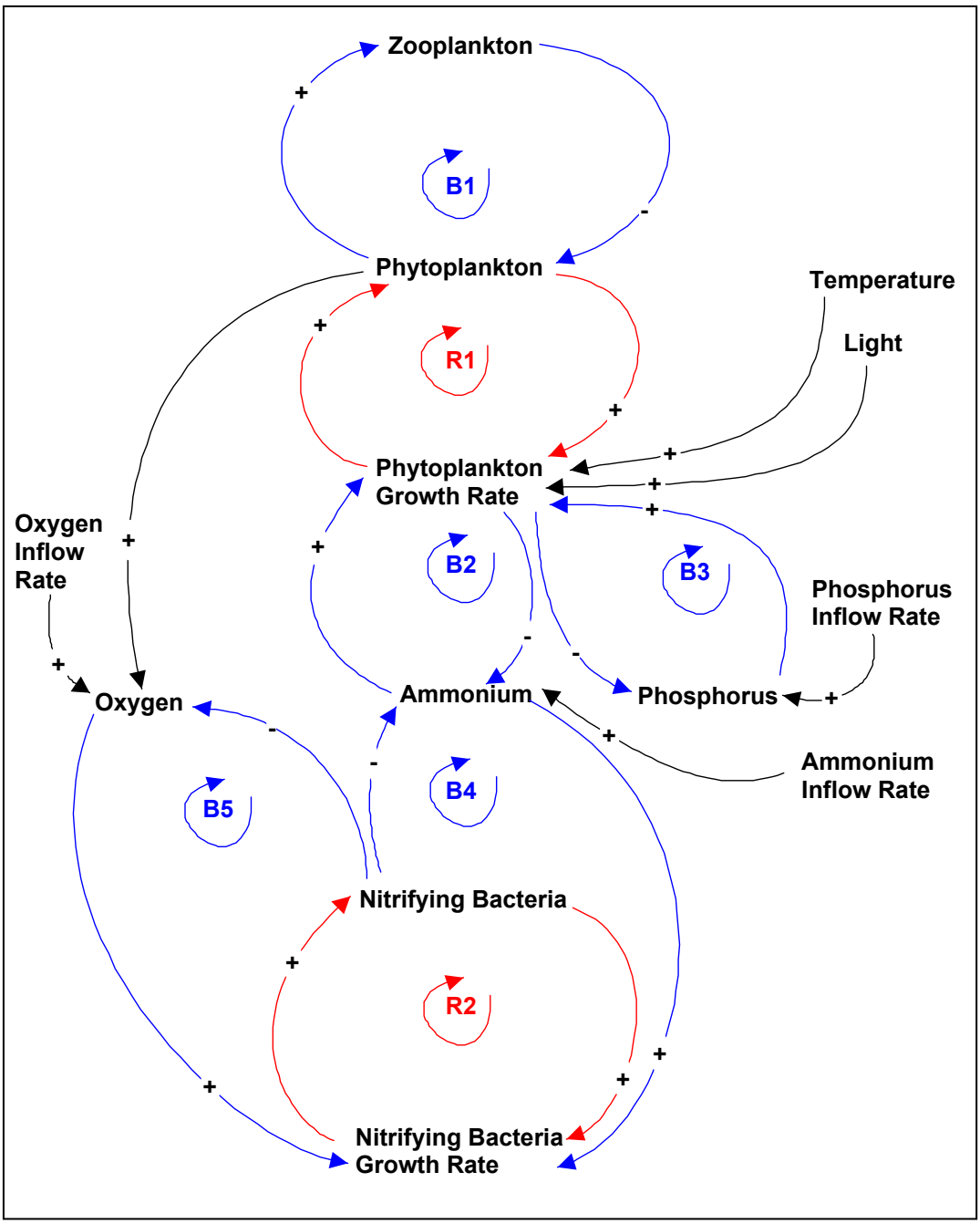


Figure 3-1: Simple causal loop diagram – the biology model (death rates not included).

thereby have an impact on the direction of the whole loop and later the connected loops.

The two reinforcing loops are crucial for the system, as they control oxygen production and oxygen consumption, respectively. The more phytoplankton there are, the more oxygen is produced. The more nitrifying bacteria there are, the higher the nitrification rate, and the more oxygen is consumed, if there are no other constraints limiting the process.

R1: Phytoplankton growth
R2: Nitrifying bacteria growth
B1: Zooplankton predation
B2: Phytoplankton growth by ammonium
B3: Phytoplankton growth by phosphorus
B4: Nitrifying bacteria growth by ammonium
B5: Oxygen consumption

Figure 3-2: The main feedback loops of the Sørfjord model.

3.2.1 Phytoplankton Growth

'R1, Phytoplankton growth' loop describes how the phytoplankton increase based on their growth rate. The more phytoplankton there are, the higher the phytoplankton growth rate. The higher the growth rate, the higher the phytoplankton level. It is important to note that if the phytoplankton growth rate is reduced, this will not reduce the amount of phytoplankton, as a growth rate will always increase a level. This is a common problem with causal loop diagrams (Richardson, 1986). The model also contains a phytoplankton death loop. For simplicity this is not shown here, but in the more specific causal loop diagram for phytoplankton in section 3.7.

There are three balancing loops controlling the phytoplankton growth loop. 'B1: Zooplankton predation' is a standard predator-prey loop (Lotka, 1956), where zooplankton eat phytoplankton, causing the phytoplankton level to be lower than it otherwise would have been. As the phytoplankton level decrease, the living conditions for zooplankton will deteriorate, causing a lower birth rate for zooplankton. When the zooplankton level decreases

the living conditions for phytoplankton will again excel, and fewer of them will be devoured by zooplankton.

'B2: Phytoplankton growth by ammonium' and 'B3: Phytoplankton growth by phosphorus' are balancing loops that control the phytoplankton level by limiting the level of nutrients they can consume. Higher levels of ammonium and phosphorus in the water will increase the phytoplankton growth rate. As the growth rate increases, ammonium and phosphorus is consumed from the water and reduced to levels lower than they otherwise would have been. When ammonium and phosphorus reach a certain lower level they become constraints on the phytoplankton growth, and thereby cause a reduction in the phytoplankton growth rate. When the growth rate decreases, less ammonium and phosphorus is used, and the substance levels may increase as long as the inflow rate is higher than the consumption.

3.2.2 Nitrification and Nitrifying Bacteria

The nitrifying bacteria grow based on 'R2: Nitrifying bacteria growth' loop. An increase in nitrifying bacteria results in a higher nitrifying bacteria growth rate than there otherwise would have been. The nitrifying bacteria growth loop is controlled by two balancing loops: 'B4: Nitrifying bacteria growth by ammonium' and 'B5: Oxygen consumption'. Nitrifying bacteria grow through the nitrification process, and use ammonium and oxygen in order to perform the process. Therefore, there is a negative relationship going from nitrifying bacteria to ammonium and oxygen. As the levels of ammonium and oxygen go down, the nitrifying bacteria growth rate will decrease. Further, the more oxygen and ammonium there are, the higher the nitrifying bacteria growth rate will be.

Balancing loops tend to go towards a goal. 'Goals are the desired state of the system, and all negative loops function by comparing the actual state to the goal, then initiating a corrective action in response to the discrepancy' (Sterman, 2000, p. 5-27). It will not be possible to determine the behavior of the system by looking at CLD's, because they only show the direction of relationships. They still make it possible to understand the direction the balancing loops will tend to go. In the Sørfjord model, the reinforcing growth loops will presumably try to grow at an exponential rate, as long as there are enough nutrients for the indicated growth rate. The balancing loops will control the growth of the reinforcing growth loops.

3.3 Conceptual Model – the Hydrodynamic Model

The hydrodynamics of a fjord are influenced by a number of factors, and some of these are listed in figure 3-3. The main factors are pressure in the atmosphere above the fjord, wind, tide, salinity of the water, and temperature variations in the water. The topography is also important in determining the water movements (O'Riordan, 1995). In the Sørfjord, fresh water from the Opo River and other smaller rivers flow into the fjord. These factors result in movements of the water within the fjord, and an exchange of water between the fjord arm (the Sørfjord), and the larger fjord which it is part of (the Hardangerfjord).

- **Pressure**
- **Wind**
- **Tide**
- **Temperature**
- **Salinity**
- **Water from the river**
- **The topography of the fjord**

Figure 3-3: Variables that influence the hydrodynamics of the Sørfjord.

Making a conceptual and formal model of the hydrodynamics of a fjord is a complex task. It is difficult to decide what level of aggregation to choose based on previous knowledge of the hydrodynamics, and the complexity needed to understand and solve the problem. In the Sørfjord model, the fjord is modeled as a channel where the water flows in and out in the same end, but at different depth layers (Nævdal, 2001). This is illustrated in figure 3-4. The water flows in at the bottom layer, and out through the middle layer. The top layer also represents an outgoing flow of water, where the water from the Opo River pushes the water out towards the ocean. The Opo River flows into the fjord at the innermost part of the fjord. Having no salinity, the water from the river remains mainly at the top, and mixes gradually with some of the fjord water.

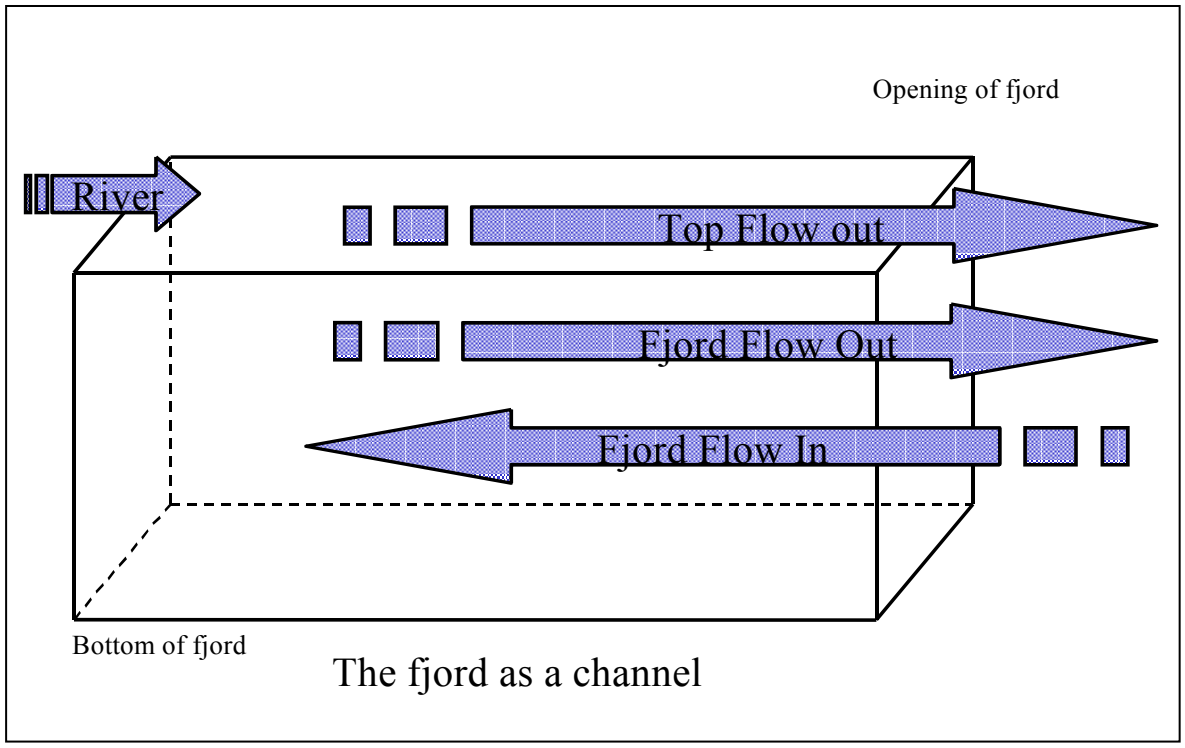


Figure 3-4: The main flows of the water in the hydrodynamic model (Nævdal, 2001, p. 1).

The Sørfjord is conceptualized containing three depth layers and a varying number of horizontal layers that can be set by the user. This divides the fjord into a number of fjord cells. Figure 3-5 presents an illustration of the cells that the fjord is divided into and the flow pattern of the water through the fjord cells.

The water from the Hardangerfjord comes in at the bottom of the fjord, in the bottom cell to the right, and flows gradually leftward through all the cells. In each cell, part of the water proceeds to the cell above, and starts flowing outward again, through all the cells until it reaches the rightmost cell in the middle depth layer, and leaves the system boundary into the Hardangerfjord. The water movement caused by the inflowing water from the Hardangerfjord, and the water that returns to the Hardangerfjord forces this. The flows are greater for the cells closer to the outer end of the fjord, because there the force of the water exchange is greater (for a further explanation, see Nævdal, 2001).

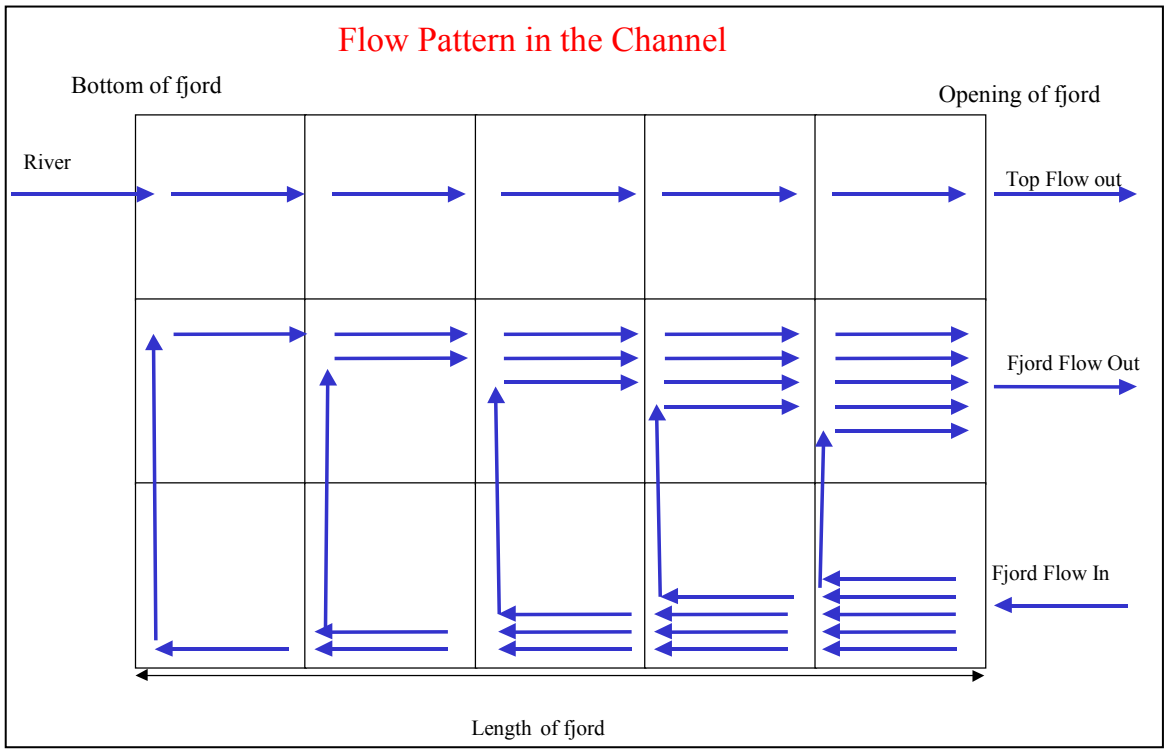


Figure 3-5: A simple illustration of the water movements of the hydrodynamic model (Nævdal, 2001, p. 3).

There is also some diffusion between the fjord cells, causing mixing of water, independent of the water exchange. Figure 3-6 illustrates this. For each substance that the water consists of, there is an exchange of substance with the neighboring fjord cells. The diffusion of each cell is determined by the concentration of the component in the fjord cell, the size of the area separating the neighbors, and a diffusion coefficient which is a diffusion velocity constant (Nævdal, 2001).

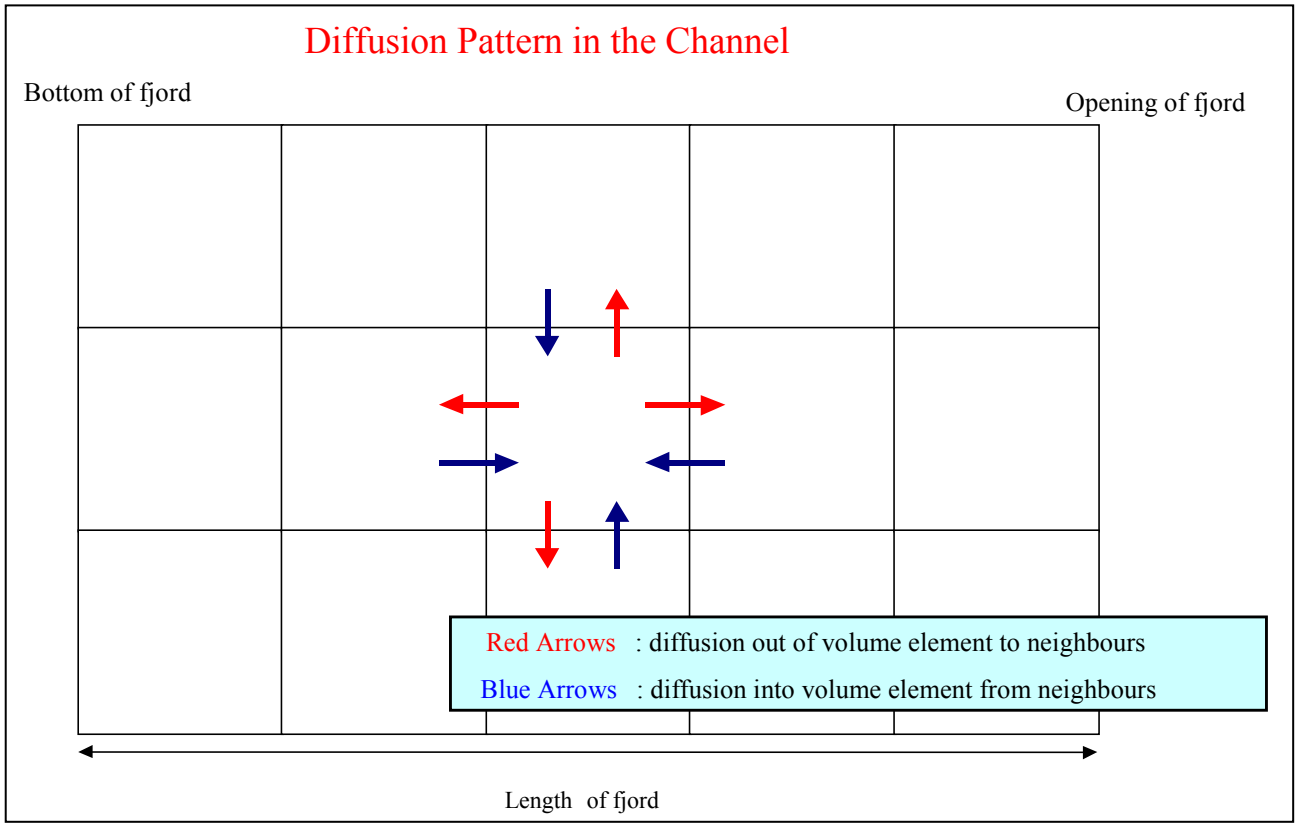


Figure 3-6: Diffusion in the hydrodynamic model (Nævdal, 2001, p. 4)

All cells that the fjord model is divided into contain the state variables in the biology part of the model, such as ammonium, bacteria, oxygen and phytoplankton. These flow with the currents of the water, and the levels may be different for each cell that the fjord model is divided into. The auxiliaries will also vary for the particular cells, because they are determined by the various levels in that cell. The seawater that flows into the rightmost cell at the bottom also contains the state variables of the biology model. The water in the Opo River only contains ammonium and oxygen.

3.4 Simple Causal Loop Diagram - The Biology and Hydrodynamic Model

There is no existing notation for showing a three-dimensional model in a causal loop diagram, and it is also problematic to do so. The main goal of causal loop diagrams are to show the feedback loops that are held responsible for the problem and showing the mental model one has of the problem (Sterman, 2000). They show a simplistic view of the causations that the system is believed to contain. We have made an attempt to construct a three-dimensional CLD, as it makes a better conceptual picture of how the formal microbiological model is integrated with the formal hydrodynamic model (see figure 3-7). The components that flow out of one cell and into another with the flow of the water are marked with arrows going in and out of the component. These variables include oxygen, ammonium, phosphorus, phytoplankton, zooplankton, and bacteria. Temperature is still considered an exogenous variable, although thermal energy could have been modeled to flow with the water, and effect the neighboring cells. This however, is not included in the model.

It is important to note that the arrows indicating flows between the fjord cells do not have a polarity, as they do not represent a direct causal relationship. It may be possible to state that they have a positive effect on the neighboring fjord cell since part of what is in one cell will flow into the nearest cell at the next time step. For instance, the more oxygen one fjord cell contains, the more will flow into the next fjord cell at the next time step. The less oxygen there is, the less will flow into the neighboring cell at the next time step. This CLD describes the processes within a fjord cell, and the arrows without polarity act as reminders that there are also flows between the fjord cells. These arrows represent the hydrodynamics of the fjord.

For the causal loop diagram to give a clearer description of the hydrodynamics of the outermost fjord cells, it should have included net inflows to all levels. This would describe the supplies and drains of various substances through the water exchange rate. For instance, new oxygen flows with the seawater into the bottommost fjord cell to the right. Oxygen in the two cells at the top furthest to the right flows out of the system and into the Hardangerfjord.

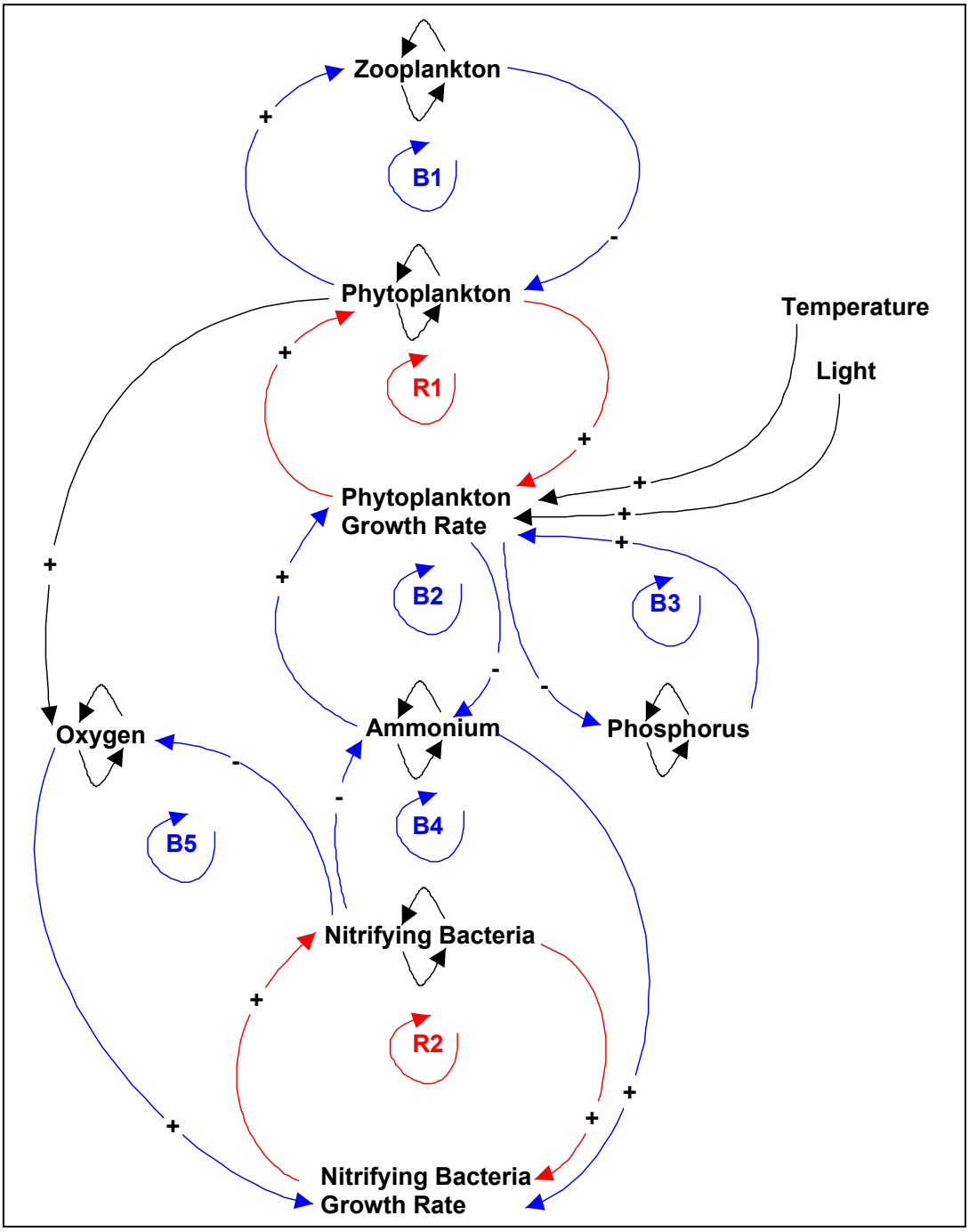


Figure 3-7: Causal loop diagram for biology and hydrodynamic model.

3.5 Conceptual Model – Nitrification

3.5.1 The Nitrogen Cycle

There are several forms of nitrogen in nature, including inorganic ammonium (NH_4^+), nitrite (NO_2^-), nitrate (NO_3^-), and ammonia (NH_3^+), as well as several other complex amines, amino acids, peptides and relatively unreactive molecular nitrogen (Brooks, 1969). The various forms of nitrogen are part of the nitrogen cycle, which is the continuous loop that inorganic nitrogen forms follow from organisms through air, soil, and water, and back into air soil and water. The main processes involved are nitrogen fixation, nitrification, decomposition, and denitrification (Merriam Webster's Collegiate Dictionary, 1996). Figure 3-8 gives a description of the fundamental processes. Nitrogen exists as nitrogen molecules (N_2) in the atmosphere. The nitrogen molecules are fixated from the atmosphere by plants and through industrial fixation. The industrial fixed nitrogen is either released directly into the ecosystem

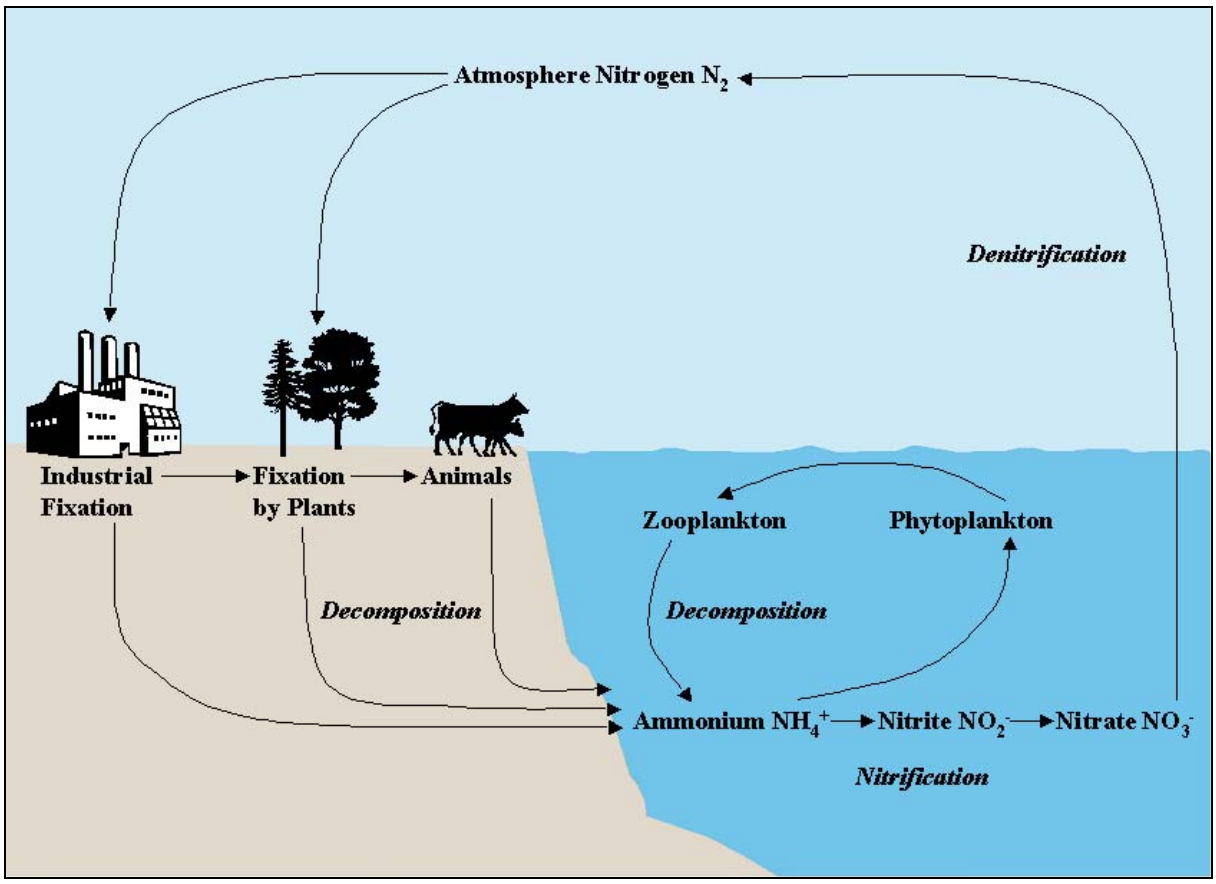


Figure 3-8: The nitrogen cycle.

as waste or fertilizers, which plants subsequently use as nutrients for growth. Animals consume the plants and the nitrogen they contain. When the plants and animals die, bacteria decompose the organic material, and the nitrogen is once more transformed into inorganic nitrogen compounds such as ammonium. The ammonium from the soil flows into the water through the ground water. Part of the ammonium is utilized as nutrients by phytoplankton and other plants in the sea. Further, the phytoplankton are consumed by zooplankton. When the zooplankton and the phytoplankton die, the matter is decomposed and the nitrogen they contain will return as inorganic ammonium to the water. Nitrifying bacteria nitrifies part of the ammonium into nitrite. Nitrite is then transformed into nitrate by similar bacteria. The nitrification process may occur both in soil and in water. Nitrate is denitrified into gaseous nitrogen that returns to the atmosphere as nitrogen molecules (N_2). The nitrogen cycle is completed when the nitrogen molecules are again fixated through industrial fixation and fixation by plants.

3.5.2 Nitrification

Nitrification is the chemical reaction that oxidizes ammonium (NH_4^+) into nitrate (NO_3^-). The process is performed in two separate steps; first, ammonium is transformed into nitrite (NO_2^-), and second, nitrite is transformed into nitrate.

It has the following chemical formulas:

$$2NH_4^+ + 3O_2 \Rightarrow 2NO_2^- + 4H^+ + 2H_2O$$

Equation 3.5-1: Nitrification from ammonium to nitrite (Rystad & Lauritzen, 1996).

$$2NO_2^- + O_2 \Rightarrow 2NO_3^-$$

Equation 3.5-2: Nitrification from nitrite to nitrate (Rystad & Lauritzen, 1996).

In the first step, two ammonium molecules and three oxygen molecules are turned into two nitrite molecules, four hydrogen molecules (H^+), and two water molecules (H_2O). Second,

two nitrite molecules and one oxygen molecule are transformed into three nitrate molecules. This process consumes large amounts of oxygen. Several types of chemoautotrophic bacteria initiate the nitrification process (Kaplan, 1983). The most common kinds are nitrosomonas and nitrobacts, responsible for each step respectively (Henriksen and Kemp, 1988). Chemoautotrophic bacteria are autotrophic bacteria that oxidize inorganic compound as a source of energy. Being autotrophic means that they need only a simple inorganic nitrogen compound for metabolic synthesis (Merriam Webster's Collegiate Dictionary, 1996). There are about 10^4 nitrifying bacteria per liter of water in the ocean (Kaplan, 1983).

3.5.3 Nitrification in the Model

Equation 3.5-3 shows the chemical formula for the nitrification process as one step, going directly from ammonium to nitrate. Two ammonium (NH_4^+) and four oxygen (O_2) molecules are transformed into four hydrogen atoms (H^+), two nitrate molecules (NO_3^-), and two water molecules (H_2O).

$$2NH_4^+ + 4O_2 \Rightarrow 2NO_3^- + 4H^+ + 2H_2O$$

Equation 3.5-3: Nitrification in one step, going directly from ammonium to nitrate.

In order to simplify the model, the nitrification process is modeled as one step, going directly from ammonium to nitrate. The intermediary step where ammonium is transformed into nitrite, and later into nitrate is omitted because the reaction happens so quickly that it is not considered necessary to include both steps in the model. Further, the water molecules and hydrogen atoms that are released during the nitrification process are also omitted form the model because they are not considered important for the behavior of the oxygen level or the hydrology of the fjord. Equation 3.5-4 shows the chemical formula for nitrification, going directly from ammonium and oxygen to nitrate, and not including hydrogen and water molecules. This is the equation that the nitrification in the model is based on.

$$2NH_4^+ + 4O_2 \Rightarrow 2NO_3^-$$

Equation 3.5-4: Nitrification in the model, omitting hydrogen atoms and water molecules.

Because the nitrification process is modeled in one step, the nitrifying bacteria are also modeled as one variable, not distinguishing between different types of nitrifying bacteria and the different parts of the process that they perform. The nitrifies grow through the nitrification process, and the rate of this process is determined by the amount of ammonium and oxygen in the water, in addition to the amount of nitrifying bacteria. Temperature and light may also influence the nitrification process (Wada and Hattori, 1991), but these factors are not included.

3.5.4 Causal Loop Diagram – Nitrification

Figure 3-9 shows a causal loop diagram of the nitrification process, and the feedback loops in the diagram are listed in figure 3-10. The model consists of one reinforcing growth loop 'R1: Nitrifying bacteria growth' which is controlled by five balancing loops. The more nitrifying bacteria there are the higher the nitrification rate. The higher the nitrification rate, the higher the nitrifying bacteria growth rate, and the higher the nitrifying bacteria level becomes. 'B1: Oxygen nitrification' shows how the oxygen level in the water influences the nitrification rate, and thereby the oxygen consumption rate. If there is a high level of oxygen in the water, the nitrification rate will be higher than otherwise. If it is lower it will limit the nitrification rate. 'B2: Ammonium nitrification' shows a similar effect of the ammonium level in the water. The more ammonium in the water, the higher the nitrification rate is. The higher the nitrification rate, the higher the ammonium consumption, and the lower the ammonium level in the water.

'B3: Nitrifying bacteria death' describes how the nitrifying bacteria die based on their death rate. The higher the death rate, the less nitrifying bacteria there are. A lower level of nitrifying bacteria will again lead to a lower death rate than there otherwise would have been. The level of oxygen in the water has an effect on the nitrifying bacteria death rate when there is a shortage of oxygen in the water. This is described by loop 'B4: Effect of oxygen level on nitrifying bacteria death'. As the oxygen level in the water approaches a certain lower limit, the bacteria death rate will increase, causing the system to contain a lower bacteria level, which again will reduce the nitrification rate and the oxygen consumption rate. The result will be that there is more oxygen in the water than there otherwise would have been, which again

will reduce the bacteria death rate. 'B5: Effect of ammonium level on nitrifying bacteria death' describes a similar relationship between the ammonium in the water and the nitrifying bacteria death rate.

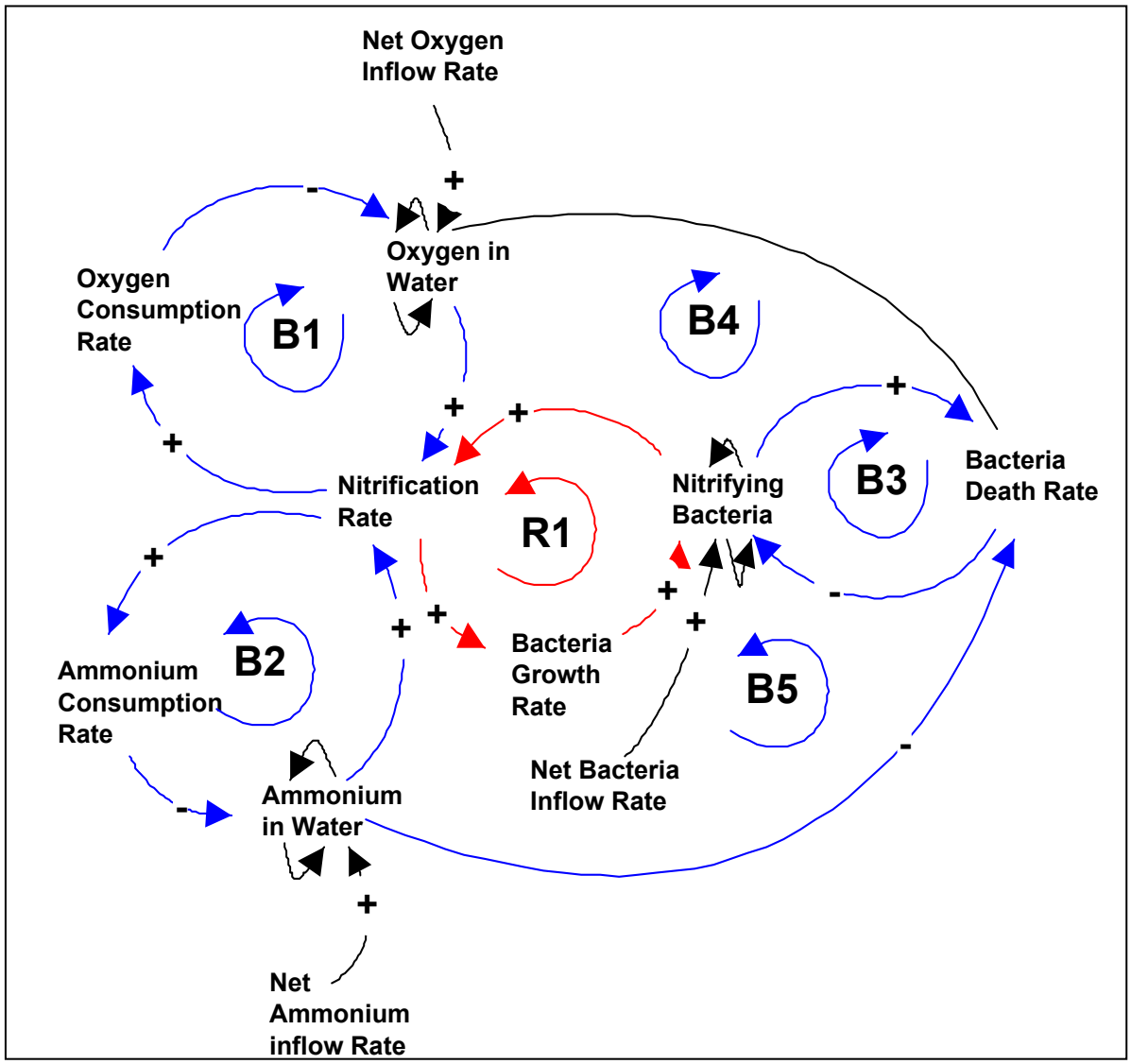


Figure 3-9: Causal loop diagram for nitrification.

R1: Nitrifying bacteria growth

B1: Oxygen nitrification

B2: Ammonium nitrification

B3: Nitrifying bacteria death
B4: Effect of oxygen level on nitrifying bacteria death
B5: Effect of ammonium level on nitrifying bacteria death

Figure 3-10: The feedback loops concerning nitrification.

In the hydrodynamic model, the state variables, ammonium, oxygen, and nitrifying bacteria are influenced by the flow of the water and diffusion between the fjord cells. Further, the levels of the outermost cells are directly affected by the net inflow rate of that level, to and from the ocean. The ammonium level in the innermost fjord cells also receives input through the nitrogen pollution from Odda Smelteverk, and ammonium in the Opo River.

3.6 Theoretical Background for Nitrification

3.6.1 Michaelis-Menten Kinetics

The bacteria growth and death rates are implemented through the use of Michaelis-Menten kinetics. The Michaelis-Menten equation was formulated by Leonor Michaelis and Maud Menten in 1913 (Crotty, 1996), and is commonly used in uptake kinetics. A maximum velocity on the growth or uptake is set, and this velocity is limited by certain other factors, such as nutrient scarcity. The effects of the limiting factors are controlled by the half saturation concentration, which is the concentration of the limiting substance that would result in half of the maximum velocity of growth. A Michaelis-Menten equation is presented in equation 3.6-1.

$$V = V_{\max} * \frac{S}{(K_s + S)}$$

Equation 3.6-1: (Valiela, 1984, p. 50).

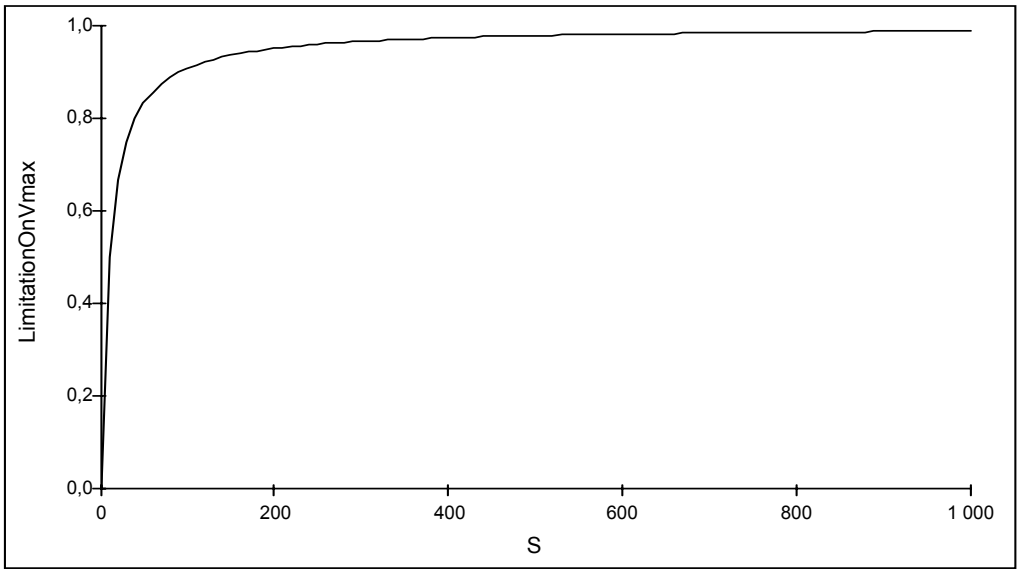
V = Velocity or growth rate
V_{max} = The maximum velocity when there are no constraints
S = Concentration of limiting substance

K_s = The half saturation concentration - the concentration of the limiting substance that results in half of the maximum velocity (V_{max})

If there are no constraints to the velocity (V), it will hold the value of the maximum velocity (V_{max}). This is because as long as the concentration of the limiting substance (S) is high, the effect of dividing the substance on the sum of the half saturation concentration (K_s) and the substance will be small, that is, close to 1. This is illustrated in equation 3.6-2. When the concentration of the limiting substance (S) approaches the half saturation concentration (K_s), the half saturation concentration will have a larger effect because the concentration will be divided by a relatively larger number than when there is a surplus of the substance. This results in a smaller number being multiplied with the maximum velocity (V_{max}), and the magnitude of the velocity (V) is reduced.

$$\frac{S}{(K_s + S)} \rightarrow 1 \text{ when } S >> K_s \qquad \textbf{Equation 3.6-2}$$

Graph 3-1 presents an example of the effect of limitation in a Michaelis-Menten equation. Here, the half saturation concentration (K_s) is set to 10. When the substance (S) has the value of 10, the velocity (V) will be half of the maximum velocity (V_{max}). Limitation on V_{max} is the limitation on the maximum velocity: S / (K_s + S). When the substance (S) has the value of 1000, there is a high level of the substance in the water, and the limitation is close to 1, that is, there is basically no limitation due to the level of the substance. As the substance (S) approaches 0, the limitation on the maximum velocity (V_{max}) also approaches 0, resulting in the velocity (V) going towards 0, as the level of the substance (S) in the water goes towards 0.



Graph 3-1: Example of limitation on V_{max}.

3.6.2 Michaelis-Menten used for the Nitrification Rate

According to Kaplan (1983) the following equation, based on Michaelis-Menten kinetics can be used to calculate an in situ nitrification rate:

$$R = a \cdot N \cdot K_{\max} \cdot \frac{\left[NH_4^+\right]}{\left(K_m + \left[NH_4^+\right]\right)}$$

Equation 3.6-3: Nitrification rate (Kaplan, 1983, p. 178)

R = The nitrification rate (Unit: mol N/hour)
a = The amount of NH_4^+ used per cell per doubling (Unit: mol/cell)
N = The number of nitrifying bacteria cells per liter of water (Unit: cells)
K_{max} = The nitrification rate at high levels of NH_4^+ (the maximum nitrification rate. Unit: 1/hour)
K_m = The half saturation constant for NH_4^+. The amount of ammonium in the water that would give half of the maximum nitrification rate
$[NH_4^+]$ = The actual ammonium level in the water

The amount of ammonium (a) that is used per cell per doubling, is multiplied by the number of nitrifying bacteria in the water (N) in order to find the total initiated nitrification rate. K_{max} is the maximum nitrification rate at high levels of ammonium, i.e. the nitrification rate when there are no other constraints. This is multiplied by the actual amount of ammonium in the water ($[NH_4^+]$) divided by the half saturation constant for ammonium (K_m) and the amount of ammonium in the water ($[NH_4^+]$). The half saturation constant defines the ammonium level that would result in half of the maximum nitrification rate. When there is a surplus of ammonium in the water the last part of the equation will approach 1, thereby making the nitrification rate close to the maximum. Contrary, when there is a deficiency, the limiting fraction will go towards 0, because the amount of ammonium in the water will be divided by a higher number. This will decrease the nitrification rate when there is less ammonium in the water.

Using this version of the Michaelis-Menten function for nitrification is not sufficient when there is a possibility for additional substances other than ammonium to limit the nitrification process. Kaplan (1983) does not take into account the oxygen level in the water. In the Sørfjord the low oxygen level is periodically a problem, and it is therefore necessary to include it as a constraint on the nitrification rate. Using this equation in the model without alterations would result in a higher nitrification rate than possible in the Sørfjord, and oxygen would continue to be depleted at a negative level after it has reached 0. The equation is therefore not sufficient to describe the problem at hand. A Michaelis-Menten equation where oxygen is included is therefore constructed. No literature references to the use of Michaelis-Menten for oxygen levels in relation to nitrification were found.

The following equation is used in the model:

$$R = a \cdot N \cdot K_{\max} \cdot \frac{\left[NH_4^+\right]}{\left(K_m + \left[NH_4^+\right]\right)} \cdot \frac{\left[O_2\right]}{\left(K_m O_2 + \left[O_2\right]\right)}$$

Equation 3.6-4: Nitrification rate as implemented in the model.

[O_2]: The actual amount of oxygen per liter of water
K_mO_2: The half saturation concentration for oxygen - the level of oxygen in the water that would result in half of the maximum nitrification rate.

The last part of the equation will have a similar effect as for ammonium. When the oxygen level in the water ([O_2]) approaches the half saturation concentration for oxygen (K_mO_2), the number multiplied into the nitrification rate will decrease, causing a lower nitrification rate. Otherwise the effect of the oxygen level will be close to 1, resulting in the nitrification rate approaching its maximum as long as there is also an ammonium surplus in the water.

3.6.3 Bacteria Production Rate

Kaplan (1983) presents a cell yield of 10^6 cells per micromol nitrogen that is nitrified per liter per day. That is the number of nitrifying bacteria cells that are formed per micromol nitrified nitrogen. Converted into microgram this will result in a cell yield of 71.39. The cell yield (c) is multiplied into the nitrification rate in order to define the bacteria production rate (N_b) per time unit in the following equation:

$$N_b = R \cdot c$$ **Equation 3.6-5: Bacteria birth rate.**

N_b = Bacteria production rate
c = Cell yield

3.6.4 Bacteria Death Rate

No reference to the bacteria death rate was found in the literature. It is therefore assumed that the bacteria have a maximum life span, and that this life span is limited by the amount of ammonium and oxygen in the water. When ammonium and oxygen is in scarcity, the death rate will increase, and the life span decrease. This is implemented using a Michaelis-Menten function similar to the one used for nitrification, where it is assumed that there is a half saturation concentration for ammonium and oxygen in the water, which results in half of the

maximum life span. The number of bacteria is divided by the average life span for each bacterium per time unit.

$$N_d = \frac{N}{\left(L_{\max} \cdot \frac{\left[NH_4^+\right]}{\left(K_m + \left[NH_4^+\right]\right)} \cdot \frac{\left[O_2\right]}{\left(K_m O_2 + \left[O_2\right]\right)} \right)}$$

Equation 3.6-6: Bacteria death rate.

N_d = The bacteria death rate

L_{max} = The maximum bacteria life span.

Under ideal conditions when there is a surplus of ammonium and oxygen in the water, the result of the limitation by ammonium and oxygen in the water will be close to 1, and the actual bacteria life span will equal the maximum bacteria life span (L_{max}). However, when ammonium or oxygen is in scarcity the bacteria will die more quickly. When for instance the oxygen level in the water ($[O_2]$) approaches the half saturation concentration for oxygen ($[K_mO_2]$), the oxygen level in the water will be divided by a fairly low number, resulting in a small fraction for the last part of the equation. This small fraction will then be multiplied by the max bacteria life span (L_{max}), resulting in a lower number for the bacteria life span. When the number of bacteria in the water (N) is divided by this lower number, more bacteria will die due to the shortage of oxygen in the water.

3.7 Conceptual Model for Phytoplankton

Phytoplankton is a free-floating microorganism and an important group of algae. There are a number of different types of phytoplankton, and the composition of the different types is mainly stated by the nutrient composition in the area. We have chosen to model all types as one group, and used data and parameters from one of the dominating types in the area. This first of all simplifies the modeling process. Another reason to do so is the lack of usable data from the area. The primary production of biomass in the Sørfjord is mainly done by photosynthesis in phytoplankton. Photosynthesis is a process with several steps. The first step is the photochemical process that transforms light energy into chemical energy. Chemical energy is then used to convert carbon dioxide (CO_2) and water (H_2O) into sugar ($C_6H_{12}O_6$) and oxygen (O_2).

$$6CO_2 + 6H_2O \Rightarrow C_6H_{12}O_6 + 6O_2$$

Equation 3.7-1: Photosynthesis process (Rystad and Lauritzen, 1996, p. 302)

With sugar as building material and supply of other nutrients we get growth in biomass (plants, algae etc.). The growth of phytoplankton is caused by partition of cells. Phytoplankton in seawater usually have an abundantly access to CO_2 and most nutrients except nitrogen and phosphorus. Because of the high supply rate of ammonium (NH_4), a form of nitrogen, phosphorus is the only growth-limiting nutrient in the inner part of the Sørfjord. Further out in the fjord the ammonium concentration seems to be normal, but the variations are large. General analysis of the composition of plankton is done, and the most known are the findings published by Redfield et al. (1963).

	Mol-ratio	Weight-ratio
Carbon (C)	106	41
Nitrogen (N)	16	7.2
Phosphorus (P)	1	1

Table 3-4: Normal chemical composition of phytoplankton. (Bjerkeng, 1994, p.31)

This ratio is normally used for estimating the density of algae in seawater, but the numbers are average and cannot be used to measure the different types of phytoplankton. The ratio shows approximately the supply and uptake of nutrients needed for an optimal growth rate. There are some variances between the different types of phytoplankton, and the availability of the different nutrients determines the composition of different types.

In this causal loop diagram we will show the most important factors that drive the growth and death of phytoplankton.

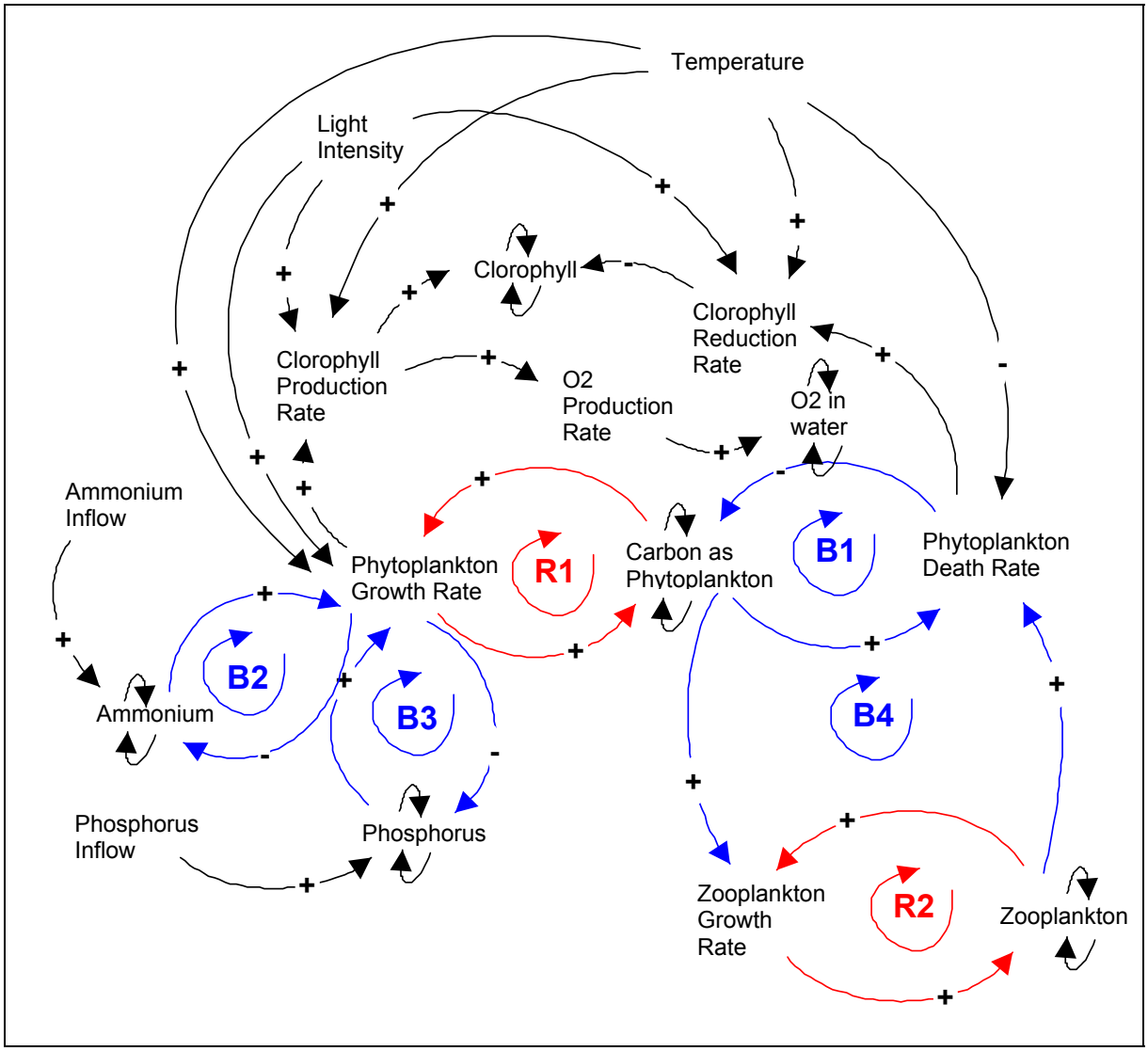


Figure 3-11: Causal loop diagram for the phytoplankton cycle.

The diagram shows some of the exogenous and endogenous factors that influence the behaviour of the system. The selection is done with the intention of giving an overview of the variables that drives the big lines of the behaviour of phytoplankton. Factors with small impact on the total system behaviour are omitted.

3.7.1 Exogenous Factors

Temperature and light are two factors that are directly influencing the growth of phytoplankton. Both are closely related to the growth rate since the growth is a result of photosynthesis, as mentioned above. Optimal light and temperature conditions give basis for an optimal growth rate. A reduction in one or both of these gives a reduction in the growth rate. In tropical areas the values for light and especially temperature can pass the optimal values, which leads to a decrease of the growth rate. This is however not the case in the Sørfjord where either light or temperature reach optimal values under any circumstances.

3.7.2 Endogenous Factors

Ammonium flow and phosphorus flow are the flows in and out between neighbour cells in the hydrodynamic model. This includes both the one-way water stream that goes trough a cell, and equalization between to neighbour cells that goes both ways. Supply of ammonium from diffusion and waste from Odda Smelteverk are also included in this flow, but are only relevant for two of the cells.

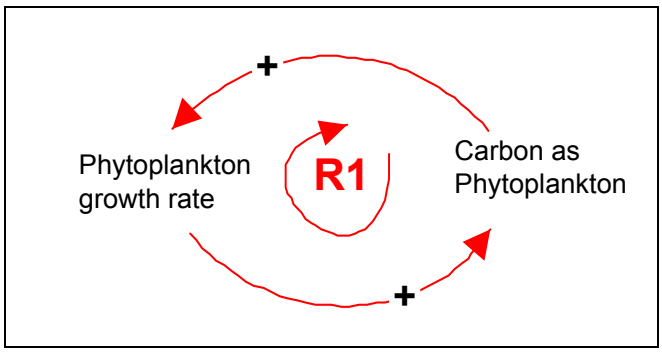


Figure 3-12: Loop R1: Phytoplankton growth rate.

When the phytoplankton growth rate increases, the number of phytoplankton (carbon as phytoplankton) increases. This leads to a further increase in the phytoplankton growth rate, and we get a reinforcing loop. Without any other limitation this would give an exponential growth of phytoplankton.

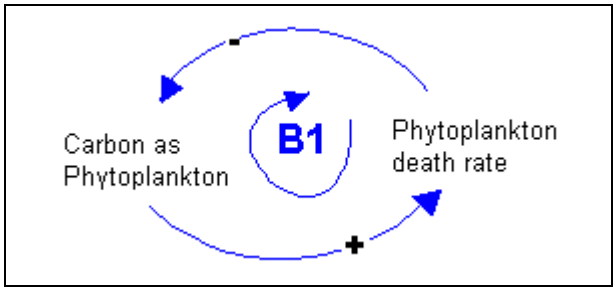


Figure 3-13: Loop B1: Phytoplankton death rate.

When the number of phytoplankton increases, the phytoplankton death rate increases as well. This is caused by the fact that the rate is a fraction of the phytoplankton stock. When we get a decrease in this stock, the death rate goes down as well. This is a balancing loop that limits the stock of phytoplankton.

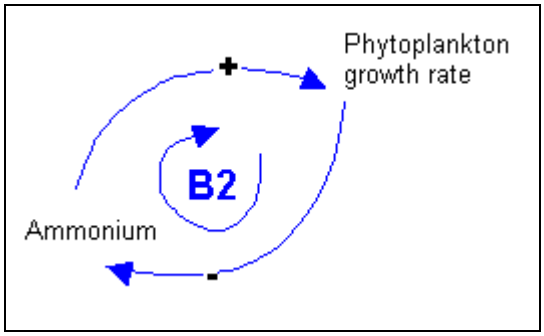


Figure 3-14: Loop B2: Ammonium consumption by phytoplankton.

An increase in the phytoplankton growth rate gives higher ammonium consumption, and thereby decreasing the ammonium stock. The availability of ammonium in the water is an important factor for phytoplankton growth, but the influence on the growth rate is limited to conditions where the amount of ammonium is below a limit. That means that this loop is only active as a limiting factor for growth under this condition. Then the decrease in ammonium

leads to a lower growth rate. Because of the vast supply of ammonium of ammonium from Odda Smelteverk to the innermost part of the fjord, this condition rarely occurs.

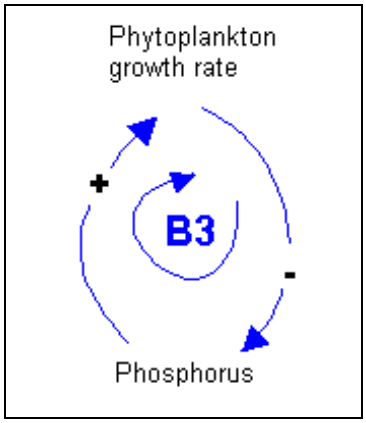


Figure 3-15: Loop B3: Phosphorus consumption by phytoplankton.

The condition is the same for phosphorus as for ammonium. An increase in the phytoplankton growth rate causes decrease in the amount of phosphorus. The feedback effect is not active unless the stock of phosphorus is below a limit. If it is, it leads to a decrease in the growth rate. Otherwise it has no effect on the growth rate.

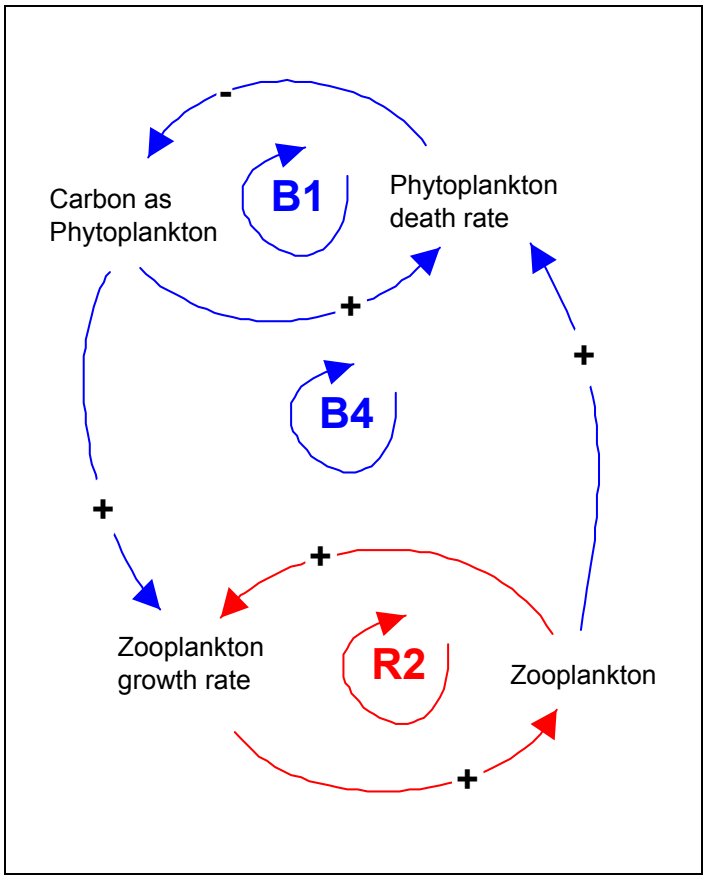


Figure 3-16: Loop B4: The phytoplankton – zooplankton relation.

Zooplankton is the most important grazers of the phytoplankton. (Nybakken, 1997, p.36) An increase of phytoplankton leads to an increase of the zooplankton growth rate. This increase in the amount of zooplankton increases the phytoplankton death rate, which reduces the amount of phytoplankton.

3.8 Theoretical Background for Phytoplankton

The uptake of nutrients, growth and death rate of phytoplankton are, as we have seen, given by a number of different factors. Based on batch experiments and analysis of phytoplankton in natural environments, researchers have found the effect of each of the factors affecting these processes. There are some small variations in the ways to estimate the rates, and as we are unqualified to evaluate these variations, we have based our model mainly on the work done by NIVA in the Oslofjord model. (Bjerkeng, 1994)

3.8.1 Factors Affecting Growth

The growth rate of phytoplankton is not similar to the uptake of nutrients form the water. These are two different processes, but they are closely related to each other. The growth rate will be presented first, and later the nutrient uptake bill be described.

The growth in phytoplankton is measured as assimilation of organic carbon per time unit, and given as:

$$\frac{d(CarbonAsPhytoplankton)}{dt} = \mu \cdot CarboneAsPhytoplankton(t) \qquad \text{Unit: day}^{-1}$$

Equation 3.8-1: Phytoplankton growth rate.

Where μ = relative growth rate Unit: day $^{-1}$

Three main factors are affecting the growth rate of phytoplankton:

Light intensity:	I	(energy/time/area)	Unit: dimensionless
Temperature:	T	(degree Celsius)	Unit: day $^{-1}$
Nutrient content in phytoplankton (NUTFAC):	q_P = Phosphorus:Carbone ratio		Unit: dimensionless
	q_N = Nitorgen:Carbone ratio		Unit: dimensionless

From this the relative growth rate can be written as:

$$\mu = \mu(T) * \mu(I) * \mu(NUTFAC)$$

Equation 3.8-2: Relative growth rate.

Each factor will be discussed below. The figure 3.5 shows how light and temperature are influencing the growth and death rates of phytoplankton.

3.8.2 Temperature Limitation

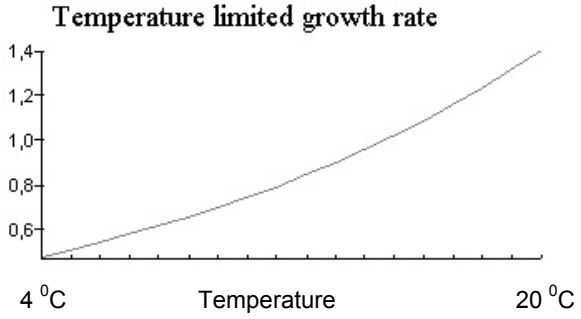
If there is sufficient light and nutrient (nitrogen and phosphorus), the growth rate will be limited by internal consumption of nutrients in the phytoplankton, and this is a function of temperature (Bjerkeng, 1994). Statistically the growth rate for biomass will increase if the average temperature increases. (Eppley, 1972) Eppley has, based on data from a number of sources, found an exponential relation between maximum specific growth rate and temperature. This has been used in several well-known models. (The Narranganset Bay model (Kremer and Nixion, 1978) and the Østersjø model (Stigebrandt and Wulff, 1987). We use Eppley's exponential function with temperature T given in ^{0}C and with temperature coefficient $k_{T,fyt.}$

$k_{T,fyt}$ =0.063 pr ^{0}C Unit: ^{o}C $^{-1}$.

$$\mu_{\max}(T) = \mu_{20} \exp\left[k_{T,fyt} \cdot (T - 20^0 C)\right]$$ Unit: day $^{-1}$

Equation 3.8-3: Temperature limitation (Bjerkeng, 1994, p.34).

This formula ignores temperatures above 20 ^{0}C, which rarely occurs in the Sørfjord. With a temperature range form 4 ^{0}C to 20 ^{0}C this gives a growth rate, based on temperature only, ranging from 0.5 to 1.4. (Figure 3.11)



Graph 3-2: Temperature limited relative growth rate between 4 and 20 ^{0}C.

3.8.3 Temperature and Nutrient Limitation Together

An internal lack of nutrients (based on the Redfield-ratio) can limit the growth. The composition of the phytoplankton is an important factor for the growth, and is used in the formulas for the growth rate. If all other conditions are optimal, the limitation caused by the lack of a single nutrient *x* can be given as Droop's formula (Droop, 1974):

$$\mu = \mu_{\max}(1 - q_0 / q)$$ Unit: day $^{-1}$

Equation 3.8-4: (Bjerkeng, 1994, p. 35)

Where q = relative nutrient content = nutrient *x*/ carbon in phytoplankton,
q_0 = lower limit for nutrient content

This can be written as:

$$\mu = \frac{\mu_{\max}}{1 + 1/K_q}$$ Unit: day $^{-1}$

Equation 3.8-5: (Bjerkeng, 1994, p. 35)

Where

$K_q = (q - q_0)/q_0$

K_q is then the relative nutrient surplus compared to the lower limit.

This formula can be extended to cover several limiting nutrients *a, b, c*..., and combined with the equation for temperature limitation we get:

$$\mu_{opt,I} = NUTFAC \cdot \mu_{\max}(T)$$ **Equation 3.8-6: (Bjerkeng, 1994, p.35)**

The combined nutrient limitation is expressed with a factor (*NUTFAC*) given with the equation

$$NUTFAC = \frac{1}{1 + \frac{1}{K_{q,a}} + \frac{1}{K_{q,b}} + \frac{1}{K_{q,c}} + \ldots}$$

Equation 3.8-7: (Bjerkeng, 1994, p.35)

If the relative surplus of all nutrients is high, *NUTFAC* goes toward one and does not influence on the growth rate. In cases where one or more of the nutrients goes toward the lower limit, *NUTFAC* is reduced and goes toward zero.

3.8.4 Light limitation

Equation 3.8-3 describes the rate for the internal transformation of nutrients, which represents phytoplankton growth. This process is driven by energy from the light. With faint light, the process can be restricted by the lack of energy.

$$\mu(I,T) = \mu_{opt,I} \cdot \Theta(I)$$ Unit: day $^{-1}$

Equation 3.8-8:Light limitation. (Bjerkeng, 1994, p. 36)

Where

$$\Theta(I) = \frac{I}{(I_k^n + I^n)^{1/n}}$$ Unit: day $^{-1}$

Equation 3.8-9: (Bjerkeng, 1994, p. 36)

The factor Θ is the relative reduction in the growth rate cased by sub optimal light intensity. The critical light intensity I_k gives the point of change between light limited growth and nutrient and temperature limited growth. I_k is a relative size that changes as the phytoplankton adapt the light intensity. The coefficient n indicates how fast the adaptation goes.

3.8.5 Critical light intensity

When phytoplankton is adapting to lower light intensity, I_k is changed. The equation below describes these changes based on experiments by Steeman-Nielsen (Steeman-Nielsen, 1975). A log-linear regression on the data from the experiments gives the relation:

$$I_k = I_0^{0.6} I_s^{0.4}$$ **Equation 3.8-10: Light intensity adjustment (Bjerkeng, 1994, p. 36)**

I_0 = constant = $345 \mu E \cdot m^{-2} \cdot s^{-1}$ (ca. 75 W/m^2 visible light)

I_s = light intensity (W/m^2)

At a constant light intensity I_s =75 W/m^2 $I_k = I_s$. This light intensity gives a turning point for light adapted growth. At lower light intensity the light limitation will dominate, and at higher intensity the temperature and nutrients will dominate. The light intensity in the model is set to 75, and is a constant. There is a decreased light intensity in the lower water layers, but without any data about the light in the Sørfjord, it is difficult to estimate any numbers. For further work with the model, real data can used to adjust the light intensity for the different layers in the water.

3.8.6 Chlorophyll

Chlorophyll is produced during the photosynthesis process and is an important factor in this process. The chlorophyll contributes in the crucial process of converting light energy into chemical energy that can be used by the algae. The production rate of chlorophyll, $\mu(CHL_p)$, is determined by the growth rate of phytoplankton, but to optimize the light utilization the production rate can be increased or decreased to adapt the changes in light intensity. The higher relative content of chlorophyll in the phytoplankton the better faint light can be utilized.

Production rate:

$$\mu(CHL_p) = \mu(I,T) \cdot CFYT \cdot Y_{I,opt}$$ **Equation 3.8-11: (Bjerkeng, 1994, 38)**

.

$$Y_{I,opt} = \frac{\mu_{opt,I}}{\max\left(5\left[w/m^2\right], I_0^{0,6} I^{0,4}\right)}$$ **Equation 3.8-12: (Bjerkeng, 1994, 148)**

The first part $\mu(I,T) \cdot CFYT$ corresponds to the growth rate for carbon in phytoplankton. $Y_{I,opt}$ is a variable changing with light intensity. Lower values for light give relatively higher production of chlorophyll to compensate for this. There is a lower limit for the light intensity where the production rate for chlorophyll is maximized compared to growth rate for phytoplankton. The limit is set to 5 *w/m*^{2} (Bjerkeng, 1994, p. 148). When the light intensity exceed the average of 75 *w/m*^{2}, the production rate decrease relatively compared to the growth rate for phytoplankton.

Reduction rate:

The reduction rate for chlorophyll, $\mu(CHL_l)$, is based on the reduction rate for carbon in phytoplankton.

$$\mu(CHL_l) = r_{fyt} \cdot CFYT \cdot Y_I$$ Unit: 1/day

Equation 3.8-13: (Bjerkeng, 1994, 37)

Where

$$Y_I = \frac{\mu_{opt,I}}{I_k}$$ **Equation 3.8-14: (Bjerkeng, 1994, 37)**

The first part, $r_{fyt} \cdot CFYT$, is similar to the reduction of carbon in phytoplankton and Y_I is a light dependent variable regulating the ratio between chlorophyll and other nutrients in phytoplankton.

3.8.7 Nutrient uptake

The nutrient uptake from the water indirectly regulates the phytoplankton growth rate. If the nutrient uptake does not meet the internal consumption, this will lead to a change in the ratio between the different nutrients in the phytoplankton. If the ratio is 'out of balance' it will normally lead to a decrease in the growth rate. After some time the nutrient balance will be restored. Occasionally the uptake is higher then the growth rate requires, which gives a surplus of nutrient. This will give an increase in the growth rate. The factors that regulate the uptake are mainly the concentration of nutrients in the water and the phytoplankton growth rate.

Two equations are used to estimate the nutrient uptake. The first equation describes the capacity based on nutrient availability in the water (based on the Michaelis-Menten equation explained in chapter 3.6.1) and the light limitations. The second one describes the optimal nutrient uptake rate based on the assumption that the nutrient balance in the phytoplankton is to be maintained. To determine the uptake rate we use the limiting of these two equations at any time (MIN($V_{Nutr,kap}$, $V_{Nutr,opt}$)).

Uptake based on capacity:

$$V_{Nutr} \le V_{Nutr,kap} = V_{Nutr,\max} \frac{Nutr}{K_{Nutr} + Nutr} (k_{Nutr,0} + k_{Nutr,I} \Theta(I))$$

Equation 3.8-15: (Bjerkeng, 1994, p.38)

V_{Nutr} = actual nutrient uptake Unit: microgram/day

$V_{Nutr,kap}$ = uptake capacity at concentration x Unit: microgram/day

$V_{Nutr,max}$ = upper limit for nutrient uptake Unit: microgram/day

$Nutr$ = concentration of available nutrients in the water Unit: microgram/liter

K_{Nutr} = half saturation concentration, Unit: microgram/liter
the concentration that gives 50 % of maximum uptake

$\Theta(I)$ = light limitation of the growth Unit: microgram/day

$k_{Nutr,0}$ and $k_{Nutr,I}$ are dimensionless coefficients for the relation between growth and uptake

Uptake based on optimal uptake:

$$V_{Nutr} \le V_{Nutr,opt} = \mu_{\max} \cdot (q_{\max} - q) + \mu \cdot q$$ **Equation 3.8-16: (Bjerkeng, 1994, p. 39)**

$V_{Nutr,opt}$ = upper limit for nutrient uptake adjusted to nutrient content in phytoplankton and the growth rate Unit: microgram/day

$\mu_{\max}$ = maximum growth rate at a given temperature Unit: microgram/day

μ = actual growth rate Unit: microgram/day

$q_{\max}$ = upper limit for nutrient content in phytoplankton Unit: microgram/microgram

q = actual nutrient content in phytoplankton compared to carbon Unit: microgram/microgram

3.8.8 Phytoplankton death rate

The death rate of phytoplankton is composed by two main factors. The first is the natural loss rate, 'Phytoplankton Loss Factors', consisting of respiration and natural death, and the other is the predation rate by zooplankton. Together these two make the total death rate for phytoplankton.

Respiration

The respiration is stated by the temperature with the same function as the growth rate, and gives a proportional loss of all components in phytoplankton. Respiration $RESP_{f,20}$ at temperature $T = 20^{o}C$ is an adjustable coefficient set to 0.04 day^{-1}. (Laws and Bannister, 1980)

$$RESP_f = RESP_{f,20} \cdot \exp\left[k_{T,fyt} \cdot (T - 20^{0}C)\right]$$ Unit: day^{-1}

Equation 3.8-17: (Bjerkeng, 1994, p. 45)

Natural death

Since the phytoplankton actually is a group of different algae it is hard to estimate this natural death. The interaction between the different species is balancing the growth and death due to the change in domination between them in a way that is too complex to model. This natural interaction creates a smooth balance and keeps the species within a frame given by the nature. This formula is an attempt to stabilize the stock within such limits that are natural and avoid unnatural behavior. The death rate will increase when the stock becomes large or the supply of nutrients is insufficient.

$$D_r = \left(D_{r,20} \cdot e^{k_t (T - 20^0 C)}\right) \cdot \left(\frac{CFYT}{CFYT_D + CFYT}\right) \cdot \left[1 - \left(1 - FD_{Nutr}\right) \cdot NUTFAC\right] \quad \text{Unit: day}^{-1}$$

Equation 3.8-18: (Bjerkeng, 1994, p. 45)

The first part of the expression gives the maximum rate as a function of temperature. One assumes that the rate increases parallel to the growth rate as temperature increases. This is caused by the acceleration of the speed of most processes with an increase of the temperature.

The second part of the equation is a saturation function for the phytoplankton stock. $CFYT_D$ is the half saturation concentration for phytoplankton and $CFYT$ is the actual amount of phytoplankton. An increase in the phytoplankton stock gives an increase in this rate because this part of the equation goes towards one.

The last part gives the effect of the nutrient content in phytoplankton. The coefficient FD_{Nutr} set the effect of this function. FD_{Nutr} is assumed to be between 0 and 1. If the value is set to 1, the nutrient content does not affect the death rate. In our model FD_{Nutr} is set to 0.5. (Bjerkeng, 1994, p.155)

Zooplankton predation rate

Grazing of phytoplankton by zooplankton is an important factor in regulating the density of phytoplankton in the water. The formulas are described in chapter 3.10. Together with respiration and natural death it represent the total reduction rate for phytoplankton.

3.9 Conceptual Model Predator-Prey

3.9.1 Causal Loop Diagram for Predator-Prey

Figure 3-17 shows a causal loop diagram of the interaction between phytoplankton and zooplankton, while the names of the feedback loops are presented in figure 3-18. The diagram consists of two reinforcing growth loops, responsible for phytoplankton and zooplankton growth respectively. Further, there are three balancing loops; two describing natural death of phytoplankton and zooplankton, and one for the predation of phytoplankton by zooplankton. Inputs in the diagram are nutrients, temperature, and light, which influence the phytoplankton growth rate.

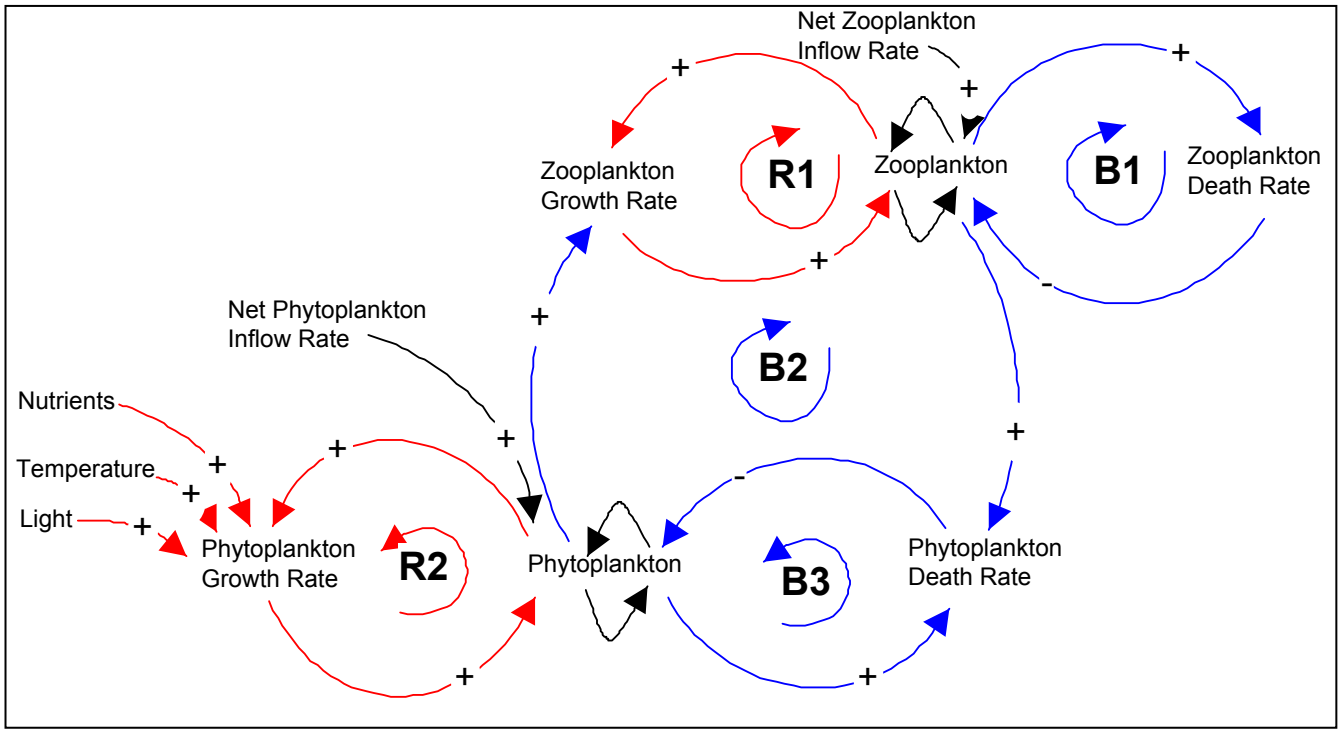


Figure 3-17: Causal loop diagram for predation.

R1: Zooplankton growth
R2: Phytoplankton growth
B1: Zooplankton death
B2: Predation of phytoplankton by zooplankton
B3: Phytoplankton death

Figure 3-18: The predator-prey feedback loops.

In a system with no zooplankton and unlimited nutrients, the phytoplankton will grow exponentially, driven by the reinforcing loop 'R2: Phytoplankton Growth'. There will also be a balancing loop 'B3: Phytoplankton death' which is the natural death rate that limits the number of phytoplankton. If a predator such as zooplankton is introduced to the system, the amount of phytoplankton will be reduced. The amount of zooplankton will influence the phytoplankton death rate, which will lead to a lower number of phytoplankton in the water. This will lead to a lower zooplankton growth rate, because the zooplankton grow based on the phytoplankton they consume. A lower zooplankton growth rate will again lead to a lower level of zooplankton than there otherwise would have been. This will again lower the phytoplankton death rate. As more phytoplankton survive, the amount of nutrients for zooplankton will increase, and they will obtain a higher growth rate. Loop 'B2: Predation of phytoplankton by zooplankton' illustrates this.

Loop 'R1: Zooplankton growth' is a reinforcing loop where the zooplankton level controls the zooplankton growth rate. The more zooplankton there are in the water, the higher the growth rate. The higher the growth rate, the higher the zooplankton level. Further, loop 'B1: Zooplankton death' controls the exponential growth of zooplankton because the more zooplankton there are in the water, the higher the death rate will be, and the lower the zooplankton level.

In the hydrodynamic model, the phytoplankton and zooplankton levels also receive input and give output through the water exchange rate. Phytoplankton and zooplankton are exchanged between the fjord cells, and with the Hardangerfjord through the flow of the water and diffusion.

3.10 Theoretical Background for Predator-Prey

3.10.1 Lotka-Volterra Equations

Lotka-Volterra equations can be used to describe systems where one organism is the primary food source for another organism above it in the food chain (Chapra, 1997). They describe how the predator and prey levels in a system vary through the interaction between them. In the Sørfjord model the Lotka-Volterra equations are used to describe predation by zooplankton on phytoplankton. First, a description of the general characteristics of the simplest version of the equations will be given. Two independent scientists, the Italian mathematician Vito Volterra and the American biologist A.J. Lotka, developed the equations at the beginning of last century (Chapra, 1997). There are a number of Lotka-Volterra equations, however only a few simple ones are described here.

Considering a system without predators, only prey, and no limitations on the growth of the prey, the following equation may be used to describe the change in the prey level over time:

$$\frac{dx}{dt} = ax$$

Equation 3.10-1: (Chapra, 1997, p. 623).

x = The number of prey
a = First order growth rate (net growth rate)

The change in the prey level over time is the amount of prey (x) multiplied by the growth rate (a). Further, in a system with only predators and no prey, the predator will be without its food source. The following equation can be used to describe the change in the predator level:

$$\frac{dy}{dt} = -cy$$

Equation 3.10-2: (Chapra, 1997, p. 623).

y = The number of predators
c = First order death rate

The amount of predators is multiplied by a death rate. The death rate is negative since predators are drawn from the level as they die. The interaction between the two, that is, the number of prey a predator encounters in a time unit, should depend on both the predator and prey densities. It is possible to show this as a product of *xy*:

$$\frac{dx}{dt} = ax - bxy$$

Equation 3.10-3: (Chapra, 1997, p. 623).

b = A parameter that quantifies the impact of the interaction on prey mortality

The prey still has the same growth function as in equation 3.10-1, but the amount of prey that die due to predation is solved by multiplying *x* and *y*, in order to find the interaction between predators and prey. Further, this is multiplied by a parameter that quantifies the impact of the interaction on prey mortality. The predation will have a positive effect on the predators, and this can be shown in the following equation:

$$\frac{dy}{dt} = -cy + dxy$$

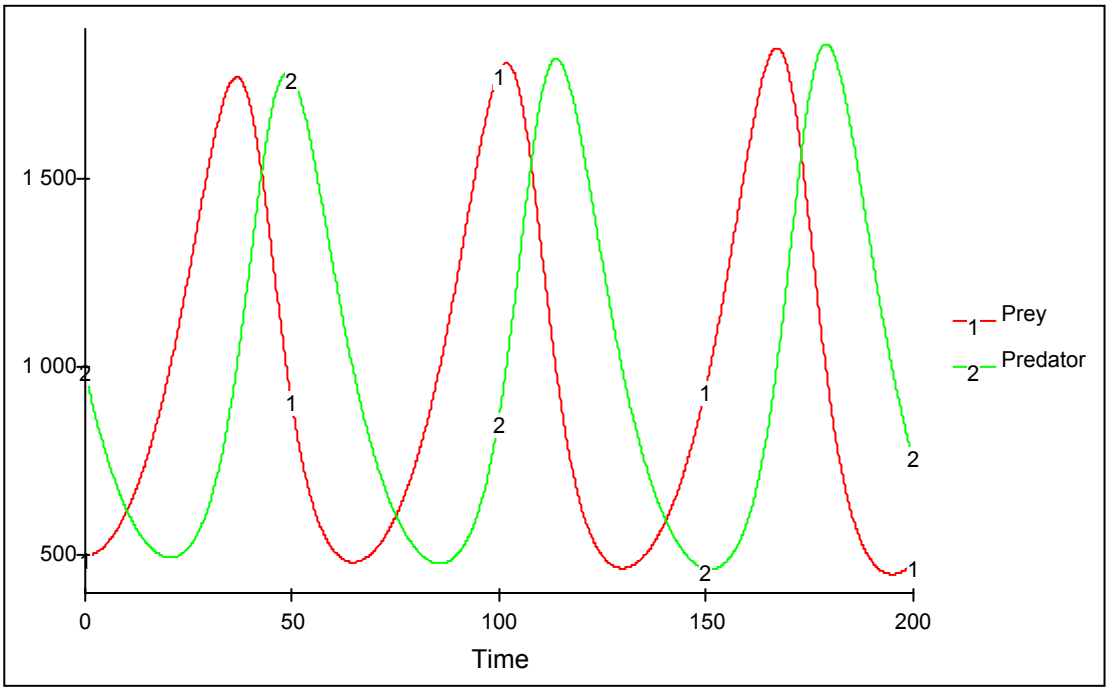
Equation 3.10-4: (Chapra, 1997, p. 623).

d = The impact on predator growth

The predator has the same function for the death rate as in equation 3.10-2, but the gain on predators from predation is solved by multiplying predators, prey, and a constant (*d*), representing the impact of predation on predator growth.

Graph 3-3 shows the behavior of a predator-prey model that is used as a basis in the Sørfjord model. It is based on the equations 3.10-1, 3.10-2, 3.10-3, and 3.10-4. The prey does not have a limitation to growth other than consumption by predators, which will decrease the prey level, and thereby reduce the prey growth rate. The predator level will increase as a result of predation, and this increase will cause the rate at which the prey increases to decrease as more are eaten. After some time there are fewer nutrients for the predator because they have consumed the prey, and the growth rate of the predator will slow down, and more die than are being born, and the prey level decreases. This will result in a net growth for the prey level.

The behavior in the graph and its relation to the causal loop diagram for predation (figure 3-17) is further explained in section 3.10.2.



Graph 3-3: An example of the behavior of a predator-prey model based on Lotka-Volterra equations.

Other more complicated equations may be used to describe the interaction between predator and prey levels in a model. Shoemaker (1977) presents a similar equation for the predation rate (equation 3.10-5). This is however only showed as an example, and not implemented in the model.

$$J(1,2) = S_p Q_1 Q_2 \alpha$$

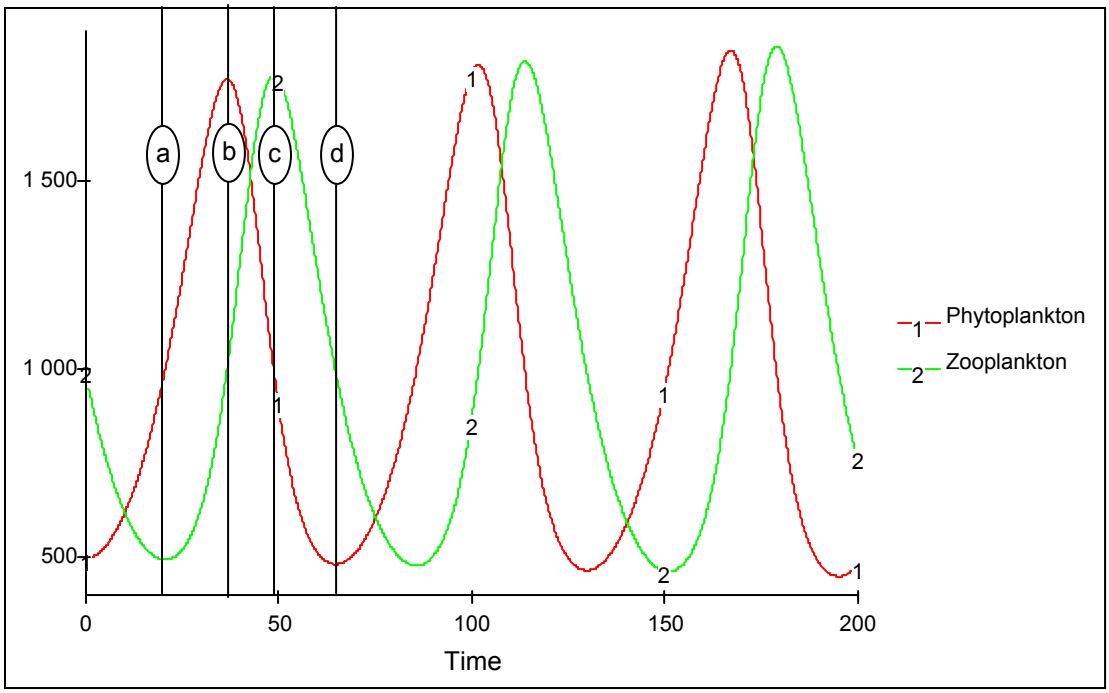
Equation 3.10-5: (Shoemaker, 1977, p. 78).

$J(1,2)$ = The predation rate
S_p = The searching rate of the predator
Q_1 = Prey population
Q_2 = Predator population
α = The fraction of encountered prey that is consumed

This equation assumes that the number of prey consumed is a certain fraction (α) of the number of prey encountered. The number of prey encountered is set by the searching rate of the predator (S_p) and the density of the number of prey (Q_1) and predators (Q_2). However, the number of prey a predator can consume is not unlimited. Shoemaker (1977) uses a Michaelis-Menten function (See section 3.6.1) to calculate the fraction of encountered prey that is consumed (α). This describes how the fraction of prey consumed varies with the density of prey in the water. This however is not described here since it is not implemented in the model.

3.10.2 Possible Behavior of Predator-Prey Based on CLD

The behavior in graph 3-4 is based on the Lotka-Volterra equations explained previously (equations 3.10-1, 3.10-2, 3.10-3, and 3.10-4). Looking at the CLD for Predator-Prey (figure 3-17), it may be possible to picture how this as a closed system can cause the behavior in graph 3-4. If the system is closed there must be enough nutrients for optimal phytoplankton growth, as well as optimal temperature and light intensity. This is only an illustration intended to describe a potential behavior of a predator-prey model based on Lotka-Volterra equations. The behavior of the graph is not extracted directly from the Sørfjord model, as the loops in this model are connected to other loops of a larger feedback system (see figures 3-1 and 3-7).



Graph 3-4: Example of possible behavior of a predator-prey model.

At the point where line ‘a’ intersects the graph, loop ‘R2: Phytoplankton growth’ dominates the behavior. The phytoplankton can grow exponentially because there are not enough zooplankton to consume the phytoplankton. However, this improves the growth conditions for the zooplankton, and gradually loop ‘B2: Predation of phytoplankton by zooplankton’ starts to get more influence, and gives a positive input to loop ‘R1: Zooplankton growth’, causing also zooplankton to grow exponentially. The phytoplankton reaches its maximum level at ‘b’, and this is when the growth rate of the zooplankton is at its maximum. After this, the phytoplankton level decreases rapidly because it is consumed by the increasing amount of zooplankton. The phytoplankton level in the water exhibits an exponential decay. The zooplankton level continues to grow, and reaches its maximum at ’c’. This is also when the death rate of the phytoplankton is the highest, and the slope of the phytoplankton the steepest. After this, there are not enough phytoplankton to maintain the zooplankton growth rate, and they die due to natural causes (in a more specific model the phytoplankton limitation may cause a higher death rate). As there are less zooplankton in the water, and ‘R1: Zooplankton growth’ becomes less dominating, the phytoplankton death rate will go down, and there will be more phytoplankton in the water than there otherwise would have been. At ‘d’, ‘R2: The phytoplankton growth’ loop again starts to dominate the system, causing an exponential growth of phytoplankton.

3.10.3 Zooplankton Growth Rate

The model contains relative simple versions of the Lotka-Volterra equations. This is because the main reason for including predation of phytoplankton by zooplankton is not for the sake of the zooplankton itself, but in order to have some kind of predation limitation on the phytoplankton level. It is therefore considered unnecessary to include a more complicated predation model. In the model a predator coefficient is included in order to show how much the zooplankton grow based on the amount of phytoplankton and zooplankton in the water. The zooplankton growth rate has the following equation:

$$Z_g = C_{fyt} * Z * p$$

Equation 3.10-6: The zooplankton growth rate.

Z_g = The zooplankton growth rate

C_{fyt} = The phytoplankton level in the water

Z = The zooplankton level in the water

p = The predator coefficient.

3.10.4 Zooplankton Death Rate

The zooplankton death rate is assumed to be a simple first-order death rate. It could be a function of the nutrient level in the water (phytoplankton), but in order to simplify the model this is not considered. The amount of zooplankton is divided by the average zooplankton life span.

$$z_d = \frac{Z}{z_1}$$

Equation 3.10-7: The zooplankton death rate.

z_d = Zooplankton death rate

z_l = Zooplankton life span

3.10.5 Predation

The predation of phytoplankton is determined by the number of phytoplankton in the water, the number of zooplankton, and the predation coefficient. The predation coefficient indicates how many phytoplankton will be eaten when there is a certain number of phytoplankton and zooplankton in the water.

$$P_{cfyt} = C_{fyt} * Z * P_c$$

Equation 3.10-8: Phytoplankton predation by zooplankton.

P_{cfyt} = Phytoplankton predation

p_c = Predation coefficient

4. Implementation

This chapter describes the implementation of the Sørfjord model in PowerSim™ Constructor 2.51. First, the implementation of the biology model is explained, then the merging between the biology and hydrodynamic model. The equations in the model are based on the equations that are described in chapter 3. The equations are listed alphabetically in Appendix A, and the stock and flow diagrams are presented in Appendix B.

4.1 Implementation of Nitrification in the Biology Model

4.1.1 Nitrification Rate

The nitrification rate is measured in microgram nitrogen per day. Kaplan (1983) uses micromol nitrogen per liter per day, and any numbers drawn from his text are converted to microgram in order to obtain unit coherence in the model.

The following equation is implemented in the model, using the Michaelis-Menten function to constrain the nitrification rate at low levels of ammonium and oxygen in the water (see equation 3.6-4):

```
aux     NitrificationRate = AmmoniumPerBacteria * NitrifyingBacteria *
        MaxNitrificationRate * (AmmoniumInWater / (AmmoniumInWater +
        AmmoniumHalfSaturation)) * (Oxygen / (Oxygen + OxygenHalfSaturation))

unit    NitrificationRate = microgram / day
```

Equation 4.1-1

4.1.2 Ammonium and Oxygen Nitrification

The nitrification rate is measured in microgram nitrogen per day. In order to determine how much ammonium and oxygen the nitrification rate corresponds to, it must be transformed into microgram of nitrified ammonium and oxygen per liter per day. Ammonium (NH_4^+) contains one nitrogen atom and four hydrogen atoms. This indicates that for each nitrogen atom that is nitrified, four hydrogen atoms are also nitrified. The weight of these hydrogen atoms must be added to the 'AmmoniumNitrificationRate' in order to determine how much ammonium that is nitrified based on the nitrification rate. The atomic weight of one nitrogen atom is 14.00674, and the weight of four hydrogen atoms is 4.03176 (Joesten & Wood, 1996). The relative relationship between the nitrogen atoms and the hydrogen atoms is found by dividing the weight of nitrogen by hydrogen: 4.03176 / 14.00674 = 0.28784. This results in the dimensionless constant:

```
const   HydrogenAddition = 0.28784

unit    HydrogenAddition = dimensionless
```

Equation 4.1-2

This implies that for each microgram of nitrogen that is nitrified, an addition of the fraction 0.28784 should be added to ammonium. This is implemented as follows:

```
aux     AmmoniumNitrificationRate = NitrificationRate + (HydrogenAddition *
        NitrificationRate)

unit    AmmoniumNitrificationRate = microgram / day
```

Equation 4.1-3

The amount of oxygen consumed per microgram of nitrified nitrogen is implemented in a similar manner. Nitrate has the chemical symbol NO_3^-. This indicates that for each nitrogen atom that is nitrified from ammonium (NH_4^+) to nitrate (NO_3^-), three oxygen atoms are consumed. The atomic weight of three oxygen atoms is 47.99820 (Joesten & Wood, 1996).

Dividing this by the weight of nitrogen results in the following calculation: 47.99820 / 14.00674 = 3.42679. This is implemented with a constant for oxygen per nitrification:

```
const   OxygenPerNitrification = 3.42679

unit    OxygenPerNitrification = dimensionless
```

Equation 4.1-4

The oxygen consumption rate caused by nitrification is implemented applying the following equation:

```
aux     OxygenConsumptionRate = NitrificationRate * OxygenPerNitrification

unit    OxygenConsumptionRate = microgram / day
```

Equation 4.1-5

The reason this equation is slightly different than the one for ammonium nitrification (equation 4.1-3), is that oxygen is not like ammonium a compound of different substances, and it does not contain nitrogen. The first part of equation 4.1-3 is therefore left out because it represents the nitrogen that is consumed in the nitrification process, which is not part of the oxygen consumption.

4.1.3 Nitrifying Bacteria

The nitrifying bacteria are measured in cells per liter (equation 4.1-6), and grow based on the nitrification rate (equation 4.1-1). Kaplan (1983) presents a cell yield of 70.39 cells microgram nitrified nitrogen (converted from 10^6 cells per micromol nitrogen). The cell yield is multiplied by the nitrification rate, in order to solve for the bacteria production rate (equation 4.1-7).

```
init    NitrifyingBacteria = 100000

flow    NitrifyingBacteria = +dt * BacteriaProductionRate
        -dt * BacteriaDeathRate

unit    NitrifyingBacteria = cells
```

Equation 4.1-6

```
aux     BacteriaProductionRate = NitrificationRate * BacteriaCellYield

unit    BacteriaProductionRate = cells/day
```

Equation 4.1-7

4.1.4 Nitrifying Bacteria Death Rate

As mentioned in section 3.6-4, references to bacteria death rates were not found in the literature. The bacteria death rate is therefore assumed to be a function of the amount of ammonium and oxygen in the water, since these factors also enhance bacteria growth (equation 4.1-8). Michaelis-Menten equations are applied in the implementation, assuming that the bacteria die more quickly as ammonium and oxygen become a scarcity. A maximum bacteria life span is set to 12 days. The bacteria level is divided by the maximum life span, which is multiplied by the half saturation equations. As the amount of ammonium and oxygen in the water is drained, the maximum bacteria life span will be multiplied by a lower number. This results in a lower bacteria life span. When the total bacteria level is divided by a lower number, more bacteria will die. The last part of the equation will be close to 1 when there is a surplus of ammonium and oxygen in the water, resulting in a bacteria life span close to the maximum.

A DIVZ1-function is implemented in the nitrifying bacteria death rate in order to prevent errors if there is no ammonium or oxygen in the water. If the equation is divided by 0, the result will be 1, causing the death rate to be at its maximum when either ammonium or oxygen lacks completely.

```
aux     BacteriaDeathRate = NitrifyingBacteria DIVZ1 (MaxBacteriaLifeSpan *
        (AmmoniumInWater / (AmmoniumInWater + AmmoniumHalfSaturation)) *
        (Oxygen / (Oxygen + OxygenHalfSaturation)))

unit    BacteriaDeathRate = cells / day
```

Equation 4.1-8

4.2 Implementation of Phytoplankton in the Biological Model

The initialisation of the stocks in the biological model is done to ensure a consistent relation between the different factors. With the amount of carbon as phytoplankton as basis, the other components of phytoplankton are initialised to fit the Redfield ratio (Chapter 3.7). Supply of nutrients is initialised to cover the consumption and shall not be a limiting factor for the growth. The oxygen is initialised to 8000 microgram per litre, which is considered as good conditions, but the supply of oxygen does not cover the potential consumption and is a limiting factor for phytoplankton growth.

4.2.1 Phytoplankton Growth Rate

Carbon as phytoplankton is used to measure the amount of phytoplankton in the water, and the growth rate is measured in microgram carbon per litre per day. The phytoplankton growth is split into three parts. Each of the main components in phytoplankton is modelled separately. Carbon is the main component and is not considered as a limiting factor in the growth. For this reason this part of the model is simpler then for the other components, ammonium and phosphorus.

To model phytoplankton in the water, the growth conditions are among the most important factors. In the model temperature, light and nutrients are implemented, and the equations below describes how these factors are implemented.

```
aux     CarbonInPhytoplanktonGrowthRate= CarbonInPhytoplankton*PhytoplanktonGrowthRate*
        EffectOfCarbonInPhytoplanktonAndWater

unit    microgram/day
```

Equation 4.2-1

4.2.2 Limitations on the growth rate

The phytoplankton growth rate is the growth rate adjusted for light, temperature and nutrient limitations. The TemperatureAndNutrientLimitedGrowthRate is the growth rate without light limitation, and by multiplying with the LightLimitation we get the phytoplankton growth rate.

aux PhytoplanktonGrowthRate= TemperatureAndNutrientLimitedGrowthRate*LightLimitation

unit 1/day

Equation 4.2-2

4.2.3 Nutrient Limitation

The nutrient limitation limits the growth rate when the surplus of either phosphorus or ammonium goes towards zero. When the surplus of nutrients is high, the conditions for optimal growth are present and the NutrientLimitation goes toward one. Then it does not influence the initial growth rate.

aux NutrientLimitation = 1/(1+(1/RelativeNitrogenSurplus)+(1/RelativePhosphorusSurplus))

unit dimensionless

Equation 4.2-3

The relative phosphorus surplus and the relative nitrogen surplus are the surplus of the two limiting nutrients in the phytoplankton. The fraction of the nutrient minus the lower limit for the nutrient compared to carbon in the phytoplankton gives the relative surplus.

aux RelativePhosphorusSurplus=(FractionPhosphorusInPhytoplankton-Lower LimitPhosphorusComparedToCarbon)/LowerLimitPhosphorusComparedToCarbon

unit dimensionless

Equation 4.2-4

Fraction phosphorus in phytoplankton compares phosphorus in phytoplankton to carbon in phytoplankton, and the same equation is used for nitrogen.

aux FractionPhosphorusInPhytoplankton=PhosphorusInPhytoplanktonDIVZ0 CarbonInPhytoplankton

unit dimensionless

Equation 4.2-5

4.2.4 Temperature Limitation

The temperature limited growth rate is the implementation of Eppley's exponential function for growth (Eppley, 1972). It is based on a maximum temperature (MAXTemperature) set to 20°C, and the growth rate increases exponentially towards 20°C. The temperature coefficient is 0,063 according to Eppley.

aux TemperatureLimitedGrowthRate =GrowthRate20C*EXP(TemperatureCoefficient*(Temperature-MAXTemperature))

unit 1/day

Equation 4.2-6

This is the temperature limited growth rate adjusted for nutrient limitation. The nutrient limitation is in the range between 0 and 1 and can be multiplied directly into the growth rate.

aux TemperatureAndNutrientLimitedGrowthRate=
NutrientLimitation*TemperatureLimitedGrowthRate

unit 1/day

Equation 4.2-7

4.2.5 Light Limitation

The light limitation influences the phytoplankton growth at low light levels. Light is measured in W/m^2. The critical light intensity marks the transition between growth limited by light and temperature. When light level goes below this transition, the LightLimitation goes towards zero and reduces the growth rate. The CriticalLightIntensity is a variable that changes during time and describes the phytoplankton's ability to utilize the light. The CoefficientN is 3 and sets the ability of phytoplankton to adapt to changing light intensities.

aux LightLimitation=
Light/(((CriticalLightIntensity^CoefficientN)+Light^CoefficientN)^(1/CoefficientN))

unit dimensionless

Equation 4.2-8

4.2.6 Relative Carbon In Phytoplankton And Water

EffectOfCarbonInPhytoplanktonAndWater is a dimensionless factor that influences the growth rate. Both carbon and phytoplankton in water are compared to the half saturation concentrations. If there is enough carbon in the water the effect is set to 1. Multiplied into the growth rate it gives no effect. Limited carbon availability in the water will be limiting on the growth, and with small amounts of phytoplankton in the water, the effect will increase. The range of the effects is 1 to 0.

aux EffectOfCarbonInPhytoplanktonAndWater=
IF(EffectOfRelativeCarbonInWater=1,1,EffectOfRelativeCarbonAsPhytoplankton*
EffectOfRelativeCarbonInWater)

unit dimensionless

Equation 4.2-9

4.2.7 Carbon in Phytoplankton Reduction

CarbonInPhytoplanktonReductionRate is the sum of natural death of phytoplankton, respiration and grazing by zooplankton. The rate is measured in microgram per litre per day. Predation rate (see Chapter 4.1.3) represents the grazing by zooplankton. The natural reduction rate

aux CarbonInPhytoplanktonReductionRate = PredationRate+(NaturalReductionRate*
CarbonInPhytoplankton)

unit microgram/day **Equation 4.2-10**

4.2.8 Phytoplankton Loss Factors

The natural reduction rate is the sum of the death rate and respiration. The predation rate is not included in this rate. The loss factors are implemented as follows:

aux NaturalReductionRate = PhytoplanktonDeathRate+Respiration

unit 1/day

Equation 4.2-11

4.2.9 Respiration

The respiration is natural loss of nutrients during the lifecycle of the phytoplankton. In the same way as the temperature-limited growth, it covers the temperature range up to 20°C, and is highly influenced by temperature. RespirationAt20C is the initial respiration rate at 20°C, and has a value of 0.04. The temperature coefficient is 0.063, and MAXTemperature is 20.

aux Respiration=RespirationAt20C*EXP(TemperatureCoefficient*(Temperature-MAXTemperature))

unit 1/day

Equation 4.2-12

4.2.10 Death Rate

Phytoplankton death rate is based on a first order death rate influenced by temperature, carbon in phytoplankton and the internal nutrient limitation in phytoplankton. All the factors are relative numbers and are multiplied into phytoplankton death rate.

aux PhytoplanktonDeathRate=
DeathAsAFunctionOfTemperature*CarbonInPhytoplanktonHalfSaturationFunction*
NutrientEffectOnDeathRate

unit 1/day

Equation 4.2-13

4.2.11 Temperature

Death as a function of temperature is the same function as for respiration, and is implemented the same way.

```
aux   DeathAsAFunctionOfTemperature=
      DeathRateAt20C*EXP(TemperatureCoefficient*(Temperature-MAXTemperature))

unit  1/day
```

Equation 4.2-14

4.2.12 Nutrients

The nutrient effect on death rate uses the same function for nutrient limitation as the growth rate, and the nutrient coefficient states the effect that nutrient limitation has on the phytoplankton death rate. The coefficient has a value of 0.5.

```
aux   NutrientEffectOnDeathRate = 1-(1-NutrientCoefficient)*NutrientLimitation

unit  dimensionless
```

Equation 4.2-15

4.2.13 Carbon

The CarbonInPhytoplanktonHalfSaturationFunction compares the actual amount of phytoplankton to the half saturation concentration, and results in a death rate as a function of the phytoplankton population

```
aux   CarbonInPhytoplanktonHalfSaturationFunction=
      CarbonInPhytoplankton/(CarbonInPhytoplanktonHalfSaturation+
      CarbonInPhytoplankton)

unit  dimensionless
```

Equation 4.2-16

4.2.14 Ammonium consumption

The ammonium consumption rate is the rate by which ammonium is consumed by phytoplankton from the water. It is based on the 'carbon in phytoplankton growth rate' and adjusted for limitations on ammonium and some other factors. Initiated ammonium consumption is the minimum of the optimal consumption based on the Redfield ratio (Redfield et al. 1963) and a light and nutrient limitation. Multiplied with the carbon growth rate this would give an optimal ratio. The 'effect of phytoplankton and ammonium in water' is a factor adjusting the ammonium uptake based on the availability of ammonium in the water. Low availability will decrease the uptake. The range is 0 to 1. 'Effect of relative ammonium on consumption rate' compensates for the amount of ammonium consumed but still not utilized in the phytoplankton. It only decreases the uptake if the stock of unutilized ammonium grows above a fixed fraction, and the factor is between 0 and 1. The 'Redfield effect on ammonium growth' is the only factor that can increase the growth rate. It compares the nutrient composition of phytoplankton with the Redfield ratio, and increases the growth rate with a factor ranging between 1 and 1.4 to close the gap.

```
aux     AmmoniumConsumptionRate=
        InitiatedAmmoniumConsumption*CarbonInPhytoplanktonGrowthRate*
        EffectOfPhytoplanktonAndAmmoniumInWater*
        EffectOfRelativeAmmoniumOnConsumptionRate*
        RedfieldEffectOnAmmoniumGrowthRate

unit    microgram/day
```

Equation 4.2-17

The 'initiated ammonium consumption' uses a MAX and a MIN function. The Max function is used to avoid negative growth rates, and the MIN function is used to decide whether to use the optimal ammonium consumption or the light and ammonium limited consumption capacity. 'Consumption capacity limited by light and ammonium in water' is maximum consumption adjusted for limitations of light and ammonium in the water. Low light intensity can reduce the ability to take up ammonium from the water. The factor also includes the effect of low concentrations of ammonium in the water. High concentrations of phytoplankton

in the water can compensate for lack of ammonium because they can utilize even very small amounts of ammonium.

```
aux     InitiatedAmmoniumConsumption=
        MAX(0,MIN(ConsumptionCapacityLimitedByLightAndAmmoniumInWater,
        OptimalAmmoniumConsumption))

unit    1/day
```

Equation 4.2-18

The 'effect of phytoplankton and ammonium in water' is a factor doing almost the same as the one above. It is implemented in the model to increase the simulation speed. Using the half saturation concentration generally solves the lack of ammonium in theoretical formulas. This gives a level where the growth rate is reduced to 50 % of the initial growth. Using this in the model causes some problems in the simulation. Since the half saturation concentration of ammonium is low, the steps in the simulation have to be very small to get the effect of this function. To increase the speed there is used some graph functions to enlarge the area of reduced growth rate.

```
aux     EffectOfPhytoplanktonAndAmmoniumInWater=
        IF(EffectOfRelativeAmmoniumInWater=1,1,EffectOfRelativeCarbonAsPhytoplankton*EffectOfR
        elativeAmmoniumInWater)

unit    dimensionless
```

Equation 4.2-19

```
aux     EffectOfRelativeAmmoniumOnConsumptionRate=
        GRAPH(RelativeAmmoniumStock,0.1,0.01,[1,0.98,0.89,0.79,0.66,0.54,0.36,0.17,0.07,0.03,0
        "Min:0;Max:1;Zoom"])

unit    dimensionless
```

Equation 4.2-20

4.2.15 Ammonium in Phytoplankton Growth Rate

The phytoplankton growth rate is the rate by which phytoplankton uses the ammonium in the stock to grow. The rate is based on the phytoplankton growth rate and adjusted for limitations and to seek the Redfield ratio. 'Redfield effect on ammonium growth rate' is a graph function

that increases the rate with a factor between 1 and 1.4 to fill the gap between the actual carbon – ammonium ratio and the Redfield ratio.

'Effect of relative ammonium on growth rate' avoids the stock of ammonium in phytoplankton to become empty. It is implemented as a graph function reducing the growth rate when the stock goes below 10% of ammonium in phytoplankton.

```
aux     AmmoniumInPhytoplanktonGrowthRate=
        PhytoplanktonGrowthRate*RedfieldEffectOnAmmoniumGrowthRate*
        EffectOfRelativeAmmoniumOnGrowthRate

unit    microgram/day
```

Equation 4.2-21

```
aux     EffectOfRelativeAmmoniumOnGrowthRate=
        GRAPH,0,0.01,[0,0.02,0.04,0.08,0.34,0.69,0.86,0.97,0.99,1,1"Min:0;Max:1;Zoom"])

unit    dimensionless
```

Equation 4.2-22

4.2.16 Ammonium in Phytoplankton Reduction Rate

The reduction rate is similar to the one for carbon as phytoplankton. The 'natural reduction rate' is calculated based on ammonium in phytoplankton and the reduction of ammonium in phytoplankton is proportional to the actual ratio between carbon and ammonium in phytoplankton.

```
aux     AmmoniumInPhytoplanktonReductionRate =
        NH4inPhytoplankton*NaturalReductionRate+AmmoniumPredation

unit    microgram/day
```

Equation 4.2-23

'Ammonium predation' is the predation of the ammonium in phytoplankton. When zooplankton grazes phytoplankton, the phytoplankton contains a certain fraction of ammonium. The 'Predation rate' is the same as for 'carbon as phytoplankton', and multiplied with the 'fraction ammonium in phytoplankton' this gives the 'ammonium predation'.

```
aux     AmmoniumPredation = PredationRate*FractionAmmoniumInPhytoplankton

unit    microgram/day
```

Equation 4.2-24

4.2.17 Phosphorus in phytoplankton

The implementation of phosphorus in phytoplankton is almost identical to the implementation of ammonium in phytoplankton. Only some constants that are individual for each nutrient are different.

4.3 Implementation of Predator-Prey in the Biology Model

4.3.1 Zooplankton Growth Rate

The predation of phytoplankton by zooplankton is implemented applying simple predator-prey equations as described in section 3.10. The zooplankton (equation 4.3-1) grow based on equation 4.3-2, where the amount of zooplankton and phytoplankton (measured as carbon in phytoplankton) is multiplied by a predator coefficient. This coefficient describes the interaction between zooplankton and phytoplankton, and the effect the level of each of them has on the zooplankton growth. The number of zooplankton is measured in cells per liter.

```
init    Zooplankton = 900

flow    Zooplankton = -dt * ZooplanktonDeathRate
        +dt * ZooplanktonGrowthRate

unit    Zooplankton = cells
```

Equation 4.3-1

```
aux     ZooplanktonGrowthRate = Zooplankton * CarbonInPhytoplankton *
        PredatorCoefficient

unit    ZooplanktonGrowthRate = cells / day
```

Equation 4.3-2

4.3.2 Zooplankton Death Rate

Zooplankton death (equation 4.3-3) is implemented as a first order death rate, where the number of zooplankton is divided by a zooplankton life span. In an alternative implementation a Michaelis-Menten equation could have been utilized, in order to make more zooplankton die when there are less phytoplankton in the water. However, for simplification it

is omitted from the model, as it is not considered necessary to describe the problem. The zooplankton life span is set to 8 days.

```
aux     ZooplanktonDeathRate = Zooplankton / ZooplanktonLifespan

unit    ZooplanktonDeathRate = cells / day
```

Equation 4.3-3

4.3.3 Phytoplankton Predation

The predation of phytoplankton by zooplankton is also implemented as a simple Lotka-Volterra equation (equation 4.3-4). The amount of phytoplankton (measured in microgram) is multiplied by the number of zooplankton cells, and a predation coefficient. The predation coefficient represents the effect the number of zooplankton and the amount of phytoplankton have on the predation rate.

```
aux     PredationRate = CarbonInPhytoplankton * Zooplankton * PredationCoefficient

unit    PredationRate = microgram / day
```

Equation 4.3-4

The predation rate is later added into the 'CarbonInPhytoplanktonReductionRate' (see section 4.2.7). This, however, only calculates predation of carbon in phytoplankton. Ammonium, phosphorus, and chlorophyll are also consumed by zooplankton. The chlorophyll reduction rate is based on the reduction rate of carbon where predation is already included (see Appendix A, equation BA11), so no alterations are needed to the chlorophyll reduction rate. Ammonium and phosphorus predation is implemented by finding the fraction of ammonium and phosphorus compared to carbon in phytoplankton (equations 4.3-5 and 4.3-6).

aux FractionAmmoniumInPhytoplankton = AmmoniumInPhytoplankton DIVZ0 CarbonInPhytoplankton

unit FractionAmmoniumInPhytoplankton = dimensionless

Equation 4.3-5

aux FractionPhosphorusInPhytoplankton = MAX (PhosphorusInPhytoplankton DIVZ0 CarbonInPhytoplankton, 0.0027)

unit FractionPhosphorusInPhytoplankton = dimensionless

Equation 4.3-6

Both equations are implemented by a DIVZ0-function in order to prevent errors if the carbon in phytoplankton level is 0. Further, the fraction phosphorus in phytoplankton is implemented by a MAX-function, where if the fraction is lower than the lower limit for phosphorus in phytoplankton (0.0027), the lower limit is the result of the equation. This is related to the phytoplankton growth rate and explained in section 4.2. There are some problems related to this equation, which are discussed in the validation chapter, section 5.3.3.

In order to find the ammonium and phosphorus predation, the fractions of ammonium and phosphorus are multiplied by the predation rate. This is then added to the ammonium and phosphorus reduction rates (see section 4.2.16). Equations 4.3-7 and 4.3-8 represent the ammonium and phosphorus predation respectively.

aux AmmoniumPredation = PredationRate * FractionAmmonuimInPhytoplankton

unit AmmoniumPredation = microgram / day

Equation 4.3-7

aux PhosphorusPredation = PredationRate * FractionPhosphorusInPhytoplankton

unit PhosphorusPredation = microgram / day

Equation 4.3-8

4.4 Implementation of the Hydrodynamic and the Biology Model

The biology model is two-dimensional and represents the structure and behavior of the oxygen production and consumption in a liter of water. A model containing only two dimensions may give a sufficient explanation of chemical and biological processes, but is inadequate for explaining the behavior of the oxygen levels in a fjord as a whole. In a real fjord system the levels of the various components will vary from location to location causing diverse behavior in different segments of the fjord. Ammonium, for example, is discharged into the water at one point, and transported with the currents to other locations. The result is that parts of the fjord have higher ammonium supplies and levels than others. Also, temperature conditions have unequal values at different depths, causing varying growth conditions for phytoplankton, and thereby affecting the zooplankton population and oxygen production of those levels.

Merging the two-dimensional biology model with a simple three-dimensional hydrodynamic model gives a better description of the oxygen problem in the fjord as a whole. The hydrodynamic model divides the fjord into fjord cells in order to differentiate between varying depths and the distance from the innermost part of the fjord (Nævdal, 2001). This is obtained by using the PowerSim™ range function. Ranges are implemented on every variable that can vary from location to location. The model contains three depth layers and four layers in the outward direction. Both, the number of layers in the outward direction, and the distance outward in the fjord, covered by the model, are alterable. The water is mixed through the currents or flow of the water and diffusion, and the various components flow with the water. Following are three important ranges in the model:

d = depth
x = xdir
k = component

'd' indicates the depth of the fjord cell or the lateral layer at which it is located. 'x' defines the position of the fjord cell in the horizontal direction, that is, in the outward direction of the fjord. References to the fjord cells are presented in figure 4-1.

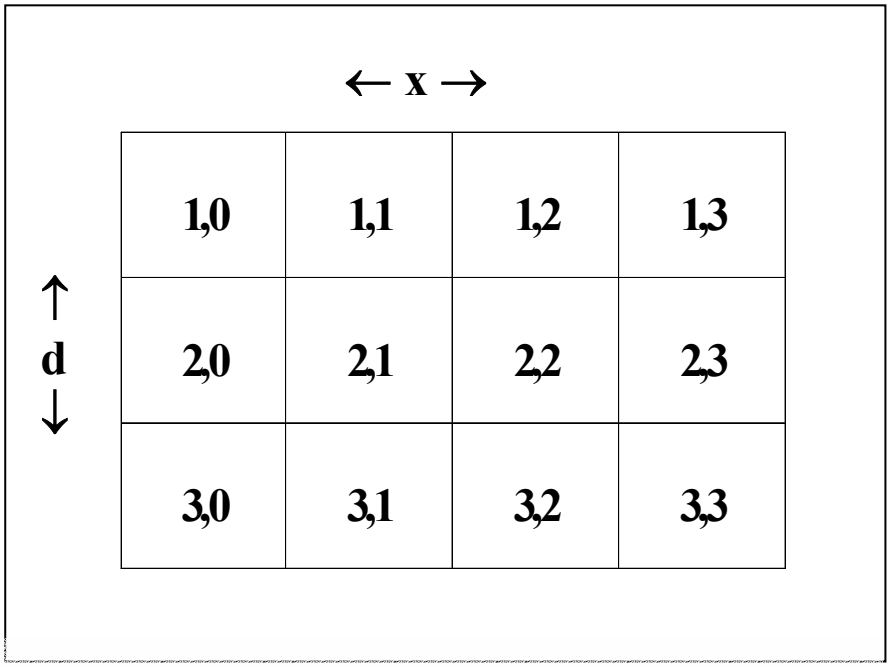


Figure 4-1: The fjord cells that the model is divided into.

Finally, 'k' defines the components, or substances that the water consists of. Originally, the hydrodynamic model contained only four components: water (H_2O), salt (NaCl), oxygen (O_2), and ammonium (NH_4^+) (Nævdal, 2001). The components in the biology model are added in order to let the levels in the biological system flow with the currents in the fjord. The following levels are included in the range 'Component':

- H2O (Water)
- O2 (Oxygen)
- NH4 (Ammonium)
- NaCl (Salt)
- Bacteria (Nitrifying bacteria)
- NH4InPhytoplankton (Ammonium in phytoplankton)
- CarbonInPhytoplankton (Carbon in phytoplankton)
- PhosphorusInPhytoplankton (Phosphorus in phytoplankton)

- NH4InStock (Ammonium in phytoplankton stock)
- PhosphorusInStock (Phosphorus in phytoplankton stock)
- Phosphorus
- Carbon
- Chlorophyll (Chlorophyll in phytoplankton)
- Zooplankton

An important level in the hydrodynamic model is the 'MassPerComponent'. This is the mass in kilos of each component or substance that each fjord cell contains. It has the following equation:

(4.4-1a) dim MassPerComponent = (k = component, d = depth, x = xdir)

(4.4-1b) init MassPerComponent = Volume (d ,x) * CompositionFjord (k, d, x) * ComponentWeight (k) * 1000

flow MassPerComponent =

(4.4-1c)

```
+ dt * (MixingOut (k, d, x - 1) | x  > FIRST(xdir); 0)
+ dt * (MixingIn (k, d, x + 1)  | x < LAST(xdir); 0)
- dt * (MixingIn (k, d, x) )
- dt * (MixingOut (k, d, x) )
+ dt * (OutgoingFlowIn (k, d, x + 1) | x < LAST (xdir) AND d
= layer3; 0)
- dt * (OutgoingFlowIn (k, d, x) |  x <> FIRST (X) ; 0 )
+ dt * (OuterExchange (k, d) | x = LAST (xdir) AND d =
LAST (depth); 0)
+ dt * (InOutBalance (k, d + 1, x) |  d = layer2; 0)
- dt * (InOutBalance (k, d, x) | d = layer3; 0)
+ dt * (OutgoingFlowOut (k, d, x - 1) | d < LAST (depth) AND
x > FIRST (xdir); 0)
- dt * (OutgoingFlowOut (k, d, x) )
- dt * (MixingUp (k, d, x) )
+ dt * (MixingUp (k, d + 1, x) | d < LAST (depth); 0)
+ dt * (VerticalMixingDown (k, d - 1, x)  | d > FIRST(depth);
0)
```

```
                    - dt * (VerticalMixingDown (k, d, x) )
                    + dt * (Opo (k, d, x) )
(4.4.1d)            - dt * (OutFlowAll (k, d, x))
(4.4.1e)            + dt * (InFlowAll (k, d, x))

        unit        MassPerComponent = kg
```

Equation 4.4-1

The dimensions are set to (k, d, x), respectively representing the component level of each type of substance in each fjord cell, for a certain depth layer, and outward layer (4.4-1a). It is initialized by the volume of each fjord cell, the composition of the fjord, that is, the relative amount of each type of component, and the weight of the component (4.4-1b). The inflows and outflows for each fjord cell is controlled by the flow of the water in the hydrodynamic model (4.4-1c), with the exception of the flows 'InFlowAll' (4.4-1d) and 'OutFlowAll' (4.4-1e)[1]. These flows result from the biological part of the model, and represent the biological processes within each fjord cell. 'InFlowAll' and 'OutFlowAll' are the only flows that are added to the original hydrodynamic model.

'MassPerComponent' is initialized through the amount of each component in the water from the ocean that flows into the fjord. This is implemented by the constant 'CompositionOcean' (equation 4.1-2). The unit of measure is dimensionless, and represents the amount of the component in kilos compared to one kilo of water.

[1] For a further explanation of the hydrodynamic flows, see Nævdal, 2001.

```
dim     CompositionOcean = (k=component,d=depth)

const   CompositionOcean = 1 | k=H2O AND d=layer1top;
        0.035| k=NaCl AND d=layer1top;
        0.000008 | k=O2 AND d=layer1top;
        0.00000001 | k=NH4 AND d=layer1top;
        0.00005 | k=Bacteria AND d=layer1top;
        0.000000014 | k=NH4inPhytoplankton AND d=layer1top;
        0.000000083 | k=CarbonInPhytoplankton AND d=layer1top;
        0.000000002 | k=PhosphorusInPhytoplankton AND d=layer1top;
        0.0000000014 | k=NH4InStock AND d=layer1top;
        0.0000000002 | k=PhosphorusInStock AND d=layer1top;
        0.0000001 | k=Phosphorus AND d=layer1top;
        0.01 | k=Carbon AND d=layer1top;
        0.000000002075 | k=Chlorophyll AND d=layer1top;
        0.000001 | k=Zooplankton AND d=layer1top;
        1 | k=H2O AND d=layer2;
        0.035 | k=NaCl AND d=layer2;
        0.000006 | k=O2 AND d=layer2;
        0.000001 | k=NH4 AND d=layer2;
        0.00005 | k=Bacteria AND d=layer2;
        0.000000014 | k=NH4inPhytoplankton AND d=layer2;
        0.000000083 | k=CarbonInPhytoplankton AND d=layer2;
        0.000000002 | k=PhosphorusInPhytoplankton AND d=layer2;
        0.0000000014 | k=NH4InStock AND d=layer2;
        0.0000000002 | k=PhosphorusInStock AND d=layer2;
        0.0000001 | k=Phosphorus AND d=layer2;
        0.01 | k=Carbon AND d=layer2;
        0.000000002075 | k=Chlorophyll AND d=layer2;
        0.000001 | k=Zooplankton AND d=layer2;
        1 | k=H2O AND d=layer3;
        0.035 | k=NaCl AND d=layer3;
        0.000005| k=O2 AND d=layer3;
        0.000001 | k=NH4 AND d=layer3;
        0.00005 | k=Bacteria AND d=layer3;
        0.000000014 | k=NH4inPhytoplankton AND d=layer3;
```

0.000000083 | k=CarbonInPhytoplankton AND d=layer3;

0.000000002 | k=PhosphorusInPhytoplankton AND d=layer3;

0.0000000014 | k=NH4InStock AND d=layer3;

0.0000000002 | k=PhosphorusInStock AND d=layer3;

0.0000001 | k=Phosphorus AND d=layer3;

0.01 | k=Carbon AND d=layer3;

0.000000002075 | k=Chlorophyll AND d=layer3;

0.000001 | k=Zooplankton AND d=layer3

unit CompositionOcean = dimensionless

Equation 4.4-2

4.4.1 InFlowAll

In merging the biology and hydrodynamic model, depth and length ranges (d,x) are put on all biological variables that will vary at different depths and lengths. All the inflows concerning the biological part of the model are gathered in the auxiliary 'InFlowAll', which represents the inflows of the various components in each fjord cell, caused by the biological processes within the fjord cell. Similarly, all outflows from the biological part of the system are gathered into 'OutFlowAll', which represents all outflows caused by the biological part of within each fjord cell. Figure 4-2 shows a stock and flow diagram of all the inflows for the different components that are gathered into one inflow variable. In PowerSim™ the double circles around the variables denote that they are implemented by ranges. Here it means that their values are calculated for each fjord cell and for the particular components that the fjord cell contains.

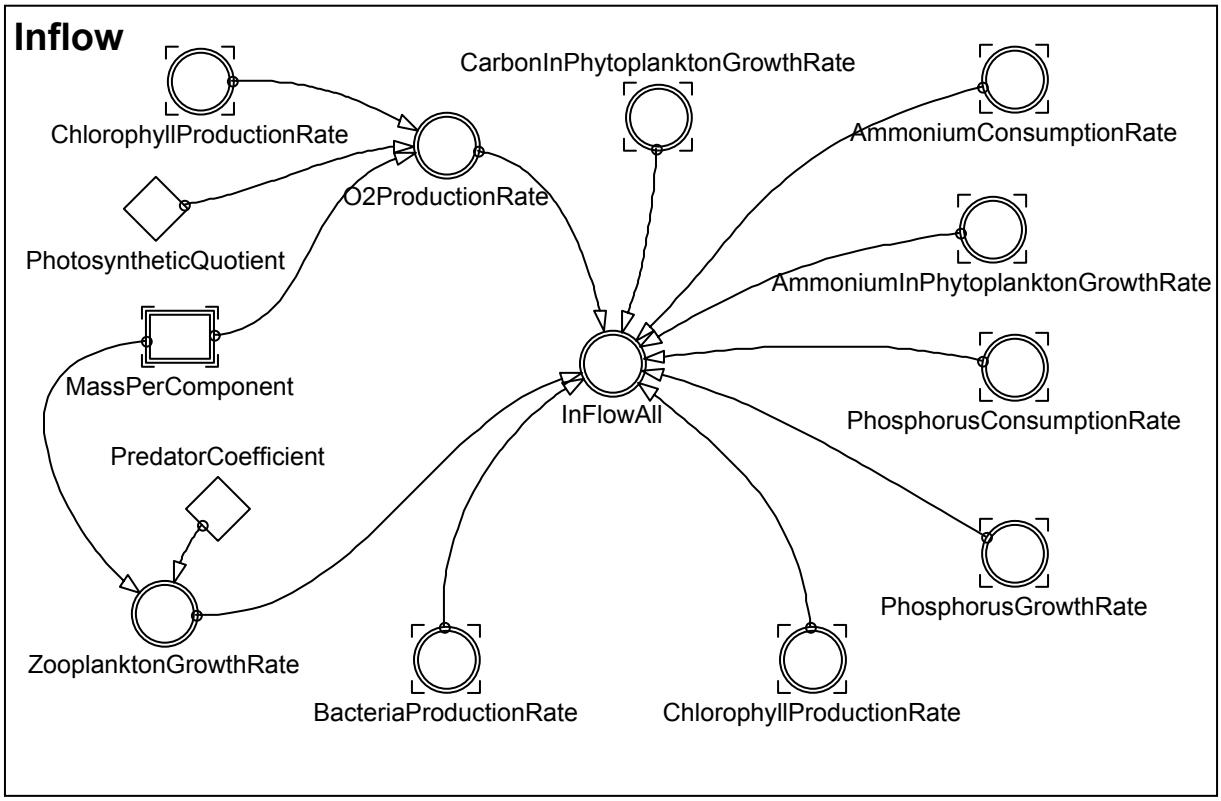


Figure 4-2: Stock and flow diagram of all the inflows from the biological part of the model.

Equation 4.4-3 shows 'InFlowAll', all inflows based on the biological part of the model.

```
(4.4-3a) dim    InFlowAll = (k = component, d = depth, x = xdir)

(4.4-3b) aux    InFlowAll = BacteriaProductionRate (d, x) | k = Bacteria;
(4.4-3c)        ZooplanktonGrowthRate (d, x) | k = Zooplankton;
(4.4-3d)        CarbonInPhytoplanktonGrowthRate (d, x) | k =
                CarbonInPhytoplankton;
                ChlorophyllProductionRate (d, x) | k = Chlorophyll;
                PhosphorusGrowthRate (d, x) | k =
                PhosphorusInPhytoplankton;
                PhosphorusConsumptionRate (d, x) | k =
                PhosphorusInStock;
                AmmoniumConsumptionRate (d, x) | k = NH4InStock;
                AmmoniumInPhytoplanktonGrowthRate (d, x) | k =
                NH4inPhytoplankton;
(4.4-3e)        O2ProductionRate (d, x) | k = O2;
                0

        unit    InFlowAll = kg/day
```

Equation 4.4-3

'InFlowAll' has the dimensions (k, d, x) in order to calculate the right inflow for a particular fjord cell, and for the right component (4.4-3a). This is where the variables caused by the biological part of the system is connected to the right component or level in the biological system. The first part of the equation is the bacteria production rate, which gives a gain to the bacteria populations in each fjord cell (4.4-3b). The three dimensions that this part of the equation involves are also defined. 'k = Bacteria' sets the component associated with the first part of the equation. This means that the equation will calculate the value of the 'BacteriaGrowthRate' for each fjord cell, and that this rate is an input to the bacteria population of the particular cell. The second part of the equation calculates the zooplankton growth rate (4.4-3c). The last part of the equation line indicates that this is a gain for each of the zooplankton populations in the particular cells. Next, the inflows concerning the various components associated with phytoplankton consumption and growth are indicated (4.4-3d). These are implemented using the same structure as for the previous part of the equation. The

oxygen production rate is implemented in the same manner (4.4-3e). The 0 at the end of the equation indicates that if there are components in 'MassPerComponent' that are not included in the equation, they will not be affected. The unit of measure is kilos per day.

4.4.2 OutFlowAll

The outflows for the various components in the fjord cells from the biological part of the model are implemented in a similar way as the inflows. Figure 4-3 is a stock and flow diagram of all the outflows for the different components gathered into one outflow variable with ranges. The 'OutFlowAll' variable is connected to 'MassPerComponent' as an outflow.

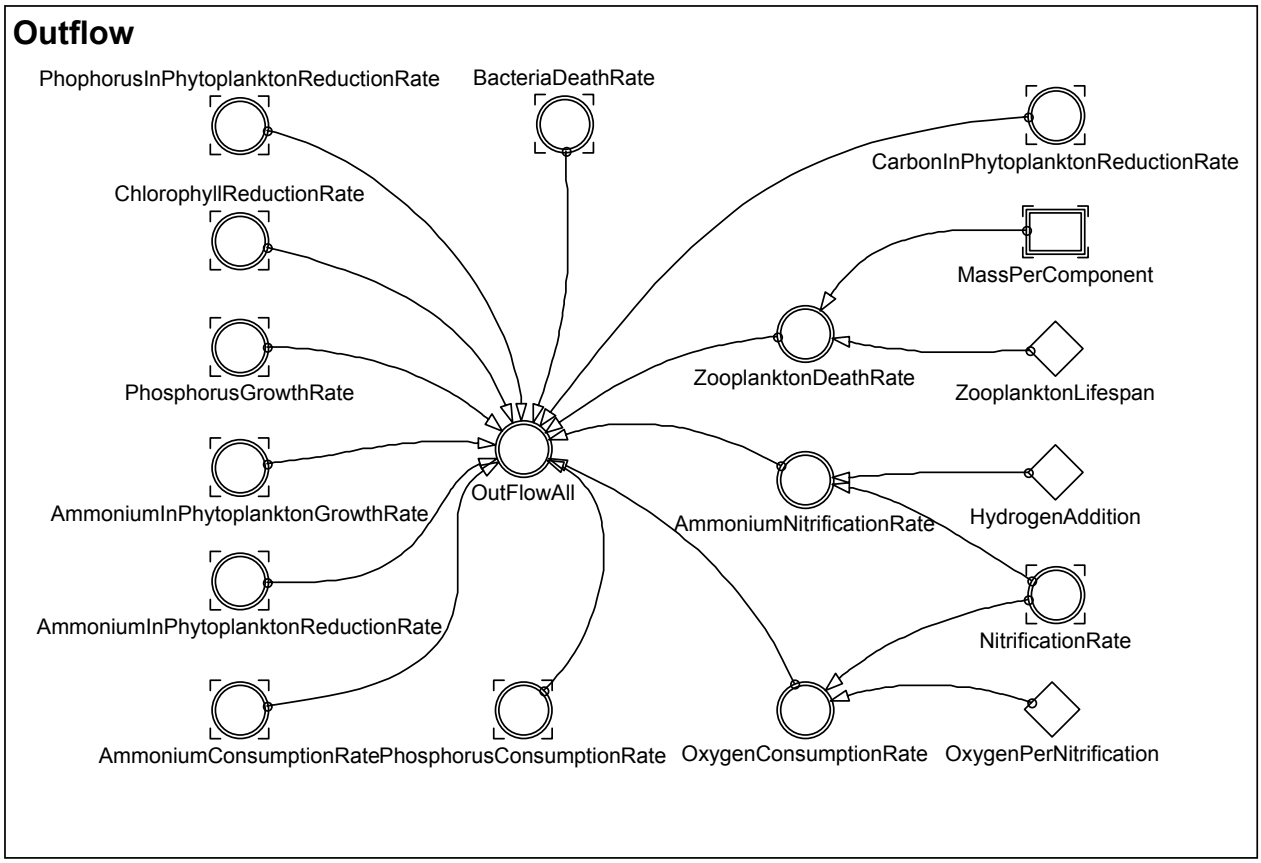


Figure 4-3: Stock and flow diagram of all outflows from the biological part of the model.

Equation 4.4-4 calculates the outflows that are drawn from the fjord cells based on the biological part of the model.

```
(4.4-4a) dim    OutFlowAll = (k = component, d = depth, x = xdir)

(4.4-4b) aux    OutFlowAll = OxygenConsumptionRate (d, x) | k = O2;
(4.4-4c)        AmmoniumNitrificationRate (d, x) +
                AmmoniumConsumptionRate (d, x) | k = NH4;
(4.4-4d)        BacteriaDeathRate (d, x) | k = Bacteria;
(4.4-4e)        ZooplanktonDeathRate (d, x) | k = Zooplankton;
(4.4-4f)        CarbonInPhytoplanktonReductionRate (d, x) | k =
                CarbonInPhytoplankton;
                ChlorophyllReductionRate (d, x) | k = Chlorophyll;
                PhosphorusGrowthRate (d, x) | k = PhosphorusInStock;
                PhosphorusConsumptionRate (d, x) | k = Phosphorus;
                PhophorusInPhytoplanktonReductionRate (d, x) | k =
                PhosporusInPhytoplankton;
                AmmoniumInPhytoplanktonGrowthRate (d, x) | k =
                NH4InStock;
                AmmoniumInPhytoplanktonReductionRate (d, x) | k =
                NH4inPhytoplankton;
                0

         unit   OutFlowAll = kg / day
```

Equation 4.4-4

The dimensions k, d, and x are set in order to calculate the outflow for the particular components of each cell (4.4-4a). The first line solves the oxygen consumption rate, i.e. the oxygen that is consumed in the nitrification process (4.4-4b). Letting k equal O2 indicates that this outflow concerns the oxygen consumption. This will subtract the oxygen consumption rate from the oxygen level of each fjord cell that the fjord is divided into. Further, the outflows caused by ammonium nitrification (4.4-4c), bacteria death (4.4-4d), zooplankton death (4.4-4e), and the phytoplankton loss factors (4.4-4f) are listed. For each, k is associated with the component in question, that is, the level that the outflow is going to be drawn from.

4.4.3 Ammonium Supply

The ammonium supply from the river and the wastewater from Odda Smelteverk are implemented by the constant 'NH4Supply' (equation 4.4-5). 900 000 kilos of ammonium are divided by 365 days in order to distribute the yearly ammonium supply on each day of the year.

```
const   NH4Supply = (900000/365)

unit    NH4Supply = kg/day
```

Equation 4.4-5

The ammonium supply is injected into the three different depth layers in the innermost part of the fjord. Equation 4.4-6 represents the apportionment of the ammonium between the depth layers. The two top layers each receive 30% of the ammonium input, while the bottom layer receives the remaining 40%.

```
dim     NH4Apportionment = (d=depth)

const   NH4Apportionment = [0.3,0.3,0.4]

unit    NH4Apportionment = dimensionless
```

Equation 4.4-6

Equation 4.4-7 represents the Opo River. The ammonium supply is multiplied by the ammonium apportionment in order to give the innermost fjord cells the right amount of ammonium input (for a further explanation of equation 4.4-7, see Nævdal, 2001).

```
dim     River = (k=component,d=depth, x=xdir)

aux     River = RiverFlow*ComponentWeight(k)*CompositionRiver(k) | d=FIRST(depth)
        AND  x=FIRST(xdir) AND k<>NH4;
        NH4Supply*NH4Apportionment(d) |  k=NH4  AND x=FIRST(xdir) ;
        0

unit    River = kg/day
```

Equation 4.4-7

4.4.4 Unit Transformation

In order to merge the biological and hydrodynamic model the units of the variables must be transformed. In the biology model all components or stock variables, except nitrifying bacteria and zooplankton, are measured in microgram per liter. Bacteria and zooplankton are measured in the number of cells per liter. The hydrodynamic model uses kilos per fjord cell. First, each variable is transformed from microgram to kilos. One microgram equals one millionth of a gram; that is, one microgram should be multiplied by10^{-6} in order to convert it into grams. There are 1000 grams in a kilo, implying that one microgram is equivalent of 10^{-9} kilos. This number is multiplied into the equations in order to convert from microgram to kilos.

The hydrodynamic model is measured in kilos of the particular component per fjord cell. The size of the cell is defined in the variable 'Volume', which is measured in cubic meters, m^3. The volume states how many cubic meters a fjord cell is, indicating that Volume must be multiplied into the equation. However, this is not sufficient. The variables imported from the biological model are now converted into kilos per liter, not the amount in the 'Volume'. It is therefore necessary to determine how many liters there are in one fjord cell. One cubic meter (1 m^3) equals to 1000 liters of water. Therefore the equation is multiplied by 1000. The result is a conversion from microgram per liter to kilos per fjord cell. Equation 4.4-8 shows a general description of the equation for the conversion from microgram per liter to kilos per fjord cell:

$$X * 10^{-9} * \text{Volume} * 1000$$

Equation 4.4-8

X = The component or substance in microgram

For example, equation 4.4-9 shows the ammonium half saturation concentration for the biological model. This is the number of micrograms per liter that would result in half of the maximum phytoplankton growth rate. It is set to 7 micrograms per liter.

```
const   AmmoniumHalfStaurationConcentration = 7

unit    AmmoniumHalfStaurationConcentration = microgram
```

Equation 4.4-9

Equation 4.4-10 represents the ammonium half saturation concentration variable that is used in the hydrodynamic model.

```
dim     AmmoniumHalfStaurationConcentration = (d=depth, x=xdir)

aux     AmmoniumHalfStaurationConcentration = 7 * (10^-9) * Volume (d,x) * 1000 *
        ComponentWeight (NH4)

unit    AmmoniumHalfStaurationConcentration = kg
```

Equation 4.4-10

Here, the dimensions (d,x) are set to let it apply for all fjord cells. The half saturation concentration is set to 7 multiplied by 10^{-9}, in order to transform 7 micrograms into kilos. Further, it is multiplied by the volumes of the respective fjord cells, and the component weight of phosphorus (which is set to 1), and 1000 or the numbers of liter per cubic meter in order to transform it into kilos per fjord cell.

For zooplankton and nitrifying bacteria this transformation would be slightly different because they are measured in cells per liter in the biology model. However, there are no variables concerning zooplankton and nitrifying bacteria where the units are transformed.

4.5 Implementation of Nitrification in the Hydrodynamic Model

4.5.1 Nitrification rate

Equation 4.5-1 shows the implementation of the nitrification process in the hydrodynamic model.

```
dim     NitrificationRate = (d=depth, x=xdir)

aux     NitrificationRate = AmmoniumPerBacteria * MassPerComponent (Bacteria,d,x)
        * (MaxNitrificationRate * (MassPerComponent (NH4,d,x) / (MassPerComponent
        (NH4,d,x) + AmmoniumHalfSaturation)) * (MassPerComponent (O2,d,x) /
        (MassPerComponent (O2,d,x) + OxygenHalfSaturation (d,x))))

unit    NitrificationRate = kg / day
```

Equation 4.5-1

The difference from the equation used in the biology model is that here, the dimensions are set to (d,x) in order to let it apply for all the fjord cells. Further, 'MassPerComponent' in the equation is defined to apply for the substance in question. When the amount of ammonium per bacteria is multiplied by 'MassPerComponent', k is defined as 'Bacteria', which means that it is to be multiplied by all bacteria levels in all fjord cells. The ammonium and oxygen levels are also part of 'MassPerComponent', and they are also defined in the same way. The nitrification rate is measured in kilos per day.

4.5.2 Ammonium and Oxygen Nitrification

In the biology model the constant 'HydrogenAddition' is multiplied into the equation in order to determine how much ammonium is nitrified at a certain nitrification rate (equation 4.1-2). This is because the nitrification rate is only measured as the amount of nitrogen, while ammonium (NH_4^+) also contains four hydrogen atoms. The additional weight of the hydrogen

atoms is added to the outflow from ammonium caused by the nitrification rate. Equation 4.5-2 represents the nitrification rate for ammonium. The unit of measure is kilos per day.

```
dim     AmmoniumNitrificationRate = (d = depth, x = xdir)

aux     AmmoniumNitrificationRate = NitrificationRate (d, x) + (HydrogenAddition *
        NitrificationRate (d, x))

unit    AmmoniumNitrificationRate = kg / day
```

Equation 4.5-2

The additional weight for hydrogen is multiplied by the nitrification rate for each fjord cell in order to solve for the amount of hydrogen that is nitrified. This is added to the nitrification rate, and the result is the amount of ammonium that is nitrified. Equation 4.5-3 shows the equation for oxygen consumption caused by nitrification, which is implemented in a similar manner.

```
dim     OxygenConsumptionRate = (d=depth, x=xdir)

aux     OxygenConsumptionRate = NitrificationRate (d,x) * OxygenPerNitrification

unit    OxygenConsumptionRate = kg / day
```

Equation 4.5-3

Since oxygen does not contain nitrogen, the nitrification rate is not added into the equation. The 'OxygenPerNitrification' variable corresponds to the amount of oxygen that is used per kilo nitrogen that is nitrified, and it is sufficient to multiply it with the nitrification rate.

4.5.3 Bacteria Growth

The nitrifying bacteria grow based on the nitrification rate. Equation 4.5-4 shows the bacteria production rate that is implemented in the hydrodynamic model. It is measured in cells per day.

```
dim     BacteriaProductionRate = (d = depth, x = xdir)

aux     BacteriaProductionRate = (NitrificationRate (d,x) * BacteriaCellYield)

unit    BacteriaProductionRate = cells / day
```

Equation 4.5-4

The bacteria cell yield is multiplied by the nitrification rate for each fjord cell. The unit used is cells per kilo nitrified nitrogen. The value of the bacteria cell yield is set to 70.39, which is the same as in the biology model, only that in the biology model the unit is cells per microgram. Thus, the transformation is not completely correct, but considered necessary to obtain a likely behavior of the hydrodynamic model. The scales for nitrifying bacteria that are used in both the biology and the hydrodynamic model as a result of the simulations are comparable. Equation 4.5-5 shows the bacteria cell yield that is implemented in the hydrodynamic model.

```
const   BacteriaCellYield = 71.39

unit    BacteriaCellYield = cells / kg
```

Equation 4.5-5

4.5.4 Bacteria Death

Equation 4.5-6 represents the bacteria death rate.

```
dim     BacteriaDeathRate = (d = depth, x = xdir)

aux     BacteriaDeathRate = MassPerComponent (Bacteria,d,x) /
        (MaxBacteriaLifeSpan DIVZ1 ((MassPerComponent (NH4,d,x) /
        (MassPerComponent (NH4,d,x) + AmmoniumHalfSaturation (d,x))) *
        (MassPerComponent (O2,d,x) / (MassPerComponent (O2,d,x) +
        OxygenHalfSaturation (d,x)))))

unit    BacteriaDeathRate = cells / day
```

Equation 4.5-6

This is implemented in a similar way as for the biology model, only the rate is set to apply for each fjord cell, setting dimensions (d,x). The bacteria level is divided the bacteria life span. The bacteria life span is determined by dividing a maximum life span by limitations to the maximum life span. The limitations are determined by the ammonium and oxygen levels in the water, compared to their half saturation. This makes the life span smaller when there is less ammonium and oxygen in the water. A DIVZ1 function is inserted in order to prevent errors when there is no ammonium or oxygen in the water.

4.6 Implementation of Phytoplankton in the Hydrodynamic Model

4.6.1 Carbon In Phytoplankton Growth Rate

The equation shows the implementation of the growth for carbon as phytoplankton in the hydrodynamic model.

```
dim   CarbonInPhytoplanktonGrowthRate = (d=depth, x=xdir)

aux   CarbonInPhytoplanktonGrowthRate(d=depth, x=xdir) =
      MassPerComponent(CarbonInPhytoplankton,d,x)*PhytoplanktonGrowthRate(d,x)*
      EffectOfCarbonInPhytoplanktonAndWater(d,x)

unit  CarbonInPhytoplanktonGrowthRate = kg/day
```

Equation 4.6-1

The dimensions d (depth) and x (xdir) refers to the actual cell in the hydrodynamic model. This gives an individual value for each cell in the system, and the value is based on other specific parameters for the actual cell. All factors in the equation are exclusively calculated for each cell based on the dimensions. The unit for the rate is kg per day.

4.6.2 Phytoplankton Growth Rate

The equation is implemented in the same way as CarbonInPhytoplanktonGrowtRate with dimensions d and x to identify each cell.

```
dim   PhytoplanktonGrowthRate = (d=depth, x=xdir)

aux   PhytoplanktonGrowthRate =
      TemperatureAndNutrientLimitedGrowthRate(d,x)*LightLimitation(d,x)
```

```
unit    PhytoplanktonGrowthRate = 1/day
```

Equation 4.6-2

The unit for phytoplankton growth rate is 1/day

4.6.3 Temperature And Light Limitation

Both the temperature limitation factor and the light limitation factor have the same dimensions, and are calculated individually for each cell. Light can be estimated for the different layers and give a unique limitation depending on the layer. The constants are equal independent of the dimensions. The unit for light limitation is dimensionless

```
dim     LightLimitation = (d=depth, x=xdir)

aux     LightLimitation = Light(d,x)/
        ((CriticalLightIntensity(d,x)^CoefficientN)+Light(d,x)^CoefficientN)^(1/CoefficientN)

unit    LightLimitation = dimensionless
```

Equation 4.6-3

As for the light limitation the temperature and nutrient limitation have the two dimensions d and x. The nutrient limitation depends on the composition for the phytoplankton in each cell, and for temperature it can be measured individually for each cell. The unit for this factor is 1/day.

```
dim     TemperatureAndNutrientLimitedGrowthRate = (d=depth, x=xdir)

aux     TemperatureAndNutrientLimitedGrowthRate =
        NutrientLimitation(d,x)*TemperatureLimitedGrowthRate(d,x)

unit    TemperatureAndNutrientLimitedGrowthRate = 1/day
```

Equation 4.6-4

4.6.4 Carbon in Phytoplankton Reduction

As the growth rate, the reduction rate has the dimensions d and x. That is the only addition from the biological model, and adjusts the rate to the actual cell. The unit for the reduction rate is kilo per day.

```
dim   CarbonInPhytoplanktonReductionRate = (d=depth, x=xdir)

aux   CarbonInPhytoplanktonReductionRate = PredationRate(d,x)+(NaturalReductionRate(d,x)*
      MassPerComponent(CarbonInPhytoplankton,d,x))

unit  CarbonInPhytoplanktonReductionRate = kg/day
```

Equation 4.6-5

4.6.5 Natural Reduction Rate

The natural reduction rate is implemented in a similar way in hydrodynamic model as in the biological model. The only extension is the dimensions. The unit is also here 1/day.

```
dim   NaturalReductionRate = (d=depth, x=xdir)

aux   NaturalReductionRate = PhytoplanktonDeathRate(d,x)+Respiration(d,x)

unit  NaturalReductionRate = 1/day
```

Equation 4.6-6

4.6.6 Respiration

The respiration is a product of the temperature in the water and depends on the actual temperature in each cell. When the dimensions d and x are included, the respiration can be calculated individually for each cell. The unit is 1/day.

dim Respiration = (d=depth, x=xdir)

aux Respiration = RespirationAt20C*EXP(TemperatureCoefficient*(Temperature(d,x)-MAXTemperature))

unit Respiration = 1/day

Equation 4.6-7

4.6.7 Death Rate

The phytoplankton death rate implemented the same way as in the biological model, but it also includes the dimensions d and x. Like the other rates this is calculated for each cell, and influence the phytoplankton stock in the cell it belongs. All factors in the equation are expanded with the dimensions d and x, and the unit is 1/day.

dim PhytoplanktonDeathRate = (d=depth, x=xdir)

aux PhytoplanktonDeathRate = DeathAsAFunctionOfTemperature(d,x)*CarbonInPhytoplanktonHalfSaturationFunction(d,x)*NutrientEffectOnDeathRate(d,x)

unit PhytoplanktonDeathRate = 1/day

Equation 4.6-8

4.6.8 Ammonium consumption

All factors in the 'ammonium consumption rate are calculated individually for each cell in the fjord. This done by adding the dimensions d and x. Except from these changes, the rate is implemented as in the biological model. The unit is kg per day.

```
dim   AmmoniumConsumptionRate = (d=depth, x=xdir)

aux   AmmoniumConsumptionRate =
      InitiatedAmmoniumConsumption(d,x)*CarbonInPhytoplanktonGrowthRate(d,x)*
      EffectOfPhytoplanktonAndAmmoniumInWater(d,x)*
      EffectOfRelativeAmmoniumOnConsumptionRate(d,x)*RedfieldEffectOnAmmoniumGrowthRate(d,x)

unit  AmmoniumConsumptionRate = kg/day
```

Equation 4.6-9

Both 'initiated ammonium consumption', 'effect of phytoplankton and ammonium in water' and 'effect of relative ammonium on consumption rate' are implemented in the same way as in the biological model. The only difference is the inclusion of the dimensions d and x, which allow the calculation of all factors individually for each cell. The unit is per day for the 'initiated ammonium consumption' and dimensionless for the two others.

```
dim   InitiatedAmmoniumConsumption = (d=depth, x=xdir)

aux   InitiatedAmmoniumConsumption =
      MAX(0,MIN(ConsumptionCapacityLimitedByLightAndAmmoniumInWater(d,x),
      OptimalAmmoniumConsumption(d,x)))

unit  InitiatedAmmoniumConsumption = 1/day
```

Equation 4.6-10

```
dim     EffectOfPhytoplanktonAndAmmoniumInWater = (d=depth, x=xdir)

aux     EffectOfPhytoplanktonAndAmmoniumInWater =
        IF(EffectOfRelativeAmmoniumInWater(d,x)=1,1,EffectOfRelativeCarbonAsPhytoplankton(d,x)*
        EffectOfRelativeAmmoniumInWater(d,x))

unit    EffectOfPhytoplanktonAndAmmoniumInWater = dimensionless
```

Equation 4.6-11

```
dim     EffectOfRelativeAmmoniumOnConsumptionRate = (d=depth, x=xdir)

aux     EffectOfRelativeAmmoniumOnConsumptionRate =
        GRAPH(RelativeAmmoniumStock(d,x),0.1,0.01,[1,0.98,0.89,0.79,0.66,0.54,0.36,0.17,0.07,0.03,0
        "Min:0;Max:1;Zoom"])

unit    EffectOfRelativeAmmoniumOnConsumptionRate = dimensionless
```

Equation 4.6-12

4.6.9 Ammonium in Phytoplankton Growth Rate

The ‘ammonium in phytoplankton growth rate’ is also implemented in the with the d and x dimensions to separate the growth rate for each cell. All the influencing factors have the same dimensions as the growth rate and are calculated individually for each cell. All other implementation is done as in the biological model. The unit is kg per day for the ‘ammonium in phytoplankton growth rate’ and the ‘effect of relative ammonium on growth rate’ is dimensionless.

```
dim     AmmoniumInPhytoplanktonGrowthRate = (d=depth, x=xdir)

aux     AmmoniumInPhytoplanktonGrowthRate =
        MassPerComponent(NH4inPhytoplankton,d,x)*PhytoplanktonGrowthRate(d,x)*
        RedfieldEffectOnAmmoniumGrowthRate(d,x)*EffectOfRelativeAmmoniumOnGrowthRate(d,x)

unit    AmmoniumInPhytoplanktonGrowthRate = kg/day
```

Equation 4.6-13

```
dim     EffectOfRelativeAmmoniumOnGrowthRate = (d=depth, x=xdir)

aux     EffectOfRelativeAmmoniumOnGrowthRate =
        GRAPH(RelativeAmmoniumStock(d,x),0,0.01,[0,0.02,0.04,0.08,0.34,0.69,0.86,0.97,0.99,1,1
        "Min:0;Max:1;Zoom"])

unit    EffectOfRelativeAmmoniumOnGrowthRate = dimensionless
```

Equation 4.6-14

4.6.10 Ammonium in Phytoplankton Reduction Rate

The implementation of 'ammonium in phytoplankton reduction rate' in the hydrodynamic model is based on the implementation in the biological model. The dimensions d and x are added to differentiate the individual rates for each cell. The rate is based on the amount of ammonium in phytoplankton in the actual cell, and the other factors are calculated individually for each cell.

```
dim     AmmoniumInPhytoplanktonReductionRate = (d=depth, x=xdir)

aux     AmmoniumInPhytoplanktonReductionRate =
        (MassPerComponent(NH4inPhytoplankton,d,x)*NaturalReductionRate(d,x))+
        AmmoniumPredation(d,x)

unit    AmmoniumInPhytoplanktonReductionRate = kg/day
```

Equation 4.6-15

```
dim     AmmoniumPredation = (d=depth, x=xdir)

aux     AmmoniumPredation = PredationRate(d,x)*FractionAmmoniumInPhytoplankton(d,x)

unit    AmmoniumPredation = kg/day
```

Equation 4.6-16

4.6.11 Phosphorus in phytoplankton

The implementation of phosphorus in phytoplankton is almost identical to the implementation of ammonium in phytoplankton. Only some constants that are individual for each nutrient are different.

4.7 Implementation of Predator-Prey in the Hydrodynamic Model

The predator-prey equations are implemented in a similar way in the hydrodynamic model as in the biology model. The basic difference is that dimensions for depth and horizontal position are added to the equations.

4.7.1 Zooplankton Growth

Equation 4.7-1 represents the growth rate for zooplankton.

```
dim     ZooplanktonGrowthRate = (d = depth, x = xdir)

aux     ZooplanktonGrowthRate = MassPerComponent (Zooplankton,d,x) *
        MassPerComponent (CarbonInPhytoplankton,d,x) * PredatorCoefficient

unit    ZooplanktonGrowthRate = cells / day
```

Equation 4.7-1

The number of zooplankton cells in each fjord cell is multiplied by the number of kilos of carbon in phytoplankton, and the predator coefficient, in order to solve for the zooplankton growth rate. The predator coefficient states the effect that the amount of zooplankton and the phytoplankton have on the zooplankton growth rate. The unit used is cells per day.

4.7.2 Zooplankton Death Rate

The zooplankton death rate, equation 4.7-2 is basically identical when implemented in the hydrodynamic model.

```
dim     ZooplanktonDeathRate = (d = depth, x = xdir)

aux     ZooplanktonDeathRate = MassPerComponent (Zooplankton, d, x) /
        ZooplanktonLifespan

unit    ZooplanktonDeathRate = cells / day
```

Equation 4.7-2

The unit of measure is converted from cells per liter per day into cells per fjord cell per day, but this does not have an effect on the equation itself since the zooplankton life span is measured in days.

4.7.3 Predation Rate

Equation 4.7-3 represents the predation rate, the rate at which phytoplankton are consumed by zooplankton.

```
dim     PredationRate = (d = depth, x = xdir)

aux     PredationRate = MassPerComponent (CarbonInPhytoplankton,d,x) *
        MassPerComponent (Zooplankton,d,x) * PredationCoefficient

unit    PredationRate = kg / day
```

Equation 4.7-3

The predation rate is calculated using basically the same simple Lotka-Volterra equations as in the biology model. The amount of phytoplankton in the various fjord cells is multiplied by the number of zooplankton and a predation coefficient. The predation coefficient states the effect that the amount of phytoplankton and zooplankton has on the phytoplankton death rate.

The predation rate is drawn directly from the amount of carbon in phytoplankton. However, phytoplankton also exist as phosphorus and ammonium, so there must be an equation that calculates how much ammonium and phosphorus in phytoplankton are predated per time unit. Equation 4.7-4 represents the amount of ammonium in phytoplankton relative to the amount of carbon. There is a similar equation for phosphorus that is not documented here (see equation HA53, Appendix A).

```
dim   FractionAmmoniumInPhytoplankton = (d = depth, x = xdir)

aux   FractionAmmoniumInPhytoplankton = MassPerComponent
      (NH4inPhytoplankton, d, x) DIVZ0 MassPerComponent
      (CarbonInPhytoplankton, d, x)

unit  FractionAmmoniumInPhytoplankton = dimensionless
```

Equation 4.7-4

The DIVZ0 function is utilized in order to control that the result of the equation is 0 when there are no phytoplankton in the water. The ammonium predation is found by multiplying the predation rate with the fraction of ammonium in phytoplankton (Equation 4.7-5).

```
dim   AmmoniumPredation = (d=depth, x=xdir)

aux   AmmoniumPredation =
      PredationRate(d,x)*FractionAmmoniumInPhytoplankton(d,x)

unit  AmmoniumPredation = kg/day
```

Equation 4.7-5

This represents the rate at which ammonium is consumed by phytoplankton. The only differences from the biology model are the ranges identifying the fjord cells and the change of unit from microgram per day, to kilos per day.

Phosphorus predation is implemented in a similar manner, presented in equation 4.7-6.

```
dim     PhosphorusPredation = (d = depth, x = xdir)

aux     PhosphorusPredation = PredationRate (d,x) *
        FractionPhosphorusInPhytoplankton (d,x)

unit    PhosphorusPredation = kg / day
```

Equation 4.7-6

5. Validation

5.1 Open Systems, Subjectivity and Model Boundaries

In Merriam Webster's Collegiate Dictionary (1996) one of several definitions of the term 'valid' states that it is something that is well grounded and supported by objective truth. Further, 'objective' is defined as a 'phenomenon, or condition in the realm of sensible experience independent of individual thought and perceptible by all observers', and 'having reality independent of the mind' (Merriam Webster's Collegiate Dictionary, 1996, p. 801). This suggests the existence of an impersonal, unbiased reality independent of individual thought, and that the aim of validation is to establish the properties of this reality. The view corresponds to a logical positivistic philosophy of science. According to this tradition there is an independent objective truth, and something is valid only if it can be verified or its accuracy or correctness can be established (Gilje & Grimen, 1993). A hypothesis is meaningless if it is not possible to prove whether it is in conformance with the objective truth. However, this only means that it must be testable in principle. It must be conceivable that the hypothesis can be tested, and that it can be verified or falsified in theory.

According to this view a system dynamic model is a theory of the variables and structure that are part of a real system. The model must be tested on how well it imitates the variables and structure of a particular segment of reality. In order to do this the system that is to be modeled must be studied to find its exact structure and the behavior caused by the structure. A model is valid if it reflects the actual conditions of the reality. From this perspective, if the output of a model does not match that of the data collected from the system, given the objectivity of the data, the model must be declared invalid. The validity of the system dynamic model can be declared if it is verified, meaning if its fit to reality is established.

The problem is that a model is only a picture of reality, not reality itself. It is intended to be an abstraction or simplified version of reality. The system that is modeled is in itself an open system with no set boundary; it is a segment of a vast reality of interconnected variables. Establishing the truth of an open system is infeasible (Oreskes, Schrader-Frechette & Belitz, 1994). Oreskes illustrates this by an example where she states that she is going to be working at home the next day. Someone checks if she actually does work at home, and find that she is

not. The hypothesis of her working at home has failed. The reason she decided not to work at home is that her mother is in the hospital. The illness of her mother is an unexpected input variable to the system, and makes her behave in an unexpected way; she is not working at home even though she intended to. It is possible to think of other unexpected input variables that may lead her to leave the house that day. She might win a vacation trip, an important meeting may come up at work, or her house may even burn down. Similar unexpected input variables may also effect systems that are illustrated by system dynamic models. Ruud (2000) developed a model of a hydroelectric power plant in Colombia. One of the power stations was removed from the model after the real power station was blown up by the guerilla. If the model previously had been used to predict the future behavior of the power plant, the behavior of the real system would turn out quite different than expected because part of the power plant was destroyed. The behavior of the system would have been different from that of the model because unexpected input variables caused by the political or criminal situation in the country influenced the real system, while these variables were not included in the model. The Sørfjord also contains unexpected input variables that may influence the behavior of the fjord. An example may be accidental spills, not just of DCD, but of other contaminating substances that may have an influence on the aquatic life, and thereby effect the oxygen levels in the fjord.

There will always be external variables that affect the behavior of the real system, but are not considered in the model. Making decisions about a real system based on a model may give a different result than expected because there are variables in the open system that are not included in the model. The setting of the model boundary is therefore a crucial but difficult task when constructing a system dynamic model (Sterman, 2000).

Since the system is open, and no boundaries exist in reality, the modeler must decide where to set the model boundaries, what to include and what to omit. In determining the model boundary and selecting which variables to include, the model is applied the subjectivity of the modeler. The chance of two different modelers or modeling teams choosing the same boundary is unlikely. The model boundary is influenced by the domain knowledge and preconceptions of the modeler, the incentives for building the model, and the people and actions surrounding the modeler. It is impossible to declare complete validity of a system

dynamic model, since the model is a relatively subjective abstraction of a segment of a real system.

The choice of model boundaries, with respect to the Sørfjord model, is presumably affected by the developers' limited previous knowledge of microbiology and hydrodynamics. It may have been easier to accept the most common theory of the origin of the problems, i.e. the pollution from Odda Smelteverk, based on the opinion of most experts and research reports. With more previous knowledge we may have been encouraged to explore other theories, and implement additional oxygen consuming variables in the model.

5.1.1 Validation with Respect to Purpose

Because the system in itself is open, and the choice of the model boundaries is subjective, declaring complete validity of a system dynamic model is impossible. The validation process is also relatively subjective. Yet, guidelines of how to assess the validity of a model may be advantageous. This can be approached by looking at the usefulness of the model (Barlas, 1996). However, what makes a model useful? The purpose of building a model is to solve some kind of problem. Thus, according to Sterman (2000), the problem defines the purpose of the model. The usefulness of the model can be assessed by an evaluation of how well the model solves the problem. During a validation process it is therefore important to keep in mind the purpose of the model, that is, the problem it is meant to solve (Forrester, 1961). This brings us back to the definition of validity at the beginning of the chapter. Another definition in Webster's Dictionary (1996) states that 'valid' is something that is 'appropriate to the end in view'. This definition corresponds to the system dynamic definition of the term valid, where the model's capability of solving the problem determines its validity. The assessment of validity of the model should be based on how well the model fulfills its purpose. The purpose therefore has to be stated clearly before assessing the usefulness of the model (Forrester, 1961).

The purpose of the Sørfjord model is to develop a relatively simple system dynamic model that identifies the main variables and structures that influence the interaction between the oxygen and nitrogen levels in a fjord, and to generate the general behavior of the oxygen levels over time. This indicates that it for example is not a decision support system, where

point prediction of the resulting oxygen levels caused by a certain amount of nitrogen pollution may be important. Based on the purpose of the model, the goal of the validation of the model should be to identify weaknesses, so that they can be corrected immediately or in a later version of the model. It is also important to establish some confidence in the model, in order to see which parts should be included and elaborated further in a later version of the model. Focus is held on the main behavior of the model, for example that the scale of the levels used in the model, cohere with data collected from the Sørfjord.

5.1.2 Validation Approaches in System Dynamics

Tests applied to models and the interpretation of the test results must be appropriate to the purpose of the model. Different modeling approaches may have different purposes embedded in them. According to Oreskes (1994) the primary value of models is heuristic. However, how it is intended to aid learning may depend on the modeling approach.

Modeling approaches can be classified into two different types based on whether they explicitly reveal the underlying structure that cause the behavior of the system, or just the behavior (Barlas, 1996). There are different validation criteria for the two kinds of modeling techniques. Purely correlational models are often called 'black box models', because they only produce input and output, and do not describe explicitly the causal structure between the variables in the system. These models are purely data driven, and validation in this respect only concerns matching the input and output of the model with that of data from the real system. Revealing the structure is not part of the purpose for this type of models.

Contrary, causal descriptive models show the relationships between the variables explicitly, in order to show how the interactions between the variables cause the specific behavior. These models are called 'white box' models (Barlas, 1996). Part of the purpose of this kind of models may be for the developer or user to learn more about the explicit relationships between variables in the system and the behavior that they generate. The validation of a causal descriptive model must therefore include testing of both the structure and the behavior that it generates. The goal of the model is to generate 'the right behavior for the right reason' (Barlas, 1996, p. 187). Optimally, the heuristic effect of the model should also be evaluated. This however, is not studied here.

System dynamic models reveal the underlying structure of the model, and are causal descriptive. Being causal descriptive implies that for each link between two variables, there should exist an actual causal relationship in the real system where the value of one variable influences the value of another. The Sørfjord model is causal descriptive, and is therefore subjected to several tests, assessing both structure and behavior.

5.2 Validation of the Sørfjord Model

In system dynamic modeling, much of the validation takes place during the modeling process (Richardson and Pugh, 1981; Balci, 1995). This also applies to the development of the Sørfjord model. However, it is difficult to do a thorough description of the validation performed during the development process, because most was performed informally without documentation. For example, if the model, during the modeling process exhibited behavior that was known not to correspond to data collected from the real system, variables were added, deleted, or altered based on the discovery. An example of validation that was performed during the modeling process is the use of the variable 'RedfieldCheck'. This made it possible to constantly surveil the phytoplankton's fit to the Redfield ratio, and alter the structure of the model when the nutrient consumption by phytoplankton did not show a reasonable behavior.

However, several formal tests may be performed during the modeling process and after the model is considered finished, in order to alter the model based on the findings. These formal tests are important as more and more confidence in the model may be accumulated as the tests are passed (Forrester and Senge, 1980). Barlas (1996) presents an overview of various validation tests for system dynamic models, and distinguishes between three major categories of tests:

- Direct structure tests
- Structure-oriented behavior tests
- Behavior pattern tests

Direct structure tests compare the structure of the model to that of the real system or descriptions of the real system in the literature. There are several ways of assessing this.

Structure-oriented behavior tests assess the validity of the model structure by looking at its behavior. Behavior pattern tests only aim to test the validity of the behavior of the model compared to the real system. These tests are not intended to be performed in a linear order. The assessment of a system dynamic model should be an iterative process (Barlas, 1996).

5.3 Direct Structure Tests

Barlas (1996) distinguishes between two kinds of direct structure tests: empirical tests and theoretical tests. Empirical tests are performed by comparing the structure and parameters of the model directly with that of the real system. This is in itself problematic because the equations that are used in the model do not actually exist in the real system. They are merely attempts to describe the system's reaction to the presence and values of certain stock variables and other influencing factors. For the Sørfjord model it is especially difficult since experiments would have had to be performed in the fjord. However, it would be possible to check for the presence of various state variables to see whether they exist in the water, but because of time limitation and that it is not considered necessary, confidence is held in data collected by researchers and findings in the literature.

Theoretical structure tests involve testing the model structure against descriptions of the real system structure as described in the literature. This does not truly imply the real system since it is only a description of the real system. Barlas (1996) distinguishes between the following theoretical direct structure tests:

- Structure confirmation test
- Parameter confirmation test
- Direct extreme condition test
- Dimensional consistency test

Richardson & Pugh (1981) also suggest applying boundary adequacy tests. Direct structure tests are implemented through formal inspections or reviews, walkthroughs, and semantic analysis (Barlas, 1996). Conference of experts is ideal in this process, but due to time constraints experts are not involved.

5.3.1 Structure Confirmation Test

A theoretical structure confirmation test compares the relationships of the model with the relationships found in the literature (Barlas, 1996). This is a difficult test to perform, but can be implemented through formal inspections/reviews, walkthroughs, and data flow analysis (see Balci 1994). For the Sørfjord model it is performed during the modeling process where literature is searched to find out more about the system (See chapter 2 and 3).

5.3.2 Parameter Confirmation Test

A parameter confirmation test checks the existence of the constant parameters in the model, as well as their numerical values (Barlas, 1996). The actual existence of the parameters is difficult to examine, but table 5-1 lists a selection of some constants that are implemented in the Sørfjord model, and their reference in the literature. The references to the equations in Appendix A are indicated after the names of the constants.

Name of constant	**Documentation**	**Value**	**Literature reference**
Respiration at 20°C (BC4 and HC34)	The phytoplankton respiration at 20°C.	0.04 / day	Bjerkeng, 1994, p.41
Bacteria cell yield (BC7)	The number of cells that are produced per microgram nitrogen that is nitrified	71.39 cells/microgram	Kaplan, 1983, p.178
Hydrogen addition (BC13 and HC11)	The fraction that is added because of the weight of the hydrogen atoms in ammonium, in order to calculate how much ammonium is nitrified compared to nitrogen.	0.28784 dimensionless	Based on numbers in Joesten & Wood, 1996
Light coefficient (Coefficient N) (BC17 and HC3)	Light coefficient for the ability of phytoplankton to adapt to changing light intensities.	3 dimensionless	Bjerkeng, 1994, p.36

Table 5-1: A selection of constants that were referred in the literature.

The bacteria cell yield was not transformed when the biology model was merged with the hydrodynamic model. This may be considered a weakness of the model, and the transformation should be discussed in a later version of the model.

Some constants were not found in the literature, and a selection of these is listed in table 5-2. These constants may be used as a basis for possible improvements of the model, both by finding reference to their existence in the system and literature, and by determining their values.

Name of constant	**Value**
Zooplankton life span (BC45 and HC40)	8 days
Predator coefficient (BC40 and HC33)	0.000015 hydrodynamic model 0.0002 biology model (Dimensionless)
Predation coefficient (BC39 and HC32)	0.0001 hydrodynamic model 0.00009 biology model (Dimensionless)
Max bacteria life span (BC24 and HC20)	12 days (this number was suggested by researchers at NIVA, but no literature references were found).

Table 5-2: A selection of constants that were not found reference to in the literature.

5.3.3 Direct Extreme Conditions Test

In direct extreme conditions tests the variables in the model are subjected to extreme values or conditions in order to investigate whether they behave in the same way that variables of the real system are expected to behave under similar conditions (Richardson and Pugh, 1981). This however, does not involve simulation, but is a static theoretical test where the equations are tested separately, not related to the rest of the model. An example of such a test is to check that a population of 0 will have 0 growth. Testing the effect of an extremely large population may be more problematic because it may be expected to deplete all resources, which again will result in an increased death rate. This however, can mostly be seen over time through simulations or by examining a number of variables.

Following are some examples of direct extreme conditions tests with low values that the Sørfjord model is subjected to. For simplicity the equations shown here are taken from the biology model, but they are implemented without change in the hydrodynamic model.

1. **If the ammonium level in the water is 0, the ammonium consumption rate by phytoplankton will also equal 0.**

If there is no ammonium in the water, ammonium consumption from the water by phytoplankton should not be possible. Equation 5.3-1 indicates the ammonium consumption rate.

```
aux     AmmoniumConsumptionRate =     InitiatedAmmoniumConsumption *
        CarbonInPhytoplanktonGrowthRate *
        EffectOfPhytoplanktonAndAmmoniumInWater *
        EffectOfRelativeAmmoniumOnConsumptionRate *
        RedfieldEffectOnAmmoniumGrowthRate
```

Equation 5.3-1

The ammonium level in the water, which is set to 0, is not part of this equation. This implies that at least one of the other variables must be 0 in order to make the ammonium consumption rate 0. Both the 'Initiated Ammonium Consumption' and the 'Effect Of Phytoplankton And Ammonium In Water' are calculated based on the ammonium level in the water. These variables will therefore be 0 when the ammonium level is 0. 0 is multiplied into the equation, resulting in an ammonium consumption rate of 0 when the ammonium level is 0.

2. **If the amount of ammonium or oxygen in the water is 0, the nitrification rate should be 0.**

Inevitably, nitrification of ammonium can only occur when the bacteria, ammonium, and oxygen levels in the water are greater than 0. This is also withheld in the equation for the nitrification rate (see equation 5.3-2). If the ammonium level in the water is 0, 0 is divided by the ammonium half saturation, resulting in 0 being multiplied into the equation. Similarly, if

the oxygen level is 0, it will be divided by the oxygen half saturation, and the resulting 0 will be multiplied into the equation causing the nitrification rate to be 0.

```
aux     NitrificationRate = AmmoniumPerBacteria * NitrifyingBacteria *
        MaxNitrificationRate * (AmmoniumInWater / (AmmoniumInWater +
        AmmoniumHalfSaturation)) * (Oxygen / (Oxygen + OxygenHalfSaturation))
```

Equation 5.3-2

Table 5-3 lists similar tests performed on all the levels in the model. The levels are taken from the biology model, but the equations were implemented in the exact same form in the hydrodynamic model.

	If level is 0	**Rate must be 0**	**Because variable multiplies 0 into the equation**
1	Carbon In Water	Carbon In Phytoplankton Growth Rate	Effect Of Carbon In Phytoplankton And Water
2	Carbon In Phytoplankton	Carbon In Phytoplankton Growth Rate	Effect Of Carbon In Phytoplankton And Water
3	Carbon In Phytoplankton	Carbon In Phytoplankton Reduction Rate	Carbon In Phytoplankton and Predation Rate
4	Ammonium In Water	Ammonium Consumption Rate	Effect Of Phytoplankton And Ammonium In Water and Initiated Ammonium Consumption
5	Ammonium In Water	Ammonium Nitrification Rate	Nitrification Rate
6	Ammonium In Stock	Ammonium In Phytoplankton Growth Rate	Effect Of Relative Ammonium On Growth Rate
7	Ammonium In Phytoplankton	Ammonium In Phytoplankton Reduction Rate	Ammonium In Phytoplankton and Ammonium Predation
8	Phosphorus In Water	Phosphorus Consumption	Effect Of Phytoplankton And

		Rate	Phosphorus In Water and Initiated Phosphorous Consumption
9	Phosphorus In Stock	Phosphorus Growth Rate	Effect Of Relative Phosphorus On Growth Rate
10	Phosphorus In Phytoplankton	Phosphorus In Phytoplankton Reduction Rate	Phosphorus In Phytoplankton and Phosphorus Predation
11	Chlorophyll	Chlorophyll Reduction Rate	Chlorophyll (implemented by IF, THEN, ELSE statement)
12	Zooplankton	Zooplankton Death Rate	Zooplankton
13	Zooplankton	Zooplankton Growth Rate	Zooplankton
14	Oxygen	Oxygen Consumption Rate	Nitrification Rate
15	Nitrifying Bacteria	Bacteria Production Rate	Nitrification Rate
16	Nitrifying Bacteria	Bacteria Death Rate	Nitrifying Bacteria
17	Nitrifying Bacteria	Nitrification Rate	Nitrifying Bacteria

Table 5-3: Theoretical extreme conditions tests.

There is a problem related to the extreme tests of 0 ammonium and phosphorus in phytoplankton (test 7 and 10), which should result in 0 ammonium in phytoplankton reduction rate and 0 phosphorus in phytoplankton reduction rate. When the test is performed with 0 ammonium in phytoplankton, the 'Ammonium In Phytoplankton Reduction Rate' passes the test because the amount of ammonium in phytoplankton is multiplied into the equation, and the ammonium predation is 0, because of 0 ammonium in phytoplankton (equation 5.3-3). However, 0 ammonium in phytoplankton results in negative outflows for carbon and phosphorus in phytoplankton (equations 5.3-4 and 5.3-5). A negative outflow is equivalent of an inflow; it adds to the stock. The negative outflow originally stems from the relative nitrogen surplus (equation 5.3-6), where the lower limit for ammonium compared to carbon is subtracted from the relative amount of ammonium in phytoplankton. The relative amount of ammonium is 0, causing the relative surplus to be negative. This negative surplus is used to calculate the natural reduction rate, which is multiplied into the phosphorus and carbon

reduction rates. It is multiplied by the amount of carbon and phosphorus in phytoplankton, resulting in a negative outflow from phytoplankton.

aux AmmoniumInPhytoplanktonReductionRate = (AmmoniumInPhytoplankton * NaturalReductionRate) + AmmoniumPredation

Equation 5.3-3

aux CarbonInPhytoplanktonReductionRate = (CarbonInPhytoplankton * NaturalReductionRate) + PredationRate

Equation 5.3-4

aux PhophorusInPhytoplanktonReductionRate = (PhosphorusInPhytoplankton * NaturalReductionRate) + PhosphorusPredation

Equation 5.3-5

aux RelativeNitrogenSurplus = (FractionAmmoniumInPhytoplankton - LowerLimitNitrogenComparedToCarbon) / LowerLimitNitrogenComparedToCarbon

Equation 5.3-6

There is a similar problem regarding the PhosphorusInPhytoplanktonReductionRate (equation 5.3-5), however slightly different. The variable for the fraction of phosphorus in phytoplankton is implemented by MAX-function in order to prevent the growth rate from going negative (equation 5.3-7). When there is no phosphorus in phytoplankton, the result of this variable will be 0.0027, which is the lower limit for phosphorus in phytoplankton. This fraction influences the natural reduction rate (equation 5.3-8) and the phosphorus predation rate (equation 3.5-9). These variables should have been 0 with no phosphorus in phytoplankton, but are not. This will result in a negative phosphorus in phytoplankton level, because the phosphorus reduction rate will be greater than 0 even though the phosphorus in phytoplankton level is 0.

```
aux     FractionPhosphorusInPhytoplankton =
        MAX (PhosphorusInPhytoplankton / CarbonInPhytoplankton, 0.0027)
```

Equation 5.3-7

```
aux     NaturalReductionRate = Respiration + PhytoplanktonDeathRate
```

Equation 5.3-8

```
aux     PhosphorusPredation = PredationRate * FractionPhosphorusInPhytoplankton
```

Equation 5.3-9

Amendments may be suggested to improve these equations in a later version of the model, however, the initialization of the model is implemented in a manner that makes it impossible for phytoplankton to exist without one of the substances (see section 4.2). This is also a fact in the real system. No phytoplankton can exist without containing carbon, ammonium (or some other nitrogen compound), and phosphorus. If there is not enough ammonium or phosphorus in the water and phytoplankton stock to initiate phytoplankton growth, the equations are constructed also not to consume carbon. Carbon consumption is based on the relative level of ammonium and phosphorus compared to carbon that the phytoplankton contain. Later, the model will be subjected to an extreme test that involves simulation where phytoplankton is initiated by 0. This explains the problem further (see section 5.4.2 and 5.4.3).

5.3.4 Dimensional Consistency Test

A dimensional consistency test is an analysis of the dimensional correctness of the model's rate equations (Forrester & Senge, 1980). The dimensions of every variable must cohere with the computation (Richardson and Pugh, 1981). This test was performed by a walkthrough of all the variables in the model, where it was ensured that they contained the correct units. The model did not at first pass this test and the mistakes were corrected in order to obtain consistency between the units in the model.

Equation 5.3-10 shows the temperature limited growth rate as it is implemented both in the biology and the hydrodynamic model, respectively. The equation in the biology model does

however not contain dimensions. It has the unit '1/day', which indicates the growth factor that changes every day, limited by the temperature in the water.

```
dim     TemperatureLimitedGrowthRate = (d=depth, x=xdir)

aux     TemperatureLimitedGrowthRate =
        GrowthRate20C*EXP(TemperatureCoefficient*(Temperature(d,x)-
        MAXTemperature))

unit    TemperatureLimitedGrowthRate = 1/day
```

Equation 5.3-10

In equation 5.3-11 the units are inserted into the equations instead of the variables and the units that are not part of the result are crossed with a line. When one over degrees is multiplied by degrees, the result is dimensionless and the unit of the temperature limited growth rate results in '1/day'. The test was performed on the other variables in the model as well, however a description of this process is not included here.

1 / day * EXP (~~1 / deg C~~ * (~~deg C~~ – ~~deg C~~)
= 1 / day

Equation 5.3-11

5.3.5 Boundary Adequacy Test

Richardson and Pugh (1981) suggest subjecting the structure of the model to a boundary adequacy test. This is to make sure that the variables and feedback effects necessary to solve the problem are included in the structure of the model. No formal theoretical boundary adequacy test is performed on the Sørfjord model, but discussions of which variables or parts of the system to include were reviewed in collaboration with various researchers during the modeling process. It was evident that there was no clear agreement of which variables were important and not. Some suggested including phytoplankton and blue mussels, while others meant that this part of the model was superfluous. Also other chemicals were suggested to be part of the model. This is further discussed in chapter 3.1 Model Boundary.

5.4 Structure-Oriented Behavior Tests

Structure-oriented behavior tests assess the structure indirectly by applying tests that should generate a certain behavior. This type of tests involves running simulations. Barlas (1996) lists several structure oriented behavior tests, and the Sørfjord model is subjected to the following:

- Extreme condition test
- Behavior sensitivity test
- Boundary adequacy test

5.4.1 Extreme Condition Tests

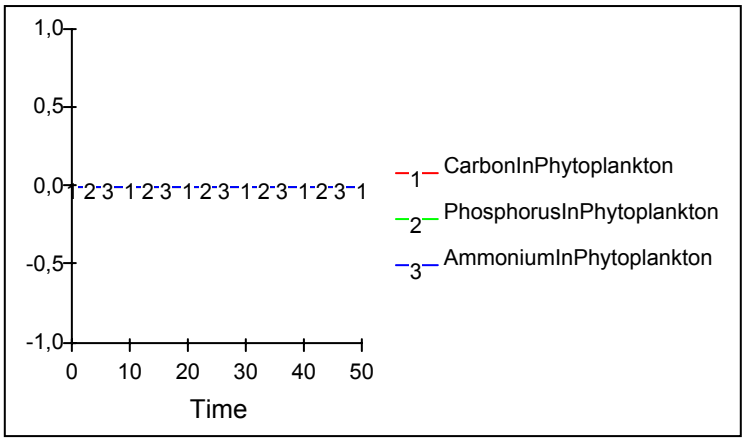
Extreme conditions testing of the structure-oriented behavior is similar to theoretical extreme conditions testing (Section 5.3.3). The model is subjected to extreme values of certain variables, in order to test whether the results are the same as expected of the real system when subjected to similar values. However, structure-oriented behavior tests involve simulations, not just testing of autonomous equations as theoretical extreme conditions tests. An example is initiating a population level with an extremely high value, or initiating it with 0. In the first example, it is expected to deplete all resources and cause an increased death rate. In the second, growth is expected to stagnate completely because the level is 0 initially, and a population cannot grow without a basis population.

In the following several extreme conditions tests are performed on the biology model, followed by similar tests of the hydrodynamic model. Subjecting both models to the same tests ensures that the transformation from the biology model to the hydrodynamic model has been successful. The extreme-conditions tests shown here are just examples of tests that the model may be subjected to. More tests are performed, however not documented here.

5.4.2 Extreme Conditions Testing of the Biology Model

1. **If there are 0 phytoplankton in the water, the phytoplankton level will continue to be 0, and the zooplankton level will eventually decline towards 0.**

A simulation is run where phytoplankton is initiated with 0 microgram carbon in phytoplankton per liter. This results in an initial value of 0 for all levels concerning phytoplankton. The rates controlling the phytoplankton levels are defined not to grow when their value is 0 (see section 4.2). Here, the problem of negative outflows, and outflows from levels with 0 microgram, mentioned in section 5.3.3 Theoretical Extreme Conditions Testing is avoided because an initialization variable (equation BC14, Appendix A) sets all variables related to phytoplankton to 0. This initialization variable controls that none of the stock variables concerning phytoplankton can be 0 initially, unless all are 0.



Graph 5-1: The behavior of phytoplankton when their initial value is 0.

Graph 5-1 shows the three substances that the phytoplankton consists of (ammonium and phosphorus in phytoplankton stock and chlorophyll are not included in the graph). These levels remain at 0 when their initial value is 0.

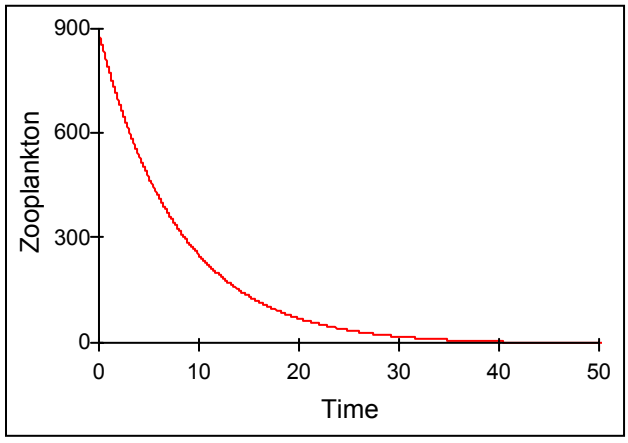
The model did not initially pass this test. This is because the growth and consumption rates were largely based on the relative amount of one substance compared to another, both in water and in phytoplankton. Although not considered 'good modeling practice' this was

solved by adding a 'DIVZ0' function that gives the variable the value of 0 whenever it is being divided by 0. This is illustrated in the equation 5.4-1

```
aux     RelativeAmmoniumStock = AmmoniumInStock DIVZ0
        AmmoniumInPhytoplankton
```

Equation 5.4-1

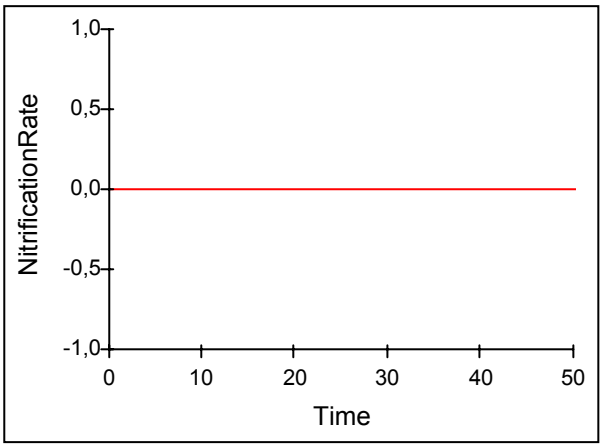
Graph 5-2 shows how the zooplankton level decreases when the phytoplankton level is 0. Zooplankton has a growth rate of 0 because there is no phytoplankton prey for them to consume. The death rate is higher in the beginning when there is a higher zooplankton level, and slows down as the zooplankton level decreases. This is because the death rate is implemented using a simple first-order death rate.



Graph 5-2: The behavior of zooplankton when the initial value of phytoplankton is 0.

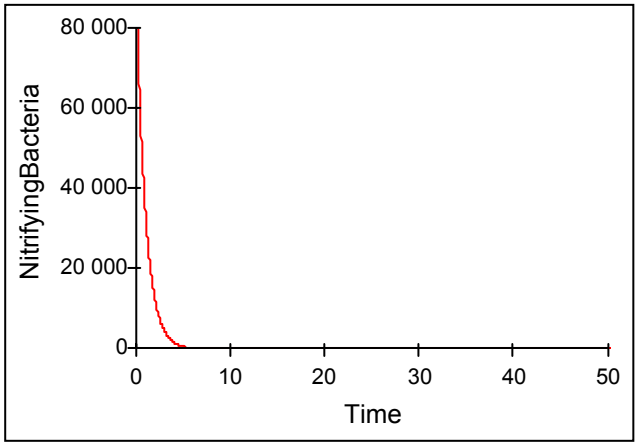
2. **If there is 0 ammonium in the water, there will be no nitrification, the nitrifying bacteria and phytoplankton will die, and gradually the zooplankton will die due to lack of phytoplankton prey.**

Graphs 5-3, 5-4, 5-5, and 5-6 show the behavior of the biology model when there is no ammonium in the water. There are three elements that are considered crucial for nitrification: nitrifying bacteria, oxygen and ammonium. When there is no ammonium in the water the nitrification process cannot take place. Therefore, the nitrification rate in graph 5-3 is 0 (equation BA17, Appendix A).



Graph 5-3: The nitrification rate when the ammonium level in the water is 0.

The nitrifying bacteria grow based on the nitrification rate. As long as the nitrification rate is 0, the bacteria growth rate is 0. Lack of ammonium in the water leads to a higher bacteria death rate than there otherwise would have been. Graph 5-4 shows that the death rate is higher in the beginning when there are more bacteria in the water, and stagnates as the bacteria level decreases.



Graph 5-4: The behavior of nitrifying bacteria when the ammonium level in the water is 0.

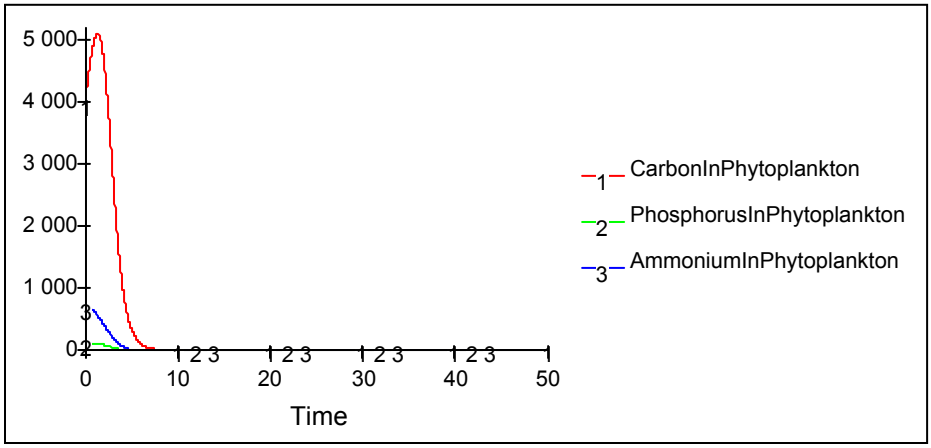
At first the model did not pass this test. The bacteria death rate is based on a simple first-order death rate in addition to a factor increasing the death rate as the amount of ammonium and oxygen is depleted from the water. When the amount of ammonium in the water is 0, the equation will be divided by 0, causing an error. This is solved by inserting a DIVZ1-function,

which divides the number of nitrifying bacteria by 1, if the result of the limitation of ammonium or oxygen is 0. Equation 5.4-2 shows the changed equation for the bacteria death rate.

```
aux     BacteriaDeathRate = NitrifyingBacteria DIVZ1 (MaxBacteriaLifeSpan *
        (AmmoniumInWater / (AmmoniumInWater + AmmoniumHalfSaturation)) *
        (Oxygen / (Oxygen + OxygenHalfSaturation)))
```

Equation 5.4-2

The result of this is that if there is no ammonium or oxygen in the water, the whole bacteria population will die in about one week. It is however unlikely that the fjord will be completely without oxygen or ammonium.

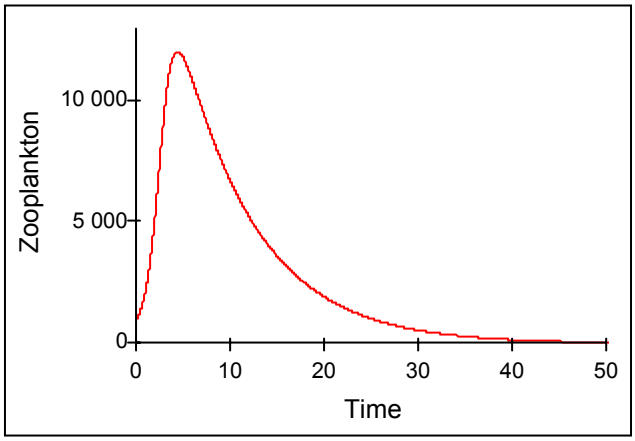


Graph 5-5: The behavior of phytoplankton when there is no ammonium in the water.

When there is no ammonium in the water there are not enough nutrients for phytoplankton to grow. Graph 5-5 shows the phytoplankton behavior when the phytoplankton level in the water is 0. In the beginning the phytoplankton continue to grow for a short time. They cannot consume ammonium from the water, but they have previously stocked up ammonium before the ammonium in the water was depleted. After a while the ammonium stocks in phytoplankton are drained, and the phytoplankton die.

Graph 5-6 shows the behavior of the zooplankton when there is 0 ammonium in the water. The zooplankton level continues to rise after the ammonium level in the water is depleted. This is because it does not react to the ammonium level, but to the phytoplankton level, which

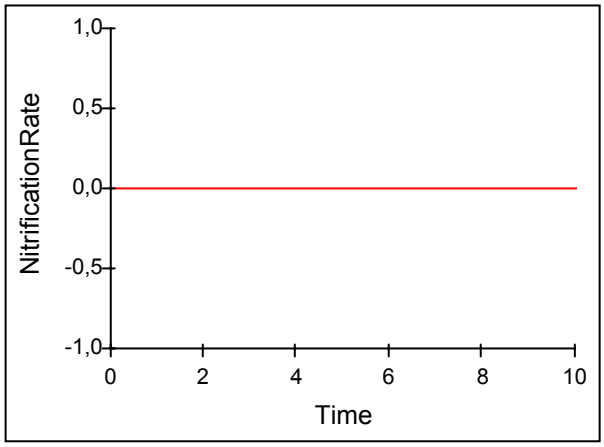
also increases initially. After a while the phytoplankton stop growing and have an increased death rate due to the lack of ammonium. When there is not enough phytoplankton for the zooplankton to consume, they gradually die out.



Graph 5-6: The behavior of zooplankton when there is no ammonium in the water.

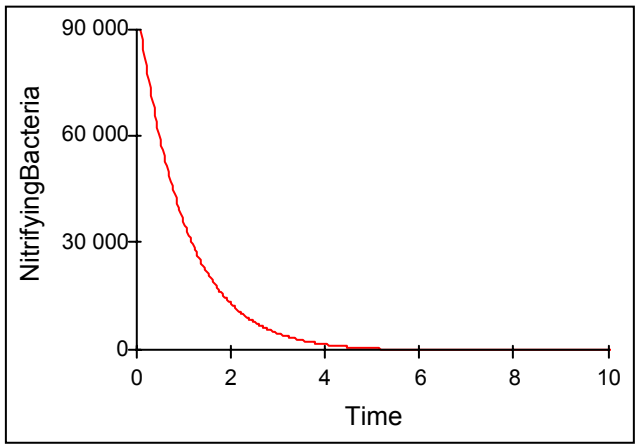
3. If there is 0 oxygen in the water, the nitrification rate will be 0, and the nitrifying bacteria will die.

As mentioned in test example 2, nitrification can only take place if there is oxygen in the water. Graph 5-7 shows that the behavior of the nitrification rate is constantly at 0 when there is no oxygen in the water.



Graph 5-7: The nitrification rate when the oxygen level in the water is 0.

It is also mentioned in test example 2 that nitrifying bacteria grow based on the nitrification rate. They will therefore die when there is no oxygen in the water. This is illustrated in graph 5-8.



Graph 5-8: The behavior of nitrifying bacteria when there is no oxygen in the water.

5.4.3 Extreme Conditions Testing of Hydrodynamic Model

The same tests that were performed on the biology model have also been performed on the hydrodynamic model to make sure that the biology model was merged with the hydrodynamic model in an adequate way. Figure 5-1 represents the fjord cells that the fjord is divided into, in order to enhance the interpretation of the simulation results.

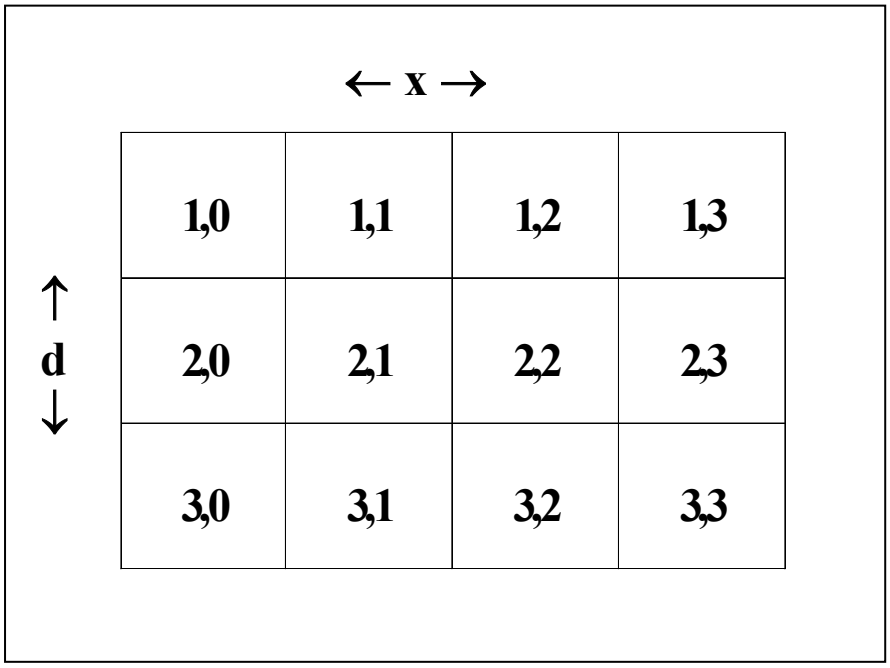
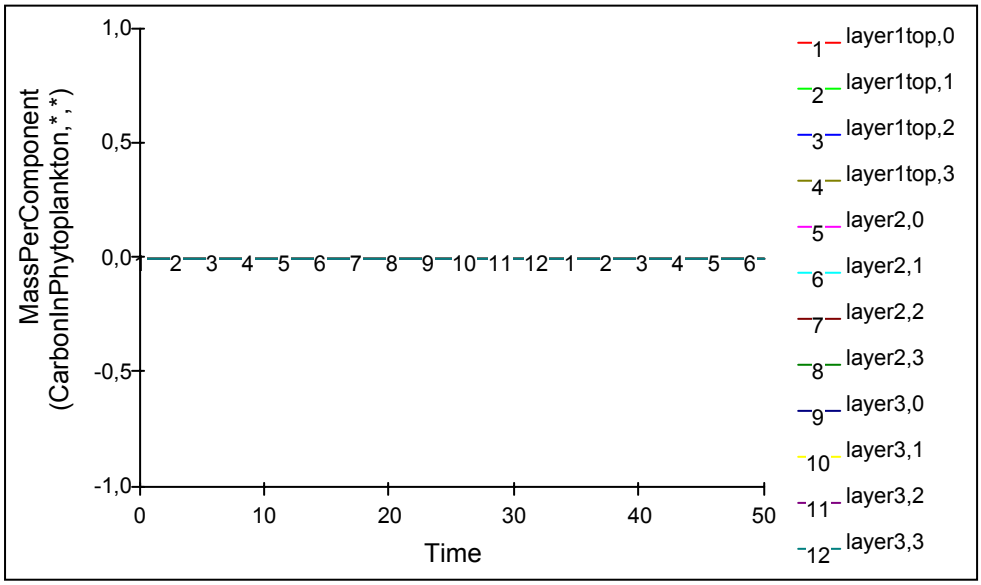


Figure 5-1: The fjord cells that the model is divided into.

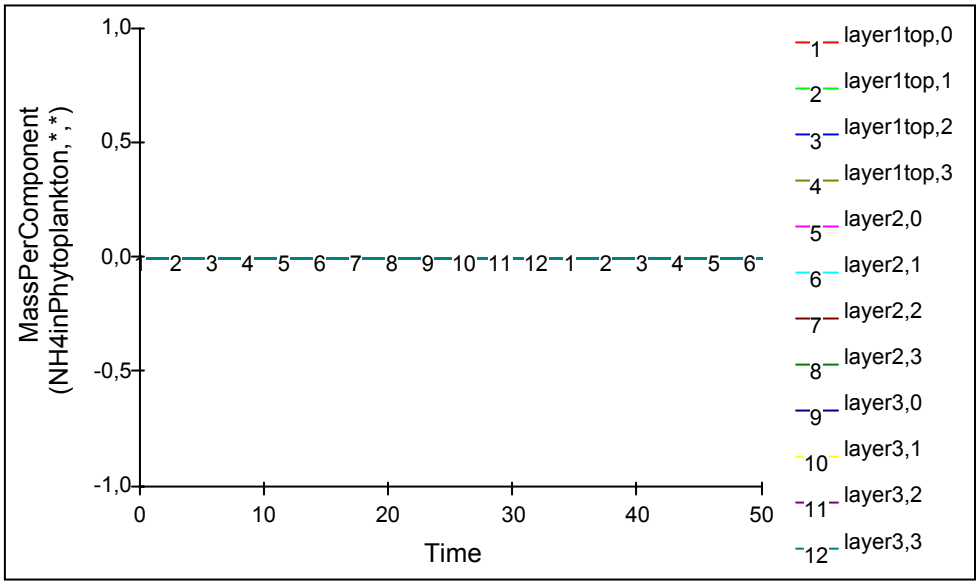
1. **If there are 0 phytoplankton in the water, the phytoplankton level will continue to be 0, and the zooplankton level will decrease because there will be no zooplankton production in the fjord, only an inflow of zooplankton through fresh seawater.**

The main aim of this test is to check that the structure ensures that phytoplankton cannot grow without an existing phytoplankton substance, that is, if the phytoplankton level is 0, it will remain 0 as long as there is no inflow of phytoplankton during the simulation. Also, the test will control that zooplankton only grow from phytoplankton predation, and that there will be no zooplankton growth in the fjord as long as there is no phytoplankton in the water. This test is performed by initializing the 'CompositionOcean' variable (equation HC5, Appendix A) by 0 for ammonium, phosphorus and carbon in phytoplankton and phytoplankton stock. This

results in no phytoplankton, either in the fjord or the water from the ocean that flows into the fjord. The simulation is run for 50 days. Graph 5-9 shows how carbon in phytoplankton remains at 0 throughout the simulation, while the behavior of ammonium in phytoplankton is presented in graph 5-10.

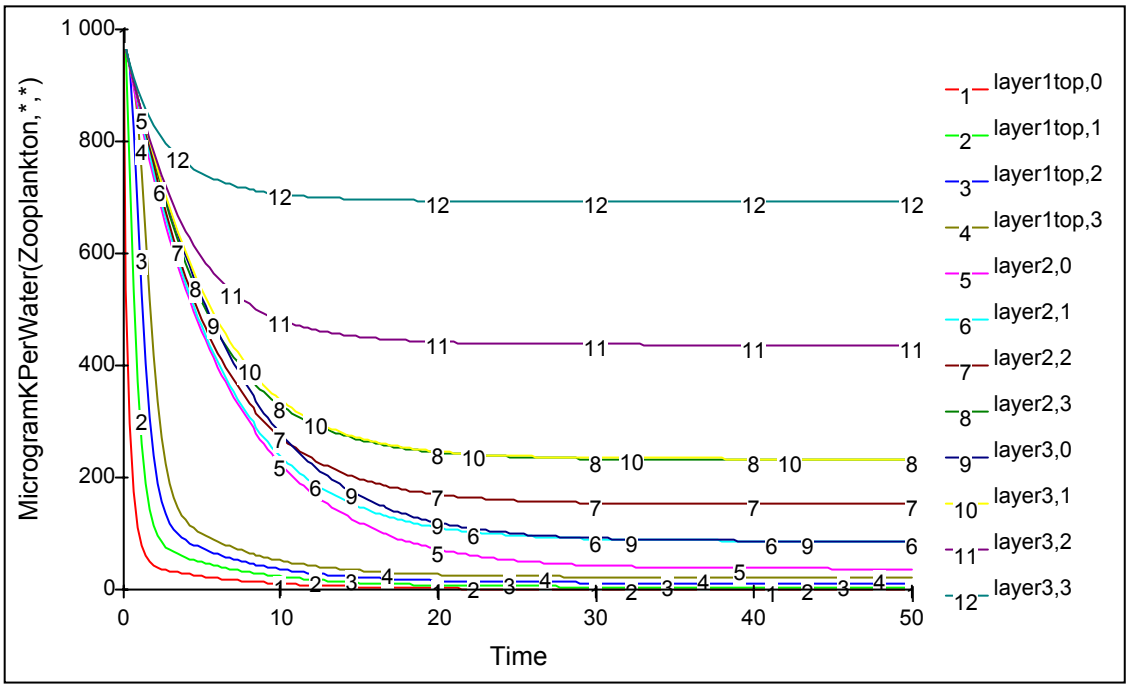


Graph 5-9: The behavior of carbon in phytoplankton when there are no phytoplankton in the water.



Graph 5-10: The behavior of ammonium in phytoplankton when there are no phytoplankton in the water.

The zooplankton levels in graph 5-11 decrease rapidly in the beginning of the simulation when there are no phytoplankton in the water. This is because the zooplankton that were in the fjord initially die based on the first order death rate, and there is no zooplankton production because there is no phytoplankton to be predated. The reason the zooplankton do not all die, but stabilize at certain levels for the various fjord cells, is that there is a constant stream of zooplankton coming into the fjord with the water exchange. The zooplankton level stabilizes at a higher level in fjord cells 3,3 and 3,2. This is where the fresh seawater, and thereby the zooplankton come in. When the zooplankton population reaches layers later in the water exchange cycle many of them have already died due to the natural death rate.

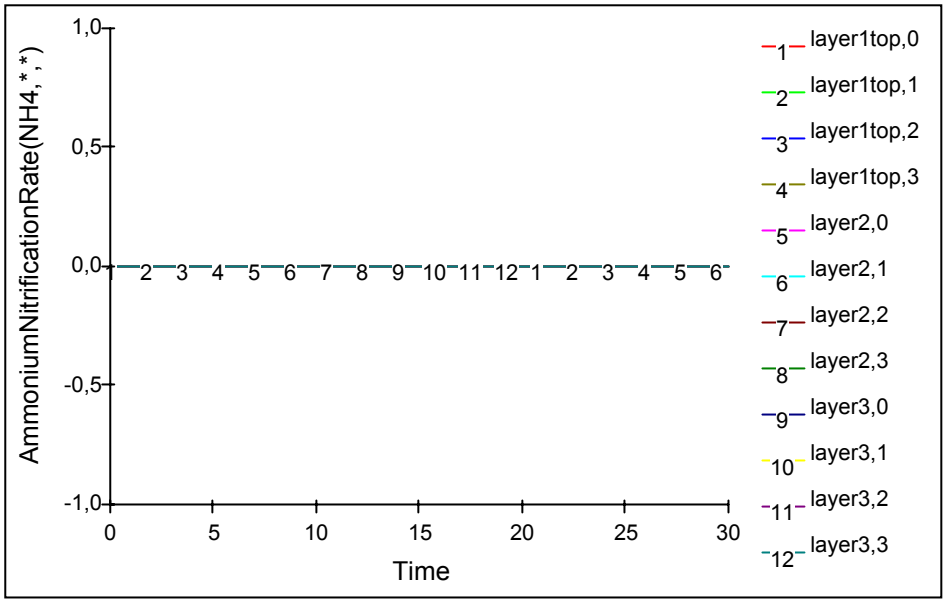


Graph 5-11: The behavior of zooplankton when there is no phytoplankton in the water.

2. **If there is 0 ammonium in the water, there will be no nitrification, the nitrifying bacteria and phytoplankton will die, and gradually the zooplankton will die due to lack of phytoplankton prey.**

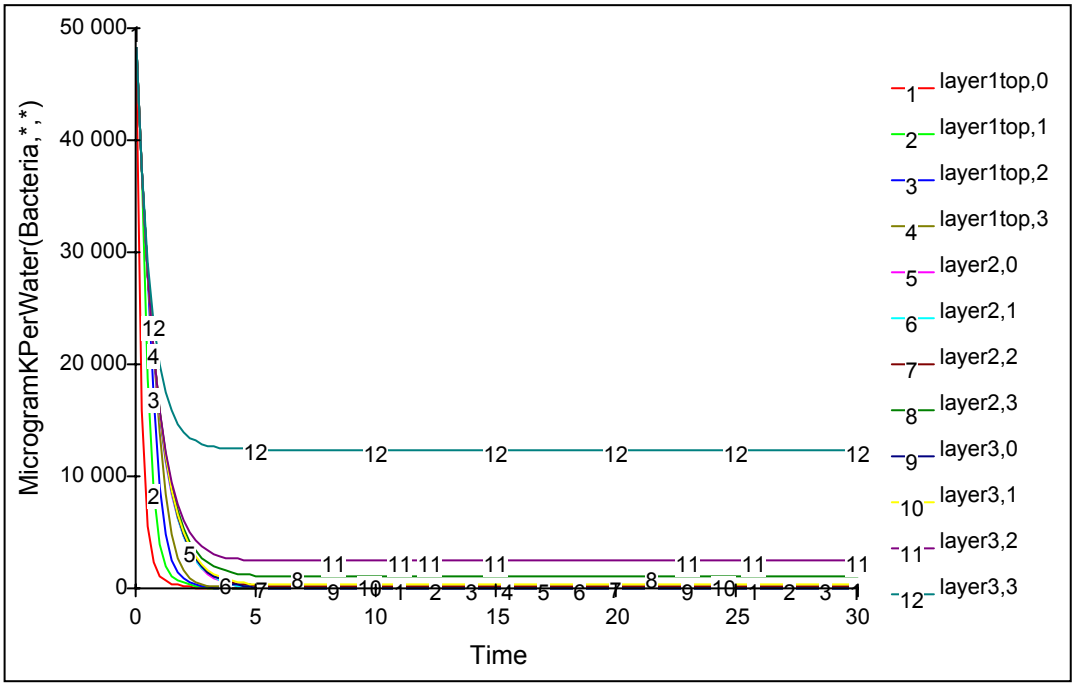
The main aim of this test is to make sure that the structure of the model does not allow nitrification when there is no ammonium in the water. The nitrifying bacteria will then die, because they only grow based on the nitrification rate. The phytoplankton will also die because they need ammonium as part of their nutrients. Eventually the zooplankton will also die due to lack of phytoplankton prey.

The test is performed by initializing the ammonium in the fjord and seawater by 0, and stopping the ammonium inflow from the industry. Graph 5-12 shows that the nitrification rate is 0 when there is no ammonium.



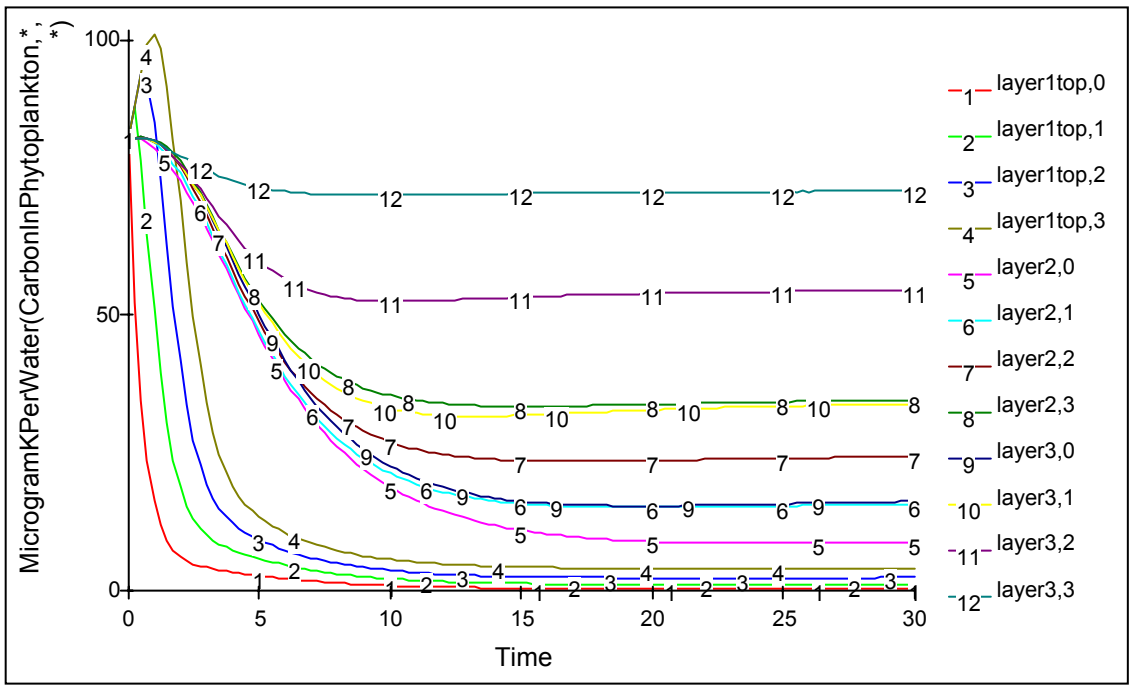
Graph 5-12: The ammonium nitrification rate when there is no ammonium in the water.

The behavior of the nitrifying bacteria when there is no ammonium in the water is shown in graph 5-13. The bacteria are initiated at a relatively high level. There is no ammonium in the water, which results in a birth rate of 0. The lack of ammonium also results in a higher death rate than there otherwise would have been. After some time the bacteria levels of the various fjord cells stabilize at a certain level, many of them close to 0. In fjord cell 3,3 the bacteria level is fairly high. This is the cell where the fresh seawater comes in, thereby causing the level to stabilize at a higher level than for the other fjord cells. By the time the bacteria flow into the next fjord cell, most of them have already died due to the lack of ammonium.



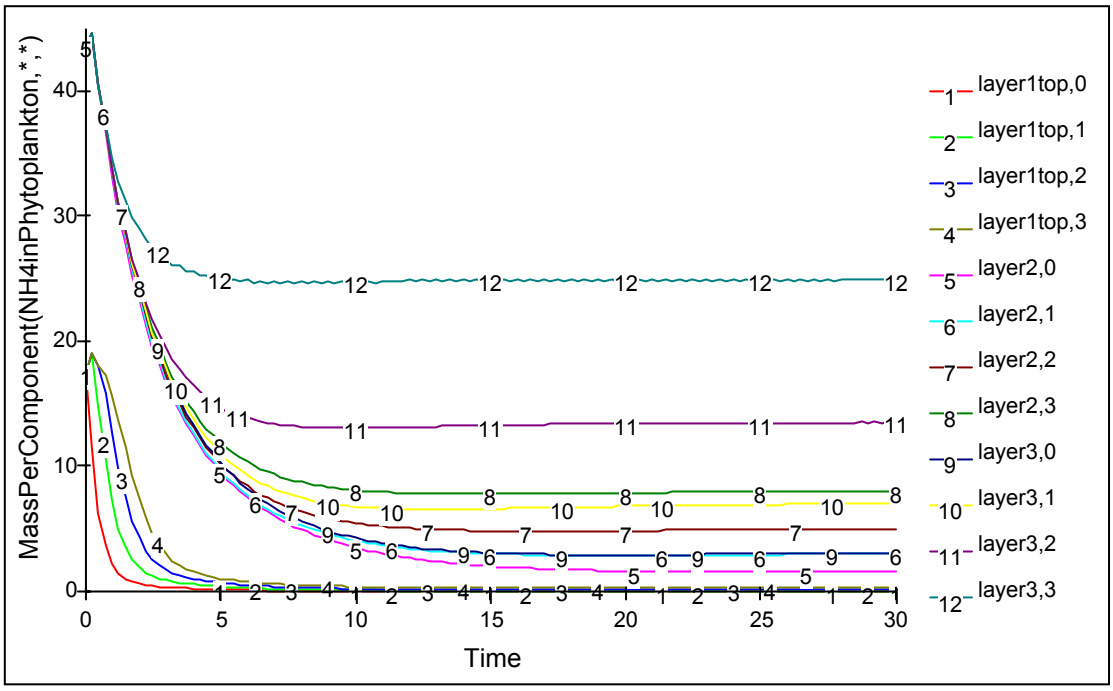
Graph 5-13: The behavior of nitrifying bacteria when there is no ammonium in the water.

Phytoplankton can continue to grow for a short period of time, even after the ammonium level in the water reaches 0. This is because they have stored some ammonium in the cells that are not yet used for growth, and because they can substitute the lack of ammonium by increasing the carbon uptake. However, after a while the phytoplankton level will start to decrease due to a lower growth rate and a higher death rate caused by a lack of nutrients. The water exchange brings in new phytoplankton, which prevents the level from reaching 0. Again, the outermost layer at the bottom (layer 3,3) contains the largest phytoplankton levels since it receives fresh seawater and thereby new phytoplankton before the outer fjord cells. This is described in graph 5-14.



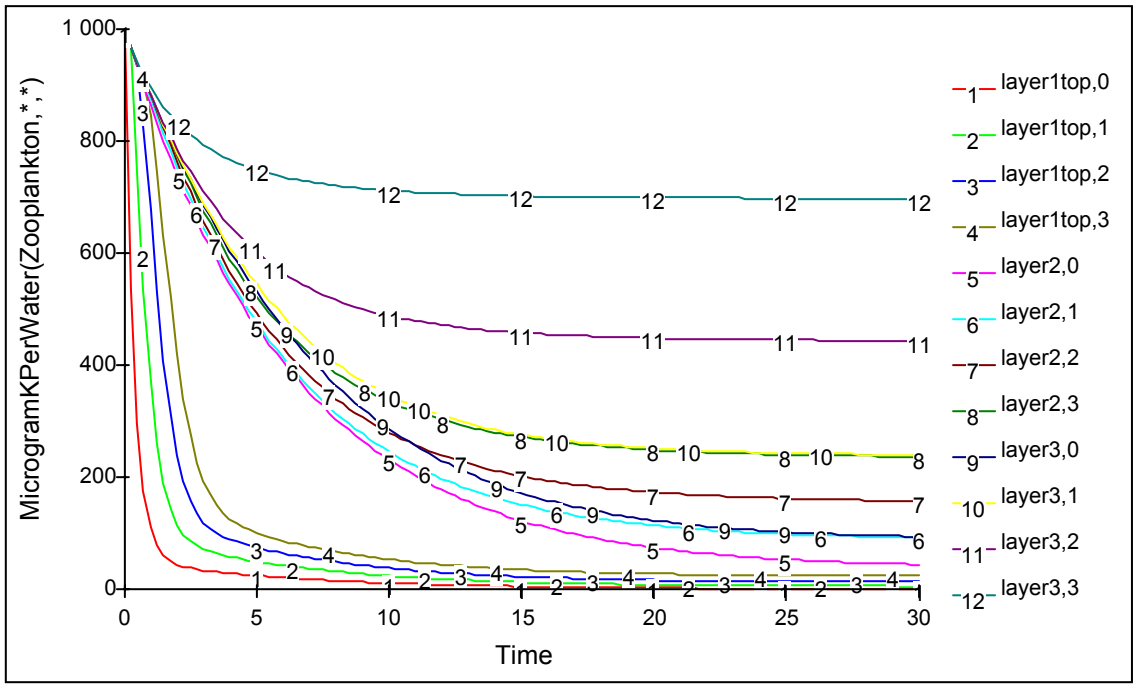
Graph 5-14: The behavior of carbon in phytoplankton when there is no ammonium in the water.

Graph 5-15 shows the behavior of ammonium in phytoplankton. Equivalent to carbon in phytoplankton, the ammonium in phytoplankton level also increases slightly in the beginning of the simulation. This however is only due to the ammonium that the phytoplankton have stocked in their cells. The ammonium in phytoplankton starts to decrease earlier than the phytoplankton because, as mentioned previously, the phytoplankton compensate for lacking ammonium by consuming more carbon. After a while, the ammonium in phytoplankton level stabilizes, and only receives input through the water exchange, which brings in new phytoplankton.



Graph 5-15: The behavior of ammonium in phytoplankton when there is no ammonium in the water.

The zooplankton in graph 5-16 exhibits a similar behavior to the phytoplankton in graph 5-14. As the phytoplankton die, there are no nutrients for the zooplankton, they die and gradually stabilize at low levels only receiving input through the water exchange.

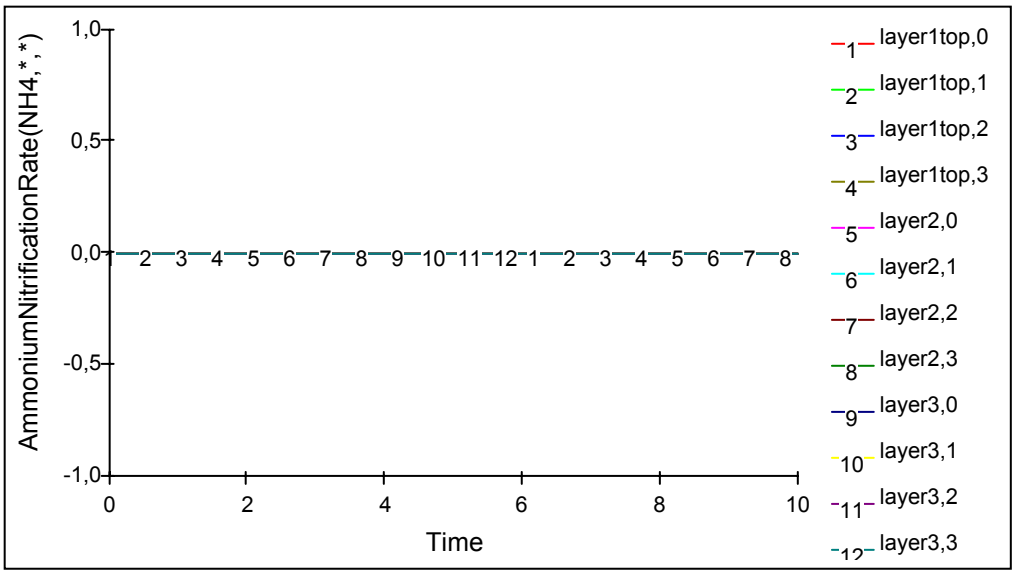


Graph 5-16: The behavior of zooplankton when there is no ammonium in the water.

3. If there is 0 oxygen in the water, the nitrification rate will be 0, and the nitrifying bacteria will die.

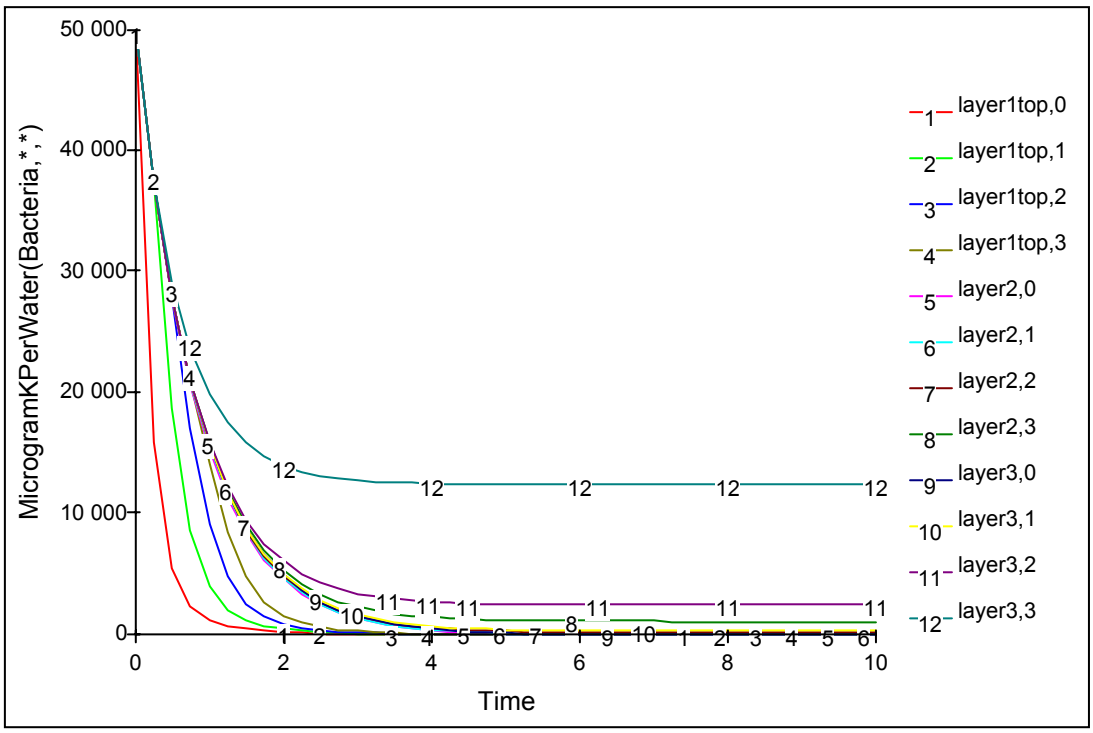
Nitrification cannot take place without the presence of oxygen. The main goal of this test is to assess whether the structure of the model generates a behavior where the nitrification rate and bacteria growth rate is 0 when the oxygen level is 0, and to make sure that the bacteria die due to lack of oxygen.

The test is performed by initiating the oxygen level by 0, letting the fresh seawater and the Opo River contain 0 oxygen, and setting the phytoplankton oxygen production to 0. Graph 5-17 shows that there is no nitrification when the oxygen level is set to 0.



Graph 5-17: The ammonium nitrification rate when there is no oxygen in the water.

Graph 5-18 displays the behavior of the nitrifying bacteria. Their growth rate is 0, and at first they have a fairly high death rate due to the lack of oxygen. After some time they stabilize at a certain level due to the inflow of bacteria from the seawater.



Graph 5-18: The behavior of the nitrifying bacteria when there is no oxygen in the water.

The extreme condition tests turned out to be necessary because some of the rate equations were not formulated adequately to pass the test. The model is now more robust, and ensures that there will be no negative levels, and no attempts at dividing variables by zero.

5.4.4 Behavior Sensitivity Test

Sensitivity analysis involves finding parameters that the model is particularly sensitive to, and evaluating whether the real system would exhibit the same sensitivity (Barlas, 1996). According to Tank-Nielsen (1980), sensitivity analysis in system dynamics is 'the study of model responses to model changes' (p.187). He states four main goals of sensitivity analysis:

- To test the effects of uncertainties in parameter values
- To gain insight about the structure and behavior of the model
- To gain insight about the structure and behavior of the real system
- To direct further work on parameter and structure

Variables of dynamic systems are often difficult to quantify. It is important to be aware of the uncertainties of values and how they affect the behavior of the model. Behavior sensitivity analysis may also help in gaining a better understanding of how the model as well as the real system works. The results of the analysis can be used to direct further work on parameter and structure. It may also establish a basis for policy suggestions.

With respect to the Sørfjord model, sensitivity analysis may be an indicator of which variables are important to do further research on, and which real system variables may be important to surveil because they have a great impact on the behavior of the fjord.

A behavior sensitivity test is performed by changing the value of a parameter, and evaluating the response in the model behavior. Here, 'parameter change' means change in variables within the system, not mere perturbations of exogenous input variables (Tank-Nielsen, 1980). An example of changes in parameter values is running the model with different initial values of a level.

Much of the sensitivity testing for the Sørfjord model has been performed without documentation during the modeling process. When a new parameter was added, its value and the effect of additions to the structure were evaluated, and it was changed to confirm with the beliefs of the structure and behavior of the real system.

The Sørfjord model is subjected to five formal sensitivity tests. In this case it is only adequate to evaluate the sensitivity of the biology model. This is because the hydrodynamic model adjusts to the constant inflow of components, and stabilize at the same level independent of the initial values of the stocks (This is also tested through simulations, however not documented here). The tests on the biology model are run by initializing specific levels by three different values for three different simulations, and comparing the results. The sensitivity of the following levels were tested with three different initial values:

1. Phytoplankton level
2. Nitrifying bacteria level
3. Zooplankton level
4. The relative amount of ammonium compared to carbon in phytoplankton

5. The relative amount of phosphorus compared to carbon in phytoplankton

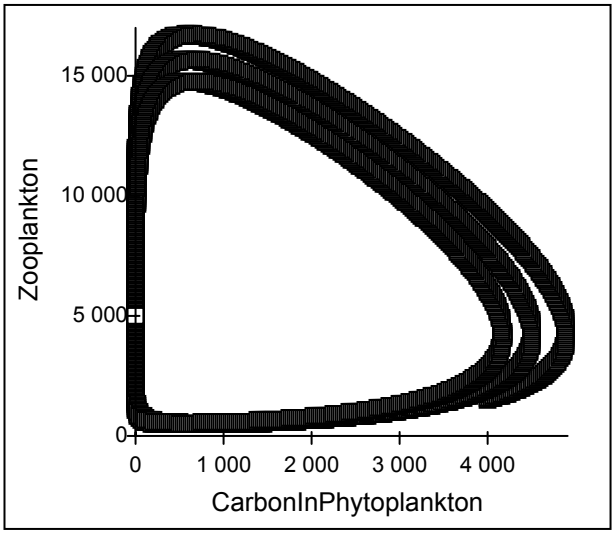
The results from the sensitivity tests that the biology model is subjected to are summarized in table 5-4. The rows of the table each represent one test. Further, the rows are divided into four columns, showing the variable or variables that the test was performed on, the initial values of the variable for each of the three simulations, comments about the results, and a list of the variables that were particularly effected. This is a relatively informal form of sensitivity testing. A more formal method may have been to calculate the results mathematically, and place the variables into predefined categories. However, these tests are only intended to direct further work on parameter values and structure, and exact measures are therefore not considered important.

Variable	**Initial values**	**Comments**	**Particular effect on variables**
1. Phytoplankton	1: 3 000 2: 4 000 3: 5 000	Very sensitive, although almost same results for simulation 2 and 3, where the phytoplankton and zooplankton died. Run 1 exhibited large oscillations, mainly caused by the predator-prey loops of phytoplankton and zooplankton. The bacteria and nitrification rate oscillated due to oscillations in the oxygen production rate by phytoplankton.	- All levels and rates
2. Nitrifying Bacteria	1: 25 000 2: 100 000 3: 175 000	Not very sensitive. Almost no effect on phytoplankton and zooplankton. Some changes in oscillations of bacteria and oxygen before stabilization.	- Oxygen - Bacteria
3. Zooplankton	1: 300 2: 900 3: 1 500	Very sensitive. Same results as test 1. Phytoplankton	- All levels and rates
4. Ammonium	1: 0.065 (just	Fairly sensitive. Simulation 3 had the	- Phytoplankton

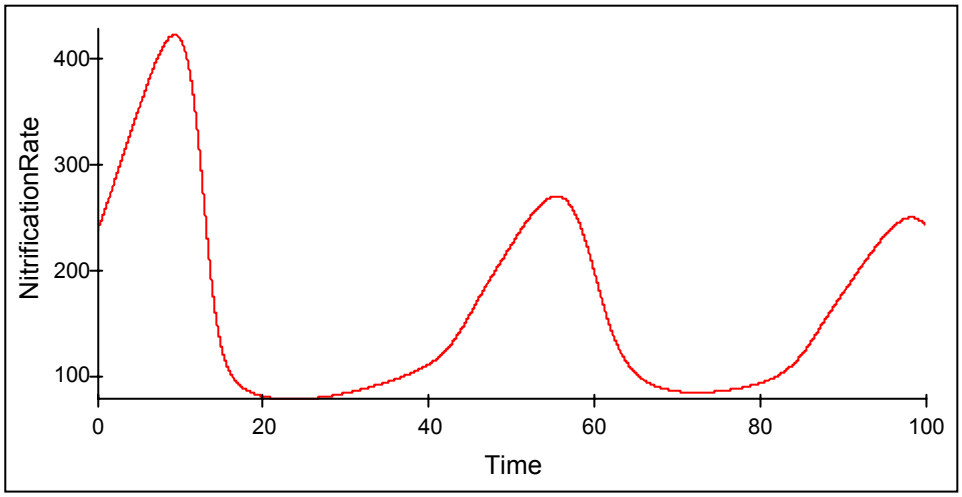
Level in Phytoplankton	above lower limit) 2: 0.1178 3: 0.17561 (Redfield)	original Redfield setting, where the phytoplankton and zooplankton died. Simulation 2 and 3 however, oscillated because the phytoplankton did not grow as fast in the beginning, causing a smaller growth rate for zooplankton, and thereby the chance for phytoplankton to start growing again before being almost completely predated by zooplankton.	- All levels and rates
5. Phosphorus Level in Phytoplankton Compared to Carbon	1: 0.0027 (lower limit) 2: 0.01085 3: 0.02439 (Redfield)	Not sensitive. Some variations in peaks before stabilization. Phytoplankton and zooplankton died out.	

Table 5-4: Sensitivity testing of the biology model.

The reason sensitivity test 2. Nitrifying Bacteria does not have a great effect on the behavior of the model, while the variables related to zooplankton and phytoplankton do, may be because the biology model is set to have the same values of variables as the hydrodynamic model. In the hydrodynamic model the inflow of new substances to the water is crucial for the sustainability of the phytoplankton and zooplankton levels. With the present initialization in the biology model, they die after some days. When the model is initialized with values that sustain both levels, the phytoplankton and zooplankton exhibit steady state oscillations, forced by the predator-prey loops. This causes a steady state oxygen production rate, which again influences the oxygen level in the water, and thereby the nitrification rate. An oscillating nitrification rate also causes the nitrifying bacteria to oscillate. Graph 5-19 shows the interaction between phytoplankton and zooplankton in a steady state oscillation, while the oscillating nitrification rate is presented in graph 5-20.



Graph 5-19: The interaction between the phytoplankton and zooplankton level in the biology model.



Graph 5-20: The nitrification rate in the biology model when the phytoplankton level oscillates.

The fact that the phytoplankton and zooplankton die makes it hard to analyze the results of the sensitivity tests, but these tests also indicate the importance of the phytoplankton and zooplankton variables in the model. It may indicate that the predator-prey loops in the real system are important, but most important of all, that in a further version of the model it is important to find out more about these loops, and the values of the variables. The behavior of the model may be changed by altering the initial values of the levels, but also the constants that influence the predation loops.

5.4.5 Boundary Adequacy Test

Boundary adequacy tests cannot only be performed theoretically, but also through simulations (Forrester and Senge, 1980). This involves removing or adding superfluous or additional structure to the model, and checking whether the behavior exhibited is equivalent to the original simulations, or whether there are great changes in behavior patterns. The Sørfjord model is subjected to several such tests. The necessity of the following structures are tested:

1. Oxygen production by phytoplankton
2. The influence of light on phytoplankton growth
3. Phytoplankton loss due to respiration
4. The influence of temperature on phytoplankton growth and death
5. The effect of zooplankton predation on phytoplankton

First, one simulation with the original structure included was run. Afterwards, the structure was removed, and the model was run again. The results from the simulations are summarized in table 5-5 and 5-6 for the biology and hydrodynamic model respectively.

Removed Structure	**Result**
1. Oxygen production (all variables concerning phytoplankton)	Almost no changes in behavior were seen. The bacteria and oxygen stabilized at the same levels.
2. Light influence on phytoplankton growth	Caused less restriction to phytoplankton growth. This lead to a higher phytoplankton growth rate in the beginning, which again lead to a higher zooplankton growth rate. The phytoplankton still died out due to the predation after about seven weeks.
3. Respiration by phytoplankton	Caused some changes in the phytoplankton for the fist ten time-steps, due to a lower phytoplankton death rate. This again caused the zooplankton to grow to a higher peak before they died because of lack of

	phytoplankton. Except for this, there were basically no changes in the behavior, and it did not affect the variables concerning nitrifying bacteria notably.
4. Temperature influence on phytoplankton growth and death	This caused a lower growth rate for phytoplankton, which again caused a lower zooplankton growth rate. The oxygen production was thereby also lower, which resulted in a lower nitrification rate. The phytoplankton died out, and the variables concerning nitrification stabilized at the same level as in the original simulation.
5. Zooplankton (phytoplankton predation)	Had a large impact on all variables in the model. Omitting zooplankton allowed for a smaller phytoplankton death rate, which again lead to higher phytoplankton level and a higher phytoplankton growth rate. This again caused a higher nitrification rate and a higher bacteria level. The system oscillated due to competition between bacteria and phytoplankton about the ammonium in the water.

Table 5-5: Boundary adequacy testing of the biology model.

The result of the boundary adequacy test for the biology model is rather difficult to interpret because when it is parameterized and initialized with the same values as in the hydrodynamic model, the phytoplankton and zooplankton die. In the hydrodynamic model the inflow of phytoplankton from the ocean helps maintain the phytoplankton and zooplankton levels throughout the simulation. The main result from this test may therefor be that in a later version of the model it may be important to establish the interaction between phytoplankton and zooplankton more accurately, identify the related variables, and try to estimate the values of the variables further.

Removed Structure	Result
1. Oxygen production (all variables concerning phytoplankton)	The oxygen level stabilized at a lower level, there were more bacteria, and much higher ammonium levels in the water.
2. Light influence on phytoplankton growth	The largest changes were observed in the phytoplankton and ammonium levels in the water. These stabilized both higher and lower for different fjord cells however, the differences were not great. There were almost no effects on oxygen in the water, zooplankton, and bacteria. Some changes were observed in the nitrification rate.
3. Respiration by phytoplankton	Largest change in zooplankton and phytoplankton levels. All levels were mainly the same.
4. Temperature influence on phytoplankton growth and death	The phytoplankton died out, which again had a large impact on all other variables.
5. Zooplankton (phytoplankton predation)	Resulted in changes in less phytoplankton death, and all levels concerning phytoplankton stabilized at higher levels. This resulted in less phosphorus and ammonium in the water, and more oxygen due to the increased production by the high phytoplankton level.

Table 5-6: Boundary adequacy testing of the hydrodynamic model.

The interpretation of the results from the boundary adequacy tests for the hydrodynamic model, which is summarized in table 5-6, is based mainly on the stabilization levels of the stock variables. In most of the tests the stabilization of the oxygen and ammonium levels were affected, which may indicate that the structure included in the model is necessary to describe the problem. Again, the removal of zooplankton had a large effect on the behavior, which may indicate the importance of examining this part of the model and system further.

5.5 Behavior Pattern Tests

The aim of behavior pattern tests is to evaluate how well the model reproduces the behavior patterns of data gathered from the real system (Barlas, 1996). Barlas suggests a formal logical procedure for performing statistical tests on behavior patterns of a model. This however, is only suitable for systems that are in relatively steady state, not for systems that exhibit transient behavior. The Sørfjord shows transient behavior due to constantly changing conditions, especially influenced by the water exchange rate. The behavior of the model of the Sørfjord, however, stabilizes at a certain level when the water exchange rate, the diffusion, and the flow from the Opo River, are constant. Limited data is gathered of the flow patterns of the fjord (Molvær, 2001), which makes it difficult to make reasonable assumptions about the changes of these variables. It is therefore difficult to use statistical methods to assess the behavioral validity of the Sørfjord model. According to Barlas (1996), graphical or visual measures of the typical behavior patterns are more suitable for behavior pattern testing of models of this kind of system. This may include an inspection of the amplitude of the peaks, the time between two peaks, minimum value, slope, number of inflection points, and the time to settle. The Sørfjord model is not subjected to formal behavior pattern tests, but the behavior of certain levels is discussed based on comparisons of data collected from the fjord. The main emphasis is put on evaluating whether the model exhibits a reasonable behavior, based on the assumptions that are made in the model in terms of stabilization levels and interaction between variables.

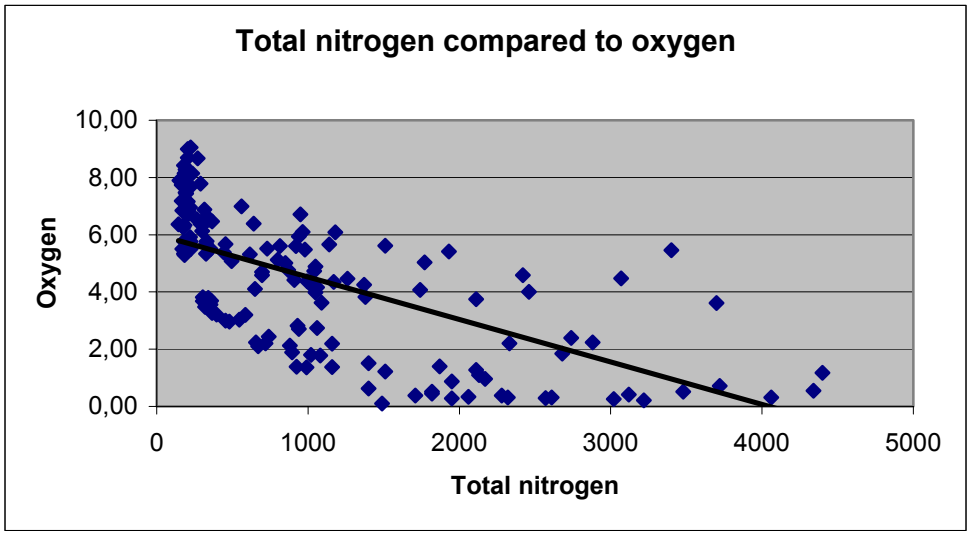
5.5.1 An Evaluation of Oxygen Levels Compared to Nitrogen Levels

The oxygen data that is gathered by NIVA is measured in ml O_2 per liter. In the Sørfjord model however, kilos are used, in addition to a variable that converts kilos into microgram per liter. In order to perform a comparison between data gathered by NIVA and data generated by the Sørfjord model, it is necessary to show the relationship between ml O_2 per liter and microgram O_2 per liter. The conversions are presented in table 5-7. The results will vary with the pressure in the water.

ml	1	2	3	4	5	6	7	8	9	10
microg	1 429	2 858	4 287	5 716	7 145	8 574	10 003	11 432	12 861	14 290

Table 5-7: Conversion from ml O_2 to microgram O_2.

Some reports on the pollution levels in the Sørfjord compare the total nitrogen levels (totN) in the water to the oxygen levels (Molvær, 1998; Molvær, 2001). The comparisons exhibit a relatively high correlation between high total nitrogen levels and low oxygen levels. Data gathered by NIVA (Molvær, 2001) are shown in graph 5-21. However, a comparison in this manner may be problematic. The total nitrogen level includes ammonium, nitrite, and nitrate. Ammonium and nitrite represent a possible future oxygen consumption rate, while nitrite and nitrate represent oxygen that has already been used. Hence, the ammonium and nitrite levels may indicate a future oxygen consumption rate, while the nitrite and nitrate levels represent a past oxygen consumption rate. A high total nitrogen level may therefore either indicate that there is a potential high oxygen consumption, or that there has already been a high oxygen consumption rate, or both. This makes it difficult to compare the total nitrogen level to the oxygen level. From a system dynamic perspective it would not be a proper comparison, because it does not represent a direct causal relationship.

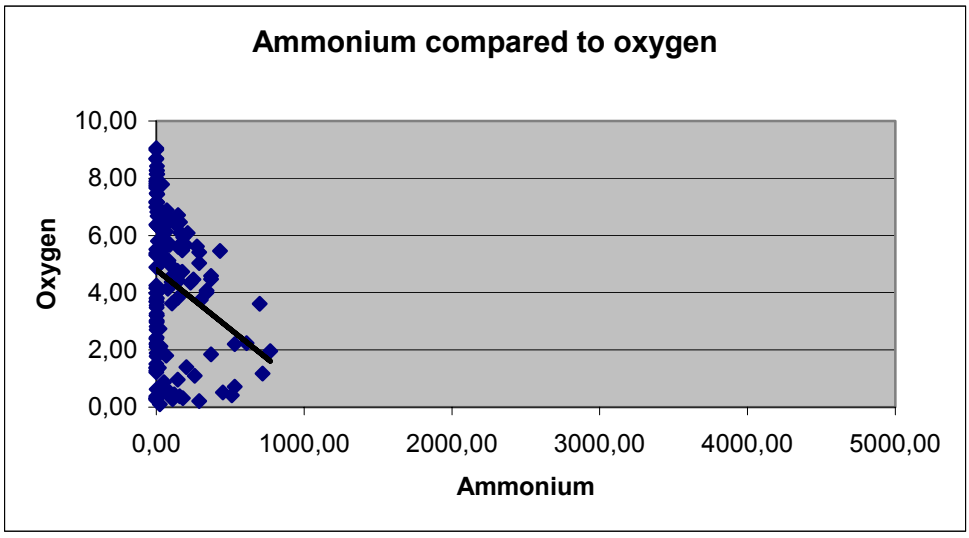


Graph 5-21: The total nitrogen level compared to oxygen in the Sørfjord (Based on data in Molvær, 2001, Vedlegg B).

Molvær (2001) also present measurements of the ammonium levels in the water. This is compared to the oxygen levels in graph 5-22. The data is gathered at three different locations

in the fjord over a period of 4.5 months. The correlation between ammonium and oxygen is not high, although there is some correlation. However, it is important to note that this does not represent a direct causal relationship. There may however be a direct causal relationship to the oxygen consumption rate. A high ammonium levels result in a high oxygen consumption rate if the ammonium does not flow out with the water exchange before it is nitrified. It is also important to note that the ammonium levels in the graph are relatively low compared to the total nitrogen levels in graph 5-21. This may indicate that the nitrite and nitrate levels in the fjord were relatively high when the data were gathered, or that there was a high inflow rate of nitrite and nitrate from the river and the ocean.

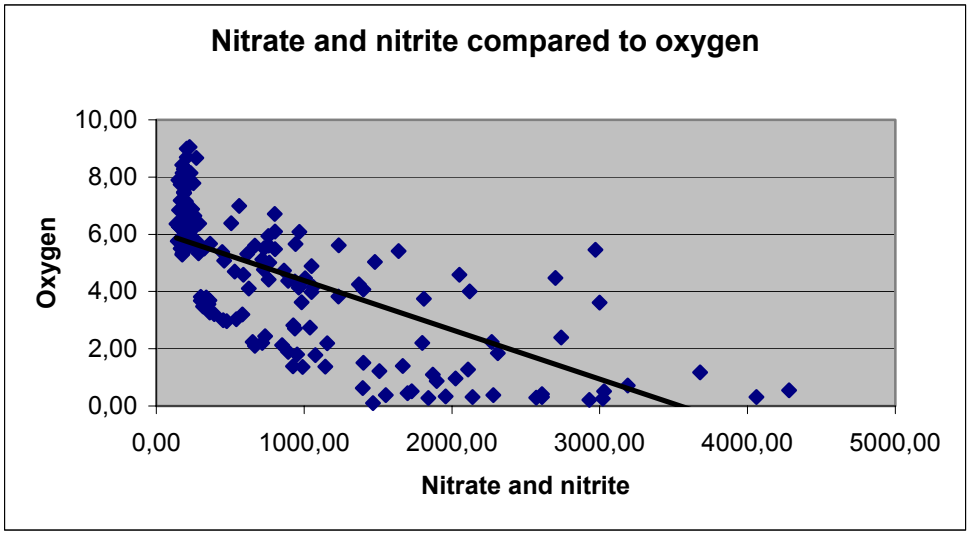
For there to be a purpose of comparing the oxygen levels to the ammonium levels there must be an analysis of the behavior of the two variables over time, since high levels of ammonium influence the future oxygen consumption, and thereby the oxygen level.



Graph 5-22: The ammonium levels compared to the oxygen levels in the Sørfjord (Based on data in Molvær, 2001, Vedlegg B).

A comparison between nitrite and nitrate, and oxygen levels is however appropriate, since nitrite and nitrate contain oxygen, and because unless the nitrite and nitrate have flown into the fjord through the water exchange or the river, they must have been produced through nitrification from ammonium to nitrite and later nitrate. Molvær (2001) does not present data of the nitrite and nitrate levels in the Sørfjord. However, it is likely that the total nitrogen measurements mainly contain ammonium, nitrite, and nitrate. Since data about the ammonium

levels are also gathered, it is here considered appropriate to subtract the ammonium level from the total nitrogen level in order to solve for an approximate of the nitrite and nitrate level. The results are presented in graph 5-23 where the data are compared to the oxygen levels.



Graph 5-23: The nitrate and nitrite levels compared to the oxygen levels in the Sørfjord (Based on data in Molvær, 2001, Vedlegg B).

Here, the correlation between the nitrogen compounds and the oxygen levels is even higher than for total nitrogen compared to oxygen (graph 5-21). This is probably because it mostly contains nitrogen that has already consumed oxygen from the water, not a potential oxygen consumption rate.

A formal comparison of the data presented in graph 5-21, 5-22, and 5-23 with the results from the Sørfjord model is problematic. First, the water samples that the data in the graphs are based on are gathered one to two months apart. It is therefore difficult to see whether a high ammonium level is followed by a high nitrate level and a low oxygen level. This means that it is difficult to see the change of behavior over time. The time units used would be one to two months, and not specific enough to study the changes properly. Further, there are characteristics of the model that inhibit some inspections that would otherwise be appropriate. This includes a comparison of the nitrate levels in the water. The ammonium that is nitrified into nitrate in the Sørfjord model is simply an output from the model, and nitrate is outside the system boundary. The inclusion of nitrate should be evaluated in a later version of the model,

because it would be easier to compare the behavior of the model to data gathered from the fjord by researchers.

However, an attempt is made to perform an informal comparison of the behavior of the oxygen levels and nitrogen levels in the model and data gathered from the fjord. The comparison must be interpreted on the basis of the previous discussion, and the main goal is to ensure a likely behavior of the system, and to ensure that the scales are within reasonable levels of data gathered from the real fjord. The ammonium and oxygen levels for each fjord cell in the Sørfjord model are compared in scatter graphs. The simulation time is set to 100 days, the water exchange rate to one fourth of the water form the Opo River, and the mixing factor is set to 250. The results are presented in figure 5-2. When comparing the behavior of the fjord to the behavior of the model, it is important to note that the scales of the graphs that are used are different. Graphs 5-21, 5-22, and 5-23 use a scale that goes from 0 to 10 ml oxygen, and 0 to 5000 microgram nitrogen compounds. The figure from the Sørfjord model uses 0 to 8000 microgram oxygen and 0 to 1700 microgram ammonium.

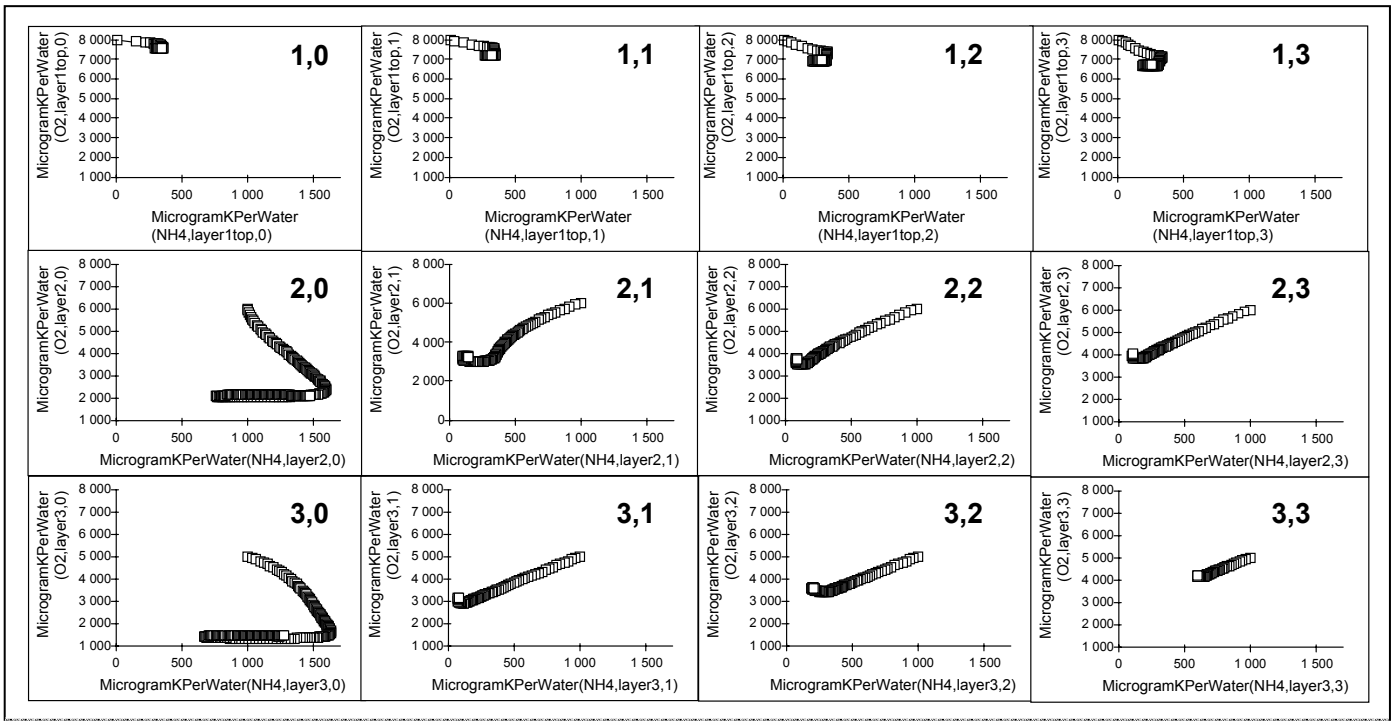


Figure 5-2: Ammonium levels compared to oxygen levels in the Sørfjord model.

In the top layer the ammonium level is generally below 500 microgram per liter, and the oxygen level between 7000 and 8000 microgram per liter. This roughly corresponds to 5-6 ml oxygen per liter, which is the unit of measure used by NIVA (Molvær, 2001). Comparing figure 5-2 with the top level in graph 5-22, it is possible to recognize certain similarities in the behavior. The oxygen levels are higher at smaller levels of ammonium. However, the oxygen levels are generally fairly high, and the ammonium levels low.

As for fjord cells 2,1 to 2,3 and 3,1 to 3,3, they exhibit an opposite behavior of that of graph 5-22. The ammonium levels in these cells are relatively high, and as the levels decrease, oxygen is consumed. However, low levels of ammonium may not necessarily indicate a low total nitrogen level, because there may be nitrate in the water.

Fjord cells 2,0 and 3,0 exhibit a different behavior pattern than that of the other cells. This is because they receive a large ammonium input from the factory. The oxygen levels are generally low in these cells, and the ammonium levels high. The part of the scale that is used, about 700-1700 microgram ammonium per liter, and an oxygen level ranging from 1000 to 6000 microgram per liter, however, roughly correspond to the ammonium levels that the data gathered by NIVA exhibit (Molvær, 2001).

As mentioned in the beginning of the section, it is difficult to compare oxygen levels to the total amount of nitrogen, because the total nitrogen represents both a potential future oxygen consumption, through nitrification from ammonium to nitrate, and oxygen that is already consumed and is part of nitrate. Further, it is also difficult to compare oxygen and ammonium levels because the oxygen level at a specific point in time does not have a direct causal relationship to the ammonium level at the same point in time. However, figure 5-2 show that the fjord cells that contain high levels of ammonium generally contain low levels of oxygen. It is possible to assume that these fjord cells also would contain high levels of nitrate, if nitrate had been included in the model, since the particular fjord cells have relatively high ammonium inputs, and low oxygen levels may be caused by ammonium nitrification.

This informal test may indicate that in order to be able test the behavior of ammonium compared to oxygen in the model, it may be necessary to gather data both of the ammonium and nitrate levels in the water, not just the total nitrogen level. It may also be important in

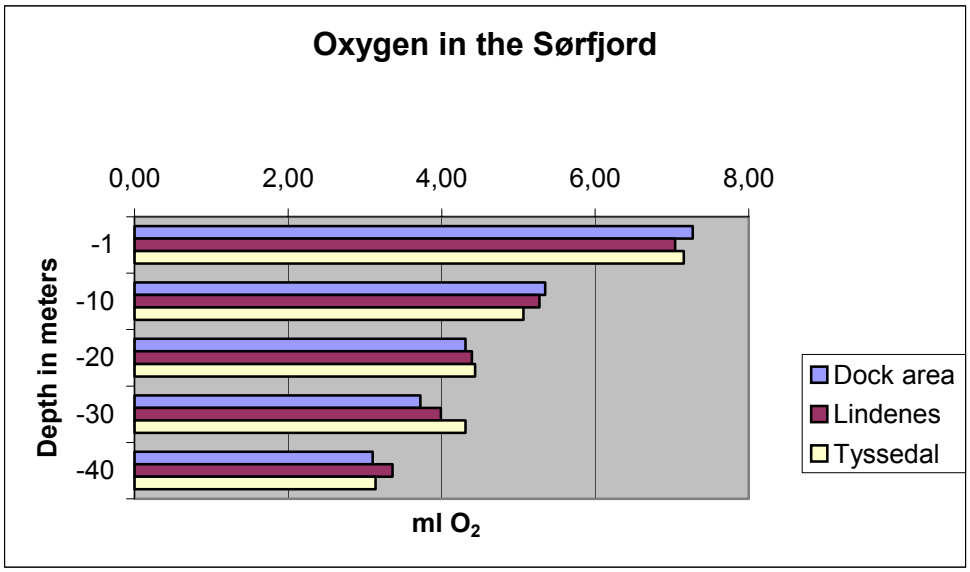
order to predict and understand more about the varying oxygen levels in the real fjord system. If the total nitrogen input from the factory to the fjord is relatively constant, the total nitrogen level may mainly be controlled by varying nitrogen input from the river, a change in the water exchange rate, and the nitrogen levels of the seawater that flows into the fjord. If this is the case, it may be important to differentiate between ammonium and nitrate. Also, frequent data gathering may be necessary in order to obtain a closer perspective of the changes in levels over a shorter period of time, to make assumptions of the rates that are influenced by the levels. In a later version of the Sørfjord model the inclusion of nitrate should also be considered.

5.5.2 An Evaluation of the Ammonium and Oxygen Levels at Different Locations and Depths

Even though direct formal tests of the behavior of the Sørfjord model are difficult to accomplish, it is possible to investigate some major variables, and check that the scale and relationship between the levels that the model utilizes coheres to that of data gathered by scientists. Both the oxygen and ammonium levels are subjected to this test. Data gathered by NIVA (Molvær, 2001), is used as a basis for the comparisons. The water samples are collected at three different locations: the dock area in Odda, Lindenes (about 3500 meters from Odda), and Tyssedal (about 5500 meters). This represents a much longer distance than the 250 meters that the Sørfjord model is set to. However, the test is only intended to check that the relationship between the levels at varying depths and lengths is appropriate.

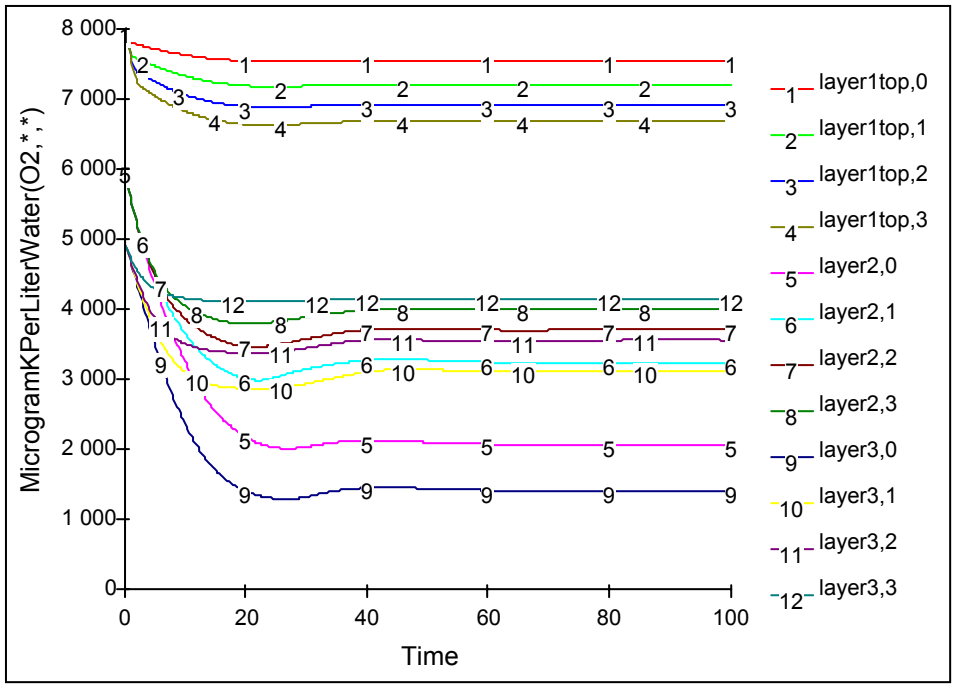
Oxygen Levels

Graph 5-24 presents data gathered by NIVA of the oxygen levels in the fjord Molvær,2001). The water samples are collected at three different locations, at five different depths, and at four different points in time. Each stack represents the average number of ml oxygen per liter, at a certain depth and location, over the four dates. The graph exhibits lower oxygen levels at greater depths. The average oxygen level is also lower in the dock area in the innermost part of the fjord than further out (with the exception of the oxygen level at 1 meter), although the differences are not great. Hence, ideally the Sørfjord model should exhibit higher oxygen levels in the top layer, and in the fjord cells further out in the fjord.



Graph 5-24: Oxygen levels in the Sørfjord at different locations and depths (Based on data in Molvær, 2001, Vedlegg B).

The Sørfjord model is run for 100 days with a constant water exchange rate of one fourth of the Opo River, and a mixing factor of 500. The results are displayed in graph 5-25. For an indication of the numbering of the various fjord cells, see figure 5-1.



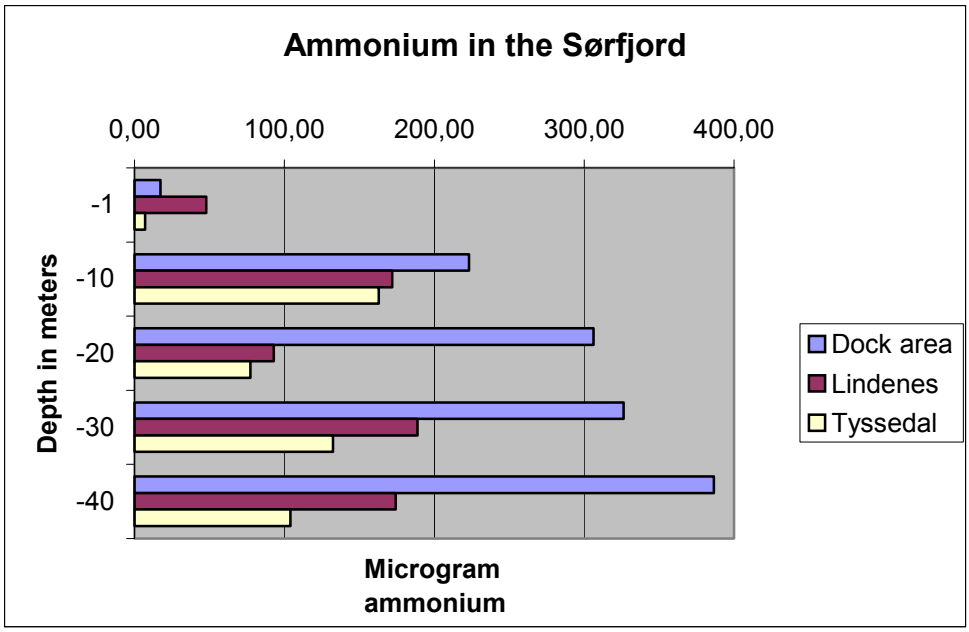
Graph 5-25: Oxygen levels in the Sørfjord model.

Series one to four represent the top layer of the fjord. The levels are mainly between 6500 and 8000 microgram oxygen per liter, which roughly corresponds to the average oxygen levels in graph 5-24. The behavior of the model coheres with the behavior of the data gathered from water samples of the fjord in that the oxygen level is fairly high at the top. In graph 5-24 the data from NIVA show that the oxygen levels gradually decrease as the depth approaches 40 meters. For the samples taken below one meter, there does not seem to be much difference in the data gathered at different locations. Other reports however, have documented increasing oxygen levels further out in the fjord, as the outmost end of the fjord is approached (Skei, 1998; Molvær, 1998).

In the Sørfjord model fjord cells 2,0 and 3,0 contain the lowest levels of oxygen. This is because these cells receive ammonium input from the pollution from Odda Smelteverk, and oxygen is consumed in the nitrification process that occurs. Further, the oxygen level gradually increases for each fjord cell in the outward direction. The oxygen level is also lowest at the bottom layer, which corresponds to the data collected by NIVA in graph 5-24. It seems as if the Sørfjord model exhibits a reasonable behavior with respect to the oxygen levels, and the relation between the oxygen levels in the different fjord cells.

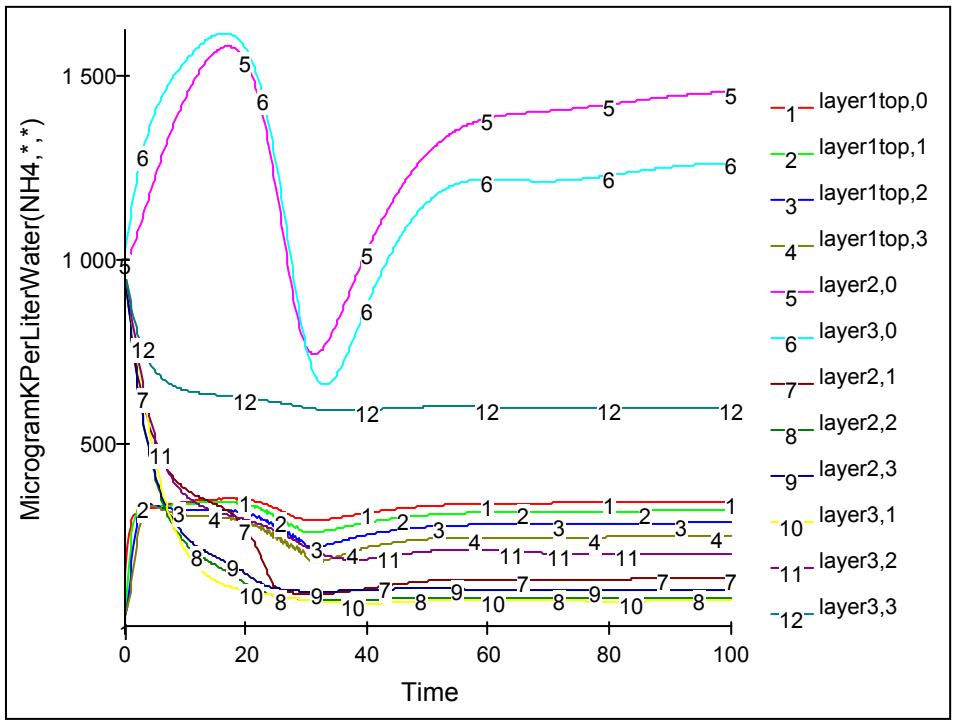
Ammonium Levels

A similar test is performed on the ammonium levels in the various fjord cells. The results of the ammonium levels that were tested by NIVA (Molvær, 2001) are summarized in graph 5-26. The ammonium levels increase as the depth increases. There is also a much higher ammonium level in the dock area in the innermost part of the fjord. This indicates that the Sørfjord model should exhibit lower ammonium levels in the top layer, and that the level should increase at increasing depths. Further, the ammonium level in the innermost fjord cells should be much higher than that of the others.



Graph 5-26: Ammonium levels in the Sørfjord (Based on data in Molvær, 2001, Vedlegg B).

Graph 5-27 shows the behavior pattern of the ammonium in the water for the different fjord cells in the Sørfjord model. The graph is based on the same simulation run as graph 5-25.



Graph 5-27: Ammonium levels in the Sørfjord model.

Layers 2,0 and 3,0 contain much higher levels of ammonium than the other fjord cells. This is because these cells receive ammonium input through pollution from the factory. This corresponds to the behavior of the data gathered by NIVA, however, the fjord cell in the middle depth layer, 2,0 contain more ammonium than fjord cell 3,0. NIVA's data shows that there is more ammonium at the bottom. The reason the behavior in the model is different is that the largest portion of the ammonium is put into these fjord cells. Also, because of the flow pattern of the water, the ammonium from fjord cell 3,0, which also receives a higher ammonium input, flows into fjord cell 2,0, causing a higher ammonium level. This could be altered in a later version of the model, with a more detailed flow pattern, or simply by discharging more ammonium into the bottom layer.

Layer 3,3 stabilizes at a fairly high ammonium level. This does not correspond to the data from the Sørfjord, where the ammonium levels decrease further out in the fjord. The behavior in the model is caused by input from the fresh seawater that has a relatively high level of ammonium.

Layers 2,1 to 2,3, 3,1 and 3,2 stabilize at relatively low levels. This is also the case for the four top layer (1,0 to1,3), however they stabilize at a higher level than that of the others. This does not correspond to the behavior of the data from the water samples, where there is a relatively low amount of ammonium at the top, and a relatively high amount at the bottom. This may be altered in a later version of the model, maybe by performing a more thorough investigation of the biological processes in the upper layer. The amount of ammonium and the speed of the water may also be studied.

Even though the model exhibits some differences from the data from NIVA in behavior patterns, it is still considered useful in order to see how the stocks in the different parts of the fjord may change due to the flow of the water and biological processes. The scale that is used is about the same as that of the data from NIVA, and this can be used as a basis for further development of the model.

6. Behavior Analysis

6.1 Introduction

Before analyzing the behavior of the model it is necessary to go through some limitations and weaknesses of the model. Both the hydrodynamic and the biological model are simplified down to a general model of a fjord including some biological factors. During the work with the model, the behavior changed dramatically from the one-dimensional implementation of the biological model to the two-dimensional hydrodynamic model. On of the reasons is the relatively stabile behavior of the water in the hydrodynamic model. All water flows are stabile over time and are contributing to stabilize the biological environment. In a real fjord the velocity and patterns of the water flows are in continues change, and the biological system are under constant influence of these changes. Exogenous factors like weather, time of year, tide water and other things are all influencing a real system in a fjord, but are very hard to implement in such a model.

The hydrodynamic model has the possibility for an accurate initialization of the parameters in each of the individual cells in the model. The lack of reliable data from the Sørfjord makes this initialization very difficult. Therefore the model has been initialized with the same values for the constants in one whole layer. In a goal seeking system this leads to a stabilization phase in the beginning of a simulation, because one is not able to initialize the system in equilibrium.

To analyze and explain the behavior of the system, the causal loop diagrams (CLD) are of great help. The CLD shows the casual relations between the different factors, and the reinforcing and balancing loops is a useful tool in understanding the behavior. The CLD's used in this analysis are described in chapter 3, where all the loops are explained in more details.

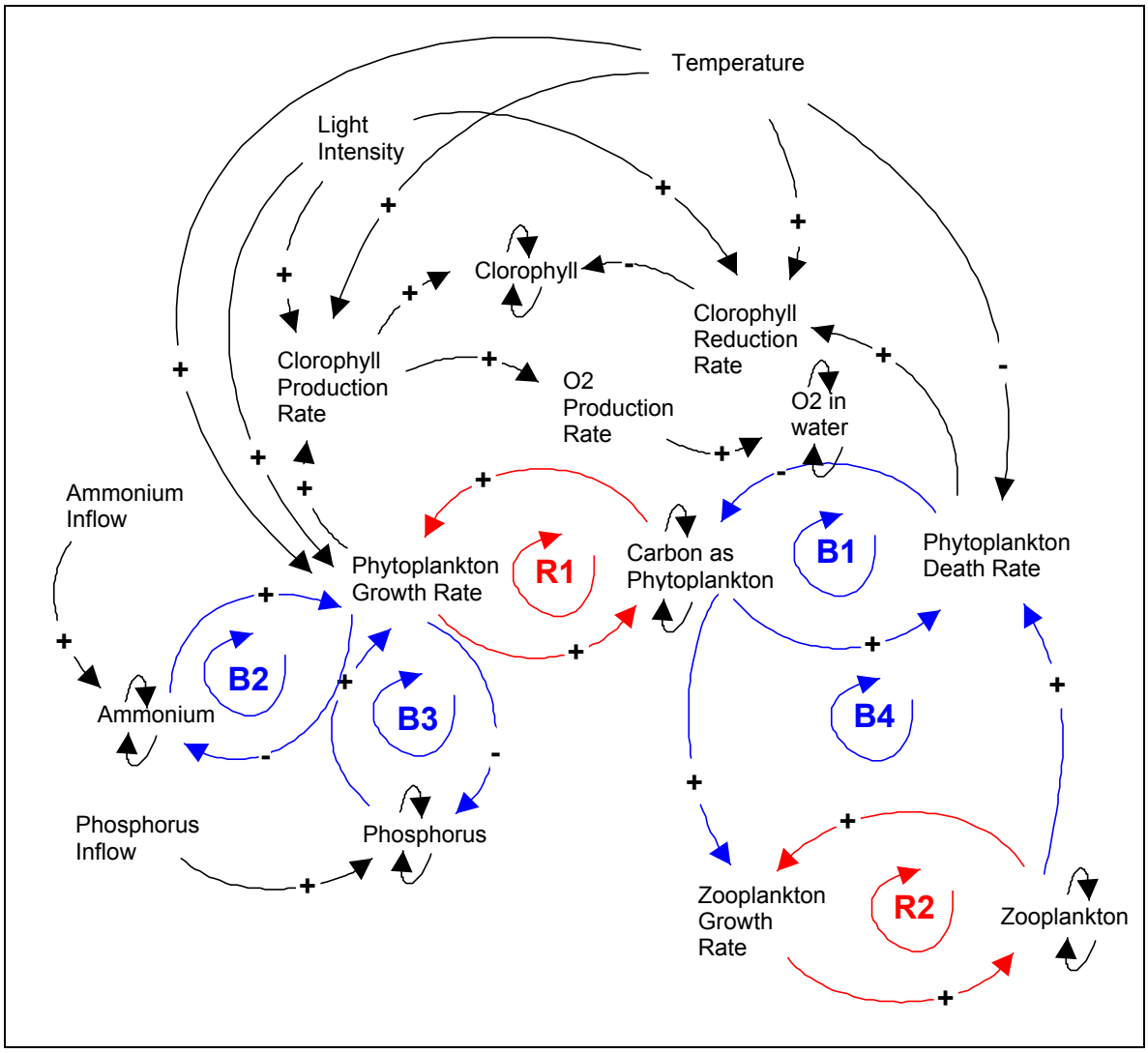


Figure 6-1: CLD for phytoplankton.

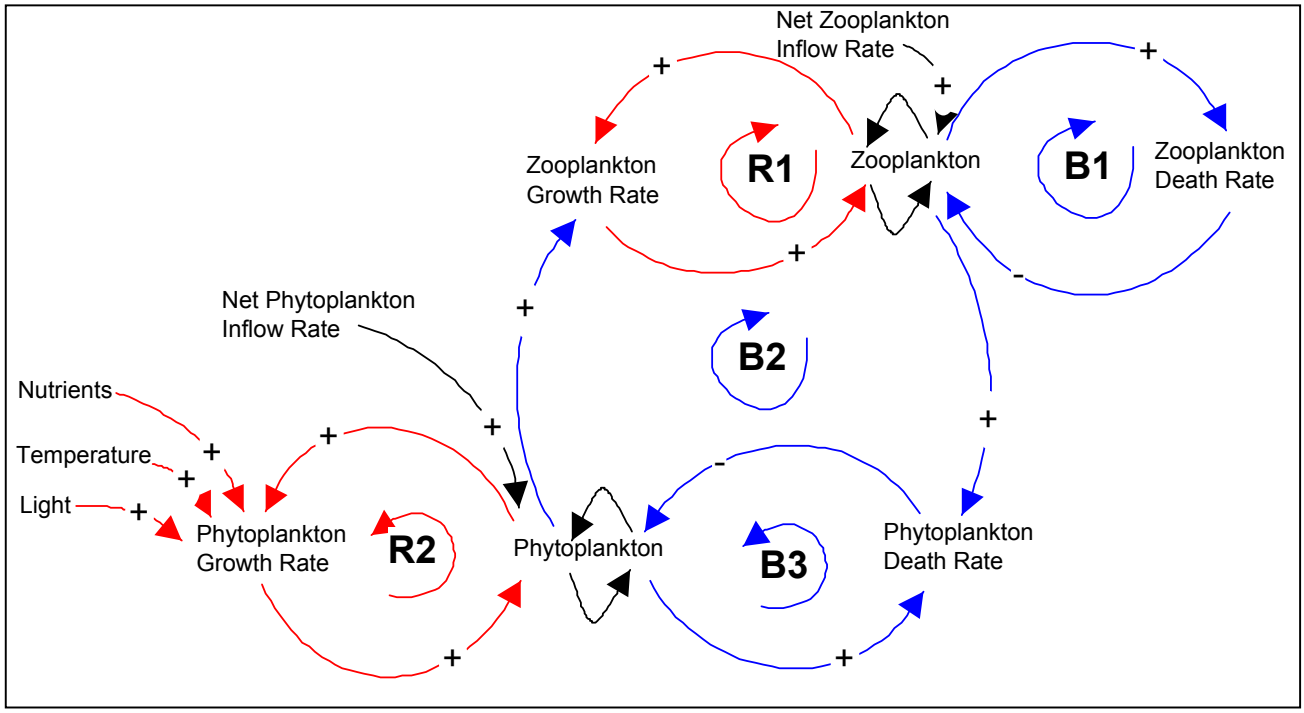


Figure 6-2: CLD for bacteria (without zooplankton)

The focus in the analysis will be on a selection of variables, based on their importance in the ecological system and their influence on the behavior of the system. Most of the graphs are normalized to better show the behavior, and to illustrate how the balancing and reinforcing loops are driving the behavior. The Y-axis is therefore not representing a comparable scale for the elements in the graph. Where the numbers are of any interest, they are showed explicit in other graphs.

6.2 Simulation

In this first simulation the initialization of the parameters is based on numbers from different reports (Molvær, 1997), (Schaanning, 1999), (Garmann & Co, 1999) from the Sørfjord. The goal is to fit the model to the Sørfjord, and to analyze the behavior that can also be seen there. Some variables are of more interest then others, and will be focused on in the analysis. The most important factors is:

- Oxygen (O^2)
- Ammonium (NH_4^+)
- Nitrifying bacteria
- Phytoplankton (Carbon as phytoplankton)
- Zooplankton.

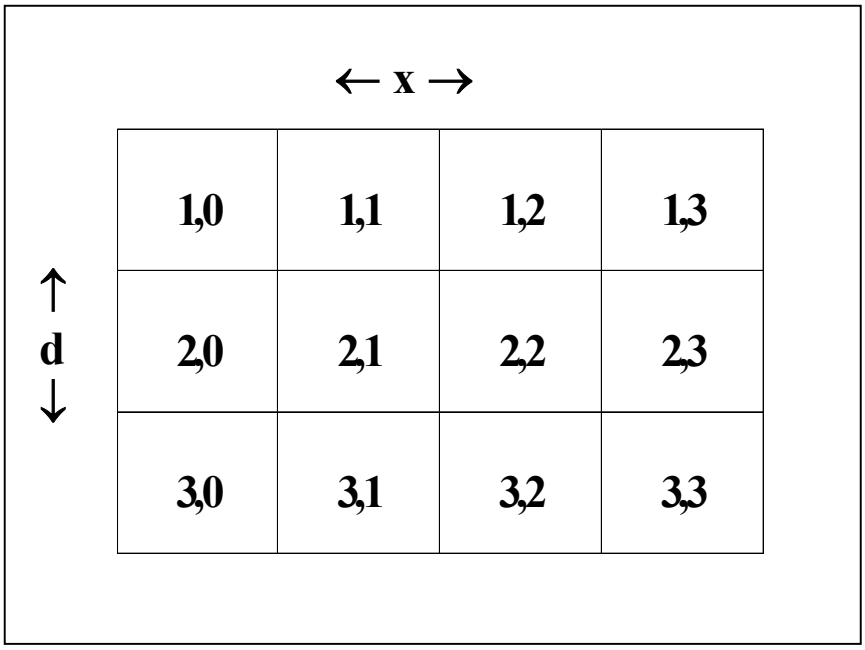
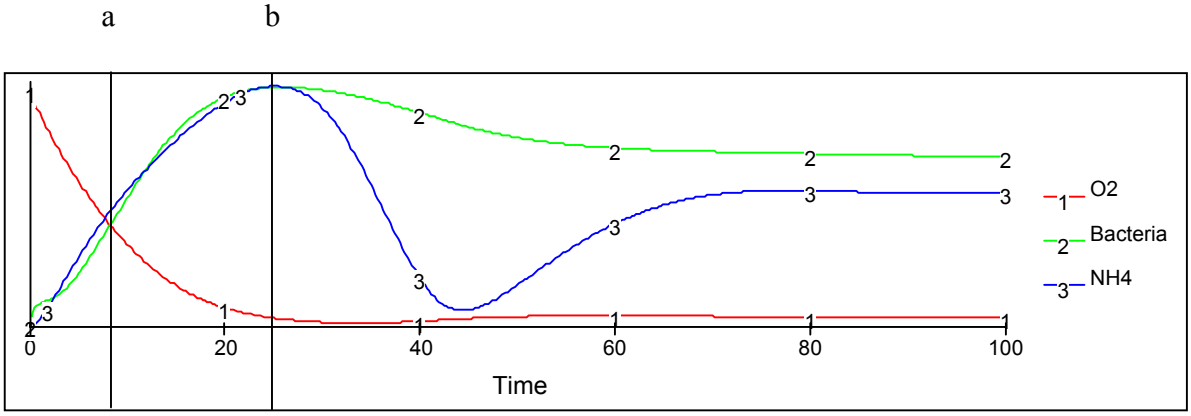


Figure 6-3: In this chapter the fjord cells in the hydrodynamic model will be referred to with the numbers used in this figure.

6.3 Simulation of Initial Conditions

In the analysis of this simulation, the first part will be used to describe and explain the behavior in a single cell. The patterns are the same for most of the cells, and it is therefore convenient to use one of the cells where the behavior is clear and distinct. This will be the inner part of the fjord where the currents are influencing the behavior least. Later a comparison of the behavior in the different cells with respect to the currents will be discussed.



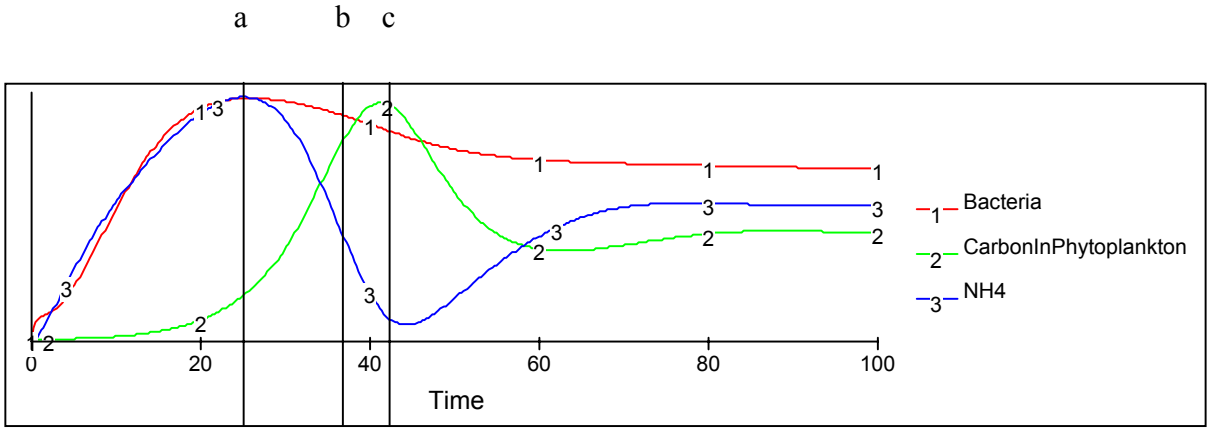
Graph 6-1: Oxygen (O^2), Bacteria and Ammonium (NH_4^+) in cell 2,0. (The graph is normalized to better show the behavior, and the Y-axis is not equal to zero for any of the variables.)

The graph shows a goal seeking behavior for all three elements after an unstable behavior in the beginning. As explained previously the bacteria consumes ammonium (NH_4^+) and oxygen (O^2), and the growth is closely related to this consumption. The increasing level of ammonium (NH_4^+) is caused by the exogenous supply form Odda Smelteverk and from the diffusion form the bottom. The initial level of ammonium (NH_4^+) in the water is set to a 'normal' level equal to what is coming in from the outer part of the fjord.

The shifts in loop dominance can explain changes in the behavior of the system. But in a complex system as this, the identification of a specific loop, or combination of loops, causing this change is complicated. Several factors are influencing on each other's behavior, and the hydrodynamic currents make it even worse to get a clear picture.

From the beginning of the simulation until the point 'b', the balancing loop 'B1: Oxygen nitrification' dominates the behavior of the oxygen. The level of oxygen is decreasing caused by an increase in the nitrification rate. The bacteria increase exponentially from the beginning until 'a'. 'R1: Nitrifying bacteria growth' is the dominating loop and causes an exponential growth. Between point 'a' and 'b' the growth decreases and the bacteria reach a top point at 'b'. The surrounding balancing loops dominate, and the reduction of oxygen both decreases the nitrification rate and increases the bacteria death rate. From point 'b' the death rate becomes larger then the growth rate, and the amount of bacteria decreases with a goal seeking behavior caused by the balancing loops 'B1: Oxygen nitrification' and 'B3: Nitrifying bacteria death'. The bacteria are adjusting to the available amount of oxygen.

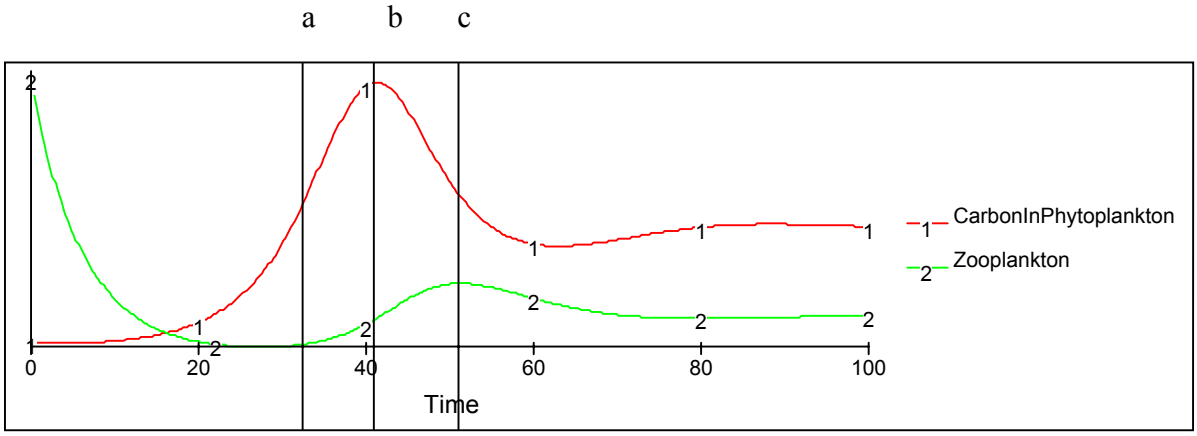
When it comes to the ammonium (NH_4^+) the phytoplankton has to be brought into the discussion.



Graph 6-2: Bacteria, ammonium (NH_4^+), and phytoplankton (CarbonInPhytoplankton) in cell 2,0.

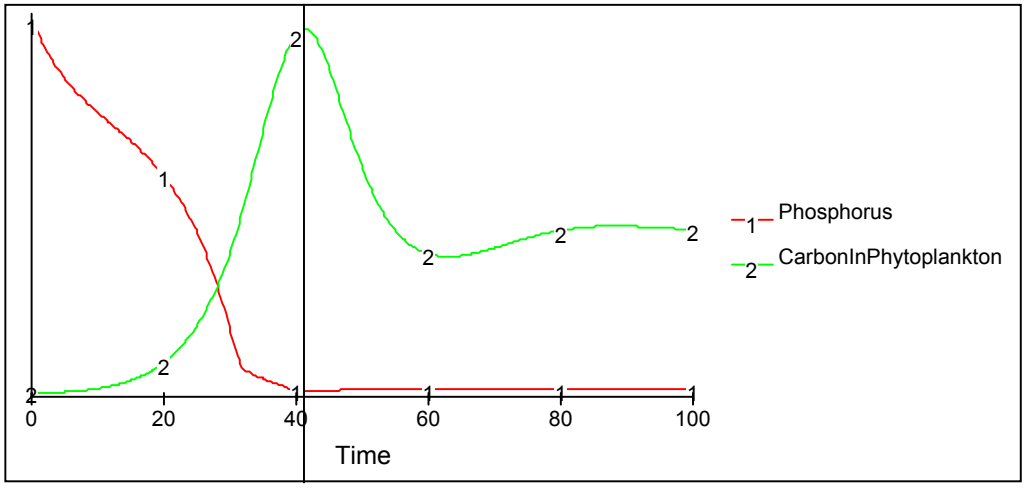
Graph 2 shows that the phytoplankton is growing exponentially until 'b'. As long as the relative amount of ammonium is high compared to the number of phytoplankton, the reinforcing loop 'R1: Phytoplankton growth' is dominating. From 'b' to 'c' the balancing loop 'B1: Phytoplankton death' are dominating and gives a decreasing growth until it reaches the top at 'c'. This is not the only factor influencing the phytoplankton at this point. Zooplankton is also an important factor. This is discussed below. At point 'a' the consumption of ammonium by phytoplankton and bacteria is getting higher then the supply of ammonium to the water, and the result is an exponential decrease of ammonium in the water. This exponential decrease continues

until ‘b’ where the growth of phytoplankton is beginning to decrease. At this point the loop ‘B2: Ammonium reduction’ is dominating.



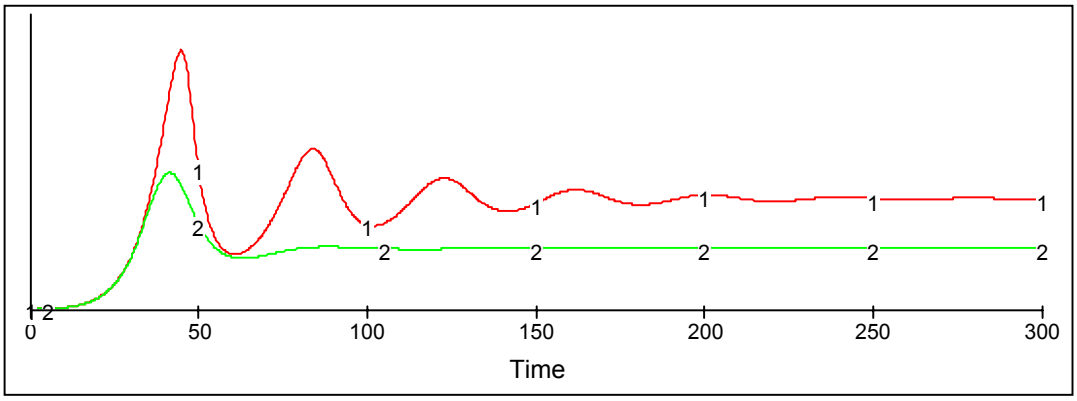
Graph 6-3: Phytoplankton and zooplankton in cell 2,0.

As described above, zooplankton is another important factor influencing the phytoplankton. Graph 5 shows the behavior of phytoplankton and zooplankton. The behavior is not far from the description in chapter 3.11 explaining a possible behavior of the predator-prey scenario. Point ‘a’ shows the time where the growth of phytoplankton goes from exponential to decreasing. Compared to the simulation in a one-dimensional model, this shift accurse some days later then the shift for zooplankton. This can be explained by the fact that other factors also are influencing the phytoplankton growth. The turning point ‘b’ is nearly corresponding with shift in dominance for the zooplankton. Between ‘a’ and ‘b’ the loop ‘R1: Zooplankton growth’ is driving the zooplankton to an exponential growth, and the loops ‘B3: Phytoplankton death’ and ‘B2: Predation of phytoplankton by zooplankton’ are forcing the phytoplankton to a decreasing growth. From ‘b’ to ‘c’ the phytoplankton decreases exponentially while the zooplankton shows a decreasing growth. For phytoplankton the loop ‘B3: phytoplankton death’ becomes dominating because the amount of zooplankton is increasing. The decreasing growth of zooplankton is caused by the ‘B1: Zooplankton death’.



Graph 6-4: Phosphorus in water compared to phytoplankton in cell 2,0.

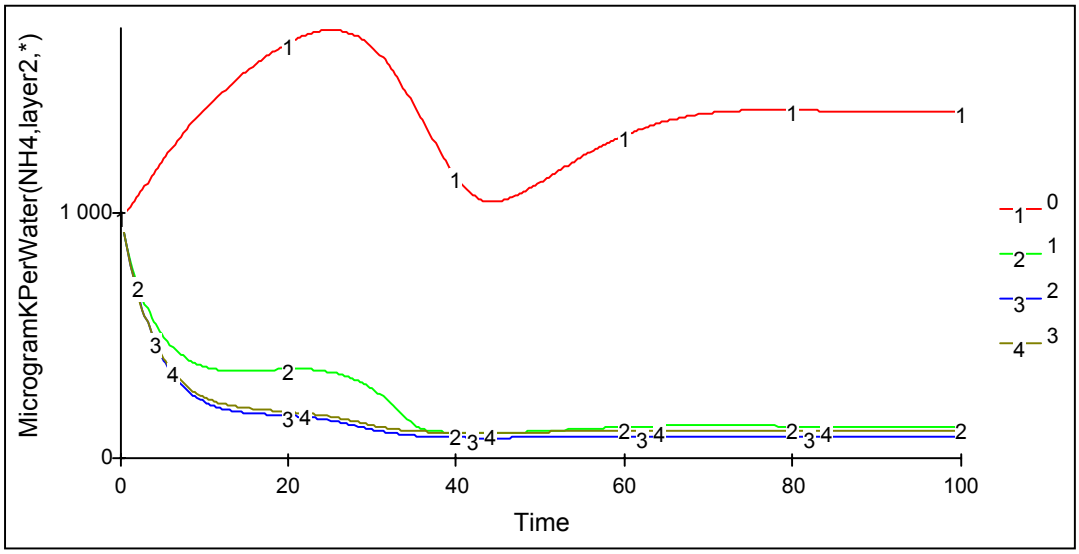
Another factor influencing the phytoplankton growth is the availability of phosphorus. The graph shows that much of the phosphorus is consumed after around 40 days. This may also be one of the reasons to the reduction in the amount of phytoplankton.



Graph 6-5: Phytoplankton with unlimited phosphorus recourses (1) and phytoplankton with limited phosphorus recourses (2).

The graph shows that the phosphorus limitation causes a faster stabilization of the behavior and a lower level of phytoplankton in the water. This indicates that the balancing loop ‘B3: Phosphorus consumption’ is a dominating loop under such conditions.

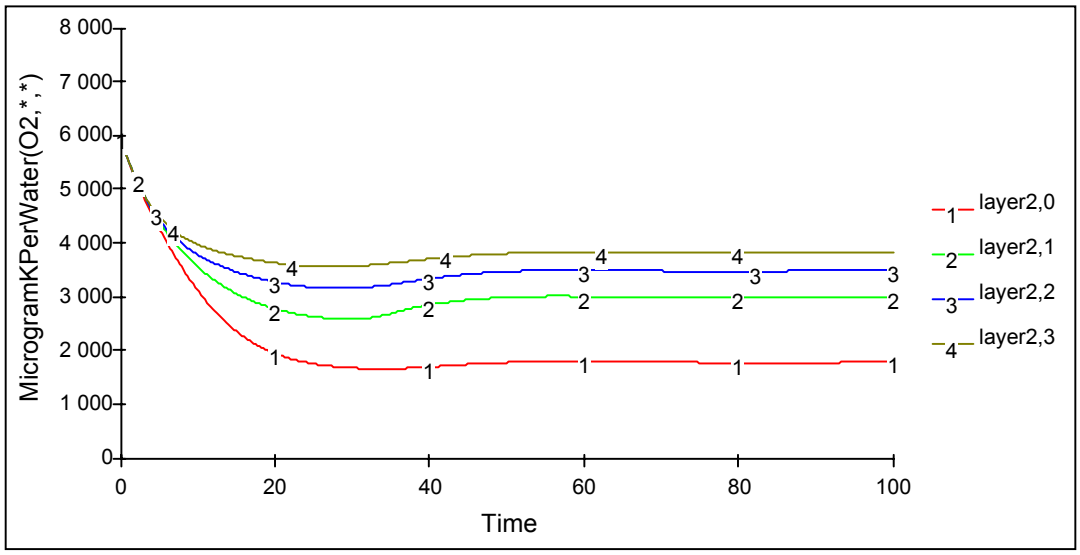
6.4 The Hydrodynamic Behavior



Graph 6-6: Ammonium (NH_4^+) in layer 2.

Graph 6-6 shows the ammonium level for the four cells in the mid-layer (layer 2). Cell 2,0 is the innermost part of the fjord where the ammonium supply from Odda Smelteverk is located. The ammonium diffusion from the bottom in cell 3,0 is located under cell 2,0. Because of the currents in the hydrodynamic model, the ammonium that is not consumed in cell 3,0 is transported to cell 2,0. From 2,0 the current goes through cell 2,1, 2,2 and 2,3. Even with a relatively large supply of ammonium to cell 2,0 the high consumption in the cell causes a relatively low transport of ammonium into the next cells.

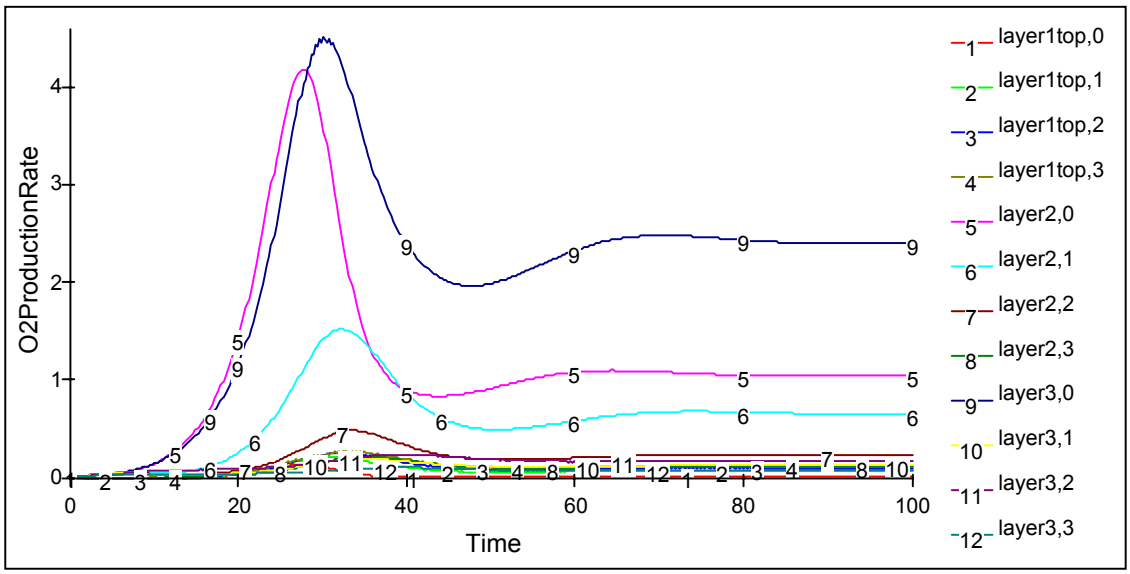
Compared to actual numbers from measuring programs in the fjord, a larger amount of the ammonium should be transported further out in the fjord. Some reasons for this difference can be the very simple hydrodynamic model, and the lack of reliable data to initialize this part of the model. Another reason that can seem obvious is too high consumption of ammonium in each cell. This might be caused by the phytoplankton that uses ammonium, and not is limited by the lack of oxygen. The light limitation might influence the phytoplankton growth more than it does in this model and the growth rate might therefore be too high, but simulations show that decreasing the growth for phytoplankton rate does not change this behavior much, and not enough to explain it.



Graph 6-7: Oxygen (O^2) in layer 2.

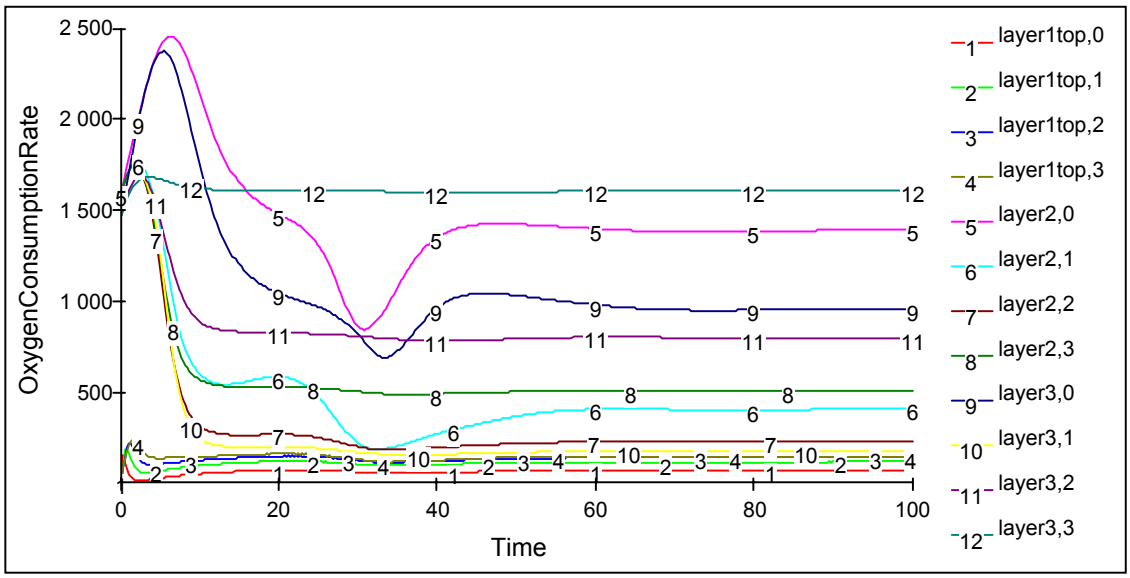
The oxygen in layer two (graph 6-7) corresponds more with data from the Sørfjord. The graph shows an increasing oxygen level with an increased distance from the nitrogen source in cell 2.0. In the two last cells, the oxygen has reached a satisfactory level.

6.5 Oxygen Production and Consumption



Graph 6-8: Oxygen production rate in kg per day.

The oxygen production is caused by the photosynthesis in the production of phytoplankton. The rates are very small and the contribution to the total oxygen level is small. Only in periods when the phytoplankton level from time to time growth to enormous amounts it may influence the system in any degree. For further development of such a model it may be considered to leave this factor out.



Graph 6-9: Total oxygen consumption by nitrification (kilo per day).

Compared with the oxygen production one can the large difference, and that the contribution from the production does not influence the behavior in any degree. The oxygen consumption is highest in the cells with a great supply of ammonium. The incoming water from the fjord to cell 3.3 (line 12) contains a high level of ammonium. The most of this ammonium is consumed in the first two cells (3.3 and 3.2). This causes a high oxygen consumption, which can be seen from line 12 and 11 in the graph. Cell 3.0 and 2.0 has also a large consumption of oxygen. This is where the ammonium supply sources are in the inner part of the fjord.

The top-layer has a high oxygen level and a high ammonium level, but the oxygen consumption is still low. This is because the velocity of the water in the top layer is higher, and the bacteria do not get a chance to grow before they leave the cell.

6.6 A Simulation With Hydrodynamic Variations

The hydrodynamic variations in a fjord have a great influence on the behavior of a biological system. Hydrodynamic variations can be seasonal changes in the supply of freshwater from rivers, human activities like building power plants that regulate the freshwater supply or exogenous factors like changes in weather that influence the velocity of the currents in the fjord. Some of these factors like freshwater supply from rivers and supply controlled by human can be predicted or planned more or less accurate. The seasonal variations in a river can be observed over some years, and form these observations one can give a statistically based forecast for the coming years. A forecast of changes in weather conditions can only be given short time ahead, and the effect of the changes on the hydrodynamic behavior is difficult to estimate.

The Sørfjord has a river, Opo, coming in the innermost part of the fjord. Observations of this river show large seasonal variations through the year (Svendsen, 1973). A normal season shows two peaks in the water supply to the fjord. The first is in June/July and is caused by the change in temperature in the surrounding mountain areas. The next peak comes around October and is caused by the increase in rainwater. This second peak is more unstable then the first one both in time and size.

In the hydrodynamic model the water exchange rate in the fjord is based on the same numbers as the freshwater supply from the river, and the supply from the outer part of the fjord is set to ¼ of the freshwater supply. This is because one assumes that an increase in the freshwater supply to the top-layer will increase the pressure on the underlying layers. Such an increase in the pressure will lead to an increase in the water exchange rate as well. Form this one assumes that there is a correlation between the supply form the river and the water exchange rate.

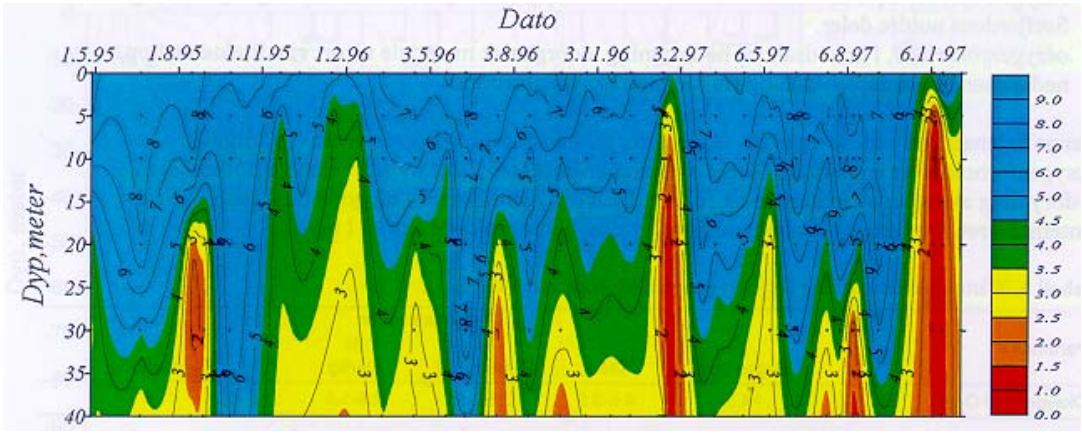


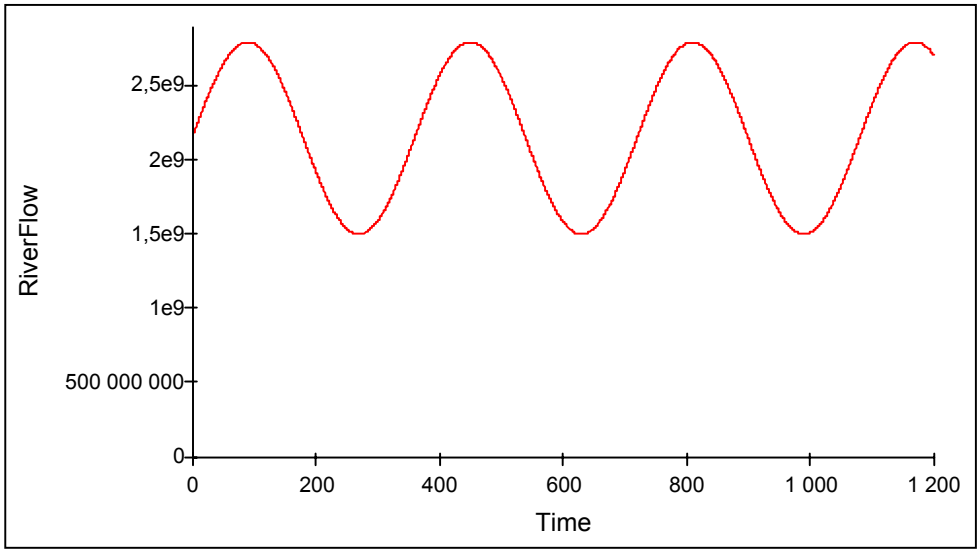
Figure 6-4: The oxygen level in the inner part of the Sørfjord, May 1995 - November 1997. (Skei,1998, NIVA 724/98, p. 24)

The figure shows the oxygen level in the inner part of the Sørfjord over 2 ½ year. The Y-axis represents the water depth, and it shows clearly that the oxygen level near the bottom is worse then in the top-layer. The conditions are varying from extremely oxygen poor to good several times during the period near the bottom. The oxygen level in the top-layer is more stabile on a higher level, but the large peaks reach to top-layer and causes lack of oxygen all the way to the surface.

A closer look at the figure shows a relative strong regularity of the peaks. The first peak comes in August 1995, and a similar peak can be found both in August 1996 and in 1997. A smaller peak occurs in November 1995, and can also be identified in 1996 and 1997. November 1997 shows extremely low oxygen level through all the water layers, but corresponds in time with the others. For 1996 and 1997 the November-peaks continue until they reach a top in February. There is also a peak around May.

Compared to the information about seasonal changes in the water supply from the river, one can assume that the high rates of fresh oxygen-rich water in June/July and October correspond with periods with satisfactory oxygen levels in the fjord. Even if figure 6-4 does not show this correlation as clearly as the frequently peaks, one can see that the peaks occur in front of, and after these two periods whit high water exchange rates.

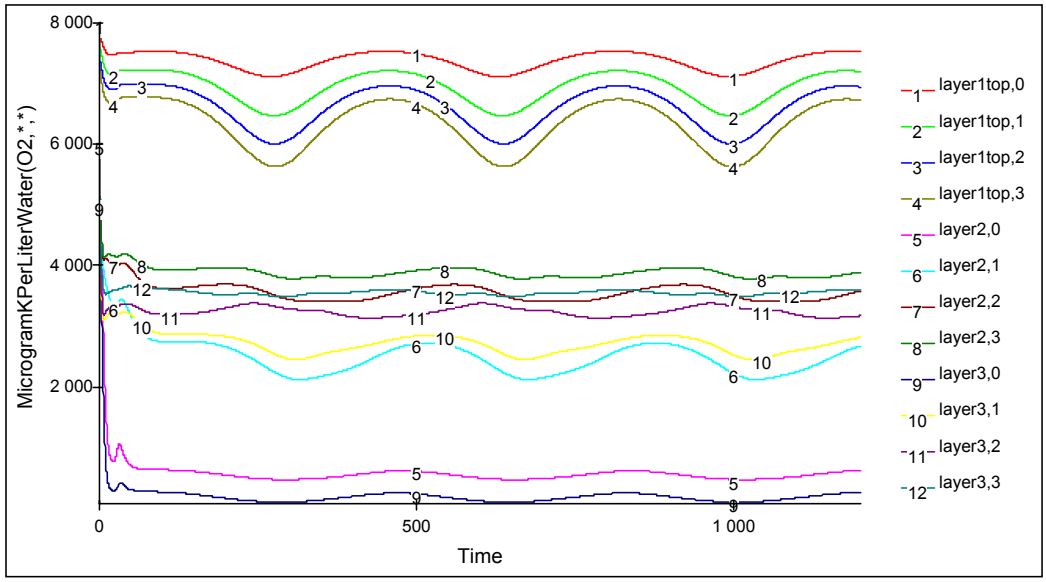
A simulation of this scenario to see the effect of the seasonal variations can be done in a simple way by including a sinus wave in the two supply rates. Letting the sinus wave go over 360 days it will only show a single peak during the year, but the low freshwater supply during the winter months will be better represented whit this single peak and single bottom.



Graph 6-10: The river flow including a sinus wave based on 30 % of the river flow.

The river flow is extended with a sinus wave to simulate one top and one bottom during a year. Since the river flow is driving the water exchange rate, this rate is extended with same sinus wave relative to the initial rate.

6.6.1 Oxygen

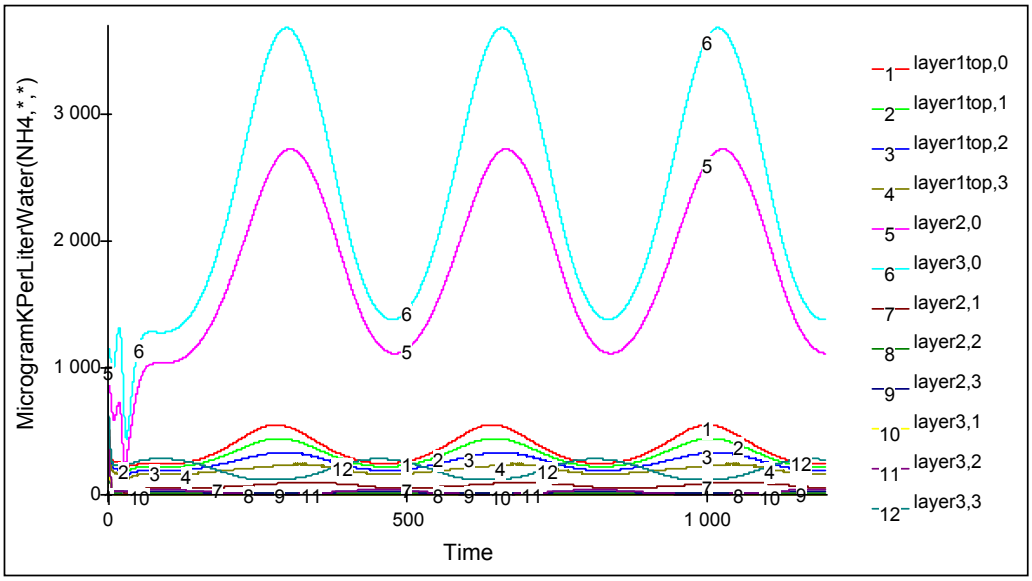


Graph 6-11: The oxygen level in the fjord with oscillating water exchange rate and river flow.

Graph 6-9 shows the oxygen level for all cells over time with oscillations in the water exchange rate and fresh water supply. Line 1 to 4 shows the oxygen level in the top-layer. The effect of the oscillations is most visible in this layer. The behavior of the oxygen level follows the exchange rate and water supply, and higher rates give higher oxygen levels. This is because the incoming water has a relatively high oxygen level. Line 1 is the innermost cell, and the oxygen level decreases further out in the fjord. This is caused mainly by water mixing with the underlying layer and oxygen consumption by bacteria. Line 5 and 9 at the bottom represent cell 2.0 and 3.0. The two inner most cells in layer 2 and 3. These cells contain the nitrogen supply sources, and the nitrification rate is high and causes a high consumption of oxygen.

The high nitrification rate in cell 2.0 and 3.0 uses most of the ammonium in the cells, and the amount of ammonium going into the next cells is not enough to generate the same oxygen consumption as in these two cells. Because of this the oxygen level gets remarkably better in the other cells.

6.6.2 Ammonium

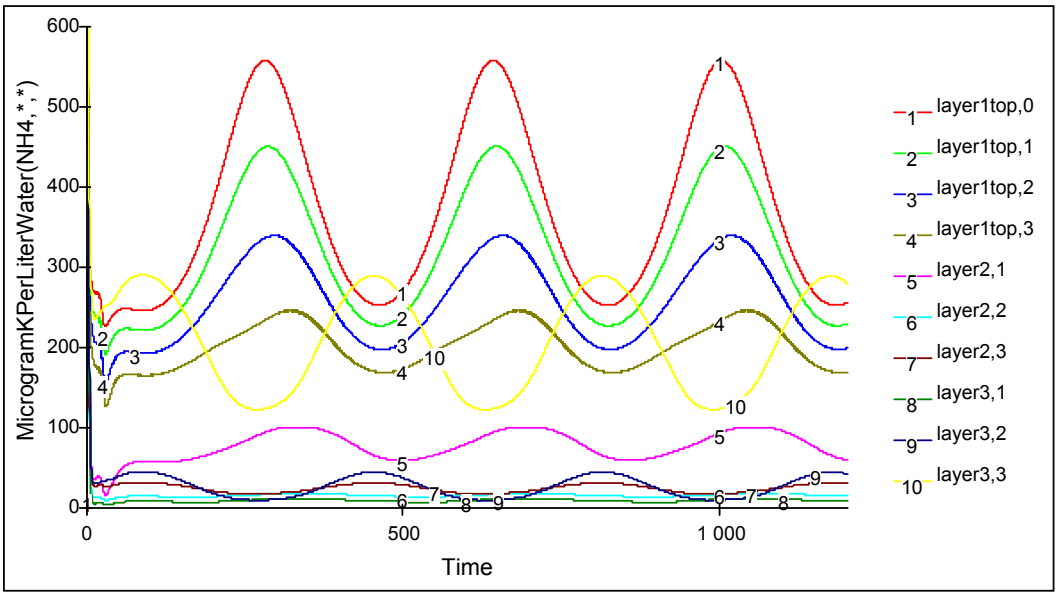


Graph 6-12: The ammonium level in the fjord with oscillating water exchange rate and river flow.

The graph shows the ammonium level in all water cells over a period of three year. Line number 5 and 6 represent the two innermost cells in layer 2 and 3. These show levels of ammonium far above the rest of the cells. This must be seen as the situation close to the supply points of ammonium in the two cells. If relatively much of the ammonium is consumed near these points, there will be a large gap form the level in these cells to the level in the surrounding cells. From this one can criticize the choice of size on the cells in the model. Making the cells smaller would reduce the gap between these two cells and the surrounding cells. The reason for not doing this is to reduce the size and complexity of the model. The consequence is that one has to compare these two innermost cells with the others with this in mind. For analysis of the behavior these two cell is gives a good view of the effect of the dominating and balancing loops because the effect is larger in these cells then in the others.

Graph 6-10 shows that the ammonium level increases in cell 2.0 and 3.0 when the water exchange rate decreases. This is because the ammonium supply to these cells is constant, and when the water exchange rate decreases, less of the ammonium in the cells are transported further

out in the fjord. The supply rates from the factories and from the diffusion from the bottom are larger than the rate whish is transporting it away.



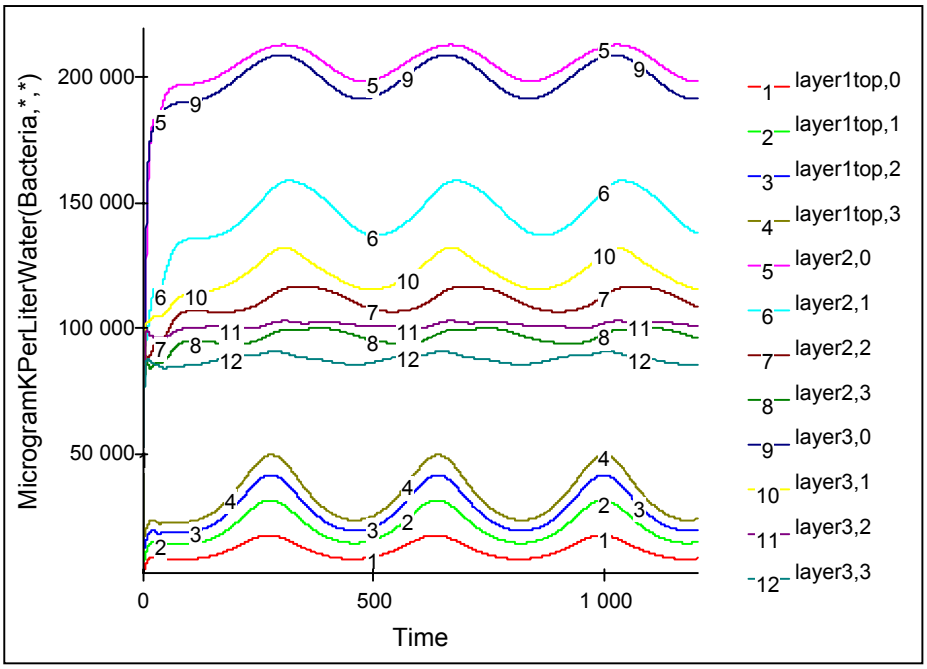
Graph 6-13: The ammonium level in the fjord except cell 2.0 and 3.0.

To get a closer look the ammonium level in the rest of the cells, cell 2.0 and 3.0 is left out in the graph. This shows that there are variations in how the ammonium levels develop during changes in the exchange rate. Line 1-4 shows the top-layers that have the same behavior as can be seen in cell 2.0 and 3.0. The ammonium supply rate from the river is constant independently from the freshwater supply rate because the ammonium comes from diffusion along the river, and is assumed not to be related to the water supply. A higher freshwater supply rate gives only a higher velocity in the top-layer outward the fjord and causes the ammonium concentration to decrease in the top-layer.

Line 9 and 10, which represent cell 3.2 and 3.3, shows an opposite behavior. The increase in the water exchange rate causes an increase in the ammonium level in these cells. Unlike the freshwater supply the incoming water from the outer parts of the fjord has a relatively constant concentration of ammonium. This means that an increase in the water exchange rate will give an increase in the supply rate of ammonium from the fjord. The effect decreases from cell 3.3 to 3.2 and almost all the ammonium is consumed before the incoming water reaches cell 3.1.

Line 5 represent cell 2.1 and is a neighbor to cell 2.0. The effect of the changes in the water exchange rate is reduced to just percents of what could be seen in cell 2.0 whit a high water exchange rate. One of the main reasons for this is that the oxygen supply also increases with the water exchange rate. Because of this most of the ammonium can be consumed in the first cell.

6.6.3 Bacteria



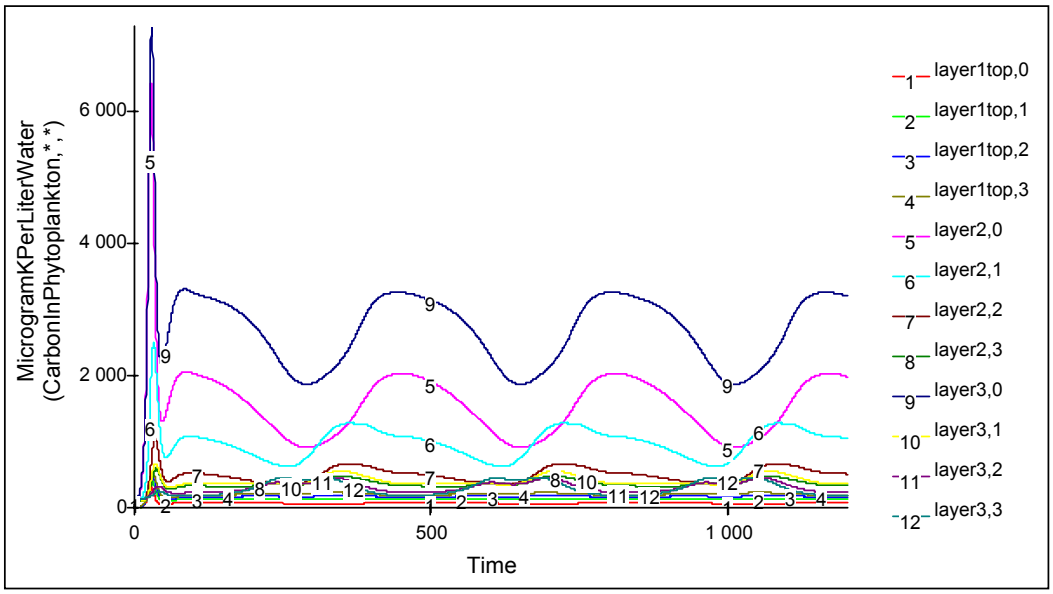
Graph 6-14: The bacteria level in the fjord.

The graph shows the bacteria level in the cells and the concentration is largest in the two innermost cells 2.0 and 3.0. Cell 2.1 also has a relatively high level of bacteria because the surplus of unconsumed ammonium from cell 2.0 is transported to cell 2.1. The level of bacteria fits the oscillations in the ammonium level and an increase in the ammonium level gives better conditions for the bacteria, and the level increases.

In the top-layer the bacteria level lays below the two underlying layers. This is because the freshwater does not contain these bacteria initially, while the incoming seawater to layer two

already contains a normal amount of bacteria. The growth of these bacteria find place when the freshwater mixes up with the seawater containing these bacteria. Because of this the bacteria level increase from the innermost cell 1.0 and out through the fjord.

6.6.4 Phytoplankton



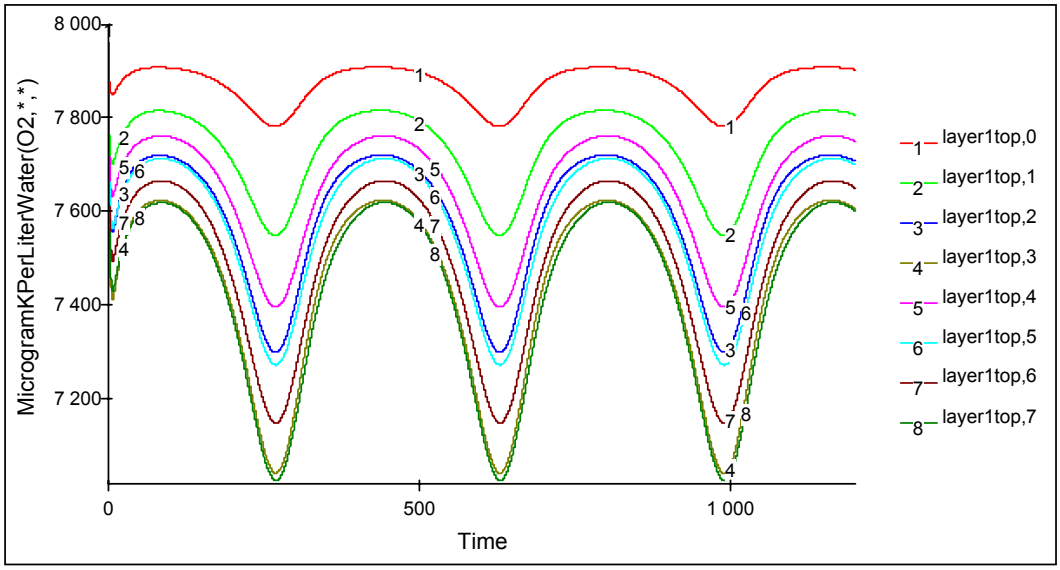
Graph 6-15: The phytoplankton level in the fjord.

The level of phytoplankton oscillates in correspondence with the water exchange rate. Only three of the cells contain phytoplankton above a normal level. These are the three cells near the supply points of ammonium: 3.0, 2.0 and 2.1. One of the reasons is the high availability of ammonium. Looking back at graph 6-11 one can see that the ammonium level is higher in all the cells in the top-layer than in cell 2.1, but the velocity of the water keeps the level down. The growth rate of phytoplankton is slower then for bacteria, so when the oxygen supply is high enough the bacteria can keep the phytoplankton level down. First when the bacteria growth is limited by oxygen, the phytoplankton can start growing.

6.7 Including Another River

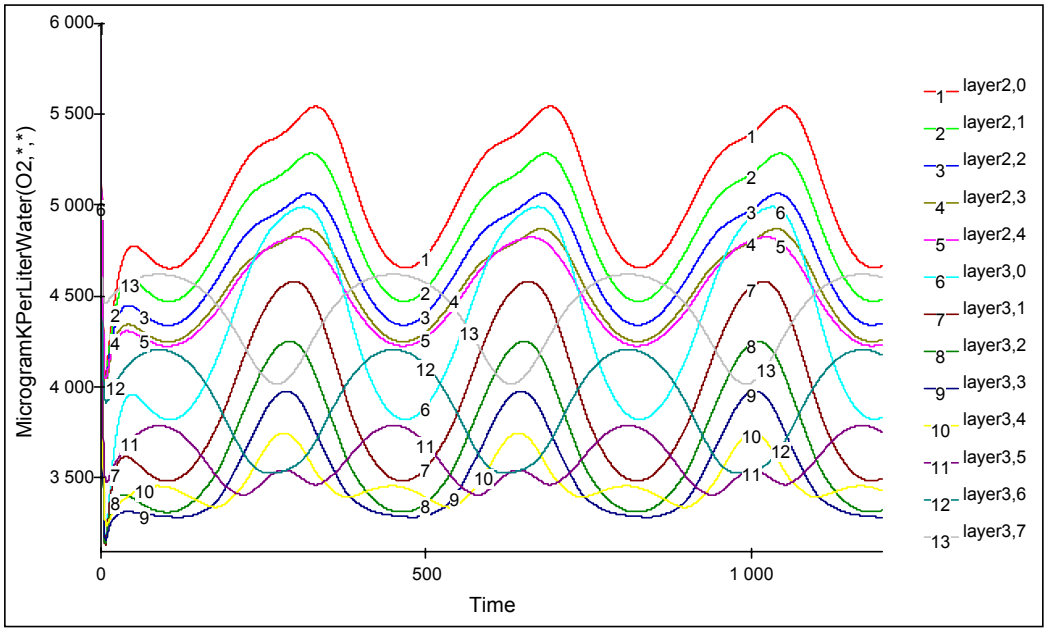
The Sørfjord has several small rivers supplying freshwater to the fjord. The hydrodynamic model initially contains only one supplying freshwater into cell 1.0. Based on the assumption that the freshwater supply is an important factor driving the incoming water exchange rate from the fjord in bottom-layer, these other rivers will also influence this rate. To see the effect of this, another freshwater supply source was included in the hydrodynamic model. The number of cell was doubled in the x-direction and the freshwater source was placed into cell 1.4. The freshwater supply was the same for both rivers, and because of this the water exchange rate coming in from the fjord was doubled. Since there are very few data about the hydrodynamic in the fjord, it is impossible to verify if this would be a correct assumption.

Now the supply of oxygen rich water was higher and the exchange rate was increased. One would expect that the conditions in the fjord became better, and that the gap between the innermost cells and the others was filled because of the increased mixing.



Graph 6-16: The oxygen level in layer 1.

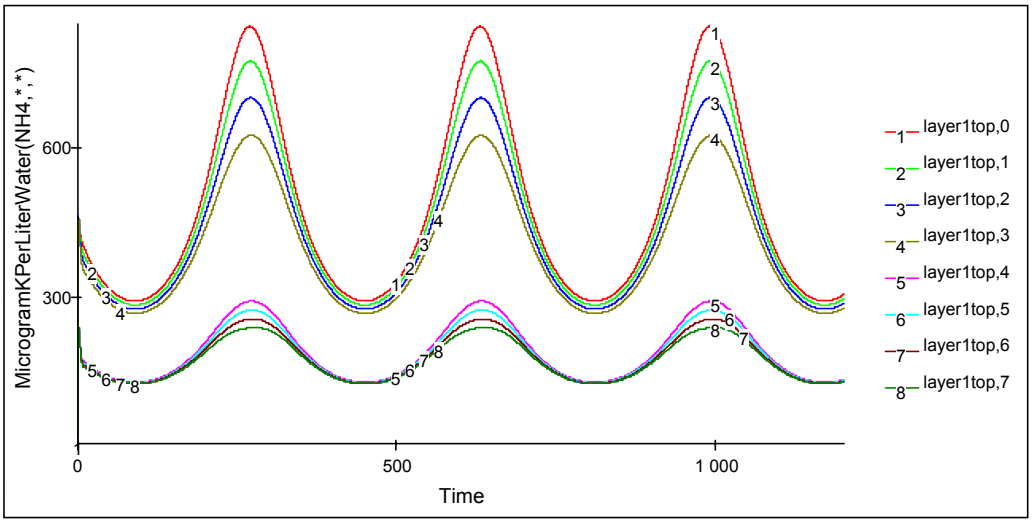
The graph shows the oxygen levels in the top-layer after including a second river. The lines 1 and 2 represent the levels in cell 1.0 and 1.1. They oscillated between 7900 –7800 microgram oxygen per liter. Below line 2 one finds line 5 that is the oxygen level in cell 1.4 where the new river comes in. It lies a little bit below the two other because the water from cell 1.3 (line 4) has a relatively low oxygen level, and it compensates for this. Compared to graph 6-9 where the oxygen level oscillates between 8000 and 6000, the oxygen level lies on a higher level in this new simulation. The reason for this can be found below in layer 2 and 3.



Graph 6-17: Oxygen levels in layer 2 and 3. (Except the cells 2.4, 2.5, 2.6 and 2.7)

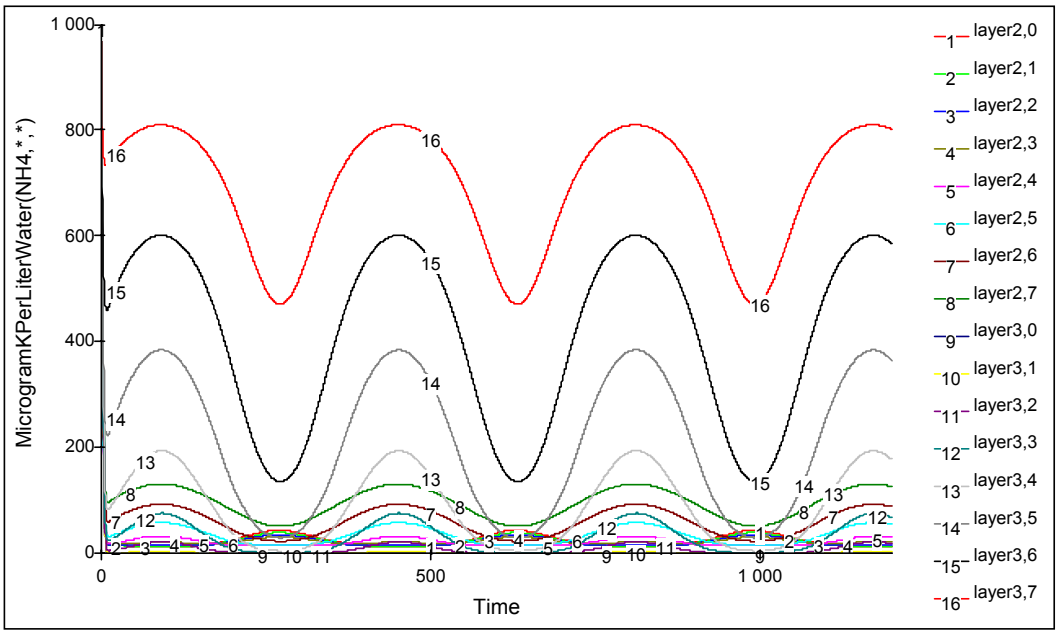
This graph shows the oxygen level in layer 2 and 3. The 4 outermost cells in layer 2 are left out for the graph. The oxygen levels in these 4 cells have small oscillations around 4200-4300.

In layer 2 and 3 the situation has changed from graph 6-9 where there were more stabile conditions and a large gap between the oxygen level in cell 2.0 and 3.0 and the rest of the cells. The oxygen level in cell 2.0 and 3.0 has increased from below 1000 microgram per liter to an oscillation between 5500 and 3800. The best oxygen level in the 2nd and 3rd layer is now found in cell 2.0. The increased velocity of the currents transports the more oxygen all the way into the innermost cells because less oxygen is consumed in the other cells. The increased water exchange rate also influence the ammonium concentration in the cell, and that is also an important factor.



Graph 6-18: The ammonium level in layer 1.

The graph shows the ammonium level in the top-layer. The ammonium level oscillates as the freshwater supply changes. A higher supply rate gives smaller concentrations of ammonium in the water as long as the ammonium supply is the same. There is a gap between cell 1.3 and 1.4 where the new water supply rate is placed. The water from the new river does not contain any ammonium, so the concentration of ammonium is reduced when the extra water is mixed with the water from cell 1.3. The small amount of ammonium in the outermost cells restricts the nitrification, so the reduction in the ammonium layer is small in these cells.



Graph 6-19: The ammonium level in layer 2 and 3.

Graph 6-17 shows the ammonium level in layer 2 and 3. When the water exchange rate increases, the ammonium level in the outermost cells in layer 3 increases. This is because the ammonium level in the incoming water from the fjord is constant. More water gives more ammonium, but it is also going faster out on the other side of the cell and into the next. The underlying reason is therefore the nitrification capacity in the cell. If consumption capacity of ammonium does not increase the same as the water exchange rate, the ammonium level will increase. During the period with a low exchange rate, most of the ammonium is consumed in the two first cells where the fjord water is coming in. (Cell 3.7 and 3.6)

In the innermost cells where the ammonium supply is located, the ammonium level is low. This is because there is enough oxygen to consume almost all the ammonium that is accumulation in the cells. The same happens in the next cells the ammonium is transported to.

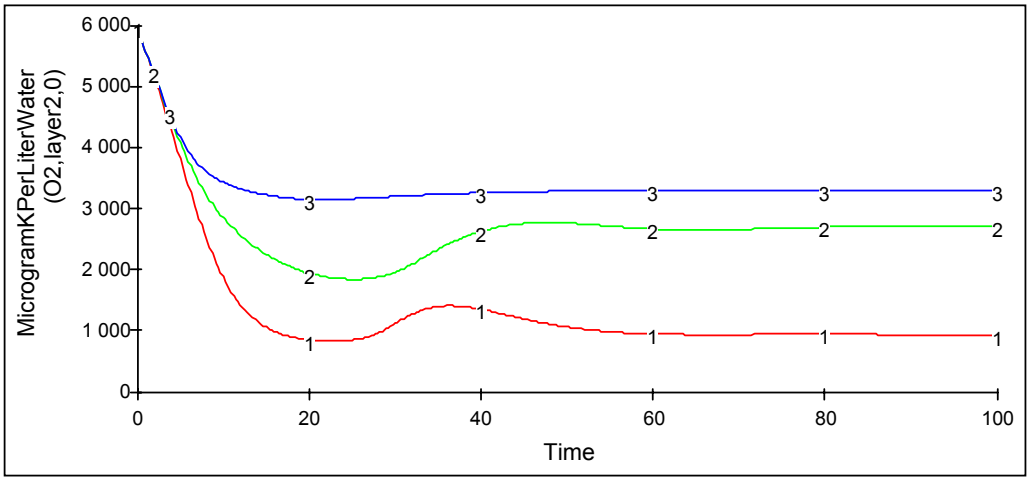
The ammonium level in the incoming fjord water may be a little bit too high compared to the situation in the Sørfjord, and it may cause a too high oxygen consumption in the outermost cells in the bottom layer.

6.8 Simulation With Changes in Nitrogen Supply

The water exchange rater is driven by factors that are impossible to control, and changing this it not an option if the goal is to change environmental stat of a fjord. In a fjord like the Sørfjord, the nitrogen supply is the factor that can be controlled by humans. In this simulation, the effect of changes in the nitrogen supply to the Sørfjord will be discussed.

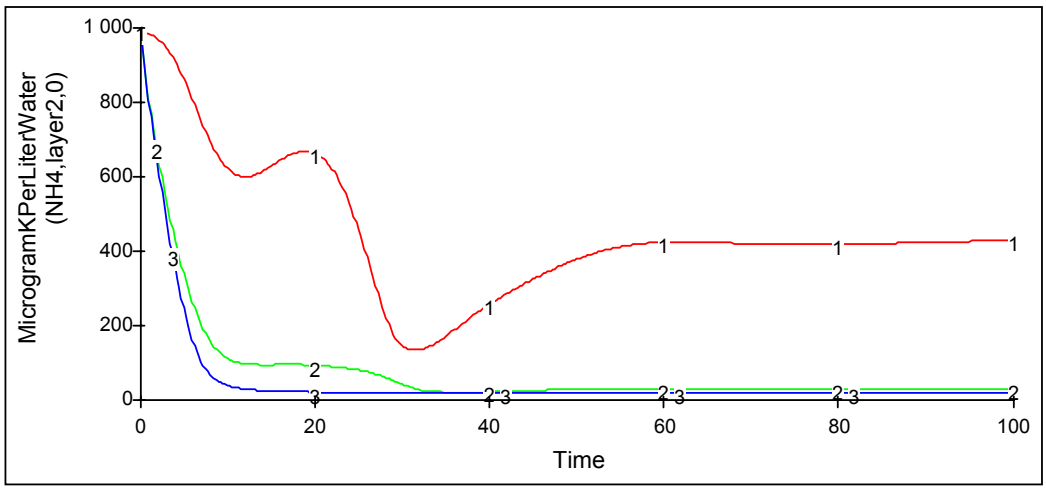
The supply of ammonium to the Sørfjord has three different sources. Of the total 900 tons of nitrogen, 30 % is supplied trough the river Opo, 30 % is coming from factories around the inner part of the fjord and the last 40 % is coming form diffusion from deposits on the bottom of the inner part of the fjord. These deposits are bi-products containing nitrogen dumped by the factories. The direct supply form the factories can be stopped, but the cost will be high. The supply from the river coming mostly from old deposits along the river and will be reduced over time. The diffusion from the deposits at the bottom will be stabile as long as the factories still are dumping the bi-products into the fjord. If this stops, the diffusion will decrease over time. (Garmann & Co, 1999).

The nitrogen supply is implemented as supply to the innermost cells in the three layers after this numbers (30, 30, 40). To see the effect of a reduction of this supply the simulation is done with reductions in the supply in the mid-layer to see the short time effect, and with reduction in both mid-layer and bottom layer to see the long time effect of a stop in supply of nitrogen.



Graph 6-20: The graph shows oxygen levels in cell 2.0 with three different nitrogen supply rates. Line 1 shows normal conditions, line 2 shows the level with a stop in supply in the mid-layer and line 3 shows the level with stop in supply in the mid-layer and a 50 % reduction of the diffusion from the bottom.

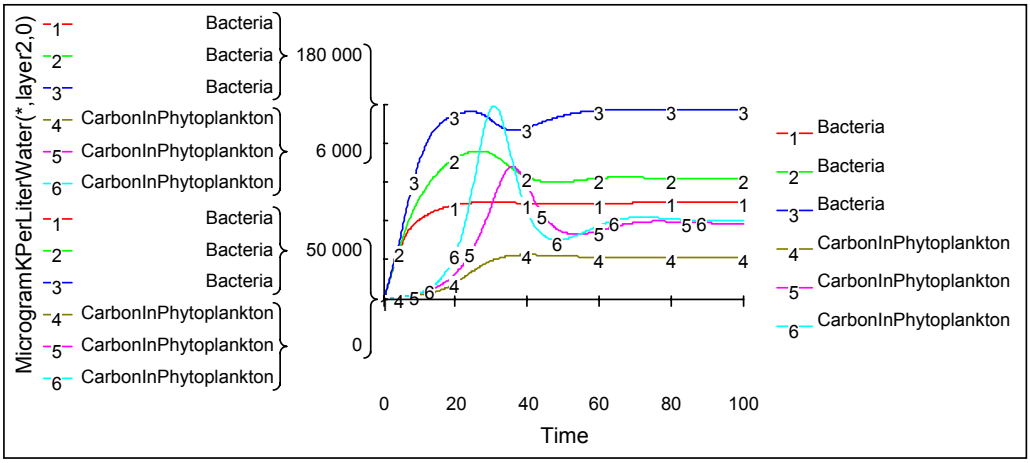
The graph shows that reduction in the supply of nitrogen has a great influence on the oxygen level. From the normal conditions (line 1) to the reduced diffusion in cell 3.0 and no supply to cell 2.0 (line 3) the oxygen level increases from a dramatically low level to an acceptable level. With oxygen levels around 1000 microgram per liter, it has a negative influences natural processes and limits biological growth. The oxygen situation can be categorized as very critical (Molvær, 1999). Under conditions with an oxygen level from 3500 microgram per liter and above, like line 3, the situation can be categorized as satisfactory. (For categorization of oxygen level, see chapter 2.1) Under such conditions the oxygen level is no longer a limiting factor for growth.



Graph 6-21: The ammonium levels for three different supply scenarios. (The same as for graph 6-13)

Under normal conditions (line 1) the ammonium level will stabilize at a relatively high level, and will not be a limiting factor for bacteria or phytoplankton growth. This causes a high nitrification rate by bacteria, and the oxygen level becomes low like graph 6-13 shows.

Line 2 and 3 in the graph shows the ammonium level when the supply from the factories is reduced. In both scenarios the ammonium level stabilize on a low level, and will be a limiting factor for nitrification and the phytoplankton growth rate.



Graph 6-22: Line 1, 2 and 3 is bacteria in cell 2.0 in different scenarios for nitrogen supply. Line 1 is the bacteria level when there is no supply to the mid-layer and 50 % reduction in the diffusion from the bottom layer. Line 2 is the bacteria level there is no supply in the mid-layer and no reduction in the bottom layer. Line 3 shows the bacteria level under normal conditions. Line 4, 5 and 6 shows the phytoplankton level for the same scenarios.

The bacteria level is stabilizing on a more moderate level when the nitrogen supply is reduced. As line 1 shows the bacteria level increases only until it reaches a stabile level. The same happens to the phytoplankton (line 4) under the same conditions. As the ammonium level indicated in graph 6-14, it reaches a bottom line already after 10-15 day. The growth of bacteria and phytoplankton decreases at the same time, and they stabilize at a level set by the lack of ammonium after a short time. It is the loop ‘B2: Ammonium consumption by phytoplankton’ that drives the increasing growth of phytoplankton. For bacteria it is the loop ‘B2: Ammonium nitrification’ that limits the growth.

7. Conclusion

The goal of the project was to identify the main variables and structures that influence the interaction between the nitrogen and oxygen levels in a fjord. This would be done through the development of a system dynamic model that generates the general behavior of the oxygen levels over time.

A biology model of the main variables concerning oxygen production and consumption was developed. This model was later merged with a simple hydrodynamic model of the main water movements, in order to represent the interaction within the biological system and the influence of the water exchange. The modeling process showed that the interaction between nitrogen and oxygen and the related variables is extremely complex. The system contains several feedback loops that influence the behavior of the oxygen levels.

7.1 Main Findings

The analysis of the model behavior supports the common theory of the close relationship between a large nitrogen supply and the low oxygen concentrations in the inner part of the Sørfjord. According to the simulations, a reduction in the nitrogen supply has a positive effect on the oxygen level. It may be reasonable to believe that the same results would be seen in the real fjord system, but there are too many uncertainties in the model to predict detailed results. The model is not intended to be used for point predictions, but to describe the general behavior of the oxygen levels, and the main feedback structure that generate them.

The model shows that the influence of the hydrodynamics on the biological conditions in the fjord is great. There is however, insufficient data of the hydrodynamics in the Sørfjord to develop a further specified hydrodynamic model representing more detailed flows of the water in the fjord. The model presented here may give insight in the complex interaction between the hydrodynamics and the biological system, but the generality of the simple hydrodynamic model does not allow for conclusions about the effect of future attempts to improve the oxygen levels.

7.2 Criticism of the Model

Several possible amendments are pointed out that may improve the model. No literature references were found regarding the nitrifying bacteria death rate and the variables related to this process. Further, the inclusion of a more specific equation representing the nitrification rate may have been advantageous. Temperature may influence the nitrification process, and no literature references were found with respect to the effect of the oxygen level on the nitrification rate.

Nitrate, which is the product of nitrified ammonium, may also have been appropriate to include in the model. This would not change the model behavior directly, but make it easier to compare the behavior of the nitrogen levels to the oxygen levels.

A main criticism of the structure of the model is that the return of substance to the water when phytoplankton and zooplankton die is not included. Instead of returning ammonium, phosphorus, and carbon to the respective levels in the model, they are merely outflows, leaving the system boundary. One of the aims in system dynamics is to locate the feedback structure in order to see how the loops interact and cause a certain behavior. By omitting the return of the substance, an important feedback loop may have been neglected, and mass balance is not maintained. Further, the oxygen consumption related to the decomposition of dead phytoplankton and zooplankton is also omitted. With respect to the structure of the model this means that both, factors that represent inflows to the ammonium level, and factors that represent outflows from the oxygen level are omitted. In a later version of the model the inclusion of these factors should be evaluated.

Whether the inclusion of the variables related to photosynthesis phytoplankton is adequate, may also be discussed. It may be that the situation in the Sørfjord is mainly influenced by the large ammonium supply, causing a large oxygen consumption, and that the change in the behavior of the oxygen levels is mainly caused by the water exchange rate. This however, requires further modeling, testing, and involvement of experts.

7.3 Experiences

Several experiences are gained during the work on the model, the simulations and the comparison to the data gathered from the fjord. The main observations are the following:

- The hydrodynamics play a major part for the behavior of the oxygen levels in the fjord. For the problem to be analyzed a model of both the biological system and the hydrodynamics is necessary.

- Available data are gathered by different sources, and the data gathering does not seem to be related to a model. This makes the interpretation of the data difficult.

- The Sørfjord model has weaknesses both when it comes to the representation of the biological system and the hydrodynamics of the fjord. If these weaknesses are to be revealed more data should be gathered. The modeling process and data gathering should be coordinated.

- The least developed structure of the model consists of the variables concerning the hydrodynamics. This is also the part where the least data is gathered from the real system.

This underlines the importance of developing a research agenda based on a common goal and cooperation in relation to improving the oxygen levels in the fjord. Collaboration between researchers and decision-makers is required, and data gathering based on a common model may be advantageous. When gathering and analyzing data it may be advantageous to think of the fjord as a feedback system of interrelated variables, in order to understand more about the processes that occur in the water, and how they are influenced by various factors. A common model may enhance this.

7.4 Further Research

Several issues may be discussed in relation to further work on the model. First, it may be appropriate to evaluate whether the inclusion of photosynthesis and phytoplankton is adequate. If it is, oxygen consumption and return of substance at phytoplankton and zooplankton death should be added to the model structure. Even if it does not represent a great change in the model behavior, part of the feedback structure that is believed to be in the real system is currently not included.

If phytoplankton is omitted, a more detailed nitrification model may be developed and integrated with a more specific hydrodynamic model. It seems that the hydrodynamics in the fjord have a large influence on the behavior of the system, and developing this part of the model further may therefore be adequate.

For further work on models that may improve system understanding and support decisions and strategies to improve the oxygen conditions in the Sørfjord, there is a need for more data and information about the hydrodynamics of the fjord. Without data to base a more specific hydrodynamic model on, an important factor may be neglected. Data of hydrodynamics requires extensive resources, but may increase the usefulness of the model.

It may also be possible to develop a more specific model, which with parameter changes based on data from other fjords, can be used to illustrate and possibly predict the behavior patterns of the oxygen levels in these other fjords as well. This may open for a comparison of the structure and behavior in the different fjords, and it may be possible to evaluate why the fjords exhibit certain behavior patterns based on the structure and parameters of the particular fjord.

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Appendix A. Equations

A.1 Equations in the Biology Model

A.1.1 Stocks in the Biology Model

BS1: init AmmoniumInPhytoplankton = (AmmoniumRedfield/CarbonRedfield)*InitialCarbon
flow AmmoniumInPhytoplankton = +dt*AmmoniumInPhytoplanktonGrowthRate
-dt*AmmoniumInPhytoplanktonReductionRate
doc AmmoniumInPhytoplankton = The amount of ammonium in phytoplankton.
unit AmmoniumInPhytoplankton = microgram

BS2: init AmmoniumInStock = AmmoniumInPhytoplankton*FractionStock
flow AmmoniumInStock = +dt*AmmoniumConsumptionRate
-dt*AmmoniumInPhytoplanktonGrowthRate
doc AmmoniumInStock = The amount of ammonium that is stored in phytoplankton, to be used for growth at a later stage.
unit AmmoniumInStock = microgram

BS3: init AmmoniumInWater = AmmoniumInPhytoplankton*10
flow AmmoniumInWater = -dt*AmmoniumNitrificationRate
+dt*AmmoniumSupplyRate
-dt*AmmoniumConsumptionRate
doc AmmoniumInWater = The amount of ammonium per liter of water.
unit AmmoniumInWater = microgram

BS4: init CarbonInPhytoplankton = InitialCarbon
flow CarbonInPhytoplankton = -dt*CarbonInPhytoplanktonReductionRate
+dt*CarbonInPhytoplanktonGrowthRate
doc CarbonInPhytoplankton = The amount of carbon in phytoplankton.
unit CarbonInPhytoplankton = microgram

BS5: init CarbonInWater = CarbonInPhytoplankton*100
flow CarbonInWater = +dt*CarbonSupplyRate
-dt*CarbonInPhytoplanktonGrowthRate
doc CarbonInWater = The amount of carbon per liter of water.
unit CarbonInWater = microgram

BS6: init Chlorophyll = InitialCarbon*RelationInitialCarbonAndChlorophyll
flow Chlorophyll = -dt*ChlorophyllReductionRate
+dt*ChlorophyllProductionRate
doc Chlorophyll = The amount of chlorophyll per liter of water. Chlorophyll is part of the phytoplankton cells.
unit Chlorophyll = microgram

BS7: init NitrifyingBacteria = 100000
flow NitrifyingBacteria = +dt*BacteriaProductionRate
-dt*BacteriaDeathRate
doc NitrifyingBacteria = The number of nitrifying bacteria per liter of water.
unit NitrifyingBacteria = cells

BS8: init Oxygen = 8000
flow Oxygen = +dt*OxygenSupplyRate
-dt*OxygenConsumptionRate
+dt*OxygenProductionRate
doc Oxygen = The amount of oxygen per liter of water.
unit Oxygen = microgram

BS9: init PhosphorusInPhytoplankton = (PhosphorusRedfield/CarbonRedfield)*InitialCarbon
flow PhosphorusInPhytoplankton = +dt*PhosphorusGrowthRate
-dt*PhophorusInPhytoplanktonReductionRate
doc PhosphorusInPhytoplankton = The amount of phosphorus in phytoplankton.
unit PhosphorusInPhytoplankton = microgram

BS10: init PhosphorusInStock = PhosphorusInPhytoplankton*FractionStock
flow PhosphorusInStock = -dt*PhosphorusGrowthRate
+dt*PhosphorusConsumptionRate
doc PhosphorusInStock = The amount of phosphorus that is stored in phytoplankton, to be used for growth at a later stage.
unit PhosphorusInStock = microgram

BS11: init PhosphorusInWater = PhosphorusInPhytoplankton*25
flow PhosphorusInWater = +dt*PhosphorusSupplyRate
-dt*PhosphorusConsumptionRate
doc PhosphorusInWater = The amount of phosphorus per liter of water.
unit PhosphorusInWater = microgram

BS12: init Zooplankton = 900
flow Zooplankton = -dt*ZooplanktonDeathRate
+dt*ZooplanktonGrowthRate
doc Zooplankton = The number of zooplankton per liter of water.
unit Zooplankton = cells

A.1.2 Auxiliaries in the Biology Model

BA1: aux AmmoniumConsumptionRate = InitiatedAmmoniumConsumption * CarbonInPhytoplanktonGrowthRate * EffectOfPhytoplanktonAndAmmoniumInWater * EffectOfRelativeAmmoniumOnConsumptionRate * RedfieldEffectOnAmmoniumGrowthRate
doc AmmoniumConsumptionRate = The amount of ammonium that is consumed from the water per time unit.
unit AmmoniumConsumptionRate = microgram/day

BA2: aux AmmoniumInPhytoplanktonGrowthRate = AmmoniumInPhytoplankton * PhytoplanktonGrowthRate * RedfieldEffectOnAmmoniumGrowthRate * EffectOfRelativeAmmoniumOnGrowthRate
doc AmmoniumInPhytoplanktonGrowthRate = The amount of ammonium that is assimilated into phytoplankton cells per time unit.
unit AmmoniumInPhytoplanktonGrowthRate = microgram/day

BA3: aux AmmoniumInPhytoplanktonReductionRate = (AmmoniumInPhytoplankton*NaturalReductionRate)+AmmoniumPredation
doc AmmoniumInPhytoplanktonReductionRate = The amount of ammonium in phytoplankton that is lost per time unit because the phytoplankton die.
unit AmmoniumInPhytoplanktonReductionRate = microgram/day

BA4: aux AmmoniumNitrificationRate = NitrificationRate+(HydrogenAddition*NitrificationRate)
doc AmmoniumNitrificationRate = The amount of ammonium that is nitrified into nitrate per time unit.
unit AmmoniumNitrificationRate = microgram/day

BA5: aux BacteriaDeathRate = NitrifyingBacteria DIVZ1 (MaxBacteriaLifeSpan*(AmmoniumInWater/(AmmoniumInWater+AmmoniumHalfSaturation))* (Oxygen/(Oxygen+OxygenHalfSaturation)))
doc BacteriaDeathRate = The number of nitrifying bacteria that dies per time unit.
unit BacteriaDeathRate = cells/day

BA6: aux BacteriaProductionRate = NitrificationRate*BacteriaCellYield
doc BacteriaProductionRate = The number of nitrifying bacteria that is formed per time unit.
unit BacteriaProductionRate = cells/day

BA7: aux CarbonInPhytoplanktonGrowthRate = CarbonInPhytoplankton*PhytoplanktonGrowthRate*EffectOfCarbonInPhytoplanktonAndWater
doc CarbonInPhytoplanktonGrowthRate = The amount of carbon that is assimilated into phytoplankton cells per liter per time unit.
unit CarbonInPhytoplanktonGrowthRate = microgram/day

BA8: aux CarbonInPhytoplanktonReductionRate = (CarbonInPhytoplankton*NaturalReductionRate)+PredationRate
doc CarbonInPhytoplanktonReductionRate = The amount of carbon in phytoplankton that is lost per time unit because the phytoplankton die.
unit CarbonInPhytoplanktonReductionRate = microgram/day

BA9: aux CarbonSupplyRate = CarbonInPhytoplanktonGrowthRate*0+100000
doc CarbonSupplyRate = The amount of carbon that flows into one liter of water per time unit.
unit CarbonSupplyRate = microgram/day

BA10: aux ChlorophyllProductionRate = CarbonInPhytoplanktonGrowthRate*OptimalLightUtilization

doc ChlorophyllProductionRate = The amount of chlorophyll that is produced by phytoplankton per time unit.
unit ChlorophyllProductionRate = microgram/day

BA11: aux ChlorophyllReductionRate = IF(Chlorophyll>CarbonInPhytoplanktonReductionRate*LightUtilization,CarbonInPhytoplanktonReductionRate*LightUtilization,Chlorophyll)
doc ChlorophyllReductionRate = The amount of chlorophyll in phytoplankton that is lost per time unit because the phytoplankton die.
unit ChlorophyllReductionRate = microgram/day

BA12: aux OxygenConsumptionRate = NitrificationRate*OxygenPerNitrification
doc OxygenConsumptionRate = The amount of oxygen that is used in the nitrification process per time unit.
unit OxygenConsumptionRate = microgram/day

BA13: aux OxygenProductionRate = PhotosyntheticQuotient*(CarbonInPhytoplankton/Chlorophyll)*ChlorophyllProductionRate
doc OxygenProductionRate = The amount of oxygen that is produced by phytoplankton in the photosynthesis process per time unit.
unit OxygenProductionRate = microgram/day

BA14: aux PhophorusInPhytoplanktonReductionRate = (PhosphorusInPhytoplankton*NaturalReductionRate)+PhosphorusPredation
doc PhophorusInPhytoplanktonReductionRate = The amount of phosphorus in phytoplankton that is lost per time unit because the phytoplankton die.
unit PhophorusInPhytoplanktonReductionRate = microgram/day

BA15: aux PhosphorusConsumptionRate = InitiatedPhosphorousConsumption*CarbonInPhytoplanktonGrowthRate*EffectOfPhytoplanktonAndPhosphorusInWater*EffectOfRelativePhosphorusOnConsumptionRate*RedfieldEffectOnPhosphorusGrowthRate
doc PhosphorusConsumptionRate = The amount of phosphorus that is consumed from the water per time unit.
unit PhosphorusConsumptionRate = microgram/day

BA16: aux PhosphorusGrowthRate = PhosphorusInPhytoplankton*PhytoplanktonGrowthRate*EffectOfRelativePhosphorusOnGrowthRate*RedfieldEffectOnPhosphorusGrowthRate
doc PhosphorusGrowthRate = The amount of phosphorus that is assimilated into phytoplankton cells per time unit.
unit PhosphorusGrowthRate = microgram/day

BA17: aux ZooplanktonDeathRate = Zooplankton/ZooplanktonLifespan
doc ZooplanktonDeathRate = The number of zooplankton that dies per time unit.
unit ZooplanktonDeathRate = cells/day

BA18: aux ZooplanktonGrowthRate = Zooplankton*CarbonInPhytoplankton*PredatorCoefficient
doc ZooplanktonGrowthRate = The number of zooplankton that is formed per time unit.
unit ZooplanktonGrowthRate = cells/day

BA19: aux AmmoniumPredation = PredationRate*FractionAmmonuimInPhytoplankton
doc AmmoniumPredation = The rate at which ammonium in phytoplankton is consumed by zooplankton.
unit AmmoniumPredation = microgram/day

BA20: aux AmmoniumRedfieldCheck = AmmoniumInPhytoplankton/(CarbonInPhytoplankton+AmmoniumInPhytoplankton+PhosphorusInPhytoplankton)*100
doc AmmoniumRedfieldCheck = The relative amount of ammonium in phytoplankton compared to carbon and phosphorus content (Redfield).
unit AmmoniumRedfieldCheck = dimensionless

BA21: aux CarbonInPhytoplanktonHalfSaturationFunction = CarbonInPhytoplankton/(CarbonInPhytoplanktonHalfSaturation+CarbonInPhytoplankton)
doc CarbonInPhytoplanktonHalfSaturationFunction = Compares the actual amount of phytoplankton to the half saturation concentration, and results in death rate as a function of the phytoplankton population.
unit CarbonInPhytoplanktonHalfSaturationFunction = dimensionless

BA22: aux CarbonRedfieldCheck = CarbonInPhytoplankton/(CarbonInPhytoplankton+PhosphorusInPhytoplankton+AmmoniumInPhytoplankton)*100
doc CarbonRedfieldCheck = The relative amount of carbon in the phytoplankton compared to ammonium and phosphorus content (Redfield).
unit CarbonRedfieldCheck = dimensionless

BA23: aux ConsumptionCapacityLimitedByLightAndAmmoniumInWater = MaxAmmoniumConsumption*(AmmoniumInWater/(AmmoniumHalfStaurationConcentration+AmmoniumInWater))*(LightCoefficient0+LightCoefficient1*LightLimitation)
doc ConsumptionCapacityLimitedByLightAndAmmoniumInWater = The amount of ammonium that the phytoplankton is capable of consuming per time unit, limited by the concentration of ammonium in water and light.
unit ConsumptionCapacityLimitedByLightAndAmmoniumInWater = 1/day

BA24: aux ConsumptionCapacityLimitedByLightAndPhosphorusInWater = MaxPhosphorusConsumption*(PhosphorusInWater/(PhosphorusHalfSaturationConcentration+PhosphorusInWater))*(LightCoefficient0+LightCoefficient1*LightLimitation)
doc ConsumptionCapacityLimitedByLightAndPhosphorusInWater = The amount of phosphorus that the phytoplankton is capable of consuming per time unit, limited by the concentration of phosphorus in water and light.
unit ConsumptionCapacityLimitedByLightAndPhosphorusInWater = 1/day

BA25: aux CriticalLightIntensity = (LightAdaptionCoefficient^0.6)*(Light^0.4)
doc CriticalLightIntensity = Dynamic critical light intensity. Marks the transition between growth limited by light and temperature.
unit CriticalLightIntensity = W/m2

BA26: aux DeathAsAFunctionOfTemperature = DeathRateAt20C*EXP(TemperatureCoefficient*(Temperature-MaxTemperature))
doc DeathAsAFunctionOfTemperature = The phytoplankton death rate caused by temperature.
unit DeathAsAFunctionOfTemperature = 1/day

BA27: aux EffectOfCarbonInPhytoplanktonAndWater = IF(EffectOfRelativeCarbonInWater=1,1,EffectOfRelativeCarbonAsPhytoplankton*EffectOfRelativeCarbonInWater)
doc EffectOfCarbonInPhytoplanktonAndWater = The effect of the carbon in phytoplankton and carbon in water. If the relative carbon in water is 1, there is a surplus of carbon, which means that carbon in water is not a constraint, and the effect will therefore be 1. If it is lower, there is

not enough carbon, and the effect of carbon in water will be multiplied by the effect of the relative carbon in phytoplankton.

unit EffectOfCarbonInPhytoplanktonAndWater = dimensionless

BA28: aux EffectOfPhytoplanktonAndAmmoniumInWater = IF(EffectOfRelativeAmmoniumInWater=1,1,EffectOfRelativeCarbonAsPhytoplankton*EffectOfRelativeAmmoniumInWater)

doc EffectOfPhytoplanktonAndAmmoniumInWater = The effect of the carbon in phytoplankton and ammonium in water. If the relative ammonium in water is 1, there is a surplus of ammonium, which means that ammonium in water is not a constraint, and the effect will therefore be 1. If it is lower, there is not enough ammonium, and the effect of ammonium in water will be multiplied by the effect of the relative carbon in phytoplankton.

unit EffectOfPhytoplanktonAndAmmoniumInWater = dimensionless

BA29: aux EffectOfPhytoplanktonAndPhosphorusInWater = IF(EffectOfRelativePhosphorusInWater=1,1,EffectOfRelativeCarbonAsPhytoplankton*EffectOfRelativePhosphorusInWater)

doc EffectOfPhytoplanktonAndPhosphorusInWater = The effect of carbon in phytoplankton and phosphorus in water. If the relative phosphorus in water is 1, there is a surplus of phosphorus, which means that phosphorus in water is not a constraint, and the effect will therefore be 1. If it is lower, there is not enough phosphorus, and the effect of phosphorus in water will be multiplied by the effect of the relative carbon in phytoplankton.

unit EffectOfPhytoplanktonAndPhosphorusInWater = dimensionless

BA30: aux EffectOfRelativeAmmoniumInWater = GRAPH(RelativeAmmoniumInWater,0,0.1,[1,1,0.97,0.93,0.85,0.72,0.44,0.18,0.06,0.01,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeAmmoniumInWater = The effect of the relative amount of ammonium in the water. As the ammonium level in the water goes down, the relative amount of ammonium in water will go towards 1, the effect of the relative ammonium level will go towards 0, because the phytoplankton are not be able to use all the ammonium in the water.

unit EffectOfRelativeAmmoniumInWater = dimensionless

BA31: aux EffectOfRelativeAmmoniumOnConsumptionRate = GRAPH(RelativeAmmoniumStock,0.1,0.01,[1,0.98,0.89,0.79,0.66,0.54,0.36,0.17,0.07,0.03,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeAmmoniumOnConsumptionRate = Regulates the consumption rate so that the phytoplankton reduce their ammonium consumption when the amount of ammonium in stock is higher than 10 percent of the ammonium in phytoplankton cells.

unit EffectOfRelativeAmmoniumOnConsumptionRate = dimensionless

BA32: aux EffectOfRelativeAmmoniumOnGrowthRate = GRAPH(RelativeAmmoniumStock,0,0.01,[0,0.02,0.04,0.08,0.34,0.69,0.86,0.97,0.99,1,1"Min:0;Max:1;Zoom"])

doc EffectOfRelativeAmmoniumOnGrowthRate = The effect of the amount of ammonium in stock compared to ammonium in phytoplankton. As the stock goes towards 10 percent of what is in phytoplankton, the effect approaches 1, and will be 1 for any percentage higher than 10. When the relation is lower than 10 percent, there will be a constraint on the growth, going towards 0 as the percentage goes towards 0.

unit EffectOfRelativeAmmoniumOnGrowthRate = dimensionless

BA33: aux EffectOfRelativeCarbonAsPhytoplankton = GRAPH(RelativeCarbonAsPhytoplankton,0,0.1,[1,1,0.99,0.97,0.83,0.64,0.4,0.13,0.05,0.02,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeCarbonAsPhytoplankton = The effect of the relative amount of carbon in phytoplankton. As the carbon level in the phytoplankton goes down, the relative amount of carbon in phytoplankton will go towards 1, the effect of relative carbon as phytoplankton will go towards 0, because the phytoplankton are not able to find all the carbon in the water if both carbon and phytoplankton are in scarcity.

unit EffectOfRelativeCarbonAsPhytoplankton = dimensionless

BA34: aux EffectOfRelativeCarbonInWater = GRAPH(RelativeCarbonInWater,0,0.1,[1,1,0.99,0.97,0.85,0.75,0.58,0.21,0.09,0.04,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeCarbonInWater = The effect of the relative amount of carbon in the water. As the carbon level in the water goes down, the relative amount of carbon in water will go towards 1. The effect of the relative amount of carbon in the water will then go towards 0, because the phytoplankton are not able to use all the carbon in the water.

unit EffectOfRelativeCarbonInWater = dimensionless

BA35: aux EffectOfRelativePhosphorusInWater = GRAPH(RelativePhosphorusInWater,0,0.1,[1,0.96,0.92,0.8,0.61,0.29,0.17,0.11,0.07,0.05,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativePhosphorusInWater = The effect of the relative amount of phosphorus in the water. As the phosphorus level in the water goes down, the relative amount of phosphorus in water will go towards 1, the effect of relative phosphorus will go towards 0, because the phytoplankton are not able to use all the phosphorus in the water.

unit EffectOfRelativePhosphorusInWater = dimensionless

BA36: aux EffectOfRelativePhosphorusOnConsumptionRate = GRAPH(RelativePhosphorusInStock,0,0.01,[1,1,1,1,1,1,1,1,1,1,1,0.97,0.92,0.75,0.58,0.44,0.23,0.14,0.08,0,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativePhosphorusOnConsumptionRate = Regulates the consumption rate so that the phytoplankton reduce their phosphorus consumption when the amount of phosphorus in stock is higher than 10 percent of the phosphorus in the phytoplankton cells.

unit EffectOfRelativePhosphorusOnConsumptionRate = dimensionless

BA37: aux EffectOfRelativePhosphorusOnGrowthRate = GRAPH(RelativePhosphorusInStock,0,0.01,[0,0.06,0.37,0.53,0.66,0.81,0.95,0.97,0.98,0.99,1"Min:0;Max:1;Zoom"])

doc EffectOfRelativePhosphorusOnGrowthRate = The effect of the amount of phosphorus in stock compared to phosphorus in phytoplankton. As the stock goes towards 10 percent of what is in phytoplankton, the effect approaches 1, and will be 1 for any percentage higher than 10. When the relation is lower than 10 percent, there will be a constraint on the growth, going towards 0 as the percentage goes towards 0.

unit EffectOfRelativePhosphorusOnGrowthRate = dimensionless

BA38: aux FractionAmmonuimInPhytoplankton = AmmoniumInPhytoplankton DIVZ0 CarbonInPhytoplankton

doc FractionAmmonuimInPhytoplankton = The amount of ammonium in phytoplankton relative to the amount of carbon.

unit FractionAmmonuimInPhytoplankton = dimensionless

BA39: aux FractionPhosphorusInPhytoplankton = MAX(PhosphorusInPhytoplankton DIVZ0 CarbonInPhytoplankton,0.0027)

doc FractionPhosphorusInPhytoplankton = The amount of phosphorus in phytoplankton relative to the amount of carbon.

unit FractionPhosphorusInPhytoplankton = dimensionless

BA40: aux InitiatedAmmoniumConsumption = MAX(0,MIN(ConsumptionCapacityLimitedByLightAndAmmoniumInWater,OptimalAmmoniumConsumption))
doc InitiatedAmmoniumConsumption = The initiated ammonium consumption per microgram carbon assimilated into phytoplankton.
unit InitiatedAmmoniumConsumption = 1/day

BA41: aux InitiatedPhosphorousConsumption = MAX(0,MIN(ConsumptionCapacityLimitedByLightAndPhosphorusInWater,OptimalPhosphorusConsumption))
doc InitiatedPhosphorousConsumption = The initiated phosphorus consumption per microgram carbon assimilated into phytoplankton.
unit InitiatedPhosphorousConsumption = 1/day

BA42: aux LightLimitation = Light/(((CriticalLightIntensity^LightCoefficient)+Light^LightCoefficient)^(1/LightCoefficient))
doc LightLimitation = Limitation on the phytoplankton growth rate due to the light intensity.
unit LightLimitation = dimensionless

BA43: aux LightUtilization = TemperatureLimitedGrowthRate/CriticalLightIntensity
doc LightUtilization = The capability of phytoplankton to utilize the light, dependent on the critical light intensity and the temperature limitation.
unit LightUtilization = 1/day

BA44: aux MaxPhosphorousInPhytoplankton = (1/41)*PhosphorusLuxus
doc MaxPhosphorousInPhytoplankton = The maximum amount of phosphorus compared to carbon that phytoplankton can contain (Redfield)
unit MaxPhosphorousInPhytoplankton = dimensionless

BA45: aux NaturalReductionRate = Respiration+PhytoplanktonDeathRate
doc NaturalReductionRate = The amount per microgram phytoplankton that is lost per time unit due to natural factors such as temperature, respiration and nutrient level in the water.
unit NaturalReductionRate = 1/day

BA46: aux NitrificationRate = AmmoniumPerBacteria*NitrifyingBacteria*MaxNitrificationRate*(AmmoniumInWater/(AmmoniumInWater+AmmoniumHalfSaturation))*(Oxygen/(Oxygen+OxygenHalfSaturation))
doc NitrificationRate = The amount of nitrogen that is nitrified per liter per day.
unit NitrificationRate = microgram/day

BA47: aux NutrientEffectOnDeathRate = 1-(1-NutrientCoefficient)*NutrientLimitation
doc NutrientEffectOnDeathRate = The effect of the nutrient level in phytoplankton on the death rate.
unit NutrientEffectOnDeathRate = dimensionless

BA48: aux NutrientLimitation = 1/(1+(1/RelativeNitrogenSurplus)+(1/RelativePhosphorousSurplus))
doc NutrientLimitation = Limitation on phytoplankton growth rate caused by the nutrient level in the phytoplankton cells.
unit NutrientLimitation = dimensionless

BA49: aux OptimalAmmoniumConsumption = MaxAmmoniumInPhytoplankton+(MaxAmmoniumInPhytoplankton-FractionAmmonuimInPhytoplankton)

doc OptimalAmmoniumConsumption = The optimal consumption of ammonium from the water, limited by the actual growth rate, temperature and the level of ammonium in phytoplankton compared to carbon.
unit OptimalAmmoniumConsumption = dimensionless

BA50: aux OptimalLightUtilization = TemperatureLimitedGrowthRate/(MAX(MinimumLightIntensity,CriticalLightIntensity))
doc OptimalLightUtilization = Light utilization by phytoplankton when the adjustment to the actual light intensity is optimal.
unit OptimalLightUtilization = 1/day

BA51: aux OptimalPhosphorusConsumption = (TemperatureLimitedGrowthRate*(MaxPhosphorousInPhytoplankton-FractionPhosphorusInPhytoplankton)+PhytoplanktonGrowthRate*FractionPhosphorusInPhytoplankton)
doc OptimalPhosphorusConsumption = The optimal consumption of phosphorus from the water, limited by the actual growth rate, temperature and the level of Phosphorous in phytoplankton compared to Carbon.
unit OptimalPhosphorusConsumption = 1/day

BA52: aux PhosphorusPredation = PredationRate*FractionPhosphorusInPhytoplankton
doc PhosphorusPredation = The rate at which phosphorus in phytoplankton is consumed by zooplankton.
unit PhosphorusPredation = microgram/day

BA53: aux PhosphorusRedfieldCheck = PhosphorusInPhytoplankton/(CarbonInPhytoplankton+AmmoniumInPhytoplankton+PhosphorusInPhytoplankton)*100
doc PhosphorusRedfieldCheck = The relative amount of phosphorus in phytoplankton compared to ammonium and carbon content (Redfield).
unit PhosphorusRedfieldCheck = dimensionless

BA54: aux PhytoplanktonDeathRate = DeathAsAFunctionOfTemperature*CarbonInPhytoplanktonHalfSaturationFunction*NutrientEffectOnDeathRate
doc PhytoplanktonDeathRate = The phytoplankton death rate controlled by temperature, nutrient level in the water, and the amount of phytoplankton.
unit PhytoplanktonDeathRate = 1/day

BA55: aux PhytoplanktonGrowthRate = TemperatureAndNutrientLimitedGrowthRate*LightLimitation
doc PhytoplanktonGrowthRate = Phytoplankton growth rate limited by temperature, light and the relative nitrogen and phosphorus content compared to carbon in phytoplankton.
unit PhytoplanktonGrowthRate = 1/day

BA56: aux PredationRate = CarbonInPhytoplankton*Zooplankton*PredationCoefficient
doc PredationRate = The rate at which phytoplankton is consumed by zooplankton.
unit PredationRate = microgram/day

BA57: aux RedfieldEffectOnAmmoniumGrowthRate = GRAPH(FractionAmmonuimInPhytoplankton,0,0.005,[1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.398,1.395,1.395,1.395,1.395,1.395,1.389,1.381,1.36,1.333,1.311,1.286,1,1,1"Min:1;Max:1.4;Zoom"])
doc RedfieldEffectOnAmmoniumGrowthRate = Makes the phytoplankton go towards Redfield by increasing the consumption rate and the growth rate when the ammonium content is lower than Redfield, and decreasing them if they are higher than Redfield.

unit RedfieldEffectOnAmmoniumGrowthRate = dimensionless

BA58: aux RedfieldEffectOnPhosphorusGrowthRate = GRAPH(FractionPhosphorusInPhytoplankton,0,0.002,[1.4,1.4,1.4,1.4,1.395,1.382,1.372,1.354, 1.337,1.318,1.293,1.258,1.002,1,1"Min:1;Max:1.4;Zoom"])
doc RedfieldEffectOnPhosphorusGrowthRate = Makes the phytoplankton go towards Redfield by increasing the consumption rate and the growth rate when the phosphorus content is lower than Redfield, and decreasing them if they are higher than Redfield.
unit RedfieldEffectOnPhosphorusGrowthRate = dimensionless

BA59: aux RelativeAmmoniumInWater = MinAmmoniumInWater DIVZ0 AmmoniumInWater
doc RelativeAmmoniumInWater = The fraction of the minimum amount of ammonium in the water compared to the actual ammonium level in the water.
unit RelativeAmmoniumInWater = dimensionless

BA60: aux RelativeAmmoniumStock = AmmoniumInStock DIVZ0 AmmoniumInPhytoplankton
doc RelativeAmmoniumStock = The fraction of ammonium in stock compared to ammonium in phytoplankton.
unit RelativeAmmoniumStock = dimensionless

BA61: aux RelativeCarbonAsPhytoplankton = MinCarbonAsPhytoplankton/CarbonInPhytoplankton
doc RelativeCarbonAsPhytoplankton = The fraction of the minimum amount of carbon in phytoplankton compared to the actual level of carbon in phytoplankton.
unit RelativeCarbonAsPhytoplankton = dimensionless

BA62: aux RelativeCarbonInWater = MinCarbonInWater/CarbonInWater
doc RelativeCarbonInWater = The fraction of the minimum amount of carbon compared to the actual carbon level in the water.
unit RelativeCarbonInWater = dimensionless

BA63: aux RelativeNitrogenSurplus = (FractionAmmonuimInPhytoplankton-LowerLimitNitrogenComparedToCarbon)/LowerLimitNitrogenComparedToCarbon
doc RelativeNitrogenSurplus = The relative surplus of nitrogen in phytoplankton compared to carbon.
unit RelativeNitrogenSurplus = dimensionless

BA64: aux RelativePhosphorousSurplus = (FractionPhosphorusInPhytoplankton-LowerLimitPhosphorousComparedToCarbon)/LowerLimitPhosphorousComparedToCarbon
doc RelativePhosphorousSurplus = The relative surplus of phosphorus in phytoplankton compared to carbon.
unit RelativePhosphorousSurplus = dimensionless

BA65: aux RelativePhosphorusInStock = PhosphorusInStock/PhosphorusInPhytoplankton
doc RelativePhosphorusInStock = The fraction of phosphorus in stock compared to phosphorus in phytoplankton.
unit RelativePhosphorusInStock = dimensionless

BA66: aux RelativePhosphorusInWater = PhosphorusHalfSaturationConcentration / PhosphorusInWater
doc RelativePhosphorusInWater = The fraction of the minimum amount of phosphorus in the water compared to the actual phosphorus level in the water.
unit RelativePhosphorusInWater = dimensionless

BA67: aux Respiration = RespirationAt20C*EXP(TemperatureCoefficient*(Temperature-MaxTemperature))
doc Respiration = The amount of phytoplankton that is lost due to respiration per time unit.
unit Respiration = 1/day

BA68: aux TemperatureAndNutrientLimitedGrowthRate = NutrientLimitation*TemperatureLimitedGrowthRate
doc TemperatureAndNutrientLimitedGrowthRate = The phytoplankton growth rate limited by temperature and the relative ammonium and phosphorus content in phytoplankton compared to carbon.
unit TemperatureAndNutrientLimitedGrowthRate = 1/day

BA69: aux TemperatureLimitedGrowthRate = GrowthRate20C* EXP(TemperatureCoefficient*(Temperature-MaxTemperature))
doc TemperatureLimitedGrowthRate = The phytoplankton growth rate limited by temperature.
unit TemperatureLimitedGrowthRate = 1/day

A.1.3 Constants in the Biology Model

BC1: const AmmoniumSupplyRate = 800
doc AmmoniumSupplyRate = The amount of ammonium that flows into one liter of water per time unit.
unit AmmoniumSupplyRate = microgram/day

BC2: const OxygenSupplyRate = 400
doc OxygenSupplyRate = The amount of oxygen that flows into one liter of water per time unit.
unit OxygenSupplyRate = microgram/day

BC3: const PhosphorusSupplyRate = 300
doc PhosphorusSupplyRate = The amount of phosphorus that flows into one liter of water per time unit.
unit PhosphorusSupplyRate = microgram/day

BC4: const AmmoniumHalfSaturation = 3.6
doc AmmoniumHalfSaturation = 1.8-3.6
The half saturation concentration for ammonium. The concentration that results in half of maximum nitrification rate and phytoplankton growth rate.
unit AmmoniumHalfSaturation = microgram

BC5: const AmmoniumHalfStaurationConcentration = 7
doc AmmoniumHalfStaurationConcentration = The level of ammonium in the water that would result in half of the maximum phytoplankton growth.
unit AmmoniumHalfStaurationConcentration = microgram

BC6: const AmmoniumPerBacteria = 2.700966*10^-5
doc AmmoniumPerBacteria = The amount of ammonium that is nitrified by each bacteria per time unit.
unit AmmoniumPerBacteria = microgram

BC7: const AmmoniumRedfield = 7.2
doc AmmoniumRedfield = The amount of ammonium compared to carbon and phosphorus, if phosphorus is 1 and carbon 41.
unit AmmoniumRedfield = dimensionless

BC7: const BacteriaCellYield = 71.39
doc BacteriaCellYield = The number of cells that are produced per microgram nitrogen that is nitrified.
unit BacteriaCellYield = cells/microgram

BC8: const CarbonInPhytoplanktonHalfSaturation = 7
doc CarbonInPhytoplanktonHalfSaturation = Half saturation value for carbon in phytoplankton. The amount of phytoplankton per liter of water that results in half of the maximum death rate caused by the amount of carbon in phytoplankton.
unit CarbonInPhytoplanktonHalfSaturation = microgram/liter

BC9: const CarbonRedfield = 41
doc CarbonRedfield = The amount of carbon compared to ammonium and phosphorus, if phosphorus is 1 and ammonium 7.2.
unit CarbonRedfield = dimensionless

BC10: const DeathRateAt20C = 0.1

doc DeathRateAt20C = The death rate caused by temperature when the temperature is 20 degrees C.
unit DeathRateAt20C = 1/day

BC11: const FractionStock = 0.1
doc FractionStock = The initial value of the phytoplankton nutrient stock compared to the value in the phytoplankton cells.
unit FractionStock = dimensionless

BC12: const GrowthRate20C = 1.4
doc GrowthRate20C = The growth rate per microgram carbon in phytoplankton at 20 degrees C.
unit GrowthRate20C = 1/day

BC13: const HydrogenAddition = 0.28784
doc HydrogenAddition = The fraction that is added because of the weight of the hydrogen atoms in ammonium in order to calculate how much ammonium is nitrified compared to nitrogen.
unit HydrogenAddition = dimensionless

BC14: const InitialCarbon = 4000
doc InitialCarbon = The initial amount of carbon per liter of water.
unit InitialCarbon = microgram

BC15: const Light = 75
doc Light = The actual light intensity in the water.
unit Light = W/m2

BC16: const LightAdaptionCoefficient = 75
doc LightAdaptionCoefficient = Coefficient for light adaption ability by phytoplankton.
unit LightAdaptionCoefficient = dimensionless

BC17: const LightCoefficient = 3
doc LightCoefficient = Light coefficient for the ability of phytoplankton to adapt to changing light intensities.
unit LightCoefficient = dimensionless

BC18: const LightCoefficient0 = 0.5
doc LightCoefficient0 = A dimensionless coefficient that combine growth and consumption.
unit LightCoefficient0 = dimensionless

BC19: const LightCoefficient1 = 0.5
doc LightCoefficient1 = A dimensionless coefficient that combine growth and consumption.
unit LightCoefficient1 = dimensionless

BC20: const LowerLimitNitrogenComparedToCarbon = 0.06
doc LowerLimitNitrogenComparedToCarbon = The lowest relative amount of nitrogen compared to carbon half saturation that is possible in phytoplankton.
unit LowerLimitNitrogenComparedToCarbon = dimensionless

BC21: const LowerLimitPhosphorousComparedToCarbon = 0.0027
doc LowerLimitPhosphorousComparedToCarbon = The lowest amount of phosphorus compared to carbon half saturation that is possible in phytoplankton.
unit LowerLimitPhosphorousComparedToCarbon = dimensionless

BC22: const MaxAmmoniumConsumption = 0.6

doc MaxAmmoniumConsumption = The maximum amount of ammonium that can be consumed by each unit of phytoplankton (measured in carbon) per time unit.
unit MaxAmmoniumConsumption = 1/day

BC23: const MaxAmmoniumInPhytoplankton = 7.2/41
doc MaxAmmoniumInPhytoplankton = The maximum amount of ammonium in phytoplankton compared to carbon (Redfield).
unit MaxAmmoniumInPhytoplankton = dimensionless

BC24: const MaxBacteriaLifeSpan = 12
doc MaxBacteriaLifeSpan = The maximum number of days a bacteria can live.
unit MaxBacteriaLifeSpan = days

BC25: const MaxNitrificationRate = 100
doc MaxNitrificationRate = 2.4 - 240 (per day)
The maximum nitrification rate per time unit.
unit MaxNitrificationRate = 1/day

BC26: const MaxPhosphorusConsumption = 0.13
doc MaxPhosphorusConsumption = The maximum amount of phosphorus that can be consumed by each unit of phytoplankton (measured in carbon) per time unit.
unit MaxPhosphorusConsumption = 1/day

BC27: const MaxTemperature = 20
doc MaxTemperature = The temperature that would give the maximum phytoplankton growth if there were no other limits to growth.
unit MaxTemperature = degC

BC28: const MinAmmoniumInWater = 70
doc MinAmmoniumInWater = The minimum amount of ammonium in water. At a lower level ammonium becomes a constraint.
unit MinAmmoniumInWater = microgram

BC29: const MinCarbonAsPhytoplankton = 1
doc MinCarbonAsPhytoplankton = The minimum amount of carbon in phytoplankton per liter of water.
unit MinCarbonAsPhytoplankton = microgram

BC30: const MinCarbonInWater = 40
doc MinCarbonInWater = The minimum amount of carbon in water.
unit MinCarbonInWater = microgram

BC31: const MinimumLightIntensity = 5
doc MinimumLightIntensity = The minimum light intensity for phytoplankton growth.
unit MinimumLightIntensity = W/m2

BC32: const NutrientCoefficient = 0.5
doc NutrientCoefficient = Coefficient that states the effect that nutrient limitation has on the phytoplankton death rate.
unit NutrientCoefficient = dimensionless

BC33: const OxygenHalfSaturation = 1000
doc OxygenHalfSaturation = The oxygen concentration in the water that would result in half of the maximum nitrification rate.
unit OxygenHalfSaturation = microgram

BC34: const OxygenPerNitrification = 3.42679
doc OxygenPerNitrification = The amount of oxygen that is nitrified per microgram nitrogen that is nitrified.
unit OxygenPerNitrification = dimensionless

BC35: const PhosphorusHalfSaturationConcentration = 30
doc PhosphorusHalfSaturationConcentration = The level of phosphorus in the water that would result in half of the maximum phytoplankton growth.
unit PhosphorusHalfSaturationConcentration = microgram

BC36: const PhosphorusLuxus = 1.1
doc PhosphorusLuxus = Phosphorus luxury consumption. The amount of phosphorus the phytoplankton can consume in addition to Redfield.
unit PhosphorusLuxus = dimensionless

BC37: const PhosphorusRedfield = 1
doc PhosphorusRedfield = The amount of phosphorus compared to carbon, if the weight of carbon is 41 (Redfield).
unit PhosphorusRedfield = dimensionless

BC38: const PhotosyntheticQuotient = 1.0
doc PhotosyntheticQuotient = 1.0-1.3
The amount of oxygen produced per microgram chlorophyll that is formed.
unit PhotosyntheticQuotient = dimensionless

BC39: const PredationCoefficient = 0.00009
doc PredationCoefficient = Coefficient that states the effect the number of zooplankton and phytoplankton has on phytoplankton growth rate.
unit PredationCoefficient = 1/day

BC40: const PredatorCoefficient = 0.0002
doc PredatorCoefficient = Coefficient that states the effect the number of zooplankton and phytoplankton has on zooplankton growth rate.
unit PredatorCoefficient = 1/day

BC41: const RelationInitialCarbonAndChlorophyll = 0.025
doc RelationInitialCarbonAndChlorophyll = The initial relative amount of chlorophyll compared to carbon in phytoplankton.
unit RelationInitialCarbonAndChlorophyll = dimensionless

BC42: const RespirationAt20C = 0.04
doc RespirationAt20C = The phytoplankton respiration at 20 degrees C.
unit RespirationAt20C = 1/day

BC43: const Temperature = 14
doc Temperature = The actual temperature in the water.
unit Temperature = degC

BC44: const TemperatureCoefficient = 0.063
doc TemperatureCoefficient = Temperature coefficient states the effect of variation in temperature.
unit TemperatureCoefficient = 1/degC

BC45: const ZooplanktonLifespan = 8
doc ZooplanktonLifespan = The zooplankton life span.

```
unit        ZooplanktonLifespan = days
```

A.2 Equations in the Hydrodynamic Model

A.2.1 Stocks in the Hydrodynamic Model

```
HS1:    dim     MassPerComponent = (k=component,d=depth, x=xdir)
        init    MassPerComponent = Volume(d,x)*CompositionFjord(k,d,x)*ComponentWeight(k)*1000
        flow    MassPerComponent = -dt*(MixingUp(k,d,x) )
                -dt*(MixingIn(k,d,x) )
                -dt*OutFlowAll
                +dt*(InFlowAll(k,d,x))
                -dt*(OutgoingFlowIn(k,d,x) |  x <> FIRST(X) ;0 )
                +dt*(OutgoingFlowIn(k,d,x+1) | x<LAST(xdir) AND d=layer3;0)
                -dt*(OutgoingFlowOut(k,d,x) )
                +dt*(OutgoingFlowOut(k,d,x-1) | d<LAST(depth) AND x>FIRST(xdir);0)
                -dt*(InOutBalance(k,d,x) | d=layer3;0)
                +dt*(InOutBalance(k,d+1,x) |  d=layer2;0)
                -dt*(MixingOut(k,d,x) )
                +dt*(MixingOut(k,d,x-1) | x > FIRST(xdir);0)
                +dt*(MixingIn(k,d,x+1)  | x<LAST(xdir);0)
                +dt*(MixingUp(k,d+1,x) | d < LAST(depth);0)
                -dt*(MixingDown(k,d,x) )
                +dt*(MixingDown(k,d-1,x)  | d>FIRST(depth);0)
                +dt*(OuterExchange(k,d) | x=LAST(xdir) AND d=LAST(depth) ;0)
                +dt*(River(k,d,x) )
        doc     MassPerComponent = The mass of each component within each fjord cell
        unit    MassPerComponent = kg

HS2:    dim     Volume = (d=depth, x=xdir)
        init    Volume = INIT(Width(x)*DD(d)*DX(x))
        flow    Volume =
        unit    Volume = m3
```

A.2.2 Auxiliaries in the Hydrodynamic Model

HA1: dim AmmoniumConsumptionRate = (d=depth, x=xdir)
aux AmmoniumConsumptionRate = InitiatedAmmoniumConsumption(d,x)*CarbonInPhytoplanktonGrowthRate(d,x)*EffectOfPhytoplanktonAndAmmoniumInWater(d,x)*EffectOfRelativeAmmoniumOnConsumptionRate(d,x)*RedfieldEffectOnAmmoniumGrowthRate(d,x)
doc AmmoniumConsumptionRate = The amount of ammonium that is consumed from the water per time unit.
unit AmmoniumConsumptionRate = kg/day

HA2: dim AmmoniumInPhytoplanktonGrowthRate = (d=depth, x=xdir)
aux AmmoniumInPhytoplanktonGrowthRate = MassPerComponent(NH4inPhytoplankton,d,x)*PhytoplanktonGrowthRate(d,x)*RedfieldEffectOnAmmoniumGrowthRate(d,x)*EffectOfRelativeAmmoniumOnGrowthRate(d,x)
doc AmmoniumInPhytoplanktonGrowthRate = The amount of ammonium that is assimilated into phytoplankton cells per liter per time unit.
unit AmmoniumInPhytoplanktonGrowthRate = kg/day

HA3: dim AmmoniumInPhytoplanktonReductionRate = (d=depth, x=xdir)
aux AmmoniumInPhytoplanktonReductionRate = (MassPerComponent(NH4inPhytoplankton,d,x)*NaturalReductionRate(d,x))+AmmoniumPredation(d,x)
doc AmmoniumInPhytoplanktonReductionRate = The amount of ammonium in phytoplankton that is lost per time unit because the phytoplankton die.
unit AmmoniumInPhytoplanktonReductionRate = kg/day

HA4: dim BacteriaDeathRate = (d=depth, x=xdir)
aux BacteriaDeathRate = MassPerComponent(Bacteria,d,x)/(MaxBacteriaLifeSpan DIVZ1 ((MassPerComponent(NH4,d,x)/(MassPerComponent(NH4,d,x)+AmmoniumHalfSaturation(d,x)))*(MassPerComponent(O2,d,x)/(MassPerComponent(O2,d,x)+OxygenHalfSaturation(d,x)))))
doc BacteriaDeathRate = The number of nitrifying bacteria that dies per time unit.
unit BacteriaDeathRate = cells/day

HA5: dim BacteriaProductionRate = (d=depth, x=xdir)
aux BacteriaProductionRate = (NitrificationRate(d,x)*BacteriaCellYield)
doc BacteriaProductionRate = The number of bacteria that is being formed per time unit.
unit BacteriaProductionRate = cells/day

HA6: dim CarbonInPhytoplanktonGrowthRate = (d=depth, x=xdir)
aux CarbonInPhytoplanktonGrowthRate = MassPerComponent(CarbonInPhytoplankton,d,x)*PhytoplanktonGrowthRate(d,x)*EffectOfCarbonInPhytoplanktonAndWater(d,x)
doc CarbonInPhytoplanktonGrowthRate = The amount of carbon that is being assimilated into phytoplankton cells per time unit.
unit CarbonInPhytoplanktonGrowthRate = kg/day

HA7: dim CarbonInPhytoplanktonReductionRate = (d=depth, x=xdir)
aux CarbonInPhytoplanktonReductionRate = PredationRate(d,x)+(NaturalReductionRate(d,x)*MassPerComponent(CarbonInPhytoplankton,d,x))
doc CarbonInPhytoplanktonReductionRate = The amount of carbon in phytoplankton that is lost each time unit because the phytoplankton die.
unit CarbonInPhytoplanktonReductionRate = kg/day

HA8: dim ChlorophyllProductionRate = (d=depth, x=xdir)
aux ChlorophyllProductionRate = CarbonInPhytoplanktonGrowthRate(d,x)*OptimalLightUtilization(d,x)
doc ChlorophyllProductionRate = The amount of chlorophyll that is produced by phytoplankton per time unit.
unit ChlorophyllProductionRate = kg/day

HA9: dim ChlorophyllReductionRate = (d=depth, x=xdir)
aux ChlorophyllReductionRate = IF(MassPerComponent (Chlorophyll,d,x) > CarbonInPhytoplanktonReductionRate(d,x)*LightUtilization(d,x), CarbonInPhytoplanktonReductionRate(d,x)*LightUtilization (d,x),MassPerComponent (Chlorophyll,d,x))
doc ChlorophyllReductionRate = The amount of chlorophyll in phytoplankton that is lost per time unit because the phytoplankton die.
unit ChlorophyllReductionRate = kg/day

HA10: dim InFlowAll = (k=component,d=depth, x=xdir)
aux InFlowAll = BacteriaProductionRate(d,x) | k=Bacteria;
ZooplanktonGrowthRate(d,x) | k=Zooplankton;
CarbonInPhytoplanktonGrowthRate(d,x) | k= CarbonInPhytoplankton;
ChlorophyllProductionRate(d,x) | k= Chlorophyll;
PhosphorusGrowthRate(d,x) | k= PhosphorusInPhytoplankton;
PhosphorusConsumptionRate(d,x) | k=PhosphorusInStock;
AmmoniumConsumptionRate(d,x) | k=NH4InStock;
AmmoniumInPhytoplanktonGrowthRate(d,x) | k=NH4inPhytoplankton;
O2ProductionRate(d,x) | k=O2;
0
doc InFlowAll = The total inflow rates that are caused by the biological system.
unit InFlowAll = kg/day

HA11: dim InOutBalance = (k=component,d=depth, x=xdir)
aux InOutBalance = RelativeAmountOfComponent(k,d,x)*WaterExchange/(INDEX(LAST(xdir)) - INDEX(FIRST(xdir)) +1) | d=layer3;0
doc InOutBalance = This is the upgoing flow from the bottom layer. The flow equals a fractional part of WaterExchange where the fraction is 1/no_of_elements in the x direction.
unit InOutBalance = kg/day

HA12: dim MixingDown = (k=component,d=depth, x=xdir)
aux MixingDown = MixingFactor(k)*AreaDebthDirection(x)*RelativeAmountOfComponent(k,d,x)*ComponentWeight (k) | d <LAST(depth);0
doc MixingDown = The diffusion from a volume to its neighbor volume in the positive x direction. The diffusion (in kg/day) is proportional to the relative concentration of the respective components (dimensionless), the component weight (kg/liter), the diffusion area in m2. This gives the MixingFactor The dimension of 1000*meter/day i.e. The dimension of velocity.
unit MixingDown = kg/day

HA13: dim MixingIn = (k=component,d=depth, x=xdir)
aux MixingIn = MixingFactor(k)*AreaXDirection(d,x)*RelativeAmountOfComponent(k,d,x)*ComponentWeight(k) | x > FIRST(x);0
doc MixingIn = The diffusion from a volume to its neighbor volume in the positive x direction.

```
        The diffusion (in kg/day) is proportional to the relative concentration of the respective
        components (dimensionless), the component weight (kg/liter), the diffusion area in m2. This
        gives the MixingFactor The dimension of 1000*meter/day i.e. The dimension of velocity.
    unit MixingIn = kg/day

HA14: dim MixingOut = (k=component,d=depth, x=xdir)
    aux MixingOut =
        MixingFactor(k)*AreaXDirection(d,x)*RelativeAmountOfComponent(k,d,x)*ComponentWeight(k)
        | x < LAST(xdir);0
    doc MixingOut = The diffusion from a volume to its neighbor volume in the positive x direction.
        The diffusion (in kg/day) is proportional to the relative concentration of the respective
        components (dimensionless), the component weight (kg/liter), the diffusion area in m2. This
        gives the MixingFactor The dimension of 1000*meter/day i.e. The dimension of velocity.
    unit MixingOut = kg/day

HA15: dim MixingUp = (k=component,d=depth, x=xdir)
    aux MixingUp =
        MixingFactor(k)*AreaDebthDirection(x)*RelativeAmountOfComponent(k,d,x)*ComponentWeight
        (k) | d >FIRST(depth);0
    doc MixingUp = The diffusion from a volume to its neighbor volume in the positive x direction.
        The diffusion (in kg/day) is proportional to the relative concentration of the respective
        components (dimensionless), the component weight (kg/liter), the diffusion area in m2. This
        gives the MixingFactor The dimension of 1000*meter/day i.e. The dimension of velocity.
    unit MixingUp = kg/day

HA16: dim OuterExchange = (k=component,d=depth)
    aux OuterExchange = WaterExchange*CompositionOcean(k,d)*ComponentWeight(k) | d=layer3;0
    doc OuterExchange = The inward flow at bottom layer from outer fjord
    unit OuterExchange = kg/day

HA17: dim OutFlowAll = (k=component,d=depth, x=xdir)
    aux OutFlowAll = OxygenConsumptionRate(d,x) | k=O2;
        AmmoniumNitrificationRate(d,x)+AmmoniumConsumptionRate(d,x) | k=NH4;
        BacteriaDeathRate(d,x) | k=Bacteria;
        ZooplanktonDeathRate(d,x) | k=Zooplankton;
        CarbonInPhytoplanktonReductionRate(d,x) | k=CarbonInPhytoplankton;
        ChlorophyllReductionRate(d,x) | k=Chlorophyll;
        PhosphorusGrowthRate(d,x) | k= PhosphorusInStock;
        PhosphorusConsumptionRate(d,x) | k=Phosphorus;
        PhophorusInPhytoplanktonReductionRate(d,x) | k=PhosphorusInPhytoplankton;
        AmmoniumInPhytoplanktonGrowthRate(d,x) | k=NH4InStock;
        AmmoniumInPhytoplanktonReductionRate(d,x) | k=NH4inPhytoplankton;
        0
    doc OutFlowAll = The total outflow rates that are caused by the biological system.
    unit OutFlowAll = kg/day

HA18: dim OutgoingFlowIn = (k=component,d=depth, x=xdir)
    aux OutgoingFlowIn = (RelativeAmountOfComponent(k,d,x)*
        WaterExchange/(INDEX(LAST(xdir)) - INDEX(FIRST(xdir)) +1))*
        SUM(j=xdir;IF(INDEX(x) > INDEX(j),1,0)) | d=layer3;
        0
    doc OutgoingFlowIn = This is the inward flow at the bottom layer. The flow decreases linearly from
        the mouth of the fjord and towards the bottom.
    unit OutgoingFlowIn = kg/day
```

```
HA19:  dim   OutgoingFlowOut = (k=component,d=depth, x=xdir)
       aux   OutgoingFlowOut = RiverFlow*RelativeAmountOfComponent(k,d,x) | d=layer1top;
             (RelativeAmountOfComponent(k,d,x)*
             WaterExchange/(INDEX(LAST(xdir)) - INDEX(FIRST(xdir)) +1))*
             SUM(j=xdir;IF(INDEX(x) >= INDEX(j),1,0)) | d=layer2;
             0
       doc   OutgoingFlowOut = This is the outgoing flow at the middle layer. The flow decreases linearly
             from the mouth of the fjord and towards the bottom.
       unit  OutgoingFlowOut = kg/day

HA20:  dim   PhophorusInPhytoplanktonReductionRate = (d=depth, x=xdir)
       aux   PhophorusInPhytoplanktonReductionRate =
             (MassPerComponent(PhosphorusInPhytoplankton,d,x)*NaturalReductionRate(d,x))+Phosphoru
             sPredation(d,x)
       doc   PhophorusInPhytoplanktonReductionRate = The amount of phosphorous in phytoplankton that
             is lost per time unit because the phytoplankton die.
       unit  PhophorusInPhytoplanktonReductionRate = kg/day

HA21:  dim   PhosphorusConsumptionRate = (d=depth, x=xdir)
       aux   PhosphorusConsumptionRate =
             InitiatedPhosphorousConsumption(d,x)*CarbonInPhytoplanktonGrowthRate(d,x)*EffectOfPhyto
             plankton AndPhosphorusInWater(d,x)*EffectOfRelativePhosphorusOnConsumptionRate(d,x)*Re
             dfieldEffectOnPhosphorusGrowthRate(d,x)
       doc   PhosphorusConsumptionRate = The amount of phosphorous that is consumed from the water
             per time unit.
       unit  PhosphorusConsumptionRate = kg/day

HA22:  dim   PhosphorusGrowthRate = (d=depth, x=xdir)
       aux   PhosphorusGrowthRate =
             MassPerComponent(PhosphorusInPhytoplankton,d,x)*PhytoplanktonGrowthRate(d,x)*EffectOf
             RelativePhosphorusOnGrowthRate(d,x)*RedfieldEffectOnPhosphorusGrowthRate(d,x)
       doc   PhosphorusGrowthRate = The amount of phosphorous that is assimilated into phytoplankton
             cells per time unit.
       unit  PhosphorusGrowthRate = kg/day

HA23:  dim   River = (k=component,d=depth, x=xdir)
       aux   River = RiverFlow*ComponentWeight(k)*CompositionRiver(k) | d=FIRST(depth) AND
             x=FIRST(xdir) AND k<>NH4;
             NH4Supply*NH4Apportionment(d) |  k=NH4  AND x=FIRST(xdir) ;
             0
       doc   River = 50 M3/sec from river gives
             50 * 3600 * 24 * 1000 kg per day
       unit  River = kg/day

HA24:  dim   AmmoniumHalfSaturation = (d=depth, x=xdir)
       aux   AmmoniumHalfSaturation = 300*(10^-9)*Volume(d,x)*1000*ComponentWeight(NH4)
       doc   AmmoniumHalfSaturation = 1.8-3.6
             The half saturation concentration for ammonium. The concentration that results in half of
             maximum nitrification rate and phytoplankton growth rate.
       unit  AmmoniumHalfSaturation = kg

HA25:  dim   AmmoniumHalfStaurationConcentration = (d=depth, x=xdir)
       aux   AmmoniumHalfStaurationConcentration = 7*(10^-
             9)*Volume(d,x)*1000*ComponentWeight(NH4)
```

doc AmmoniumHalfStaurationConcentration = The level of ammonium in the water that would result in half of the maximum phytoplankton growth.
unit AmmoniumHalfStaurationConcentration = kg

HA26: dim AmmoniumNitrificationRate = (d=depth, x=xdir)
aux AmmoniumNitrificationRate = NitrificationRate(d,x)+(HydrogenAddition*NitrificationRate(d,x))
doc AmmoniumNitrificationRate = The amount of ammonium that is nitrified into nitrate per time unit.
unit AmmoniumNitrificationRate = kg/day

HA27: dim AmmoniumPredation = (d=depth, x=xdir)
aux AmmoniumPredation = PredationRate(d,x)*FractionAmmoniumInPhytoplankton(d,x)
doc AmmoniumPredation = The rate at which ammonium in phytoplankton is consumed by zooplankton.
unit AmmoniumPredation = kg/day

HA28: dim AmmoniumRedfieldCheck = (k=component,d=depth, x=xdir)
aux AmmoniumRedfieldCheck = MassPerComponent(NH4inPhytoplankton,d,x)/(MassPerComponent(NH4inPhytoplankton,d,x)+MassPerComponent(CarbonInPhytoplankton,d,x)+MassPerComponent(PhosphorusInPhytoplankton,d,x))
doc AmmoniumRedfieldCheck = The relative amount of ammonium in phytoplankton compared to carbon and phosphorus content (Redfield).
unit AmmoniumRedfieldCheck = dimensionless

HA29: dim AreaDebthDirection = (x=xdir)
aux AreaDebthDirection = Width(x)*DX(x)
doc AreaDebthDirection = Area of volume element, in depth direction
unit AreaDebthDirection = m2

HA30: dim AreaXDirection = (d=depth, x=xdir)
aux AreaXDirection = Width(x)*DD(d)
doc AreaXDirection = The area of the volume rectangle in the x-direction
unit AreaXDirection = m2

HA31: dim CarbonInPhytoplanktonHalfSaturation = (d=depth, x=xdir)
aux CarbonInPhytoplanktonHalfSaturation = 7*(10^-9)*Volume(d,x)*ComponentWeight(CarbonInPhytoplankton)*1000
doc CarbonInPhytoplanktonHalfSaturation = Half saturation value for carbon in phytoplankton. The amount of phytoplankton per liter of water that results in half of the maximum death rate caused by the amount of carbon in phytoplankton.
unit CarbonInPhytoplanktonHalfSaturation = kg

HA32: dim CarbonInPhytoplanktonHalfSaturationFunction = (d=depth, x=xdir)
aux CarbonInPhytoplanktonHalfSaturationFunction = MassPerComponent(CarbonInPhytoplankton,d,x)/(CarbonInPhytoplanktonHalfSaturation(d,x)+MassPerComponent(CarbonInPhytoplankton,d,x))
doc CarbonInPhytoplanktonHalfSaturationFunction = Compares the actual amount of phytoplankton to the half saturation concentration, and results in death rate as a function of the phytoplankton population.
unit CarbonInPhytoplanktonHalfSaturationFunction = dimensionless

HA33: dim CarbonRedfieldCheck = (k=component,d=depth, x=xdir)
aux CarbonRedfieldCheck = MassPerComponent(CarbonInPhytoplankton,d,x)/(MassPerComponent(CarbonInPhytoplankton

```
          ,d,x)+MassPerComponent(NH4inPhytoplankton,d,x)+MassPerComponent(PhosphorusInPhytop
          lankton,d,x))
     doc  CarbonRedfieldCheck = The relative amount of carbon in phytoplankton compared to
          ammonium and phosphorus content (Redfield).
     unit CarbonRedfieldCheck = dimensionless

HA34: dim  CompositionFjord = (k=component,d=depth, x=xdir)
     aux  CompositionFjord = CompositionOcean(k,d)
     doc  CompositionFjord = The fjord is given an initial composition equal to the outer fjord composition
     unit CompositionFjord = dimensionless

HA35: dim  CompositionOutward = (k=component,d=depth)
     aux  CompositionOutward = OuterExchange(k,d) DIVZ0 OuterExchange(H2O,d)

HA36: dim  ConsumptionCapacityLimitedByLightAndAmmoniumInWater = (d=depth, x=xdir)
     aux  ConsumptionCapacityLimitedByLightAndAmmoniumInWater =
          MaxAmmoniumConsumption*(MassPerComponent(NH4,d,x)/(AmmoniumHalfStaurationConcen
          tration(d,x)+(MassPerComponent(NH4,d,x)))*(LightCoefficient0+LightCoefficient1*LightLimitatio
          n(d,x)))
     doc  ConsumptionCapacityLimitedByLightAndAmmoniumInWater = The amount of ammonium that
          the phytoplankton is capable of consuming per time unit, limited by the concentration of
          ammonium in the water and light.
     unit ConsumptionCapacityLimitedByLightAndAmmoniumInWater = 1/day

HA37: dim  ConsumptionCapacityLimitedByLightAndPhosphorusInWater = (d=depth, x=xdir)
     aux  ConsumptionCapacityLimitedByLightAndPhosphorusInWater =
          MaxPhosphorusConsumption*(MassPerComponent(Phosphorus,d,x)/(PhosphorusHalfSaturatio
          nConcentration(d,x))+(MassPerComponent(Phosphorus,d,x)))*(LightCoefficient0+LightCoefficie
          nt1*LightLimitation(d,x))
     doc  ConsumptionCapacityLimitedByLightAndPhosphorusInWater = The amount of phosphorus that
          the phytoplankton is capable of consuming per time unit, limited by the concentration of
          phosphorus in water and light.
     unit ConsumptionCapacityLimitedByLightAndPhosphorusInWater = 1/day

HA38: dim  CriticalLightIntensity = (d=depth, x=xdir)
     aux  CriticalLightIntensity = (LightAdaptionCoefficient^0.6)*(Light^0.4)
     doc  CriticalLightIntensity = Dynamic critical light intensity. Marks the transition between growth
          limited by light and temperature.
     unit CriticalLightIntensity = W/m2

HA39: dim  DeathAsAFunctionOfTemperature = (d=depth, x=xdir)
     aux  DeathAsAFunctionOfTemperature =
          DeathRateAt20C*EXP(TemperatureCoefficient*(Temperature(d,x)-MAXTemperature))
     doc  DeathAsAFunctionOfTemperature = The phytoplankton death rate caused by temperature.
     unit DeathAsAFunctionOfTemperature = 1/day

HA40: dim  DX = (x=xdir)
     aux  DX = FjordLength/(INDEX(LAST(xdir)) - INDEX(FIRST(xdir)) + 1)
     doc  DX = Defines the size ov volume element in x direction
     unit DX = meter

HA41: dim  EffectOfCarbonInPhytoplanktonAndWater = (d=depth, x=xdir)
     aux  EffectOfCarbonInPhytoplanktonAndWater =
          IF(EffectOfRelativeCarbonInWater(d,x)=1,1,EffectOfRelativeCarbonAsPhytoplankton(d,x)*Effect
          OfRelativeCarbonInWater(d,x))
```

doc EffectOfCarbonInPhytoplanktonAndWater = The effect of the carbon in phytoplankton and carbon in water. If the relative carbon in water is 1, there is a surplus of carbon, which means that carbon in water is not a constraint, and this effect will therefore be 1. If it is lower, there is not enough carbon, and the effect of carbon in water will therefore be multiplied by the effect of the relative carbon in phytoplankton.

unit EffectOfCarbonInPhytoplanktonAndWater = dimensionless

HA42: dim EffectOfPhytoplanktonAndAmmoniumInWater = (d=depth, x=xdir)

aux EffectOfPhytoplanktonAndAmmoniumInWater = IF(EffectOfRelativeAmmoniumInWater(d,x)=1,1,EffectOfRelativeCarbonAsPhytoplankton(d,x)*EffectOfRelativeAmmoniumInWater(d,x))

doc EffectOfPhytoplanktonAndAmmoniumInWater = The effect of the carbon in phytoplankton and ammonium in water. If the relative ammonium in water is 1, there is a surplus of ammonium, which means that ammonium in water is not a constraint, and this effect will therefore be 1. If it is lower, there is not enough ammonium, and the effect of ammonium in water will therefore be multiplied by the effect of the relative carbon in phytoplankton.

unit EffectOfPhytoplanktonAndAmmoniumInWater = dimensionless

HA43: dim EffectOfPhytoplanktonAndPhosphorusInWater = (d=depth, x=xdir)

aux EffectOfPhytoplanktonAndPhosphorusInWater = IF(EffectOfRelativePhosphorusInWater(d,x)=1,1,EffectOfRelativeCarbonAsPhytoplankton(d,x)*EffectOfRelativePhosphorusInWater(d,x))

doc EffectOfPhytoplanktonAndPhosphorusInWater = The effect of the carbon in phytoplankton and phosphorus in water. If the relative phosphorus in water is 1, there is a surplus of phosphorus, which means that phosphorus in water is not a constraint, and this effect will therefore be 1. If it is lower, there is not enough phosphorus, and the effect of phosphorus in water will therefore be multiplied by the effect of the relative carbon in phytoplankton.

unit EffectOfPhytoplanktonAndPhosphorusInWater = dimensionless

HA44: dim EffectOfRelativeAmmoniumInWater = (d=depth, x=xdir)

aux EffectOfRelativeAmmoniumInWater = GRAPH(RelativeAmmoniumInWater(d,x),0,0.1,[1,1,0.97,0.93,0.85,0.72,0.44,0.18,0.06,0.01,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeAmmoniumInWater = The effect of the relative amount of ammonium in the water. As the ammonium level in the water goes down, the relative amount of ammonium in the water will go towards 1. The effect will then go towards 0, because you will not be able to use all the ammonium in the water.

unit EffectOfRelativeAmmoniumInWater = dimensionless

HA45: dim EffectOfRelativeAmmoniumOnConsumptionRate = (d=depth, x=xdir)

aux EffectOfRelativeAmmoniumOnConsumptionRate = GRAPH(RelativeAmmoniumStock(d,x),0.1,0.01,[1,0.98,0.89,0.79,0.66,0.54,0.36,0.17,0.07,0.03,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeAmmoniumOnConsumptionRate = Regulates the consumption rate so that the phytoplankton reduce their ammonium consumption when the amount of ammonium in stock is higher than 10 percent of the ammonium in phytoplankton cells.

unit EffectOfRelativeAmmoniumOnConsumptionRate = dimensionless

HA46: dim EffectOfRelativeAmmoniumOnGrowthRate = (d=depth, x=xdir)

aux EffectOfRelativeAmmoniumOnGrowthRate = GRAPH(RelativeAmmoniumStock(d,x),0,0.01,[0,0.02,0.04,0.08,0.34,0.69,0.86,0.97,0.99,1,1"Min:0;Max:1;Zoom"])

doc EffectOfRelativeAmmoniumOnGrowthRate = The effect of the amount of ammonium in stock compared to phosphorous in phytoplankton. As the stock goes towards 10 % of what is in phytoplankton, the effect approaches 1, and will remain 1 for any percentage higher that 10.

When the percentage is higher that 10 %, there will be a constraint on growth, going towards 0 as the percentage of stick goes towards 0.

unit EffectOfRelativeAmmoniumOnGrowthRate = dimensionless

HA47: dim EffectOfRelativeCarbonAsPhytoplankton = (d=depth, x=xdir)

aux EffectOfRelativeCarbonAsPhytoplankton = GRAPH(RelativeCarbonAsPhytoplankton(d,x),0,0.1,[1,1,0.99,0.97,0.83,0.64,0.4,0.13,0.05,0.02,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeCarbonAsPhytoplankton = The effect of the relative amount of carbon in phytoplankton. As the carbon level in the phytoplankton goes down, the relative amount of carbon in phytoplankton will go towards 1. The effect will then go towards 0 because you will not be able to find all the carbon in the water if there are few phytoplankton and not enough carbon in the water.

unit EffectOfRelativeCarbonAsPhytoplankton = dimensionless

HA48: dim EffectOfRelativeCarbonInWater = (d=depth, x=xdir)

aux EffectOfRelativeCarbonInWater = GRAPH(RelativeCarbonInWater(d,x),0,0.1,[1,1,0.99,0.97,0.85,0.75,0.58,0.21,0.09,0.04,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativeCarbonInWater = The effect of the relative amount if carbon in the water. As the carbon level in the water goes down, the relative amount of carbon in the water will go towards 1. The effect will then go towards 0 because you will not be able to use all the carbon in the water.

unit EffectOfRelativeCarbonInWater = dimensionless

HA49: dim EffectOfRelativePhosphorusInWater = (d=depth, x=xdir)

aux EffectOfRelativePhosphorusInWater = GRAPH(RelativePhosphorusInWater(d,x),0,0.1,[1,0.96,0.92,0.8,0.61,0.29,0.17,0.11,0.07,0.05,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativePhosphorusInWater = The effect of the relative amount of phosphorus in the water. As the phosphorus level in the water goes down, the relative amount of phosphorus in water will go towards 1, the effect of relative phosphorus will go towards 0, because the phytoplankton are not able to use all the phosphorus in the water.

unit EffectOfRelativePhosphorusInWater = dimensionless

HA50: dim EffectOfRelativePhosphorusOnConsumptionRate = (d=depth, x=xdir)

aux EffectOfRelativePhosphorusOnConsumptionRate = GRAPH(RelativePhosohorusStock(d,x),0,0.01,[1,1,1,1,1,1,1,1,1,1,1,0.97,0.92,0.75,0.58,0.44,0.23,0.14,0.08,0,0"Min:0;Max:1;Zoom"])

doc EffectOfRelativePhosphorusOnConsumptionRate = Regulates the consumption rate so that the phytoplankton reduce their phosphorus consumption when the amount of phosphorus in stock is higher than 10 percent of the phosphorus in the phytoplankton cells.

unit EffectOfRelativePhosphorusOnConsumptionRate = dimensionless

HA51: dim EffectOfRelativePhosphorusOnGrowthRate = (d=depth, x=xdir)

aux EffectOfRelativePhosphorusOnGrowthRate = GRAPH(RelativePhosohorusStock(d,x),0,0.01,[0,0.06,0.37,0.53,0.66,0.81,0.95,0.97,0.98,0.99,1"Min:0;Max:1;Zoom"])

doc EffectOfRelativePhosphorusOnGrowthRate = The effect of the amount of phosphorus in stock compared to phosphorus in phytoplankton. As the stock goes towards 10 percent of what is in phytoplankton, the effect approaches 1, and will be 1 for any percentage higher than 10. When the relation is lower than 10 percent, there will be a constraint on the growth, going towards 0 as the percentage goes towards 0.

unit EffectOfRelativePhosphorusOnGrowthRate = dimensionless

HA52: dim FractionAmmoniumInPhytoplankton = (d=depth, x=xdir)
aux FractionAmmoniumInPhytoplankton = MassPerComponent(NH4inPhytoplankton,d,x) DIVZ0 MassPerComponent(CarbonInPhytoplankton,d,x)
doc FractionAmmoniumInPhytoplankton = The amount of ammonium in phytoplankton relative to the amount of carbon.
unit FractionAmmoniumInPhytoplankton = dimensionless

HA53: dim FractionPhosphorusInPhytoplankton = (d=depth, x=xdir)
aux FractionPhosphorusInPhytoplankton = MassPerComponent(PhosphorusInPhytoplankton,d,x) DIVZ0 MassPerComponent(CarbonInPhytoplankton,d,x)
doc FractionPhosphorusInPhytoplankton = The amount of phosphorus in phytoplankton relative to the amount of carbon.
unit FractionPhosphorusInPhytoplankton = dimensionless

HA54: dim InitiatedAmmoniumConsumption = (d=depth, x=xdir)
aux InitiatedAmmoniumConsumption = MAX(0,MIN(ConsumptionCapacityLimitedByLightAndAmmoniumInWater(d,x),OptimalAmmoniumConsumption(d,x)))
doc InitiatedAmmoniumConsumption = The initiated ammonium consumption per microgram carbon assimilated into phytoplankton.
unit InitiatedAmmoniumConsumption = 1/day

HA55: dim InitiatedPhosphorousConsumption = (d=depth, x=xdir)
aux InitiatedPhosphorousConsumption = MAX(0,MIN(ConsumptionCapacityLimitedByLightAndPhosphorusInWater(d,x),OptimalPhosphorusConsumption(d,x)))
doc InitiatedPhosphorousConsumption = The initiated phosphorus consumption per microgram carbon assimilated into phytoplankton.
unit InitiatedPhosphorousConsumption = 1/day

HA56: dim LightLimitation = (d=depth, x=xdir)
aux LightLimitation = Light(d,x)/((CriticalLightIntensity(d,x)^CoefficientN)+Light(d,x)^CoefficientN)^(1/CoefficientN)
doc LightLimitation = Limitation on the phytoplankton growth rate due to the light intensity.
unit LightLimitation = dimensionless

HA57: dim LightUtilization = (d=depth, x=xdir)
aux LightUtilization = TemperatureLimitedGrowthRate(d,x)/CriticalLightIntensity(d,x)
doc LightUtilization = The capability of phytoplankton to utilize the light, dependent on the critical light intensity and the temperature limitation.
unit LightUtilization = 1/day

HA58: aux MaxPhosphorousInPhytoplankton = (1/41)*PhosphorusLuxus
doc MaxPhosphorousInPhytoplankton = The maximum amount of phosphorus compared to carbon that phytoplankton can contain (Redfield)
unit MaxPhosphorousInPhytoplankton = dimensionless

HA59: dim MicrogramKPerLiterWater = (k=component,d=depth, x=xdir)
aux MicrogramKPerLiterWater = MassPerComponent(k,d,x)/(10^-9)/Volume(d,x)/ComponentWeight(k)/1000

HA60: dim MinAmmoniumInWater = (d=depth, x=xdir)
aux MinAmmoniumInWater = 70*(10^-9)*Volume(d,x)*ComponentWeight(NH4)*1000
doc MinAmmoniumInWater = The minimum amount of ammonium in water. At a lower level ammonium becomes a constraint.

unit MinAmmoniumInWater = kg

HA61: dim MinCarbonAsPhytoplankton = (d=depth, x=xdir)
aux MinCarbonAsPhytoplankton = 1*(10^-9)*Volume(d,x)*ComponentWeight(CarbonInPhytoplankton)*1000
doc MinCarbonAsPhytoplankton = The minimum amount of carbon in phytoplankton per liter of water.
unit MinCarbonAsPhytoplankton = kg

HA62 dim MinCarbonInWater = (d=depth, x=xdir)
aux MinCarbonInWater = 40*(10^-9)*Volume(d,x)*ComponentWeight(Carbon)*1000
doc MinCarbonInWater = The minimum amount of carbon in water.
unit MinCarbonInWater = kg

HA63: dim NaturalReductionRate = (d=depth, x=xdir)
aux NaturalReductionRate = PhytoplanktonDeathRate(d,x)+Respiration(d,x)
doc NaturalReductionRate = The amount per microgram phytoplankton that is lost per time unit due to natural factors such as temperature, respiration and nutrient level in the water.
unit NaturalReductionRate = 1/day

HA64: dim NitrificationRate = (d=depth, x=xdir)
aux NitrificationRate = AmmoniumPerBacteria*MassPerComponent(Bacteria,d,x)*(MaxNitrificationRate*(MassPerComponent(NH4,d,x)/(MassPerComponent(NH4,d,x)+AmmoniumHalfSaturation))*(MassPerComponent(O2,d,x)/(MassPerComponent(O2,d,x)+OxygenHalfSaturation(d,x))))
doc NitrificationRate = Kilos of nitrogen that are nitrified per day.
unit NitrificationRate = kg/day

HA65: dim NotNegative = (k=component,d=depth, x=xdir)
aux NotNegative = LIMIT(MassPerComponent(k,d,x),0,1e99)
doc NotNegative = This is useful in debugging phase. Should be removed in final implementation. OBS! gives side effects.

HA66: dim NutrientEffectOnDeathRate = (d=depth, x=xdir)
aux NutrientEffectOnDeathRate = 1-(1-NutrientCoefficient)*NutrientLimitation(d,x)
doc NutrientEffectOnDeathRate = The effect of the nutrient level in phytoplankton on the death rate.
unit NutrientEffectOnDeathRate = dimensionless

HA67: dim NutrientLimitation = (d=depth, x=xdir)
aux NutrientLimitation = 1/(1+(1/RelativeNitrogenSurplus(d,x))+(1/RelativePhosphorusSurplus(d,x)))
doc NutrientLimitation = Limitation on phytoplankton growth rate caused by the nutrient level in the phytoplankton cells.
unit NutrientLimitation = dimensionless

HA68: dim O2ProductionRate = (d=depth, x=xdir)
aux O2ProductionRate = PhotosyntheticQuotient*(MassPerComponent(CarbonInPhytoplankton,d,x) DIVZ0 MassPerComponent(Chlorophyll,d,x))*ChlorophyllProductionRate(d,x)
doc O2ProductionRate = The amount of oxygen that is produced by phytoplankton in the photosynthesis process per time unit.
unit O2ProductionRate = kg/day

HA69: dim OptimalAmmoniumConsumption = (d=depth, x=xdir)
aux OptimalAmmoniumConsumption = MaxAmmoniumInPhytoplankton+(MaxAmmoniumInPhytoplankton-FractionAmmoniumInPhytoplankton(d,x))

doc OptimalAmmoniumConsumption = The optimal consumption of ammonium from the water, limited by the actual growth rate, temperature and the level of ammonium in phytoplankton compared to carbon.
unit OptimalAmmoniumConsumption = dimensionless

HA70: dim OptimalLightUtilization = (d=depth, x=xdir)
aux OptimalLightUtilization = TemperatureLimitedGrowthRate(d,x)/(MAX(MinimumLightIntensity,CriticalLightIntensity(d,x)))
doc OptimalLightUtilization = Light utilization by phytoplankton when the adjustment to the actual light intensity is optimal.
unit OptimalLightUtilization = 1/day

HA71: dim OptimalPhosphorusConsumption = (d=depth, x=xdir)
aux OptimalPhosphorusConsumption = (TemperatureLimitedGrowthRate(d,x)*(MaxPhosphorousInPhytoplankton-FractionPhosphorusInPhytoplankton(d,x))+PhytoplanktonGrowthRate(d,x)*FractionPhosphorusInPhytoplankton(d,x))
doc OptimalPhosphorusConsumption = The optimal consumption of phosphorus from the water, limited by the actual growth rate, temperature and the level of Phosphorous in phytoplankton compared to Carbon.
unit OptimalPhosphorusConsumption = 1/day

HA72: dim OxygenConsumptionRate = (d=depth, x=xdir)
aux OxygenConsumptionRate = NitrificationRate(d,x)*OxygenPerNitrification
doc OxygenConsumptionRate = The amount of oxygen that is used in the nitrification process per time unit.
unit OxygenConsumptionRate = kg/day

HA73: dim OxygenHalfSaturation = (d=depth, x=xdir)
aux OxygenHalfSaturation = 4000*(10^-9)*Volume(d,x)*ComponentWeight(O2)*1000
doc OxygenHalfSaturation = The oxygen concentration in the water that would result in half of the maximum nitrification rate.
unit OxygenHalfSaturation = kg

HA74: dim PhosphorusHalfSaturationConcentration = (d=depth, x=xdir)
aux PhosphorusHalfSaturationConcentration = 5*(10^-9)*Volume(d,x)*ComponentWeight(Phosphorus)*1000
doc PhosphorusHalfSaturationConcentration = The level of phosphorus in the water that would result in half of the maximum phytoplankton growth.
unit PhosphorusHalfSaturationConcentration = kg

HA75: dim PhosphorusPredation = (d=depth, x=xdir)
aux PhosphorusPredation = PredationRate(d,x)*FractionPhosphorusInPhytoplankton(d,x)
doc PhosphorusPredation = The rate at which phosphorus in phytoplankton is consumed by zooplankton.
unit PhosphorusPredation = kg/day

HA76: dim PhosphorusRedfieldCheck = (k=component,d=depth, x=xdir)
aux PhosphorusRedfieldCheck = MassPerComponent(PhosphorusInPhytoplankton,d,x)/(MassPerComponent(PhosphorusInPhytoplankton,d,x)+MassPerComponent(CarbonInPhytoplankton,d,x)+MassPerComponent(NH4inPhytoplankton,d,x))
doc PhosphorusRedfieldCheck = The relative amount of phosphorus in phytoplankton compared to carbon and ammonium content (Redfield).
unit PhosphorusRedfieldCheck = dimensionless

HA77: dim PhytoplanktonDeathRate = (d=depth, x=xdir)
aux PhytoplanktonDeathRate =
DeathAsAFunctionOfTemperature(d,x)*CarbonInPhytoplanktonHalfSaturationFunction(d,x)*NutrientEffectOnDeathRate(d,x)
doc PhytoplanktonDeathRate = The phytoplankton death rate controlled by temperature, nutrient level in the water, and the amount of phytoplankton.
unit PhytoplanktonDeathRate = 1/day

HA78: dim PhytoplanktonGrowthRate = (d=depth, x=xdir)
aux PhytoplanktonGrowthRate =
TemperatureAndNutrientLimitedGrowthRate(d,x)*LightLimitation(d,x)
doc PhytoplanktonGrowthRate = Phytoplankton growth rate limited by temperature, light and the relative nitrogen and phosphorus content compared to carbon in phytoplankton.
unit PhytoplanktonGrowthRate = 1/day

HA79: dim PredationRate = (d=depth, x=xdir)
aux PredationRate =
MassPerComponent(CarbonInPhytoplankton,d,x)*MassPerComponent(Zooplankton,d,x)*PredationCoefficient
doc PredationRate = The rate at which phytoplankton is consumed by zooplankton.
unit PredationRate = kg/day

HA80: dim RedfieldEffectOnAmmoniumGrowthRate = (d=depth, x=xdir)
aux RedfieldEffectOnAmmoniumGrowthRate =
GRAPH(FractionAmmoniumInPhytoplankton(d,x),0,0.005,[1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.4,1.398,1.395,1.395,1.395,1.395,1.395,1.389,1.381,1.36,1.333,1.311,1.286,1,1,1"Min:1;Max:1.4;Zoom"])
doc RedfieldEffectOnAmmoniumGrowthRate = Makes the phytoplankton go towards Redfield by increasing the consumption rate and the growth rate when the ammonium content is lower than Redfield, and decreasing them if they are higher than Redfield.
unit RedfieldEffectOnAmmoniumGrowthRate = dimensionless

HA81: dim RedfieldEffectOnPhosphorusGrowthRate = (d=depth, x=xdir)
aux RedfieldEffectOnPhosphorusGrowthRate =
GRAPH(FractionPhosphorusInPhytoplankton(d,x),0,0.002,[1.4,1.4,1.4,1.4,1.395,1.382,1.372,1.354,1.337,1.318,1.293,1.258,1.002,1,1"Min:1;Max:1.4;Zoom"])
doc RedfieldEffectOnPhosphorusGrowthRate = Makes the phytoplankton go towards Redfield by increasing the consumption rate and the growth rate when the phosphorus content is lower than Redfield, and decreasing them if they are higher than Redfield.
unit RedfieldEffectOnPhosphorusGrowthRate = dimensionless

HA82: dim RelativeAmmoniumInWater = (d=depth, x=xdir)
aux RelativeAmmoniumInWater = MinAmmoniumInWater(d,x) DIVZ0 MassPerComponent(NH4,d,x)
doc RelativeAmmoniumInWater = The fraction of the minimum amount of ammonium in the water compared to the actual ammonium level in the water.
unit RelativeAmmoniumInWater = dimensionless

HA83: dim RelativeAmmoniumStock = (d=depth, x=xdir)
aux RelativeAmmoniumStock = MassPerComponent(NH4InStock,d,x) DIVZ0 MassPerComponent(NH4inPhytoplankton,d,x)
doc RelativeAmmoniumStock = The fraction of ammonium in stock compared to ammonium in phytoplankton.
unit RelativeAmmoniumStock = dimensionless

HA84: dim RelativeAmountOfComponent = (k=component,d=depth, x=xdir)
aux RelativeAmountOfComponent = MassPerComponent(k,d,x)/SUM(k=H2O;MassPerComponent(H2O,d,x))
doc RelativeAmountOfComponent = The relative mass of each component within a fjord cell.
unit RelativeAmountOfComponent = dimensionless

HA85: dim RelativeCarbonAsPhytoplankton = (d=depth, x=xdir)
aux RelativeCarbonAsPhytoplankton = MinCarbonAsPhytoplankton(d,x) DIVZ0 MassPerComponent(CarbonInPhytoplankton,d,x)
doc RelativeCarbonAsPhytoplankton = The fraction of the minimum amount of carbon in phytoplankton compared to the actual level of carbon in phytoplankton.
unit RelativeCarbonAsPhytoplankton = dimensionless

HA86: dim RelativeCarbonInWater = (d=depth, x=xdir)
aux RelativeCarbonInWater = MinCarbonInWater(d,x) DIVZ0 MassPerComponent(Carbon,d,x)
doc RelativeCarbonInWater = The fraction of the minimum amount of carbon compared to the actual carbon level in the water.
unit RelativeCarbonInWater = dimensionless

HA87: dim RelativeMixDown = (k=component,d=depth, x=xdir)
aux RelativeMixDown = MixingDown(k,d,x) DIVZ0 MixingDown(H2O,d,x)

HA88: dim RelativeMixUp = (k=component,d=depth, x=xdir)
aux RelativeMixUp = MixingUp(k,d,x) DIVZ0 MixingUp(H2O,d,x)

HA89: dim RelativeNitrogenSurplus = (d=depth, x=xdir)
aux RelativeNitrogenSurplus = (FractionAmmoniumInPhytoplankton(d,x)-LowerLimitNitrogenComparedToCarbon)/LowerLimitNitrogenComparedToCarbon
doc RelativeNitrogenSurplus = The relative surplus of nitrogen in phytoplankton compared to carbon.
unit RelativeNitrogenSurplus = dimensionless

HA90: dim RelativeOutgoingFlow = (k=component,d=depth, x=xdir)
aux RelativeOutgoingFlow = OutgoingFlowOut(k,d,x) DIVZ0 OutgoingFlowOut(H2O,d,x)

HA91: dim RelativePhosohorusStock = (d=depth, x=xdir)
aux RelativePhosohorusStock = MassPerComponent(PhosphorusInStock,d,x) DIVZ0 MassPerComponent(PhosphorusInPhytoplankton,d,x)
doc RelativePhosohorusStock = The fraction of phosphorus in stock compared to phosphorus in phytoplankton.
unit RelativePhosohorusStock = dimensionless

HA92: dim RelativePhosphorusInWater = (d=depth, x=xdir)
aux RelativePhosphorusInWater = PhosphorusHalfSaturationConcentration DIVZ0 MassPerComponent(Phosphorus,d,x)
doc RelativePhosphorusInWater = The fraction of the minimum amount of phosphorus in the water compared to the actual phosphorus level in the water.
unit RelativePhosphorusInWater = dimensionless

HA93: dim RelativePhosphorusSurplus = (d=depth, x=xdir)
aux RelativePhosphorusSurplus = (FractionPhosphorusInPhytoplankton(d,x)-LowerLimitPhosphorusComparedToCarbon)/LowerLimitPhosphorusComparedToCarbon
doc RelativePhosphorusSurplus = The relative surplus of phosphorus in phytoplankton compared to carbon.
unit RelativePhosphorusSurplus = dimensionless

HA94: dim Respiration = (d=depth, x=xdir)
aux Respiration = RespirationAt20C*EXP(TemperatureCoefficient*(Temperature(d,x)-MAXTemperature))
doc Respiration = The amount of phytoplankton that is lost due to respiration per time unit.
unit Respiration = 1/day

HA95: dim TemperatureAndNutrientLimitedGrowthRate = (d=depth, x=xdir)
aux TemperatureAndNutrientLimitedGrowthRate = NutrientLimitation(d,x)*TemperatureLimitedGrowthRate(d,x)
doc TemperatureAndNutrientLimitedGrowthRate = The phytoplankton growth rate limited by temperature and the relative ammonium and phosphorus content in phytoplankton compared to carbon.
unit TemperatureAndNutrientLimitedGrowthRate = 1/day

HA96: dim TemperatureLimitedGrowthRate = (d=depth, x=xdir)
aux TemperatureLimitedGrowthRate = GrowthRate20C*EXP(TemperatureCoefficient*(Temperature(d,x)-MAXTemperature))
doc TemperatureLimitedGrowthRate = The phytoplankton growth rate limited by temperature. (1/day)*dimensionlessEXP
unit TemperatureLimitedGrowthRate = 1/day

HA97: aux TotalMass = ARRSUM(MassPerComponent)
doc TotalMass = Total mass for checking purpose
unit TotalMass = kg

HA98: dim TotalMassPerComponent2 = (k=component)
aux TotalMassPerComponent2 = SUM(d=depth,x=xdir;MassPerComponent(k,d,x))
doc TotalMassPerComponent2 = Total mass pro component, for checking purpose
unit TotalMassPerComponent2 = kg

HA99: dim ZooplanktonDeathRate = (d=depth, x=xdir)
aux ZooplanktonDeathRate = MassPerComponent(Zooplankton,d,x)/ZooplanktonLifespan
doc ZooplanktonDeathRate = The number of zooplankton that dies per time unit.
unit ZooplanktonDeathRate = cells/day

HA100 dim ZooplanktonGrowthRate = (d=depth, x=xdir)
aux ZooplanktonGrowthRate = MassPerComponent(Zooplankton,d,x)*MassPerComponent(CarbonInPhytoplankton,d,x)*PredatorCoefficient
doc ZooplanktonGrowthRate = The number of zooplankton that is formed per time unit.
unit ZooplanktonGrowthRate = cells/day

A.2.3 Constants in the Hydrodynamic Model

```
HC1:  const  AmmoniumPerBacteria = 2.700966*10^-5*1
      doc    AmmoniumPerBacteria = The amount of ammonium that is nitrified by each bacteria per time
             unit.
      unit   AmmoniumPerBacteria = kg

HC2:  const  BacteriaCellYield = 71.39
      doc    BacteriaCellYield = The number of cells that are produced per microgram nitrogen that is
             nitrified.
      unit   BacteriaCellYield = cells/kg


HC3:  const  CoefficientN = 3
      doc    CoefficientN = Light coefficient for the ability of phytoplankton to adapt to changing light
             intensities.
      unit   CoefficientN = dimensionless

HC4:  dim    ComponentWeight = (k=component)
      const  ComponentWeight = 1 | k=H2O;
             1 | k=NaCl;
             1.4 | k=O2;
             1.25 | k=NH4;
             1 | k=Bacteria;
             1 | k=NH4inPhytoplankton;
             1 | k=CarbonInPhytoplankton;
             1 | k=PhosphorusInPhytoplankton;
             1 | k=NH4InStock;
             1 | k=PhosphorusInStock;
             1 | k=Phosphorus;
             1 | k=Carbon;
             1 | k=Chlorophyll;
             1 | k=Zooplankton
      doc    ComponentWeight = Component weight
      unit   ComponentWeight = kg/liter

HC5:  dim    CompositionOcean = (k=component,d=depth)
      const  CompositionOcean = 1 | k=H2O AND d=layer1top;
             0.035| k=NaCl AND d=layer1top;
             0.000008 | k=O2 AND d=layer1top;
             0.00000001 | k=NH4 AND d=layer1top;
             0.00005 | k=Bacteria AND d=layer1top;
             0.000000014 | k=NH4inPhytoplankton AND d=layer1top;
             0.000000083 | k=CarbonInPhytoplankton AND d=layer1top;
             0.000000002 | k=PhosphorusInPhytoplankton AND d=layer1top;
             0.0000000014 | k=NH4InStock AND d=layer1top;
             0.0000000002 | k=PhosphorusInStock AND d=layer1top;
             0.0000001 | k=Phosphorus AND d=layer1top;
             0.01 | k=Carbon AND d=layer1top;
             0.000000002075 | k=Chlorophyll AND d=layer1top;
             0.000001 | k=Zooplankton AND d=layer1top;
             1 | k=H2O AND d=layer2;
             0.035 | k=NaCl AND d=layer2;
```

```
                0.000006 | k=O2 AND d=layer2;
                0.000001 | k=NH4 AND d=layer2;
                0.00005 | k=Bacteria AND d=layer2;
                0.000000014 | k=NH4inPhytoplankton AND d=layer2;
                0.000000083 | k=CarbonInPhytoplankton AND d=layer2;
                0.000000002 | k=PhosphorusInPhytoplankton AND d=layer2;
                0.0000000014 | k=NH4InStock AND d=layer2;
                0.0000000002 | k=PhosphorusInStock AND d=layer2;
                0.0000001 | k=Phosphorus AND d=layer2;
                0.01 | k=Carbon AND d=layer2;
                0.000000002075 | k=Chlorophyll AND d=layer2;
                0.000001 | k=Zooplankton AND d=layer2;
                1 | k=H2O AND d=layer3;
                0.035 | k=NaCl AND d=layer3;
                0.000005| k=O2 AND d=layer3;
                0.000001 | k=NH4 AND d=layer3;
                0.00005 | k=Bacteria AND d=layer3;
                0.000000014 | k=NH4inPhytoplankton AND d=layer3;
                0.000000083 | k=CarbonInPhytoplankton AND d=layer3;
                0.000000002 | k=PhosphorusInPhytoplankton AND d=layer3;
                0.0000000014 | k=NH4InStock AND d=layer3;
                0.0000000002 | k=PhosphorusInStock AND d=layer3;
                0.0000001 | k=Phosphorus AND d=layer3;
                0.01 | k=Carbon AND d=layer3;
                0.000000002075 | k=Chlorophyll AND d=layer3;
                0.000001 | k=Zooplankton AND d=layer3
        doc     CompositionOcean = Relative amount of each component in the outer neighborhood of the
                channel (Ocean)
                Dimension kg/kg  --> dimensionless
        unit    CompositionOcean = dimensionless

HC6:    dim     CompositionRiver = (k=component)
        const   CompositionRiver = 1 | k=H2O;
                0.00| k=NaCl;
                0.000008| k=O2;
                0.000000 | k=NH4;
                0 | k=Bacteria;
                0 | k=NH4inPhytoplankton;
                0 | k=CarbonInPhytoplankton;
                0 | k=PhosphorusInPhytoplankton;
                0 | k=NH4InStock;
                0 | k=PhosphorusInStock;
                0 | k=Phosphorus;
                0 | k=Carbon;
                0 | k=Chlorophyll;
                0 | k=Zooplankton
        doc     CompositionRiver = Relative component composition in River
        unit    CompositionRiver = kg/liter

HC7:    dim     DD = (d=depth)
        const   DD = 5| d=FIRST(depth) ; 12.5
        doc     DD = Defines the size of volume element in depth direction
        unit    DD = meter

HC8:    const   DeathRateAt20C = 0.1
```

doc DeathRateAt20C = The death rate caused by temperature when the temperature is 20 degrees C.
unit DeathRateAt20C = 1/day

HC9: const FjordLength = 1000
doc FjordLength = Length of channel
unit FjordLength = meter

HC10: const GrowthRate20C = 1.4
doc GrowthRate20C = The growth rate per day of phytoplankton at 20 degrees C.
unit GrowthRate20C = 1/day

HC11: const HydrogenAddition = 0.28784
doc HydrogenAddition = The fraction that is added because of the weight of the hydrogen atoms in ammonium in order to calculate how much ammonium is nitrified compared to nitrogen.
unit HydrogenAddition = dimensionless

HC12: dim Light = (d=depth, x=xdir)
const Light = 75
doc Light = The actual light intensity in the water.60+SINWAVE(40, 365)
unit Light = W/m2

HC13: const LightAdaptionCoefficient = 75
doc LightAdaptionCoefficient = Coefficient for light adaption ability by phytoplankton.
unit LightAdaptionCoefficient = dimensionless

HC14: const LightCoefficient0 = 0.5
doc LightCoefficient0 = A dimensionless coefficient that combines growth and consumption.
unit LightCoefficient0 = dimensionless

HC15: const LightCoefficient1 = 0.5
doc LightCoefficient1 = A dimensionless coefficient that combines growth and consumption.
unit LightCoefficient1 = dimensionless

HC16: const LowerLimitNitrogenComparedToCarbon = 0.06
doc LowerLimitNitrogenComparedToCarbon = The lowest relative amount of nitrogen compared to carbon half saturation that is possible in phytoplankton.
unit LowerLimitNitrogenComparedToCarbon = dimensionless

HC17: const LowerLimitPhosphorusComparedToCarbon = 0.0027
doc LowerLimitPhosphorusComparedToCarbon = The lowest amount of phosphorus compared to carbon half saturation that is possible in phytoplankton.
unit LowerLimitPhosphorusComparedToCarbon = dimensionless

HC18: const MaxAmmoniumConsumption = 0.6
doc MaxAmmoniumConsumption = The maximum amount of ammonium that can be consumed by each unit of phytoplankton (measured in carbon) per time unit.
unit MaxAmmoniumConsumption = 1/day

HC19: const MaxAmmoniumInPhytoplankton = 7.2/41
doc MaxAmmoniumInPhytoplankton = The maximum amount of ammonium in phytoplankton compared to carbon (Redfield).
unit MaxAmmoniumInPhytoplankton = dimensionless

HC20: const MaxBacteriaLifeSpan = 12

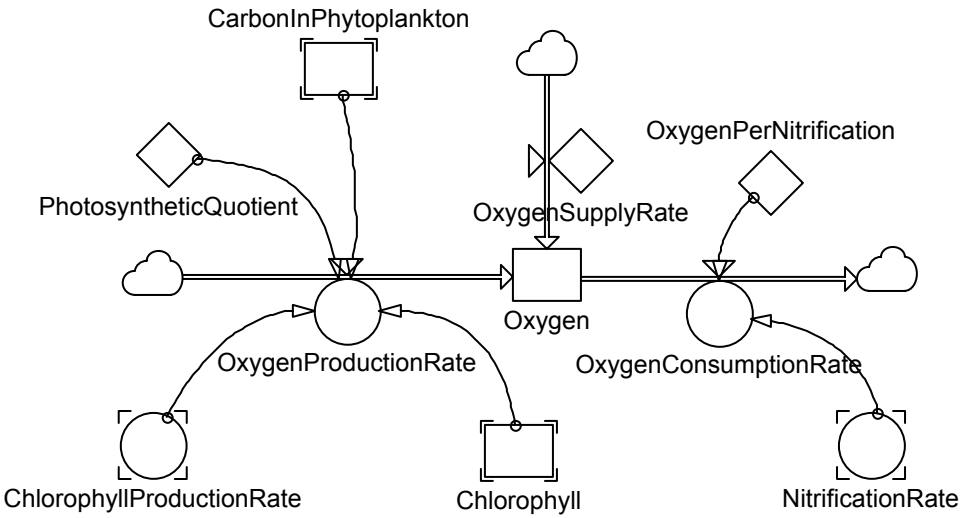
```
        doc     MaxBacteriaLifeSpan = The maximum number of days a bacteria can live.
        unit    MaxBacteriaLifeSpan = days

HC21:   const   MaxNitrificationRate = 240
        doc     MaxNitrificationRate = The maximum amount of nitrogen that can be nitrified per time unit.
        unit    MaxNitrificationRate = 1/day

HC22:   const   MaxPhosphorusConsumption = 0.13
        doc     MaxPhosphorusConsumption = The maximum amount of phosphorus that can be consumed by
                each unit of phytoplankton (measured in carbon) per time unit.
        unit    MaxPhosphorusConsumption = 1/day

HC23:   const   MAXTemperature = 20
        doc     MAXTemperature = The temperature that would give the maximum phytoplankton growth if
                there were no other limits to growth.
        unit    MAXTemperature = degC

HC24:   const   MinimumLightIntensity = 5
        doc     MinimumLightIntensity = The minimum light intensity for phytoplankton growth.
        unit    MinimumLightIntensity = W/m2

HC25:   dim     MixingFactor = (k=component)
        const   MixingFactor = 500
        doc     MixingFactor = The diffusion from a volume to its neighbor volume in the positive x direction.
                The diffusion (in kg/day) is proportional to the relative concentration of the respective
                components (dimensionless), the component weight (kg/liter), the diffusion area in m2. This
                gives the MixingFactor The dimension of 1000*meter/day i.e. the dimension of velocity.
        unit    MixingFactor = 1000*m/day

HC26:   dim     NH4Apportionment = (d=depth)
        const   NH4Apportionment = [0.3,0.3,0.4]
        doc     NH4Apportionment = The relative amount of total NH4 injected into each layer at the bottom
                end of the channel
        unit    NH4Apportionment = dimensionless

HC27:   const   NH4Supply = 900000/365
        doc     NH4Supply = 900 ton per year
        unit    NH4Supply = kg/day

HC28:   const   NutrientCoefficient = 0.5
        doc     NutrientCoefficient = Coefficient that states the effect that nutrient limitation has on the
                phytoplankton death rate.
        unit    NutrientCoefficient = dimensionless

HC29:   const   OxygenPerNitrification = 3.42679
        doc     OxygenPerNitrification = The amount of oxygen that is nitrified per microgram nitrogen that is
                nitrified.
        unit    OxygenPerNitrification = dimensionless

HC30:   const   PhosphorusLuxus = 1.1
        doc     PhosphorusLuxus = Phosphorus luxury consumption. The amount of phosphorus the
                phytoplankton can consume in addition to Redfield.
        unit    PhosphorusLuxus = dimensionless

HC31:   const   PhotosyntheticQuotient = 0.001
```

```
        doc     PhotosyntheticQuotient = The amount of oxygen produced per chlorophyll that is formed.
        unit    PhotosyntheticQuotient = dimensionless

HC32:   const   PredationCoefficient = 0.0001
        doc     PredationCoefficient = Coefficient that states the effect the number of zooplankton and
                phytoplankton has on phytoplankton growth rate.
        unit    PredationCoefficient = 1/day

HC33:   const   PredatorCoefficient = 0.000015
        doc     PredatorCoefficient = Coefficient that states the effect the number of zooplankton and
                phytoplankton has on phytoplankton growth rate.
        unit    PredatorCoefficient = 1/day

HC34:   const   RespirationAt20C = 0.04
        doc     RespirationAt20C = The phytoplankton respiration at 20 degrees C.
        unit    RespirationAt20C = 1/day

HC35:   const   RiverFlow = (25* 3600 * 24 * 1000 )
        doc     RiverFlow = 50 M3/second = 50 metric tons pr second
        unit    RiverFlow = kg/day

HC36:   dim     Temperature = (d=depth, x=xdir)
        const   Temperature = 14 | d=layer1top AND x=0;
                14 | d=layer1top AND x=1;
                14 | d=layer1top AND x=2;
                14 | d=layer1top AND x=3;
                10 | d=layer2 AND x=0;
                10 | d=layer2 AND x=1;
                10 | d=layer2 AND x=2;
                10 | d=layer2 AND x=3;
                10 | d=layer3 AND x=0;
                10 | d=layer3 AND x=1;
                10 | d=layer3 AND x=2;
                10 | d=layer3 AND x=3;
                10
        doc     Temperature = The actual temperature in the water.
        unit    Temperature = degC

HC37:   const   TemperatureCoefficient = 0.063
        doc     TemperatureCoefficient = Temperature coefficient states the effect of variation in temperature.
        unit    TemperatureCoefficient = 1/degC

HC38:   const   WaterExchange = 50* 3600 * 24 * 1000/4
        doc     WaterExchange = The in-going flow of seawater at the bottom lawyer
        unit    WaterExchange = kg/day

HC39:   dim     Width = (x=xdir)
        const   Width = 1000
        doc     Width = With of channel
        unit    Width = meter

HC40:   const   ZooplanktonLifespan = 8
        doc     ZooplanktonLifespan = The zooplankton life span.
        unit    ZooplanktonLifespan = days
```

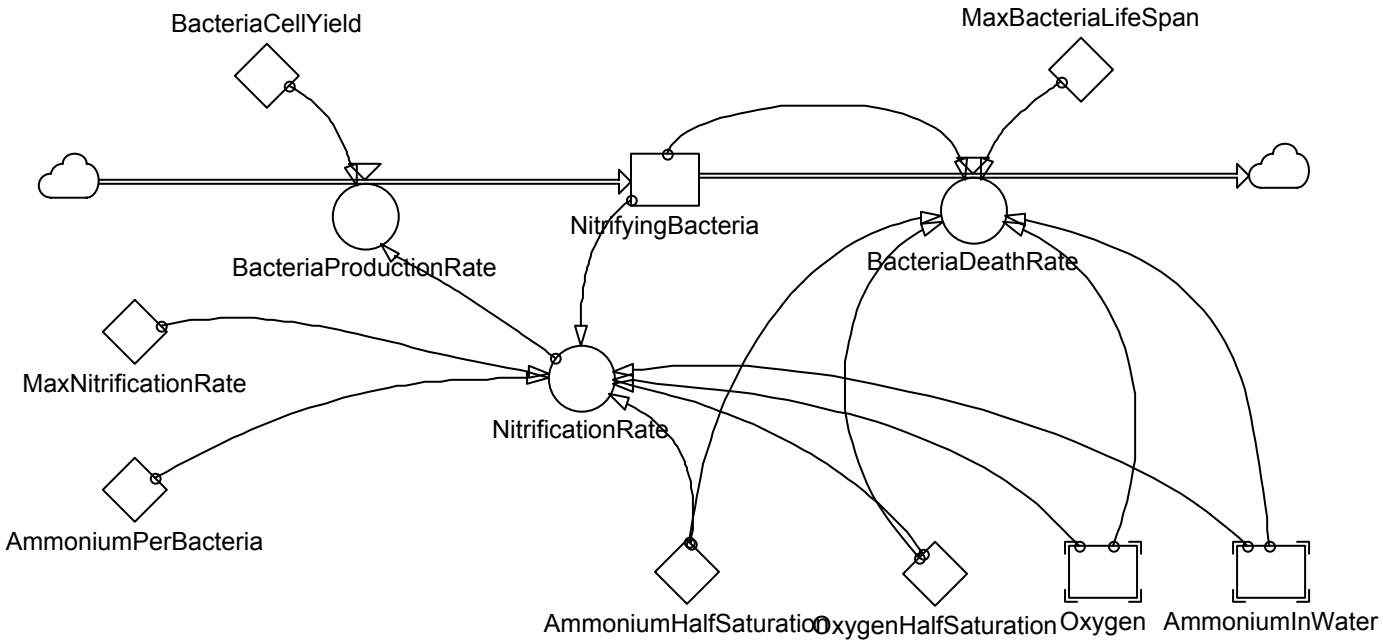
Appendix B. Stock and Flow Diagrams

B.1 Stock and Flow Diagrams of the Biology Model

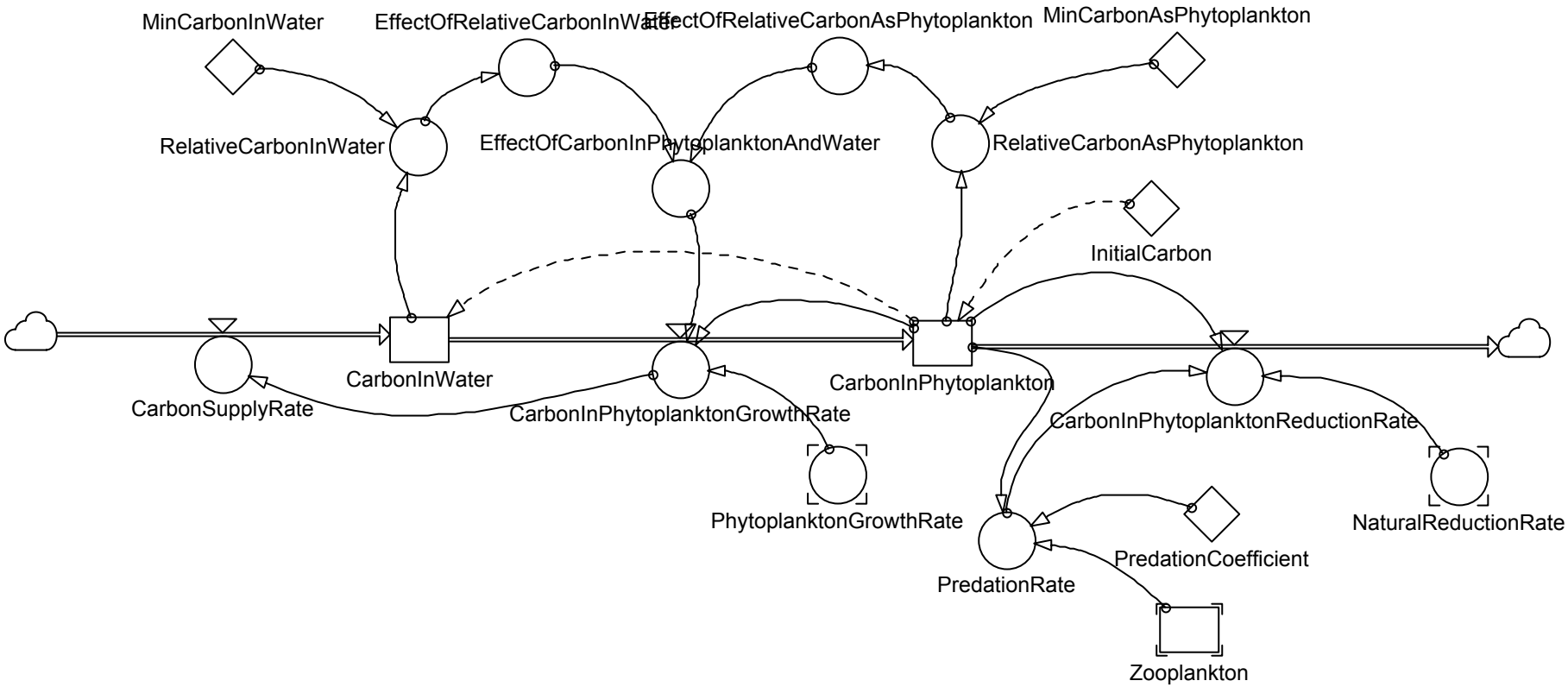
B.1.1 Oxygen



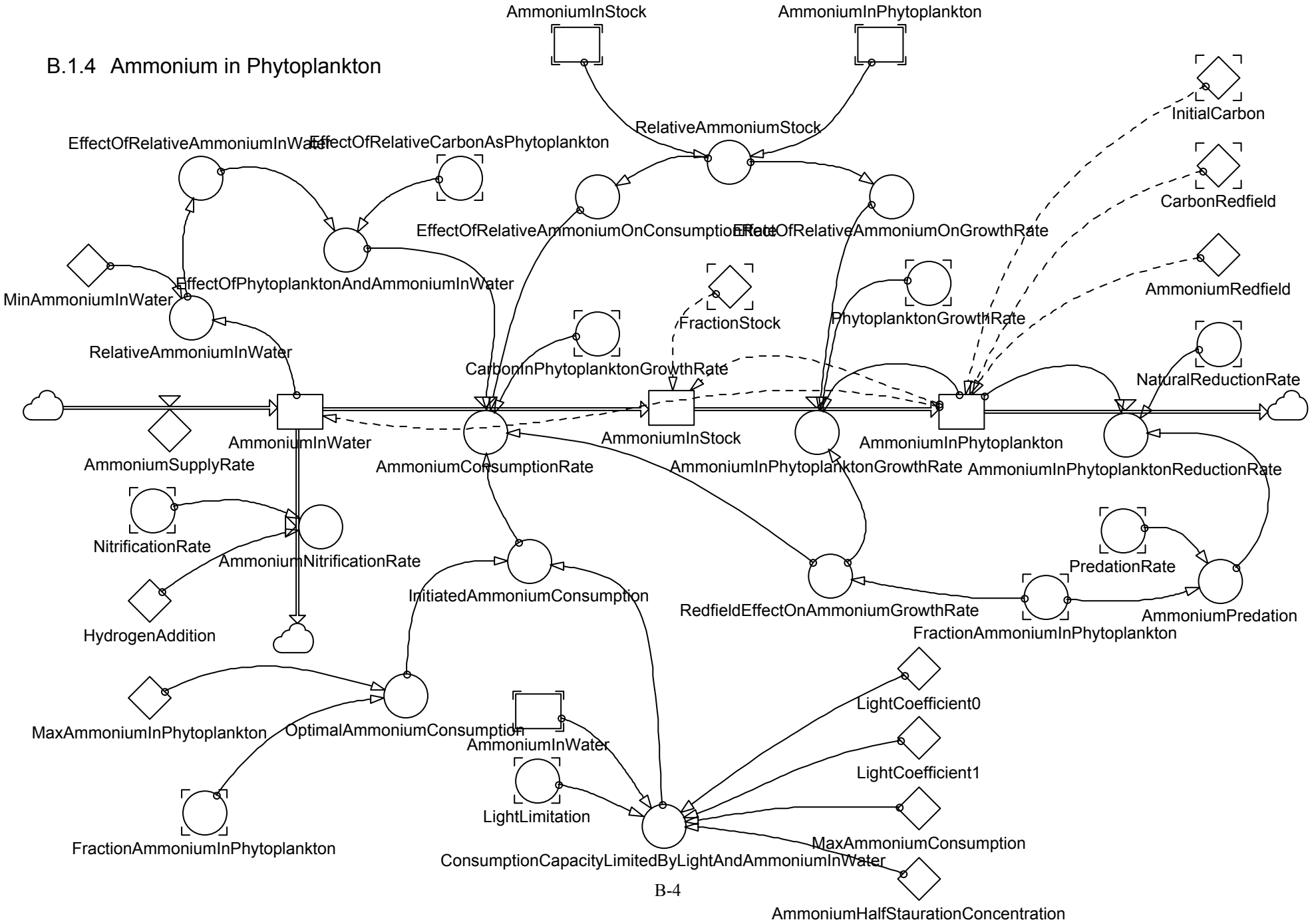
B.1.2 Nitrification



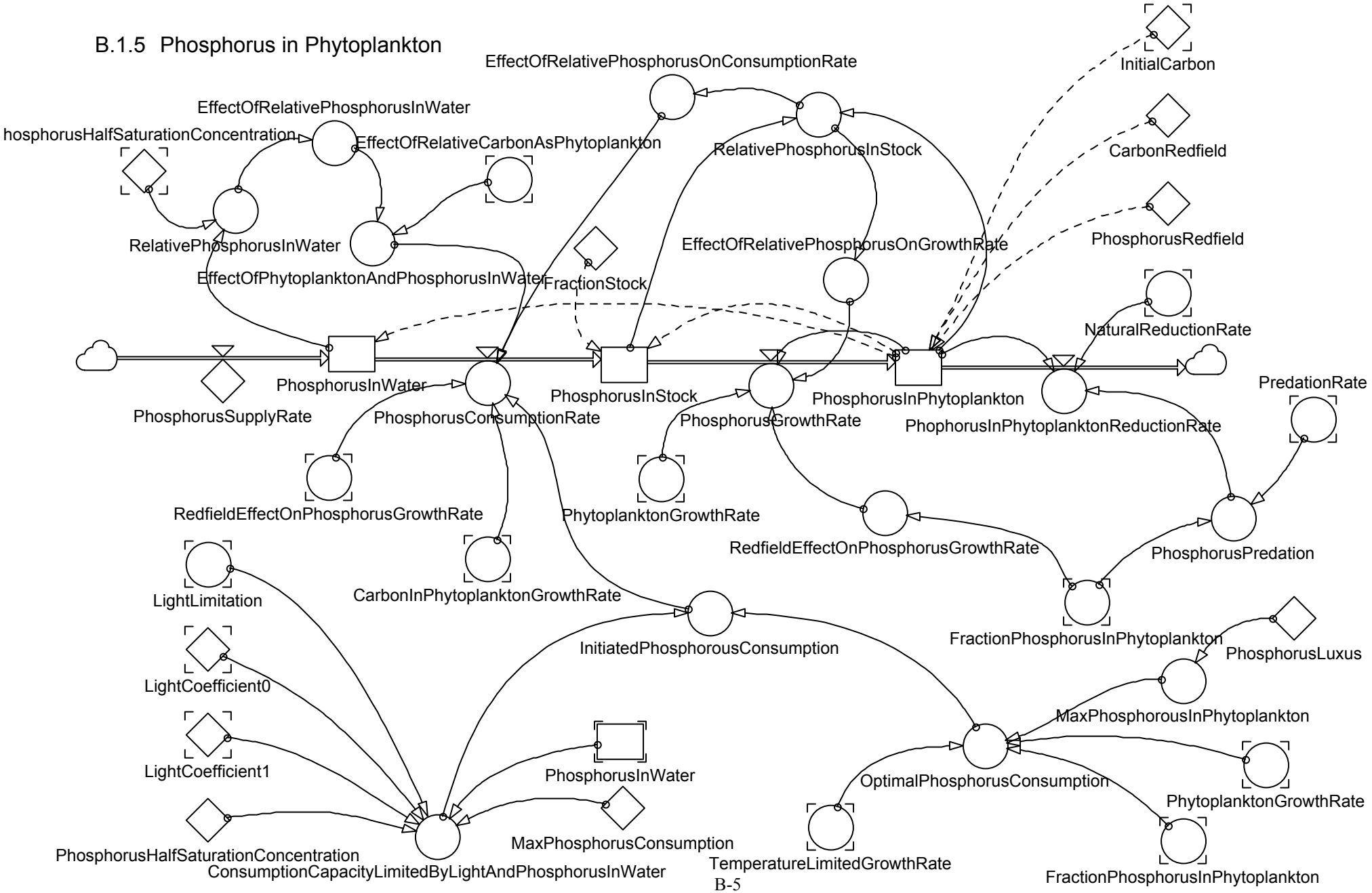
B.1.3 Carbon in Phytoplankton



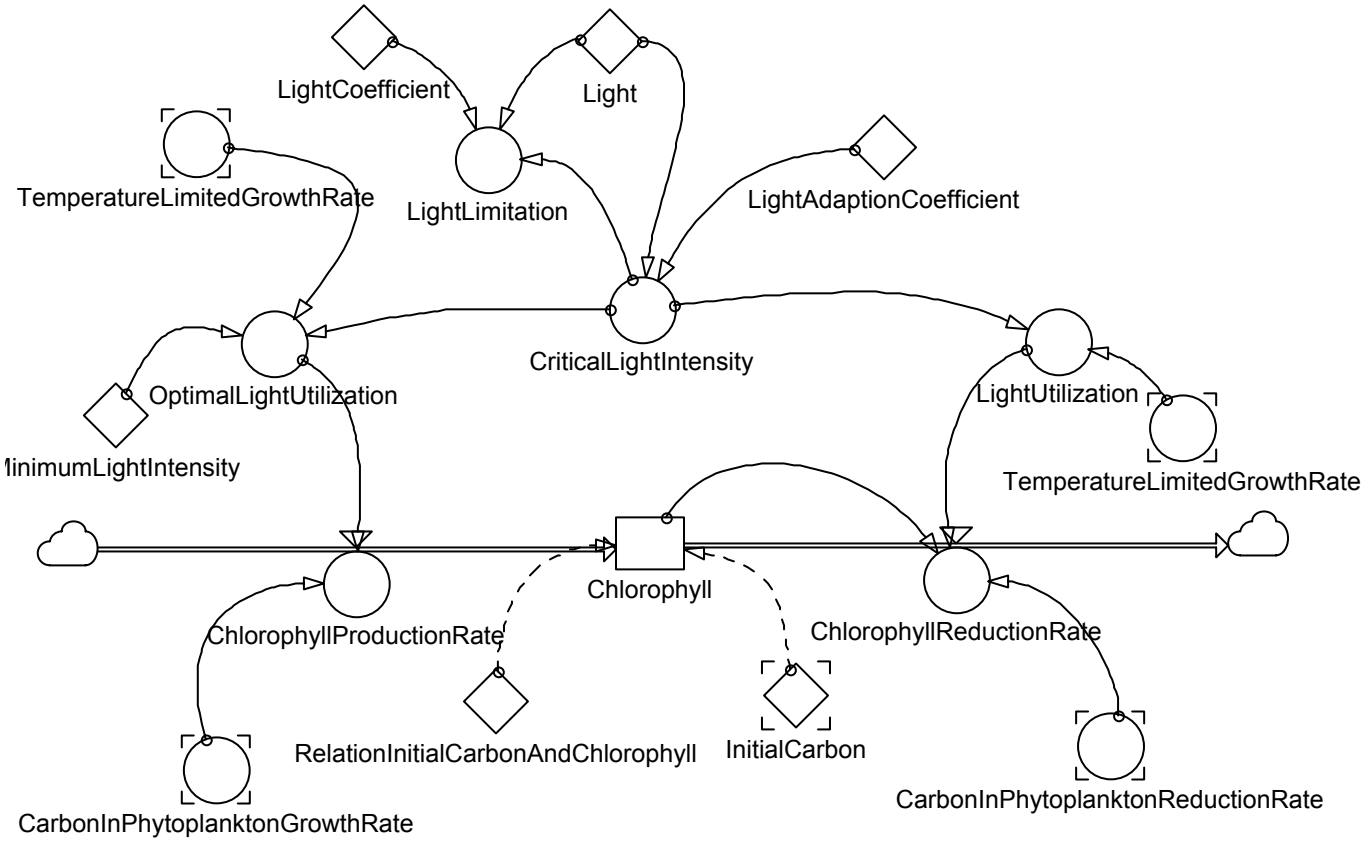
B.1.4 Ammonium in Phytoplankton



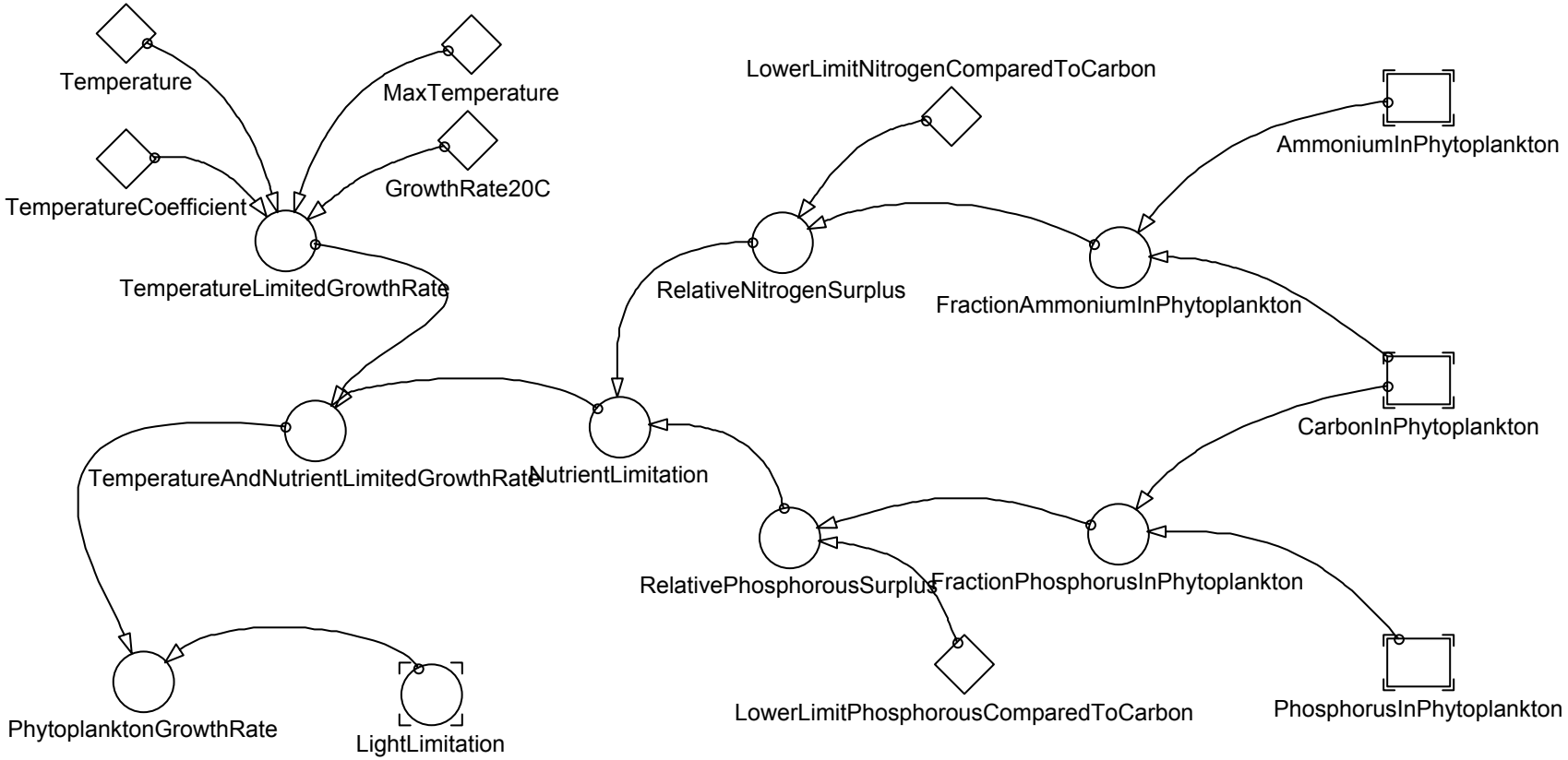
B.1.5 Phosphorus in Phytoplankton



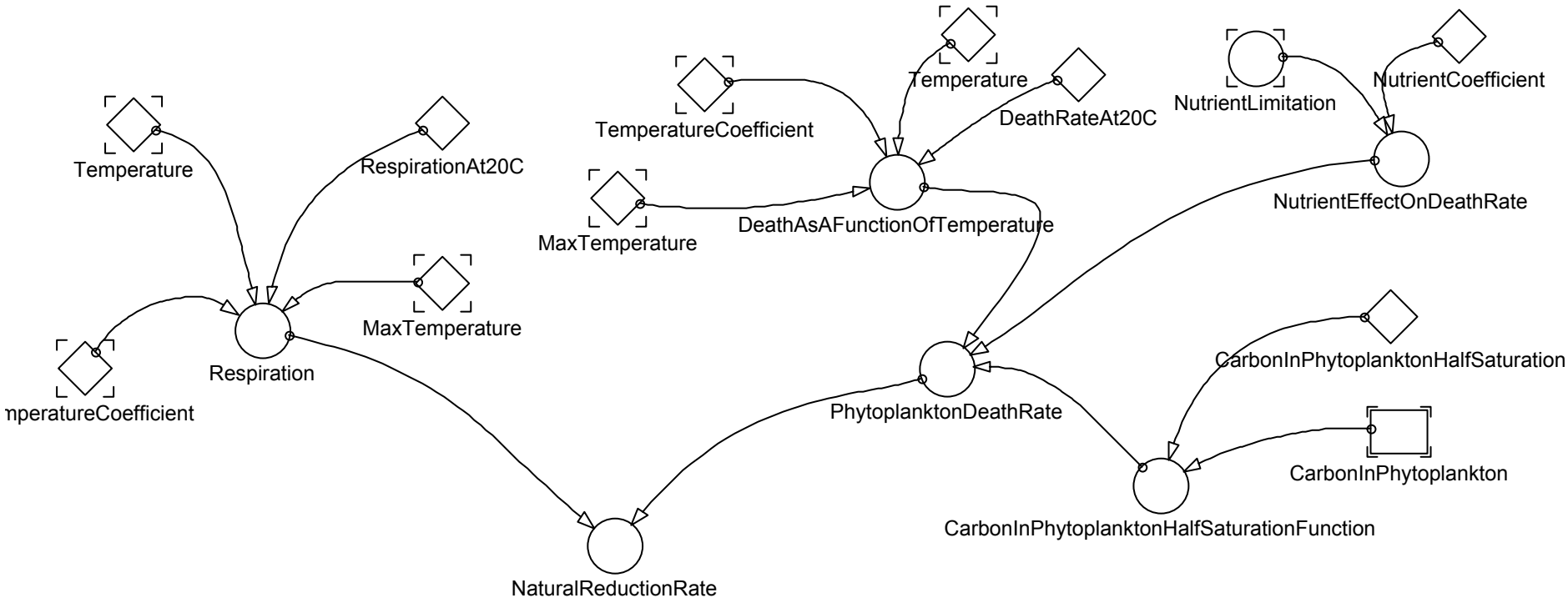
B.1.6 Photosynthesis



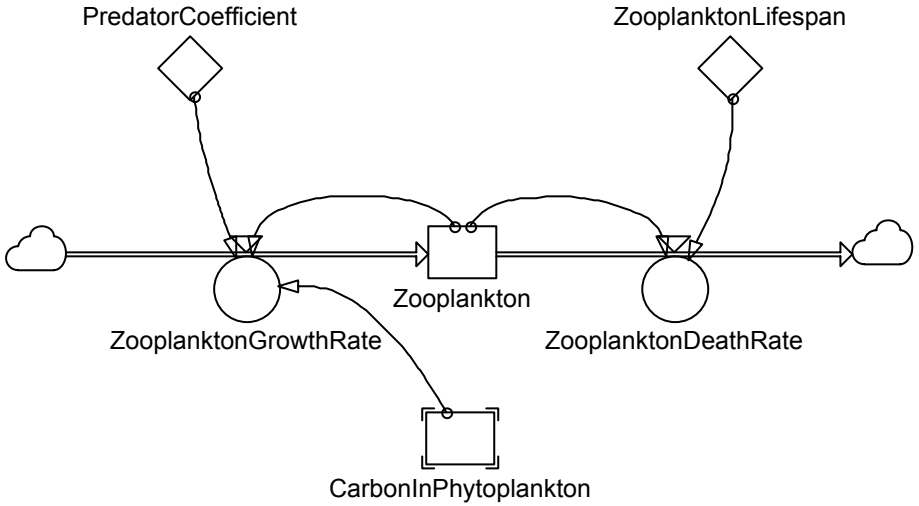
B.1.7 Phytoplankton Growth



B.1.8 Phytoplankton Death

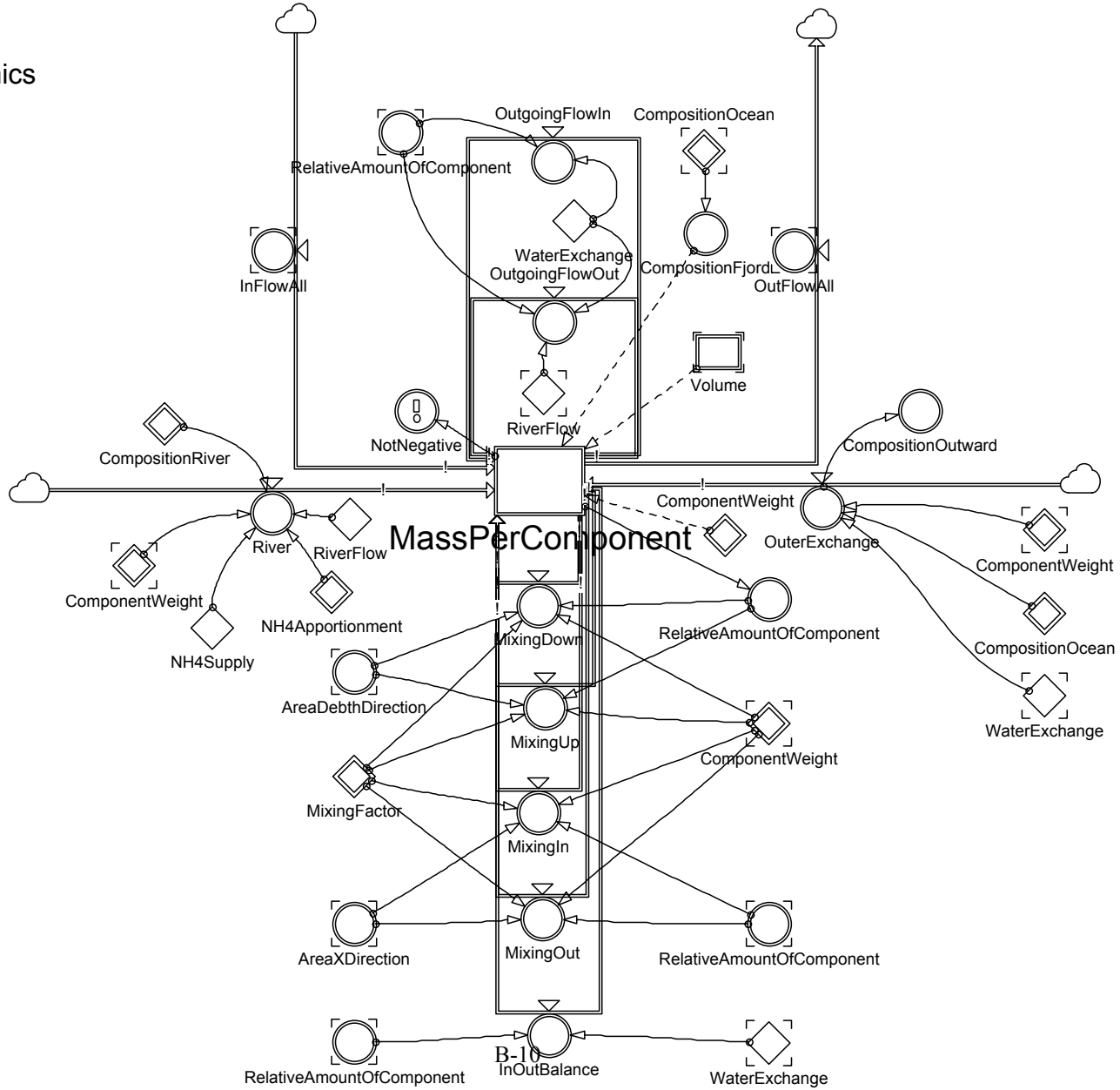


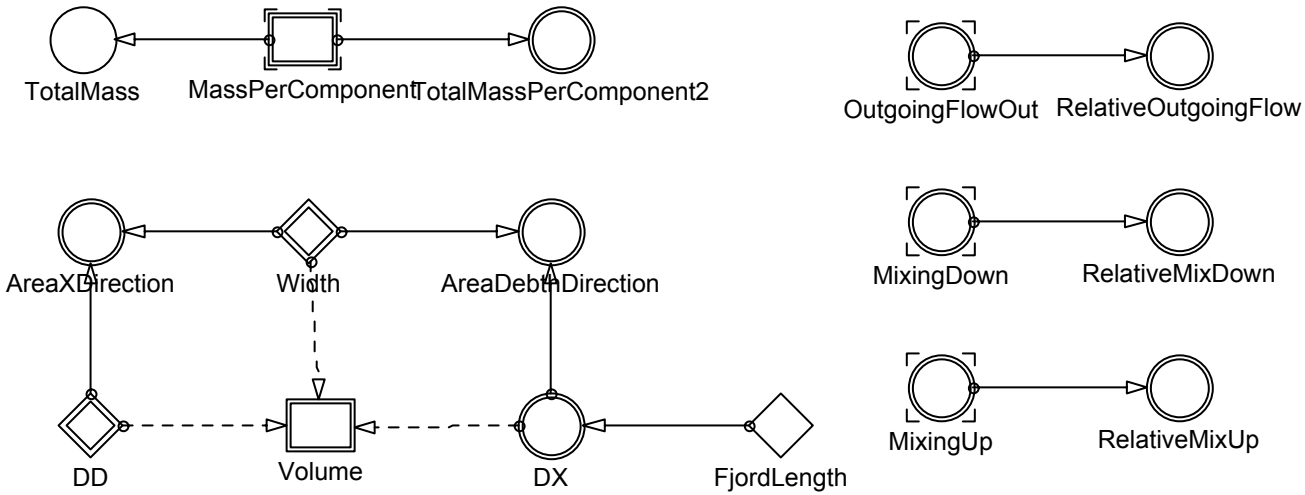
B.1.9 Zooplankton



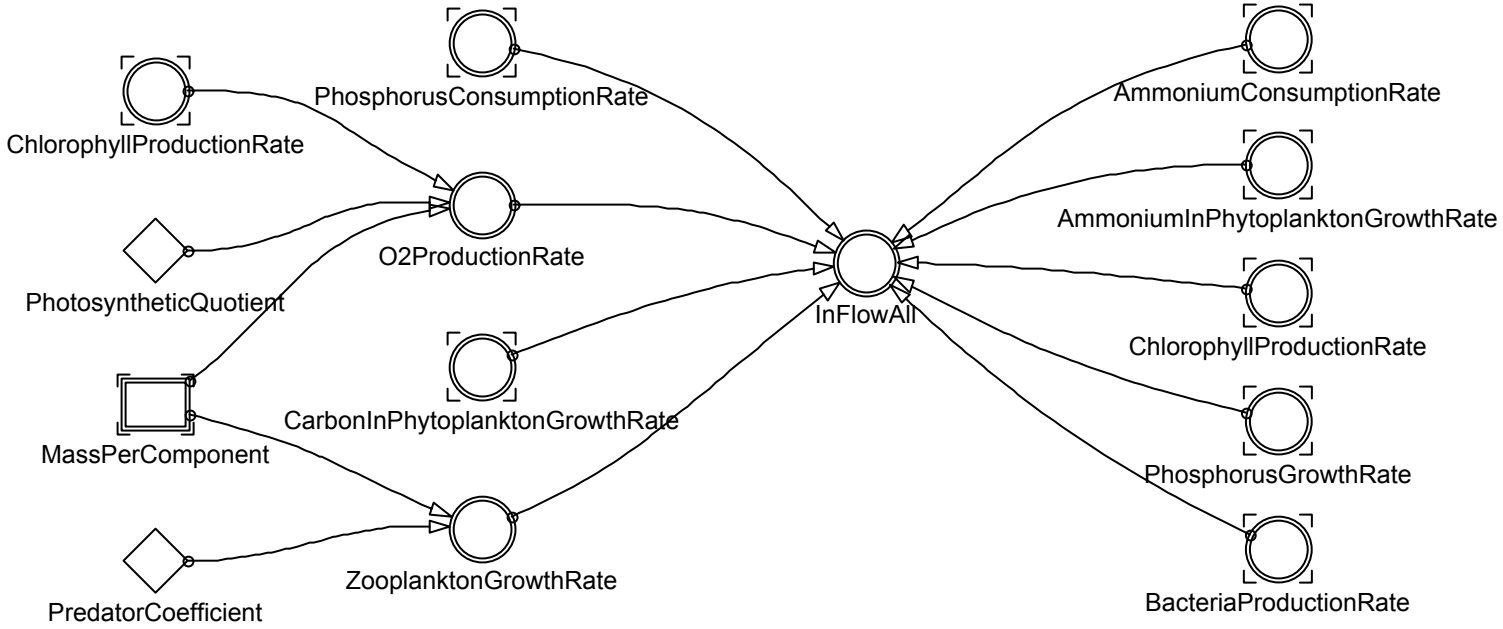
B.2 Stock and Flow Diagrams of the Hydrodynamic Model

B.2.1 The Hydrodynamics

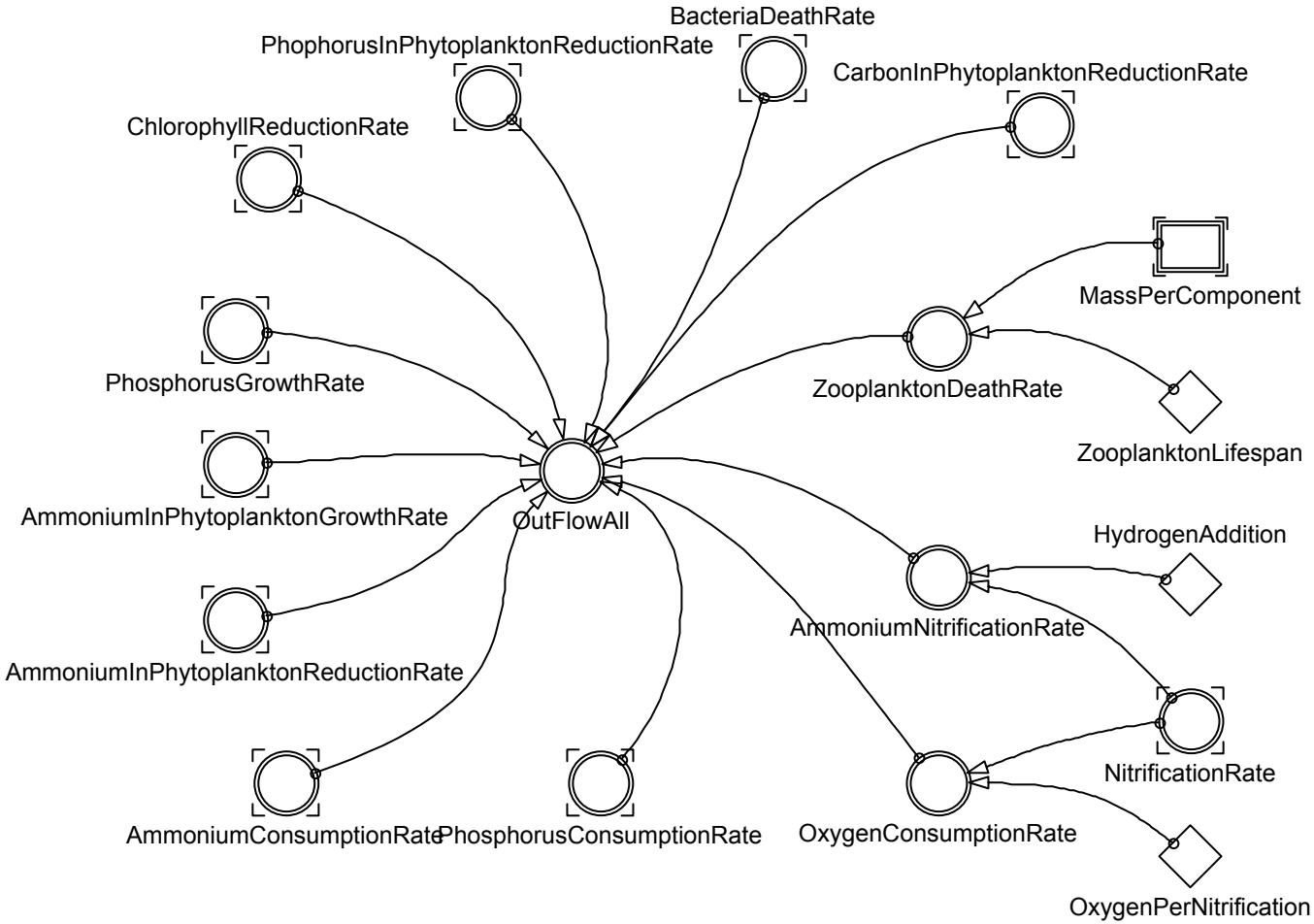



TotalMass
MassPerComponent
TotalMassPerComponent2
OutgoingFlowOut
RelativeOutgoingFlow
AreaXDirection
Width
AreaDepthDirection
MixingDown
RelativeMixDown
DD
Volume
DX
FjordLength
MixingUp
RelativeMixUp

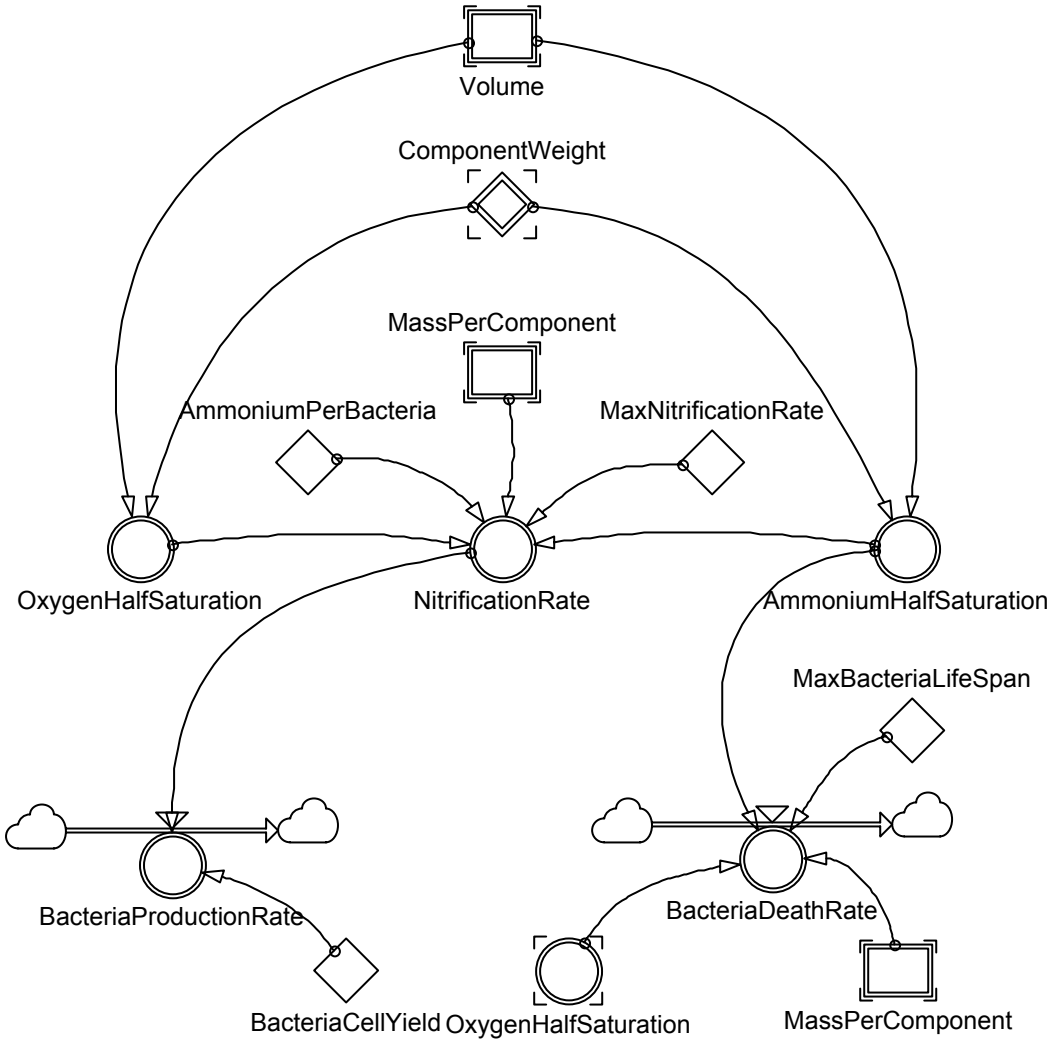
B.2.2 InFlowAll



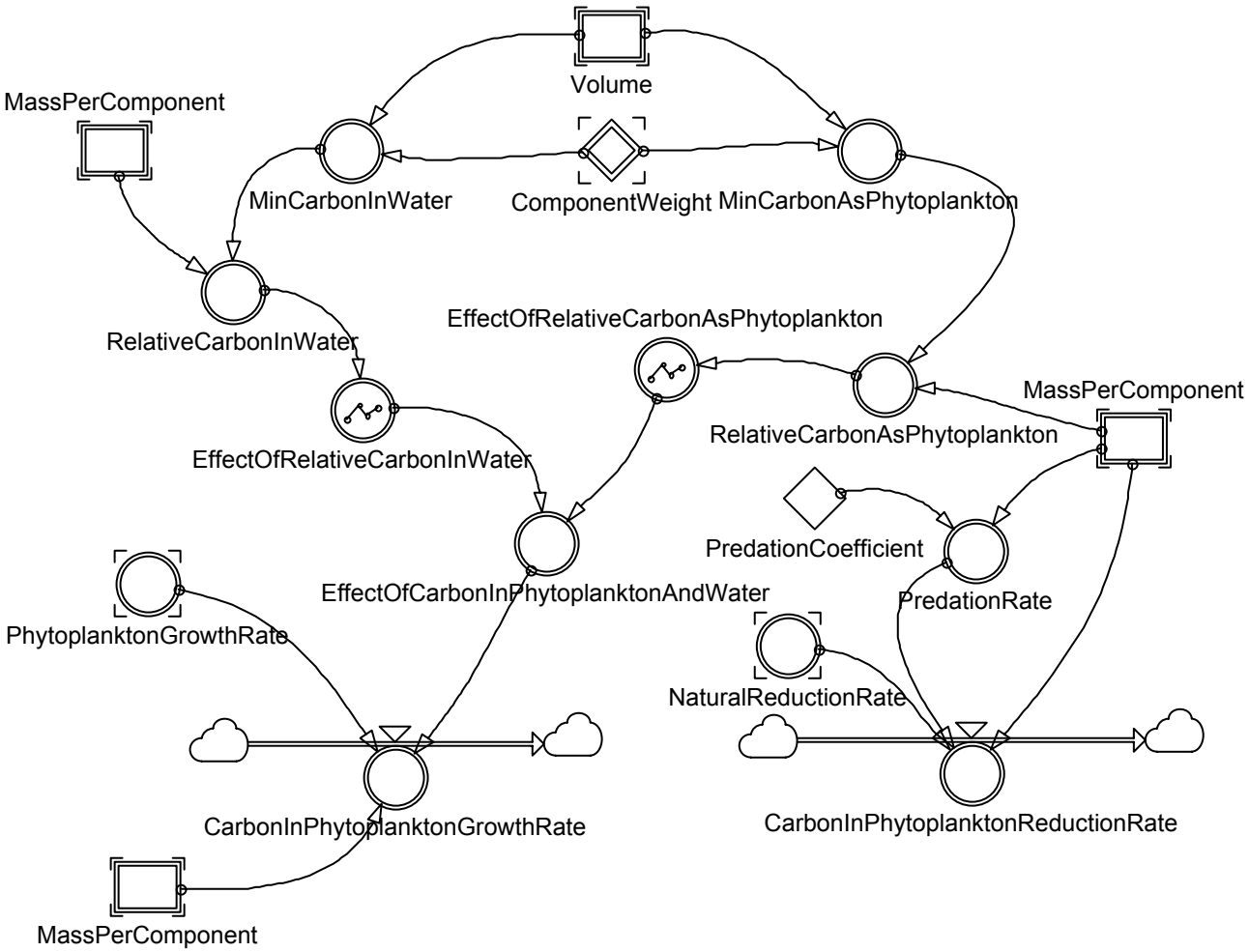
B.2.3 OutFlowAll



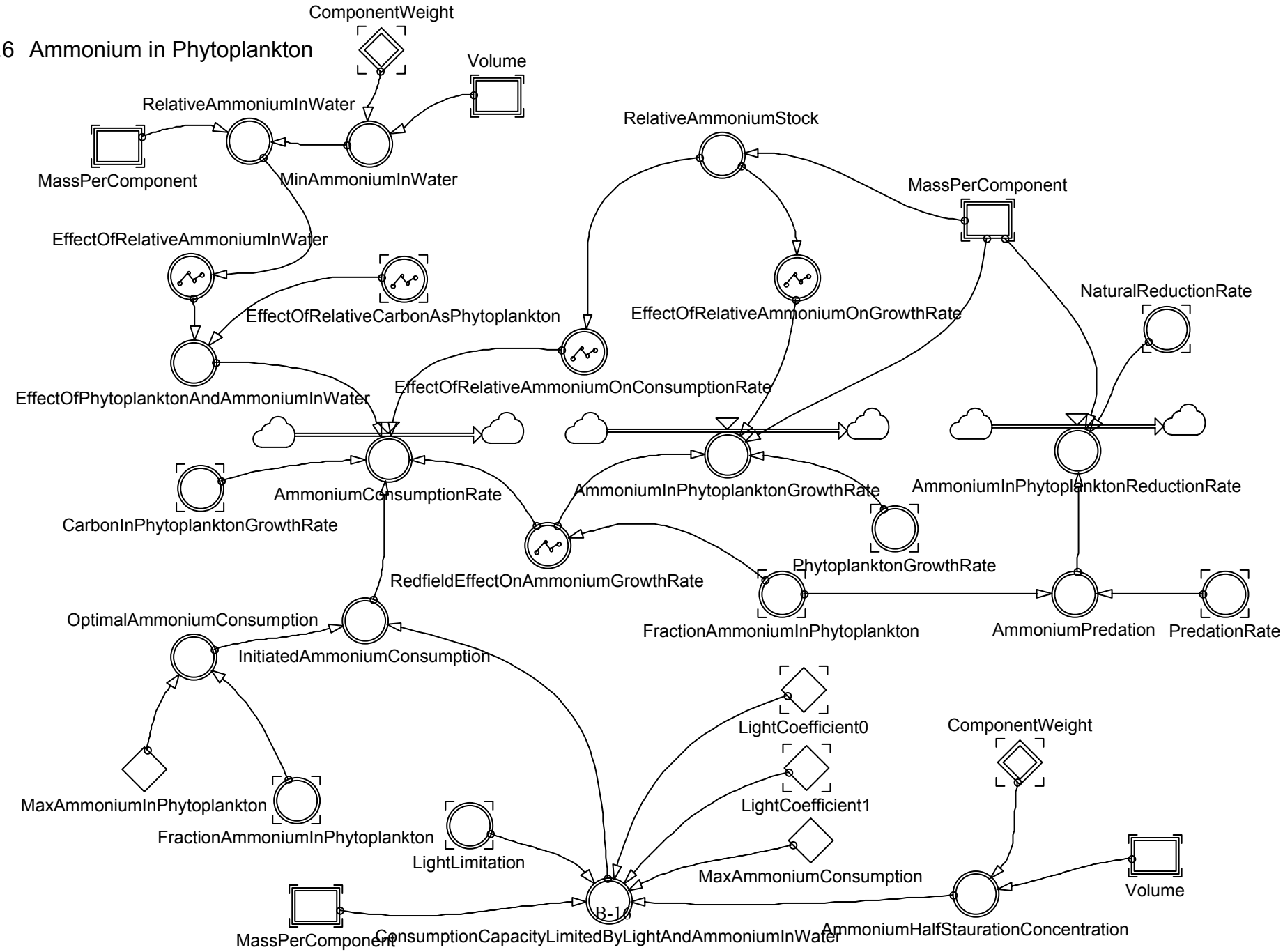
B.2.4 Nitrification



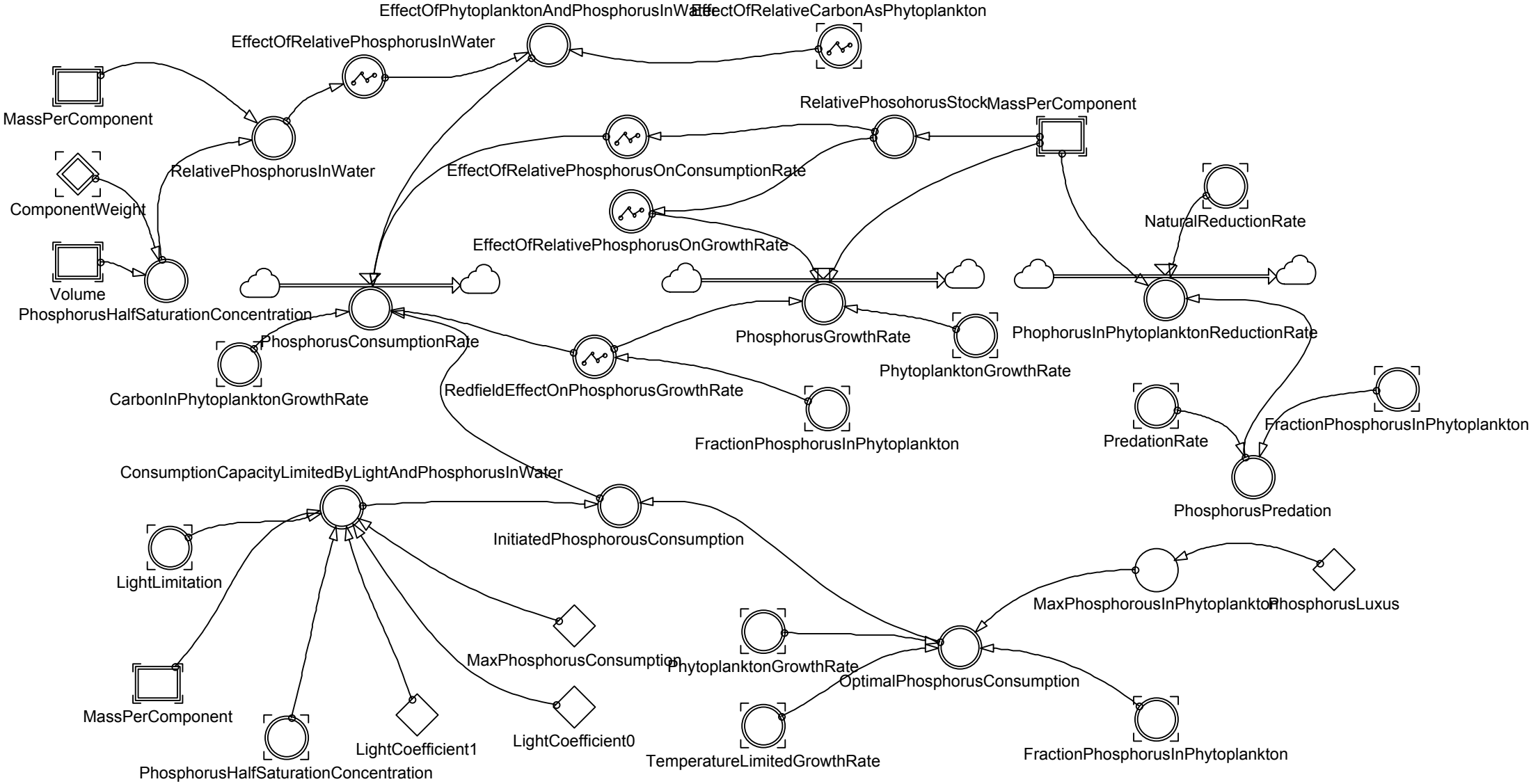
B.2.5 Carbon in Phytoplankton



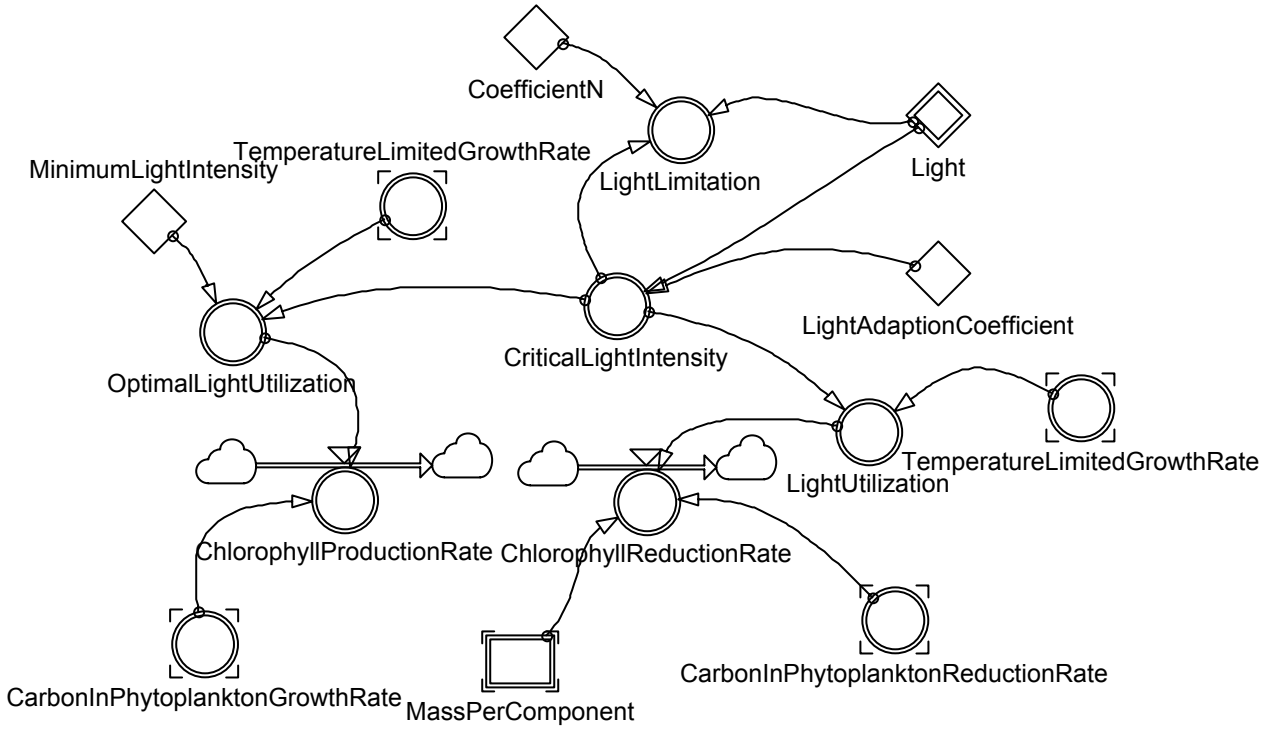
B.2.6 Ammonium in Phytoplankton



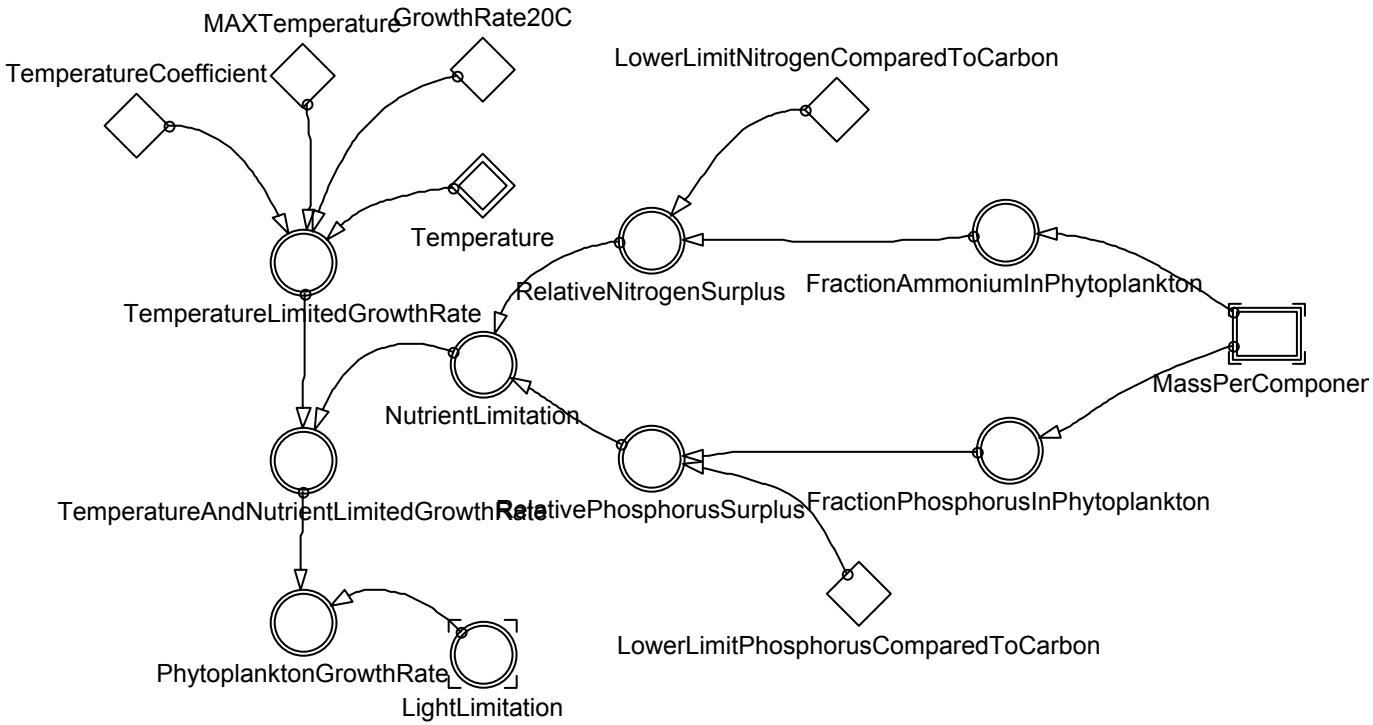
B.2.7 Phosphorus in Phytoplankton



B.2.8 Photosynthesis



B.2.9 Phytoplankton Growth



B.2.10 Phytoplankton Death