Department of APPLIED MATHEMATICS

Kinetic Theory for Evolution of a Plasma in External Electromagnetic Fields toward a State characterized by Balance of Forces Transverse to the Magnetic Field

by

Alf H. Øien

Report No. 40 April 1973

UNIVERSITY OF BERGEN
Bergen, Norway

Kinetic Theory for Evolution of a Plasma in External Electromagnetic Fields toward a State characterized by Balance of Forces Transverse to the Magnetic Field

by

Alf H. Øien

Report No.40 April 1973

Two parameters, ε and α, related to weak inhomogenity and mass ratio between electrons and ions are considered in the kinetic and macroscopic equations for a two component, weakly coupled, weakly inhomogeneous electron - ion plasma in electromagnetic fields. For certain orderings between these parameters $(\varepsilon << \alpha)$ the plasma components evolve toward local equilibrium of the same temperature before the typical transport processes set in. In this report we consider an extreme such ordering. Distribution functions and macroscopic functions are expanded in both parameters.

Solutions of equations are discussed and we show in particular that when solutions of kinetic eqations are assumed bounded on short effective collision time scales the plasma evolves so that forces transverse to the magnetic field in the mass velocity equation balance on longer time scales.

Introduction.

The transport equations of Chapman and Cowling [1] for ionized gases, we can derive by use of the multiple time scale method of Sandri [2], Frieman [3] and Su [4], starting from the following set of parametrized kinetic and macroscopic equations (cf. Appendix 1).

$$\frac{\partial f_i}{\partial t_o} + \varepsilon_1\Big(\underline{c}_o\cdot\frac{\partial f_i}{\partial \underline{r}} + \underline{C}_i\cdot\frac{\partial f_i}{\partial \underline{r}} + \underline{F}_i\cdot\frac{\partial f_i}{\partial \underline{C}_i} + \frac{e_i}{m_i}\underline{c}_o\times\underline{H}\cdot\frac{\partial f_i}{\partial \underline{C}_i} - \underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\cdot\frac{\partial f_i}{\partial \underline{C}_i} - \frac{\partial f_i}{\partial \underline{C}_i}\underline{C}_i:\frac{\partial \underline{c}_o}{\partial \underline{r}}\Big) - \frac{\partial \underline{c}_o}{\partial t_o}\cdot\frac{\partial f_i}{\partial \underline{C}_i} + \frac{e_i}{m_i}\underline{C}_i\times\underline{H}\cdot\frac{\partial f_i}{\partial \underline{C}_i} = \sum_j C_{ij}[f_i f_j] \qquad (1)$$

$$\frac{\partial \rho_i}{\partial t_o} + \varepsilon_1\frac{\partial}{\partial \underline{r}}\cdot(\rho_i\underline{c}_o) + \varepsilon_1\frac{\partial}{\partial \underline{r}}\cdot(\rho_i\,\overline{\underline{C}}_i) = 0 \qquad (2)$$

$$\rho\Big(\frac{\partial \underline{c}_o}{\partial t_o} + \varepsilon_1\underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\Big) = \varepsilon_1\sum_i \rho_i\underline{F}_i + \varepsilon_1\rho_e\underline{c}_o\times\underline{H} + \underline{j}\times\underline{H} - \varepsilon_1\frac{\partial}{\partial \underline{r}}\cdot\underset{\approx}{p} \qquad (3)$$

$$\frac{3}{2}nk\Big(\frac{\partial T}{\partial t_o} + \varepsilon_1\underline{c}_o\cdot\frac{\partial T}{\partial \underline{r}}\Big) = \varepsilon_1\,\frac{3}{2}kT\frac{\partial}{\partial \underline{r}}\cdot\sum_i n_i\overline{\underline{C}}_i + \varepsilon_1\sum_i \rho_i\overline{\underline{C}}_i\cdot\underline{F}_i + \varepsilon_1\underline{j}\cdot(\underline{c}_o\times\underline{H}) - \varepsilon_1\underset{\approx}{p}:\frac{\partial \underline{c}_o}{\partial \underline{r}} - \varepsilon_1\frac{\partial}{\partial \underline{r}}\cdot\underline{q} \qquad (4)$$

Besides, the following a priori condition has to be fulfilled

$$\sum_i \rho_i\overline{\underline{C}}_i = 0 \qquad (5)$$

Indexes i and j refer to types of particles. Later we shall be concerned with a fully ionized plasma of electrons and one type of ions only. Instead of Boltzmann collision terms we shall use Fokker - Planck (FP) collision terms in the kinetic equations. $\underline{F}_i$ and $\frac{e_i}{m_i}\underline{c}_o \times \underline{H}$ are accellerations due to an electric field and a magnetic field $\underline{H}$. The definitions of mass densities ρ_i, charge density ρ_e, mass velocity $\underline{c}_o$, conduction current $\underline{j}$, thermal flow vector $\underline{q}$, temperature T and pressure tensor $\underset{\sim}{P}$ are the same as in [1]. $\overline{\underline{C}}_i$ is the average peculiar velocity of particles of type i. ε_1 is a small parameter which partly can be interpreted as λ/L where λ is the effective mean free path of particles and L is the scale of inhomogenities, and partly as the ratio $|\underline{c}_{o\perp}|/|\underline{C}_i|$ where $\underline{c}_{o\perp}$ is the mass velocity perpendicular to the magnetic field. However, this ordering between $\underline{c}_{o\perp}$ and $\underline{C}_i$ in the equations of Chapman and Cowling is made only in the magnetic force terms. Therefore the parametrization of Chapman and Cowling is only formally consistent and the range of validity of the final equations obtained is not so clear. To clear up this Naze Tjøtta and Øien, [5], [6] and [7] introduced two small parameters ε and α in the kinetic and macroscopic equations. ε relates to weak inhomogenities and weak (electric) fields and $\alpha = (m_1/m_2)^{\frac{1}{2}}$, the square root mass ratio between electrons and ions. Relating these two parameters

to each other a new consistent ordering between all terms in the equations was obtained. The evolution of the gas through its various stages was studied by use of the multiple time scale method. The range of validity of resulting equations for the models studied was clear. These models were:

a: $\alpha \sim \varepsilon$, $\alpha^2 \sim \varepsilon\alpha \sim \varepsilon^2$, etc.

b: $\varepsilon = 0$, α finite

c: $\alpha = 0$, ε finite

Model c discribes the evolution of electrons on a background of immobile ions.The hydrodynamic stage was discribed by two variables, the density and the temperature.
The results for model b are easily obtained from the results for model a . For model a the short time scale is τ_{20} , the effective time between electron - electron collisions. On the long time scale $\tau_{22} \sim \tau_{20}/\alpha^2$ macroscopic equations similar, but not identical to those of Chapman and Cowling were obtained. The relaxation of temperature for electrons and ions also takes place on this long time scale. This feature also differs from the theory of Chapman and Cowling since they consider electrons and ions to have the same temperature. If the gas is homogeneous and no electric field is present, on the τ_{22} time scale the mass velocity perpendicular to the magnetic field dies away and the temperatures of electrons and ions relax against each other.

In this report we study a model where the relaxation of temperature for electrons and ions takes place before the typical transport phenomena due to inhomogenities and electric fields set in. We study an extreme model having this feature:

$$d: \quad \varepsilon < \alpha^n \quad , \quad \text{all} \quad n \geq 0 \quad \text{as} \quad \alpha \to 0 \quad .$$

Distribution functions and macroscopic quantities are sought as two parameter expansions. Also the time derivative is expanded in this way. The sets of equations to zeroth and first order in the parameter ε are written down in section 1 . In section 2 we study these sets of equations as time goes to infinity on the short time scale. The equations to zeroth order in ε are essentially the equations for model b . Using a H - theorem we easily see that on the short time scale electrons and ions evolve toward local equilibrium having then the same temperature to this order, and the perpendicular mass velocity to this order, allowed to be of the same order as the thermal velocity of ions, dies away. As time goes to infinity on the short time scale, from the equations to first order in ε we easily obtain continuity equations and equations for the temperature and mass transport vector parallel to the magnetic field of the same form as in [1] (cf. Appendix 1) except that only the parallel mass transport vector is seen in these equations to this order of approximation. Difficulty arises when trying to solve the kinetic equations and the equation for the perpen-

In this report we study a model where the relaxation of temperature for electrons and ions takes place before the typical transport phenomena due to inhomogeneities and electric fields set in. We study an extreme model having this feature.

$$d: \quad \epsilon < \alpha^n \; , \quad \text{all } n \geq 0 \text{ as } \alpha \to 0$$

Distribution functions and macroscopic quantities are sought as two parameter expansions. Also the time derivative is expanded in this way. The sets of equations to zeroth and first order in the parameter ϵ are written down in section 1. In section 2 we study these sets of equations as time goes to infinity on the short time scale. The equations to zeroth order in ϵ are essentially the equations for model b. Using a H - theorem we easily see that on the short time scale electrons and ions evolve toward local equilibrium having then the same temperature to this order and the perpendicular mass velocity to this order, allowed to be of the same order as the thermal velocity of ions, dies away. As time goes to infinity on the short time scale, from the equations to first order in ϵ we easily obtain continuity equations and equations for the temperature and mass transport vector parallel to the magnetic field of the same form as in [1] (cf. Appendix 1) except that only the parallel mass transport vector is seen in these equations to this order of approximation. Difficulty arises when trying to solve the kinetic equations and the equation for the perpendicular mass transport vector to first order in ε . On the short time scale these equations are strongly coupled, and because of this we have not succeeded in deriving their general form as time goes to infinity on the short time scale. To bypass this difficulty we make the plausible assumption in section 2 that the short time derivative of distribution functions to first order in ε goes to zero as the short time variable goes to infinity. This is a sufficient condition to avoid too strong growth of the distribution functions to first order in ε , but not necessary conditions for the distribution functions to be bounded on the short time scale. Consistent with this assumption we assume that the short time derivative of the perpendicular mass transport vector to first order in ε (of the order of ε times the thermal velocity of electrons) also goes to zero as the short time variable goes to infinity. This gives a balance of transverse forces at the end of the short time scale. With these assumptions we obtain a set of equations for the distribution functions to first order in ε at the end of the short time scale, corresponding to the equations of Chapman and Cowling. The balance of transverse forces determines the perpendicular mass transport vector. In section 3 we partly derive what is assumed in section 2: We derive up to a certain order of approximation that the gas evolves so that time derivatives of bounded distribution functions do vanish as time grows, and in particular that the gas evolves into a state where the transverse forces balance each other. To reach this dicular mass transport vector to first order in ε. On the short time scale these conditions are barely satisfied, and because of this we have not succeeded in deriving their general form as time goes to infinity on the short time scale. To bypass this difficulty we make the plausible assumption in section 2 that the short time derivative of distribution functions to first order in ε goes to zero as the short time variable goes to infinity. This is a sufficient condition to avoid too strong growth of the distribution functions to first order in ε, but not necessary conditions for the distribution functions to be bounded on the short time scale. Consistent with this assumption we assume that the short time derivative of the perpendicular mass transport vector to first order in ε (of the order of ε times the thermal velocity of electrons) also goes to zero as the short time variable goes to infinity. This gives a balance of transverse forces at the end of the short time scale. With these assumptions we obtain a set of equations for the distribution functions to first order in ε at the end of the short time scale, corresponding to the equations of Chapman and Cowling. The balance of transverse forces determines the perpendicular mass transport vector. In section 3 we partly derive what is assumed in section 2: We derive up to a certain order of approximation that the gas evolves so that time derivatives of bounded distribution functions go to zero as time grows, and in particular that the gas evolves into a state where the transverse forces balance each other. To reach this conclusion we expand the sets of equations to zeroth and first order in ε in the parameter α . These equations show the above feature as time grows on the τ_{20} , $\tau_{21} \sim \tau_{20}/\alpha$ and $\tau_{22} \sim \tau_{20}/\alpha^2$ time scales.

1. Basic equations and assumptions

The notations of [1] are used almost throughout. As in [5] , [6] and [7] we study a two component inhomogeneous electron - ion plasma in an external electromagnetic field. The gas is assumed to be weakly coupled, [2], [3] and [4] so that collisions between particles give rise to Fokker-Planck (FP) collision terms only. The temperatures T_1 and T_2 of electrons and ions are initially not equal, but of equal orders of magnitude

$$\tfrac{1}{2}m_1\bar{c}_1^2 \sim \tfrac{1}{2}m_2\bar{c}_2^2$$

Thus
$$\frac{\bar{c}_2}{\bar{c}_1} \sim \left(\frac{m_1}{m_2}\right)^{\frac{1}{2}} = \alpha << 1$$

(Subscripts 1 and 2 refer to electrons and ions respectively).
This is a basic assumption. Besides we have $n_1 \sim n_2$ and $e_1 \sim e_2$ and assuming for mean particle velocities

$$|\underline{\bar{c}}_1| \sim \bar{c}_1$$

$$|\underline{\bar{c}}_2| \sim \alpha\bar{c}_1$$

for the mass transport vector $\underline{c}_o$ we have

$$|\underline{c}_o| = \frac{1}{\rho}|n_1m_1\underline{\bar{c}}_1 + n_2m_2\underline{\bar{c}}_2| \sim \alpha\bar{c}_1$$

In the same way we estimate peculiar velocities $\underline{C}_i$, i.e.

$$|\underline{C}_1| \sim \bar{c}_1$$

$$|\underline{C}_2| \sim \alpha\bar{c}_1$$

Besides the small parameter α we also introduce

$$\varepsilon \sim \frac{\bar{c}_1 \tau_{20}}{L} << 1$$

Here τ_{20} is the effective time between electron-electron collisions and L is the characteristic length scale for all (weak) inhomogenities. For the electric and magnetic fields $\underline{E}$ and $\underline{H}$ we assume that

$$\frac{\tau_{20} |\underline{F}_1|}{|\underline{C}_1|} \sim \varepsilon$$

where $\underline{F}_i = e_i/m_i \underline{E}$ are particle accellerations, and

$$\tau_{20} \Omega_1 \sim 1$$

where $\Omega_i = e_i/m_i H$ are gyrofrequencies. Both $\underline{E}$ and $\underline{H}$ are assumed to be stationary and uniform on the scales we consider, and we neglect the generation of an electromagnetic field by the evolution of the plasma itself. This assumption may be omitted, cf. Appendix 2 .

Since the distribution functions f_1 and f_2 are functions of the peculiar velocities $\underline{C}_1$ and $\underline{C}_2$, besides position vector $\underline{r}$ and time t , we must be aware of the a priori condition, [1] ,

$$\int d\underline{C}_1 f_1(\underline{r},\underline{C}_1,t) m_1 \underline{C}_1 + \int d\underline{C}_2 f_2(\underline{r},\underline{C}_2,t) m_2 \underline{C}_2 = 0$$

Till now assumptions necessary for parametrizing the kinetic equations have been made. When parametrizing the moment equations we assume velocity moments vary on scales the same as the scales for corresponding powers of velocity, i.e.

$$|\overline{\underline{C}}_1| \sim |\underline{C}_1| \sim \overline{c}_1$$

$$|\overline{\underline{C}}_2| \lesssim |\underline{C}_2| \sim \alpha\overline{c}_1$$

$$|m_i\overline{\underline{C}_i\underline{C}_i}| \sim m_i\overline{C_i^2} \sim m_i\overline{c}_i^2$$

$$|\tfrac{1}{2}m_i\overline{C_i^2\underline{C}_i}| \sim m_i\overline{C_i^2C}_i$$

Consequently we parametrize the apriori condition in the following way

$$\alpha\int d\underline{C}_1 f_1 m_i \underline{C}_1 + \int d\underline{C}_2 f_2 m_2 \underline{C}_2 = 0 \tag{6}$$

which also shows necessary in order to avoid a break down of the multiple time scale method when we consider evolution from an initial state far from equilibrium. Kinetic and macroscopic equations are parametrized as follows(t is a time variable on a scale with unit τ_{20})

$$\frac{\partial f_1}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial f_1}{\partial \underline{r}} + \varepsilon\underline{C}_1\cdot\frac{\partial f_1}{\partial \underline{r}} + \varepsilon\underline{F}_1\cdot\frac{\partial f_1}{\partial \underline{C}_1} -$$

$$- \alpha\left(\frac{\partial \underline{c}_o}{\partial t} + \varepsilon\,\alpha\underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\right)\cdot\frac{\partial f_1}{\partial \underline{C}_1} + \alpha\,\frac{e_1}{m_1}\,\underline{c}_o \times \underline{H}\cdot\frac{\partial f_1}{\partial \underline{C}_1} + \frac{e_1}{m_1}\underline{C}_1 \times \underline{H}\cdot\frac{\partial f_1}{\partial \underline{C}_1} -$$

$$- \varepsilon\,\alpha\,\frac{\partial f_1}{\partial \underline{C}_1}\,\underline{C}_1 : \frac{\partial \underline{c}_o}{\partial \underline{r}} = \tag{7}$$

$$= \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1}\cdot\int d\underline{C}_1'\,\underset{\sim}{\Phi}^{11}\,(\underline{C}_1 - \underline{C}_1')\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1'}\right) f_1(\underline{C}_1)f_1(\underline{C}_1') +$$

$$+ \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1}\cdot\int d\underline{C}_2\,\underset{\sim}{\Phi}^{12}\,(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2}\right) f_1(\underline{C}_1)f_2(\underline{C}_2)\ .$$

$$\frac{\partial f_2}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial f_2}{\partial \underline{r}} + \varepsilon\,\alpha\,\underline{C}_2\cdot\frac{\partial f_2}{\partial \underline{r}} + \varepsilon\,\alpha\,\underline{F}_2\cdot\frac{\partial f_2}{\partial \underline{C}_2} -$$

$$- \left(\frac{\partial \underline{c}_o}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\right)\cdot\frac{\partial f_2}{\partial \underline{C}_2} + \alpha^2\,\frac{e_2}{m_2}\,\underline{c}_o \times \underline{H}\cdot\frac{\partial f_2}{\partial \underline{C}_2} +$$

$$+ \alpha^2\,\frac{e_2}{m_2}\,\underline{C}_2 \times \underline{H}\cdot\frac{\partial f_2}{\partial \underline{C}_2} - \varepsilon\,\alpha\,\frac{\partial f_2}{\partial \underline{C}_2}\,\underline{C}_2 : \frac{\partial \underline{c}_o}{\partial \underline{r}} = \tag{8}$$

$$= \frac{\alpha}{m_2}\,\frac{\partial}{\partial \underline{C}_2}\cdot\int d\underline{C}_2'\,\underset{\sim}{\Phi}^{22}\,(\underline{C}_2 - \underline{C}_2')\cdot\left(\frac{1}{m_2}\,\frac{\partial}{\partial \underline{C}_2} - \frac{1}{m_2}\,\frac{\partial}{\partial \underline{C}_2'}\right)f_2(\underline{C}_2)f_2(\underline{C}_2') +$$

$$+ \frac{\alpha}{m_2}\,\frac{\partial}{\partial \underline{C}_2}\cdot\int d\underline{C}_1\,\underset{\sim}{\Phi}^{21}\,(\alpha\underline{C}_2 - \underline{C}_1)\cdot\left(\frac{\alpha}{m_2}\,\frac{\partial}{\partial \underline{C}_2} - \frac{1}{m_1}\,\frac{\partial}{\partial \underline{C}_1}\right)f_1(\underline{C}_1)f_2(\underline{C}_2)\,.$$

Here $\underset{\sim}{\Phi}^{ij}(\underline{w})$ are tensors taking into account the weak interaction between particles. For suitable cut-offs they take Landau's form [8]. Note that α also appears inside some collision terms.

$$\frac{\partial \rho}{\partial t} + \varepsilon\,\alpha\,\frac{\partial}{\partial \underline{r}}\cdot(\rho\underline{c}_o) = 0 \tag{9}$$

$$\rho\left(\frac{\partial \underline{c}_o}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial \underline{c}_o}{\partial \underline{r}}\right) = -\varepsilon\,\alpha\,\frac{\partial}{\partial \underline{r}}\cdot\sum_{i=1}^{2} n_i m_i \overline{\underline{C}_i\underline{C}_i} + \alpha^2\rho_e\underline{c}_o \times \underline{H} +$$

$$+ \alpha n_1 e_1\overline{\underline{C}}_1 \times \underline{H} + \alpha^2 n_2 e_2\overline{\underline{C}}_2 \times \underline{H} + \varepsilon\,\alpha\sum_{i=1}^{2}\rho_i\underline{F}_i \tag{10}$$

$$\frac{3}{2} nk\left(\frac{\partial T}{\partial t} + \varepsilon \alpha \underline{c}_o \cdot \frac{\partial T}{\partial \underline{r}}\right) = \varepsilon \frac{3}{2} kT \frac{\partial}{\partial \underline{r}} \cdot (n_1 \overline{\underline{C}}_1) +$$

$$+ \varepsilon \alpha \frac{3}{2} kT \frac{\partial}{\partial \underline{r}} \cdot (n_2 \overline{\underline{C}}_2) + \varepsilon \rho_1 \underline{F}_1 \cdot \overline{\underline{C}}_1 + \varepsilon \alpha \rho_2 \underline{F}_2 \cdot \overline{\underline{C}}_2 +$$

$$+ \alpha n_1 e_1 \overline{\underline{C}}_1 \cdot (\underline{c}_o \times \underline{H}) + \alpha^2 n_2 e_2 \overline{\underline{C}}_2 \cdot (\underline{c}_o \times \underline{H}) - \varepsilon \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_1 - \tag{11}$$

$$- \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_2 - \varepsilon \alpha \sum_{i=1}^{2} n_i m_i \overline{\underline{C}_i \underline{C}_i} : \frac{\partial \underline{c}_o}{\partial \underline{r}}$$

We also take into account the following moment equations for each gas component:

$$\frac{\partial \rho_1}{\partial t} + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_1 \underline{c}_o) + \varepsilon \frac{\partial}{\partial \underline{r}} \cdot (\rho_1 \overline{\underline{C}}_1) = 0 \tag{12}$$

$$\frac{\partial \rho_2}{\partial t} + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_2 \underline{c}_o) + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_2 \overline{\underline{C}}_2) = 0 \tag{13}$$

$$\frac{\partial}{\partial t} (\rho_1 \overline{\underline{C}}_1) + \varepsilon \alpha \underline{c}_o \cdot \frac{\partial}{\partial \underline{r}} (\rho_1 \overline{\underline{C}}_1) + \varepsilon \alpha \rho_1 \overline{\underline{C}}_1 \frac{\partial}{\partial \underline{r}} \cdot \underline{c}_o +$$

$$+ \varepsilon \frac{\partial}{\partial \underline{r}} \cdot (\rho_1 \overline{\underline{C}_1 \underline{C}_1}) - \varepsilon \rho_1 \underline{F}_1 + \alpha \rho_1 \left(\frac{\partial \underline{c}_o}{\partial t} + \varepsilon \alpha \underline{c}_o \cdot \frac{\partial \underline{c}_o}{\partial \underline{r}}\right) -$$

$$- \alpha n_1 e_1 \underline{c}_o \times \underline{H} - n_1 e_1 \overline{\underline{C}}_1 \times \underline{H} + \varepsilon \alpha \rho_1 \overline{\underline{C}}_1 \cdot \frac{\partial \underline{c}_o}{\partial \underline{r}} = \tag{14}$$

$$= - \int d\underline{C}_1 d\underline{C}_2 \underset{\sim}{\Phi}^{12} (\underline{C}_1 - \alpha \underline{C}_2) \cdot \left(\frac{1}{m_1} \frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2} \frac{\partial}{\partial \underline{C}_2}\right) f_1(\underline{C}_1) f_2(\underline{C}_2)$$

$$\frac{\partial}{\partial t}(\rho_2\overline{\underline{C}}_2) + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial}{\partial\underline{r}}(\rho_2\overline{\underline{C}}_2) + \varepsilon\,\alpha\,\rho_2\overline{\underline{C}}_2\frac{\partial}{\partial\underline{r}}\cdot\underline{c}_o +$$

$$+\varepsilon\,\alpha\,\frac{\partial}{\partial\underline{r}}\cdot(\rho_2\overline{\underline{C}_2\underline{C}_2}) - \varepsilon\,\alpha\,\rho_2\underline{F}_2 + \rho_2\left(\frac{\partial\underline{c}_o}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial\underline{c}_o}{\partial\underline{r}}\right) -$$

$$-\alpha^2 n_2 e_2\underline{c}_o\times\underline{H} - \alpha^2 n_2 e_2\overline{\underline{C}}_2\times\underline{H} + \varepsilon\,\alpha\,\rho_2\overline{\underline{C}}_2\cdot\frac{\partial\underline{c}_o}{\partial\underline{r}} = \tag{15}$$

$$= -\alpha\int d\underline{C}_1 d\underline{C}_2\ \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{\alpha}{m_2}\frac{\partial}{\partial\underline{C}_2} - \frac{1}{m_1}\frac{\partial}{\partial\underline{C}_1}\right)f_1(\underline{C}_1)f_2(\underline{C}_2)\ .$$

$$\frac{3}{2}n_1 k\left(\frac{\partial T_1}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial T_1}{\partial\underline{r}}\right) - \frac{3}{2}kT_1\frac{\partial}{\partial\underline{r}}\cdot(n_1\overline{\underline{C}}_1) + \varepsilon\frac{\partial}{\partial\underline{r}}\cdot\underline{q}_1 -$$

$$-\varepsilon\,\rho_1\underline{F}_1\cdot\overline{\underline{C}}_1 + \alpha\,\rho_1\overline{\underline{C}}_1\cdot\left(\frac{\partial\underline{c}_o}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial\underline{c}_o}{\partial\underline{r}}\right) -$$

$$-\alpha\,n_1 e_1\overline{\underline{C}}_1\cdot\underline{c}_o\times\underline{H} + \varepsilon\,\alpha\,\rho_1\overline{\underline{C}_1\underline{C}_1} : \frac{\partial\underline{c}_o}{\partial\underline{r}} = \tag{16}$$

$$= -\alpha\int d\underline{C}_1 d\underline{C}_2\underline{C}_2\cdot\underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial\underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial\underline{C}_2}\right)f_1(\underline{C}_1)f_2(\underline{C}_2)\ .$$

$$\frac{3}{2}n_2 k\left(\frac{\partial T_2}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial T_2}{\partial\underline{r}}\right) - \varepsilon\,\alpha\,\frac{3}{2}kT_2\frac{\partial}{\partial\underline{r}}\cdot(n_2\overline{\underline{C}}_2) +$$

$$+\varepsilon\,\alpha\frac{\partial}{\partial\underline{r}}\cdot\underline{q}_2 - \varepsilon\,\alpha\,\rho_2\underline{F}_2\cdot\overline{\underline{C}}_2 + \rho_2\overline{\underline{C}}_2\cdot\left(\frac{\partial\underline{c}_o}{\partial t} + \varepsilon\,\alpha\,\underline{c}_o\cdot\frac{\partial\underline{c}_o}{\partial\underline{r}}\right) -$$

$$-\alpha^2 n_2 e_2\overline{\underline{C}}_2\cdot(\underline{c}_o\times\underline{H}) + \varepsilon\,\alpha\,\rho_2\overline{\underline{C}_2\underline{C}_2} : \frac{\partial\underline{c}_o}{\partial\underline{r}} = \tag{17}$$

$$= -\alpha\int d\underline{C}_1 d\underline{C}_2\underline{C}_2\cdot\underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{\alpha}{m_2}\frac{\partial}{\partial\underline{C}_2} - \frac{1}{m_1}\frac{\partial}{\partial\underline{C}_1}\right)f_1(\underline{C}_1)f_2(\underline{C}_2)\ .$$

The models studied in [5] and [6] were

a) $\varepsilon \sim \alpha$, $\varepsilon^2 \sim \varepsilon\alpha \sim \alpha^2$ etc.

b) $\varepsilon = 0$, $\alpha \neq 0$ but small

c) $\alpha = 0$, $\varepsilon \neq 0$ but small .

We shall now study a model which we shall characterize by

d) $(0<)\ \varepsilon < \alpha^n$ all $n \geqq 0$ as $\alpha \to 0$.

This is an extreme model having the property that electrons and ions reach the same temperature before the typical transport processes set in. We first expand the distribution functions f_1 and f_2 and macroscopic functions in series in the parameter ε , for instance

$$f_i = f_i^{(0)} + \varepsilon f_i^{(1)} + \dots$$

$$\rho = \rho^{(0)} + \varepsilon\, \rho^{(1)} + \dots$$

where $$\rho^{(k)} = \sum_{i=1}^{2} \int f_i^{(k)}(\underline{C}) m_i d\underline{C}_i$$

There is no such connection between expansions for f_i and the mass transport velocity $\underline{c}_o$. However,

we seek $\underline{c}_o$ as

$$\underline{c}_o = \underline{c}_o^{(0)} + \varepsilon\, \underline{c}_o^{(1)} + \ldots.$$

Here we note that $|\underline{c}_o|/\overline{c}_1 \sim |\underline{c}_o^{(0)}|/\overline{c}_1 \sim \alpha$.

However $|\underline{c}_o^{(1)}|/\overline{c}_1 \sim \varepsilon$ so that $|\underline{c}_o^{(1)}|/|\underline{c}_o^{(0)}| \sim \varepsilon/\alpha$.

This must be taken into account in the equations to first and higher orders in the parameter ε .

*

We introduce time variables t_i , $i = 2,3, \ldots,$ on time scales $\tau_i = \dfrac{\tau_2}{\varepsilon^{i-2}}$. Therefore the time derivative is expanded as follows

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_2} + \varepsilon \frac{\partial}{\partial t_3} + \varepsilon^2 \frac{\partial}{\partial t_4} + \ldots..$$

We substitute these expansions into Eqs(6) - (17) and separate in zeroth, first, second order etc in ε .
In each of these sets of equations we have the parameter α .
We can expand functions in each set in series of the parameter α and in each set separate in zeroth, first, second order etc in the parameter α . Thus, in all, we make a double expansion of functions in series of parameters ε and α , for instance

$$f_i = f_i^{(00)} + \alpha\, f_i^{(01)} + \alpha^2 f_i^{(02)} + \ldots.. + \\ + \varepsilon \left[f_i^{(10)} + \alpha\, f_i^{(11)} + \ldots \right] + \varepsilon^2 \left[f_i^{(20)} + \ldots. \right] +$$

Similarly the time derivative in all is expanded as follows

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_{20}} + \alpha \frac{\partial}{\partial t_{21}} + \alpha^2 \frac{\partial}{\partial t_{22}} + \ldots\ldots +$$

$$+ \varepsilon \left[\frac{\partial}{\partial t_{30}} + \alpha \frac{\partial}{\partial t_{31}} + \ldots \right] + \varepsilon^2 \left[\frac{\partial}{\partial t_{40}} + \ldots \right] + \ldots$$

consistent with introduction of time variables t_{ij}

$i = 2,3, \ldots$ and $j = 0,1,2, \ldots$ on time scales $\tau_{ij} = \frac{\tau_i}{\alpha^j}$.

The Φ^{ij} functions appearing in the collision integrals of Eqs.(7) and (8) are expanded in terms of α by use of Taylors formula, assumed distributional convergent.
Each set of equations to each order in ε and α should be solved eliminating all secular terms that arise.

*

We end this section writing down kinetic and macroscopic equations including the a priori condition to zeroth and first order in the parameter ε . Note the differences between this set of equations and the corresponding set of equations of Chapman and Cowling (cf.Appendix 1).
All ( ) in superscripts are omitted from now on.

Zeroth order equations in the parameter ε

$$\alpha \int d\underline{C}_1 f_1^o m_1 \underline{C}_1 + \int d\underline{C}_2 f_2^o m_2 \underline{C}_2 = 0 \tag{18}$$

$$\frac{\partial f_1^o}{\partial t_2} - \alpha \frac{\partial \underline{c}_o^o}{\partial t_2} \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} + \alpha \frac{e_1}{m_1} \underline{c}_o^o \times \underline{H} \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} + \frac{e_1}{m_1} \underline{C}_1 \times \underline{H} \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} =$$
$$= FP_{11}\left[f_1^o(\underline{C}_1) f_1^o(\underline{C}_1')\right] + FP_{12}\left[f_1^o(\underline{C}_1) f_2^o(\underline{C}_2)\right] \tag{19}$$

$$\frac{\partial f_2^o}{\partial t_2} - \frac{\partial \underline{c}_o^o}{\partial t_2} \cdot \frac{\partial f_2^o}{\partial \underline{C}_2} + \alpha^2 \frac{e_2}{m_2} \underline{c}_o^o \times \underline{H} \cdot \frac{\partial f_2^o}{\partial \underline{C}_2} + \alpha^2 \frac{e_2}{m_2} \underline{C}_2 \times \underline{H} \cdot \frac{\partial f_2^o}{\partial \underline{C}_2} =$$
$$= \alpha\, FP_{22}\left[f_2^o(\underline{C}_2) f_2^o(\underline{C}_2')\right] + \alpha\, FP_{21}\left[f_2^o(\underline{C}_2) f_1^o(\underline{C}_1)\right] \tag{20}$$

$$\frac{\partial \rho^o}{\partial t_2} = 0 \tag{21}$$

$$\rho^o \frac{\partial \underline{c}_o^o}{\partial t_2} = \alpha^2 \rho_e^o \underline{c}_o^o \times \underline{H} + \alpha n_1^o e_1 \overline{\underline{C}}_1^o \times \underline{H} + \alpha^2 n_2^o e_2 \overline{\underline{C}}_2^o \times \underline{H} \tag{22}$$

$$\frac{3}{2} n^o k \frac{\partial T^o}{\partial t_2} = \alpha n_1^o e_1 \overline{\underline{C}}_1^o \cdot (\underline{c}_o^o \times \underline{H}) + \alpha^2 n_2^o e_2 \overline{\underline{C}}_2^o \cdot (\underline{c}_o^o \times \underline{H}) \tag{23}$$

$$\frac{\partial \rho_1^o}{\partial t_2} = 0 \tag{24}$$

$$\frac{\partial \rho_2^o}{\partial t_2} = 0 \tag{25}$$

$$\frac{\partial}{\partial t_2}(\rho_1^o\overline{\underline{C}}_1^o) + \alpha\ \rho_1^o\ \frac{\partial \underline{c}_o^o}{\partial t_2} - \alpha\ n_1^o e_1 \underline{c}_o^o \times \underline{H} - n_1^o e_1 \overline{\underline{C}}_1^o \times \underline{H} =$$

$$= -\int d\underline{C}_1 d\underline{C}_2 \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2}\right) f_1^o(\underline{C}_1) f_2^o(\underline{C}_2) \tag{26}$$

$$\frac{\partial}{\partial t_2}(\rho_2^o\overline{\underline{C}}_2^o) + \rho_2^o\ \frac{\partial \underline{c}_o^o}{\partial t_2} - \alpha^2 n_2 e_2 \underline{c}_o^o \times \underline{H} - \alpha^2 n_2 e_2 \overline{\underline{C}}_2^o \times \underline{H} =$$

$$= -\ \alpha\int d\underline{C}_1 d\underline{C}_2 \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2} - \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1}\right) f_1^o(\underline{C}_1) f_2^o(\underline{C}_2) \tag{27}$$

$$\frac{3}{2} n_1^o k\ \frac{\partial T_1^o}{\partial t_2} + \alpha\ \rho_1^o \overline{\underline{C}}_1^o \cdot \frac{\partial \underline{c}_o^o}{\partial t_2} - \alpha\ n_1^o e_1 \overline{\underline{C}}_1^o \cdot (\underline{c}_o^o \times \underline{H}) =$$

$$= -\ \alpha\int d\underline{C}_1 d\underline{C}_2 \underline{C}_2 \cdot \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2}\right) f_1^o(\underline{C}_1) f_2^o(\underline{C}_2) \tag{28}$$

$$\frac{3}{2} n_2^o k\ \frac{\partial T_2^o}{\partial t_2} + \rho_2^o\ \overline{\underline{C}}_2^o \cdot \frac{\partial \underline{c}_o^o}{\partial t_2} - \alpha^2\ n_2^o e_2 \overline{\underline{C}}_2^o \cdot (\underline{c}_o^o \times \underline{H}) =$$

$$= -\ \alpha\int d\underline{C}_1 d\underline{C}_2 \underline{C}_2 \cdot \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2} - \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1}\right) f_1^o(\underline{C}_1) f_2^o(\underline{C}_2) \tag{29}$$

First order equations in the parameter ε

$$\alpha \int d\underline{C}_1 f_1^1 m_1 \underline{C}_1 + \int d\underline{C}_2 f_2^1 m_2 \underline{C}_2 = 0 \tag{30}$$

$$\frac{\partial f_1^1}{\partial t_2} + \frac{\partial f_1^o}{\partial t_3} + \alpha\, \underline{c}_o^o \cdot \frac{\partial f_1^o}{\partial \underline{r}} + \underline{C}_1 \cdot \frac{\partial f_1^o}{\partial \underline{r}} + \underline{F}_1 \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} - \alpha \left(\frac{1}{\alpha} \frac{\partial \underline{c}_o^1}{\partial t_2} + \right.$$

$$\left. + \frac{\partial \underline{c}_o^o}{\partial t_3} + \alpha\, \underline{c}_o^o \cdot \frac{\partial \underline{c}_o^o}{\partial \underline{r}} \right) \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} - \alpha \frac{\partial \underline{c}_o^o}{\partial t_2} \cdot \frac{\partial f_1^1}{\partial \underline{C}_1} + \alpha \frac{e_1}{m_1} \underline{c}_o^o \times \underline{H} \cdot \frac{\partial f_1^1}{\partial \underline{C}_1} +$$

$$+ \frac{e_1}{m_1} \underline{c}_o^1 \times \underline{H} \cdot \frac{\partial f_1^o}{\partial \underline{C}_1} + \frac{e_1}{m_1} \underline{C}_1 \times \underline{H} \cdot \frac{\partial f_1^1}{\partial \underline{C}_1} - \alpha \frac{\partial f_1^o}{\partial \underline{C}_1} : \frac{\partial \underline{c}_o^o}{\partial \underline{r}} = \tag{31}$$

$$= FP_{11}\left[f_1^o(\underline{C}_1) f_1^1(\underline{C}_1') + f_1^1(\underline{C}_1) f_1^o(\underline{C}_1') \right] +$$

$$+ FP_{12}\left[f_1^o(\underline{C}_1) f_2^1(\underline{C}_2) + f_1^1(\underline{C}_1) f_2^o(\underline{C}_2) \right]$$

$$\frac{\partial f_2^1}{\partial t_2} + \frac{\partial f_2^o}{\partial t_3} + \alpha\, \underline{c}_o^o \cdot \frac{\partial f_2^o}{\partial \underline{r}} + \alpha\, \underline{C}_2 \cdot \frac{\partial f_2^o}{\partial \underline{r}} + \alpha\, \underline{F}_2 \cdot \frac{\partial f_2^o}{\partial \underline{C}_2} - \left(\frac{1}{\alpha} \frac{\partial \underline{c}_o^1}{\partial t_2} + \right.$$

$$\left. + \frac{\partial \underline{c}_o^o}{\partial t_3} + \alpha\, \underline{c}_o^o \cdot \frac{\partial \underline{c}_o^o}{\partial \underline{r}} \right) \cdot \frac{\partial f_2^o}{\partial \underline{C}_2} - \frac{\partial \underline{c}_o^o}{\partial t_2} \cdot \frac{\partial f_2^1}{\partial \underline{C}_2} + \alpha^2 \frac{e_2}{m_2} \underline{c}_o^o \times \underline{H} \cdot \frac{\partial f_2^1}{\partial \underline{C}_2} +$$

$$+ \alpha \frac{e_2}{m_2} \underline{c}_o^1 \times \underline{H} \cdot \frac{\partial f_2^o}{\partial \underline{C}_2} + \alpha^2 \frac{e_2}{m_2} \underline{C}_2 \times \underline{H} \cdot \frac{\partial f_2^1}{\partial \underline{C}_2} - \alpha \frac{\partial f_2^o}{\partial \underline{C}_2} \underline{C}_2 : \frac{\partial \underline{c}_o^o}{\partial \underline{r}} =$$

$$= \alpha\, FP_{22}\left[f_2^o(\underline{C}_2) f_2^1(\underline{C}_2') + f_2^1(\underline{C}_2) f_2^o(\underline{C}_2') \right] + \tag{32}$$

$$+ \alpha\, FP_{21}\left[f_2^o(\underline{C}_2) f_1^1(\underline{C}_1) + f_2^1(\underline{C}_2) f_1^o(\underline{C}_1) \right]$$

$$\frac{\partial \rho^1}{\partial t_2} + \frac{\partial \rho^o}{\partial t_3} + \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho^o \underline{c}_o^o) = 0 \tag{33}$$

$$\begin{aligned}
&\rho^{o}\left(\frac{1}{\alpha}\frac{\partial \underline{c}_{o}^{1}}{\partial t_2} + \frac{\partial \underline{c}_{o}^{o}}{\partial t_3} + \alpha\, \underline{c}_{o}^{o}\cdot\frac{\partial \underline{c}_{o}^{o}}{\partial \underline{r}}\right) + \rho^{1}\frac{\partial \underline{c}_{o}^{o}}{\partial t_2} = -\,\alpha\,\frac{\partial}{\partial \underline{r}}\cdot\sum_{i=1}^{2} n_i^{o} m_i \overline{\underline{C}_i\underline{C}_i}^{o} + \\
&+ \alpha\, \rho_e^{o}\underline{c}_{o}^{1}\times\underline{H} + \alpha^2\, \rho_e^{1}\underline{c}_{o}^{o}\times\underline{H} + \alpha\, n_1^{o}e_1\overline{\underline{C}}_1^{1}\times\underline{H} + \alpha\, n_1^{1}e_1\overline{\underline{C}}_1^{o}\times\underline{H} + \\
&+ \alpha^2\, n_2^{o}e_2\overline{\underline{C}}_2^{1}\times\underline{H} + \alpha^2\, n_2^{1}e_2\overline{\underline{C}}_2^{o}\times\underline{H} + \alpha\sum_{i=1}^{2}\rho_i^{o}\,\underline{F}_i
\end{aligned} \tag{34}$$

$$\begin{aligned}
&\tfrac{3}{2}n^{o}k\left(\frac{\partial T^{1}}{\partial t_2} + \frac{\partial T^{o}}{\partial t_3} + \alpha\, \underline{c}_{o}^{o}\cdot\frac{\partial T^{o}}{\partial \underline{r}}\right) + \tfrac{3}{2}n^{1}k\,\frac{\partial T^{o}}{\partial t_2} = \tfrac{3}{2}kT^{o}\,\frac{\partial}{\partial \underline{r}}\cdot(n_1^{o}\overline{\underline{C}}_1^{o}) + \\
&+ \alpha\,\tfrac{3}{2}kT^{o}\,\frac{\partial}{\partial \underline{r}}\cdot(n_2^{o}\overline{\underline{C}}_2^{o}) + \rho_1^{o}\underline{F}_1\cdot\overline{\underline{C}}_1^{o} + \alpha\,\rho_2^{o}\underline{F}_2\cdot\overline{\underline{C}}_2^{o} + n_1^{o}e_1\overline{\underline{C}}_1^{o}\cdot(\underline{c}_{o}^{1}\times\underline{H}) + \\
&+ \alpha\, n_1^{o}e_1\overline{\underline{C}}_1^{1}\cdot(\underline{c}_{o}^{o}\times\underline{H}) + \alpha\, n_1^{1}e_1\overline{\underline{C}}_1^{o}\cdot(\underline{c}_{o}^{o}\times\underline{H}) + \alpha\, n_2^{o}e_2\overline{\underline{C}}_2^{o}\cdot(\underline{c}_{o}^{1}\times\underline{H}) + \\
&+ \alpha^2\, n_2^{o}e_2\overline{\underline{C}}_2^{1}\cdot(\underline{c}_{o}^{o}\times\underline{H}) + \alpha^2\, n_2^{1}e_2\overline{\underline{C}}_2^{o}\cdot(\underline{c}_{o}^{o}\times\underline{H}) - \frac{\partial}{\partial \underline{r}}\cdot\underline{q}_1^{o} - \alpha\,\frac{\partial}{\partial \underline{r}}\cdot\underline{q}_2^{o} - \\
&- \alpha\sum_{i=1}^{2} n_i^{o}m_i\overline{\underline{C}_i\underline{C}_i}^{o} : \frac{\partial \underline{c}_{o}^{o}}{\partial \underline{r}}
\end{aligned} \tag{35}$$

$$\frac{\partial \rho_1^{1}}{\partial t_2} + \frac{\partial \rho_1^{o}}{\partial t_3} + \alpha\,\frac{\partial}{\partial \underline{r}}\cdot(\rho_1^{o}\underline{c}_{o}^{o}) + \frac{\partial}{\partial \underline{r}}\cdot(\rho_1^{o}\overline{\underline{C}}_1^{o}) = 0 \tag{36}$$

$$\frac{\partial \rho_2^{1}}{\partial t_2} + \frac{\partial \rho_2^{o}}{\partial t_3} + \alpha\,\frac{\partial}{\partial \underline{r}}\cdot(\rho_2^{o}\underline{c}_{o}^{o}) + \alpha\,\frac{\partial}{\partial \underline{r}}\cdot(\rho_2^{o}\overline{\underline{C}}_2^{o}) = 0 \tag{37}$$

$$\frac{\partial}{\partial t_2}(\rho_1\overline{\underline{C}}_1)^1 + \frac{\partial}{\partial t_3}(\rho_1\overline{\underline{C}}_1)^o + \alpha\ \underline{c}_o^o\cdot\frac{\partial}{\partial \underline{r}}(\rho_1^o\overline{\underline{C}}_1^o) + \alpha\ \rho_1^o\overline{\underline{C}}_1^o\ \frac{\partial}{\partial \underline{r}}\cdot\underline{c}_o^o +$$

$$+ \frac{\partial}{\partial \underline{r}}\cdot(\rho_1^o\overline{\underline{C}_1\underline{C}_1}^o) - \rho_1^o\underline{F}_1 + \alpha\ \rho_1^o\left(\frac{1}{\alpha}\ \frac{\partial \underline{c}_o^1}{\partial t_2} + \frac{\partial \underline{c}_o^o}{\partial t_3} + \alpha\ \underline{c}_o^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}}\right) +$$

$$+ \alpha\ \rho_1^1\ \frac{\partial \underline{c}_o^o}{\partial t_2} - n_1^o e_1\underline{c}_o^1 \times \underline{H} - \alpha\ n_1^1 e_1\underline{c}_o^o \times \underline{H} - n_1^o e_1\overline{\underline{C}}_1^1 \times \underline{H} -$$

$$- n_1^1 e_1\overline{\underline{C}}_1^o \times \underline{H} + \alpha\ \rho_1^o\overline{\underline{C}}_1^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}} = \tag{38}$$

$$= -\int d\underline{C}_1 d\underline{C}_2 \underset{\sim}{\Phi}^{12}(\underline{C}_1-\alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\ \frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\ \frac{\partial}{\partial \underline{C}_2}\right)$$

$$\left(f_1^o(\underline{C}_1)f_2^1(\underline{C}_2) + f_1^1(\underline{C}_1)f_2^o(\underline{C}_2)\right)$$

$$\frac{\partial}{\partial t_2}(\rho_2\overline{\underline{C}}_2)^1 + \frac{\partial}{\partial t_3}(\rho_2\overline{\underline{C}}_2)^o + \alpha\ \underline{c}_o^o\cdot\frac{\partial}{\partial \underline{r}}(\rho_2^o\overline{\underline{C}}_2^o) + \alpha\ \rho_2^o\overline{\underline{C}}_2^o\ \frac{\partial}{\partial \underline{r}}\cdot\underline{c}_o^o +$$

$$+ \alpha\ \frac{\partial}{\partial \underline{r}}\cdot(\rho_2^o\overline{\underline{C}_2\underline{C}_2}^o) - \alpha\ \rho_2^o\underline{F}_2 + \rho_2^o\left(\frac{1}{\alpha}\ \frac{\partial}{\partial t_2}\ \underline{c}_o^1 + \frac{\partial}{\partial t_3}\ \underline{c}_o^o + \alpha\ \underline{c}_o^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}}\right) +$$

$$+ \rho_2^1\ \frac{\partial \underline{c}_o^o}{\partial t_2} - \alpha\ n_2^o e_2\underline{c}_o^1 \times \underline{H} - \alpha^2\ n_2^1 e_2\underline{c}_o^o \times \underline{H} - \alpha^2 n_2^o e_2\overline{\underline{C}}_2^1 \times \underline{H} -$$

$$- \alpha^2 n_2^1 e_2\overline{\underline{C}}_2^o \times \underline{H} + \alpha\ \rho_2^o\overline{\underline{C}}_2^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}} = \tag{39}$$

$$= -\alpha\int d\underline{C}_1 d\underline{C}_2 \underset{\sim}{\Phi}^{12}(\underline{C}_1-\alpha\underline{C}_2)\cdot\left(\frac{\alpha}{m_2}\ \frac{\partial}{\partial \underline{C}_2} - \frac{1}{m_1}\ \frac{\partial}{\partial \underline{C}_1}\right)$$

$$\left(f_2^o(\underline{C}_2)f_1^1(\underline{C}_1) + f_2^1(\underline{C}_2)f_1^o(\underline{C}_1)\right.$$

$$\frac{3}{2}n_1^o k\left(\frac{\partial T_1^1}{\partial t_2} + \frac{\partial T_1^o}{\partial t_3} + \alpha\, \underline{c}_o^o \cdot \frac{\partial T_1^o}{\partial \underline{r}}\right) + \frac{3}{2}n_1^1 k\, \frac{\partial T_1^o}{\partial t_2} - \frac{3}{2}kT_1^o\, \frac{\partial}{\partial \underline{r}}\cdot(n_1^o\overline{\underline{C}}_1^o) +$$

$$+ \frac{\partial}{\partial \underline{r}}\cdot\underline{q}_1^{\,\prime} - \rho_1^o\underline{F}_1\cdot\overline{\underline{C}}_1^o + \alpha(\rho_1\overline{\underline{C}}_1)^o\cdot\left(\frac{1}{\alpha}\frac{\partial \underline{c}_o^1}{\partial t_2} + \frac{\partial \underline{c}_o^o}{\partial t_3} + \alpha\, \underline{c}_o^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}}\right) +$$

$$+ \alpha(\rho_1\overline{\underline{C}}_1)^1\cdot\frac{\partial \underline{c}_o^o}{\partial t_2} - n_1^o e_1\overline{\underline{C}}_1^o\cdot(\underline{c}_o^1 \times \underline{H}) - \alpha\, n_1^o e_1\overline{\underline{C}}_1^1\cdot(\underline{c}_o^o \times \underline{H}) -$$

$$- \alpha\, n_1^1 e_1\overline{\underline{C}}_1^o\cdot(\underline{c}_o^o \times \underline{H}) + \alpha\, \rho_1^o\overline{\underline{C}_1\underline{C}_1}^o : \frac{\partial \underline{c}_o^o}{\partial \underline{r}} = \qquad (40)$$

$$= -\alpha\int d\underline{C}_1 d\underline{C}_2 \underline{C}_2 \cdot \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1} - \frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2}\right)$$

$$\left(f_1^o(\underline{C}_1)f_2^1(\underline{C}_2) + f_1^1(\underline{C}_1)f_2^o(\underline{C}_2)\right)$$

$$\frac{3}{2}n_2^o k\left(\frac{\partial T_2^1}{\partial t_2} + \frac{\partial T_2^o}{\partial t_3} + \alpha\, \underline{c}_o^o\cdot\frac{\partial T_2^o}{\partial \underline{r}}\right) + \frac{3}{2}n_2^1 k\, \frac{\partial T_2^o}{\partial t_2} - \alpha\, \frac{3}{2}kT_2^o\, \frac{\partial}{\partial \underline{r}}\cdot(n_2^o\overline{\underline{C}}_2^o) +$$

$$+ \alpha\, \frac{\partial}{\partial \underline{r}}\cdot\underline{q}_2^o - \alpha\, \rho_2^o\underline{F}_2\cdot\overline{\underline{C}}_2^o + (\rho_2\overline{\underline{C}}_2)^o\cdot\left(\frac{1}{\alpha}\frac{\partial \underline{c}_o^1}{\partial t_2} + \frac{\partial \underline{c}_o^o}{\partial t_3} + \alpha\, \underline{c}_o^o\cdot\frac{\partial \underline{c}_o^o}{\partial \underline{r}}\right) +$$

$$+ (\rho_2\overline{\underline{C}}_2)^1\cdot\frac{\partial \underline{c}_o^o}{\partial t_2} - \alpha\, n_2^o e_2\overline{\underline{C}}_2^o\cdot(\underline{c}_o^1 \times \underline{H}) - \alpha^2 n_2^o e_2\overline{\underline{C}}_2^1\cdot(\underline{c}_o^o \times \underline{H}) -$$

$$- \alpha^2 n_2^1 e_2\overline{\underline{C}}_2^o\cdot(\underline{c}_o^o \times \underline{H}) + \alpha\, \rho_2^o\overline{\underline{C}_2\underline{C}_2}^o : \frac{\partial \underline{c}_o^o}{\partial \underline{r}} = \qquad (41)$$

$$= -\alpha\int d\underline{C}_1 d\underline{C}_2 \underline{C}_2 \cdot \underset{\sim}{\Phi}^{12}(\underline{C}_1 - \alpha\underline{C}_2)\cdot\left(\frac{\alpha}{m_2}\frac{\partial}{\partial \underline{C}_2} - \frac{1}{m_1}\frac{\partial}{\partial \underline{C}_1}\right)$$

$$\left(f_2^o(\underline{C}_2)f_1^1(\underline{C}_1) + f_2^1(\underline{C}_2)f_1^o(\underline{C}_1)\right)$$

2. Solutions in the limit $t_2 = \infty$.

In this section we study the equations to zeroth and first order in ε in the limit $t_2 = \infty$, [9]. We do not need then to expand functions and time derivatives in the parameter α .

The solutions of the zeroth order equations can be discussed by studying the H - function, [1]

$$H = \int f_1^o(\underline{C}_1)\ln f_1^o(\underline{C}_1)\, d\underline{C}_1 + \int f_2^o(\underline{C}_2)\ln f_2^o(\underline{C}_2)\, d\underline{C}_2$$

H is bounded below and

$$\frac{\partial H}{\partial t_2} \leqq 0$$

the sign of equality holding if and only if ([10]for a one component gas)

$$f_1^o(\underline{C}_1) = \exp\left[-\tfrac{1}{2}A_1C_1^2 + \underline{B}_1\cdot\underline{C}_1 + D_1\right]$$

$$f_2^o(\underline{C}_2) = \exp\left[-\tfrac{1}{2}A_2C_2^2 + \underline{B}_2\cdot\underline{C}_2 + D_2\right]$$

and

$$\frac{1}{m_1}\frac{\partial}{\partial\underline{C}_1}\left(-\tfrac{1}{2}A_1C_1^2 + \underline{B}_1\cdot\underline{C}_1 + D_1\right) - \frac{1}{m_2}\frac{\partial}{\partial\underline{C}_2}\left(-\tfrac{1}{2}A_2C_2^2 + \underline{B}_2\cdot\underline{C}_2 + D_2\right) =$$

$$= \mu\cdot(\underline{C}_1 - \underline{C}_2)$$

Here μ (a scalar function) A_i ,

$\underline{B}_i$ and D_i , i = 1,2 , are independent of $\underline{C}_1$ and $\underline{C}_2$.

The last relation is fulfilled if and only if

$$\frac{A_1}{m_1} = \frac{A_2}{m_2} \tag{42}$$

$$\frac{\underline{B}_1}{m_1} = \frac{\underline{B}_2}{m_2} \tag{43}$$

A_i , $\underline{B}_i$ and D_i , i = 1,2 , can be expressed by the first moments :

$$A_i = \frac{\frac{3}{2}m_i}{\frac{3}{2}kT_i^o - \frac{1}{2}m_i\overline{\underline{C}}_i^{o\,2}}$$

$$\underline{B}_i = \frac{\frac{3}{2}m_i\overline{\underline{C}}_i^o}{\frac{3}{2}kT_i^o - \frac{1}{2}m_i\overline{\underline{C}}_i^{o\,2}}$$

$$e^{D_i} = n_i^o\left(\frac{A_i}{2\pi}\right)^{3/2}\exp\left[-\frac{B_i^2}{2A_i}\right]$$

The a priori condition to zeroth order , Eq. (18) , together with Eqs (42) and (43) give that

$$\overline{\underline{C}}_1^o = \overline{\underline{C}}_2^o = 0 \tag{44}$$

$$T_1^o = T_2^o \; (= T^o) \tag{45}$$

in the limit $t_2 = \infty$, and the distribution functions then take Maxwellian forms ,

$$f_{iM}^o = n_i^o \left(\frac{m_i}{2\pi k T^o} \right)^{3/2} \exp\left(- \frac{m_i C_i^2}{2kT^o} \right) \tag{46}$$

(The subscript "M" is used occasionally to denote quantities in "local" equilibrium).

Substituting Eqs. (44) and (46) into Eqs. (23), (28) and (29) show that

$$\frac{\partial T^o}{\partial t_2} \; , \; \frac{\partial T_i^o}{\partial t_2} \; , \quad i = 1,2 \; , \text{ all} \to 0 \quad \text{as} \quad t_2 \to \infty \tag{47}$$

We now study the kinetic equations to zeroth order in the limit $t_2 = \infty$. We assume

$$\frac{\partial f_i^o}{\partial \underline{C}_i} \to \frac{\partial f_{iM}^o}{\partial \underline{C}_i}$$

$$\frac{\partial f_i^o}{\partial t_2} \to \frac{\partial f_{iM}^o}{\partial t_2}$$

as $t_2 \to \infty$, and from the last relation it follows, using Eqs. (24) and (25) and Eq. (47) , that

$$\frac{\partial f_i^o}{\partial t_2} \to \frac{\partial T^o}{\partial t_2} \cdot \frac{\partial f_{iM}^o}{\partial T^o} = 0 \qquad \text{as } t_2 \to \infty \; .$$

Substituting from Eq.(22), Eqs.(19) and (20) in the limit $t_2 = \infty$ become

$$\left(-\alpha^2 \frac{\rho_e^o}{\rho^o} \underline{c}_o^o \times \underline{H} + \frac{e_1}{m_1} \underline{c}_o^o \times \underline{H}\right) \cdot \underline{C}_1 = 0$$

$$\left(-\frac{\rho_e^o}{\rho^o} \underline{c}_o^o \times \underline{H} + \frac{e_2}{m_2} \underline{c}_o^o \times \underline{H}\right) \cdot \underline{C}_2 = 0$$

showing that the perpendicular ($\perp$) mass transport vector in the limit $t_2 = \infty$ vanishes,

$$\underline{c}_{o\perp}^o = 0 \tag{48}$$

(while the parallel ($\|$) mass transport vector $\underline{c}_{o\|}^o$ may be non-vanishing).

Thus the equations to zeroth order in the parameter ε, which correspond to the . equations in the absence of an external electric field and inhomogenities, describe the evolution into a local equilibrium state in space and time where electrons and ions are of equal temperature and where only a parallel mass transport to zeroth order may exist. Before this local state thermal energy of electrons and ions has grown due to the damping of $\underline{c}_{o\perp}^o$, and electrons and ions have exchanged energy. This is in agreement with "fine structure solutions" (expanding in the parameter α) obtained in [5], [6], [7] and in section 3 of this report.

From the equations to first order in ε we find easily that ρ^1 , ρ_i^1 , $\underline{c}_{o\|}^1$, T^1 and T_i^1 , i = 1,2 , have transients on the τ_2 time scale, for instance

$$\rho_1^1(t_2 , ..) = \rho_1^1(t_2 = 0 , t_3 , ..) - \int_0^{t_2} d\tau\left[\alpha \frac{\partial}{\partial \underline{r}}\cdot\left(\rho_1^o \underline{c}_{o\perp}^o(\tau)\right) + \right.$$

$$\left. + \frac{\partial}{\partial \underline{r}}\cdot\left(\rho_1^o \overline{\underline{C}}_1^o(\tau)\right)\right]$$

and as non secularity conditions we obtain

$$\frac{\partial \rho^o}{\partial t_3} + \alpha \frac{\partial}{\partial \underline{r}}\cdot(\rho^o \underline{c}_{o\|}^o) = 0 \tag{49}$$

$$\frac{\partial \rho_i^o}{\partial t_3} + \alpha \frac{\partial}{\partial \underline{r}}\cdot(\rho_i^o \underline{c}_{o\|}^o) = 0 \ , \quad i = 1,2 \ . \tag{50}$$

$$\frac{3}{2} n^o k\left(\frac{\partial T^o}{\partial t_3} + \alpha \ \underline{c}_{o\|}^o \cdot \frac{\partial T^o}{\partial \underline{r}} \right) = - \alpha \ \underset{\sim}{I} \ p^o : \frac{\partial \underline{c}_{o\|}^o}{\partial \underline{r}} \tag{51}$$

The equation for the parallel mass transport vector becomes

$$\rho^o \left(\frac{\partial \underline{c}_{o\|}^o}{\partial t_3} + \alpha \ \underline{c}_{o\|}^o \cdot \frac{\partial \underline{c}_{o\|}^o}{\partial \underline{r}} \right) = - \alpha \frac{\partial p^o}{\partial \underline{r}_\|} + \alpha \sum_{i=1}^{2} \rho_i^o \ \underline{F}_{i\|} \tag{52}$$

Equations (49) - (52) are similar to corresponding equations of Chapman and Cowling [1] (cf.Appendix 1, Eqs.A12-A14) except that it is the mass transport vector parallel to the magnetic field that dominates in Eqs. (49)-(52).

We now study the kinetic equations, Eqs.(31) and (32), and the transverse part of Eq.(34). These equations are strongly coupled as to the evolution on the τ_2 time scale. However, the problem to find the behaviour of f_1^1, f_2^1 and $\underline{c}_{o\perp}^1$ as $t_2 \to \infty$ we here simplify as follows: We seek bounded solutions of Eqs.(31) and (32) which obey

$$\begin{aligned} \frac{\partial f_1^1}{\partial t_2} &\to 0 \ , \ \text{as} \ \ t_2 \to \infty \\ \frac{\partial f_2^1}{\partial t_2} &\to 0 \ , \ \text{as} \ \ t_2 \to \infty \end{aligned} \tag{53}$$

We note that the conditions Eq.(53) alone are neither necessary nor sufficient for f_1^1 and f_2^1 to be bounded as $t_{20} \to \infty$. Chapman and Cowling implicitly make similar assumptions concerning f_1^1 and f_2^1 (cf.Appendix 1, Eq.A15). In the limit $t_2 = \infty$ f_1^1 and f_2^1 from Eqs.(31) and (32) will then be functionals of n_1^o, n_2^o, T^o, $\underline{c}_{o\|}^o$ and $\underline{c}_{o\perp}^1$. All except $\underline{c}_{o\perp}^1$ we have found are either independent of t_2, Eqs.(24) and (25) and the parallell part of Eq.(22), or relax according to Eq.(47) as $t_2 \to \infty$. To be consistent with the assumptions above we also assume that

$$\frac{\partial \underline{c}_{o\perp}^1}{\partial t_2} \to 0 \quad \text{as} \quad t_2 \to \infty \tag{54}$$

This assumption will now give a balance of transverse forces in the limit $t_2 = \infty$: From the transverse part

of Eq.(34) using the results from zeroth order equations and the assumption Eq.(54) we get

$$- \frac{\partial p^o}{\partial \underline{r}_\perp} + \rho_e^o \underline{c}_o^1 \times \underline{H} + n_1^o e_1 \underline{\overline{C}}_1^1 \times \underline{H} + \alpha\, n_2^o e_2 \underline{\overline{C}}_2^1 \times \underline{H} + \sum_{i=1}^{2} \rho_i^o \underline{F}_i = 0 \qquad (55)$$

In this equation we find $\underline{\overline{C}}_1^1$ and $\underline{\overline{C}}_2^1$ from the solutions of the kinetic equations to first order in the limit $t_2 = \infty$. Eq.(55)then determines $\underline{c}_{o\perp}^1$ in this limit. We note here that in the next section we partly derive what is assumed above concerning the time derivatives of f_1^1, f_2^1 and $\underline{c}_{o\perp}^1$ as $t_2 \to \infty$.

Making use of the results to zeroth order and also Eqs.(50), (51) and (52) and the assumptions above (f_1^1 and f_2^1 bounded and Eqs.(53) and (54)), the kinetic equations to first order in ε, Eqs.(31) and (32) in the limit $t_2 = \infty$ can be written

$$f_{1M}^o\left(\frac{m_1 C_1^2}{2kT^o} - \frac{5}{2}\right)\frac{1}{T^o}\frac{\partial T^o}{\partial \underline{r}}\cdot\underline{C}_1 + f_{1M}^o\left(\frac{\partial}{\partial \underline{r}_\parallel}\ell n p_1^o - \frac{\rho_1^o}{p_1^o \rho^o}(\rho^o - \alpha^2 \rho_1^o)(\underline{F}_{1\parallel} - \alpha^2 \underline{F}_{2\parallel}) - \alpha^2 \frac{\rho_1^o}{p_1^o \rho^o}\frac{\partial p^o}{\partial \underline{r}_\parallel}\right)\cdot\underline{C}_1 - f_{1M}^o \frac{e_1}{kT^o}\left(\frac{m_1}{e_1}\underline{F}_{1\perp} + \underline{c}_o^1 \times \underline{H} - \frac{kT^o}{e_1}\frac{\partial}{\partial \underline{r}_\perp}\ell n p_1^o\right)\cdot\underline{C}_1 + \alpha\, f_{1M}^o \frac{m_1}{kT^o}\underline{C}_1{}^o\underline{C}_1 : \frac{\partial \underline{c}_{o\parallel}^o}{\partial \underline{r}} = \qquad (56)$$

$$= - \frac{e_1}{m_1}\underline{C}_1 \times \underline{H}\cdot\frac{\partial f_{1M}^1}{\partial \underline{C}_1} + FP_{11}\left[f_{1M}^o(\underline{C}_1) f_{1M}^1(\underline{C}_1') + f_{1M}^1(\underline{C}_1) f_{1M}^o(\underline{C}_1')\right] + FP_{12}\left[f_{1M}^o(\underline{C}_1) f_{2M}^1(\underline{C}_2) + f_{1M}^1(\underline{C}_1) f_{2M}^o(\underline{C}_2)\right]$$

$$\alpha\, f^o_{2M}\left(\frac{m_2C_2^2}{2kT^o} - \frac{5}{2}\right)\frac{1}{T^o}\frac{\partial T^o}{\partial \underline{r}}\cdot\underline{C}_2 + f^o_{2M}\left(\alpha\,\frac{\partial}{\partial \underline{r}_\parallel}\,\ell n p^o_2 -\right.$$

$$- \alpha(\rho^o - \rho^o_2)\frac{\rho^o_2}{p^o_2\rho^o}(\alpha^2\,\underline{F}_{2\parallel} - \underline{F}_{1\parallel}) - \alpha\,\frac{\rho^o_2}{\rho p^o_2}\frac{\partial p^o}{\partial \underline{r}_\parallel}\left.\right)\cdot\underline{C}_2 -$$

$$- \alpha\, f^o_{2M}\,\frac{e_2}{kT^o}\left(\frac{m_2}{e_2}\,\underline{F}_{2\perp} + \underline{c}^1_o \times \underline{H} - \frac{kT^o}{e_2}\frac{\partial}{\partial \underline{r}_\perp}\,\ell n p^o_2\right)\cdot\underline{C}_2 +$$

$$+ \alpha\, f^o_{2M}\,\frac{m_2}{kT^o}\,\underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_{o\parallel}}{\partial \underline{r}} = \tag{57}$$

$$= -\alpha^2\,\frac{e_2}{m_2}\,\underline{C}_2 \times \underline{H}\cdot\frac{\partial f^1_{2M}}{\partial \underline{C}_2} + \alpha\,FP_{22}\Big[f^o_{2M}(\underline{C}_2)f^1_{2M}(\underline{C}'_2) +$$

$$+ f^1_{2M}(\underline{C}_2)f^o_{2M}(\underline{C}'_2)\Big] + \alpha\,FP_{21}\Big[f^o_{2M}(\underline{C}_2)f^1_{1M}(\underline{C}_1) + f^1_{2M}(\underline{C}_2)f^o_{1M}(\underline{C}_1)\Big]$$

These equations together with the constraint Eq.(30) in the limit $t_2 = \infty$ correspond to the "second order kinetic equations" of Chapman and Cowling.

Eqs(30), (56) and (57) can be solved by successive approximations expanding in the parameter α . Leaving out terms of order α in Eqs.(30), (56) and (57) we obtain

$$\int d\underline{C}_2\; f^1_{2M}(\underline{C}_2)m_2\underline{C}_2 = 0 \tag{58}$$

$$f^o_{1M}\left[\left(\frac{m_1C_1^2}{2kT^o} - \frac{5}{2}\right)\frac{1}{T^o}\frac{\partial T^o}{\partial \underline{r}} - \frac{e_1}{kT^o}\left(\frac{m_1}{e_1}\underline{F}_1 + \underline{c}^1_o \times \underline{H} - \right.\right.$$

$$\left.\left. - \frac{kT^o}{e_1}\frac{\partial}{\partial \underline{r}}\ell np^o_1\right)\right]\cdot\underline{C}_1 = \qquad (59)$$

$$= -\frac{e_1}{m_1}\underline{C}_1 \times \underline{H}\cdot\frac{\partial f^1_{1M}}{\partial \underline{C}_1} + FP_{11}\left[f^o_{1M}(\underline{C}_1)f^1_{1M}(\underline{C}'_1) + f^1_{1M}(\underline{C}_1)f^o_{1M}(\underline{C}'_1)\right] +$$

$$+ D^o_1\left[f^1_{1M}\right]$$

$$f^o_{2M}\left(\frac{m_2C_2^2}{2kT^o} - \frac{5}{2}\right)\frac{1}{T^o}\frac{\partial T^o}{\partial \underline{r}}\cdot\underline{C}_2 + f^o_{2M}\frac{m_2}{kT^o}\underline{C}^o_2\underline{C}_2 : \frac{\partial \underline{c}^o_{o\parallel}}{\partial \underline{r}} =$$

$$(60)$$

$$= FP_{22}\left[f^o_{2M}(\underline{C}_2)f^1_{2M}(\underline{C}'_2) + f^1_{2M}(\underline{C}_2)f^o_{2M}(\underline{C}'_2)\right]$$

Here $D^o_1 = \frac{n^o_2}{m^2_1}\frac{\partial}{\partial \underline{C}_1}\cdot\left(\underset{\sim}{\Phi}^{12}(\underline{C}_1)\cdot\frac{\partial}{\partial \underline{C}_1}\right)$ is a diffusion operator.

To obtain Eq.(60) use is also made of Eq.(39) in the limit $t_2 = \infty$. Eqs.(59) and (60) are of the same form as Eqs.(2.50) and (2.86) of [5] or Eqs.(10) and (15) of [6], see also [7].

Using the Landau form for the tensor $\underset{\sim}{\Phi}^{ij}$,

$$\underset{\sim}{\Phi}^{ij}(\underline{w}) = \kappa\frac{w^2\underset{\sim}{I} - \underline{w}\underline{w}}{w^3}, \quad \kappa = 2\pi e_i^2e_j^2\ell n\left[\frac{3\lambda_DkT^o}{2e_1^2}\right], \text{ where } \underset{\sim}{I}$$

is the unit tensor and λ_D the Debye length, Leversen and Naze Tjøtta [11] have proved existence of solutions of Eqs. (59) and (60) with certain properties. The solution of Eq. (59) contains two arbitrary parameters, and imposing

$$n_1^1 = 0$$

$$T_1^1 = 0$$

in the limit $t_2 = \infty$, these parameters vanish. The solution of Eq.(60) contains 5 arbitrary parameters so that Eq.(58) can be fulfilled too. To see the form of the solution of Eq.(59), which is sufficient in the discussion later, we substitute

$$\nu_1 (f^o_{1M} - f_1) \tag{61}$$

for the collision terms of Eq.(7) when terms of order α are left out. Here ν_1 , assumed constant, is a measure of electron-electron and electron-ion effective collision frequency, with ions at rest. The solution of Eq.(59) thus takes the form, [5]

$$f^1_{1M} = - f^o_{1M}\left[\frac{1}{\nu_1}\underline{C}_{1\parallel} + \frac{\frac{1}{\nu_1}}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\left(\underline{C}_{1\perp} - \frac{\Omega_1}{\nu_1 H}\underline{C}_1 \times \underline{H}\right)\right]\cdot\underline{h}^1_1 \tag{62}$$

where $f^o_{1M}\,\underline{h}^1_1\cdot\underline{C}_1$ is the left hand side of Eq.(59). From Eq.(62) we derive for the homogeneous case

$$n^o_1\underline{\overline{C}}^1_{1\perp} = \frac{\frac{1}{\nu_1}}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\left[\frac{n^o_1 e_1}{m_1}\left(\frac{m_1}{e_1}\underline{F}_1 + \underline{c}^1_o \times \underline{H}\right) - \right.$$

$$\left. - \frac{n_1 e_1}{m_1}\frac{\Omega_1}{\nu_1}\frac{\underline{H}}{H}\times\left(\frac{m_1}{e_1}\underline{F}_1 + \underline{c}^1_o \times \underline{H}\right)\right] \tag{63}$$

Leaving out terms of order α in the balance of transverse forces relation Eq.(55) and substituting from Eq.(63) we obtain the form of $\underline{c}^1_{o\perp}$ in the limit $t_2 = \infty$ for the homogeneous case ,

$$\underline{c}^1_{o\perp} = \frac{n_1^o e_1 \dfrac{\frac{1}{\nu_1}}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\left(\dfrac{e_1}{m_1}\sum_{i=1}^{2}\rho_i^o \underline{F}_{i\perp} - \rho_e^o \underline{F}_{1\perp}\right)}{\rho_e^{o2} - \dfrac{\left(\frac{\Omega_1}{\nu_1}\right)^2}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\left(2\rho_e^o n_1^o e_1 - n_1^{o2} e_1^2\right)} + \tag{64}$$

$$+ \frac{\left(\rho_e^o - n_1^o e_1 \dfrac{\left(\frac{\Omega_1}{\nu_1}\right)^2}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\right)\left(n_1^o m_1 \dfrac{\left(\frac{\Omega_1}{\nu_1}\right)^2}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\underline{H}\times\underline{F}_1 - \underline{H}\times\sum_{i=1}^{2}\rho_i^o \underline{F}_i\right)}{H^2\left[\rho_e^{o2} - \dfrac{\left(\frac{\Omega_1}{\nu_1}\right)^2}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\left(2\rho_e^o n_1^o e_1 - n_1^{o2} e_1^2\right)\right]} -$$

$$- \frac{n_1^{o2} e_1 m_1 \dfrac{\left(\frac{\Omega_1}{\nu_1}\right)^2}{\left[1+\left(\frac{\Omega_1}{\nu_1}\right)^2\right]^2}\underline{H}\times\underline{F}_1}{H^2\left[\rho_e^{o2} - \dfrac{\left(\frac{\Omega_1}{\nu_1}\right)^2}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2}\left(2\rho_e^o n_1^o e_1 - n_1^{o2} e_1^2\right)\right]}$$

The first part vanishes when $\underline{F}_i$ arises from an electric field.

From Eq.(64) we get

$$\underline{c}^1_{o\perp} \rightarrow \frac{m_2 \underline{F}_2 \times \underline{H}}{e_2 H^2} \quad \text{as} \; \frac{\Omega_1}{\nu_1} \rightarrow \infty \;, \text{ i.e.}$$

the drift in collisionless theory for ions in static and uniform fields. The expression Eq.(64) is a collisional counterpart to this result. The electron drift follows from Eq.(63).

3. Evolution into a state of balance of transverse forces.

In this section we expand in α the sets of equations to zeroth and first order in ε. Then we shall in more detail see the evolution of distribution functions and macroscopic quatities. Assuming bounded solutions of all kinetic equations we shall derive to some extent the assumptions Eq. (53) and (54) and show that the gas evolves into a state where forces transverse to the magnetic field balance in the mass transport equation.

We obtain equations to zeroth order in ε and zeroth, first, second orders etc. in α from the equations (18)-(29) or from the equations of [5] putting $\frac{\partial}{\partial \underline{r}}$ and $\underline{F}_i$, i=1,2, all equal to zero. We will discuss some of these equations up to third order in α here. Since ρ^o, ρ_i^o and $\underline{c}_{o\parallel}^o$ are all independent of t_2 we do not expand them in α for simplicity here.
The equations to zeroth order in ε and zeroth order in α show that f_2^{oo}, $\underline{c}_o^{oo}$, T^{oo}, T_i^{oo}, i = 1,2, and $\overline{\underline{C}}_2^{oo}$ are all independent of t_{20} while $\overline{\underline{C}}_1^{oo}$ evolves according to the evolution of f_1^{oo} on the τ_{20} time scale. A H-theorem can be established which shows that

$$f_1^{oo} \rightarrow f_{1M}^{oo} = n_1^o \left(\frac{m_1}{2\pi k T_1^{oo}}\right)^{3/2} \exp\left(-\frac{m_1 C_1^2}{2kT_1^{oo}}\right) \quad \text{as} \quad t_{2o} \rightarrow \infty \quad (65)$$

Accordingly $\underline{\overline{C}}_1^{oo} \rightarrow 0$ as $t_{2o} \rightarrow \infty$.

From <u>the equations to zeroth order in ε and first in α</u> we get from the mass transport equation

$$\underline{c}_{o\perp}^{o1}(t_{2o}, t_{21}, \ldots) = \underline{c}_{o\perp}^{o1}(t_{2o}=0, t_{21}, \ldots) - t_{2o} \frac{\partial \underline{c}_{o\perp}^{oo}}{\partial t_{21}} + \\ + \frac{n_1^o e_1}{\rho^o} \int_o^{t_{2o}} d\tau \underline{\overline{C}}_1^{oo}(\tau) \times \underline{H}$$

and eliminating the secular term we get that $\underline{c}_{o\perp}^{oo}$ is also independent of t_{21} and

$$\underline{c}_{o\perp}^{o1}(t_{2o}, t_{21}, \ldots) = \underline{c}_{o\perp}^{o1}(t_{2o}= 0, t_{21}, \ldots) + \\ + \frac{n_1^o e_1}{\rho^o} \int_o^{t_{2o}} d\tau \underline{\overline{C}}_1^{oo}(\tau) \times \underline{H} \ ,$$

i.e. $\underline{c}_{o\perp}^{o1}$ has a transient on the τ_{2o} time scale. Similarly we find that T^{oo} , T_1^{oo} and T_2^{oo} are also independent of t_{21} , T_2^{o1} is independent of t_{2o} , while T^{o1} , T_1^{o1} , $\underline{\overline{C}}_2^{o1}$ and f_2^{o1} all have transients on the τ_{2o} time scale.

A H-theorem can again be established showing that

$$f_2^{oo} \to f_{2M}^{oo} = n_2^o \left(\frac{m_2}{2\pi k T_2^{oo}} \right)^{3/2} \exp\left(- \frac{m_2 C_2^2}{2kT_2^{oo}} \right) \text{ as } t_{21} \to \infty \qquad (66)$$

Here use has been made of the a priori condition to zeroth order in α .

The electron kinetic equation to first order is of the form

$$\frac{\partial f_1^{o1}}{\partial t_{2o}} + \frac{e_1}{m_1} \underline{C}_1 \times \underline{H} \cdot \frac{\partial f_1^{o1}}{\partial \underline{C}_1} - FP_{11}\left[f_{1M}^{oo}(\underline{C}_1) f_1^{o1}(\underline{C}_1') + f_1^{o1}(\underline{C}_1) f_{1M}^{oo}(\underline{C}_1') \right] -$$

$$- D_1^o\left[f_1^{o1}(\underline{C}_1) \right] = - \frac{e_1}{m_1} \underline{c}_o^{oo} \times \underline{H} \cdot \frac{\partial f_{1M}^{oo}}{\partial \underline{C}_1} + G^{o1}(t_{2o}) \qquad (67)$$

where $G^{o1}(t_{2o}) \to 0$ as $t_{2o} \to \infty$. Some assumptions concerning commutation of differential operations and "limit as $t_{2o} \to \infty$ " operations have been made.

In the appendixes of [5], [7] and [12] it is shown that every bounded solution of Eq.(67) evolves as $t_{2o} \to \infty$ toward the solution of

$$- \frac{e_1}{m_1} \underline{C}_1 \times \underline{H} \cdot \frac{\partial f_{1M}^{o1}}{\partial \underline{C}_1} + FP_{11}\left[f_{1M}^{oo}(\underline{C}_1) f_{1M}^{o1}(\underline{C}_1') + f_{1M}^{o1}(\underline{C}_1) f_{1M}^{oo}(\underline{C}_1') \right] +$$

$$+ D_1^o\left[f_{1M}^{o1}(\underline{C}_1) \right] = \frac{e_1}{m_1} \underline{c}_o^{oo} \times \underline{H} \cdot \frac{\partial f_{1M}^{oo}}{\partial \underline{C}_1} \qquad (68)$$

which has the same form as Eq.(59). Substituting for the collision terms the relaxation term Eq.(61) we get that the solutions of Eq.(67) evolve toward a solution of the form Eq.(62) with $- e_1/kT_1^{oo} \underline{c}_o^{oo} \times \underline{H}$ instead of $\underline{h}_1^1$.

From <u>the equations to zeroth order in ε and second in α</u> we derive from the mass transport equation, eliminating the secular terms when integrating on the t_{2o} time scale

$$\rho^o\left(\frac{\partial \underline{c}^{o1}_{o\perp}}{\partial t_{21}}(t_{2o}=0,t_{21},\ ..) + \frac{\partial \underline{c}^{oo}_{o\perp}}{\partial t_{22}}\right) = \rho^o_e\, \underline{c}^{oo}_o \times \underline{H} - n^o_1 e_1 \overline{\underline{C}}^{o1}_{1M} \times \underline{H}$$

$$\underline{c}^{o2}_{o\perp}(t_{2o},t_{21},..) = \underline{c}^{o2}_{o\perp}(t_{2o}=0,t_{21},..) +$$

$$+ \frac{n^o_1 e_1}{\rho^o}\int_o^{t_{2o}} d\tau\left(\overline{\underline{C}}^{o1}_1(\tau) - \overline{\underline{C}}^{o1}_{1M}\right) \times \underline{H}$$

We have assumed that $\frac{\partial \underline{c}^{o1}_{o\perp M}}{\partial t_{21}} = \frac{\partial \underline{c}^{o1}_{o\perp}}{\partial t_{21}}(t_{2o},t_{21},\ ..) =$ $= \frac{\partial \underline{c}^{o1}_{o\perp}}{\partial t_{21}}(t_{2o}=0,t_{21},\ ..)$. Integrating on the τ_{21} time scale we see that $\underline{c}^{o1}_{o\perp}(t_{2o}=0)$ is independent of t_{21} .

For $\underline{c}^{oo}_{o\perp}$ and likewise for T^{oo} , T^{oo}_1 and T^{oo}_2 we get the following equations on the τ_{22} time scale

$$\rho^o\,\frac{\partial \underline{c}^{oo}_o}{\partial t_{22}} = \rho^o_e\, \underline{c}^{oo}_o \times \underline{H} + n^o_1 e_1 \overline{\underline{C}}^{o1}_{1M} \times \underline{H} \tag{69}$$

$$\frac{3}{2}n^o k\frac{\partial T^{oo}}{\partial t_{22}} = n^o_1 e_1 \overline{\underline{C}}^{o1}_{1M}\cdot(\underline{c}^{oo}_o \times \underline{H}) \tag{70}$$

$$\frac{3}{2}n_1^o k \frac{\partial T_1^{oo}}{\partial t_{22}} = n_1^o e_1 \bar{\underline{C}}_{1M}^{o1} \cdot (\underline{c}_o^{oo} \times \underline{H}) - \tag{71}$$

$$- \frac{n_1^o n_2^o}{m_2}\left(\frac{m_1}{2\pi k}\right)^{3/2}\left(\frac{T_1^{oo}-T_2^{oo}}{T_1^{oo\ 5/2}}\right)\int d\underline{C}_1 \underset{\sim}{\Phi}^{12}(\underline{C}_1) : I \exp\left(-\frac{m_1 C_1^2}{2kT_1^{oo}}\right)$$

$$\frac{3}{2}n_2^o k \frac{\partial T_2^{oo}}{\partial t_{22}} = \frac{n_1^o n_2^o}{m_2}\left(\frac{m_1}{2\pi k}\right)^{3/2}\left(\frac{T_1^{oo}-T_2^{oo}}{T_1^{oo\ 5/2}}\right)\int d\underline{C}_1 \underset{\sim}{\Phi}^{12}(\underline{C}_1) :$$

$$: \underset{\sim}{I} \exp\left(-\frac{m_1 C_1^2}{2kT_1^{oo}}\right) \tag{72}$$

In general Eqs.(69)-(72) are a coupled set of equations describing the relaxation of $\underline{c}_{o\perp}^{oo}$, T^{oo} , T_1^{oo} and T_2^{oo} on the τ_{22}-time scale. We simplify using for $\bar{\underline{C}}_{1M}^{o1}$ an expression corresponding to Eq.(63). Eq.(69) then can be solved and shows that $\underline{c}_{o\perp}^{oo}$ oscillates and is damped away exponentially on the τ_{22} time scale having solutions proportional to (5) $e^{\pm i\omega_{\pm} t_{22}}$, $\omega_{\pm} = \eta \pm i\frac{\delta H^2}{\rho^o}$,

$$\delta = \frac{n_1^o e_1^2}{m_1}\frac{\frac{1}{\nu_1}}{1+\left(\frac{\Omega_1}{\nu_1}\right)^2} > 0 \ , \quad \eta = \frac{H}{\rho^o}\left(\rho_e^o - \delta\frac{\Omega_1 H}{\nu_1}\right).$$

This is in agreement with Eq.(48). Due to this damping Eq.(70) shows how the temperature grows, and Eqs.(71) and (72) describe how T_1^{oo} and T_2^{oo} relax against each other. This should be compared with Eq.(45). Due to the damping of $\underline{c}_{o\perp}^{oo}$ also f_{1M}^{o1} from Eq.(68) damps away as $t_{22} \to \infty$ (when n_1^{o1} and T_1^{o1} are set equal to zero in the limit $t_{2o} = \infty$).

The ion kinetic equation to this order shows that f_2^{o2} has a transient on the τ_{2o} time scale. On the τ_{21} time scale the equation is of the form,

$$\frac{\partial f_2^{o1}}{\partial t_{21}} - FP_{22}\left[f_{2M}^{oo}(\underline{C}_2)f_2^{o1}(\underline{C}_2') + f_2^{o1}(\underline{C}_2)f_{2M}^{oo}(\underline{C}_2')\right] = H^{o1}(t_{21}) \qquad (73)$$

where $H^{o1}(t_{21}) \to 0$ as $t_{21} \to \infty$, [5]. Using the appendix of [12] shows that every bounded f_2^{o1} evolves toward the solution of

$$FP_{22}\left[f_{2M}^{oo}(\underline{C}_2)f_2^{o1}(\underline{C}_2') + f_2^{o1}(\underline{C}_2)f_{2M}^{oo}(\underline{C}_2')\right] = 0 \qquad (74)$$

as $t_{21} \to \infty$.

Using the a priori condition Eq.(18) to first order in α in the limits $t_{2o} \to \infty$ and $t_{21} \to \infty$ and also $n_2^{o1} = 0$ and $T_2^{o1} = 0$ in the same limits give

$$f_{2M}^{o1} = 0 \qquad (75)$$

From the electron kinetic equation to this order we get, using the appendixes of [5], [7] and [12] and the solution of Eq.(68) and Eq.(75), that every bounded f_1^{o2} evolves toward the solution of

$$\frac{\partial f_{1M}^{oo}}{dt_{22}} + \frac{e_1}{m_1}\underline{c}_o^{o1} \times \underline{H}\cdot\frac{\partial f_{1M}^{oo}}{\partial \underline{C}_1} + \frac{e_1}{m_1}\underline{c}_o^{oo} \times \underline{H}\cdot\frac{\partial f_{1M}^{o1}}{\partial \underline{C}_1} -$$

$$- FP_{11}\left[f_{1M}^{o1}(\underline{C}_1)f_{1M}^{o1}(\underline{C}_1')\right] - \frac{1}{m_1 m_2}\frac{\partial}{\partial \underline{C}_1}\cdot\int d\underline{C}_2\left(\underline{C}_2\cdot\frac{\partial \underset{\sim}{\Phi}^{12}}{\partial \underline{C}_1}(\underline{C}_1)\right)\cdot\frac{\partial f_{2M}^{oo}}{\partial \underline{C}_2} f_{1M}^{oo} -$$

$$- \frac{1}{m_1^2}\frac{\partial}{\partial \underline{C}_1}\cdot\int d\underline{C}_2\left(\frac{\underline{C}_2\underline{C}_2}{2} : \frac{\partial^2 \underset{\sim}{\Phi}^{12}(\underline{C}_1)}{\partial \underline{C}_1 \partial \underline{C}_1}\right)\cdot f_{2M}^{oo}\frac{\partial f_{1M}^{oo}}{\partial \underline{C}_1} =$$

$$= - \frac{e_1}{m_1}\underline{C}_1 \times \underline{H}\cdot\frac{\partial f_1^{o2}}{\partial \underline{C}_1} + FP_{11}\left[f_{1M}^{oo}(\underline{C}_1)f_1^{o2}(\underline{C}_1') + f_1^{o2}(\underline{C}_1)f_{1M}^{oo}(\underline{C}_1')\right] +$$

$$+ D_1^o\left[f_1^{o2}\right] \qquad (76)$$

as t_{2o} and t_{21} go to infinity (in the limit $t_{2o} = \infty$ only, f_2^{oo} instead of f_{2M}^{oo}), or, rewriting terms on the left hand side, f_1^{o2} evolves toward the solution of

$$f_{1M}^{oo}\left(\frac{m_1 C_1^2}{2kT_1^{oo}} - \frac{3}{2}\right)\frac{1}{T_1^{oo}}\frac{\partial T_1^{oo}}{\partial t_{22}} + \frac{e_1}{m_1}\underline{c}_o^{o1} \times \underline{H}\cdot\frac{\partial f_{1M}^{oo}}{\partial \underline{C}_1} + \frac{e_1}{m_1}\underline{c}_o^{oo} \times \underline{H}\cdot\frac{\partial f_{1M}^{o1}}{\partial \underline{C}_1} -$$

$$- FP_{11}\left[f_{1M}^{o1}(\underline{C}_1)f_{1M}^{o1}(\underline{C}_1')\right] - \kappa\frac{2n_2^{oo}}{m_1 m_2}\frac{T_1^{oo}-T_2^{oo}}{T_1^{oo}}\frac{\partial}{\partial \underline{C}_1}\cdot\left(\frac{\underline{C}_1}{C_1^3}f_{1M}^{oo}\right) = \qquad (77)$$

$$= - \frac{e_1}{m_1}\underline{C}_1 \times \underline{H}\cdot\frac{\partial f_1^{o2}}{\partial \underline{C}_1} + FP_{11}\left[f_{1M}^{oo}(\underline{C}_1)f_1^{o2}(\underline{C}_1') + f_1^{o2}(\underline{C}_1)f_{1M}^{oo}(C_1')\right] +$$

$$+ D_1^o\left[f_1^{o2}\right] .$$

Here we have used Landau's form for the tensor $\underset{\sim}{\Phi}^{12}$ (cf.p. 30) To prove existence of solutions of Eq.(77) whose left hand side has terms of zeroth, first and higher order anistropy

in velocity space, we may follow the lines of [11].

We also need the mass transport equation to zeroth order in ε and third in α,

$$\rho^o\left(\frac{\partial \underline{c}_o^{o3}}{\partial t_{2o}} + \frac{\partial \underline{c}_o^{o2}}{\partial t_{21}} + \frac{\partial \underline{c}_o^{o1}}{\partial t_{22}} + \frac{\partial \underline{c}_o^{oo}}{\partial t_{23}}\right) = \rho_e^o \, \underline{c}_o^{o1} \times \underline{H} + n_1^o e_1 \underline{\overline{C}}_1^{o2} \times \underline{H} + n_2^o e_2 \underline{\overline{C}}_2^{o1} \times \underline{H} \qquad (78)$$

Integrating on the τ_{2o} time scale, eliminating secular terms, using that $\underline{\overline{C}}_2^{o1} \to 0$ as $t_{2o} \to \infty$ according to the a priori condition and assuming

$$\frac{\partial \underline{c}_o^{o2}}{\partial t_{21}}(t_{2o}=\infty) = \frac{\partial \underline{c}_o^{o2}}{\partial t_{21}}(t_{2o}, t_{21}, \ldots) = \frac{\partial \underline{c}_o^{o2}}{\partial t_{21}}(t_{2o}=0, t_{21}, \ldots)$$

we get

$$\underline{c}_o^{o3}(t_{2o}, t_{21}, \ldots) = \underline{c}_o^{o3}(t_{2o}=0, t_{21}, \ldots) + \int_0^{t_{2o}} d\tau \Big[\frac{\partial}{\partial t_{22}}\left(\underline{c}_o^{o1}(t_{2o}=\infty) - \underline{c}_o^{o1}(\tau)\right) + \frac{\rho_e^o}{\rho^o}\left(\underline{c}_o^{o1}(\tau) - \underline{c}_o^{o1}(t_{2o}=\infty)\right) \times \underline{H} + \frac{n_1^o e_1}{\rho^o}\left(\underline{\overline{C}}_1^{o2}(\tau) - \underline{\overline{C}}_1^{o2}(t_{2o}=\infty)\right) \times \underline{H} + \frac{n_2^o e_2}{\rho^o} \underline{\overline{C}}_2^{o1}(\tau) \times \underline{H}\Big]$$

$$\rho^{o}\left(\frac{\partial \underline{c}_{o}^{o2}}{\partial t_{21}}(t_{2o}=\infty) + \frac{\partial \underline{c}_{o}^{o1}}{\partial t_{22}}(t_{2o}=\infty) + \frac{\partial \underline{c}_{o}^{oo}}{\partial t_{23}}\right) =$$

$$= \rho_{e}^{o}\underline{c}_{o}^{o1}(t_{2o}=\infty) \times \underline{H} + n_{1}^{o}e_{1}\overline{\underline{C}}_{1M}^{o2} \times \underline{H}$$

Integrating here on the τ_{21} time scale and eliminating secular terms again show that $\underline{c}_{o}^{o2}(t_{2o}=\infty)$ (and from the above assumption $\underline{c}_{o}^{o2}(t_{2o}, \, ..\,)$ and $\underline{c}_{o}^{o2}(t_{2o}=0)$) is independent of t_{21} and $\underline{c}_{oM}^{o1}$ obeys

$$\rho^{o}\,\frac{\partial \underline{c}_{oM}^{o1}}{\partial t_{22}} = -\,\rho^{o}\,\frac{\partial \underline{c}_{o}^{oo}}{\partial t_{23}} + \rho_{e}^{o}\underline{c}_{oM}^{o1} \times \underline{H} + n_{1}^{o}e_{1}\overline{\underline{C}}_{1M}^{o2} \times \underline{H} \qquad (79)$$

$\overline{\underline{C}}_{1M}^{o2}$ we find from the solution of Eq.(77), and the expression depends on $\underline{c}_{o}^{o1}$ in the same way as $\overline{\underline{C}}_{1M}^{o1}$ depends on $\underline{c}_{o}^{oo}$. Consequently the "homogeneous" equation for $\underline{c}_{o}^{o1}$ corresponding to Eq.(79) is the same as the equation for $\underline{c}_{o}^{oo}$ and (using for $\overline{\underline{C}}_{1}^{o2}$ an expression corresponding to Eq.(63))therefore has solutions proportional to $e^{\pm i\omega_{\pm}t_{22}}$. The inhomogeneous term vary like $e^{\pm i\omega_{\pm}t_{22}}$ $\left(\text{the term } \frac{\partial \underline{c}_{o}^{oo}}{\partial t_{23}}\right)$. To avoid $\underline{c}_{oM}^{o1}$ to vary like $t_{22}e^{\pm i\omega_{\pm}t_{22}}$ (which is secular compared to $\underline{c}_{oM}^{oo}$) we set

$$\frac{\partial \underline{c}_{o}^{oo}}{\partial t_{23}} = 0$$

$\underline{c}_{oM}^{o1}$ is now damped away like $\underline{c}_{o}^{oo}$ on the τ_{22}-time scale.

We now expand some equations to first order in ε in terms of the parameter α .

Equations to <u>first order in ε and zeroth order in α</u> are

$$\int d\underline{C}_2 f_2^{10} m_2 \underline{C}_2 = 0 \tag{80}$$

$$\frac{\partial f_1^{10}}{\partial t_{20}} + \frac{\partial f_1^{00}}{\partial t_{30}} + \underline{C}_1 \cdot \frac{\partial f_1^{00}}{\partial \underline{r}} + \underline{F}_1 \cdot \frac{\partial f_1^{00}}{\partial \underline{C}_1} - \frac{\partial \underline{c}_o^{10}}{\partial t_{20}} \cdot \frac{\partial f_1^{00}}{\partial \underline{C}_1} + \frac{e_1}{m_1} \underline{c}_o^{10} \times$$

$$\times \underline{H} \cdot \frac{\partial f_1^{00}}{\partial \underline{C}_1} + \frac{e_1}{m_1} \underline{C}_1 \times \underline{H} \cdot \frac{\partial f_1^{10}}{\partial \underline{C}_1} = FP_{11}\Big[f_1^{00}(\underline{C}_1) f_1^{10}(\underline{C}_1') + \tag{81}$$

$$+ f_1^{10}(\underline{C}_1) f_1^{00}(\underline{C}_1') \Big] + D_1^{00}\Big[f_1^{10} \Big] + D_1^{10}\Big[f_1^{00} \Big]$$

where $D_1^{ij} = \dfrac{n_2^{ij}}{m_1^2} \dfrac{\partial}{\partial \underline{C}_1} \cdot \left(\underset{\sim}{\Phi}^{12}(\underline{C}_1) \cdot \dfrac{\partial}{\partial \underline{C}_1} \right)$,

$$\frac{\partial \rho^{10}}{\partial t_{20}} + \frac{\partial \rho^{o}}{\partial t_{30}} = 0 \tag{82}$$

$$\rho^{o} \frac{\partial \underline{c}_o^{10}}{\partial t_{20}} = 0 \tag{83}$$

$$\frac{3}{2} n^{o} k \left(\frac{\partial T^{10}}{\partial t_{20}} + \frac{\partial T^{00}}{\partial t_{30}} \right) = \frac{3}{2} k T^{00} \frac{\partial}{\partial \underline{r}} \cdot (n_1^{o} \overline{\underline{C}}_1^{00}) + \rho_1^{o} \underline{F}_1 \cdot \overline{\underline{C}}_1^{00} +$$

$$+ n_1^{o} e_1 \overline{\underline{C}}_1^{00} \cdot (\underline{c}_o^{10} \times \underline{H}) - \frac{\partial}{\partial \underline{r}} \cdot q_1^{00} \tag{84}$$

$$\frac{\partial \rho_1^{10}}{\partial t_{20}} + \frac{\partial \rho_1^{o}}{\partial t_{30}} + \frac{\partial}{\partial \underline{r}}\cdot(\rho_1^o \overline{\underline{C}}_1^{oo}) = 0 \tag{85}$$

$$\frac{\partial \rho_2^{10}}{\partial t_{20}} + \frac{\partial \rho_2^{o}}{\partial t_{30}} = 0 \tag{86}$$

$$\frac{3}{2}n_1^o k\left(\frac{\partial T_1^{10}}{\partial t_{20}} + \frac{\partial T_1^{oo}}{\partial t_{30}} \right) = \frac{3}{2}kT_1^{oo} \frac{\partial}{\partial \underline{r}}\cdot(n_1^o \overline{\underline{C}}_1^{oo}) + \rho_1^o \underline{F}_1 \cdot \overline{\underline{C}}_1^{oo} -$$

$$- \rho_1^o \overline{\underline{C}}_1^{oo} \cdot \frac{\partial \underline{c}_o^{10}}{\partial t_{20}} + n_1^o e_1 \overline{\underline{C}}_1^{oo} \cdot (\underline{c}_o^{10} \times \underline{H}) - \frac{\partial}{\partial \underline{r}}\cdot q_1^{oo} \tag{87}$$

Eqs.(82)-(87) show that ρ^{10} , $\underline{c}_o^{10}$ and ρ_2^{10} are t_{2o} independent while ρ_1^{10} , T^{10} and T_1^{10} all have transients on the τ_{2o} time scale. ρ^o , ρ_1^o , ρ_2^o , T^{oo} and T_1^{oo} are all independent of t_{3o} . The electron kinetic equation Eq.(81) is of the form

$$\frac{\partial f_1^{10}}{\partial t_{20}} + \frac{e_1}{m_1} \underline{C}_1 \times \underline{H} \cdot \frac{\partial f_1^{10}}{\partial \underline{C}_1} - FP_{11}\Big[f_{1M}^{oo}(\underline{C}_1) f_1^{10}(\underline{C}_1') +$$

$$+ f_1^{10}(\underline{C}_1) f_{1M}^{oo}(\underline{C}_1') \Big] - D_1^{oo}\Big[f_1^{10}\Big] = f_{1M}^{oo}\Big[\Big(\frac{m_1 C_1^2}{2kT_1^{oo}} - \frac{5}{2}\Big)\frac{1}{T_1^{oo}} \frac{\partial T_1^{oo}}{\partial \underline{r}} -$$

$$- \frac{e_1}{kT_1^{oo}}\Big(\frac{m_1}{e_1} \underline{F}_1 + \underline{c}_o^{10} \times \underline{H} - \frac{kT_1^{oo}}{e_1} \frac{\partial}{\partial \underline{r}} \ell n p_1^{oo}\Big)\Big] \cdot \underline{C}_1 + G^{10}(t_{2o}) \tag{88}$$

Here $G^{10}(t_{2o}) \to 0$ as $t_{2o} \to \infty$. Using the appendixes of [5],

[7] and [12] we find that every bounded solution of this equation evolves toward the solution of Eq.(59) as $t_{2o} \to \infty$ with obvious changes of superscripts and T_1 in place of T .

Equations to <u>first order in ε and first in α</u> are, using results obtained above,

$$\int d\underline{C}_1 f_1^{10} m_1 \underline{C}_1 + \int d\underline{C}_2 f_2^{11} m_2 \underline{C}_2 = 0 \tag{89}$$

$$\frac{\partial f_2^{10}}{\partial t_{2o}} + \frac{\partial f_2^{oo}}{\partial t_{3o}} - \left(\frac{\partial \underline{c}_o^{11}}{\partial t_{2o}} + \frac{\partial \underline{c}_o^{10}}{\partial t_{21}} + \frac{\partial \underline{c}_o^{oo}}{\partial t_{3o}} \right) \cdot \frac{\partial f_2^{10}}{\partial \underline{C}_2} = 0 \tag{90}$$

$$\frac{\partial \rho^{11}}{\partial t_{2o}} + \frac{\partial \rho^{10}}{\partial t_{21}} + \frac{\partial \rho^{o}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho^o \underline{c}_o^{oo}) = 0 \tag{91}$$

$$\rho^o \left(\frac{\partial \underline{c}_o^{11}}{\partial t_{2o}} + \frac{\partial \underline{c}_o^{10}}{\partial t_{21}} + \frac{\partial \underline{c}_o^{oo}}{\partial t_{3o}} \right) = 0 \tag{92}$$

$$\frac{3}{2} n^o k \left(\frac{\partial T^{11}}{\partial t_{2o}} + \frac{\partial T^{10}}{\partial t_{21}} + \frac{\partial T^{o1}}{\partial t_{3o}} + \frac{\partial T^{oo}}{\partial t_{31}} + \underline{c}_o^{oo} \cdot \frac{\partial T^{oo}}{\partial \underline{r}} \right) + \frac{3}{2} n^{1o} k \frac{\partial T^{o1}}{\partial t_{2o}} =$$

$$= \frac{3}{2} k T^{o1} \frac{\partial}{\partial \underline{r}} \cdot (n_1^o \overline{\underline{C}}_1^{oo}) + \frac{3}{2} k T^{oo} \frac{\partial}{\partial \underline{r}} \cdot (n_1^o \overline{\underline{C}}_1^{o1}) + \rho_1^o \underline{F}_1 \cdot \overline{\underline{C}}_1^{o1} + \tag{93}$$

$$+ n_1^o e_1 \overline{\underline{C}}_1^{oo} \cdot (\underline{c}_o^{11} \times \underline{H}) + n_1^o e_1 \overline{\underline{C}}_1^{o1} \cdot (\underline{c}_o^{10} \times \underline{H}) + n_1^o e_1 \overline{\underline{C}}_1^{10} \cdot (\underline{c}_o^{oo} \times \underline{H}) +$$

$$+ n_1^{1o} e_1 \overline{\underline{C}}_1^{oo} \cdot (\underline{c}_o^{oo} \times \underline{H}) - \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_1^{o1} - \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_2^{oo} - \sum_{i=1}^{2} n_i^o m_i \overline{\underline{C}_i \underline{C}_i}^{oo} : \frac{\partial \underline{c}_o^{oo}}{\partial \underline{r}}$$

$$\frac{\partial \rho_1^{11}}{\partial t_{20}} + \frac{\partial \rho_1^{10}}{\partial t_{21}} + \frac{\partial \rho_1^{o}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^o \underline{c}_o^{oo}) + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^o \overline{\underline{C}}_1^{o1}) = 0 \qquad (94)$$

$$\frac{\partial \rho_2^{11}}{\partial t_{20}} + \frac{\partial \rho_2^{10}}{\partial t_{21}} + \frac{\partial \rho_2^{o}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_2^o \underline{c}_o^{oo}) = 0 \qquad (95)$$

$$\frac{3}{2} n_1^o k \left(\frac{\partial T_1^{11}}{\partial t_{20}} + \frac{\partial T_1^{10}}{\partial t_{21}} + \frac{\partial T_1^{o1}}{\partial t_{30}} + \frac{\partial T_1^{oo}}{\partial t_{31}} + \underline{c}_o^{oo} \cdot \frac{\partial T_1^{oo}}{\partial \underline{r}} \right) +$$

$$+ \frac{3}{2} n_1^{1o} k \frac{\partial T_1^{o1}}{\partial t_{20}} = \frac{3}{2} k T_1^{o1} \frac{\partial}{\partial \underline{r}} \cdot (n_1^o \overline{\underline{C}}_1^{oo}) + \frac{3}{2} k T_1^{oo} \frac{\partial}{\partial \underline{r}} \cdot (n_1^o \overline{\underline{C}}_1^{o1}) +$$

$$+ \rho_1^o \underline{F}_1 \cdot \overline{\underline{C}}_1^{o1} + n_1^o e_1 \overline{\underline{C}}_1^{oo} \cdot (\underline{c}_o^{11} \times \underline{H}) + n_1^o e_1 \overline{\underline{C}}_1^{o1} \cdot (\underline{c}_o^{1o} \times \underline{H}) + \qquad (96)$$

$$+ n_1^o e_1 \overline{\underline{C}}_1^{1o} \cdot (\underline{c}_o^{oo} \times \underline{H}) + n_1^{1o} e_1 \overline{\underline{C}}_1^{oo} \cdot (\underline{c}_o^{oo} \times \underline{H}) - \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_1^{o1} -$$

$$- \rho_1^o \overline{\underline{C}_1 \underline{C}_1}^{oo} : \frac{\partial \underline{c}_o^{oo}}{\partial \underline{r}}$$

$$\frac{3}{2} n_2^o k \left(\frac{\partial T_2^{1o}}{\partial t_{2o}} + \frac{\partial T_2^{oo}}{\partial t_{3o}} \right) = 0 \qquad (97)$$

These equations show that f_2^{10} , ρ^{11} , ρ_2^{11} , $\underline{c}_o^{11}$ and T_2^{1o} are all independent of t_{2o} , while ρ_1^{11} , T^{11} and T_1^{11} all have a t_{2o} transient. $\underline{c}_o^{1o}$ is independent of t_{21} and f_2^{oo} , $\underline{c}_o^{oo}$ and T_2^{oo} are independent of t_{3o} . The variations on the τ_{21} - and τ_{22} time scales in Eq.(91) and Eqs. (93) - (96) may be absorbed by

ρ^{1o} , ρ_1^{1o} , ρ_2^{1o} , T_1^{1o} and T^{1o} (using that $t_{22} = \alpha t_{21}$), for instance

$$\rho_1^{1o}(t_{2o}, t_{21}, \dots) = \rho_1^{1o}(t_{2o}=0, t_{21}=0) - \int_0^{t_{2o}} \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^o \overline{\underline{C}}_1^{oo}(\tau))\, d\tau -$$

$$- \int_0^{t_{21}} \frac{\partial}{\partial \underline{r}} \cdot \left[\rho_1^o \underline{c}_{o\perp}^{oo}(\alpha\lambda) + \rho_1^o \overline{\underline{C}}_1^{o1}(\alpha\lambda) \right] d\lambda$$

the first integral comming from the equation to first order in ε and zeroth in α. The term $\rho_1^{1o}(t_{2o}=0, t_{21}=0)$ may vary on the τ_{22} time scale. In the limits $t_{2o} = \infty$, $t_{21} = \infty$ and $t_{22} = \infty$ we obtain the equations

$$\frac{\partial \rho^o}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho^o \underline{c}_{o\|}^{oo}) = 0 \tag{98}$$

$$\frac{\partial \rho_1^o}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^o \underline{c}_{o\|}^{oo}) = 0 \tag{99}$$

$$\frac{\partial \rho_2^o}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_2^o \underline{c}_{o\|}^{oo}) = 0 \tag{100}$$

$$\frac{3}{2} n^o k \left(\frac{\partial T^{oo}}{\partial t_{31}} + \underline{c}_{o\|}^{oo} \cdot \frac{\partial T^{oo}}{\partial \underline{r}} \right) = - \mathrm{I}p^{oo} : \frac{\partial \underline{c}_{o\|}^{oo}}{\partial \underline{r}} \tag{101}$$

$$\frac{3}{2}n_1^o k\left(\frac{\partial T_1^{oo}}{\partial t_{31}} + \underline{c}_{o\|}^{oo}\cdot\frac{\partial T_1^{oo}}{\partial \underline{r}}\right) = - Ip_1^{oo} : \frac{\partial \underline{c}_{o\|}^{oo}}{\partial \underline{r}} \qquad (102)$$

Eqs.(98) - (101) correspond to Eqs.(49) - (51)

Equations to <u>first order in ε and second in α</u> are

$$\frac{\partial f_2^{11}}{\partial t_{2o}} + \frac{\partial f_2^{1o}}{\partial t_{21}} + \frac{\partial f_2^{o1}}{\partial t_{3o}} + \frac{\partial f_2^{oo}}{\partial t_{31}} + \underline{c}_o^{oo}\cdot\frac{\partial f_2^{oo}}{\partial \underline{r}} + \underline{C}_2\cdot\frac{\partial f_2^{oo}}{\partial \underline{r}} + \underline{F}_2\cdot\frac{\partial f_2^{oo}}{\partial \underline{C}_2} -$$

$$- \left(\frac{\partial \underline{c}_o^{12}}{\partial t_{2o}} + \frac{\partial \underline{c}_o^{11}}{\partial t_{21}} + \frac{\partial \underline{c}_o^{1o}}{\partial t_{22}} + \frac{\partial \underline{c}_o^{o1}}{\partial t_{3o}} + \frac{\partial \underline{c}_o^{oo}}{\partial t_{31}} + \underline{c}_o^{oo}\cdot\frac{\partial \underline{c}_o^{oo}}{\partial \underline{r}}\right)\cdot\frac{\partial f_2^{oo}}{\partial \underline{C}_2} -$$

$$- \frac{\partial \underline{c}_o^{o1}}{\partial t_{2o}}\cdot\frac{\partial f_2^{1o}}{\partial \underline{C}_2} + \frac{e_2}{m_2}\underline{c}_o^{1o}\times\underline{H}\cdot\frac{\partial f_2^{oo}}{\partial \underline{C}_2} - \frac{\partial f_2^{oo}}{\partial \underline{C}_2}\underline{C}_2 : \frac{\partial \underline{c}_o^{oo}}{\partial \underline{r}} = \qquad (103)$$

$$= FP_{22}\left[f_2^{oo}(\underline{C}_2)f_2^{1o}(\underline{C}_2') + f_2^{1o}(\underline{C}_2)f_2^{oo}(\underline{C}_2')\right] - \frac{1}{m_1 m_2}\frac{\partial}{\partial \underline{C}_2}\cdot$$

$$\cdot\int d\underline{C}_1 \underset{\sim}{\Phi}^{12}(\underline{C}_1)\cdot\frac{\partial}{\partial \underline{C}_1}\left(f_2^{oo}(\underline{C}_2)f_1^{1o}(\underline{C}_1) + f_2^{1o}(\underline{C}_2)f_1^{oo}(\underline{C}_1)\right)$$

$$\rho^o\left(\frac{\partial \underline{c}_o^{12}}{\partial t_{2o}} + \frac{\partial \underline{c}_o^{11}}{\partial t_{21}} + \frac{\partial \underline{c}_o^{1o}}{\partial t_{22}} + \frac{\partial \underline{c}_o^{o1}}{\partial t_{3o}} + \frac{\partial \underline{c}_o^{oo}}{\partial t_{31}} + \underline{c}_o^{oo}\cdot\frac{\partial \underline{c}_o^{oo}}{\partial \underline{r}}\right) +$$

$$+ \rho^{1o}\frac{\partial \underline{c}_o^{o1}}{\partial t_{2o}} = - \frac{\partial}{\partial \underline{r}}\cdot\sum_{i=1}^{2} n_i^o m_i \overline{\underline{C}_i\underline{C}_i}^{oo} + \rho_e^o \underline{c}_o^{1o}\times\underline{H} + n_1^o e_1 \overline{\underline{C}}_1^{1o}\times\underline{H} + \qquad (104)$$

$$+ n_1^{1o} e_1 \overline{\underline{C}}_1^{oo}\times\underline{H} + \sum_{i=1}^{2}\rho_i^o \underline{F}_i$$

$$\frac{3}{2}n_2^o k\left(\frac{\partial T_2^{11}}{\partial t_{20}} + \frac{\partial T_2^{10}}{\partial t_{21}} + \frac{\partial T_2^{01}}{\partial t_{30}} + \frac{\partial T_2^{00}}{\partial t_{31}} + \underline{c}_o^{00}\cdot\frac{\partial T_2^{00}}{\partial \underline{r}}\right) =$$

$$= -\frac{\partial}{\partial \underline{r}}\cdot\underline{q}_2^{00} - \rho_2^o\overline{\underline{C}_2\underline{C}_2}^{00} : \frac{\partial \underline{c}_o^{00}}{\partial \underline{r}} \qquad (105)$$

The parallel part of Eq.(104) is

$$\rho^o\left(\frac{\partial \underline{c}_{o\|}^{12}}{\partial t_{20}} + \frac{\partial \underline{c}_{o\|}^{11}}{\partial t_{21}} + \frac{\partial \underline{c}_{o\|}^{10}}{\partial t_{22}} + \frac{\partial \underline{c}_{o\|}^{o}}{\partial t_{31}} + \underline{c}_o^{00}\cdot\frac{\partial}{\partial \underline{r}}\,\underline{c}_{o\|}^{o}\right) =$$

$$= -\frac{\partial}{\partial \underline{r}}\cdot\sum_{i=1}^{2} n_i^o m_i\overline{\underline{C}_i\underline{C}_i}_{\|}^{00} + \sum_{i=1}^{2}\rho_i^o\underline{F}_{i\|} \qquad (106)$$

remembering that $\underline{c}_{o\|}^{o}$ is not expanded in α . We see that $\underline{c}_{o\|}^{12}$ has a transient on the τ_{20} time scale and $\underline{c}_{o\|}^{11}$ has a transient on the τ_{21} time scale :

$$\underline{c}_{o\|}^{12}(t_{20}, \ldots) = \underline{c}_{o\|}^{12}(t_{20}=0, t_{21}, \ldots) - \int_0^{t_{20}} d\tau\,\frac{\partial}{\partial \underline{r}}\cdot$$

$$\cdot\left(n_1^o m_1\overline{\underline{C}_1(\tau)\underline{C}_1(\tau)}^{00} - I\,p_1^{00}\right) \qquad (107)$$

$$\underline{c}_{o\|}^{11}(t_{21}, \ldots) = \underline{c}_{o\|}^{11}(t_{21}=0, t_{22}, \ldots) - \int_0^{t_{21}} d\lambda\,\frac{\partial}{\partial \underline{r}}\cdot$$

$$\cdot\left(n_2^o m_2\overline{\underline{C}_2(\lambda)\underline{C}_2(\lambda)}^{00} - I\,p_2^{00}\right) \qquad (108)$$

In the limits $t_{2o} = \infty$ and $t_{21} = \infty$ Eq.(106) reduces to

$$\rho^o\left(\frac{\partial \underline{c}^{1o}_{o\parallel}}{\partial t_{22}} + \frac{\partial \underline{c}^{o}_{o\parallel}}{\partial t_{31}} + \underline{c}^{oo}_{o}\cdot\frac{\partial}{\partial \underline{r}}\,\underline{c}^{o}_{o\parallel}\right) = -\sum_{i=1}^{2}\frac{\partial p^{oo}_{i}}{\partial \underline{r}_{\parallel}} + \sum_{i=1}^{2}\rho^o_i\underline{F}_{i\parallel} \qquad (109)$$

As $t_{22} \to \infty$, $\underline{c}^{oo}_{o\perp} \to 0$ and $T^{oo}_{i} \to T^{oo}$. Due to this $\underline{c}^{1o}_{o\parallel}$ has a transient on the τ_{22} time scale and in the limit $t_{22} = \infty$ Eq.(109) becomes

$$\rho^o\left(\frac{\partial \underline{c}^{o}_{o\parallel}}{\partial t_{31}} + \underline{c}^{o}_{o\parallel}\cdot\frac{\partial}{\partial \underline{r}}\,\underline{c}^{o}_{o\parallel}\right) = -\frac{\partial p^{oo}}{\partial \underline{r}} + \sum_{i=1}^{2}\rho^o_i\,\underline{F}_{i\parallel} \qquad (110)$$

which corresponds to Eq.(52).

The transverse part of Eq.(104) shows that $\underline{c}^{12}_{o\perp}$ has a transient on the τ_{2o} time scale and $\underline{c}^{11}_{o\perp}$ a transient on the τ_{21} time scale. In the limits $t_{2o} = \infty$ and $t_{21} = \infty$ the transverse part is

$$\rho^o\left(\frac{\partial \underline{c}^{1o}_{o\perp}}{\partial t_{22}} + \frac{\partial \underline{c}^{o1}_{o\perp}}{\partial t_{3o}} + \frac{\partial \underline{c}^{oo}_{o\perp}}{\partial t_{31}} + \underline{c}^{oo}_{o}\cdot\,\frac{\partial}{\partial \underline{r}}\,\underline{c}^{oo}_{o\perp}\right) = -\sum_{i=1}^{2}\frac{\partial p^{oo}_{i}}{\partial \underline{r}_{\perp}} +$$

$$+\ \rho^o_e\underline{c}^{1o}_{o} \times \underline{H} + n^o_1 e_1\underline{C}^{1o}_{1} \times \underline{H} + \sum_{i=1}^{2}\rho^o_i\underline{F}_{i\perp} \qquad (111)$$

$\overline{\underline{C}}_1^{10}$ we find from Eq.(88) in the limit $t_{20} = \infty$. Thus $\overline{\underline{C}}_1^{10}$ depends on $\underline{c}_o^{10}$ in the same way as $\overline{\underline{C}}_1^{o1}$ depends on $\underline{c}_o^{oo}$ or $\overline{\underline{C}}_1^{o2}$ depends on $\underline{c}_o^{o1}$, and therefore the "homogeneous" equation corresponding to Eq.(111) for $\underline{c}_o^{10}$ is the same as the equation for $\underline{c}_o^{oo}$. The inhomogeneous term of Eq.(111) for $\underline{c}_o^{10}$ tends toward a nonzero limit as $t_{22} \to \infty$. Using for $\overline{\underline{C}}_1^{10}$ a simplified expression corresponding to Eq.(63) and for $\underline{c}_o^{oo}$ and $\underline{c}_o^{o1}$ expressions obtained from Eqs.(69) and (79) in the same way, we easily solve Eq.(111) and find that $\underline{c}_o^{10}$ evolves toward a limit as $t_{22} \to \infty$ which we can find setting the right hand side of Eq.(111) in the limit $t_{22} = \infty$ equal to zero, i.e.

$$- \sum_{i=1}^{2} \frac{\partial p_i^{oo}}{\partial \underline{r}_\perp} + \rho_e^o \underline{c}_o^{10} \times \underline{H} + n_1^o e_1 \overline{\underline{C}}_1^{10} \times \underline{H} + \sum_{i=1}^{2} \rho_i^o \underline{F}_{i_\perp} = 0 \qquad (112)$$

So, if further $\underline{c}_{o\perp}^{o2}$, $\underline{c}_{o\perp}^{o3}$ etc. like $\underline{c}_{o\perp}^{oo}$ and $\underline{c}_{o\perp}^{o1}$ all tend to zero as $t_{22} \to \infty$ we have shown that for bounded distribution functions, $\underline{c}_{o\perp}$ up to terms of order ε evolves so that balance of transverse forces is established as $t_{22} \to \infty$, using simplified expressions for $\overline{\underline{C}}_1^{o1}$, $\overline{\underline{C}}_1^{o2}$ etc. However, this result is expected to hold using for $\overline{\underline{C}}_1^{o1}$, $\overline{\underline{C}}_1^{o2}$ etc. exact expressions obtained from the solution of Eqs.(68) , (77) etc.

From Eq.(105) we see that T_2^{11} is independent of t_{2o} while T_2^{1o} may absorbe, apart from the variation on the τ_{21} time scale, also the variations on the τ_{22} time scale. In the limits $t_{2o} = \infty$, $t_{21} = \infty$ and $t_{22} = \infty$ we obtain

$$\frac{3}{2}n_2^o k \left(\frac{\partial T_2^{oo}}{\partial t_{31}} + \underline{c}^o_{o\|} \cdot \frac{\partial T_2^{oo}}{\partial \underline{r}}\right) = - \mathrm{I}p_2^{oo} : \frac{\partial \underline{c}^o_{o\|}}{\partial \underline{r}} \qquad (113)$$

which when added to Eq.(102) corresponds to Eq.(51) .

Eq.(103) shows that f_2^{11} has a transient on the τ_{2o} time scale and substituting from Eq.(39) to second order in α for the terms in paranteses on the left hand side of Eq.(103) and also from Eqs.(100) and (113) we derive that in the limit $t_{2o} = \infty$ Eq.(103) reduces to

$$\frac{\partial f_2^{1o}}{\partial t_{21}} - \mathrm{FP}_{22}\left[f_{2M}^{oo}(\underline{C}_2) f_2^{1o}(\underline{C}_2') + f_2^{1o}(\underline{C}_2) f_{2M}^{oo}(\underline{C}_2')\right] =$$

$$= - f_{2M}^{oo}\left(\frac{m_2 C_2^2}{2kT^{oo}} - \frac{5}{2}\right)\frac{1}{T^{oo}}\frac{\partial T^{oo}}{\partial \underline{r}} \cdot \underline{C}_2 - f_{2M}^{oo}\frac{m_2}{kT^{oo}}\underline{C}_2^o\underline{C}_2 : \qquad (114)$$

$$: \frac{\partial \underline{c}^o_{o\|}}{\partial \underline{r}} + H^{1o}(t_{21}, t_{22})$$

where $H^{1o}(t_{21}, t_{22}) \to 0$ as $t_{21} \to \infty$ and $t_{22} \to \infty$.

Again using the appendix of[12], every bounded solution of Eq.(114) evolves toward the solution of

$$FP_{22}\left[f_{2M}^{oo}(\underline{C}_2)f_2^{1o}(\underline{C}_2') + f_{2M}^{1o}(\underline{C}_2)f_{2M}^{oo}(\underline{C}_2')\right] =$$

$$= f_{2M}^{oo}\left(\frac{m_2C_2^2}{2kT^{oo}} - \frac{5}{2}\right)\frac{1}{T^{oo}}\frac{\partial T^{oo}}{\partial \underline{r}}\cdot \underline{C}_2 + f_{2M}^{oo}\frac{m_2}{kT^{oo}}\underline{C}_2^o\underline{C}_2 \ : \ \frac{\partial \underline{c}_{o\|}^{c}}{\partial \underline{r}}$$

as $t_{21} \to \infty$ and $t_{22} \to \infty$, the $\partial/\partial t_{21}$ of Eq.(114) also taking account of the variation on the τ_{22} time scale. This result for the ion distribution function to first order in ε and zeroth in α corresponds to the result of Eq.(60) in the same way as Eq.(88) for the electron distribution function to the same order in the limits $t_{2o} = \infty$, $t_{21} = \infty$ and $t_{22} = \infty$, corresponds to Eq.(59).

This ends our partial derivation of assumptions Eqs.(53) and (54).

Appendix 1.

We here briefly derive the equations of Chapman and Cowling [1] from Eqs. (1)-(5) by use of the multiple time scale method.

The distribution functions and macroscopic quantities are expanded in terms of the parameter ε_1 , for instance

$$f_1 = f_1^{(o)} + \varepsilon_1 f_1^{(1)} + \varepsilon_1^2 f_1^{(2)} + \ldots\ldots.$$

Later we leave out the paranteses in the superscripts.

Also the time derivative is expanded in this way

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_o} + \varepsilon_1 \frac{\partial}{\partial t_1} + \varepsilon_1^2 \frac{\partial}{\partial t_2} + \ldots\ldots\ldots$$

consistent with introduction of time variables t_i on time scales $\tau_i = \frac{\tau_o}{\varepsilon_1^i}$, $i = 0,1,2, \ldots.$, where τ_o , roughly speeking, is the time scale on which the gas evolves toward local thermodynamic equilibrium.

The equations to zeroth order are

$$\sum_i \rho_i^o \underline{\overline{C}}_i^o = 0 \tag{A1}$$

$$\frac{\partial f_i^o}{\partial t_o} - \frac{\partial \underline{c}_o^o}{\partial t_o} \cdot \frac{\partial f_i^o}{\partial \underline{C}_i} + \frac{e_i}{m_i} \underline{C}_i \times \underline{H} \cdot \frac{\partial f_i}{\partial \underline{C}_i} = \sum_j C_{ij}\left[f_i^o f_j^o\right] \tag{A2}$$

$$\frac{\partial \rho_i^o}{\partial t_o} = 0 \qquad \text{(A3)}$$

$$\rho^o \frac{\partial \underline{c}_o^o}{\partial t_o} = \underline{j}^o \times \underline{H} \qquad \text{(A4)}$$

$$\frac{3}{2} n^o k \frac{\partial T^o}{\partial t_o} = 0 \qquad \text{(A5)}$$

A H - theorem can be established showing that

$$f_i^o \to n_i^o \left(\frac{m_i}{2\pi k T^o}\right)^{3/2} \exp\left(- \frac{m_i C_i^2}{2kT^o}\right) \quad \text{as } t_o \to \infty \qquad \text{(A6)}$$

Thus $\underline{c}_o^o$ transverse to $\underline{H}$, $\underline{c}_{o\perp}^o$, obeys $\frac{\partial \underline{c}_{o\perp}^o}{\partial t_o} \to 0$ as $t_o \to \infty$,or $\underline{c}_{o\perp}^o$ has a transient on the τ_o time scale ,

$$\underline{c}_o^o(t_o , \ldots.) = \underline{c}_o^o(t_o{=}0, t_1 , \ldots) + \frac{1}{\rho^o} \int_o^{t_o} \underline{j}^o(\tau) \times \underline{H} \, d\tau$$

ρ_i^o and T^o are independent of t_o .

<u>The equations to first order</u> are (they correspond to the second order equations of Chapman and Cowling)

$$\sum_i \rho_i^o \, \underline{C}_i^1 + \sum_i \rho_i^1 \, \underline{C}_i^o = 0 \qquad \text{(A7)}$$

$$\frac{\partial f_i^1}{\partial t_o} + \frac{\partial f_i^o}{\partial t_1} + \underline{c}_o^o \cdot \frac{\partial f_i^o}{\partial \underline{r}} + \underline{C}_i \cdot \frac{\partial f_i^o}{\partial \underline{r}} + \underline{F}_i \cdot \frac{\partial f_i^o}{\partial \underline{C}_i} + \frac{e_i}{m_i} \underline{c}_o^o \times \underline{H} \cdot \frac{\partial f_i^o}{\partial \underline{C}_i} -$$

$$- \frac{\partial f_i^o}{\partial \underline{C}_i} \underline{C}_i : \frac{\partial \underline{c}_o^o}{\partial \underline{r}} - \frac{\partial \underline{c}_o^o}{\partial t_o} \cdot \frac{\partial f_i^1}{\partial \underline{C}_i} - \left(\frac{\partial \underline{c}_o^1}{\partial t_o} + \frac{\partial \underline{c}_o^o}{\partial t_1} + \underline{c}_o^o \cdot \frac{\partial \underline{c}_o^o}{\partial \underline{r}} \right) \cdot \frac{\partial f_i^o}{\partial \underline{C}_i} +$$

$$+ \frac{e_i}{m_i} \underline{C}_i \times \underline{H} \cdot \frac{\partial f_i^1}{\partial \underline{C}_i} = C_{ij}\left[f_i^o f_j^1 + f_i^1 f_j^o \right] \tag{A8}$$

$$\frac{\partial \rho_i^1}{\partial t_o} + \frac{\partial \rho_i^o}{\partial t_1} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_i^o \underline{c}_o^o) + \frac{\partial}{\partial \underline{r}} \cdot (\rho_i^o \overline{\underline{C}}_i^o) = 0 \tag{A9}$$

$$\rho^o \left(\frac{\partial \underline{c}_o^1}{\partial t_o} + \frac{\partial \underline{c}_o^o}{\partial t_1} + \underline{c}_o^o \cdot \frac{\partial \underline{c}_o^o}{\partial \underline{r}} \right) + \rho^1 \frac{\partial \underline{c}_o^o}{\partial t_o} = \sum_i \rho_i^o \underline{F}_i + \rho_e^o \underline{c}_o^o \times \underline{H} +$$

$$+ \underline{j}^1 \times \underline{H} - \frac{\partial}{\partial \underline{r}} \cdot \underset{\sim}{P}^o \tag{A10}$$

$$\frac{3}{2} n^o k \left(\frac{\partial T^1}{\partial t_o} + \frac{\partial T^o}{\partial t_1} + \underline{c}_o^o \cdot \frac{\partial T^o}{\partial \underline{r}} \right) = \frac{3}{2} k T^o \frac{\partial}{\partial \underline{r}} \cdot \sum_i n_i^o \overline{\underline{C}}_i^o +$$

$$+ \sum_i \rho_i^o \overline{\underline{C}}_i^o \cdot \underline{F}_i + \underline{j}^o \cdot (\underline{c}_o^o \times \underline{H}) - \underset{\sim}{P}^o : \frac{\partial \underline{c}_o^o}{\partial \underline{r}} - \frac{\partial}{\partial \underline{r}} \cdot \underline{q}^o \tag{A11}$$

From Eq.(A9) we see that ρ_i^1 has a transient on the τ_o time scale due to the evolution of $\underline{c}_{o\perp}^o$ and $\overline{\underline{C}}_i^o$, and in the limit $t_o = \infty$ we obtain

$$\frac{\partial \rho_i^o}{\partial t_1} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_i^o \underline{c}_{oM}^o) = 0 \tag{A12}$$

where M means expressions in the limit $t_o = \infty$. Similarly Eq.(A.11) shows that T^1 has a transient on the τ_o time scale due to the transients of $\underline{c}^o_{o\perp}$, $\overline{\underline{C}}^o_i$, $\underline{j}^o$, $\underset{\sim}{P}^o$ and $\underline{q}^o$. In the limit $t_o = \infty$ we get

$$\frac{3}{2}n^o k\left(\frac{\partial T^o}{\partial t_1} + \underline{c}^o_{oM} \cdot \frac{\partial T^o}{\partial \underline{r}}\right) = - p^o \frac{\partial}{\partial \underline{r}} \cdot \underline{c}^o_{oM} \qquad \text{(A13)}$$

From the parallel version of Eq.(A.10) we get that $\underline{c}^1_{o\|}$ has a transient on the τ_o time scale due to the transients of $\underline{c}^o_{o\perp}$ and $\underset{\sim}{P}^o$ and in the limit $t_o = \infty$ we obtain

$$\rho^o\left(\frac{\partial \underline{c}^o_{o\|}}{\partial t_1} + \underline{c}^o_{oM} \cdot \frac{\partial}{\partial \underline{r}} \underline{c}^o_{o\|}\right) = \sum_i \rho^o_i \underline{F}_{i\|} - \frac{\partial p^o}{\partial \underline{r}_{\|}} \qquad \text{(A14)}$$

From Eq.(A8) we seek bounded solutions for f^1_i which obey

$$\frac{\partial f^1_i}{\partial t_o} \to 0 \quad \text{as} \quad t_o \to \infty \qquad \text{(A15)}$$

Then from Eq.(A.10) we get that $\underline{c}^1_{o\perp}$ has a transient on the τ_o time scale due to the transients of $\underline{c}^o_{o\perp}$, $\underline{j}^1$ and $\underset{\sim}{P}^o$, and in the limit $t_o = \infty$ we have

$$\rho^o\left(\frac{\partial \underline{c}^o_{oM\perp}}{\partial t_1} + \underline{c}^o_{oM} \cdot \frac{\partial \underline{c}^o_{oM\perp}}{\partial \underline{r}}\right) = \sum_i \rho^o_i \underline{F}_{i\perp} + \rho^o_e \underline{c}^o_{oM} \times \underline{H} +$$

$$+ \underline{j}^1_M \times \underline{H} - \frac{\partial p^o}{\partial \underline{r}_{\perp}} \qquad \text{(A16)}$$

Here we find $\underline{j}_M^1$ from f_i^1 in the limit $t_o = \infty$. The equation for f_{iM}^1 is, substituting from Eqs.(A.14) and (A.16)

$$\frac{\partial f_{iM}^o}{\partial t_1} + \underline{c}_{oM}^o \cdot \frac{\partial f_{iM}^o}{\partial \underline{r}} + \underline{C}_i \cdot \frac{\partial f_{iM}^o}{\partial \underline{r}} + \underline{F}_i \cdot \frac{\partial f_{iM}^o}{\partial \underline{C}_i} + \frac{e_i}{m_i} \underline{c}_{oM}^o \times \underline{H} \cdot \frac{\partial f_{iM}^o}{\partial \underline{C}_i} -$$

$$- \frac{\partial f_{iM}^o}{\partial \underline{C}_i} \underline{C}_i : \frac{\partial \underline{c}_{oM}^o}{\partial \underline{r}} - \frac{1}{\rho^o}\left(\sum_j \rho_j^o \underline{F}_j + \rho_e^o \underline{c}_{oM}^o \times \underline{H} + \underline{j}_M^1 \times \underline{H} - \frac{\partial p^o}{\partial \underline{r}}\right) \cdot \frac{\partial f_{iM}^o}{\partial \underline{C}_i} +$$

$$+ \frac{e_i}{m_i} \underline{C}_i \times \underline{H} \cdot \frac{\partial f_{iM}^1}{\partial \underline{C}_i} = \sum_j C_{ij}\left[f_{iM}^o f_{jM}^1 + f_{iM}^1 f_{jM}^o \right]$$

or, substituting further from Eqs.(A.6), (A.12) and (A.13)

$$f_{iM}^o \left(\frac{m_i C_i^2}{2kT^o} - \frac{5}{2} \right) \frac{1}{T^o} \frac{\partial T^o}{\partial \underline{r}} \cdot \underline{C}_i + f_{iM}^o \frac{m_i}{kT^o}\left[\frac{1}{\rho^o}\left(\sum_j \rho_j^o \underline{F}_j + \rho_e^o \underline{c}_{oM}^o \times \underline{H} - \frac{\partial p^o}{\partial \underline{r}} \right) - \right.$$

$$\left. - \underline{F}_i + \frac{kT^o}{m_i} \frac{\partial}{\partial \underline{r}} \ell n p_i^o - \frac{e_i}{m_i} \underline{c}_{oM}^o \times \underline{H} \right] \cdot \underline{C}_i + f_{iM}^o \frac{m_i}{kT^o}\left(\underline{C}_i \underline{C}_i - \frac{1}{3} \underset{\sim}{I} C^2 \right) : \frac{\partial \underline{c}_{oM}^o}{\partial \underline{r}} =$$

$$= \sum_j C_{ij}\left[f_{iM}^o f_{jM}^1 + f_{iM}^1 f_{jM}^o \right] - \frac{e_i}{m_i} \underline{C}_i \times \underline{H} \cdot \frac{\partial f_{iM}^1}{\partial \underline{C}_i} - \qquad \text{(A.17)}$$

$$- f_{iM}^o \frac{m_i}{\rho^o kT^o} \underline{C}_i \cdot \left(\underline{j}_M^1 \times \underline{H} \right)$$

Eqs. (A.12), (A.13), (A.14), (A.16) and (A.17) are the "second order" equations of Chapman and Cowling [1]. Eq.(A.7) in the limit $t_o = \infty$, i.e.

$$\sum_i \rho_i^o \, \underline{\overline{C}}_{iM}^1 = 0 \qquad \text{(A.18)}$$

has to be fulfilled when solving Eq.(A.17) for f_{iM}^1.

Appendix 2.

Till now the electromagnetic field has been assumed stationary and uniform, imposed on the plasma by external means. However, the generation of a field by the evolution of the plasma itself may also be taken into account with only small modifications of the results obtained in sections 2 and 3. For such a model we study the kinetic and macroscopic equations of the previous sections together with the Maxwell equations for the total electric field $\underline{E}$ and magnetic field $\underline{B}$:

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{E} = 4\pi c^2 \rho_e \qquad (A.19)$$

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{B} = 0 \qquad (A.20)$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{E} = - \frac{\partial \underline{B}}{\partial t} \qquad (A.21)$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{B} = 4\pi(\rho_e \underline{c}_o + \underline{j}) + \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \qquad (A.22)$$

As in section 1 we assume that

$$\frac{\bar{c}_1 \tau_2}{L} \sim \varepsilon$$

$$\frac{E_o}{\bar{c}_1 B_o} \sim \varepsilon \qquad \text{(A.23)}$$

$$\tau_2 \sim \frac{1}{\Omega_1} = \frac{m_1}{e_1 B_o}$$

where E_o and $B_o = H$ are the magnitude of the externally imposed fields. New assumptions on the sources of the fields now have to be made: Since the fields shall not be stronger than before the sources must be weak to a certain extent. Also to solve the Maxwell equations requires additional assumptions. We therefore parametrize the Maxwell equations as we did with the kinetic and macroscopic equations and make the assumptions when we need to make them.

The order of magnitude of the terms in Eq.(A.19) thus become

$$\frac{E_L}{L} : 4\pi c^2 \rho_e$$

Here E_L is the part of $\underline{E}$ that varies in space. We further get, dividing by $\frac{E_L}{L}$:

$$1 : \frac{4\pi c^2 n_1 e_1^2}{m_1} \cdot \frac{m_1 \rho_e}{n_1 e_1^2} \cdot \frac{L}{E_L}$$

and using the relations Eq.(A.23) and $\omega_{p1} = \left(\frac{4\pi c^2 n_1 e_1^2}{m_1} \right)^{\frac{1}{2}}$, the electron plasma frequency,

$$1 : \left(\frac{\omega_{p1}}{\Omega_1}\right)^2 \frac{1}{\varepsilon^2} \left(\frac{E_o}{E_L}\right) \left(\frac{\rho_e}{n_1 e_1}\right)$$

Since for the model we consider $\omega_{p1} >> \Omega_1$ and we allow $E_L \sim E_o$, for a balance of the terms of Eq.(A.19) we must have

$$\rho_e \sim (\Omega_1/\omega_{p1})^2 \cdot \varepsilon^2 n_1 e_1$$

i.e. charge neutrality to a certain extent.

In a similar way we estimate the terms in Eq.(A.21)

$$\frac{E_L}{L} : \frac{B_t}{\tau_2}$$

where B_t is the part of $\underline{B}$ that varies in time. We further get

$$1 : \frac{B_t}{B_o} \frac{1}{\varepsilon^2}$$

We have used the relations Eq.(A.23) and $E_L \sim E_o$.
For the terms in Eq.(A.21) to balance we have $B_t \sim \varepsilon^2 B_o$.

The order of magnitude of the terms in Eq.(A.22) becomes

$$\frac{B_L}{L} : 4\pi n_1 e_1 \bar{c}_1 : \frac{1}{c^2} \frac{E_t}{\tau_2}$$

where B_L and E_t denote the parts of $\underline{B}$ and $\underline{E}$ that vary in space and time. We have set $|\underline{j}| \sim n_1 e_1 \bar{c}_1$. We further get

$$1 : \left(\frac{\omega_{p1}}{\Omega_1}\right)^2 \frac{1}{\varepsilon} \frac{B_o}{B_L} \left(\frac{\bar{c}_1}{c}\right)^2 : \left(\frac{\bar{c}_1}{c}\right)^2 \left(\frac{E_t}{E_o}\right)\left(\frac{B_o}{B_L}\right)$$

We assume that $\left(\frac{\omega_{p1}}{\Omega_1}\right)^2 \frac{1}{\varepsilon} \left(\frac{\bar{c}_1}{c}\right)^2 << 1$. More specifically, let $\left(\frac{\omega_{p1}}{\Omega_1}\right)^2 \frac{1}{\varepsilon} \left(\frac{\bar{c}_1}{c}\right)^2 \sim \varepsilon^2$. Then $B_L \sim \varepsilon^2 B_o$ for the first and second term to be of equal order of magnitude. The third term will always be vanishing small when $E_t \sim E_o$, which is the order of magnitude of E_t that we can permit. We shall therefore neglect the last term (displacement current) of Eq.(A.22), and the approximative Maxwell equations for the gas model are

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{E} = 4\pi c^2 \rho_e \qquad (A.19')$$

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{B} = 0 \qquad (A.20')$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{E} = - \frac{\partial \underline{B}}{\partial t} \qquad (A.21')$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{B} = 4\pi(\varepsilon^2 \alpha \rho_e \underline{c}_o + n_1 e_1 \underline{\bar{C}}_1 + \alpha n_2 e_2 \underline{\bar{C}}_2) \qquad (A.22')$$

where $\underline{E}$ and $\underline{B}$ are sought as

$$\underline{E}(\underline{r},t) = \underline{E}^o(\underline{r},t) + \varepsilon \underline{E}^1(\underline{r},t) + \ldots$$

$$\underline{B}(\underline{r},t) = \underline{H} + \varepsilon^2 \underline{B}^2(\underline{r},t) + \ldots$$

Making the multiple time scale expansion of $\frac{\partial}{\partial t}$, we obtain as the lowest order sets of equations in the parameter ε :

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{E}^0 = 4\pi c^2 \rho_e^0$$

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{B}^2 = 0$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{E}^0 = - \frac{\partial \underline{B}^2}{\partial t_2} \tag{A.24}$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{B}^2 = 4\pi(n_1^0 e_1 \underline{\overline{C}}_1^0 + \alpha n_2^0 e_2 \underline{\overline{C}}_2^0)$$

and

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{E}^1 = 4\pi c^2 \rho_e^1$$

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{B}^3 = 0$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{E}^1 = - \left(\frac{\partial \underline{B}^3}{\partial t_2} + \frac{\partial \underline{B}^2}{\partial t_3} \right) \tag{A.25}$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{B}^3 = 4\pi(n_1^0 e_1 \underline{\overline{C}}_1^1 + \alpha n_2^0 e_2 \underline{\overline{C}}_2^1) + 4\pi(n_1^1 e_1 \underline{\overline{C}}_1^0 + \alpha n_2^1 e_2 \underline{\overline{C}}_2^0)$$

Necessary conditions for existence of solutions are

$$\frac{\partial}{\partial \underline{r}} \cdot (n_1^0 e_1 \underline{\overline{C}}_1^0 + \alpha n_2^0 e_2 \underline{\overline{C}}_2^0) = 0$$

$$\frac{\partial}{\partial \underline{r}} \cdot (n_1^0 e_1 \underline{\overline{C}}_1^1 + \alpha n_2^0 e_2 \underline{\overline{C}}_2^1) + \frac{\partial}{\partial \underline{r}} \cdot (n_1^1 e_1 \underline{\overline{C}}_1^0 + \alpha n_2^1 e_2 \underline{\overline{C}}_2^0) = 0 \tag{A.26}$$

which also can be sufficient conditions.

The sets of equations (A.24) and (A.25) now have to be solved together with the zeroth and first order equations of section 1, Eqs.(18)-(29) and Eqs.(30)-(41). The zeroth order equations (18)-(29) are uncoupled with the Maxwell equations, so the evolution of the zeroth order distribution functions and macroscopic quatities are the same as before. The first order equations (30)-(41) are coupled with the zeroth order Maxwell equations (A.24). Due to the zeroth order evolution the electric field $\underline{E}^{0}$ and magnetic field $\underline{B}^{2}$ show a transient variation on the τ_2 - time scale. The transient of $\underline{E}^{0}$ give rise to new transients in the first order kinetic and macroscopic equations in addition to the transients studied before. The transient of $\underline{B}^{2}$ is too small to be seen in these equations. In the limit $t_2 = \infty$, however, the first order kinetic and macroscopic equations are the same as before, the electric field $\underline{E}^{0}$ now obeying the Poisson equation.

References.

[1] S. Chapman and T.G. Cowling, "The Mathematical Theory of Non-Uniform Gases" (Cambridge University Press, Cambridge, (1958)),Chap. 18.

[2] G. Sandri, Ann.Phys. (N.Y.) 24 , 332 and 380 (1963).

[3] E.A. Frieman, J. Math.Phys. 4, 410 (1963).

[4] C.H. Su, J. Math.Phys. 5, 1273 (1964).

[5] J. Naze Tjøtta and A.H. Øien, Rep. No. 20, Dept.Appl. Math. Bergen (1969).

[6] J. Naze Tjøtta and A.H. Øien, Proceedings of the Ninth International Conference on Phenomena in Ionized Gases 1969, p. 363, Bucharest, Romania, Sept.1-6,1969.

[7] J. Naze Tjøtta and A.H. Øien, J.Math.Phys. November or December 1973.

[8] L. Landau, Physik. Z. Sovjetunion 10, 154 (1936).

[9] A.H. Øien, Proceedings of the Tenth International Conference on Phenomena in Ionized Gases 1971, p. 248, Oxford, England, Sept. 13-18, 1971.

[10] D.C. Montgomery and D.A. Tidman, Plasma Kinetic Theory (Mc Graw-Hill, New York 1964).

[11] T. Leversen and J. Naze Tjøtta, Rep. No. 29, Dept. Appl. Math., Bergen (1971).

[12] A.H. Øien and J. Naze Tjøtta, Phys. of Fluids 14, 2373 (1971) (see also Rep. No. 18, Dept. Appl. Math. Bergen (1969)).