

# Individual-Based Modelling As a Tool for Sustainable Red Deer Management

by  
Helene Wilson

Master of Science Thesis in  
Applied and Computational Mathematics



Department of Mathematics  
University of Bergen

June 2019



## Abstract

Being able to predict how a population of red deer evolve over time for different hunting strategies, can be helpful for wildlife management in their decision-making as means for achieving a sustainable red deer population. This is an area where mathematical models turn out to be quite applicable. The goal of this thesis is to implement an individual-based model for simulating results of different hunting strategies. A thorough presentation of the model setup will be provided before different cases of hunting strategies are simulated. The influence from each strategy on the red deer population will be analyzed. In addition to these case studies, a study of the impact from the different parameters in the model will be presented. There are many possibilities of further studies with an individual-based model as the one implemented in this thesis. Examples such as the method of Monte Carlo Markov Chains and parameter estimation will be discussed at the end of the thesis.

According to our results, hunting and the choice of hunting strategy turn out to be crucial for how the population of red deer evolve. Choosing the best fitted hunting strategy for every red deer habitat, could be vital for the quality of the red deer's lives. The simulations in this thesis indicate a higher probability of dying from environmental causes, such as starvation, illness or stress, if the hunting strategy fails or hunting is disregarded completely. Since we humans have such a large say in the matter of wildlife management, we need do what we can for preserving the nature and its wildlife. Hopefully, this thesis can be a positive addition to wildlife management and further research.

## Acknowledgements

First, I would like to express my gratitude towards Guttorm Alendal, for being my supervisor and giving me the opportunity to do research within a field of my own interest. This project would not have been possible without your enthusiasm and encouragement. I am grateful for all the interesting discussions we had regarding subjects as the use of mathematics and modelling in biology, wildlife in Norway, red deer and my thesis.

I would also like to thank the Norwegian Red Deer Centre and its employees, for the partnership throughout this project. Especially thanks to Johan Trygve Solheim and Magnus Frøyen for your support and help, and for letting me present my thesis at Viltseminaret 2019 in Florø. In addition, I would like to thank you for your hospitality at Svanøy, and for giving me an experience I will never forget during the 2018 red deer hunting season. Also, thanks to Jørgen Henden for reading through a draft of the thesis and providing useful comments.

Thanks to associate professor Stein Joar Hegland at Western Norway University of Applied Sciences for useful tips at the beginning of this project. Thanks to the wildlife manager in Bergen, Caroline Christie, for your help and interest in my thesis. I would also like to give my thanks to Shirin Fallahi for discussions and help regarding MCMC and parameter estimations.

Last, but not least, a huge thanks to all my friends at the university during these five years. For every coffee break, every trip around the forth floor, every mental breakdown in our office and for every supporting hug when I needed it the most; it would not have been the same without you! Thank you!



Helene Wilson  
June 2019

# Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
1.1	Introduction . . . . .	2
1.2	Red Deer . . . . .	4
1.3	Norwegian Red Deer Centre . . . . .	5
1.4	Thesis Outline . . . . .	6
<b>2</b>	<b>Method</b>	<b>7</b>
2.1	Individual-Based Model . . . . .	7
2.2	Mathematical and Statistical Tools . . . . .	10
2.3	R As a Programming Language . . . . .	13
<b>3</b>	<b>The Red Deer Population Model</b>	<b>14</b>
3.1	Outline of the IBM . . . . .	14
3.2	Full Model Description . . . . .	17
<b>4</b>	<b>Case Studies and Analysis</b>	<b>23</b>
4.1	Equal Number of Harvested Game . . . . .	24
4.2	Mainly Hunting Stags . . . . .	35
4.3	Mainly Hunting Hinds . . . . .	41
4.4	Mainly Hunting Young Individuals . . . . .	48
4.5	Result of No Hunting . . . . .	55
4.6	Sensitivity Analysis of Parameters . . . . .	57
<b>5</b>	<b>Discussion and Further Work</b>	<b>65</b>
5.1	Case Studies . . . . .	65
5.2	Further Work . . . . .	68
<b>6</b>	<b>Conclusion</b>	<b>72</b>
	<b>Appendices</b>	<b>76</b>
<b>A</b>	<b>Table of Parameters</b>	<b>77</b>

**B Represent Data Visually****78**

# Chapter 1

## Introduction

### 1.1 Introduction

Hunting has been an important part of human's life for centuries. Even though today's society do not depend on harvesting game for survival, hunting still generates income and resources for a lot of people in Norway. To avoid over- or under-exploitation of the harvested species, it is crucial to have a wildlife management giving restrictions on the number of prey and the length of hunting season, among other things (Samdal et al., 2003).

To be able to provide the best possible wildlife management, we need knowledge and research regarding how our decisions will influence the harvested species' populations and behavior. This is what inspired the theme of this master thesis. Can we use mathematical modelling as a research and consulting tool for wildlife management?

Here, the focus is on the red deer. However, wildlife management goes far beyond managing populations of red deer. Other species as reindeer, moose, roe deer, predators as wolves and bears and even fisheries and life in the ocean are of concern for the wildlife management in Norway. The size of the species populations, in what direction they evolve and how much impact humans have on their habitats through land use, pollution or harvesting, are just a few examples of questions that can be studied. Mathematical modelling could be used as an approach for providing some answers to these questions.

The focus of this thesis will be how the number of harvested red deer impacts how the population evolves over time. Even though some harvest models already exist (Samdal et al., 2003; Meisingset, 2008), the population density of red deer have

increased rapidly during the last decades (Solberg et al., 2017). This could be a consequence of weaknesses in the models or wildlife management who avoid consulting such models when deciding harvest strategies for an area. Providing a new model can only help strengthen the research field and help wildlife management in their decision-making regarding a sustainable future for the red deer.

The arguments behind the choice of harvest strategy is often a result of how the red deer population in that particular area preferably should evolve. A harvest model should provide answers regarding what the trend in the population will be, based upon different harvest strategies. Different interests related to red deer management, due to the people living in that area, could impact the choice of harvest strategy.

When dealing with wildlife management for red deer, two opposite interests often cause problems; on one side, browsing damages in farming due to hungry cervids could cause a general desire of lowering the population of red deer from farmers (Groven et al., 2019)<sup>1</sup>. An increasing population could cause even more damage for farming, due to competition between individuals for the best food resources, or the lack of food resources elsewhere. This is something wildlife management need to take into consideration when choosing hunting strategy for areas close to farms.

On the other side, we have those who make a living by hunting and providing hunting experiences for others. For a lot of people, hunting can be both recreation and a food resource, and this can be good business for some. In a survey conducted by Andersen and Dervo (2019), big game hunting (this includes other game as well, such as moose, roe deer and reindeer) resulted in an estimated consumption of 1 470 MNOK in 2018. They also calculated the consumption for a 10-year perspective, estimating big game hunting to 1 470 - 3 150 MNOK annually. Making sure the populations of different game remain large enough for more business, is in the best interest of the people who make a living by hunting.

Trying to find the most sustainable hunting strategy, dealing with different interests from the society, keeping the populations at both a business reasonable and sustainable level and including the welfare of the animals itself, is a challenge. The goal of this thesis is to implement an individual-based model for predicting how different hunting strategies can impact red deer populations, to test if the concept of an individual-based model can contribute to a more sustainable red deer management.

---

<sup>1</sup><https://www.nrk.no/sognogfjordane/bonder-fortvilar-over-at-hjort-gjer-skade-pa-marka-1.14508652>



## 1.2 Red Deer

Red deer (*Cervus elaphus*) in Norway can be traced back to 2500 b.c. (Meisingset, 2008), and they live south of Saltfjell in Nordland County. The majority of red deer can be found in the western part of Norway, but the traveling red deer are slowly spreading throughout the country (Solberg et al., 2012). The female red deer is called a hind, stag is the male and the offspring is called a calf.

October is the peak of the rut, and the dominant stags with the largest antlers get to mate first, forcing the younger and smaller stags to wait for a possible mating opportunity later in the season. The hinds give birth to one calf (on a rare occasion two) after about eight months pregnancy, in May to mid June (Bjørneraas, 2012). However, if the mating is late, causing late breeding, the calf get less time to feed and grow before the winter (Meisingset, 2008).

The lifespan varies between hinds and stags. For hinds, the probability of dying by natural causes increases at the age of 17-19, but for stags the increase happens already at the age of 12-13. However, the main cause of death for red deer in Norway, is hunting (Meisingset, 2008). According to Statistics Norway, Norwegian hunters harvested 43 800 red deer during the 2018 hunting season (SSB, 2019). This was an 10 % increase from the year before, and could be interpreted as a consequence from the increasing population density of red deer, as the trend has been for the last couple of years (Solberg et al., 2017).

The increasing population density could be a result from changes in hunting strategies, expansion in the living area of red deer and forestry (Miljødirektoratet, 2017). We need to do much more research regarding possible consequences due to an increasing population. How does this increase affect the ecosystem? How should we manage our wildlife to avoid higher death rates, browsing damages and spreading of diseases?

Especially the situation regarding CWD (Chronic Wasting Disease) found in reindeer, causes concerns for the future of cervids in Norway (Hansen et al., 2017). In March 2016, CWD was detected in the wild reindeer population located in Nordfjella, the mountain area between Lærdal and Gol in Norway. To avoid a national disaster, The Norwegian Food Safety Authority decided to cull the entire population of approximate 1400 individuals before the disease spread. CWD can spread to other cervids as well, and a larger population of red deer could increase the risk of spreading such diseases. Wildlife management and research will play an important part for the future and survival of red deer in Norway.

### 1.3 Norwegian Red Deer Centre

Norwegian Red Deer Centre is located at the island of Svanøy (61°30'N, 5°05'E) in the county of Sogn og Fjordane. The centre focus on how to exploit deer as a resource in the best possible way. They have great facilities for research, and much knowledge related to questions regarding management, farmed deer and the biology of deer among other things. The centre was included early on as a partner in the project, as a great source of help regarding deer-related questions.



Figure 1.1: Norwegian Red Deer Centre © Johan Trygve Solheim (Solheim, 2010).

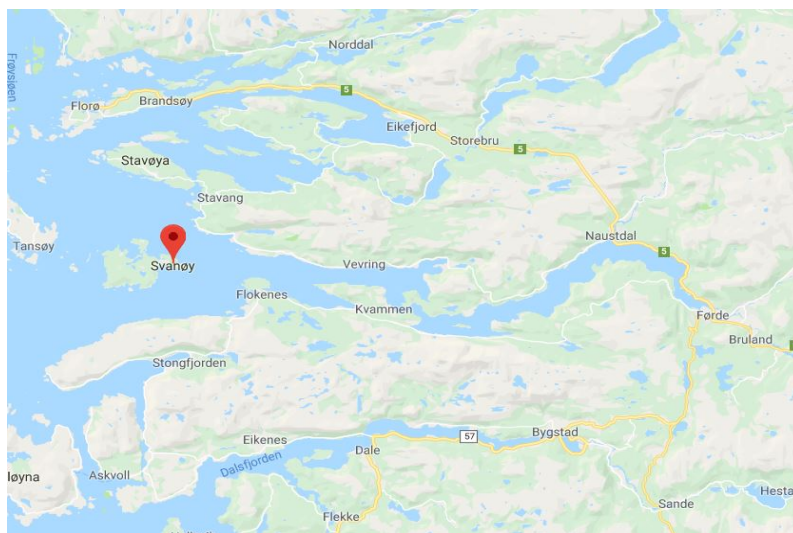


Figure 1.2: Svanøy, location of Norwegian Red Deer Centre (GoogleMaps, 2019).

---

The centre have a large deer farm, including both red deer (*Cervus elaphus*) and fallow deer (*Dama dama*), and they offer guided tours in the park. In addition, the centre is an idealistic foundation and they arrange courses in deer farming, seminars, red deer hunting and cooking classes. The centre was founded in 2000, and have since then been established as one of Norway's leading research facilities for deer and wildlife management consultants.

## 1.4 Thesis Outline

The second chapter will present the choice of model for this thesis, how to describe such a model and what mathematical and statistical tools we will use for analyzing the model's output. In the third chapter, a full description of the implementation of the model, based upon information from the second chapter, will be presented. The fourth chapter will present results from different hunting strategies, discuss and analyze what could cause these different outcomes and look into the sensitivity of some of the parameters used. Chapter 5 will discuss more the findings of the fourth chapter, and possible further work as parameter estimation, Markov Chain Monte Carlo and expansions of the model. At last, a conclusion will be described in the sixth chapter of this thesis.

# Chapter 2

## Method

Population modelling is an important tool within theoretical ecology (Case, 2000). P. H. Leslie (1945) introduced the use of matrices as a means of tracking population changes. This is a popular method often used in ecology, and scientists as H. Caswell (2006) have contributed to develop the use further. Differential equations, as the logistic equation published by Pierre François Verhulst in 1838 (Bacaër, 2011), is another example of how to mathematically describe evolving populations.

However, these methods do not take individuality into account. One possible consequence is ending up with a too simplified model when trying to describe a system of individuals as one unit. Another problem with these models, is their deterministic approach. Events in nature are stochastic phenomena, and should be treated in such a way. Using a deterministic model, the output is determined by the parameter values and initial conditions, giving rise to a lot of uncertainty in the output.

If we choose a stochastic individual-based model on the other hand, these problems can be avoided. This chapter will give an introduction to individual-based modelling, what advantages such a model holds, brief description of its disadvantages and a formulation of mathematical and statistical tools used for analyzing the results from the model.

### 2.1 Individual-Based Model

An individual-based model (IBM) is used to simulate individuals in a population or in systems of populations. The individuals in the model could represent animals, bacteria, humans or other organisms. This type of model contains also sets of phenotypic traits (e.g. weight, age and sex) the individuals possess, and a history of interactions between the individuals and the environment (Huston et al., 1988;

DeAngelis and Mooij, 2005; Grimm et al., 2005). This type of modelling is also often referred to as agent-based modelling.

### 2.1.1 Why IBM?

When describing an ecological system, mathematical models have a tendency to assume that many individuals can be represented by one variable, and they usually disregard the individuals' locations (Huston et al., 1988). This way, the results from the model do not take individual variation into consideration, which is an important tenet in biology.

IBM is a modelling approach where complexity of individuals and the interaction between them are an important part of the simulation (DeAngelis and Grimm, 2014). Each individual have their own characteristics, and it is possible to simulate how one individual can affect a population and the population's collective behavior (van der Vaart et al., 2015).

One of the advantages with an individual-based models is that even though it can describe complex systems, the model is easy to implement. Another main advantage of using IBM instead of a classical model, is the possibility of including many details. DeAngelis and Mooij (2005) describes five different categories where IBM have a better resolution than classical models:

**Variability in space:** Individuals can live and move between different environments, and local impacts on the environment that may affect the individuals, are taken into account.

**Life cycle details:** An IBM can describe individual and variability in life cycles in finer detail than classical models.

**Phenotypic traits variation and behavior:** IBMs have a clear advantage when it comes to dealing with more than one or two features, so it is possible to implement complex behavior in the model.

**Experience and learning:** Learning involves memories, and this can be difficult to implement in a classical model. For example a game-theory approach would be preferable to implement in an IBM, instead of in a classical model.

**Genetics and evolution:** IBMs can handle genetic changes within a population, and is therefore a better option when it comes to mimicking real life situations.

Even though an IBM possesses many advantages, the disadvantages should also be mentioned. First of all, a more complex model requires longer simulation time and

more robust computers which are able to perform such simulations and preferably parallel computing. Second, an IBM could be computed without using any mathematical equations as the framework. This could cause problems when trying to analyze the model, since one can not use general mathematical analysis directly to a system of equations. Lastly, reproducing someone else's IBM is often impossible, since there exists no general guidelines for describing an IBM. This last issue, could however be solved by the ODD (Overview, Design Concepts and Details) protocol, described in the next section.

## 2.1.2 The ODD Protocol

As mentioned in DeAngelis and Mooij (2005), there is no absolute definition of an individual-based model. However, we can find suggested guidelines and protocols for describing IBMs. Grimm et al. (2005) proposed an idea on how to describe IBMs, and an international workshop held in Bergen in 2004, led to the ODD protocol (Grimm et al., 2006). This Overview, Design Concepts and Details protocol contain elements one should try to include when giving the reader an understanding of the IBM implemented.

First of all, the purpose of the model should be clear from the start. This will give the reader a better understanding of what we want to accomplish with the model. Next, the properties of the model's entities and the scales should be described. This include individuals and their characteristics, as age and sex, and what time steps the model uses.

The reader should also get a clear understanding of processes included in the model and the order they are executed, as the order of execution can impact the end results. This brings us to the design concept of the IBM, another element of the ODD protocol. Design concepts can provide a common framework for the IBM community, where concepts as interaction, observation and prediction should be described.

The last elements of the ODD protocol, are initialization, input and sub models. The initial conditions of the model should be stated clearly. Were these conditions chosen arbitrarily or based on data? What about the inputs? The sub models will represent the initial conditions, inputs and outputs, and the processes from the model in a more detailed way; by mathematical terms or in a full model description. A description of the IBM in this thesis will be presented later on.

## 2.2 Mathematical and Statistical Tools

A system where randomness is involved for the development of future outcomes is called a stochastic process (Pinsky and Karlin, 2011). When modelling such natural phenomena, the use of statistics can help describing the complexity. This is not only the case for modelling, but also if one wants to analyze the results from the model. Since we are dealing with probability at each time step, we need to take every possible outcome into account. This is where a Monte Carlo sampling approach is useful. Further, performing different types of analysis, such as sensitivity analysis, will provide a basis for interpreting the results from the model.

### The Monte Carlo Methods

The idea behind Monte Carlo simulations are to repeat an operation or sampling enough times to obtain a distribution of possible outcomes (Kroese et al., 2014). This method has become an essential part of scientific computing, because of its many applications. Even though the idea already appeared in 1777 as the "Buffon's needle" problem by Georges Louis Leclerc Comte de Buffon (Dörrie, 1965), a more systematic approach became relevant at the same time as the development of electronic computing (Liu, 2001).

As described in Kroese et al. (2014), typical uses of the Monte Carlo method are sampling, estimation and optimization. This thesis will focus on the Monte Carlo method used for sampling. Since our model's output is based upon random sampling at each time step, we will end up with different results for each run. As an example, one simulation could experience a larger number of births at each time step, increasing the number of individuals more than other simulations could experience. If we observe an ensemble of many realizations, we make sure to have covered a large number of different possibilities. Such an ensemble of realizations can be thought of as an ergodic dynamical system if the averaged over time is the same as the averaged over the state space (van Lith, 2001). All of these realizations will then be analyzed by using different tools as described next.

### Some Statistics

Graphic representation and visual summaries of data provide insights early on, and could give preliminary indications of the data's contents. Different types of plots which are often used to visualize and analyze data, are described in Appendix B. If we want a more formal analysis, different types of calculations are required. What type of calculations we perform depend on the data set and the information we want to obtain from it. The definitions presented in this section are taken directly from Pinsky and Karlin (2011) and Devore and Berk (2012).

By the use of Monte Carlo sampling, we obtain a large number of different outcomes from our model. To be able to interpret these results, we start by using basic statistical analysis as a tool for better understanding the information presented. As we will describe later on, the main result from the IBM is how the population evolve over time based upon different hunting strategies. The sample space of each experiment for  $i = 1, \dots, I$  number of simulations, is

$$\mathcal{S} = \{s_1, s_2, \dots, s_I\},$$

where each event  $s_i \in \mathcal{S}$  represent one possible sample. For every  $s_i \in \mathcal{S}$ , we have a subset  $\mathcal{N} = \{n_1, n_2, \dots, n_t\}$  describing the number of individuals at time  $T = 1, \dots, t$ .

As written in Pinsky and Karlin (2011), a stochastic process can be defined as

**Definition 2.1.** A *stochastic process* is a family of random variables  $X_t$ , where  $t$  is a parameter running over a suitable index set  $T$ . In a common situation, the index  $t$  corresponds to discrete units of time, and the index set is  $T = \{0, 1, 2, \dots\}$ . For that case,  $X_t$  could represent the observations of some characteristics of a certain population. ┘

$X_t$  for our case will be the sample space  $\mathcal{S}$  with index set  $T = \{0, 1, 2, \dots, t\}$  describing discrete units of time. Since  $\mathcal{S}$  will contain a large number of different results, useful calculations to perform are a measure of the mean, variance and standard deviation as defined in Devore and Berk (2012).

**Definition 2.2.** The *sample mean*  $\bar{x}$  of observations  $x_1, x_2, \dots, x_n$  is given by

$$\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j.$$

┘

**Definition 2.3.** The *sample variance* is given by

$$s_x^2 = \frac{1}{n-1} \sum (x_j - \bar{x})^2$$

with the *standard deviation of the sample*

$$s_x = \sqrt{s_x^2}.$$

┘



In addition of being a stochastic process, most IBMs also have a Markov structure (Kattwinkel and Reichert, 2017). This is the case if the future states of the model only depend on the current states, and not the past, as described by Pinsky and Karlin (2011).

**Definition 2.4.** A **Markov process**  $\{X_t\}$  is a stochastic process with the property that, given the value of  $X_t$ , the values of  $X_v$  for  $v > t$  are not influenced by the values of  $X_u$  for  $u < t$ . A **discrete-time Markov chain** is a Markov process whose state space is a finite or countable set, and whose (time) index is  $T = \{0, 1, 2, \dots\}$ . In formal terms, the Markov property is

$$P\{X_{n+1} = j | X_0 = i_0, \dots, X_{n-1} = i_{n-1}, X_n = i\} = P\{X_{n+1} = j | X_n = i\}$$

for all time points  $n$  and all states  $i_0, \dots, i_{n-1}, i, j$ . ┘

In words, this means that the probability of any future state, depends only on the current state, when the current state is known. We will return to the use of this property when constructing the individual-based model used in this thesis.

## Sensitivity Analysis

The dynamics of a model such as the one in this thesis, are influenced by the stochasticity caused by random sampling. Because of stochasticity, there will be some sort of uncertainty and variability in the results. To be able to give a correct interpretation of the results, we need an understanding of which parameters could be causing large variability (Cariboni et al., 2007). Sensitivity analysis is a great approach for answering questions related to what input factors influence the results more than others.

One simple sensitivity analysis to perform, is changing one parameter at a time (One-Factor-At-a-Time Method) and keeping the rest of the parameters fixed (Masada and Carmel, 2008; Razavi and Gupta, 2015). This is a local sensitivity analysis, and it will detect the effects from one single parameter. For observing how sensitive each parameter can be, the changed result will be compared to a reference value. If there is no data available, the reference value could be hard to obtain. For that case, the focus will be the change in the mean value and the standard deviation of the result.

Since we are working with a stochastic model, the output of the model can vary for each simulation. Once again we see the importance of Monte Carlo sampling; a large number of simulations will make sure we account for as many outcomes as possible. A large number of simulations, will ensure the most realistic mean value.

However, different output distributions could have the same mean, but different variance. Therefore, we need to assess standard deviation as well (Cariboni et al., 2007; Massada and Carmel, 2008).

As the results will show later on, our simulations will approach a quasi steady state after some years, with variability fluctuations around a steady mean. Even though it could be counter-intuitive for a stochastic model to end up in a quasi steady state, an observation of the system over a long periode of time can indicate an over-all stability for the system (Brock, 1967). This will be evident later on, when we represent the outputs of our model graphically. When performing stability analysis, the quasi steady state area will be the area of interest to reduce uncertainty in the comparisons.

## 2.3 R As a Programming Language

To be able to simulate the IBM, we need to use a programming language. R is an open-source project and because of its availability, the program provides many possibilities. Volunteers around the world contribute to develop the program continuously, and it is possible to find all the information needed on The R Foundation's web page <https://www.r-project.org/> or in various Internet forums. It is mostly used by statisticians for data analysis and good graphic representation of data (Hothorn and Everitt, 2014).

R is based upon the *S* language by John Chambers and research colleagues (Chambers, 1998; Becker et al., 1988). The idea was to create a programming language for data analysis, and this became the root of R (Hothorn and Everitt, 2014). R is free and therefore available to everyone, and is widely used when teaching statistics courses at universities. Not only used by students, but researches within a whole range of different fields tend to use R for statistical analysis.

R have been used to develop software packages in many different disciplines, as astrophysics, climate science, chemistry and oceanography (Tippmann, 2015). Epidemiology and genetics are also among research areas where R is often the chosen programming language, and one can find packages in R for simulating an individual-based model. The model in this thesis on the other hand, has been built from scratch by the use of some built-in functions which will be introduced along the way.

## Chapter 3

# The Red Deer Population Model

Based upon the ODD protocol as mentioned earlier, this chapter describes the details behind the individual-based model created for this thesis. First we will provide a short outline of the IBM, before giving the full model description.

### 3.1 Outline of the IBM

The purpose of this IBM is to simulate how hunting affects a population of red deer over time. We want to see how certain harvest strategies influence the population. Do we get an increase or decrease in the population size? How about the distribution of each sex? These are some of the questions a model such as this one should be able to answer.

Our individuals in the population represent red deer. They are assigned an integer representing age, and a sex, either hinds or stags. The time step in the model is discrete time steps of one year at a time, and we can choose how many years we want to run the model. In this first version of the model, we do not take spatial variability into consideration, so impact from the environment is only a part of the carrying capacity described later on.

The IBM consists of different functions describing the biological features in a red deer's life. The model have functions describing growth, reproduction and death. The latter either by natural causes or through hunting. These functions differ in nature. Whereas growth is a constant function, reproduction and death are based upon stochasticity. An example of interaction between the individuals is the reproduction function, where a requirement of at least one sexually mature stag present in the population are needed for a sexually mature hind to reproduce. The states of all individuals are stored for each time step.

The order of execution of each function in the model is not arbitrary. The first function is the growth function, in the sense of aging. Each individual turns one year older at the beginning of each year. Next function is reproduction, since red deer give birth late spring/early summer. In Norway, the red deer hunting season is from 1st of September until 23rd of December. The hunting function could be the appropriate choice after reproduction, but we also need to involve the natural death function. We assume the natural death to come before the hunting season, but we will study the effects of this order later on.

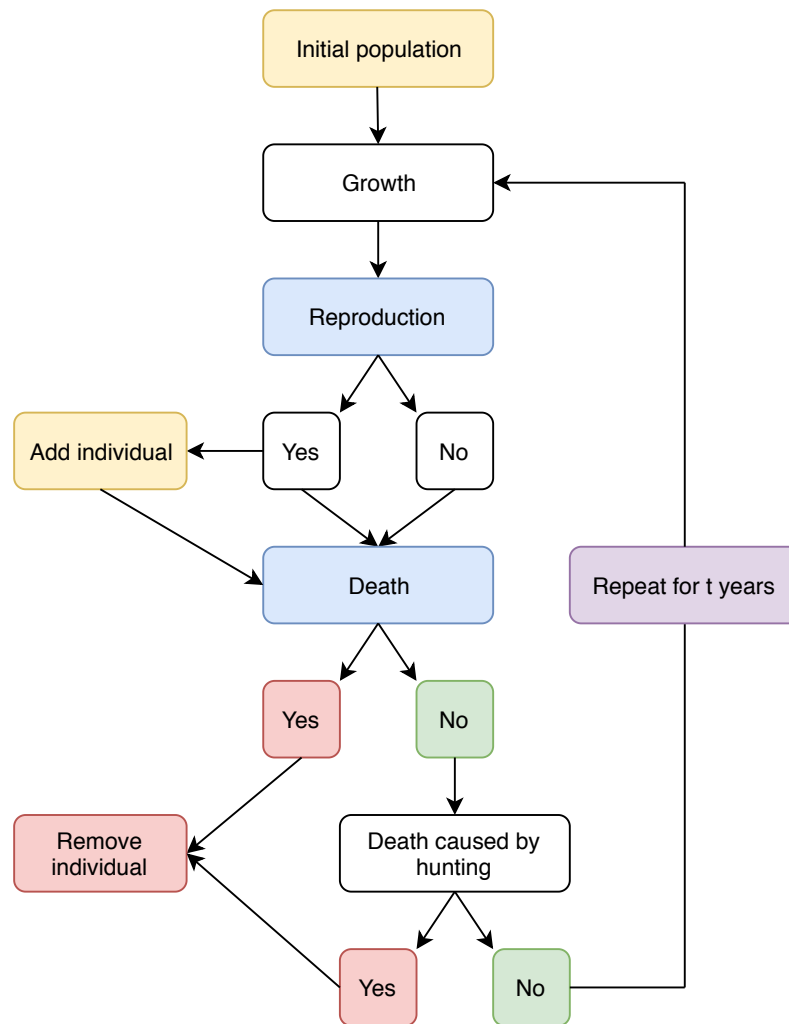


Figure 3.1: Flow chart of the IBM, a schematic representation of life events for the red deer in the model. The model starts with the initial population, before we follow each individual through each life event for  $t$  years.

As mentioned earlier, the growth function is kept constant. The individuals turn one year older for each year. In the hunting function, a constant number of individuals to harvest yearly, are given in advance. The choice of which individuals we harvest, can be arbitrary. Reproduction and death function are stochastic functions, whose outcome are based upon probabilities. The future state (i.e. individuals at each time step) is only based upon the current one, and this is where the discrete-time Markov chain as described before, appears in our model.

The initial population is chosen arbitrary by a random sampling function in R called *sample*. This function generates a vector by sampling  $n$  given samples from a specified data set or elements. We use this approach to create a *data frame*, where the first column represent the age and the second column the sex of each individual in the population. This data frame is now our starting population, and we will store new information regarding the individuals in the same data frame. The distribution of sex and age in the initial population will be random due to the sample function. By the use of *set seed*, the random starting population will be reproduced for each simulation. It is possible to choose the initial population if one have specific data describing the age and sex distribution in the population. Otherwise, the approach applied here could be a good replacement.

For each simulation, we have the following initial population of 100 individuals, with a 50/50 distribution of hinds and stags. The age distribution for each sex is:

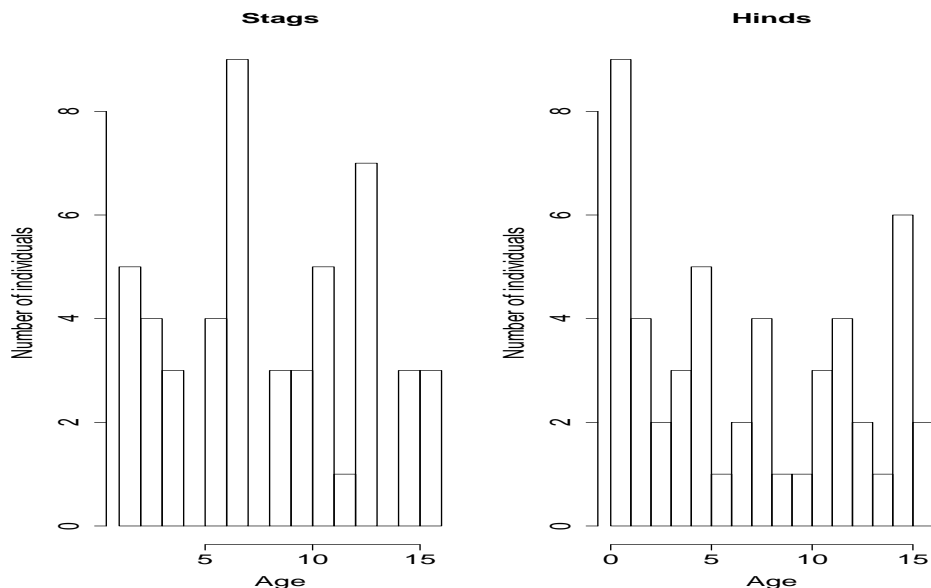


Figure 3.2: Initial distribution of age for each sex in the population. This initial starting population is random, but will be the same for each simulation.

Since the purpose of this IBM is to see how populations are affected by different hunting strategies, this is the important input in the model. Before the hunt starts, a given number of individuals to hunt are stated for five different groups of individuals. These groups are based upon age and sex of the individuals. In Norway, it is common to divide the red deer into calves, young hinds, young stags, hinds and stags, so this is the approach for our model as well. As an example, we could have the following hunting strategy:

*Table 3.1: A hunting strategy example. From each group, 5 individuals can be harvested yearly. For calves, one can not differ between each sex because both hinds and stags have equal looks at this age.*

Parameter	Individual	Total
$h_c$	Calves	5
$h_{yh}$	1-year-old hinds	5
$h_{ys}$	1-year-old stags	5
$h_h$	Hinds	5
$h_s$	Stags	5

In Chapter 4, we analyze what happens to a population for different hunting strategies.

## 3.2 Full Model Description

In this section, the full model is described. As a starting point for the R program, Petzoldt (2003) was used as inspiration. One can find the full parameter listing and description in Appendix A.

### Growth Function

Since we are trying to simulate how a population of red deer evolve over a given number of years, we need algorithms to describe certain biological features. First of all, is the growth function updating each individual's age each year it is alive.

---

**Algorithm 1:** Growth function

---

**Input:** Age of each individual

**Output:** Updated age

```

1 for each individual do
2   |  $i_a := i_a + 1$ 
3 end

```

---

## Reproduction Function

According to Solberg et al. (2012), 1-year-old hinds in the west of Norway have approximately a 30% chance of reproducing. Hinds above this age, have on the other side a 90% chance of reproducing. In this model, we assume this to be true for hinds up to the age of 12. If a hind does reproduce, it is a 48/52 % chance the offspring is either female/male (Meisingset, 2008). It is also a requirement to have at least one sexually mature male red deer present in the population (otherwise reproduction is impossible). This function compares the probability of reproduction against a random number between 0 and 1 to decide whether that particular hind will reproduce or not.

---

### Algorithm 2: Reproduction function

---

**Input:** Individuals

**Output:** Population included the new individuals

```

1 for each individual do
2   if  $i_a = 1$  &  $i_f$  is TRUE then
3     if at least one  $i_m$  with  $i_a \geq 1$  present in population then
4       individual reproduces if random number  $< p_r = 0.30$ 
5         if reproducing is TRUE then
6           sex of offspring is based upon  $p_{o,f} = 0.48$  and  $p_{o,m} = 0.52$ ,
7           then add new individual to population
8         end
9       end
10    else if  $1 < i_a < 12$  &  $i_f$  is TRUE then
11      if at least one  $i_m$  with  $i_a \geq 1$  present in population then
12        individual reproduces if random number  $< p_r = 0.90$ 
13          if reproducing is TRUE then
14            sex of offspring is based upon  $p_{o,f} = 0.48$  and  $p_{o,m} = 0.52$ ,
15            then add new individual to population
16          end
17        end
18      end
19    end
20  end

```

---

## Death Function

Another important biological feature of a red deer's life, is death. Depending only on the age of the individual, each individual is assigned a probability of dying by

natural causes. Based upon Meisingset (2008), we start with

$$p_{i,d} = \begin{cases} 0.15 & \text{if } i_a = 0 \\ 0.03 + \left(\frac{0.05}{14}(i_a - 1)\right) & \text{if } 0 < i_a < 16 \\ 0.08e^{2.47(i_a - 16)} & \text{if } i_a \geq 16 \end{cases} \quad (3.1)$$

where  $p_{i,d}$  is the probability of dying for that specific individual. For calves, this is approximately 15%. For older red deer, the probability is somewhere between 3%-8%, so we use a linear function from  $i_a = 1$  to  $i_a = 15$  to find this probability. After the age of 15, the probability of dying is increasing exponentially from approximately 8% at the age of 16 to 95% at the age of 17.

However, these death rates do not take carrying capacity into consideration. We can assume that a larger population with the same living conditions, will experience different death rates because of events as more diseases spreading or decreasing access to food. This will vary between different habitats and living areas. The next algorithm will calculate a new  $p_{i,d}$  with the influence from carrying capacity taken into consideration.

---

**Algorithm 3:** Mortality affected by carrying capacity

---

**Input:** Individuals,  $p_{i,d}$

**Output:** Updated  $p_{i,d}$

$$1 \quad p_{i,d} := p_{i,d} + \frac{c}{2}(1 + \tanh(a(i_{now} - i_{max})))$$


---

The mortality function updates  $p_{i,d}$  by using a hyperbolic tangent function. This function is assumed to simulate logistic growth with carrying capacity. If our population reaches maximum  $i_{max}$ , or exceeds this maximum, the probability of dying will increase from the age based probability by the addition from the carrying capacity function. The different variables are described in Table 3.2 and Table A.1 in Appendix A.



Table 3.2: Variables in the carrying capacity function.

Variable	Description
$c$	Maximum impact from carrying capacity
$p_{i,d}$	Death probability of given individual
$a$	Chosen slope of carrying capacity curve
$i_{now}$	Number of individuals present <i>now</i>
$i_{max}$	Chosen maximum of individuals

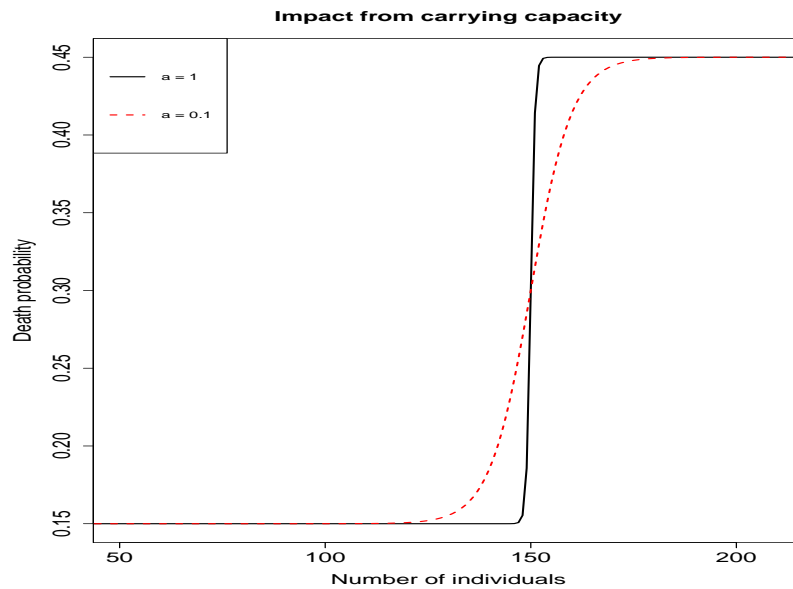


Figure 3.3: Two cases of how carrying capacity can impact death probability for calves. For different choices of  $a$ , the impact will occur at different times.

Even though the number of individuals present could be far from maximum  $i_{max}$ , the death probability could still experience a small increase (most likely not significant, this is depended upon our chosen slope in the carrying capacity). The reasoning behind these choices is based upon the size of our initial population and how much impact we want from the carrying capacity function. As a demonstration, Figure 3.3 presents different cases of how carrying capacity can impact the death probability

of a calf.

---

**Algorithm 4:** Death function
 

---

**Input:** Individuals

**Output:** Individuals who survived

```

1 for each individual do
2   if  $i_a = 0$  then
3     individual dies and is removed from population if random number  $< p_{i,d}$ 
4     if individual dies is FALSE then
5       individual survives
6     end
7   else if  $0 < i_a < 16$  then
8     individual dies and is removed from population if random number  $< p_{i,d}$ 
9     if individual dies is FALSE then
10      individual survives
11     end
12   else if  $i_a \geq 16$  then
13     individual dies and is removed from population if random number  $< p_{i,d}$ 
14     if individual dies is FALSE then
15       individual survives
16     end
17 end

```

---

By the use of the mortality function and the different  $p_{i,d}$ , we create a death function for our population. This function compares each individual's probability of dying against a random number between 0 and 1 and decides whether it survives or not. The probability of dying is based upon both  $p_{i,d}$  and the mortality function. If an individual dies, it is removed from the population. The output from this function is the surviving individuals.

## Hunting Function

The final function is the hunting function. This function divides the individuals into groups based upon age and sex, as described in Section 3.1. From these groups, we randomly sample a given number of individuals to harvest by the sample function and we do this procedure every year. However, if we try to harvest too many individuals from one group, a limit  $l$  given in advance will make sure we do not harvest more individuals than present in the population. In other words, we can not hunt any individuals from a given group, if there are less than  $l$  individuals present in that group. This will prevent overexploitation and the number of individuals harvested could vary between years. We will investigate the impact of such a limit

later on.

---

**Algorithm 5:** Hunting function
 

---

**Input:** Individuals

**Output:** Individuals who survived

```

1 if  $\Sigma(i_a = 0) > l$  then
2   | harvest  $h_c$  number of calves, remove these individuals from population
3 end
4 if  $\Sigma(i_a = 1 \ \&\& \ i_f \text{ is } TRUE) > l$  then
5   | harvest  $h_{yh}$  number of young hinds, remove these individuals from
   | population
6 end
7 if  $\Sigma(i_a = 1 \ \&\& \ i_m \text{ is } TRUE) > l$  then
8   | harvest  $h_{ys}$  number of young stags, remove these individuals from
   | population
9 end
10 if  $\Sigma(i_a > 1 \ \&\& \ i_f \text{ is } TRUE) > l$  then
11   | harvest  $h_h$  number of hinds, remove these individuals from population
12 end
13 if  $\Sigma(i_a > 1 \ \&\& \ i_m \text{ is } TRUE) > l$  then
14   | harvest  $h_s$  number of stags, remove these individuals from population
15 end

```

---

## Main Algorithm

All these functions put together, form an IBM. We choose a number of years we want to follow one population. This will be one simulation. Given the same initial values, we run the IBM a given number of times to simulate different possible outcomes.

---

**Algorithm 6:** Main algorithm - IBM
 

---

**Input:** Starting population

**Output:** Sample space  $\mathcal{S}$

```

1 for  $i = 1, \dots, I$  do
2   | for  $T = 1, \dots, t$  do
3     | run growth, reproduction, death and hunting algorithms in this order
4     | end
5 end

```

---

## Chapter 4

# Case Studies and Analysis

One of the main challenges for wildlife management is deciding the number of individuals in a population one should harvest to achieve the best possible outcome when dealing with population size. The best possible outcome will be different for each area and possible hunting strategies varies. In this chapter we will use the IBM described in Chapter 3, to test how these hunting strategies will affect the population and analyze the different distributions of individuals for each result.

For the first four cases, we will have the same initial values as input, listed in Table 4.1. Afterwards, we will change some of these values to analyze the sensitivity of the parameters and how much they will impact the end result.

*Table 4.1: Initial values for the first four hunting strategies, as previously described in Chapter 3. We will analyze how much impact these choices for  $c$ ,  $a$ ,  $i_{max}$  and  $l$  have on the result at the end of this chapter.*

Parameter	Description	Value
$\mathcal{S}$	Sample space	500
$c$	Maximum impact from carrying capacity	0.3
$a$	Slope of carrying capacity curve	1
$i_{max}$	Maximum number of individuals	150
$l$	Hunting limit	10
$i_{init}$	Initial number of individuals	100

## 4.1 Equal Number of Harvested Game

The first hunting strategy we want to consider, is the case of annually harvesting five individuals from the different classes defined in Table 4.2. As described before, we start with the same initial conditions for every simulation.

Table 4.2: First hunting strategy of harvesting equal number of game.

Parameter	Individual	Total
$h_c$	Calves	5
$h_{yh}$	1-year-old hinds	5
$h_{ys}$	1-year-old stags	5
$h_h$	Hinds	5
$h_s$	Stags	5

As we can see from Figure 4.1, we mainly end up with two different quasi steady states. The darker spots reveal where multiple simulations occur, so a darker area means more than one outcome will have this result. After about 15 years, the majority of the simulations have stabilized in the sense that they are fluctuating around two approximate steady means (with some exceptions, as we can see from Figure 4.1).

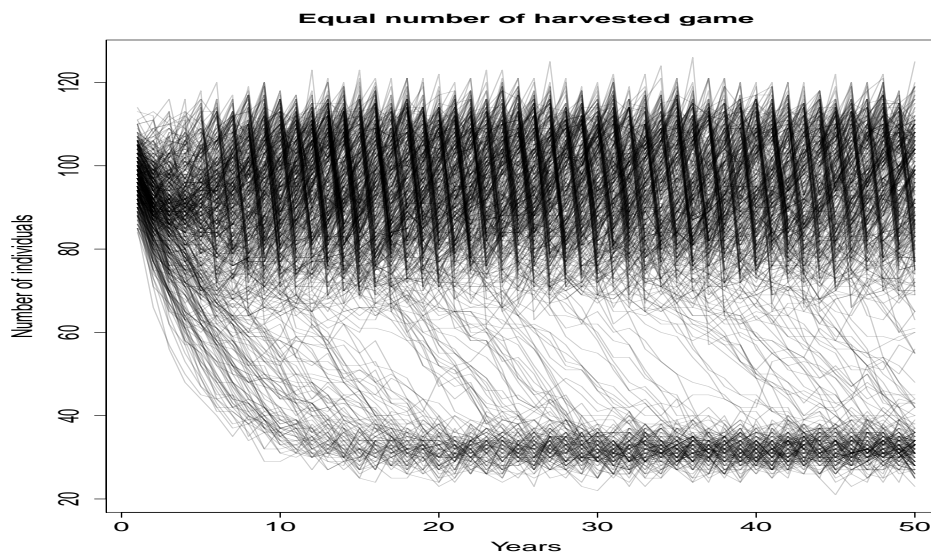


Figure 4.1: Result of 500 simulations. The plot reveals two situations occurring for this hunting strategy. A large cloud with a population around 100 individuals and a smaller collection of simulations around 30 individuals in the population.

We separate the two cases in the following way:

$$\begin{cases} 125/500 \text{ simulations:} & \text{average population} < 70 \text{ individuals} \\ 375/500 \text{ simulations:} & \text{average population} \geq 70 \text{ individuals} \end{cases}$$

As we can observe from Figure 4.1, it appears to be few simulations "recovering" from dropping to a lower level, in the sense that almost all simulations either stay around 100 individuals or drop down to 30. This implies 25% of 500 different outcomes could result in a much lower population than the rest. We examine what could be different in these two cases by analyzing the sex and age distribution for each case.

Even though we have a few exceptions, the area between 40 and 50 years will be our choice of quasi steady state. From Figure 4.1 it is quite clear that almost all simulations have converged towards one of the two means by that time. Figure 4.2 show the quasi steady states with its mean populations and standard deviations. As we can observe, the standard deviation for both cases appear to increase when approaching year 50. This could be a consequence of some simulations still experiencing drop in the number of individuals (recall Figure 4.1). So far, this hunting strategy appears to be causing much variability in the number of individuals.

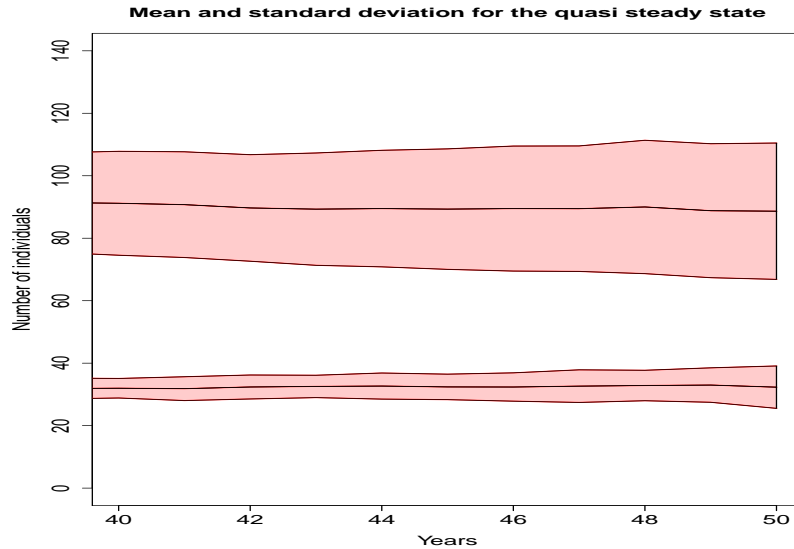
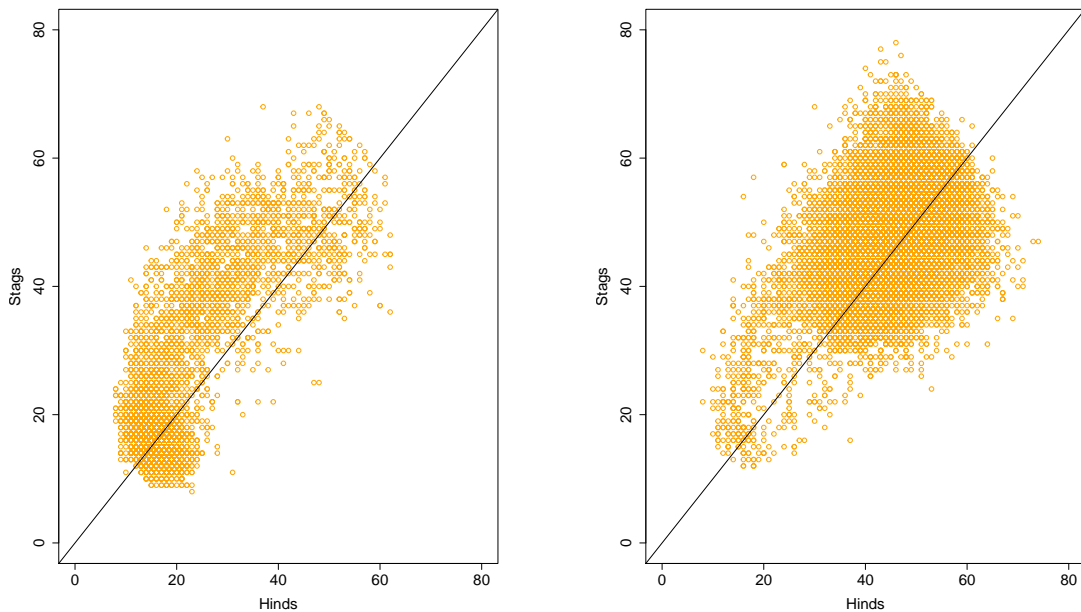


Figure 4.2: Mean and standard deviation of both cases in the quasi steady state area. Both cases appear to have an approximately steady mean with standard deviation somewhere between 2-15 individuals.

Next, we look more into details of what age and sex distribution the two distributions consist of within the whole simulation period. This can result in some uncertainty, as the lower populations did at one point have at least 100 individuals in the population, before dropping to a lower level. However, we do this for all case studies, to make sure we cover every result. In addition, we will observe the quasi steady state for each case study, as this indicate what we can expect if we continue with the same hunting strategy for a long period of time. For some case studies, we will also look into the age and sex distribution for the quasi steady state.

First, let us examine the sex distribution in both cases. We can create a plot for each case, with the ratio between all hinds one year against all stags the same year. These ratios could show a trend if plotted in a scatter plot. In addition, we add a linear function to the plots. This function will give an indication of what sex we have a larger frequency of. Every point along the linear function have equal number of hinds and stags, and points above or under the linear function represent a majority of either sex.

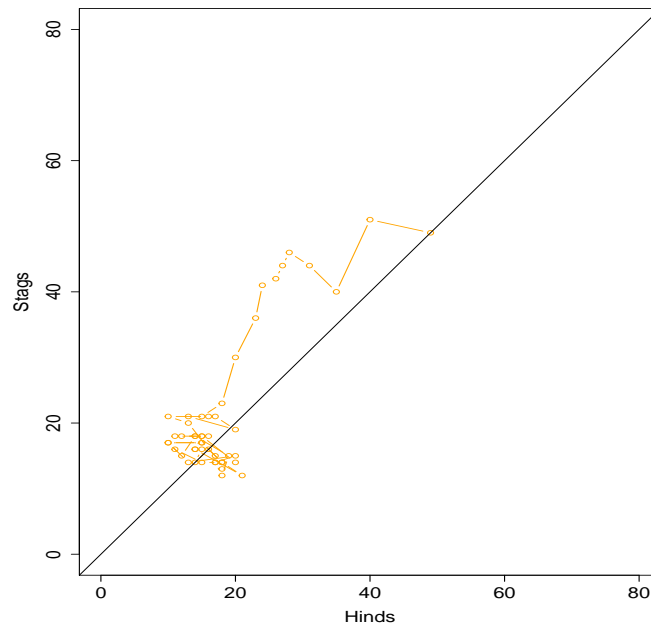


(a) Sex distribution for mean around 30 individuals. As the plot reveals, the simulations start with an almost equal amount of hinds and stags, before a majority of stags dominate, ending in an almost equal distribution at last.

(b) Sex distribution for mean around 100 individuals. This case have a more equal distribution of stags and hinds, with a small majority of stags rather than hinds.

Figure 4.3: The sex distribution during the whole period for both cases.

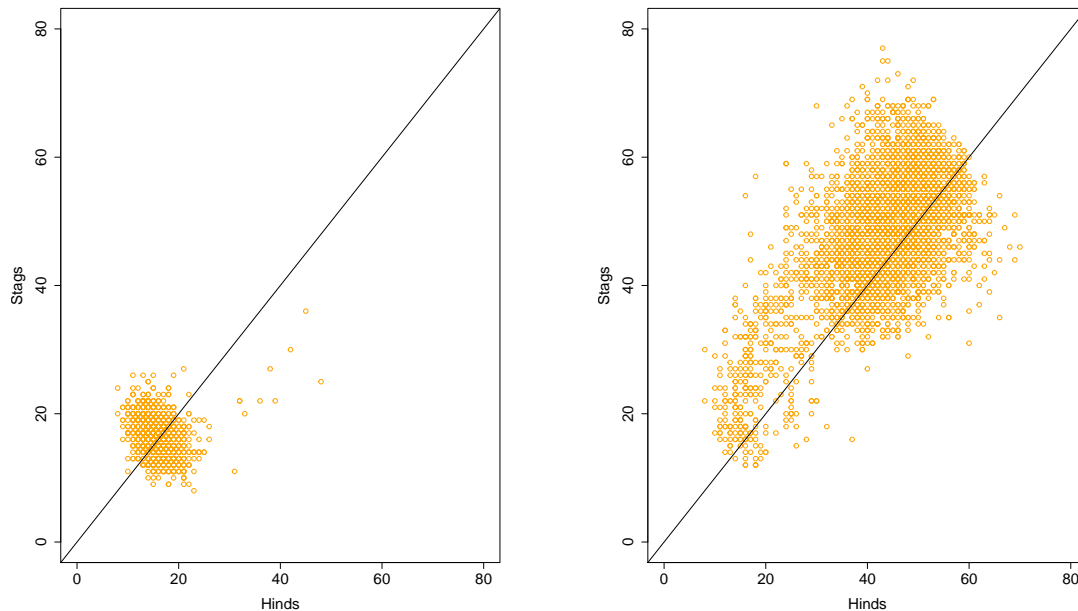
Figure 4.3 show the sex distributions for both the upper mean population and the lower mean population during the whole period of 50 years. Figure 4.3 (a) show an interesting trend. As we already have observed from Figure 4.1, we end up with two quasi steady states. The sex distribution from Figure 4.3 (a) can provide some of the explanation of why this could occur. We observe points containing approximately 50 hinds and 50 stags, which are most likely from the start of each realization. Before dropping to the lower numbers of hinds and stags, we observe an overwhelming majority of stags during a transient period. Figure 4.4 show this trend for one realization.



*Figure 4.4: Realization of one simulation, revealing how the sex distribution in the population evolve over time.*

After a transient period where stags dominate, the realization stabilize around approximate 30-40 individuals. As observed in Figure 4.5 (a), the distribution of sex is almost equally divided the ten last years. Figure 4.5 (b) shows how the sex distribution for the higher mean population are approximately the same the ten last years, as we could observe for the whole period in Figure 4.3 (b). For both cases however, we can observe some variability in the results. This is a consequence of not all simulations having stabilized yet (recall Figure 4.1).





(a) Sex distribution for mean around 30 individuals. By now, the populations have stabilized around an approximately equal distribution of sex, as the higher mean case have.

(b) Sex distribution for mean around 100 individuals. This case have approximately the same sex distribution during all 50 years.

Figure 4.5: The sex distribution from year 40 to 50.

The slight majority of stags for each case could be a consequence of the higher probability of giving birth to a male red deer as described in Chapter 3. On the other side, the probability of giving birth to a male red deer is equal for all simulations. It could be possible that other factors influence this drop in number of individuals as well. Could hunting and reproduction also be a part of why 25% of the realizations drop in number of individuals? This will be analyzed later in this section.

What can we observe from the age distribution during the ten last years? The initial population started with a diverse age distribution. After 40 years of equal number of harvested individuals from each group, one could assume the age distribution to remain diverse, as there is no indication so far of the opposite. Figure 4.6 shows the age distribution for both cases the ten last years. As we can observe, we have a young population in both cases.

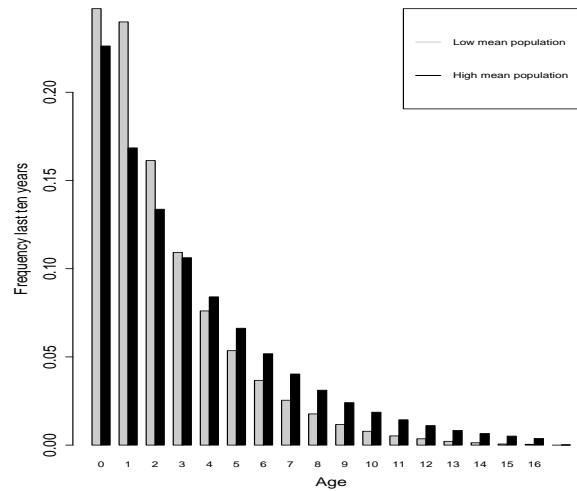
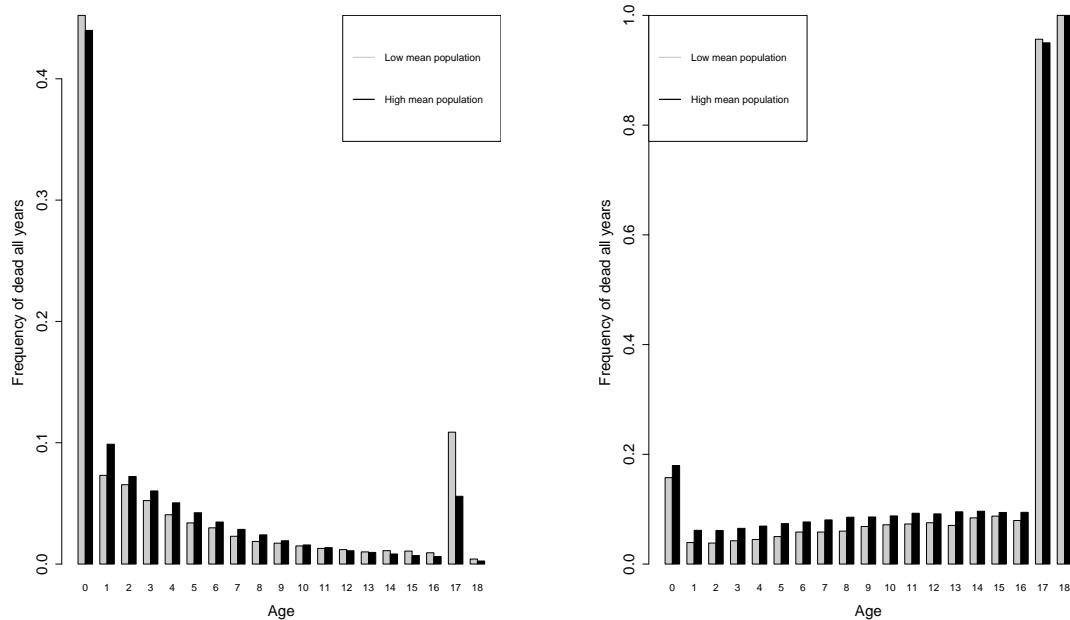


Figure 4.6: The normalized age distribution for both cases during the ten last years in the simulation period. We observe a young population for both cases.

We have already observed how the sex distribution is almost equal for both cases. According to Figure 4.6, the median age is approximately 2 years in both cases. This could indicate:

- A majority of older red deer dying of natural causes, leaving a young population of red deer behind.
- The red deer do not get the opportunity to grow older because of the hunting strategy.
- A too low hunting pressure against calves and young red deer with this hunting strategy (if one wants a more diverse age distribution for the population).

This can easily be checked by looking at the number of red deer dying of both natural causes and hunting.



(a) Normalized age distribution of individuals dying of natural causes for both cases. This figure compares the number of dead individuals in each age class against all dead individuals. The majority of the individuals are young.

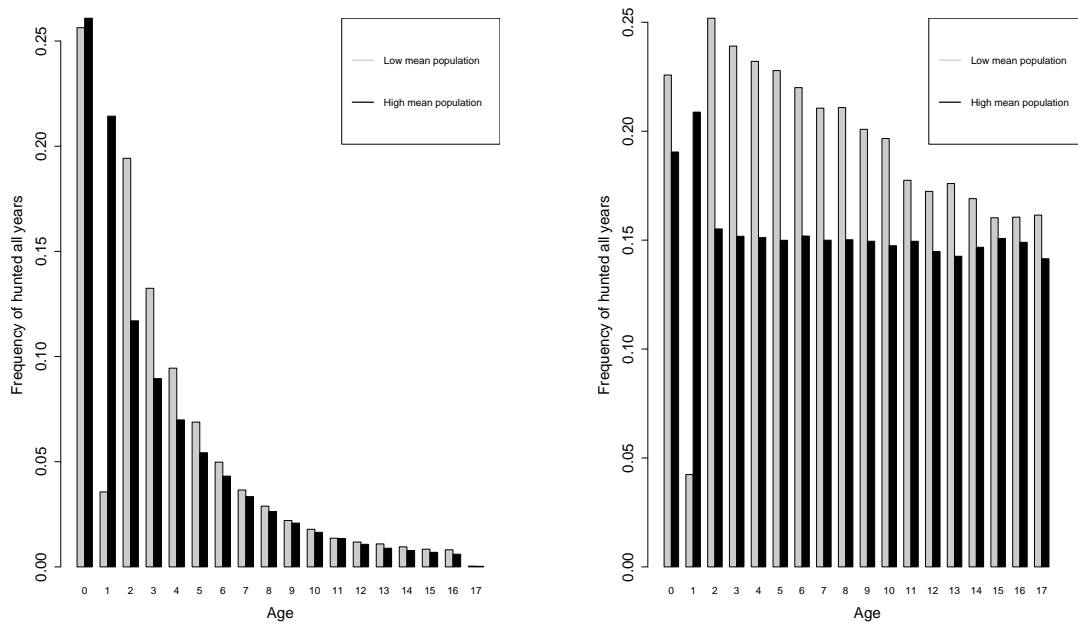
(b) All dead individuals per age class for each case, normalized against all individuals in the population. It could appear to be a small impact from the carrying capacity function.

Figure 4.7: Figures describing the amount of individuals dying of natural causes per age class, when compared to both the total number of dead individuals and all individuals in the population during all 50 years.

We start by looking at the number of individuals dying of natural causes. Since we have a young population in both cases, it is no surprise that the majority of red deer dying of natural causes are young. This we can observe from Figure 4.7 (a), where almost 50% of all the dead individuals are calves in both cases. Figure 4.7 (b) show the amount of dead individuals when compared to the entire population. We know from before the death by natural causes probability for each age,  $p_{i,d}$ , and if we look at Figure 4.7 (b), these probabilities are reflected in the amount of dead individuals. Since the number of dead individuals are not much higher than expected from the given death probabilities, the carrying capacity have a small impact on the death rates for these cases. This could indicate an advantage with this hunting strategy; it takes the carrying capacity into consideration and makes sure the individuals do not suffer too much from environmental impacts.

However, since the death function in the IBM comes before the hunting strategy,

the amount of individuals dying of natural causes could also indicate poor choices of values for the different parameters in the carrying capacity. Then again, Figure 4.7 (b) is the overall amount of individuals who died of natural causes, and this could be a result of a positive working hunting strategy (in the sense that the individuals do not get influenced by the environment in a bad way). In the sensitivity study later on, we will look more into this question.



(a) All harvested individuals per age class, compared to the total number of harvested individuals. The majority are calves, but something interesting happens for 1-year-olds in the low mean population.

(b) All harvested individuals for each age class normalized against all the individuals in the population (after individuals who died of natural causes have been removed from the population).

Figure 4.8: Figures describing the amount of harvested individuals per age class, when compared to both the total number of harvested individuals and all individuals in the population during all 50 years.

Figure 4.8 (a) shows the normalized distribution of harvested individuals in each age class. The majority are calves, followed by 1-year-olds for the high mean population case and 2-year-olds for the low mean population case. The latter result is interesting. As the majority of the individuals in both populations are young, one could assume a large number of 1-year-olds also being harvested for the low mean case. One possible cause of this not being the case could be the chosen limit of individuals before we can hunt,  $l$ . As a consequence, the number of harvested 2-year-olds is high. Also,

a high number of 1-year-olds causes a low median age for the population, and the 1-year-old hinds have a much lower reproduction rate than the older ones. This could be one of the reasons we get a lower mean population in addition to the high mean population.

In Figure 4.8 (b) all harvested individuals in each age class are normalized against all individuals in the population for each age class. The percentage of harvested individuals is different for the two cases. For the lower mean population, the percentage of harvested individuals is higher than for the higher mean population. For the higher mean population, we see an approximate harvest of 15% for each age class, except for the younger and the oldest red deer. Since the overall number of individuals is lower in the lower mean population, followed by a lower number of simulations, the percentage of harvested individuals will be higher than for the high mean population, as the plot reveals.

With our initial conditions, this hunting strategy provides two quasi steady populations. One population which have approximately the same mean as the initial population, and one population with a much lower number of individuals. We have seen a large difference in the sex distribution for the two cases. However, what could be the reason that 25% of the simulations end up with a much lower number of individuals?

If we go back to Figure 4.1, we observe that the majority of simulations "dropping" in number of individuals, occur during the ten first years. Next, we will try to highlight what is causing this separation by analyzing the ten first years. From before, we know that death by natural causes do not have a large impact on the population for the two different population means. This could indicate that the hunting function or the reproduction function are responsible for causing two completely different outcomes for the population. We analyze the number of individuals being born in both cases, looking at the average number of young hinds and older hinds reproducing each year during the first ten years.

The dashed line in Figure 4.9 reveals a lower number of individuals being born each year in the lower mean population. This could also be a result of the sex distribution for the lower mean population; fewer hinds result in fewer calves being born. We have already observed how stags dominate during the first ten years for the low mean population. On the other side, is reproduction the only function impacting the number of individuals? What about a possible impact from the hunting function?

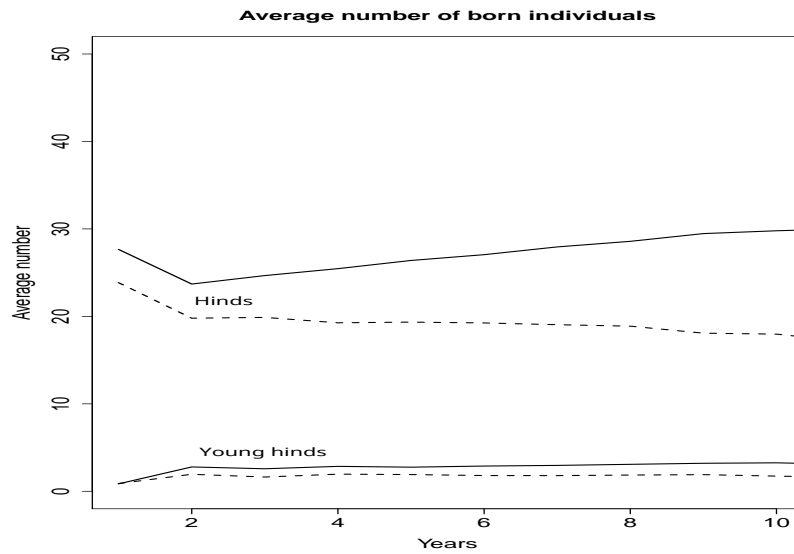
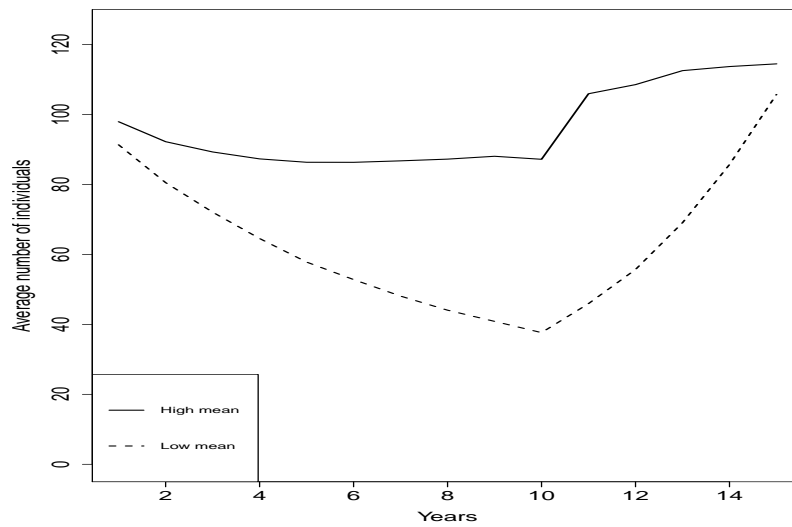


Figure 4.9: Average number of young hinds and older hinds reproducing for both cases the ten first years. The dashed line represent the low mean population, and the solid line represent the higher mean population. Since older hinds have a much higher reproduction probability, they are responsible for the majority of the reproduction.

As observed from Figure 4.8 (b), the hunting pressure is higher for the lower population mean. This could be due to the fact that we always harvest the same amount of individuals. As an example, five individuals harvested from a population consisting of 50 red deer would give a much higher harvest percentage than five individuals from a population with 100 red deer. However, what happens if we stop hunting completely after ten years?

If we decide to stop hunting completely after ten years, one would expect an increase in the population if hunting has a large impact on the number of individuals. As observed in Figure 4.10, the average number of individuals in both cases drastically changes after ten years. The dashed line represent the lower mean population, and as both Figure 4.1 and 4.2 showed, the lower mean population drops to a level of just below 40 individuals on average. However, if we stop hunting, the number of individuals increase towards the same level as the high mean population. Also, the higher mean population will increase because of no hunting. The "new" population mean appears to stabilize around 120 individuals. This is a consequence of the carrying capacity function having much more impact on the death probability for a larger number of individuals, and we will revisit this topic in Section 4.5.



*Figure 4.10: Average number of individuals for both populations. After ten years the hunting stops completely, drastically impacting the population in both cases.*

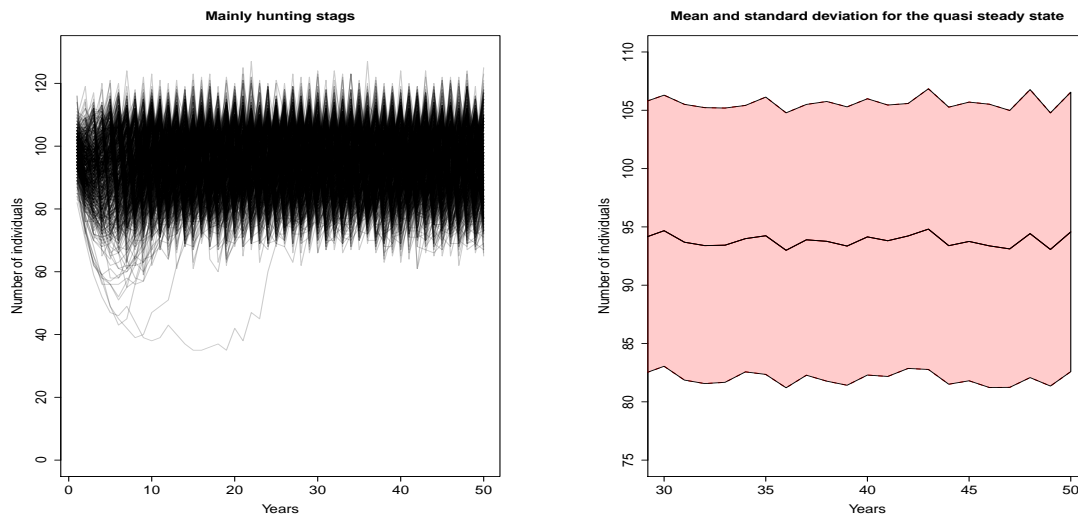
Harvesting equal number of individuals can as observed in this case study, cause two different outcomes for the population. Even though the majority of simulations obtain a population around 100 individuals on average, one should know about the possibility of ending up with a population around 40 individuals. A higher hunting pressure and a low reproduction rate occurring at the same time, could cause a low population of red deer. This consequence should be taken into consideration when deciding upon a hunting strategy for an area. However, a larger impact from the environment (expressed via the carrying capacity function) should also be taken into consideration, as it did not influence the death probability much in this case study.

## 4.2 Mainly Hunting Stags

Because of their powerful antlers and large body size, stags are often the sex hunters want to harvest. This could cause problems for the population in regards to the number of individuals and the rut (Samdal et al., 2003). Analyzing what will happen to a population of red deer if the main focus is harvesting stags, is the next case we will study. For this hunting strategy, the majority of the harvested individuals will be young and older stags. Some calves and hinds will be harvested, because it is not common to only harvest one sex overall. As for the case of the equal number of harvested individuals, we have the same initial values for this hunting strategy.

Table 4.3: The number of possible individuals to harvest from each group with this hunting strategy.

Parameter	Individual	Total
$h_c$	Calves	5
$h_{yh}$	1-year-old hinds	3
$h_{ys}$	1-year-old stags	8
$h_h$	Hinds	2
$h_s$	Stags	7



(a) 500 simulations over 50 years. Almost every simulation obtain a number between 70 and 120 individuals at each given year.

(b) After 30 years, we can see a quasi steady state for this hunting strategy. The mean population is located just below the initial number of individuals.

Figure 4.11: The result of 500 simulations. With a couple of exceptions, we end up with a population around 95 individuals.



Unlike the result from the previous hunting strategy, we end up with only one approximate population this time. Figure 4.11 (b) shows a mean population around 95 individuals, almost the same as the initial population. The initial population on the other hand, has a diverse age distribution (recall Figure 3.2). As observed from the box plot in Figure 4.12, we end up with a young population with this hunting strategy as well. Since the hunting pressure is especially high for stags, the low median age for stags could be explained by the hunting strategy; the stags do not get the opportunity to grow older, because they are harvested before this. The median age for hinds is also low. However, the hinds have a bit larger spread in age than stags, which could be a consequence of the much lower hunting pressure against hinds.

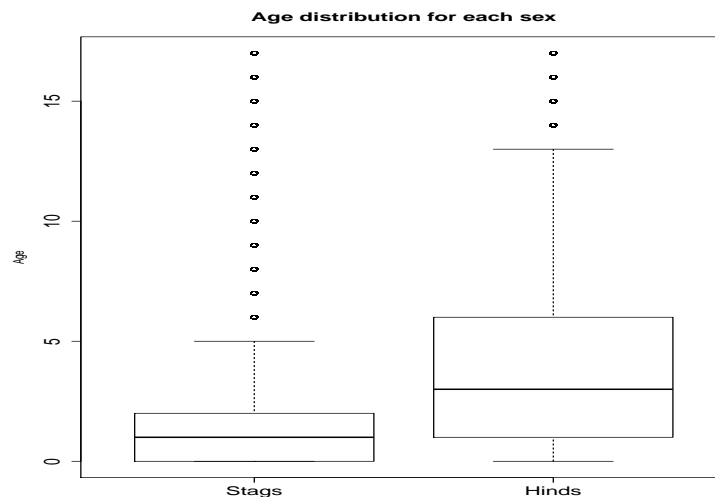


Figure 4.12: The age distribution of each sex for the whole simulation period. The stags have a lower median age and less spread in the age than hinds. This could be due to an overall low number of stags in the population, or a too high hunting pressure against stags.

Since this hunting strategy mainly focus on harvesting stags, it is interesting to observe the sex distribution for the different simulations. We can beforehand assume a majority of hinds, as the hunting pressure is much lower against hinds. Figure 4.13 describes the amount of hinds versus stags for this case. As expected, the majority of individuals are hinds. This could describe why hinds have a wider spread in age than stags; hinds have a better chance of growing older before they are harvested or die of natural causes.

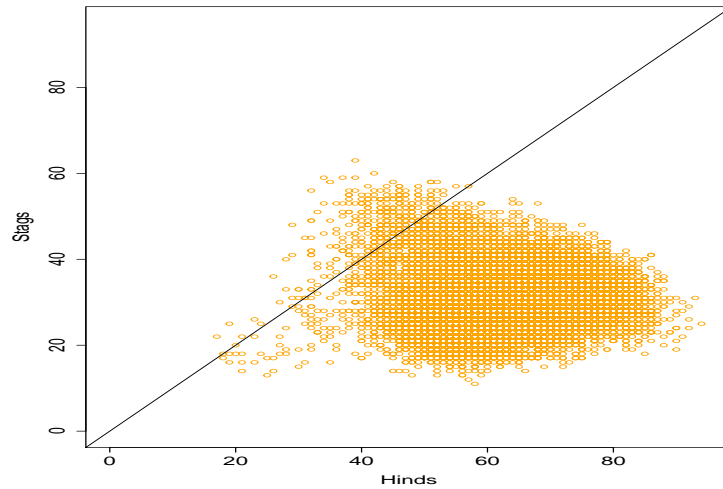


Figure 4.13: The distribution of hinds and stags in the populations during the whole simulation period when applying the mainly hunting stags strategy. Clearly, the majority of red deer are hinds. This is most likely a result of the much higher hunting pressure against stags.

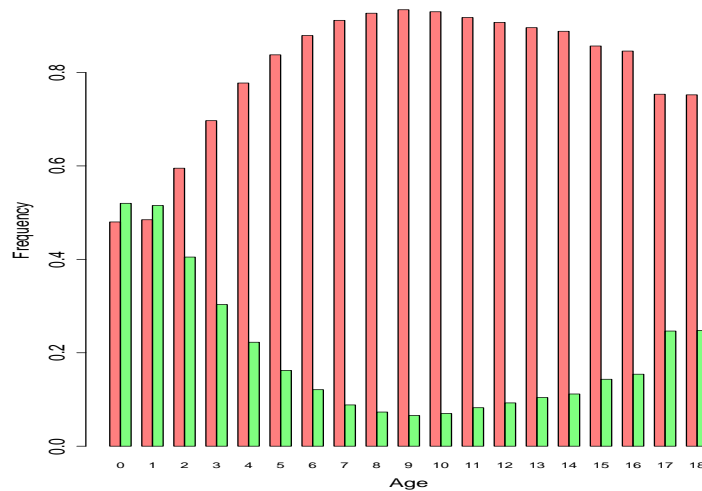


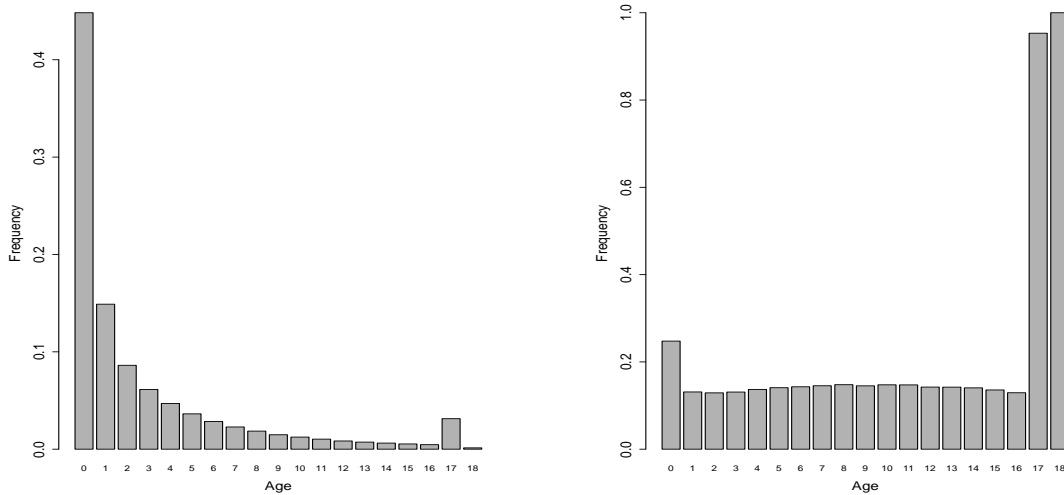
Figure 4.14: All individuals who died of natural causes, divided by age and sex. We see a majority of hinds (red) dying rather than stags (green).

Since this hunting strategy causes a high number of hinds in the population, we can assume a high number of hinds dying of natural causes. If we plot the number of hinds and stags dying of natural causes in each age class, we see a drastically higher number of hinds than stags. Figure 4.14, shows the amount of dead individuals

divided by sex and age. Only male calves and young stags have a higher relative frequency of dying than hinds, which could be a consequence of the majority of male red deer being calves or young stags. Also, we observe that the oldest red deer dying of natural causes are mainly hinds.

If we look at the number of individuals dying of natural causes from each age class compared to the overall number of dead individuals in Figure 4.15 (a), the majority are young red deer (not considering the sex). This makes sense when compared to Figure 4.12, as most of the red deer in the population are young.

If we consider the amount of individuals dying of natural causes normalized against the entire population, as in Figure 4.15 (b), we can once again observe the given death rates for each age. Contrary to what the previous hunting strategy showed, this case have a larger percentage of individuals dying in each age class. More than 20% of all the calves died as opposed to the given 15% from  $p_{i,d}$  in Eq. 3.1. This indicates an impact from carrying capacity with this hunting strategy, and could be a consequence of a too low hunting pressure against hinds. Hinds give birth to new individuals and the population increase towards a level where environmental impacts are larger.

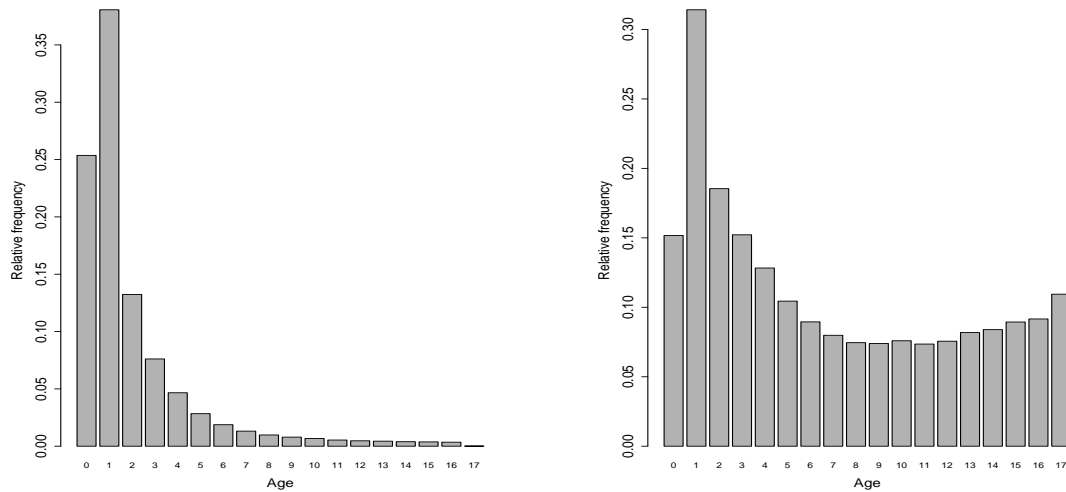


(a) Normalized age distribution of the dead individuals. The majority are young red deer, which makes sense when compared to the overall young population.

(b) All dead individuals per age class, normalized against all individuals in the population. The given death probabilities are reflected, with an increase from carrying capacity.

Figure 4.15: Figures describing individuals dying of natural causes during the whole simulation period.

The main focus of this case study is how this hunting strategy impacts the population. We have already seen how the carrying capacity influence this case as opposed to the previous case study. This could be a result of the choices made with this hunting strategy. If we observe the frequency of individuals being harvested from the entire population in Figure 4.16 (b), we have for most age classes a lower hunting pressure overall than we did for the high mean population in the first case study (Figure 4.8 (b)). Comparing the high mean case from the previous hunting strategy and the population in this case, we observe an approximate equal sized population. However, hunting mainly stags result in more individuals dying of natural causes, which could be interpreted as much larger environmental impacts.



(a) Normalized age distribution of harvested individuals. The majority of the harvested red deer are young.

(b) Harvested individuals in each age class, normalized against the entire population. The majority of the harvested individuals are young.

Figure 4.16: The normalized distribution of harvested individuals, when compared to the total number of harvested individuals and the entire population for the whole simulation period.

In Figure 4.16 (a), the amount of harvested individuals per age class compared to all harvested individuals, can be observed. As expected, the majority of the harvested individuals are young. It is also interesting to observe how greatly this hunting strategy differs between each sex. In Figure 4.17 it is no surprise that the majority of the harvested individuals are stags (green). For red deer between the age of 7-12, hinds (red) are actually the most harvested individual. This is probably due to the age distribution.

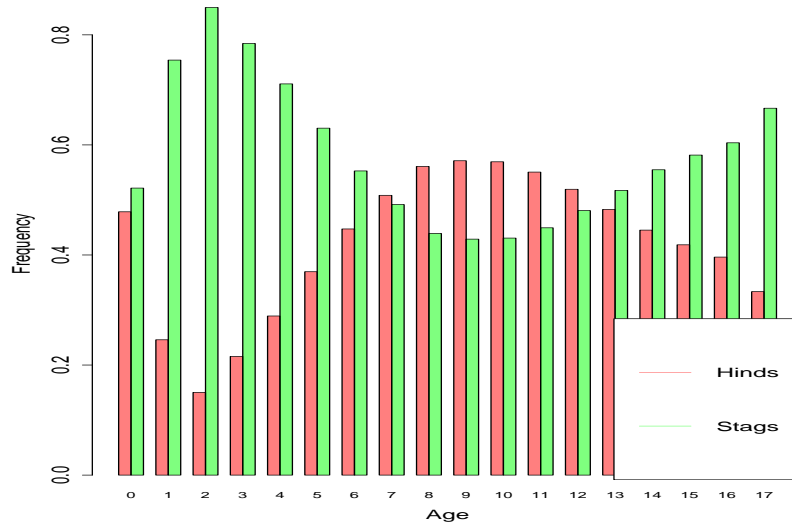


Figure 4.17: From this plot, we can observe harvested individuals in each age class divided by sex. The majority of the harvested red deer are stags (green), but hinds (red) dominate from the age of 7-12.

If the purpose of the hunting strategy is being able to harvest large stags with big antlers over time, this type of hunting strategy is the wrong way to go. The stags do not get the opportunity to grow old enough for the body size and antlers to become large. As we can see from the distribution of individuals, the majority of the red deer will be young or hinds. A young population will result in small sized individuals, as they need time to grow larger. We will address this in Chapter 5. For hinds, this type of hunting strategy keeps the hunting pressure against them at a lower level. This results in a higher natural death probability, as the carrying capacity function impacts the death rates.

Many hinds result in many calves being born, as it according to the model setup only requires one sexually mature male in the population before females can reproduce. However, a larger number of calves being born results in more calves dying of natural causes. As we observed for the hinds, this is due to the carrying capacity function and could be interpreted as more calves dying of starvation or illness. If we want to lower the number of hinds in the population, we need to increase the hunting pressure against hinds. This is the main focus of the next case study.

### 4.3 Mainly Hunting Hinds

By now, we have seen hunting strategies where the number of hinds have been essential for what direction the population and its size have evolved. What happens if we simulate a hunting strategy where the majority of the harvested individuals are hinds? As before, we have the same initial values for each simulation. This hunting strategy is however the opposite of the previous one; instead of harvesting mainly stags, we focus on harvesting mainly hinds. In Table 4.4, the amount of possible individuals to harvest from each group are listed.

Table 4.4: The yearly possible number of harvested individuals.

Parameter	Individual	Total
$h_c$	Calves	5
$h_{yh}$	1-year-old hinds	8
$h_{ys}$	1-year-old stags	3
$h_h$	Hinds	7
$h_s$	Stags	2

In the first case study, we ended up with two different approximate populations. The population with the lower mean number of individuals consisted of mostly stags. Based upon this, we can expect a result with a lower number of individuals present in this population, than we got for the hunting mainly stags case.

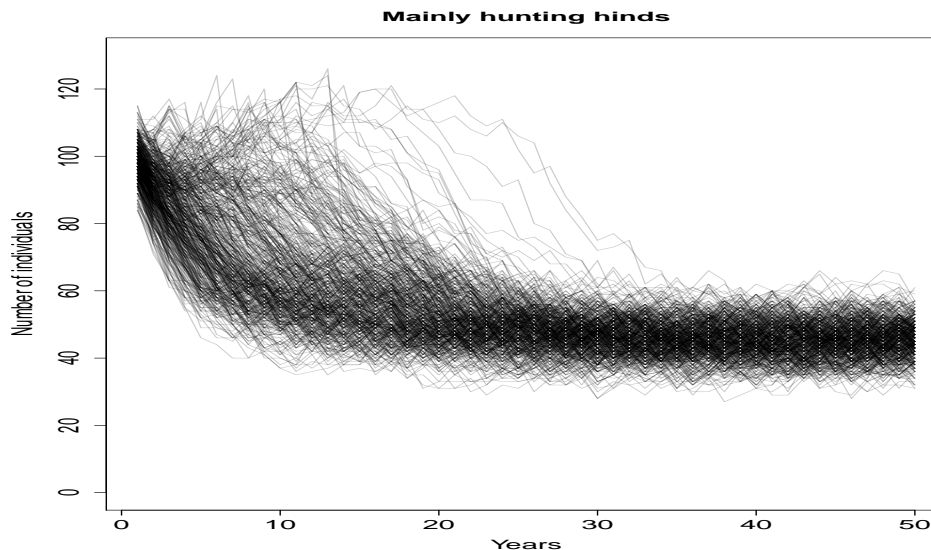


Figure 4.18: The result of 500 simulations over a period of 50 years. As opposed to the previous case study, this population drops to a lower level of individuals.

The expectation of a lower number of individuals turns out to be true, as observed in Figure 4.18. After approximate 20 years of time, all of the simulations have dropped in population size. The number of individuals appear to stabilize between 40 and 60. Especially between 30 and 50 years of time, the simulations are at a quasi steady state. This is the area in Figure 4.19. From this result, we can assume an initial population of 100 individuals ending up with a much lower number of individuals if applying this type of hunting strategy.

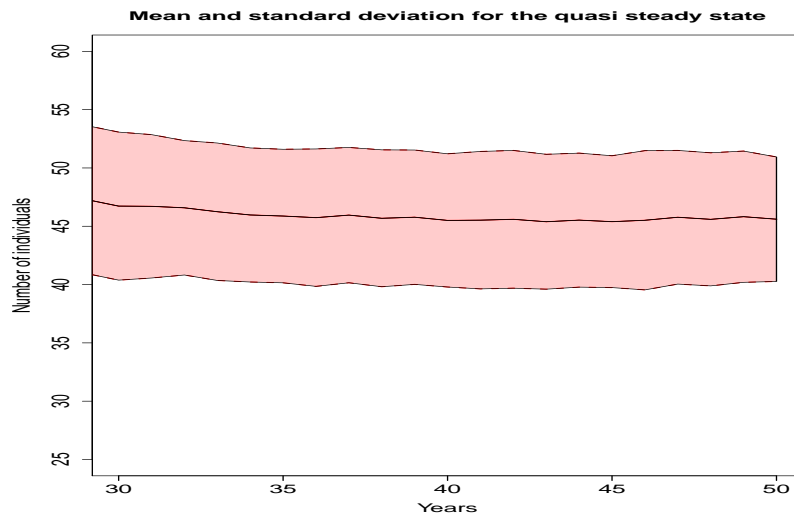


Figure 4.19: The quasi steady state for the mainly harvesting hinds hunting strategy.

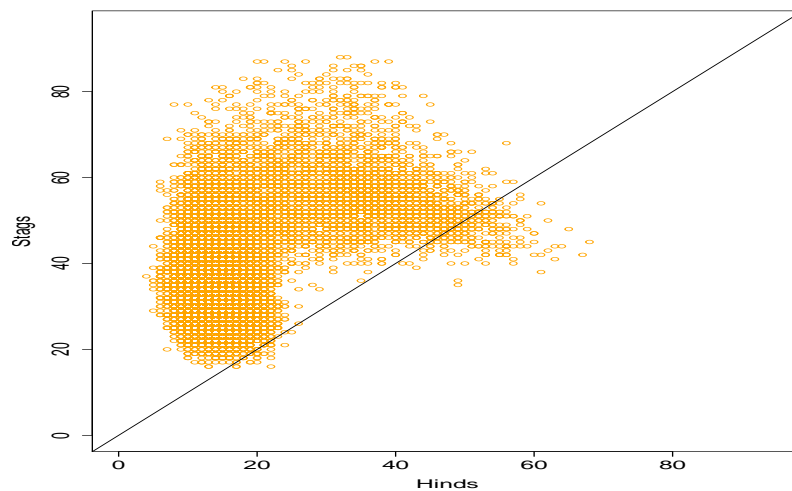
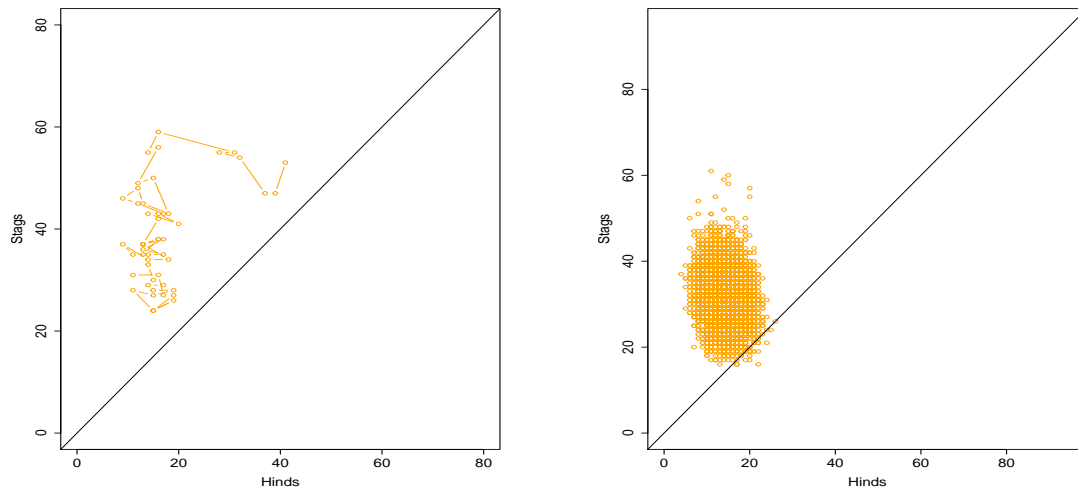


Figure 4.20: The sex distribution for the whole simulation period when applying a hunting strategy where the majority of the harvested individuals are hinds.

Due to the expected higher hunting pressure against hinds with this hunting strategy, we have already made the assumption of a sex distribution where the majority of the red deer are stags. This assumption is confirmed in Figure 4.20, where we can observe a clear majority of stags. However, this figure represent the entire simulation period. As we observed for the low mean population in the first case study, the sex distribution stabilized in the quasi steady state area. If we plot how the sex distribution evolves during the whole simulation period for one realization with this hunting strategy, we can observe a similar trend.

From Figure 4.21 (a) we observe how each time step have a majority of stags. At the end of the first year, the population have approximately 90 individuals. When the realization approaches a total of 50 individuals, it appears to be stabilizing. This assumption is confirmed by Figure 4.21 (b), where we observe the sex distribution in the quasi steady state area.



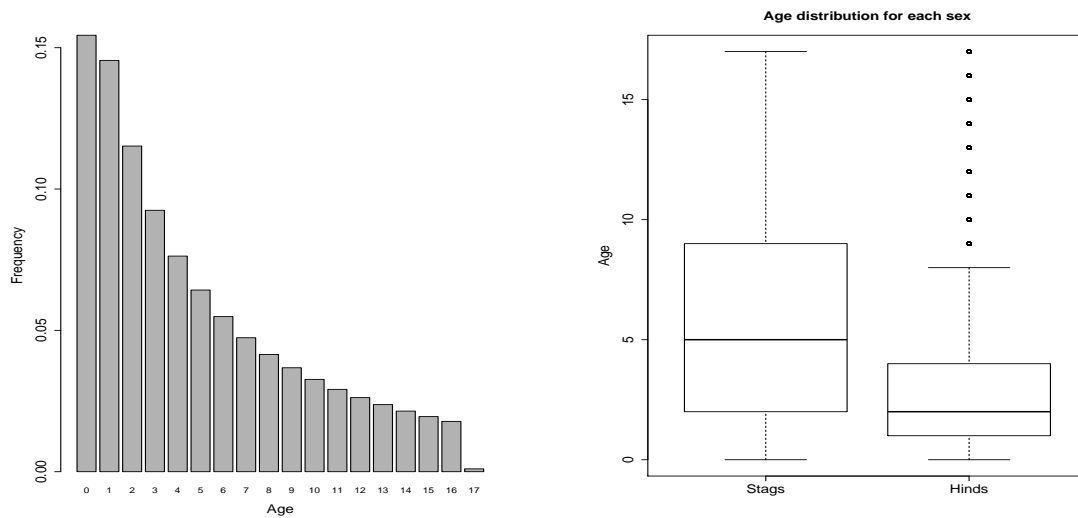
(a) One realization of the sex distribution over the whole simulation period.

(b) The sex distribution for the quasi steady state area between year 30-50.

Figure 4.21: Figures describing how the distribution of sex evolve over time, and stabilize in the quasi steady state.

Next, we want to analyze the age distribution when applying this hunting strategy. As we could observe in the last case study, a high hunting pressure against stags resulted in a low median age for stags. We could assume the same happening in this case, just for hinds instead of stags. Figure 4.22 reveals a more diverse age distribution within the population, in contrast to previous hunting strategies.





(a) The normalized age distribution with this hunting strategy. Even though the main result is a young population, the result is a more overall even distribution of age.

(b) The boxplot reveals how the age of the individuals are distributed between each sex. Stags have a more diverse age distribution than hinds.

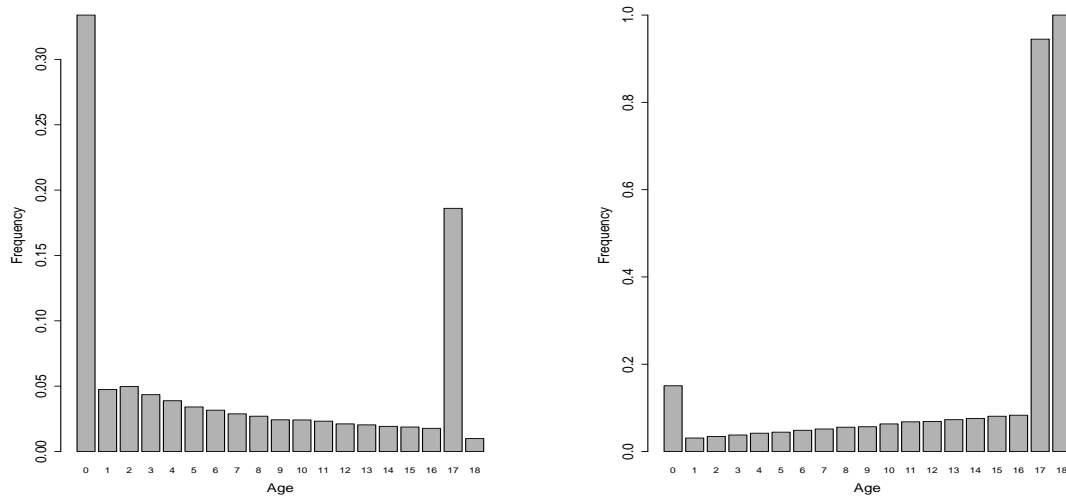
Figure 4.22: The age distribution for mainly harvesting hinds hunting strategy over the whole simulation period.

Even though the majority of red deer are young for this case as well, both stags and hinds have a higher median age than we have seen before. In the first case study, we got a low population of individuals with a low median age. This case on the other hand, have a low number of individuals, but a better distribution of age. Since the hunting pressure is higher against hinds with this hunting strategy than in the previous case study, we observe the opposite of what we did for the stags. Now it is the majority of hinds who do not get the opportunity to grow old, but stags do.

For the last hunting strategy, carrying capacity increased the natural death probability. As observed from Figure 4.23 (b), only the probabilities from Eq. 3.1 are reflected. This indicates a small to none impact from the environment via the carrying capacity function. This could be a consequence of the low population of red deer, which is again a consequence of having a focus of mainly hunting hinds. Figure 4.23 (a) show the normalized age distribution for the individuals who died of natural causes compared to the total number of individuals dying of natural causes. The majority are calves. As a contrast to the previous case study of mainly hunting stags, this hunting strategy have 17-year-old red deer as the second largest group of individuals dying of natural causes.

However, the low mean population in the first case study also had 17-year-olds as

the second largest group of dead individuals. Here we find a similarity between the result from this hunting strategy and the low mean population from equal number of harvested game, even though the sex distribution and age distribution are different.

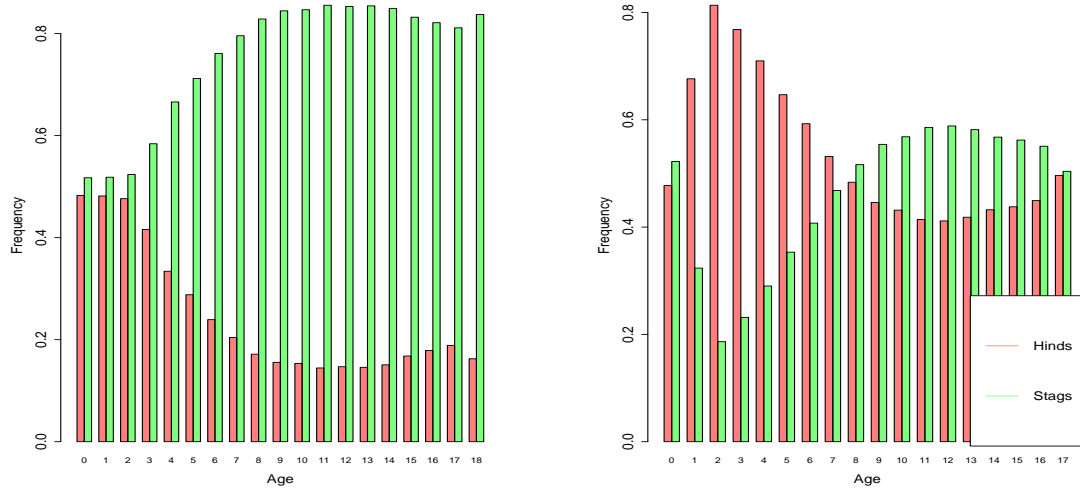


(a) The normalized age distribution of death by natural causes.

(b) The frequency of individuals who died of natural causes compared to all individuals in the population. The given probabilities from Eq. 3.1 are reflected.

Figure 4.23: The amount of dead individuals divided by age classes and sex for the whole simulation period.

For the mainly harvesting stags hunting strategy, the majority of dead individuals of natural causes were hinds. With this hunting strategy, we can observe the opposite. In figure 4.24 (a) the majority of the dead individuals of natural causes are stags. This makes sense since the hunting pressure is lower against stags in this case study. Because the focus in this hunting strategy is mainly hunting hinds, we expect the majority of the harvested individuals to be hinds. From figure 4.24 (b), we can observe the distribution of hinds and stags harvested from each age class. For red deer younger than 8 years, hinds dominate as the harvested sex. For the older red deer, stags are dominating. As mentioned before, this is probably a consequence of more stags growing older than hinds.



(a) The individuals who died of natural causes in each age class, divided by sex. The majority are stags (green) as expected, since the focus of this hunting strategy is harvesting hinds.

(b) The majority of the young harvested red deer are hinds. However, older harvested red deer are stags. This is most likely due to the age distribution, recall Figure 4.22 (b).

Figure 4.24: How the number of dead individuals by natural causes and by hunting differ between each sex.

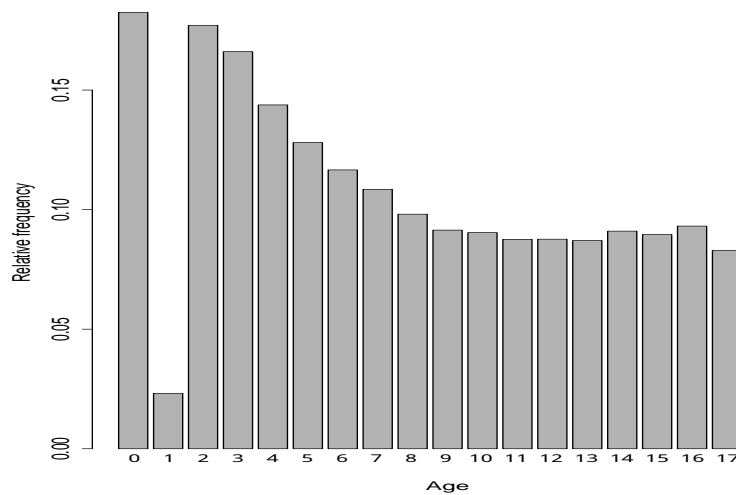


Figure 4.25: Normalized age distribution of harvested individuals, compared to the entire population for the whole simulation period. Something interesting happens to 1-year-olds for this case as well.

Figure 4.25 show how the overall hunting pressure from this hunting strategy. For red deer 8 years and older, the hunting pressure is almost stable at just below 10 %. For the younger red deer, the hunting pressure is higher. Another similarity between this hunting strategy and the lower mean population from the first case study, is the low number of harvested 1-year-olds. Again, this could be a result of the hunting limit  $l$  given in advance. If we harvest many calves, we may not have more than the given limit of both young hinds and young stags present in the population each year, resulting in few to none 1-year-olds being harvested. As a contrast to the high number of harvested 2-year-olds in the first case study, the amount of harvested 2-year-olds in this study, is not much larger than the rest.

The low number of harvested 1-year-olds could also be a consequence of the more overall diverse distribution of age for this hunting strategy. The first two case studies got a lower median age than this hunting strategy did. If we want to make sure more red deer get the opportunity to grow older, perhaps a hunting strategy as this one should be the preferred choice.

Whereas the hunting strategy of harvesting mainly stags resulted in a large population of individuals, this hunting strategy got the opposite result. The initial population "dropped" to an approximate mean number of individuals around 40-55 individuals. If the main goal of the hunting strategy is to lower the population of individuals, one should choose a hunting strategy where the focus is harvesting hinds, according to these results.

It is also interesting to see how carrying capacity did not impact the death probability for this hunting strategy. This could be the case due to two reasons as mentioned before:

- A hunting strategy making sure the environmental impacts are not larger than necessary.
- The choices we made in advance, regarding maximum number of individuals, the slope of the carrying capacity function and its maximum impact, are too unrealistic when the goal is to simulate environmental impacts.

How much impact the values for the parameters in the carrying capacity function have on the end result, will be analyzed later on.

By now, all of our case studies have resulted in relatively young populations of red deer. What will happen if we choose a hunting strategy where the main focus is harvesting a majority of the calves? This will be the case studied in the next section.

## 4.4 Mainly Hunting Young Individuals

So far, we have seen three different types of hunting strategies. The last hunting strategy we will consider, is the case of harvesting 50% of all calves in the population every year, as long as we have at least ten calves present in the population. This is the same limit as before, and the rest of the initial values are as described in Table 4.1. In addition, we will harvest a couple of more hinds than stags. The different numbers of possible harvested individuals are listed in Table 4.5.

Table 4.5: The yearly possible number of harvested individuals.

Parameter	Individual	Total
$h_c$	Calves	50%
$h_{yh}$	1-year-old hinds	4
$h_{ys}$	1-year-old stags	3
$h_h$	Hinds	4
$h_s$	Stags	2

Based upon previous observations, we can assume a couple of possible results in advance. First of all, since every previous case consisted of a young population, this case should end up with a more diverse age distribution and a higher median age. This is because the focus of the hunting strategy is mainly calves. Second, as we harvest a couple of more hinds than stags, this should combined with harvesting calves, cause a drop in the number of individuals from the initial population.

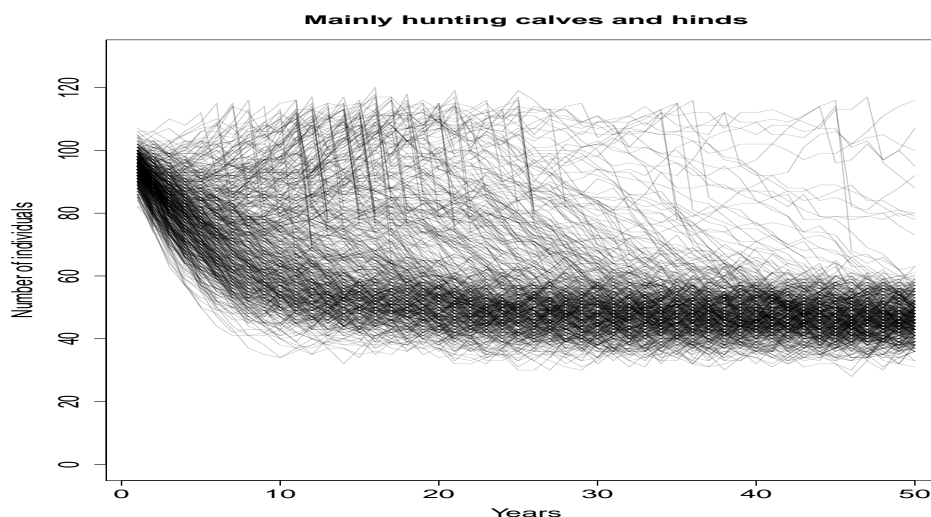


Figure 4.26: Result of 500 simulations. The majority of the simulations end up with a population between 40 and 60 individuals.

By first glance, it seems like we end up with a result similar to figure 4.18 in figure 4.26. The majority of the simulations drop to a population between 40 and 60 individuals. For the previous case study of hunting hinds, the majority of the simulations dropped after 20 years, which appears to be almost the same for this case. For some of the simulations however, it takes more than 20 years before they drop in number of individuals and a few simulations appear to never drop from the initial number of individuals. This only applies to a couple of the simulations (of 500 simulations in total), so the years between 30 and 50 describe the quasi steady state for this case.

If we compare the quasi steady state in figure 4.27 to the one we got for mainly hunting hinds, the approximate mean is a bit higher for this case. Also, the standard deviation is larger for this case than observed in the previous hunting strategy. This could be due to the fact that not all simulations dropped to a lower level of individuals, as all simulations did for the hunting hinds case. So our initial assumption of a lower population was roughly met with this hunting strategy.

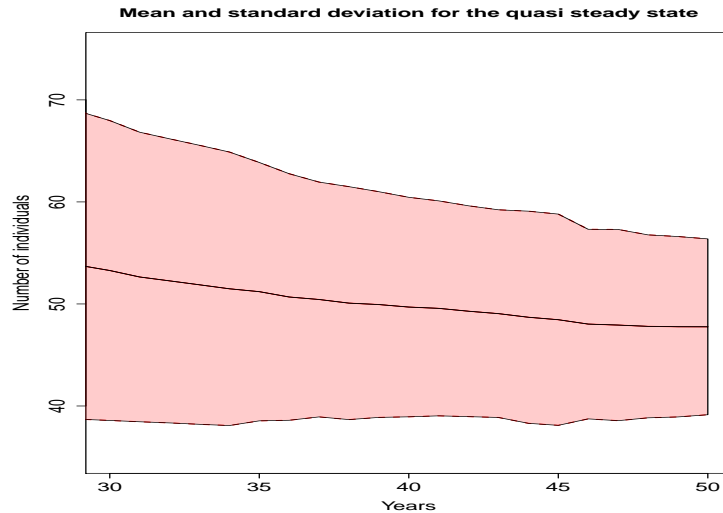
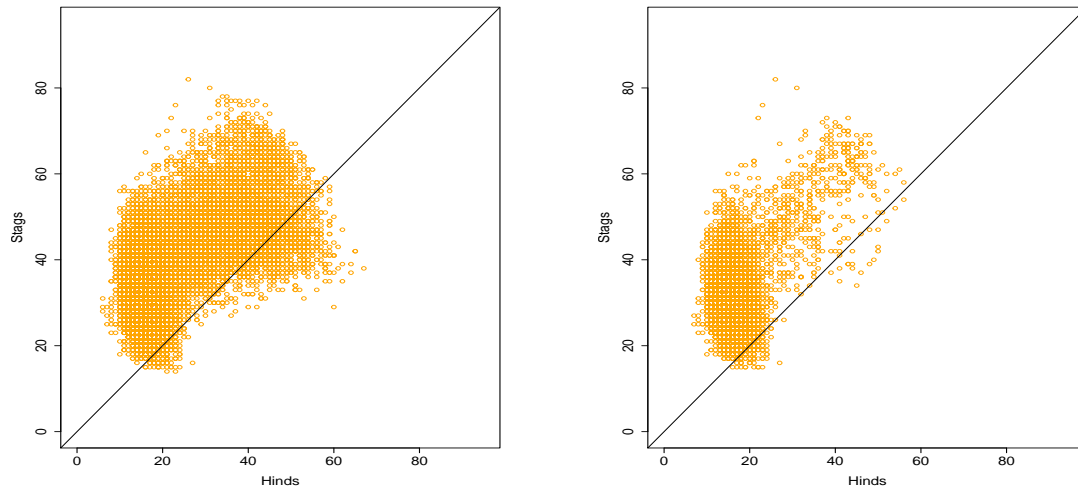


Figure 4.27: The quasi steady state for mainly hunting young individuals. It is similar to the mainly hunting hinds case we observed previously, but with larger standard deviation.

Since this hunting strategy harvest more hinds than stags, one could assume a sex distribution consisting of more stags. With this in mind, we can observe from figure 4.28 (a) a majority of stags. The sex distribution appears to go through a transient period as we observed for previous case studies as well. Figure 4.28 (b) show the sex distribution the last 20 years. It appears to stabilize around 50-60 individuals, where the majority are stags. If we look at one realization, as the one in Figure 4.29,

we can follow how the sex distribution evolves over the whole simulation period. This realization shows the trend from Figure 4.28 (a).



(a) The sex distribution for the whole simulation period.

(b) The sex distribution for the 20 last years. It appears to stabilize around 50-60 individuals in total, with a majority being stags.

Figure 4.28: The sex distribution for both the whole simulation period and the quasi steady state.

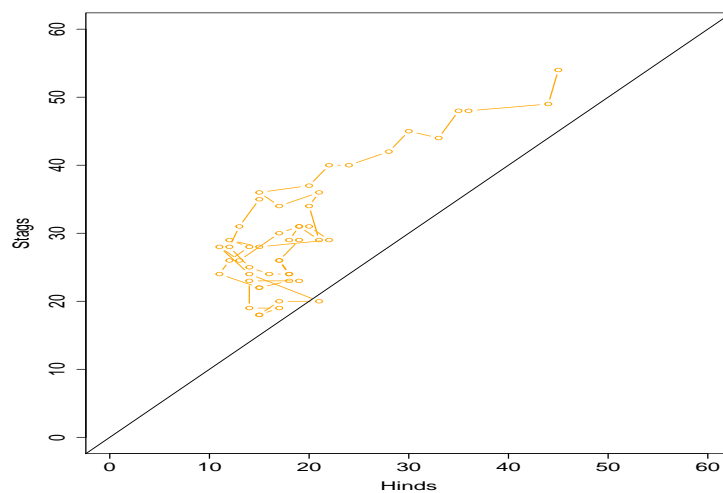
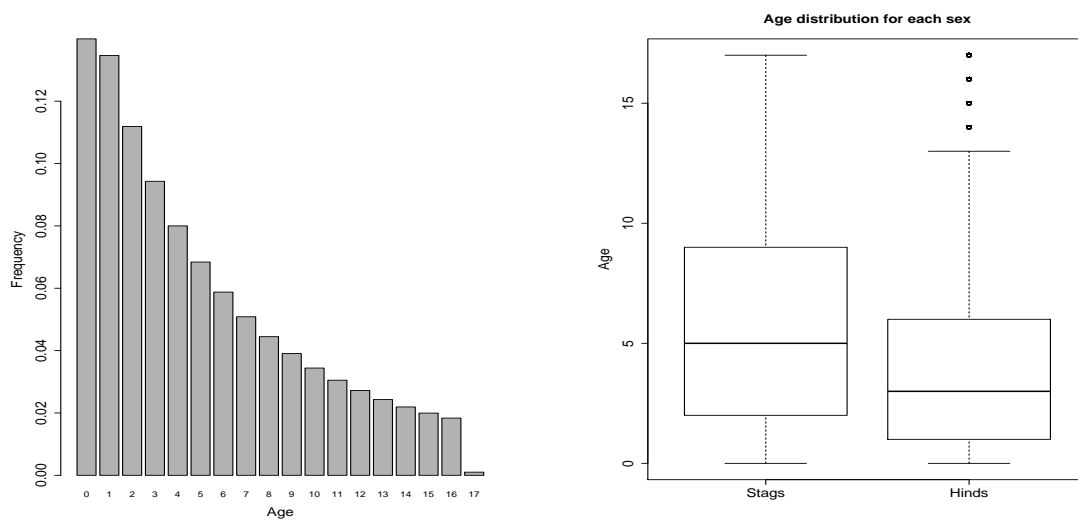


Figure 4.29: One realization describing how the sex distribution evolves over the whole simulation period.

The age distribution for this hunting strategy is quite similar to the one we got in the previous case study. The box plot in Figure 4.30 (b) reveals more variation in the age for hinds than we observed in previous case study (recall Figure 4.22 (b)). This could be a result of not as high hunting pressure against hinds with this hunting strategy. Our initial assumption about the median age for this case study is roughly met. Even though the relative frequency per age class is higher for calves and 1-year-olds, the median age from 4.30 (b) is 5 years for stags and 3 years for hinds.



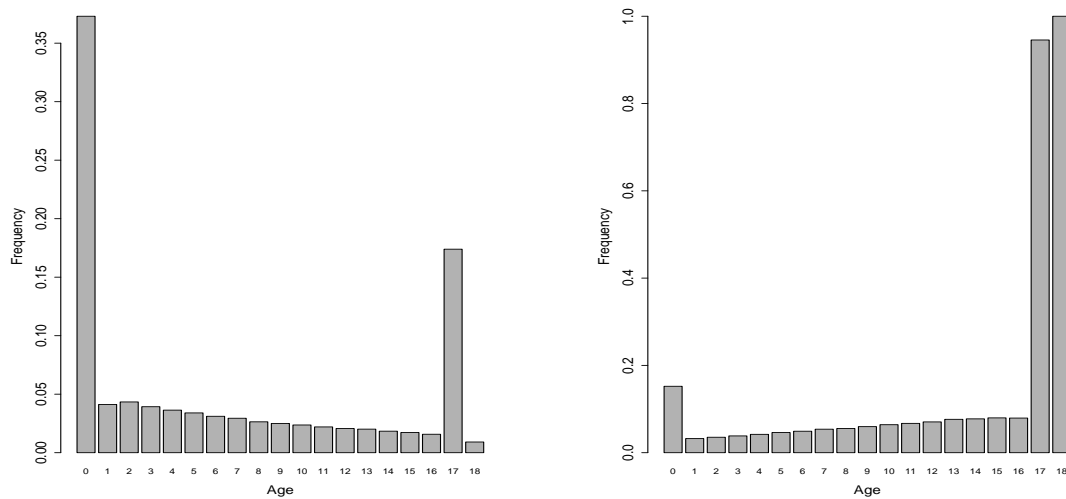
(a) The normalized age distribution. The majority of the red deer are young, but the amount of young individuals are not much larger than the rest of the individuals.

(b) A box plot presenting the age distribution for each sex. The majority are stags, which result in a more diverse age distribution for stags than hinds. The overall distribution of age is much more diverse with this hunting strategy.

Figure 4.30: The age distribution for this hunting strategy during the whole simulation period.

The population could be considered a young one, but the overall age is older than the previous results. As we have seen before, a young population result in a majority of young individuals dying of natural causes. This is also the case for this hunting strategy, even though the majority of the harvested individuals are young. From Figure 4.31 (a) we observe how calves and 17-year-olds represent the majority of all individuals who died of natural causes. If we compare the number of individuals who died of natural causes to the entire population, we can once again observe the given probabilities from Eq. 3.1. This indicates a small to none impact from the carrying capacity with this hunting strategy.



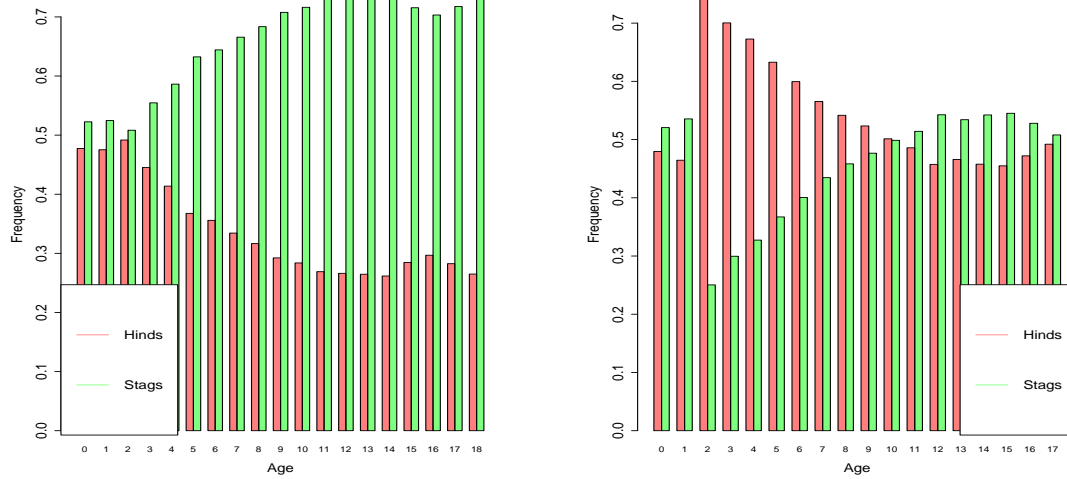


(a) The normalized age distribution of individuals who died of natural causes. The majority of these individuals are calves.

(b) The frequency of individuals who died of natural causes compared to all individuals in the population. The given probabilities from Eq. 3.1 are reflected, as we have observed for other hunting strategies as well.

Figure 4.31: The normalized age distribution of individuals who died of natural causes, compared to both the total number of individuals who died and the entire population during the whole simulation period.

Figure 4.32 (a) show the distribution of sex for the individuals who died of natural causes. We observe that stags are the dominant sex dying of natural causes for every age class. This could be a consequence of the higher hunting pressure against hinds. How the hunting pressure differs between hinds and stags, are presented in Figure 4.32 (b). For calves and young red deer, the majority of the harvested individuals are stags. However, for red deer between 2-10 years, hinds are the most harvested sex. The oldest red deer harvested are stags. Since the hunting pressure is higher for younger hinds, this could mean fewer hinds getting the opportunity to grow older, resulting in more older stags being harvested. An explanation for higher hunting pressure against male calves and 1-year-olds, could be the higher probability of giving birth to a male calf, resulting in a larger portion of male calves being hunted.



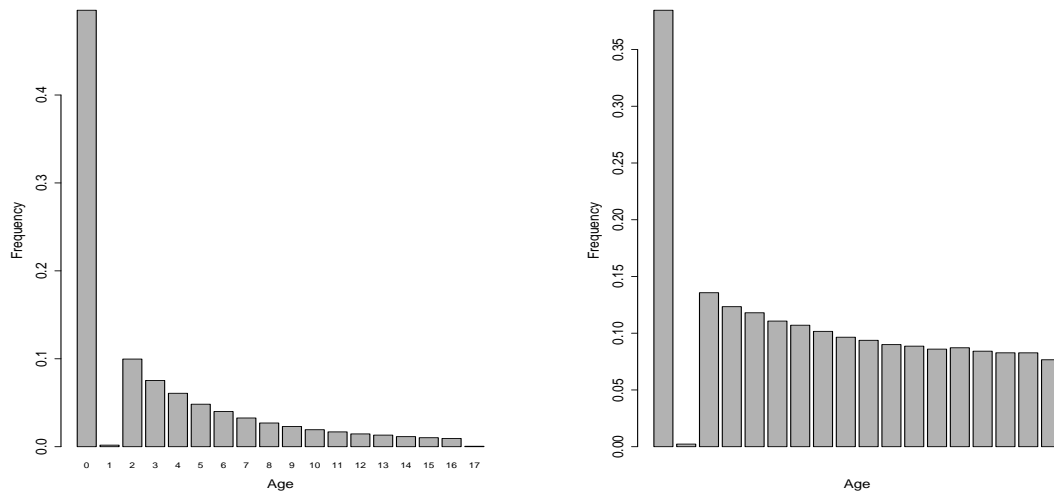
(a) How the dead individuals per age class are divided between each sex. The majority of the individuals dying of natural causes are stags.

(b) How the harvested individuals per age class are divided between each sex. For red deer under the age of ten, the majority are hinds. The older red deer harvested are mainly stags.

Figure 4.32: The distribution of individuals who died of natural causes or hunting per age class, divided between each sex for the whole simulation period.

Figure 4.33 (a) reveals how 50% of all the harvested individuals are calves. This is most likely due to our hunting strategy. The overwhelming majority of harvested calves, results in few 1-year-olds being harvested. This could as assumed before, be a consequence of the hunting limit  $l$  from Table 4.1. Since we harvest many calves each year, the number of 1-year-olds may not be more than the hunting limit for neither hinds or stags. The result is then a low hunting pressure against 1-year-olds. However, this could be a reason why this hunting strategy ends in a more diverse age distribution.

For the older red deer, the hunting pressure is rather low per age class which could be due to the overall low number of individuals in the population. If we compare the number of harvested individuals to all individuals in the population as described by Figure 4.33 (b), red deer from the age of 2 and older, have an almost steady harvest rate of around 10 % for each age class. The rate for calves and 1-year-olds stand out from the rest. Since this hunting strategy focus on mainly harvesting calves, it is not surprise that approximately 40 % of all calves are harvested.



(a) The normalized age distribution of harvested individuals.

(b) The age distribution of harvested individuals normalized against the entire population.

Figure 4.33: The cases of normalized age distribution of harvested individuals and normalized age distribution of harvested individuals compared to the entire population for the whole simulation period.

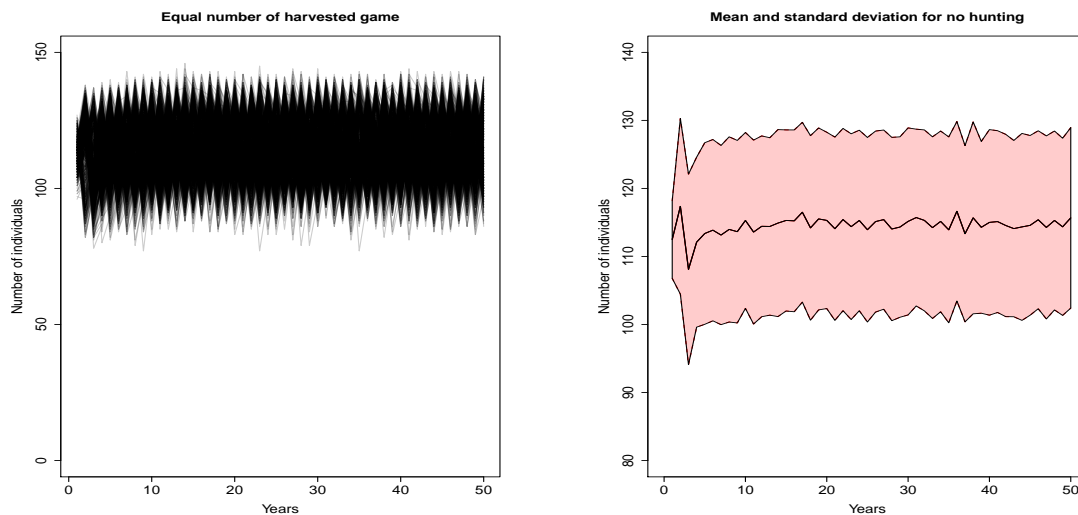
The hunting strategy of harvesting mainly calves and a couple of hinds more per stag, has resulted in a population with a more diverse age distribution than previous cases. Also, the carrying capacity function had a small to none impact on the end result. As discussed before, this could be due to either a well working hunting strategy, making sure the environmental impacts remain small or it could be a consequence of the chosen values for our carrying capacity being too unrealistic.

The last results we looked at for this case, were the amount of harvested individuals in the entire population and from which age classes this hunting strategy harvested most individuals. As predicted, calves were the major harvested group of individuals. Except for 1-year-olds, the rest of the harvested in each age class appeared to stabilize around 10 % of all individuals.

From the results so far, this hunting strategy could appear to be working well for many reasons: we get a diverse age distribution, small impact from the environment and the hunting pressure against each age class seem to stabilize. Next, we will observe what happens if we disregard hunting completely and how this can affect the population of red deer.

## 4.5 Result of No Hunting

As the main focus of this thesis is testing how different hunting strategies affects a population of red deer, it can be interesting to observe how our model works if we disregard hunting. We would expect an increase in the number of individuals, as we already observed in Figure 4.10 when we stopped hunting after ten years. By the use of the same initial conditions before, and just removing the hunting function completely from the IBM, we simulate a last case study. The result is presented in Figure 4.34, which reveals a mean population just above 110 individuals when hunting is disregarded.



(a) The result of 500 simulations. We end up with a much larger mean population than for any other case studies.

(b) The mean population is just above 110 individuals.

Figure 4.34: The number of individuals are much higher than before, if we disregard hunting completely.

We can assume a larger amount of individuals dying in this case study, as we do not include hunting as means for keeping the population at a sustainable level. To make sure the population from each simulation do not increase beyond capability, the carrying capacity function impacts the death probability much more in this case study than before. This is a direct consequence of disregarding hunting. As Figure 4.35 shows, the amount of individuals dying in each age class are overall higher than for any other case, when normalizing the age distribution of dead individuals against the entire population.

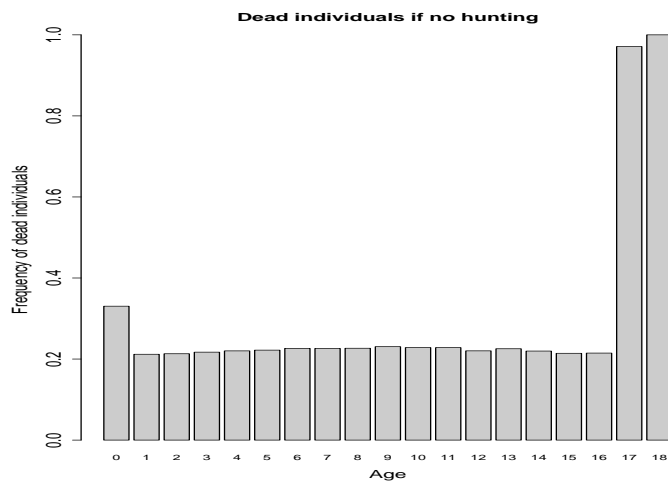


Figure 4.35: The age distribution of dead individuals normalized against the entire population during the whole simulation period. We see a much higher death rate in this case study than for any other case study.

Previously we have observed death rates reflecting the given death probabilities from Eq. 3.1, without much impact from carrying capacity. For this case on the other hand, red deer have a much higher death rate than observed before. This indicates an impact from carrying capacity and could be interpreted as larger populations resulting in less food resources, more competition or even stress for the individuals, causing an overall higher probability of dying.

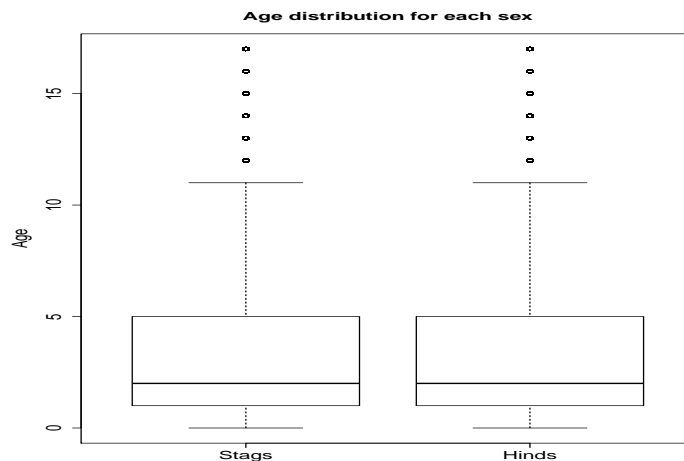


Figure 4.36: The age distribution for each sex during the whole simulation period. The age distribution appear to be equal for both hinds and stags.

The age distribution for the whole simulation period reveals a low median age for both hinds and stags. Since this case study disregards hunting, the low median age can be a consequence of many individuals being born each year or the much higher death probability for older individuals than we have observed before. Since the death probability is not based upon sex, we can assume a small majority of the individuals being stags. This is due to the higher probability of giving birth to a male calf, as described in Chapter 3.

This case study present a possible result of what could happen if we stop hunting completely. The choices we have made for the parameters in the carrying capacity function are also an important part of how this case study turned out. Now we want to analyze the sensitivity of the parameters and their impact on the end result.

## 4.6 Sensitivity Analysis of Parameters

Collecting data and reference values for the sensitivity analysis from the real world come at high costs. As mentioned in the sensitivity analysis section in Chapter 2, if one do not have a reference value for comparison, the focus will be change in mean value and standard deviation. This is an alternative to conduction expensive experiments. The new mean and standard deviation will be compared to the original mean when applying the initial values used for the case studies in this chapter. The quasi steady state for one of the previous hunting strategies will be used as our reference. In addition to analyzing the values in Table 4.6, we will also change the order of two functions in the IBM for observing how this could impact the end result. At last, we will observe what happens if we change a parameter in the middle of the simulation period.

*Table 4.6: Initial values for parameters we will conduct sensitivity analysis on.*

Parameter	Description	Value
$c$	Maximum impact from carrying capacity	0.3
$a$	Slope of carrying capacity curve	1
$i_{max}$	Maximum number of individuals	150
$l$	Hunting limit	10

As observed from the four different case studies with different hunting strategies, the hunting strategy of harvesting mainly stags appeared to have the majority of simulations occurring at almost the same area for every year. This is the reason why we choose this hunting strategy for the sensitivity analysis. We will then compare the new results we get from changing one parameter at a time, to the quasi steady state from Figure 4.11 (b).

## Slope of Carrying Capacity

First, we start by changing the slope  $a$  of the carrying capacity function. Recall the carrying capacity function from Chapter 3, which updates the probability of dying of natural causes for that particular individual,  $p_{i,d}$ .

$$p_{i,d} := p_{i,d} + \frac{c}{2}(1 + \tanh(a(i_{now} - i_{max}))). \quad (4.1)$$

Figure 3.3 demonstrated how a different slope in the carrying capacity function could impact the death probability. For this case, we compare the result when applying the initial slope,  $a = 1$ , to the two different cases of  $a = 0.5$  and  $a = 0.1$ . Figure 4.37 show the result of the mean population for the three different slopes, with the standard deviation for each case. The different colors red, black and blue represent respectively  $a = 1$ ,  $a = 0.5$  and  $a = 0.1$ .

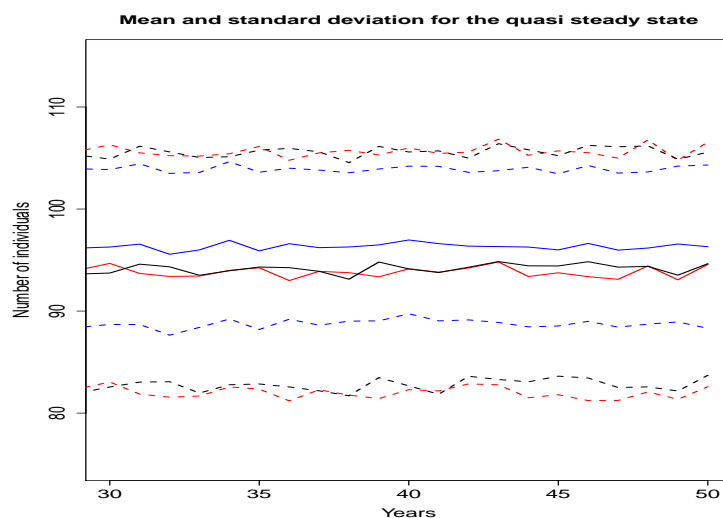


Figure 4.37: Comparison of the different values of the slope  $a$  in the quasi steady state. The dashed lines represent the standard deviations and the solid lines are the mean populations. The different colors represent different slopes: red ( $a = 1$ ), black ( $a = 0.5$ ) and blue ( $a = 0.1$ ).

As we can see in Figure 4.37, the populations are approximately the same for each case. However, the standard deviation for the case where  $a = 0.1$  is smaller than in the other two cases. A lower value for standard deviation is an indication of less variation. Based upon the way we defined the carrying capacity function, for  $a = 0.1$  the impact in the mortality should be larger than for the other two values of  $a$ . This is probably a part of the reason we get less variability with  $a = 0.1$ ; a much less dramatic increase overall in death probability result in less fluctuations

each year. Since the differences in mean value and the standard deviation are so small for different choices of the slope  $a$ , we can assume this parameter to have a small impact on the end result.

## Maximum Impact from Carrying Capacity

The next parameter we want to analyze, is the parameter deciding the maximum impact from carrying capacity. As the initial choice, we set  $c = 0.3$ . This could be a too low choice, so we compare this to two higher choices,  $c = 0.5$  and  $c = 0.8$ . For a larger value of  $c$ , we expect the death probability to increase greatly if the number of individuals present are high enough for the carrying capacity to have an impact.

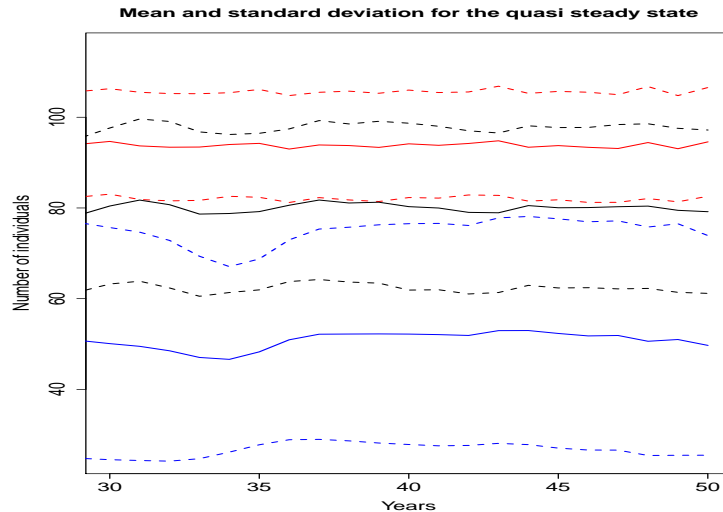


Figure 4.38: The choice of  $c$  have a large impact to the mean populations and standard deviations. The different colors red, black and blue represent respectively  $c = 0.3$ ,  $c = 0.5$  and  $c = 0.8$ . The dashed lines represent standard deviation and the solid lines the mean population.

As observed in Figure 4.38, our assumption of a larger  $c$  influencing the mean population appears to be true. We can observe a larger value of  $c$  resulting in a lower mean population and an increasing standard deviation. The difference in standard deviation for  $c = 0.3$  and  $c = 0.5$  appear to be almost equal, but the standard deviation for  $c = 0.8$  are much larger than for the other two. This implies much variation in the number of individuals and could be a consequence of the much increased death probability with such a choice of  $c$ . This parameter have a much larger impact on the end result, as we can observe from Figure 4.38.



## Maximum Individuals

At what point the carrying capacity function starts to impact the death probabilities, also depends upon the choice of maximum individuals  $i_{max}$ . For the case studies previously in this chapter, we have used  $i_{max} = 150$  individuals. If we change it to a higher number of individuals, we can assume an increase in the mean population. This is because the carrying capacity had a small impact for  $i_{max} = 150$ , recall Figure 4.15 (b). For a higher value of  $i_{max}$ , the carrying capacity will not impact the death probabilities before the population reach a higher number of individuals. If we choose  $i_{max} = 120$ , we can assume the opposite happening, as 120 individuals are much closer to the initial population of 100 individuals, than the initial  $i_{max}$  of 150 are.

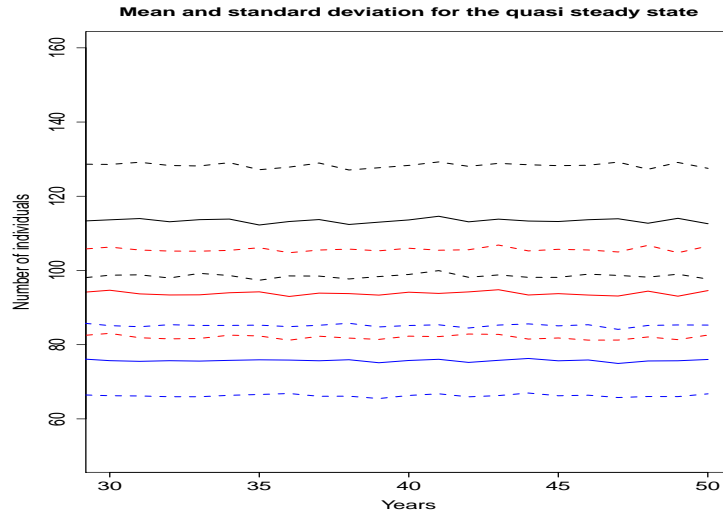


Figure 4.39: Changing the maximum number of individuals result in different mean populations and standard deviations. Black represent  $i_{max} = 180$ , red for  $i_{max} = 150$  and blue represent  $i_{max} = 120$ . The dashed lines are standard deviations and the solid lines represent the mean populations.

Observing the different mean population in Figure 4.39, our assumptions appear to be true. If we set  $i_{max}$  to 180, the mean population increases from 100 individuals to stabilizing around just below 120 individuals. If we set  $i_{max}$  to 120, the mean population stabilize around 80 individuals instead. The standard deviation for the larger mean population appear to be a bit larger than for the other two cases, indicating more variation in the simulations. The results changed when  $i_{max}$  changed, implying that  $i_{max}$  is a sensitive parameter in the sense that its value could impact the end result greatly.

## Hunting Limit

Before being able to harvest the given number of individuals in each harvest group, we had to check if the number of individuals from each group were larger than the hunting limit  $l$ . This is to make sure we did not harvest more individuals than present from each group. However, the limit could impact the end result. For all previous case studies, we have used  $l = 10$  for each harvest group. Now we want to see what happens if we change this limit for each group.

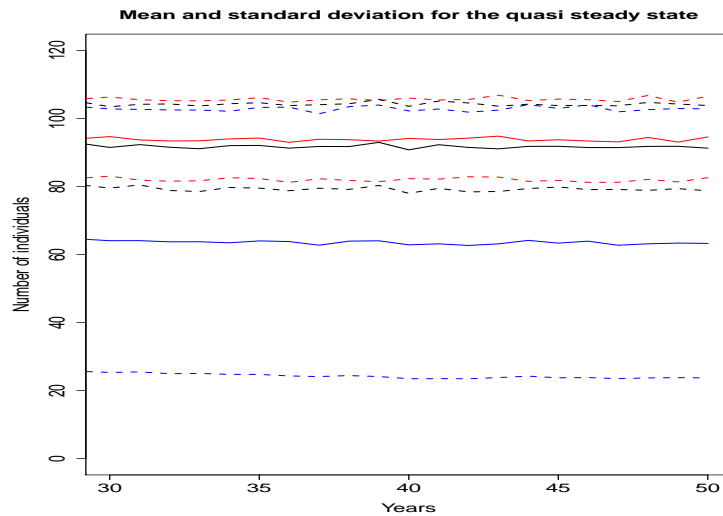


Figure 4.40: The dashed lines represent standard deviations and the solid lines represent mean populations. Red represent the initial choice of limit  $l = 10$ , black for  $l = 8$  and the blue if the limit  $l$  is the same as the different  $h_c, h_{yh}, h_{ys}, h_h$  and  $h_s$ .

The red lines in Figure 4.40 are from the initial choice of  $l = 10$ . As we can observe from the figure, changing it to  $l = 8$  does not make much different for the mean population and the standard deviation. However, if we set the limit  $l$  to match each number of possible individuals to harvest from each group ( $h_c = 5, h_{yh} = 3, h_{ys} = 8, h_h = 2$  and  $h_s = 7$ ), the change is major. The standard deviations are much greater and the mean population is lower than observed for the other cases. This is an indication of the large impact the hunting limit have on our end results. How we define this limit and if such a limit should be included in the first place, are important matters to consider.

## Changing Order of Two Functions

The next case we will study, is if the order of execution for two of the functions in the IBM have an impact on the end result. We tried to order the functions as close

as possible to how they would occur in the real world; each year start by aging one year, then reproduction occurs, followed by natural death and at last the hunting season. The latter two events however, are most likely happening at the same time in the real world. In this IBM, we made a choice of which one should occur first and we went for the natural death function (recall the flow chart in Figure 3.1). What happens if the hunting function comes before the natural death function?

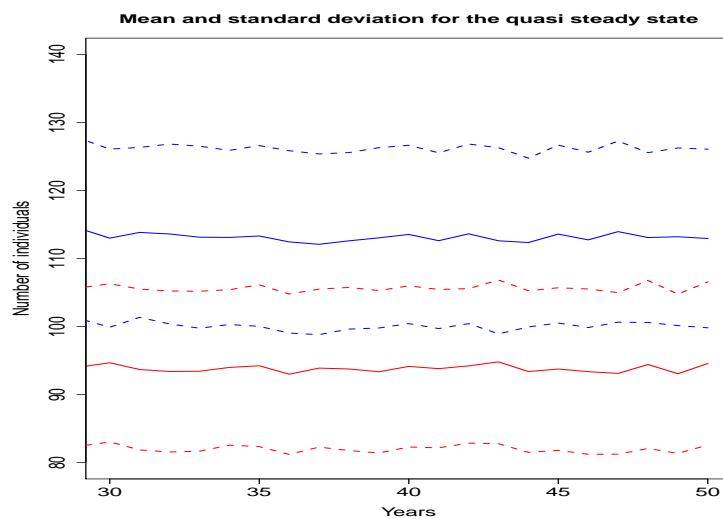


Figure 4.41: If we change the order of two functions in the IBM, so that hunting occur before natural death, we can observe a large change in the mean population represented by the blue solid line. The standard deviations, represented by dashed lines, are approximately the same for both cases. The red lines are the result from the initial choice of order.

As we can observe from Figure 4.41, the order of execution matters for the end result. If we put the hunting function before the natural death function, we end up with a population with a greater mean population, but with the same standard deviation. The blue lines represent the new, greater mean population and standard deviations for the new order of execution. We harvest equal amount of individuals each year (as long as the condition regarding hunting limit  $l$  is met). If the death function occur after hunting, the number of individuals dying of natural causes will be less than before, giving rise to a larger population. This is due to the fact that the death function is based upon a percentage of the number of individuals present at that exact time. This is something one should be aware of and take into consideration when designing a model such as this IBM to simulate a natural phenomenon.

## Change in Parameter After 25 Years

Nature is an unstable living area, as it can change rapidly due to impact from climate, weather, humans or other factors. One interesting case to study, is how a change in one parameter after some time can influence a population of red deer. This change represent an unforeseen event influencing the red deer's lives. For the sake of comparison, we will make such a change in both the hunting mainly stags strategy and in the case of no hunting. After 25 years, the parameter  $i_{max}$  will change from 150 individuals to 120 individuals.

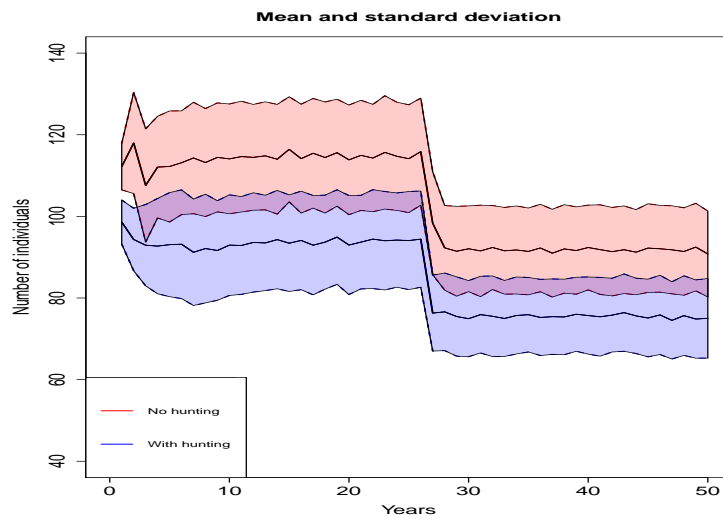


Figure 4.42: The mean population and standard deviations of both the hunting mainly stags case and the no hunting case. If we change the maximum number of individuals after some time, we can see a rapid change in the populations. The change in the mean population appear to be larger for the no hunting case.

Figure 4.42 show how much the populations change if we change the parameter  $i_{max}$  halfway through the simulation period. The red color represent the case of no hunting. As observed from the case studies, no hunting results in a larger population of red deer as opposed to the case studies with hunting. When we change  $i_{max}$ , the change in the number of individuals is larger for the no hunting case. The mean drop from 115 individuals to 92 individuals for the no hunting case, which is 5 individuals more on average than for the hunting case (which dropped from 94 to 76 individuals).

Such unforeseen events could cause good and bad years for the red deer, in the sense of changing food access or a potential outbreak of an epidemic. A change in the parameters could illustrate the population's resilience, which describes how the population responds to disturbances. We are only illustrating this by changing the

parameter once, but this could also be set to occur at different times with varying impact and length of presence. As the no hunting case gets more influenced by the change in one parameter, one interpretation could be that without hunting, the population is more sensitive to environmental impacts. Once again we see the importance of hunting for impacting the population of red deer in a positive way.

## Chapter 5

# Discussion and Further Work

So far, we have examined how hunting can affect a population of red deer in different ways by the use of an individual-based model. Also, we have tested the sensitivity of some parameters in the IBM. This chapter will focus on what the different hunting strategies could imply for the populations in a more general sense, and discuss possible further work for the IBM implemented in this thesis.

### 5.1 Case Studies

If there is no data available regarding the number of individuals in a population or the distribution of sex and age, a model such as this one could be used to predict general trends when applying different hunting strategies. The results from this model will indicate what wildlife management can expect when deciding upon which hunting strategy to apply towards a population of red deer.

We start by looking at the number of individuals, and it is no surprise that the hunting strategy of harvesting mainly hinds resulted in a lower mean population of red deer, as fewer hinds result in fewer calves being born. However, the hunting strategy of harvesting mainly young individuals and a couple of hinds more per stag, also resulted in almost the same low mean population of red deers as the hunting mainly hinds strategy.

Whereas the hunting mainly hinds case did not have much variability in the simulations, the mainly hunting young individuals case did. Recall Figure 4.26 from the mainly hunting young individuals case, where some of the simulations never dropped in number of individuals. These exceptions cause a larger standard deviation in the results and should be taken into consideration when choosing a hunting strategy.

Another fascinating result was the transient period some case studies experienced.

The low mean population in the first case study of equal number of harvested individuals, ended up with the same sex distribution as the higher mean population, but at a different population size. For the mainly hunting young individuals case, the transient period also led to a reduction in the population, before stabilizing at a lower level. Experiencing such a transient period appear to have a large impact on how the population will evolve over time.

For each hunting strategy the age distribution have been one of the interesting points to examine. According to Solberg et al. (2017), the age distribution amongst the individuals can provide insight related to sex distribution and the different hunting pressure between each sex. Also, the age distribution can give indications of the weight and fitness of the red deer.

As observed from the simulations in Chapter 4, and mentioned in both Samdal et al. (2003) and Solberg et al. (2012), a high hunting pressure against a population of red deer can result in a young population of individuals. An overall low age distribution can cause issues for the red deer. One example is how weight is related to age. Younger individuals have not had enough time to grow larger in size and a low weight can make it hard for the red deer to survive through the winter. Also, this could make them more susceptible to interactions and catch diseases more easily.

Another problem with an overall young population of red deer is how this could influence the rut and when calves are born during the spring. Younger stags have a tendency to wait for the older and larger stags to be done with the rut before they try to mate. If the mating happens later in the season, this will cause the hinds to give birth later in the spring and the calves might not get enough time to gain weight before the winter comes around (Samdal et al., 2003).

An overexploitation of either sex could also cause issues regarding the age distribution and fitness of the red deer. For both the hunting mainly hinds case and especially the hunting mainly stags case, the age distributions revealed a young population. According to Samdal et al. (2003), a high hunting pressure against hinds should result in a lower number of individuals as observed in our case study. Also, if we harvest the hinds with best fitness, this will lower the quality of fitness for the generations to come. The same will happen for an overexploitation of stags, as fewer stags in the population mean less competition between the stags. Then one could disregard survival of the fittest, since low competition could result in survival of the less strong individuals as well.

The hunting function have a crucial role for how the population evolves. As observed, the population will increase if we do not hunt to keep the number of individuals at a more sustainable level. For a larger number of individuals, the carrying capacity function increase the mortality of the red deer. This is because the carrying capacity mimic environmental impacts as less food resources, diseases or even less living area

for the red deer; all possible consequences of an increasing population. Although hunting is one factor impacting the population of red deer, one should be aware of other factors as well. These factors could be environmental impacts as already discussed, but also the model setup. Consequences from the design of the IBM, are what the sensitivity analysis tried to highlight.

The sensitivity analysis studied the amount of impact from changing some parameters to the end result. The extent of how much influence each parameter have, varies. As a consequence, these parameters should be chosen with care. Take the hunting limit  $l$  as an example. Changing this limit from  $l = 10$  to  $l = 8$  did not impact the end result much. However, when we set the limit for each hunting group to be equal the number of individuals to harvest from each group, the changes were major. The mean population changed to a lower number of individuals and the standard deviation increased greatly.

The idea behind the hunting limit was to make sure we did not harvest more individuals than present in each group. However, it is possible that the consequences of this limit have been greater than expected. If we recall Figure 4.8 (a), the number of harvested 1-year-olds were low for the lower mean population. This is most likely due to the hunting limit; for most years, the amount of 1-year-olds have been less than the limit, so the number of harvested 1-year-olds is low.

The case mentioned from the first hunting strategy, with few harvested 1-year-olds, resulted in a much higher number of harvested 2-year-olds. How this IBM is designed, no individuals from one group will be harvested a given year if the number of individuals present are lower than the limit. This is a bit different from the real world. Few individuals observed could result in few being harvested, as the probability of being shot decrease when the individuals are less visible. Also, not every area manage to harvest all the given individuals. So the hunting limit was created as a compromise. However, one could have made a hunting function based upon harvesting a percentage of the individuals each year. This would probably impact the end result, as the amount of harvested individuals would differ each year if the number of individuals fluctuated.

The last study we performed in the sensitivity analysis, was changing one parameter in the middle of the simulation period. Changing  $i_{max}$  from 150 to 120 rapidly changed the number of individuals both in the hunting case and the no hunting case. As we observed, the case of no hunting changed more than the case with hunting. This is once again an argument for how important hunting can be for obtaining a sustainable red deer population.



## 5.2 Further Work

We mentioned in Chapter 2 how complex the design of an IBM could be. The model built in this thesis is still limited in features and has a great potential for further expansions. Also as mentioned in the thesis outline, one could proceed with more statistical methods for dealing with the uncertainties in the model. In this section, we will describe different options for further work and extensions to The Red Deer Population Model.

### 5.2.1 Markov Chain Monte Carlo

In Chapter 2 and 3, we briefly discussed the Markov chain and implementation of the Markovian structure in this IBM. As described in Gilks et al. (1995), Markov Chain Monte Carlo (MCMC) is basically Monte Carlo integration using Markov chains. The idea is to construct a Markov chain with a stationary distribution and then draw samples by Monte Carlo sampling to generate an estimate for the expectation.

In other words, the IBM in this thesis creates a sequence of random variables at each time step where the next state is sampled from a distribution only depended on the current state of the chain. This sequence is the Markov chain in the model. The main state this IBM describes at each time step is the number of individuals that given year. Since we use Monte Carlo sampling, we will have a large number of different possible states for each time step.

By using one of the MCMC methods, we can exclude those states who are less likely to be true. For instance, at each time step, we compare the number of individuals against a proposed distribution, and if the current state is far from the proposed distribution, one could assume it to be less likely. The state is then rejected and the process is repeated, until a state is accepted. Then we move on to the next time step and repeat the process until we accept the state of that time step.

Recall the result from harvesting mainly young individuals. The different simulations in Figure 4.26 could perhaps be more centered around one expected mean if a MCMC method was implemented in the model, instead of the large variation one can observe now. This is just an example of where the implementation of a MCMC method could provide less uncertainty in this model.

### 5.2.2 Parameter Estimation

An individual-based model such as this one, predicts the number of individuals present each year based upon the parameters given in advance. However, if one knew the output beforehand, it is possible to reconstruct the model to give an estimation of the parameters instead. If one had data set consisting of observations

of the different individuals in the population, it could be possible to do an operation with the IBM "in reverse". This way, we could get a better estimation of parameters such as natural death probabilities, rather than the approximations the model in this thesis are based upon.

As an example, one could use data from <https://www.hjorteviltregisteret.no/> or the local wildlife management regarding the number of harvested individuals and observed individuals per hunting day. The latter one could be used as a measurement of approximate population size. However, one need to take the uncertainty in these measurements into account before including them in the process. With these data over a given time period, it could be possible to perform a parameter estimation.

*Table 5.1: Harvested red deer at Svanøy from 2014 until 2017.*

	<b>Calves</b>	<b>Young Hinds</b>	<b>Young Stags</b>	<b>Hinds</b>	<b>Stags</b>	<b>Total</b>
<b>2014</b>	7	2	2	5	4	20
<b>2015</b>	12	2	4	6	4	28
<b>2016</b>	8	6	3	5	4	26
<b>2017</b>	10	3	7	7	0	27

*Table 5.2: Total sum of observed red deer for all hunting days at Svanøy and the total number of hunting days each hunting season from 2014 until 2017.*

	<b>2014</b>	<b>2015</b>	<b>2016</b>	<b>2017</b>
<b>Hunting Days</b>	55	87	92	71
<b>Sum Observed</b>	216	174	362	332

As described in Kattwinkel and Reichert (2017), one possible approach for parameter estimation is using a Particle Markov Chain Monte Carlo (PMCMC) algorithm. Some IBMs have a "hidden Markov structure", implying that the underlying Markov structure of the states is hidden. This mean observations are only available if the individuals reach a certain level of size or age.

Even though hidden states are not the case for this IBM, the same approach of parameter estimation could be used. The algorithm starts by choosing an initial parameter set and then approximate the likelihood of the parameters based upon observations. After this, the MCMC method, e.g. Metropolis acceptance, can be used to accept or reject the approximated likelihood. If the Markov chain of parameters is sufficiently long, the method stops.

### 5.2.3 Possible Expansions of the IBM

One of the advantages by using an IBM, is the possibility of including interactions between the individuals and the environment (DeAngelis and Mooij, 2005). As mentioned in Section 1.2, the red deer are no strangers to traveling and some even choose to swim across fjords for exploring new territories. Migration would be a natural next addition to the model in this thesis. This could for instance be migration between habitats on the main land or migration between islands.

Norway is a country consisting of many islands, so migration between some of these could be a good place to start. For instance Svanøy, Stavøy and Tansøy, the three island next to each other in Figure 1.2. One could try to simulate how different hunting strategies affect the migration for the red deer. Questions as if whether a higher hunting pressure impacts migration or not, could be something such an IBM would try to answer by running different scenarios.

Interaction between individuals do not necessarily only include migration. Another important part of the red deer lives is the hierarchy between the individuals in a population. One dominant stag could have a harem consisting of two to four sexually mature hinds (Meisingset, 2008). Competition between stags for domination in the population or the harem, often occur. This is where a game theory approach could be useful.

Game theory aims to explain interactions between individuals, were each individual (player) have an approach of its own to achieve a set goal, with different outcomes (win/lose). As described in DeAngelis and Mooij (2005), a game theory approach could be implemented in an IBM. This way, the individuals can learn from encounters with others and the hierarchy in the population could change due to different outcomes from the "game" between individuals.

Combining a game theory approach with migration in the model, could give rise to other possible studies. One could look into the effects of low access to food. Could the lack of food resources increase the competition between individuals, and if so, could this increase the probability of migration to another place in the search of food? Could the decision of migrating increase or decrease the death probability for that particular individual?

Another example of a possible expansion is adding weight of each individual to the model. A combination of sex, age and weight would impact both migration and the competition. Larger individuals could have a better chance of surviving a swim across a fjord or becoming the dominant stag in a population. Also, weight would impact reproduction and the death probabilities, as larger individuals have a higher probability of reproducing and a lower probability of dying (Solberg et al., 2012).

The last example of an expansion that would improve this IBM, is dividing the year

into seasons. We tried to order the functions in the IBM according to when during the year they would occur. If seasons were implemented in the model, one could try to simulate the impact from late breeding or how survival during the winter can rely on fitness of the individual. Also, if seasons were implemented in the model, maybe one could observe at what time during the year the majority of red deer migrate.

Further additions to an IBM will result in more possibilities regarding different studies to perform. As mentioned in Chapter 2, the complexity of an IBM could be limited due to simulation time and the computers performing the simulations. Which expansions one wants to add to the model, should be reflected by the studies one wants to perform.

# Chapter 6

## Conclusion

From the case studies in this thesis, we can conclude that different hunting strategies have a large impact on how red deer populations evolve over time. What could be the correct choice for achieving a sustainable red deer population, can differ between areas and depend on the red deer population already settled in that area. It is important to be aware of variability in the results from the different hunting strategies when deciding upon which strategy to apply in each given area.

When designing an IBM, questions regarding complexity of the model, arise. This should depend on the natural phenomenon the IBM is supposed to simulate. No matter how complex the model turns out to be, different parameters or order of execution of functions could impact the end result in different ways. Sensitivity and variability should be taken into consideration when implementing an IBM.

Being able to provide the best possible wildlife management, models such as this one could give insight beforehand regarding different consequences related to different hunting strategies. The future of red deer, and even wildlife in general, depend on the decisions we humans make. If we want to preserve the nature and its wildlife, we better explore all methods available to be able to provide the most sustainable and optimal management possible. According to the UN report from IPBES<sup>1</sup>, the rate of extinction of many species is accelerating and wildlife as we know it, is threatened. Hopefully, this thesis can be a positive addition to wildlife management and further research.

All models are wrong,  
but some are useful.

---

*George E. P. Box*

---

<sup>1</sup><https://www.un.org/sustainabledevelopment/blog/2019/05/nature-decline-unprecedented-report/>

# Bibliography

- Andersen, O. and Dervo, B. K. (2019). Jegernes og fiskernes forbruk av varer og tjenester i norge i 2018. *NINA Rapport*, 1605.
- Bacaër, N. (2011). *A Short History of Mathematical Population Dynamics*. Springer.
- Becker, R., Chambers, J., and Wilks, A. (1988). *The new S language*. Chapman and Hall/CRC Press.
- Bjørneraas, K. r. (2012). *Klauvvilt i norsk natur - historie, biologi og forvaltning*. Akademika Forlag, Trondheim.
- Brock, T. D. (1967). The ecosystem and the steady state. *Bioscience*, 17(3):166–169.
- Cariboni, J., Gatelli, D., Liska, R., and Saltelli, A. (2007). The role of sensitivity analysis in ecological modelling. *Ecological modelling*, 203(1-2):167–182.
- Case, T. J. (2000). *Illustrated guide to theoretical ecology*. Oxford University Press.
- Caswell, H. (2006). Matrix population models. *Encyclopedia of Environmetrics*, 3.
- Chambers, J. M. (1998). *Programming with data: A guide to the S language*. Springer Science & Business Media.
- DeAngelis, D. L. and Grimm, V. (2014). Individual-based models in ecology after four decades. *F1000Prime reports*, 6.
- DeAngelis, D. L. and Mooij, W. M. (2005). Individual-based modeling of ecological and evolutionary processes. *Annu. Rev. Ecol. Evol. Syst.*, 36:147–168.
- Devore, J. L. and Berk, K. N. (2012). *Modern mathematical statistics with applications*. Springer.
- Dörrie, H. (1965). *100 Great Problems of Elementary Mathematics: Their History and Solution*. Courier Corporation.
- Gilks, W. R., Richardson, S., and Spiegelhalter, D. (1995). *Markov chain Monte Carlo in practice*. Chapman and Hall/CRC.

- GoogleMaps (2019). Svanøy. <https://www.google.com/maps/place/Svan%C3%B8y/@61.4911858,5.0715476,13z> (visited 23.04.19).
- Grimm, V., Berger, U., Bastiansen, F., Eliassen, S., Ginot, V., Giske, J., Goss-Custard, J., Grand, T., Heinz, S. K., Huse, G., et al. (2006). A standard protocol for describing individual-based and agent-based models. *Ecological modelling*, 198(1-2):115–126.
- Grimm, V., Railsback, S. F., and Railsback, V. (2005). *Individual-based Modeling and Ecology*. Princeton University Press.
- Grov, B., Helle, M., and Skram, O. W. (2019). Bønder fortvilar over at hjort gjer skade på marka. <https://www.nrk.no/sognogfjordane/bonder-fortvilar-over-at-hjort-gjer-skade-pa-marka-1.14508652> (visited 18.04.19 NRK Sogn og Fjordane).
- Hansen, H., Kapperud, G., Mysterud, A., Solberg, E. J., Strand, O., Tranulis, M. A., Ytrehus, B., Gulbrandsen, M., Grahek-Ogden, D., Eckner, K. F., et al. (2017). Cwd in norway—a state of emergency for the future of cervids (phase ii). opinion of the panel on biological hazards. *VKM Report*.
- Hothorn, T. and Everitt, B. S. (2014). *A handbook of statistical analyses using R*. Chapman and Hall/CRC.
- Huston, M., DeAngelis, D., and Post, W. (1988). New computer models unify ecological theory. *BioScience*, 38(10):682–691.
- Kattwinkel, M. and Reichert, P. (2017). Bayesian parameter inference for individual-based models using a particle markov chain monte carlo method. *Environmental modelling & software*, 87:110–119.
- Kroese, D. P., Brereton, T., Taimre, T., and Botev, Z. I. (2014). Why the monte carlo method is so important today. *Wiley Interdisciplinary Reviews: Computational Statistics*, 6(6):386–392.
- Leslie, P. H. (1945). On the use of matrices in certain population mathematics. *Biometrika*, pages 183–212.
- Liu, J. S. (2001). *Monte Carlo strategies in scientific computing*. Springer Verlag.
- Massada, A. B. and Carmel, Y. (2008). Incorporating output variance in local sensitivity analysis for stochastic models. *ecological modelling*, 213(3-4):463–467.
- Meisingset, E. L. (2008). *Alt om hjort: biologi, jakt, forvaltning*. Tun Forlag.
- Miljødirektoratet (2017). Hjortevilt. <https://www.miljostatus.no/Hjortevilt/> (visited 20.03.19).

- Petzoldt, T. (2003). R as a simulation platform in ecological modelling. *R News*, 3(3):8–16.
- Pinsky, M. A. and Karlin, S. (2011). *An Introduction to Stochastic Modeling*. Elsevier Inc.
- Razavi, S. and Gupta, H. V. (2015). What do we mean by sensitivity analysis? the need for comprehensive characterization of “global” sensitivity in earth and environmental systems models. *Water Resources Research*, 51(5):3070–3092.
- Samdal, B., Veiberg, V., and Knutsen, S. (2003). *Målrettet hjorteforvaltning. Bedre ressursutnytting*. Norges Skogeierforbund, Norges Bondelag og Landbruksforlaget.
- Solberg, E. J., Strand, O., Veiberg, V., Andersen, R., Heim, M., Rolandsen, C. M., Langvatn, R., Holmstrøm, F., Solem, M. I., Eriksen, R., et al. (2012). Hjortevilt 1991-2011. oppsummeringsrapport fra overvåkingsprogrammet for hjortevilt. *NINA rapport*.
- Solberg, E. J., Strand, O., Veiberg, V., Andersen, R., Heim, M., Rolandsen, C. M., Solem, M. I., Holmstrøm, F., Jordhøy, P., Nilsen, E. B., et al. (2017). Hjortevilt 1991–2016. oppsummeringsrapport fra overvåkingsprogrammet for hjortevilt. *NINA rapport*.
- Solheim, J. T. (2010). Galleri norsk hjortesenter. <http://www.triggerfoto.no/#!album-33-23> (visited 23.04.19).
- SSB (2019). Nytt rekordår for hjortejakta - nedgang for elgjakta. <https://www.ssb.no/jord-skog-jakt-og-fiskeri/artikler-og-publikasjoner/nytt-rekordar-for-hjortejakta-nedgang-for-elgjakta> (visited 20.03.19).
- Tippmann, S. (2015). Programming tools: Adventures with r. *Nature News*, 517(7532):109.
- van der Vaart, E., Beaumont, M. A., Johnston, A. S., and Sibly, R. M. (2015). Calibration and evaluation of individual-based models using approximate bayesian computation. *Ecological Modelling*, 312:182–190.
- van Lith, J. (2001). Ergodic theory, interpretations of probability and the foundations of statistical mechanics. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 32(4):581–594.



# Appendices

# Appendix A

## Table of Parameters

Table A.1: Parameter listing and description for The Red Deer Population Model.

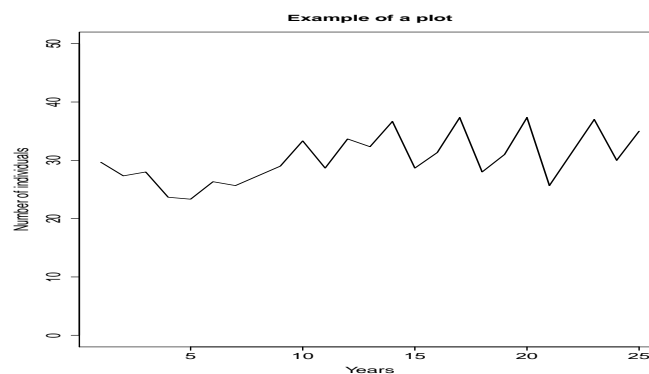
Parameter	Description	Value	Reference
$i_{init}$	Initial number of individuals	100	Section 3.1
$i_a$	Age of individual	Varies	Figure 3.2
$i_f$	Individual is female	<i>True</i>	Section 3.1
$i_m$	Individual is male	<i>True</i>	Section 3.1
$p_r$	Probability of reproducing	$0.3 \vee 0.9$	(Solberg et al., 2012)
$p_{o,f}$	Probability the offspring is female	0.48	(Meisingset, 2008)
$p_{o,m}$	Probability the offspring is male	0.52	(Meisingset, 2008)
$p_{i,d}$	Probability of dying for specific individual	Varies	Eq. 3.1
$c$	Maximum impact from carrying capacity	0.3	Table 3.2
$a$	Slope of carrying capacity curve	1	Table 3.2
$i_{now}$	Number of individuals present <i>now</i>	Varies	Table 3.2
$i_{max}$	Maximum number of individuals	150	Table 3.2
$l$	Hunting limit	10	Section 3.2
$T$	Years simulated	$1, \dots, t$	Section 2.2
$\mathcal{S}$	Sample space	$1, \dots, I$	Section 2.2
$h_c$	Harvest number of calves	Varies	Table 3.1
$h_{yh}$	Harvest number of young hinds	Varies	Table 3.1
$h_{ys}$	Harvest number of young stags	Varies	Table 3.1
$h_h$	Harvest number of hinds	Varies	Table 3.1
$h_s$	Harvest number of stags	Varies	Table 3.1

## Appendix B

# Represent Data Visually

If you want to investigate your dataset, graphical representation is a good place to start. There are different types of techniques if you want to plot your dataset and the different approaches could give you different answers. Here is a couple of examples of graphical techniques.

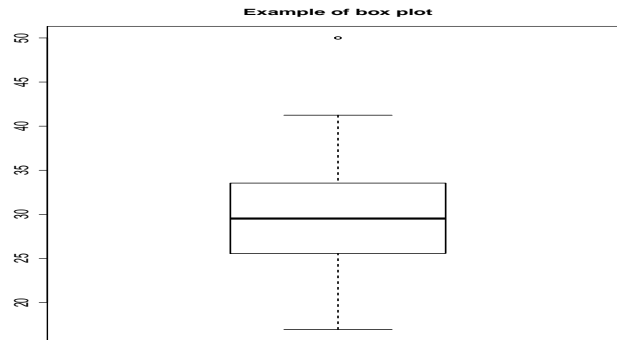
**Plot of a function.** When you have a function where an input value  $x$  provides an output value  $y$ , one could plot each  $x$  and the resulting  $y$ . As an example, one could present this in a plot as a line with the number of individuals as a function of each year. This is a great way to see the development of number of individuals over a period of time.



*Figure B.1: Example of a plot where the  $x$ -axis could represent years and the  $y$ -axis number of individuals that given year.*

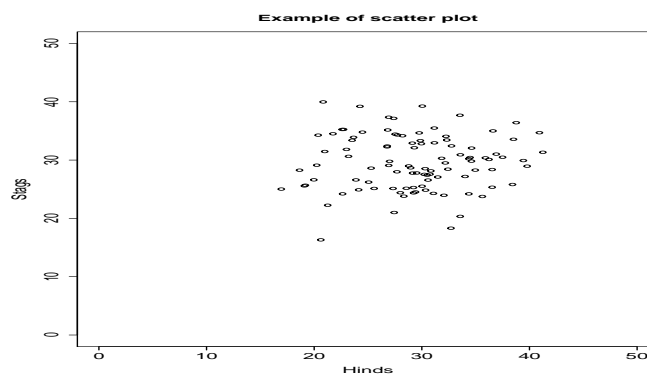
**Box plot.** The box plot is a great way of representing samples of data. This way, a lot of information can be interpreted from one plot. A box plot will show you the median of the samples, how much the samples varies from the median and the

interquartile range (middle 50% of the samples). Also, the maximum and minimum of the samples will be shown by the use of whiskers. If the samples have any outliers, these will also be present in the boxplot. If you want to compare samples taken at different times, boxplots next to each other can graphically reveal possible differences.



*Figure B.2: Example of a box plot. We can observe two outliers, a maximum and minimum, median and the interquartile range.*

**Scatter plot.** A scatter plot is usually displaying two variables in a dataset against each other, where each point could be a Cartesian coordinate. Each point is usually represented by a dot and could give you indications about how your variables evolve over time. Are the variables increasing or decreasing as time pass? Do they have a linear relationship? These are just a few examples of what information a scatter plot provide.



*Figure B.3: Example of a scatter plot where each dot could represent the number of hinds and the number of stags in the same year.*

**Histogram.** A histogram is a way of representing data by the use of bars to show

the frequency of the data. This could appear to be same as a bar plot, but a histogram is a distribution of quantitative data and the width of each bar do not need to be equal. Since the width can differ, we call this the range of values.

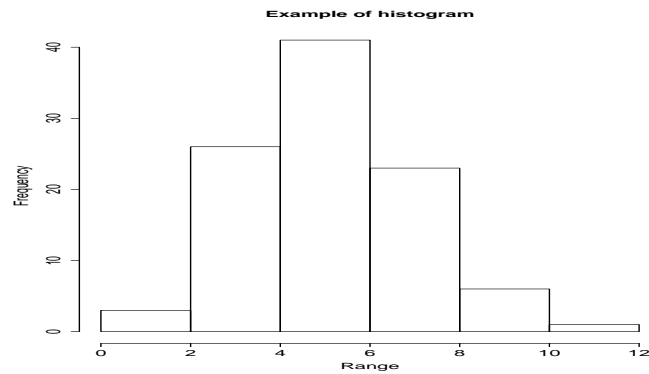


Figure B.4: Example of a histogram. The different frequencies describe the number of times the values in the different ranges occur.

**Bar plot.** A bar plot is similar to the histogram, but a bar plot presents categorical data. Each bar represent one category and the width of each bar should be the same, as a bar plot is not used when describing a range of values. The bars can be used to compare data from different categories.

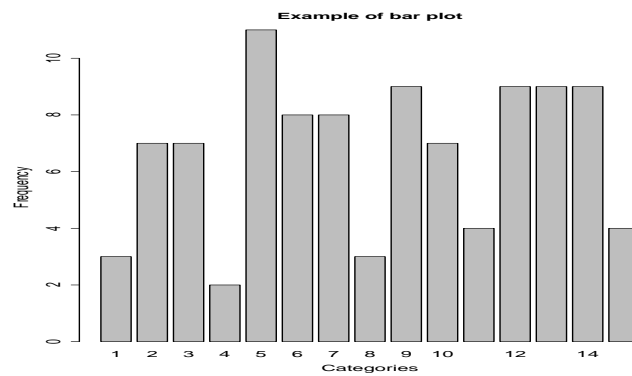


Figure B.5: Example of a bar plot. Each bar represent the frequency of each category.