Disagreement about logic^{*}

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July 12, 2019

Abstract

What do we disagree about when we disagree about logic? On the face of it, classical and nonclassical logicians disagree about the laws of logic and the nature of logical properties. Yet, sometimes the parties are accused of talking past each other. The worry is that if the parties to the dispute do not mean the same thing with 'if', 'or', and 'not', they fail to have genuine disagreement about the laws in question. After the work of Quine, this objection against genuine disagreement about logic has been called the meaning-variance thesis. We argue that the meaning-variance thesis can be endorsed without blocking genuine disagreement. In fact, even the type of revisionism and nonapriorism championed by Quine turns out to be compatible with meaning-variance.

1 The meaning-variance thesis

Suppose that two logicians, Astrid and Beatrice, are arguing about a logical law. Astrid says that the law is valid, Beatrice that it is invalid. Both of them have reasons for their beliefs. They believe what they believe because they subscribe to different theories about logical properties such as validity, truth, and provability. At least on the face of it Astrid and Beatrice are capable of having a rational debate about their beliefs. They offer arguments, objections, and replies. Although they ultimately stick to their respective positions, they feel the pull of each other's arguments. Every now and then Astrid relies on the very law in dispute when arguing for her position. Beatrice rejects these arguments. Nonetheless, they don't give up on finding ways of convincing the other person that they in fact have the superior position. Despite the apparent stalemate, Astrid and Beatrice both think the debate is worthwhile.

The situation, I believe, is familiar in the philosophy of logic. Classical and nonclassical logicians disagree about logical laws, and they offer reasons for their

^{*}Acknowledgements: This research is funded by the Research Council of Norway (RCN), grant 251218, and Deutsche Forschungsgemeinschaft (DFG) grant GZ HJ 5/1-1, AOBJ 617612. Previous versions of the paper were presented at the 'Engineering Logical Concepts' workshop at the University of Oslo, at the 2nd Buenos Aires-MCMP workshop, LMU Munich, and at the 'Disagreement Within Philosophy' conference at the University of Bonn. I am very grateful to the audiences for many helpful comments.

positions. Sometimes the arguments rely on contentious principles, but not always. The participants, myself included, believe that the debate is important and productive. Nonetheless there is a well-known claim to the effect that there is no genuine disagreement in cases such as that of Astrid and Beatrice. Instead they are involved in a mere verbal dispute. Classical and nonclassical logicians are not engaged in a substantive debate about the nature of logical laws, but are simply attaching different meanings to the same expressions. Once the parties are clear on what they mean by locutions such as 'and', 'not', 'valid', 'proof', the conversation can proceed with the dispute resolved.

This claim has been called the *meaning-variance thesis*, and it is attributed to Quine (1960a; 1986).¹ In a pivotal section, entitled 'Change of logic, change of subject', Quine considers a 'popular extravaganza', a discussion between a logician who rejects the law of non-contradiction and a counterpart who warns against the consequences:

My view of the dialogue is that neither party knows what he is talking about. They think that they are talking about negation, ' \sim ', 'not'; but surely the notion ceased to be recognisable as negation when they took to regarding some conjunctions of the form ' $p.\sim p$ ' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject. (Quine 1986, 81)

Elsewhere Quine puts the point in a different way:

[T]he departure from the law of the excluded middle would count as evidence of revised usage of 'or' and 'not'. ... For the deviating logician the words 'or' and 'not' are unfamiliar, or defamiliarized. (Quine 1960a, 362)

So when the nonclassical logician rejects a central classical law, say, for negation, the use of the expression has been revised. '[A]lternative logics are inseparable practically from mere change in usage of logical words.' (ibid., 355)

In one sense it is trivial that rejecting a classical logical law will lead the nonclassical logician to revise the use of an expression. If one rejects the law of excluded middle, then the use of disjunction should reflect that. But Quine's claim appears to be stronger. He says that neither party knows what they are talking about. The nonclassical logician 'only changes the subject'. Nonclassical logic cannot be separated from 'mere change in usage'.

Quine (1986, 81) gives an example to motivate the thesis. He asks us to consider a deviant logician who has simply swapped around the logical laws of conjunction and disjunction. In that case, he suggests, it would be 'nonsense' to think that the deviant logician really disagreed with us 'on points of logical doctrine'. He continues:

 $^{^{1}}$ As far as I know the name is due to Haack (1978, 224).

There is no residual essence of conjunction and alternation in addition to the sounds and notations and the laws in conformity with which a man uses those sounds and notations. (ibid.)

Although Quine doesn't talk about change of meaning explicitly, others have interpreted him as saying that a change of logical laws involves a change of meaning. The thinking is that the laws that govern a logical expression—or at least some of them—also fix the meaning of the expression. Thus, if a nonclassical logician rejects modus ponens for the conditional, the resulting nonclassical conditional differs in meaning from the conditional of classical logic.

Haack (1974, 8) explicitly attributes an argument of this form to Quine. Before quoting Quine as an example, she points out that some authors have claimed that classical and nonclassical logics are not genuine rivals, 'because their apparent incompatibility with classical logic is explicable as resulting from *change of meaning* of the logical constants'. Dummett (1978a) also construes Quine's argument as an argument about meaning:

[Quine] begins by remarking that it is impossible for anyone to deny a law of classical logic: for, if he fails to accept some formula which a classical logician would take to be a formulation of such a law, this failure would establish a conclusive ground for saying that he was not attaching to the logical constants appearing in the formula the same meanings as those attached by the classical logician, and hence he had not denied anything held by the classical logician, but merely changed the subject. (Dummett 1978a, 270)

The worry is that there is no proposition that the classical and nonclassical logician disagree about. Despite the apparent disagreement about, say, whether the law of excluded middle is valid, the parties are expressing their opinion about different propositions. The classical logician attributes classical validity to a proposition formed with their negation and disjunction, while the nonclassical logician attributes nonclassical invalidity to a proposition formed with their negation and disjunction formed with their negation and disjunction. The result, Haack warns, is that 'there is no real conflict between [nonclassical] and classical logic' (Haack 1974, 8).

2 Quine's revisionism and gradualism

Both Haack and Dummett argue that Quine's meaning-variance thesis is at odds with the view about logic he championed in his influential 'Two Dogmas of Empiricism' (Quine 1951). Here Quine claims that no theoretical statement is immune to revision, not even the laws of logic. This is Quine's *revisionism*:

Conversely, by the same token, no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle? (Quine 1951, 40)

Quine's question is meant to suggest that there is only a difference in degree between an allegedly *a priori* theory such as logic and the theories of the natural sciences. This is Quine's *gradualism*:

The kinship I speak for is rather a kinship with the most general and systematic aspects of natural science, farthest from observation. Mathematics and logic are supported by observation only in the indirect way that those aspects of natural science are supported by observation; namely, as participating in an organized whole which, way up at its empirical edges, squares with observation. I am concerned to urge the empirical character of logic and mathematics no more than the unempirical character of theoretical physics; it is rather their kinship that I am urging, and a doctrine of gradualism. (Quine 1986, 100)

This kinship between logic and the empirical sciences have been echoed by a number of more recent authors, for example Maddy (2002; 2014), Priest (2006; 2014; 2016), Russell (2014; 2015; (year?)), Williamson (2017), and Hjortland (2017a; 2017b). Although they differ on the details, all these authors reject the privileged epistemological status of logic, be it foundationalism, apriorism, obviousness or self-evidence. Instead they subscribe to a view where logical theories are supported by arguments similar to those used for theories in other sciences. That doesn't mean that logic is directly empirical in the sense that its laws are justified by induction, but it does mean that logical theories answer to a posteriori evidence through their connections with empirical theories.

According to revisionism, there can be genuine disagreement about logical theories, and furthermore, there can be arguments that support rational theorychoice. So if Quine favours a form of anti-exceptionalism, how can that be squared with his meaning-variance thesis? Dummett's answer is that Quine has simply abandoned his position from 'Two Dogmas' in his later writings (e.g. Quine 1960b; 1986). 'In the meantime,' Dummett writes, 'Quine himself has totally reversed his position, as may be seen from the chapter on 'Variant Logics' in *Philosophy of Logic*' (Dummett 1978b, 270). Haack also concludes that Quine has abandoned his earlier position:

The attack in 'Two Dogmas' on synonymy etc. would threaten an account of logical truths as analytic because true in virtue of the meaning of the logical constants. Now in Word and Object Quine renews his sceptical attack on meaning notions, but makes an exception in the case of the logical connectives, which, he claims, do have determinate meaning ...; and this paves the way for his acceptance ... of a meaning-variance argument to the effect that the theorems of deviant and classical logics are, alike, true in virtue of the meaning of the (deviant or classical) connectives. (Haack 1978, 237)

If Haack and Dummett's interpretation is correct, Quine simply abandoned a key component of his revisionism when he gave his meaning-variance thesis.² If Dummett is right, and it is '*impossible* for anyone to deny a law of classical logic' (Dummett 1978b, 270, my emphasis), then how can one simultaneously insist that logic is not only revisable, but rationally revisable?

Hence, revisionists have a reason to reject the meaning-variance argument. Indeed, many commentators have found fault with Quine's argument for the thesis, regardless of whether or not they agree on Quine's particular gradualist picture.³ In the next section, we will look at some prominent objections to the meaning-variance thesis.

3 Against meaning-variance

Recall Astrid and Beatrice, the two logicians who are in a dispute about a logical law. Suppose that their dispute is about modus ponens (in the form $A, A \rightarrow B \models B$). Astrid claims that it is valid, Beatrice disputes it. Suppose further that modus ponens is a meaning-determining principle for Astrid's conditional. Since modus ponens is meaning-determining, it is valid simply in virtue of the meaning of Astrid's conditional. The worry is that Beatrice, in disputing modus ponens, doesn't succeed in rejecting the law that Astrid supports. For that law is expressed with Astrid's conditional, and if Beatrice disputes it, she is simply rejecting some other law, expressed with her own Beatrice-conditional (for which modus ponens doesn't hold). There is, therefore, no single proposition—the logical law—that Astrid accepts and Beatrice rejects.

If it is indeed true that Beatrice does not succeed in disputing the law that Beatrice accepts, and, moreover, genuine disagreement is propositional disagreement, then there is no genuine disagreement between Astrid and Beatrice. The same line of reasoning will carry over to disputes about any logical law that is meaning-determining. This is what leads Haack (1974, 8) to conclude that there is 'no real conflict' between the parties of the dispute. She outlines the argument as follows:

- (a) if there is change of meaning of the logical constants, there is no real conflict between Deviant and classical logic,
- (b) if there is Deviance, there is change of meaning of the logical constants,

 \mathbf{SO}

(c) there is no real conflict between Deviant and classical logic.(ibid.)

²Other authors have also attributed two mutually exclusive positions to Quine. Williamson (2007, 97) says that 'Quine's epistemological holism in "Two Dogmas" undermines his notorious later claim about the deviant logician's predicament'. Arnold & Shapiro (2007) investigate the conflict between what they call the 'logic-friendly Quine' and the 'radical Quine'.

³Of course, Dummett himself is a revisionist about logic. More about that below.

However, it is unclear what Haack means by 'no real conflict'. So in what follows, we will work with a reformulation of the argument better suited to our purposes:

- P1 If two parties, A and B, are in a dispute over a basic logical law S involving logical expressions $\circ_1, ..., \circ_n$, then their dispute arises wholly in virtue of A and B's disagreement about the meaning of $\circ_1, ..., \circ_n$.
- P2 If A and B are in a dispute over S, and the dispute arises wholly in virtue of their disagreement about the meaning of an expression \circ in S, then their dispute is verbal.
- C If two parties, A and B, are in a dispute over a basic logical law S involving logical expressions $\circ_1, ..., \circ_n$, then their dispute is verbal.

The first premise (P1) corresponds very roughly to Haack's premise (b), but the formulation is different in a couple of ways. First, P1 talks only about disputes over *basic logical laws*. The point is that two logicians could be in a dispute about whether or not something is a logical law (a theorem), agree on all axioms and primitive proof rules, but nonetheless reach different conclusions because one of them made an error. In such cases, one would hope that the error could be spotted, leading to genuine agreement. Those are clearly not the sort of cases Quine's meaning-variance thesis is supposed to apply to. Thus, P1 limits the disputes to those about a subclass of logical laws.

Furthermore, note that basic logical laws can refer to logical truths (e.g. the law of non-contradiction $\models \neg(\varphi \land \neg\varphi)$) and to valid arguments (e.g. modus ponens in the form $\varphi, \varphi \rightarrow \psi \models \psi$), but also to valid meta-arguments, that is, arguments from a set of arguments to an argument (e.g. conditional proof, classical *reductio*).⁴ This is important because many of the actual disputes between classical and nonclassical logics are about meta-arguments. In fact, certain logics agree on the validity of all arguments, but disagree about meta-arguments.

Second, in Haack's (b) we only learn that logical 'deviance' leads to 'change in meaning'. This is misleading. As Priest (2006) points out, even if we accept the difference in meaning, it doesn't follow that the classical logician keeps the meaning fixed, whereas the nonclassical (i.e. deviant) logician 'changes' the meaning. The classical logician certainly is not entitled to claim that their logical concepts are the standard logical concepts, for example in the sense that natural language expressions pick out the classical concepts. That is precisely one of the claims that has been disputed by nonclassical logicians. Rather, the two parties of the dispute enter the dispute with different meanings attached to the same logical expression.

$$\begin{array}{ccc} [\varphi]^u & [\neg\varphi]^u \\ \vdots \\ \frac{\psi}{\varphi \to \psi} & (u) & \frac{1}{\varphi} & (u) \end{array}$$

⁴Conditional proof and classical *reductio* respectively:

Third, P1 says that the difference in meaning is due to the expressions involved in the logical law. For example, if the law of excluded middle $(\varphi \lor \neg \varphi)$ is basic, a dispute could arise in virtue of a disagreement about \lor or \neg . However, the dispute could also arise from a disagreement about the meaning of meta-theoretic expressions. The law that the dispute is about is better expressed as:

(1)
$$\varphi \vee \neg \varphi$$
 is valid.

In that case, the two parties to the dispute could be assigning the same meaning to \lor and \neg , but different meanings to the logical expression 'valid'.⁵ After all, classical and nonclassical logics mean different things by these expressions, just like they do with the run of the mill connectives. They might agree on theory-relative terms, such as 'classically valid' or 'intuitionistically provable', but they do not seem to agree on the meaning of 'valid' and 'provable' simpliciter.⁶ The result is that Quine's meaning-variance thesis arguably applies not only to logical connectives and quantifiers, but also to meta-theoretic logical expressions (e.g. 'valid', 'consistent', 'provable'). It is convenient, therefore, to talk more broadly about logical laws as including such meta-theoretical claims, even if that is unorthodox terminology.

Just as P1 corresponds to Haack's premise (b), P2 roughly corresponds to Haack's (a). However, P2 avoids saying that there is 'no real conflict' between classical and nonclassical logics, and instead claims that disputes about basic logical laws are *verbal disputes*. In fact, P2 is a slightly modified version of an account of verbal disputes due to Chalmers (2011). The account does not say anything about logical disputes in particular, but gives a sufficient condition for a dispute in general to be verbal. For the purposes of the meaning-variance thesis, we will simply assume that Chalmers' account is correct. What counts as a verbal dispute is certainly an interesting discussion, but it is beyond the scope of this paper.

For our purposes it is therefore the first premise that will receive most of the attention. If P1 turns out to be false, then at least this type of argument for the meaning-variance thesis is blocked. This is indeed the strategy that a number of authors have used in their objection to Quine's claim. What is required, then, is a counterexample to the the conditional of P1: a case where two parties A and B are in dispute over a basic logical law S, but where their dispute doesn't arise wholly in virtue of A and B's disagreement about the meaning of the logical expressions involved.

Suppose that modus ponens is a basic logical laws for the conditional, and that A and B are in dispute over modus ponens. First, note that if B rejects modus ponens, it is compatible with P1 that B believes that the dispute does not arise wholly in virtue of a disagreement about meaning. She might insist

 $^{{}^{5}}A$ simple example is a pair of many-valued systems with the same truth-functions attached to the logical connectives, but where validity is defined in terms of different preservation properties. See Hjortland (2012) for a more detailed example.

 $^{^{6}}$ See Field (2015) for a discussion about this distinction.

that the meaning she assigns to the conditional is the same as A, despite the fact that she rejects modus ponens. All that P1 requires is that the dispute is, in fact, a result of a disagreement about meaning, even if B fails to realize it. She might have been convinced, for example, by McGee's (1985) counterexample to modus ponens, but still insist that she shares the same meanings as A. Of course, it could turn out that what McGee's counterexample shows is that, contrary to previous claims, modus ponens isn't a basic logical law for the conditional; indeed, that it isn't a logical law at all! But that is besides the point here. An objection to P1 would have to satisfy the antecedent, namely that S is a basic logical law.

One way to achieve this is to divorce basic logical laws from meaningdetermining laws. The law in dispute is only said to be a basic law, so if modus ponens is a basic logical law for the conditional (in the sense that it is primitive), but without being meaning-determining, then rejecting it is compatible with full agreement on meaning. In other words, P1 can be resisted.

An early proponent of this objection is Putnam (1976). In keeping with Quine's revisionism, Putnam suggests that quantum mechanics gives us reason to revise classical logic in favour quantum logic.⁷ In the latter system, the law of distributivity (i.e. $(\varphi \land (\psi \lor \chi)) \rightarrow ((\varphi \land \psi) \lor (\varphi \land \chi)))$ is rejected, and so the question of whether we are changing the meaning of disjunction arises. However, Putnam believes that the revision is compatible with sameness of meaning. He observes that even if there is subtle change of the primitive proof rules for disjunction, there is a substantial number of shared theorems between classical and quantum logic. For Putnam, the problem is that 'we do not posses a notion of 'change in the meaning' refined enough to handle this issue' (ibid., 190):

Only if it can be made out that [distributivity] is 'part of the meaning' of 'or' and/or 'and' (which? and how does one decide?) can it be maintained that quantum mechanics involves a 'change in the meaning' of one or both of these connectives. (ibid.)

In a similar vein, Field (2008, 17) also rejects the meaning-variance thesis because of doubts about the very distinction between meaning-determining and non-meaning-determining laws:

The question [of meaning change] is clear only to the extent that we know how to divide up such firmly held principles into those that are "meaning constitutive" or "analytic" and those which aren't, and this is notoriously difficult.

The worry is echoed by Shapiro (2014, 91):

My tentative conclusion ... is that the talk of whether the meaning of the logical terminology is the same or different in different

⁷Putnam later changes his mind about quantum logic (cf. Putnam 1981; 2012).

contexts/theories is itself context-sensitive and, moreover, interestrelative—in part because such talk is vague. So I am skeptical of the very question of whether the classicist and intuitionist, for example, talk past each other—if we insist that this is a sharp and objective matter, to be determined once and for all.

A related criticism of meaning-variance is that although there are meaningdetermining basic logical laws, not all basic laws are meaning-determining. Elsewhere I have called this view *minimalism* about the meaning of logical connectives (Hjortland 2012; 2014). Roughly put, the minimalist acknowledges that a proper subset of the logical laws are meaning-determining, but allows for non-verbal disputes about the remaining laws. The minimalist view has an early formulation in the work of Putnam (1957) and Haack (1974), but has been given a more precise formulation by Paoli (2003; 2007; 2014) and Restall (2002). Here is Restall (2002, 11):

If any set of rules is sufficient to pick out a single meaning for the connective, take that set of rules and accept those as meaning determining. The other rules are important when it comes to giving an account of a kind of logical consequence, but they are not used to determine meaning.

Again, such a division of labour between logical laws will allow for disputes that are non-verbal, though only if the disputes are about non-meaning-determining laws. The difficulty is to identify a non-arbitrary divide between the basic laws that contribute to the meaning and the ones that do not.

Paoli has developed an answer to this question by applying a useful distinction from structural proof theory. In sequent calculus rules there are two types of proof rules: *operational rules*, that is, left and right proof rules that govern specific logical expressions, and *structural rules*, that is, proof rules that govern the deducibility relation independent of the type of formula in question (e.g. weakening, contraction).⁸ Paoli's proposal is that whereas operational rules are meaning-determining, structural rules are not. With the distinction in place, the meaning theory allows for non-verbal disputes between logical theories that differ only with respect to structural properties. That includes linear logics, affine logics, and a number of relevant logics.

An immediate worry is that some standard disputes, such as that between classical and intuitionistic logic, won't count as genuine disagreements on this account. But, since classical and intuitionistic logic can be formalized in sequent calculus in such a way that the only difference is the cardinality of the succedent, it has been suggested that even this dispute would count as a genuine disagreement under the proposal.⁹ However, such a fix raises a more general concern. If we anchor the meaning-determining versus non-meaning-determining distinction to the operational versus structural distinction, we need the latter distinction to

 $^{^8 \}mathrm{See}$ Negri & von Plato (2001) for an introduction.

⁹The point is anticipated by Haack (1974).

be sufficiently stable. The problem is that what counts as structural is highly system-dependent. Different sequent calculi introduce structural operators in the proof theory in order to add structural rules. In standard sequent calculus we have right- and left-side commas, while generalizations like hypersequents or labelled sequent calculus allow for new structural rules. Bunch theory with intensional and extensional premise-combinators and bilateral systems further load the structural language with operators. Such systems simply shift the various logical properties of connectives over to the structural language.

In addition, even if the structural rules can be claimed to be independent of the meaning of the specific connectives, there is still the question of whether or not a change in structural rules brings about a shift in the meaning in metatheoretic expressions such as 'valid' or 'provable'. If that is the case, and the disputes are about the validity of, say, the law of excluded middle, we are no closer to an account of nonverbal disputes.¹⁰

Let us return to the argument for the meaning-variance thesis. Are there other strategies for resisting the premise P1? Another influential criticism of the idea that there are meaning-constitutive principles is due to Williamson (2007). He formulates a number of objections against so-called understanding-assent links (UA links). He denies that there are logical laws that a speaker must assent to in order to qualify as understanding a connective. More generally, let S be a sentence that expresses such a principle. Then consider the following UA link:

(2) Necessarily, everyone who understands the sentence S assents to it.

Contraposed, everyone who dissents to S fails to understand it. Williamson objects to (2) and a number of related UA links. To see one reason why consider the following example. Suppose that modus ponens is the principle in question. Recall that McGee (1985) thinks there are counterexamples to modus ponens, and therefore rejects the validity of the law. By the UA link this entails that McGee does not understand the English language expression "if". Williamson thinks this conclusion is simply false:

In conversation, [McGee] appears to understand it perfectly well. By ordinary standards, he *does* understand it. Before he had theoretical doubts about modus ponens, he understood it. Surely his theoretical doubts did not make him cease to remember what it means. Moreover, his doubts derive from taking at face value a natural pattern of native speaker reactions to an ingeniously chosen case. If he counts as not understanding "if," so do millions of other native speakers of English. (ibid., 94)

If it turns out that the premise P1 entails the UA link, Williamson's argument would offer a strategy for blocking the argument for the meaning-variance thesis. If we have reason to reject the UA link, we would also have reason to reject P1.

¹⁰ See Hjortland (2014) for a more detailed criticism of minimalism, and Paoli (2014) for some promising replies on behalf of the minimalist.

However, in the current formulation P1 does not entail the UA link. In fact, while it does require that one party dissents to a logical principle, it does not require that the party therefore fails to understand any of the involved logical expressions. It simply says that the dispute arises wholly in virtue of their disagreement about the meaning of the expressions.

Of course, one could object that what Quine *really* had in mind was something stronger than P1, and indeed a premise that entails the UA link. That is not entirely uncalled for given Quine's verdict that 'neither party knows what he is talking about' in disputes about logical principles. At other points, however, Quine makes comments that point to the more modest formulation. It is a formulation, I argue, that allows us accept the meaning-variance thesis without jettisoning genuine disagreement and the revisionism he endorses in his earlier work.

4 Revisionism and meaning-variance reconciled

Not everyone has concluded that there is a dissonance between Quine's revisionism and the meaning-variance thesis. Although Priest (2006) disagrees with Quine's classicism, he suggests that there is more of a continuity between Quine's views than he is given credit for:

It would be wrong, however, to suppose that Quine takes the meaning variance of connectives across different logics to be an argument against his earlier view concerning the revisability of logic. (Priest 2006, 168)

Priest points out that rather than rejecting his old position, Quine (1986) seems to open for the possibility that meaning-variance is not at odds with rational revision of logical laws. Only a few pages after his infamous 'change of logic, change of subject' quip, Quine is again discussing revision of a classical law, but now in a more concessive manner:

[W]hoever denies the law of excluded middle changes the subject. This is not to say that he is wrong in so doing. In repudiating 'p or $\sim p$ ' he is indeed giving up classical negation, or perhaps alternation, or both; and he may have his reasons. (Quine 1986, 83, my emphasis)

The contrast with the more polemical comments from earlier in the same chapter is striking. Rather than accusing the parties of speaking past each other, he now suggests that rejecting classical laws could have a rationale, even if it involves giving up classical logical concepts.

But if revisionism is possible despite meaning-variance, what is the nature of the revision and what is the nature of the underlying disagreement? One possibility is that the disagreement is simply *about* meaning. By raising the disagreement to a metalinguistic level, we can own the meaning-variance thesis and still save genuine disagreement. Granted, in that case the impression that the disagreement was *about* the law of excluded middle is misleading.

That logical disputes are sometimes about meaning should be no surprise. Some standard cases of disputes in the philosophy of logic are overtly about meaning. Some discussions about logical laws are about how natural language expressions, for instance 'not', 'if', 'all', are actually used. Semantic projects in the spirit of Montague (1973) and Davidson (1967) are typically attempts at using formal systems as tools in natural language semantics.¹¹ Similarly, critics of classical logic have pointed out that the logical laws governing, say, material implication or the boolean negation simply do not accurately reflect how we use their natural language counterparts. When relevantists refer to the postive and negative paradoxes, and when McGee gives a natural language counterexample to modus ponens, they are relying on natural language data. So, if the project is to give a formal semantics for certain natural language expressions (i.e. the logical expressions), the dispute is clearly about the meaning of these expressions.

But although formal logics are routinely used for this purpose, not all logical disputes are about the semantics of natural language. After all, classical and many nonclassical logics are not really concerned with the complexities of natural language usage. Instead, some logicians are more concerned with the logic of the concepts we *ought* to use than the logic of the concepts we actually use. The approach varies from those who merely suggest that scientific purposes call for a light modification of natural language (e.g. Quine's regimentation) to those who insist that a formal language should at least partly replace natural language in order to achieve sufficient precision (e.g. Frege's *Begriffschrift*). Shapiro (2006, 48) calls this the *normative orientation* towards logic.

A well-known example of the normative orientation is the type of intuitionist revisionism promoted by Dummett (1991). According to Dummett, the classical negation concept semantically misfires, and should therefore be replaced by intuitionistic negation. The argument is explicitly about meaning. In fact, Dummett's contention is precisely that a revisionist argument for logic must proceed via meaning-theoretic considerations. He identifies conditions that the inference rules for logical expressions must satisify in order to count as meaningful (e.g. conservativeness, harmony), and argues that classical negation fails the test. Both the conditions that Dummett introduce and the claim that they aren't satisfied by classical concepts have been contended in the literature, but the details are not important here.¹² The point is that Dummett's revisionism is not only compatible with the meaning-variance thesis, but premised on the very idea that revision of logic springs out of a disagreement about which concepts are legitimate. The disagreement is metalinguistic, but definitely not 'merely verbal'. Compare it, for example, with Quine's case of merely verbal dispute where someone has simply swapped the terms ' \lor ' and \land ' in the logical laws. The latter dispute results from a notational variant, while Dummett's revisions are motivated by meaning-theoretic arguments. Even if we ultimately reject his arguments for intuitionism, the revisionary strategy cannot be written off as a linguistic confusion.

 $^{^{11}}$ Arguably the logic-as-modelling approach in Cook (2002) and Shapiro (1998; 2006) is a variant of the semantic approach, but one on which the logical systems allow for more idealization.

¹²See for example Read (2000).

5 Revision of concepts

Dummett's revisionism is actually an example of a philosophical methodology that has received more attention recently. Instead of merely offering conceptual analysis of crucial concepts such as personal identity, justice, truth, and knowledge, some philosophers engage in conceptual *engineering*.¹³ They do not merely seek to elucidate the concepts we actually possess, but to investigate which concepts we *ought* to use for certain purposes. When concepts do their job well, they help us represent the world and express our thoughts. But concepts can also be unhelpful. They can be too imprecise or nonexpressive. By some accounts they can also be inconsistent or unethical. Since concepts can be deficient in a variety of ways, the argument goes, we are sometimes better off with new concepts. Ultimately, those who promote conceptual engineering think that we sometimes ought to introduce replacement concepts.

Here are some examples. The folk concept mass is muddled. It runs together two theoretical concepts, weight (measured in newtons) and mass (measured in kilograms). The folk concept fish is unsuited for the theoretical purposes of biology, where it has been replaced by the typological concept pisces and then by concepts suited for phylogenetic classification. When greater precision is called for, the everyday concepts of likelihood and chance are dispensed with in favour of the mathematically precise concept of probability.

These are cases where deficient concepts have supposedly been replaced by more sophisticated, theoretical alternatives. In other cases conceptual engineering doesn't involve conceptual re-engineering of already existing concepts.¹⁴ Instead concepts that bear little similarity with already applied concepts are introduced to fulfil some purpose. Concepts such as *quark* in physics or *DNA* in biology do not serve as improvements of deficient concepts, but allow for the formulation of improved theories.

What about concepts in logical theories? When Dummett (1973, 454) talks about deficient concepts, he mentions the expression 'Boche', a slur used by the French against Germans. The example is supposed to anticipate his analysis of allegedly deficient logical concept, first the trivializing connective 'tonk' (Prior 1960), and ultimately the problems with classical negation. For Dummett, 'rejection or revision of concepts' is called for when the the inference rules associated with the concept fail to 'harmonize'. Although his diagnosis of the defects of logical concepts is idiosyncratic, it is natural to think of this and related projects in the philosophy of logic as cases of conceptual engineering.¹⁵

Later philosophers of logic have been even more explicitly engaged in what they consider conceptual engineering. The most widely discussed case is likely

¹³See Scharp (2013), Burgess & Plunkett (2013), Cappelen (2018), and Eklund (2017a) for some recent approaches to conceptual engineering as a methodology. Tanswell (2018) has also explored how conceptual engineering can be applied in set theory. The label 'conceptual engineering' is due to Blackburn (1999).

 $^{^{14}\}mathrm{According}$ to Brun (2016), conceptual re-engineering is closely related to Carnap's notion of explication. (See also Dutilh Novaes & Reck 2017.)

 $^{^{15}}$ Tennant (1987; 1997) and Brandom (1994) are two philosophers who have followed Dummett in relevant respects.

the concept of truth. A number of authors have defended *inconsistency theories*, according to which the everyday concept of truth is inconsistent (e.g. Azzouni 2007, Burgess & Burgess 2011, Eklund 2002; 2007, Scharp 2007; 2013, Patterson 2007; 2009, Scharp & Shapiro 2017). Although the authors disagree about whether or not the alleged inconsistency is a reason to replace the everyday concept of truth, at least some inconsistency theorists have developed and defended replacement concepts. These concepts are designed to preserve consistency in classical logic, while simultaneously staying faithful to a number of the expressive roles traditionally assigned to the truth predicate.¹⁶

In order to be able to count concepts, and not only theories as inconsistent, inconsistency theories agree that concepts are governed by constitutive principles. A familiar example from logic is that standard proof rules are constitutive principles of logical concepts: conditional proof and modus ponens are constitutive of the conditional; disjunction introduction and proof by cases are constitutive of disjunction, etc. According to the inconsistency theorists, what sets the everyday concept of truth apart is that its constitutive principles are inconsistent. Hence, analogously with Dummett's complaints about tonk and classical negation, inconsistency theories identify problems in the very concept of truth. The core idea is that the concept is governed by the naive T-schema or some unrestricted inference rules, such as T-in and T-out.¹⁷ From that assumption, they argue that since the semantic paradoxes lead to contradiction, the involved concept of truth is inconsistent.

Of course, other accounts of the paradoxes can also explain the contradictions, by attributing the problem to a bad *theory* of truth as opposed to a bad concept of truth. It can still be acknowledged that the unrestricted truth principles must be rejected, but it is denied that they have a privileged connection to the very concept of truth. Eklund (2017b) also points out that the semantic paradoxes do not unequivocally point to principles of truth as the culprit, even if these are indeed constitutive. After all, some logical principles, for example of negation or implication, are also involved. Indeed, the naive truth principles on their own do not lead to inconsistency. Classical logic or some suitably strong nonclassical logic is also required, together with a theory of syntax (e.g. arithmetic). So, even if we accept that the unrestricted inference rules are constitutive, the paradoxes need not be a reason for revising the concept of truth. It might equally well suggest a revision of logical concepts.

That brings us to the more general point. Inconsistency theories are just a special case of theories that promote conceptual engineering in logic broadly speaking. The methodology can be extended in two ways, both of which are arguably already implicit in Dummett's work. First, inconsistency need not be the only relevant conceptual flaw that motivates a revision. For Dummett,

$$\frac{\varphi}{T({}^{r}\varphi^{})} \qquad \frac{T({}^{r}\varphi^{})}{\varphi}$$

¹⁶The most detailed suggestion for a concrete replacement can be found in Scharp (2013).

¹⁷The *T*-schema is the biconditional $T({}^{r}\varphi{}^{r}) \leftrightarrow \varphi$, where ${}^{r}\varphi{}^{r}$ is the name of the sentence φ . The corresponding proof rules in natural deduction are sometimes called *T*-in and *T*-out:

inconsistency is only a limit case. Less damaging features of the constitutive principles (e.g. nonconservativeness) might also suggest that a logical concept is flawed. Second, conceptual engineering could be applied to logical concepts that are more central to logical theorizing than truth.

Consider for example the material implication in classical logic. Although it resembles the English language indicative conditional in a number of respects, it is certainly a minority view that the material implication is an accurate semantic theory of its natural language counterpart. We routinely pause in introductory logic classes to warn our students about straightforwardly translating from natural language 'if' to the the material implication in the formal language. The reason, of course, is that the material implication is at best an idealization of any natural language concept. We trade in the nuances of the indicative conditional for a number of theoretical advantages. It has a simple semantics and it is interdefinable with other connectives. But most of all the material implication contributes to a highly successful formal theory of mathematical proofs.

In fact, the conceptual engineering methodology need not be limited to connectives and quantifiers. Even the concept of validity, arguably the most central logical concept, can be the target of engineering. In fact, just like inconsistency theorists have been inspired by Tarski's criticism of truth in natural language, we could take Tarski's dismissive remarks about the ordinary concept of consequence as an invitation to think of his model-theoretic definition as a conceptual re-engineering:

With respect to the clarity of its content the common concept of consequence is in no way superior to other concepts of everyday language. Its extension is not sharply bounded and its usage fluctuates. Any attempt to bring into harmony all possible vague, sometimes contradictory, tendencies which are connected with the use of this concept, is certainly doomed to failure. (Tarski, 1936, p. 409)

Tarski is not trying to account for any concept of consequence that we actually employ in natural languages. Instead he is defining a theoretical concept with the advantage of the precision offered by the model-theoretic framework. Its success is not only due to the fact that Tarski captures important intuitions about logical consequence, but also its utility for mathematical and philosophical purposes.

Eklund (2017c), who supports conceptual engineering in other cases, has suggested it has application in logic as well. According to him, we can best make sense of the philosophical interest in logical pluralism vs monism if we consider the issue as one about which language we ought to have for certain theoretical purposes:

Doubts about the significance of the actual language project point us toward the normative project. The normative project – what is the best type of language like? what is the best truth predicate like? the best negation? – promises to have the significance that the actual language project lacks. (Eklund 2017c, 5) What is the upshot of the conceptual engineering approach for the question of disagreement? Given conceptual engineering we can trade in the first-order dispute about logical laws for a metalinguistic (or meta-conceptual) disagreement about whether to adopt, say, intuitionistic negation instead of classical negation. The latter dispute is genuine disagreement, even if it counts as verbal dispute in the sense that it arises wholly in virtue of the parties' dispute about meanings. It is just that the disagreement is not about the actual meaning of a natural language expression, but about which concept we ought to use in our theorizing. On this picture, Astrid and Beatrice's dispute about the law of excluded middle cannot be dismissed as insubstantial, but it nonetheless allows for meaning-variance.

6 Theory-choice and concept-choice

The conceptual engineering methodology offers a way to accept the meaningvariance thesis without giving up on genuine disagreement about logic. Yet, we set out to do more than simply find an approach to logical theorizing that would vindicate meaning-variance. We also want an approach that combines the meaning-variance thesis with Quine's revisionism and gradualism. Suppose that Dummett's intuitionism is indeed a case of conceptual engineering. Although Dummett and the Quine of 'Two Dogmas' share the view that logic can be revised, their methodologies remain far apart. Dummett's brand of revisionism is a revision of concepts, justified by meaning-theoretic arguments. The resulting justification of basic logical laws is an *a priori* justification. Since Quine rejects apriorism, his revisionism, in contrast, is designed precisely to explain how logical theories can be justified on a nonapriori basis.

The worry, then, is that conceptual engineering is in conflict with Quinean revisionism about logic. However, even if Dummett's revisionism cannot be reconciled with nonapriorism, it does not mean that revision of concepts has no place in a Quinean epistemology of logic. For as we have seen above (in section 4), Quine thinks that you can have perfectly good reasons for rejecting a logical concept (e.g. classical negation). Those reasons would not be a meaning-theoretic argument like Dummett's, but rather a matter of theory-choice. What is more, Quine's gradualism leads him to conclude that the mechanism of theory-choice in logic ought to be similar to that of the sciences.

Even though Quine (1986) appears dismissive of nonclassical logic, the very chapter where he accuses the deviant logician of 'changing the subject' is followed by a detailed discussion of the advantages and disadvantages of concrete nonclassical theories (ibid., 83–9). His emphasis is on how these theories and classical logic compare wholesale with respect to a number of theoretical virtues (e.g. simplicity, the maxim of minimal mutilation, unification with other theories, expressive power, deductive strength). He readily admits that classical logic has issues both with semantic paradoxes and vagueness, but maintains that the cost of rejecting classical laws in favour of a nonclassical alternative is too high. In Quine (1981) he goes further in listing the drawbacks of a classical theory of vagueness, but concludes that its simplicity still outweighs the advantage a

nonclassical logic affords. In sum, he favours an abductive argument for classical logic.

For Quine, there is no conflict between abductive arguments for logic and the meaning-variance thesis. In revising our theories we not only change the laws, but we also introduce new concepts. These concepts should be better suited for the theoretical task, and they are only warranted when they make valuable—or even indispensable—contributions to the theory. Put differently, theory-choice and concept-choice happen simultaneously. In line with his confirmational holism, it is theories and not individual logical laws that receive evidential support. If we reject one theory of logic for another, we not only reject laws of logic, but we replace the theoretical concepts of one logic with another. That is compatible with there being no identifiable set of laws that together form the constitutive principles for a concept in the theory, and thus no identifiable truths of the theory that count as conceptual truths.

Moreover, this revision of concepts is not a feature unique to logical theories. Theory-choice and concept-choice are indistinguishable in general, be it in physics, economics, or logic. Let us return to conceptual engineering for a moment. Recall that many of the examples of allegedly successful applications of the methodology come from outside philosophy (cf. Brun 2016). Eklund (2014, 293) also underlines that there is nothing exceptional about philosophy in this respect:

[W]hile philosophers often have been concerned with our actual concepts or the properties or relations they stand for, philosophers should also be asking themselves whether these really are the best tools for understanding the relevant aspects of reality, and in many cases consider what preferable replacements might be. Philosophers should be engaged in conceptual engineering. Compare: when physicists study reality they do not hold on to the concepts of folk physics but use concepts better suited to their theoretical purposes. Why should things stand differently with what philosophers study?

Revision of concepts is not a specifically logical or philosophical methodology; it is ubiquitous throughout the sciences. Hence, if meaning-variance is a worry for genuine disagreement in logic, it ought to be one in physics as well. In fact, in her discussion of Quine's meaning-variance thesis in logic, Haack points out that a corresponding meaning-variance thesis has been promoted in the philosophy of science (Feyerabend 1962). Yet, most philosophers would not conclude from this that there is no genuine disagreement between rival theories in chemistry and economics. If the meaning-variance prevents genuine disagreement in logic, that would have to be because the rival parties cannot have rational dispute about concept-choice, for example because of incommensurability. However, the meaning-variance thesis in itself does not provide a reason for such a dramatic conclusion. It also seems unlikely that Quine accepted this conclusion, given his explicit assessment of the advantages and disadvantages of classical and nonclassical logics respectively. But, most importantly, the conclusion is belied by what happens in actual debates about logical theories. The classicist and the nonclassicist do not fail to understand each other's concepts. Williamson and Field understand paracomplete and classical concepts perfectly well, but assess the theories they are embedded in differently.

Conclusion

Quine's meaning-variance thesis should not be considered a threat against genuine logical disagreement. The thesis is compatible with revisionism, in fact a form of metalinguistic revisionism already part of many disputes about logic. There is no real conflict between Quine's revisionism and his meaning-variance thesis. Concept-choice is an integrated part of theory-choice, in logic and elsewhere in the sciences.

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