The Temple of Athena Alea at Tegea: Revisiting Design-Unit Derivation from Building Measurements

Jari Pakkanen

Deriving the length of a possibly used design-unit from architectural measurements is a complex statistical problem. The method used in the paper is based on cosine quantogram analysis, and the relevance of the obtained results is calculated by computer simulations: it can be used to criticise previous attempts of defining the foot-unit of the late classical temple of Athena Alea at Tegea and to show how a statistically valid result can be obtained. The Parthenon is used as an example to demonstrate that it is feasible to use small building detail dimensions as the analysed data set, even though this does not produce a significant result at Tegea. One alternative is to use full block dimensions, and the statistical analysis strongly supports that a design-unit of ca. 99 mm (corresponding to one third of a foot of 297-8 mm) was used at Tegea.

1. Introduction

The discussion of ancient Greek foot-units and architectural modules has been going on almost as long as scholarly work on the buildings has been conducted. One conclusion was apparently reached in 1961, when W.B. Dinsmoor argued that only two foot-units were generally used in Greek architecture: the 'Ionic foot' of ca. 294 mm and the 'Doric foot' of ca. 326 mm.[1] This is not, however, generally accepted by all scholars. The scepticism is perhaps best worded by J.J. Coulton: "As far as measurement is concerned, the assumption that only two foot-standards were used throughout the Greek world needs to be proved, not just accepted, and the chaotic situation in other branches of Greek metrology suggests that this is unfounded."[2] In this paper the only preliminary assumption on the lengths of possibly used Greek design-units is that they should fall within

1. Dinsmoor 1961.
2. Coulton 1974, 62.

the range 50 – 400 mm for full building blocks and 4 – 25 mm for detailed mouldings.[3] I have chosen to use the term "design-unit" in the title of this paper rather than foot-unit since methodologically it makes no difference whether the possibly used basic units in Greek architecture are related to a foot-unit or some other conceivable module, such as the column spacing or the triglyph width.

Traditional studies on Greek metrology make very little use of statistical methods, even though their advantages are quite easy to see. They make analysis of large sets of measurement data feasible and assessment of the probability of the reached conclusions possible. I think we can make an even stronger statement: deriving design-units from building dimensions is a statistical problem, and studies which do not employ proper methodology are in serious danger of reaching false conclusions. A statistical method called cosine quantogram analysis is used in this study: it can demonstrably be used to determine the size of unknown unit-lengths in measurement data.[4]

The most important single ancient source on Greek classical foot-standards is Herodotos: from this fifth-century historian we learn that different foot lengths were in use, and something about the relationships between different units.[5] The Greek foot was divided into four palms and a palm into four dactyls or finger-widths. Contrary to the well documented Roman foot,[6] the lengths of suggested Greek units are usually derived from analyses of building dimensions. Some indications on the lengths of the used standards may possibly be derived from two preserved metrological reliefs[7] and by combining the information of a length given in an ancient inscription with the actual measurement of the dimension.[8]

H. Bankel has proposed a system for defining the length of the Greek foot-standards which he calls the "metrological scale".[9] Interestingly, one of his case studies was based on the analysis of the late classical temple of Athena Alea at Tegea.[10] We shall have a closer look at this method and use cosine quantogram analysis to show why Bankel's analysis does not succeed in finding a possibly used foot-unit at Tegea. Alternative approaches to the question are based on a large set of moulding measurements and the full dimensions of a set of building

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3. On why these limits are chosen, see Section 2 on the method.
4. Pakkanen 2002.
5. Hdt. 1.60, 1.178, 2.149, 2.168, 6.127.
6. See *e.g.* Rottländer 1993.
7. Michaelis 1883; Dekoulakou-Sideris 1990; Slapšak 1993; Wilson Jones 2000.
8. See *e.g.* Haselberger 1983, 115-21, and Pakkanen 2002.
9. Bankel 1983.
10. Bankel 1984.

blocks. In order to demonstrate that studying the measurements of architectural details can be a statistically valid metrological approach, I will make use of the Parthenon as a parallel case study.

2. Cosine quantogram analysis and computer simulations

Data selection is perhaps the basic question behind ancient metrology: which building elements can be used in the study of metrological units?[11] One possibility is using dimensions of individual blocks, and there is also inscriptional evidence to support this.[12] The building blocks had to be ordered to size from the quarries, but they were always left with an extra layer of stone in order to protect them during transport and to allow for final fitting of the blocks on the building site. It is therefore possible that the dimensions do not exactly reflect the length of a design-unit. Another possible option is to study small building details: carving of the mouldings in classical marble buildings is very precise, and *if* the mouldings were designed and executed using fractions of dactyls, we could reasonably expect to derive the length of the design-unit from these details.

The exploratory statistical method used in this paper is based on cosine quantogram analysis; after this initial analysis, Monte Carlo computer simulations must be used to test the probability of the obtained results.[13] The analysis is based on the hypothesis that a building dimension X can be expressed in terms of an integral multiple M times a design-unit, or quantum, q plus a small error component e:

$$X = Mq + e.$$

From a statistical point of view it is irrelevant whether the error e is the result of ancient Greek design methods and execution or modern measurement, but it is significant that e should be notably smaller than q. By computer simulations it can be shown that an error of ±10 mm – quite usual in Greek architecture[14] – does not prevent detecting a design-unit of the size of ca. 80 mm, or a quarter of a 'Doric' foot.[15] If smaller units were employed in building design and execution, it is quite unlikely that they could be discovered in a metrological

11. Cf. Fieller 1993, 286.

12. *E.g.* the Erechtheion building block inventory of 409/08 B.C. (*IG* I^3 474).

13. On the method more in detail, see Kendall 1974 and Pakkanen 2002. Cosine quantogram analysis has been employed in connection with ancient architecture *e.g.* by Rottländer 1996, but he does not use Monte Carlo simulations to validate the results.

14. Coulton 1975, 94.

15. Pakkanen 2002, 502-3.

analysis of relatively large building dimensions: in order to give some scope for a smaller error than ±10 mm, I have used the range 50 – 400 mm for the unit-length in the final section of this study. Since the discrepancies in the sizes of mouldings are much smaller, due to the size of the elements themselves, a range of 4 – 25 mm is used in Section 4 for detailed mouldings.

In order to analyse how accurately dimension X can be expressed in terms of unit q, X is divided by q and the remainder e is studied: the closer to 0 or q it is, the better unit q fits the dimension. The amount of clustering around any q within the tested unit range can be calculated by using the formula

$$f(q) = \sqrt{2/\mathrm{N}\sum_{i=1}^{\mathrm{n}} \cos(2\pi\varepsilon_i/\mathrm{q})},$$

where N is the number of building dimensions. The cosine gives a value of 1 for the exactly fitting measurements and -1 for those least fitting: therefore, the largest value of the score $f(q)$ gives the most probable candidate q for the unit. Computer simulations still have to be used to determine if the function score $f(q)$ is high enough to indicate a statistically significant 'true' unit. In the Monte Carlo simulations random data sets are created from non-quantal distributions; these are analysed in the same way as the original data to determine whether peaks as high as or higher than the original arise from the distributions.[16]

3. Bankel's analysis of the unit-length at Tegea

Bankel's metrological scale is a graphic method where the length of the possibly used foot-unit is in centimetres on the left and the length of the various building elements in corresponding dactyls on the right (Fig. 1): for example, the lower column diameter at Tegea, 1.555 m, expressed as dactyls of a foot-unit of 300 mm, is very close to 83. If all the elements were designed and executed as multiples of the dactyl in question, the dimensions would fall neatly on the same line. This is not the case, and the closest candidate is, according to Bankel, the 'Ionic foot' of 294 mm. One drawback of the method is immediately apparent: as a graphical method it is time-consuming to construct the complicated tables, the number of analysed elements is necessarily limited, and the analysis of the results

16. I have implemented the computer programs used in the cosine quantogram analyses, Monte Carlo simulations, and kernel density estimations on top of Survo MM, the Windows version of the statistical program; very warm thanks are due to S. Mustonen for providing a copy of the program. C.C. Beardah's MATLAB routines were used for calculating the optimal window widths of the kernel density estimates.

can be quite subjective. These problems involved in the metrological scale can be avoided by using a numerical method instead of a graphical one.

The results of the cosine quantogram analysis of Bankel's data can be presented as a single curve (Fig. 2): the quantum score $f(q)$ calculated from the measurements (see Column 2 in Table 1) is here plotted against q. The higher the peak, the more likely it is that q is a 'true design-unit'. The studied range for q is very large, 9 – 400 mm; as stated above, we cannot expect to discover a quantum in the lower part of the range, but the range below 50 mm is included in this initial analysis in order to take into account the small dimensions regarded as relevant by Bankel: 9.2 mm is half a dactyl of Bankel's foot-unit of 294 mm. The sub-division scores of this unit are marked with small circles in Fig. 2, and the fit to the measurements is by no means convincing: the first three correspond to a half-dactyl, a dactyl and 1/8 foot, all with a score of 1.5 or less; the next three, at quarter-, half- and full foot mark, are at local maximum points of the curve, but their scores are not any better. There is an impressive local maximum of 4.0 at 29.4 mm, exactly one tenth of Bankel's foot-unit, but it is to the left of our unit detection limit of 50 mm and could therefore be a result of trying to fit a too small unit to the data. The highest peak to the right of 50 mm is at 60.1 mm with a quantum score of 2.9. However, Monte Carlo computer simulations of non-quantal replica data sets indicate that only a peak with a height of 3.4 or greater is significant at 5% significance level, [17] so no 'true quantum' can be detected in the data.

The analysis can be taken one step further by substituting some new measurements for the ones given by Bankel: the five slightly different dimensions in Column 3 of Table 1 are the result of recent fieldwork at the temple site.[18] The cosine quantogram curve of the partially new set is plotted in Fig. 3. As we see, changes of a few millimetres in only a part of the measurements are enough to make the height of the original peak at 29.4 mm collapse, and to the right of the 50 mm limit the curve follows very closely the shape of the curve in Fig. 2.

Statistical analysis indicates that no single design-unit can be derived from Bankel's small selection of building measurements, and, in the case of Tegea, the validity of the metrological scale method can be shown to be questionable.

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17. Two non-quantal data models based on Bankel's data were created using kernel density estimation using normal-scale and dpi-3 window-widths (h = 346.4 and 386.1), and for each distribution 1000 Monte Carlo simulations were run: the 5% significance level for the first data model was determined as 3.37 and the second as 3.35. On kernel density estimation and data modelling, see Pakkanen 2002, 502.

18. On the recent study of the temple, see Pakkanen 1998.

4. Deriving unit-lengths from moulding dimensions

Recently, M. Korres has suggested that the length of the Parthenon foot-standard could be obtained from small building details. (Fig. 4) He re-measured the mouldings of the building and suggests that they were designed and executed using quarter-dactyls of a foot-unit of 294 mm.[19] Korres' first suggestion is strongly supported by a quantogram analysis of 35 measurements.[20] (Fig. 5) The first peak at 4.61 mm is exceptionally prominent with a height of 6.5, and it corresponds to a quarter-dactyl of a 295 mm foot; the second peak with a score of 4.3 at 9.24 mm is a half-dactyl of a 296 mm long foot-unit. It is extremely unlikely that either one of these peaks could be a result of a coincidence: in the 2000 computer simulations based on corresponding non-quantaldata sets there was only one single simulation which produced a peak higher than the lower quantogram peak of 4.3.[21] The length of the unit derived from the Parthenon mouldings, 295-6 mm, is a millimetre or two longer than the 'standard Ionic foot' of 294 mm. More significantly, Korres' observation confirms that smaller subdivisions than half a dactyl were also employed in Greek building, even though there is no indication in inscriptions or other literary sources that any fractions of a dactyl less than a half were actually used.[22]

The 71 moulding measurements used in the analysis of the temple of Athena Alea at Tegea are listed in Table 2, and the resulting quantogram curve is plotted in Fig. 6. The method does not produce a clear result at Tegea, since the highest peak at ca. 6.0 mm reaches only 2.2: statistically significant scores at the 5% level should have a value of at least 3.4.[23] There are several possible explanations why no clear pattern emerges:

1) the mouldings were not designed using any particular unit,

2) they were designed using a certain unit but in the subsequent execution the original design was not followed meticulously, or

19. Korres 1994, 62-5.

20. The data used in the metrological analysis are given in Fig. 5 (the measurements are given in centimetres).

21. The maximum peak scores of the two simulation runs of 1000 each were 4.2 and 4.5. The kernel density distributions were created using Korres' moulding data (normal-scale and dpi –3 window-widths $h = 14.31$ and 10.74): the 5% significance level for the first model was 3.30 and the second 3.25.

22. Coulton 1975, 92-3.

23. Two kernel density distributions used in the Monte Carlo simulations were based on the data in Table 2 with window-widths $h = 15.44$ and 8.05: the 5% significance levels were 3.39 and 3.35.

3) the French block detail measurements of the early 20th century are not precise enough for deriving a design-unit.

None of the above alternatives can definitively be ruled out, but I will return to the question in Section 6.

5. Deriving a unit-length from block dimensions

In a previous study I have used the Erechtheion measurements and the inventory of 409/08 B.C. (*IG* I^3 474) to demonstrate that cosine quantogram analysis can produce statistically significant results based even on a relatively small sample of 19 dimensions.[24] For the temple of Athena Alea I have chosen to use a larger set of block dimensions and to include reliably recorded full widths, depths and heights of different types of krepis, capital, entablature and cella wall blocks (see Table 3). I have not repeated duplicate dimensions for the same type of blocks: for example, in the case of stylobate blocks the height and depth of two blocks are identical, so I have included all the relevant dimensions of the first block but only the length of the second block in the data set. Repetition of the same dimensions would very likely increase the peak heights in the quantogram plot, so there could be a danger of accepting results of the analysis as statistically significant even when they are not.

The 55 block dimensions used in the analysis are listed in Table 3. The cosine quantogram curve based on the data is quite interesting (Fig. 7): there is a single notable peak with a height of 3.72 at ca. 99 mm. A peak of this height is statistically very significant: in the 2000 computer simulations there were only 28 random peaks higher than this, so the quantogram score is not quite significant at 1% level, but nearly so.[25] One probable interpretation of this peak is that the architectural design-unit at Tegea was one third of a foot ca. 297-8 mm long.[26] Metrologically the result is very important since it is the first statistically valid indication that a foot-unit in the region of the traditional 'Ionic' foot could have actually been employed at Tegea, as has been suggested by several scholars.[27]

24. See Pakkanen 2002, 502.

25. The kernel density distributions used in two simulation runs of 1000 each were based on the data of Table 3 (window-widths $h = 236.0$ and 173.3); the 5% significance levels were 3.36 and 3.40.

26. The precise location of the peak is 99.16 mm: 3 x 99.16 = 297.48 mm.

27. See Bankel 1984, 413–5.

6. Building design, execution and unit derivation

In light of the statistically significant result derived from block data it is worthwhile to return to analysis of moulding dimensions. Even though the highest peak in Fig. 6 cannot be easily explained in terms of the detected design-unit, the local maximum at 9.3 mm clearly corresponds to half a dactyl of the defined foot-standard. A closer study of the dimensions in Table 2 indicates that 41 out of the 71 measurements fit this half-dactyl with a discrepancy of ±2 mm or less, and they suffice to give the weak signal visible in Fig. 6. Thus it is quite likely that a subdivision of the same measure-unit was used in the design of the major block dimensions as well as of the details in the mouldings.

The reason why no statistically significant dimension is detected in the latter data set is at least partially due to the execution and nature of craftsmanship of the temple. The capability of the masons is perhaps best illustrated by the arris repair on one of the column drums where two of the three carved pieces are still in their original places: no lead or dowels were used, only the perfect carving of the surfaces keeps the pieces together.[28] The masons did not, however, use their skills to slavishly copy Skopas' architectural designs. For example, no two capitals are exactly similar: visually they are unmistakably from the same building, but a study of their dimensions and proportions demonstrates the slight variations between them.[29] These variations were not only tolerated but even encouraged. This is most clearly manifest in the refinements, the slight variations from true horizontals and verticals.[30] One unintended result of the irregularity observable in Greek buildings in general is that it makes the work of architectural archaeologists a challenge, but it is also a factor behind the persistent modern fascination with these buildings.

7. Conclusions

Cosine quantogram analysis is a useful tool in the study of Greek architectural design and metrology. It can be used to analyse the shortcomings of non-statistical methods such as the metrological scale, but more importantly, when combined with Monte Carlo computer simulations, it can reveal how significant the results of various design-unit derivations are. In this paper it was demonstrated that even though the moulding measurements of the temple of Athena Alea

28. Pakkanen 1998, 28-30.
29. Pakkanen 1998, 31-40.
30. Pakkanen 1998, 41-7, 62-7.

at Tegea do not produce a statistically significant result, the method can be used to verify that a unit of ca. 295-6 mm was used in the design of Parthenon mouldings. However, analysis of a relatively large set of full block dimensions gives strong statistical support that a unit of ca. 99 mm was used in the architectural design of the Tegea temple. In general, I do not think that the importance of using proper quantitative methods in the study of Greek architectural design-units can be over-emphasized.

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Table 1. Temple of Athena Alea, Tegea. Building dimensions.

1.	2. Bankel's dimensions (in mm)	3. New measurements (in mm)
Lower column diameter	1555	1550
Upper column diameter	1209	1205
Abacus width	1616	1613
Capital height	589	596
Drum height	1470	
Metope width	1088	
Triglyph width	710	
Corner triglyph width	726	
Regula width	185	
Architrave depth	1436	1430
Architrave height	968	
Triglyph height	1088	
Geison height	295	
Distance of column centre from stylobate edge	821	
Entablature height	2351	

Table 2. Temple of Athena Alea, Tegea. Moulding dimensions used in plotting Fig. 5. Source: Ch. Dugas *et al.*, *Le sanctuaire d'Aléa Athéna à Tégée au IVe siècle*, Paris 1924 (numbers of plates and illustrations refer to this publication).

1.	2. Dimensions (in mm)
Fig. 13	78, 32, 61
Fig. 15	71, 73
Fig. 16A	37, 42, 18, 85, 62
Fig. 16B	23, 28, 34
Pl. 52B	83, 26, 57
Pl. 53	110, 67
Pl. 54A	72, 26, 59
Pl. 55	18, 138, 28, 90, 24, 27, 65, 25
Pl. 56	18, 122, 27, 75, 21, 155, 39, 66, 20, 57.5, 83
Pl. 58	18, 23, 37, 149, 143, 88, 18, 35
Pl. 64	85, 50, 76, 33, 55, 29, 68, 23, 38, 42
Pl. 78B	54, 21, 83, 43
Pl. 79	54, 85, 86, 36, 114, 46, 23, 70, 36

Table 3. Temple of Athena Alea, Tegea. Full block dimensions used in plotting Fig. 7. Source: Ch. Dugas *et al.*, *Le sanctuaire d'Aléa Athéna à Tégée au IVe siècle*, Paris 1924 (numbers of plates and illustrations refer to this publication).

1.	2.	3. Dimensions (in mm)
Euthynteria blocks	Pl. 29	1676 x 902 x 297; 1202 (length)
First step block	Pl. 30	1803 x 1465 x 348
Foundation block	Pl. 31	1392 x 1400 x 366
Stylobate blocks	Pls. 32-33	1642 x 1642 x 380; 1814 (length)
Capital	Pl. 35	1616 x 589
Architrave block	Pl. 38	788 x 968
Architrave backer	Pl. 40	718 (depth)
Frieze block	Pl. 41	1848 x 1023 x 1088
Geison block	Pl. 44	1790 x 482; 672 (distance between roof-beam cuttings)
Sima block	Pl. 46	1346 x 288
Roof tile	Pl. 48	671 (width)
Epikranitis blocks	Pls. 52 & 54	402 (height); 534 (height); 766 x 520
Pteron beam	Pl. 53	1002 x 400
Ceiling coffer block	Pl. 55	795 (width)
Pronaos capital	Pl. 57	1402 x 509
Pronaos architrave block	Pl. 58	884 x 677
Pronaos frieze block	Pl. 59	993 x 768
Toichobate blocks	Pls. 62 & 64	1728 x 1490 x 372; 938 x 295
Orthostate block	Pl. 66	1791 x 683 x 1278
Wall block	Pl. 70	897 x 893 x 385
Wall epikranitis block	Pl. 79	1187 x 480 x 375

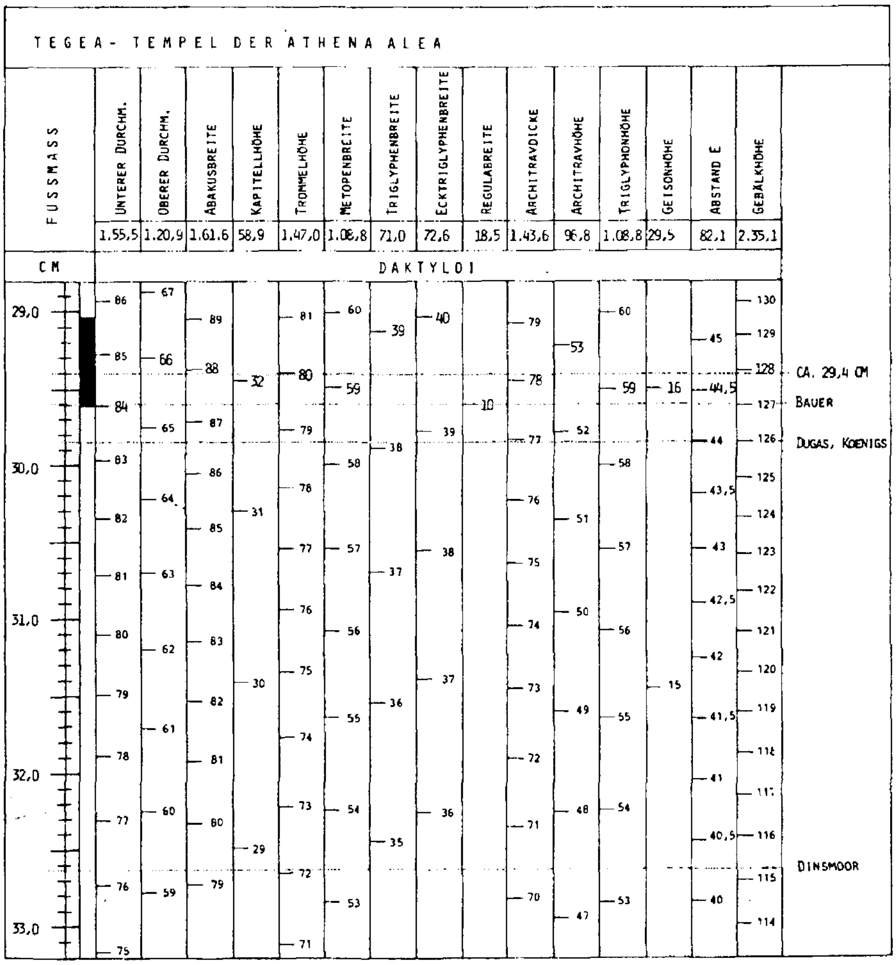

Fig. 1. Temple of Athena Alea, Tegea. “Metrological scale”. (After Bankel 1984, fig. 1)

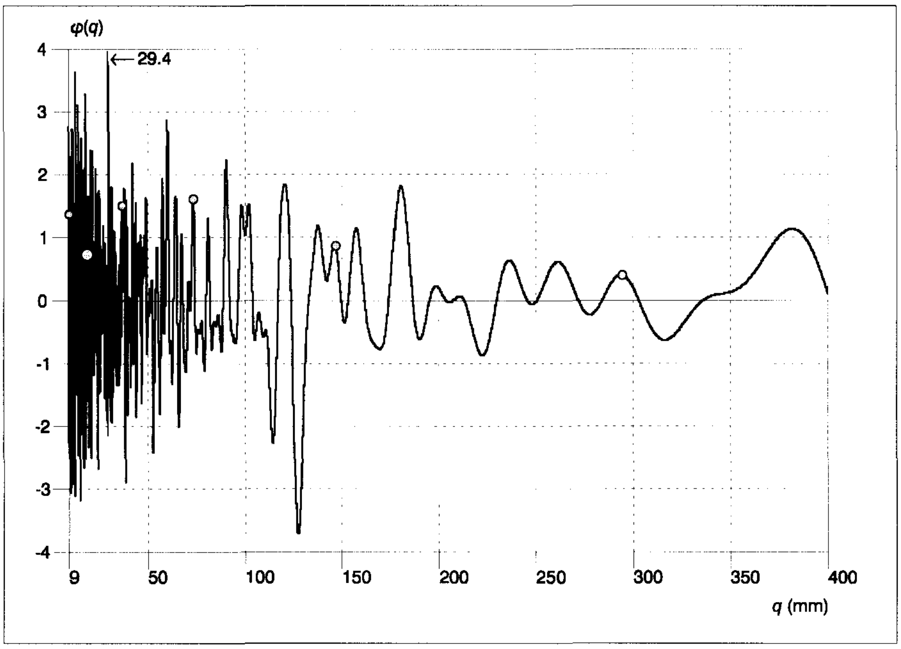

Fig. 2. Temple of Athena Alea, Tegea. Cosine quantogram analysis of Bankel's data. The small, grey circles mark Bankel's foot-unit of 294 mm and its sub-divisions.

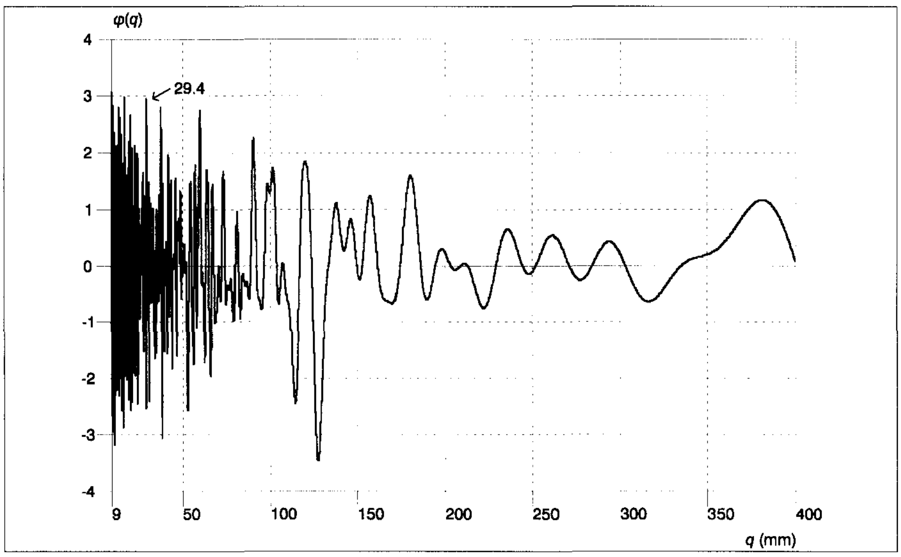

Fig. 3. Temple of Athena Alea, Tegea. Cosine quantogram analysis with five new measurements.

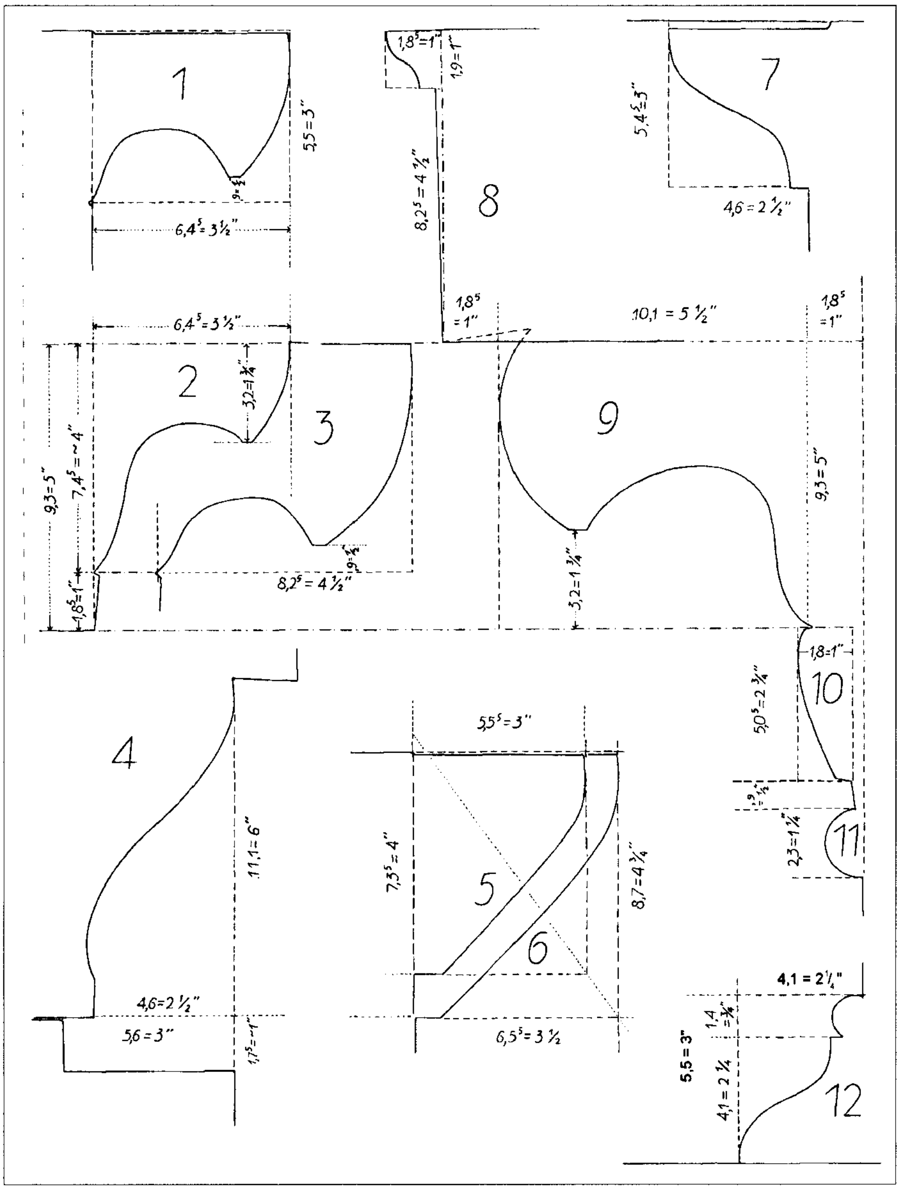

Fig. 4. Parthenon, Athens. Moulding profiles with dimensions used in cosine quantogram analysis. (After Korres 1994, fig. 4; two dimensions of profile 12 are corrected in the figure.)

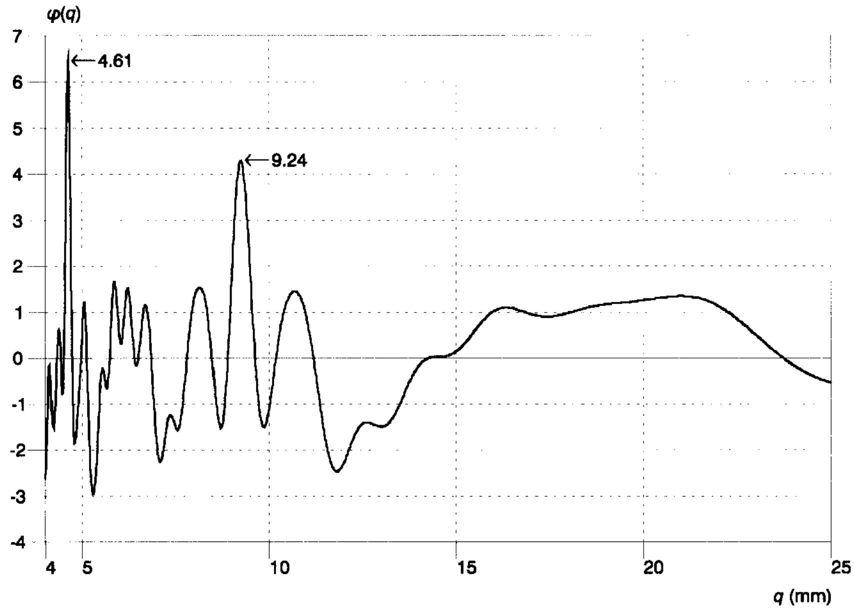

Fig. 5. Parthenon, Athens. Cosine quantogram analysis of moulding dimensions.

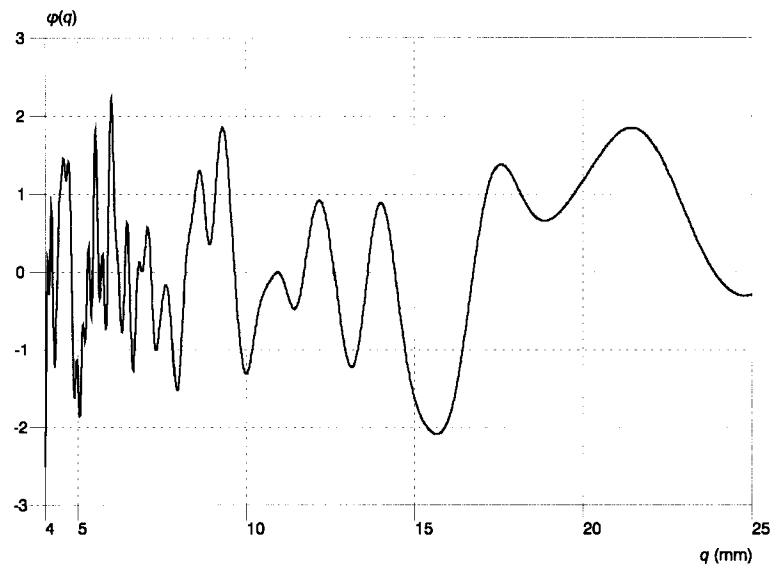

Fig. 6. Temple of Athena Alea, Tegea. Cosine quantogram analysis of moulding dimensions.

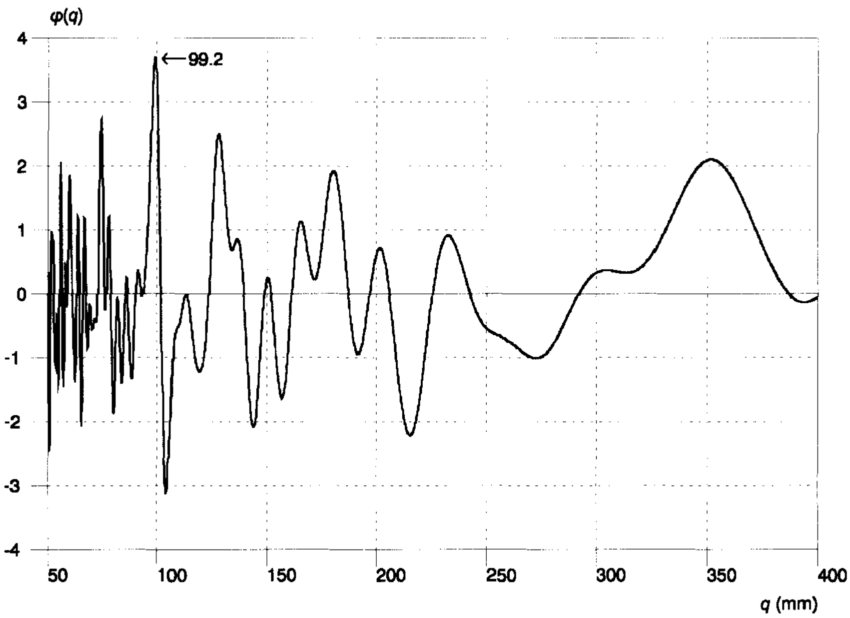

Fig. 7. Temple of Athena Alea, Tegea. Cosine quantogram analysis of building block dimensions.