Nonhydrostatic pressure in ocean models with focus on wind driven internal waves

Philosophiae Doctor Thesis

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Preface

This thesis is submitted for the degree of Philosophical Doctor in applied mathematics at the Department of Mathematics, University of Bergen, Norway. This work started in August 2005 and is a part of the project Nonhydrostatic Ocean Circulation Models founded by The Research Council of Norway through the MARE programme grant 164501/S40. My working place has been at the Computational Mathematical Unit (CMU), Bergen Center for Computational Science (BCCS) at UNIFOB. My main supervisor has been Prof. Jarle Berntsen at the Mathematical Department and my second supervisor Prof. Guttorm Alendal at CMU.

The main objective of this thesis has been to study the implementation of nonhydrostatic pressure in numerical ocean models and the influence of nonhydrostatic pressure effects on physical phenomena in ocean and lakes. Focusing on the processes involved in the generation, propagation, and degeneration of internal waves, the influence of nonhydrostatic pressure with a varying grid resolution has been studied. These studies may serve as a more general understanding of how nonhydrostatic pressure effects may influence the results in a numerical ocean model.

The thesis contains a general background in Part I and articles and reports in Part II. In Part I, I have tried to set the main objectives of this thesis in a context of the challenges that the humans, and more specific ocean scientists and numerical modellers, are facing in the near future. Part II consists of focused studies of the main objectives and are presented in two papers and one report,

Paper A: Numerical studies of wind forced internal waves with a nonhydrostatic model. J. Bergh and J. Berntsen. Accepted for publication in *Ocean Dynamics*, 2009.

Paper B: The surface boundary condition in nonhydrostatic ocean models. J. Bergh and J. Berntsen. Under revision for publication in *Ocean Dynamics*, 2009.

Paper C: Numerical studies of nonhydrostatic pressure effects on wind forced boundary layer dynamics. J. Bergh and J. Berntsen. *BCCS Technical report series*, Report No.25, October 2009.

These papers have been written under supervision from, and in collaboration with, Prof. Jarle Berntsen.

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The international community here in Bergen, with people from all over the world, has given me a good, interesting and enjoyable life here in Bergen. A rainy Monday lunch would be so much more grey without you, my friends! The time I spent with you all, at lunch, in the pub, in the cafe, in the park, in the mountains, in the ski-slope, at dinners, at parties and the daily ”hello, how are you?” has filled my life with happy and memorable moments. Thank you all, for being just the persons you are!

I am very thankful to my family for supporting me, and helping me out in many ways during my many years of studying. I would not be able to do this without you.

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Contents

Part I

General Background

Chapter 1

Introduction

Figure 1.1: Overlooking the ocean from a cave in Mosselbay, South Africa, where findings from Humans indicate an early use of marine resources [McBrearty and Stringer, 2007]. The foto is taken from the Mosselbay Archaeology Project, Arizona State University.

1.1 The human and the ocean

Since the early stages in the human evolution, the ocean and ocean resources have been essential. Exploiting marine resources are together with producing complex technology and manipulating symbols all symptomatic of modern human activity [McBrearty and Stringer, 2007]. During the Middle Stone Age, some 285 to 45 thousand years (kyr) ago, the modern human Homo sapiens evolved in Africa. With the predominantly glacial stage in the period from 195 kyr ago to 130 kyr ago, the African continent was much cooler and drier. There are evidence that humans expanded their diet to include marine resources during this period, probably due to the tough inland climate [Marean et al., 2007]. The early Homo sapiens was most probably dependent on sea food, rich on polyunsaturated fatty acids, to develop a large, complex, metabolically expensive brain [Broadhurst et al., 2002]. Also later, the human has been highly dependent on the prevailing climate state and the related mean sea level of the world ocean. In the beginning of the last ice age, around 50 kyr ago, the mean sea level of the world ocean was much lower than today. The first Homo sapiens are believed to have migrated from the African continent during this period, crossing the Gate of Grief in the Red Sea when the sea level was 70 m lower than today. The slowing rate of the sea level rise around 7000 years BP, following the last deglaciation, made it possible for the early civilisations to settle in lowland coastal regions, river-lands, and estuaries.

Today the modern societies are focused around water resources and especially in coastal areas. Approximately half of the worlds population live within 60 km from the coast and more than half of the 30 larges cities are situated at the coast. With an accelerating world population and increasing urbanisation, the management of the coastal land and sea areas are crucial for the future of humanity.

1.2 Climate change

The world is facing an indisputable climate change. The last IPCC (Intergovernmental Panel on Climate Change) report [Solomon et al., 2007] state that the total temperature increase from 1850-1899 to 2001-2005 is 0.76 °C (0.57 °C to 0.95 °C) and that the warming trend increases. The temperature in the second half of the 20th century was likely the highest in at least the past 1300 years. Observations since 1961 show that the average temperature of the global ocean above 3000 m has increased and that the ocean is absorbing more than 80 % of the heat added to the climate system. Thermal expansion of the ocean, due to increased mean temperature, together with melting glaciers lead to a sea level rise. The global sea level rose at an average of 1.8 mm (1.3 mm to 2.3 mm) per year over the period from 1961 to 2003, accelerating over the last 10 years. The IPCC projections of future climate change indicate a warming in the atmosphere of 0.1 °C to 0.2 °C per decade for the next two decades. The projections also predict a sea level rise at the end of the 21st century (2090-2099) in the range from 0.2 m to 0.6 m. The projections of the sea level rise do not include rapidly dynamical changes in the ice flow. Resent research indicate that dynamical thinning on the margins of the Greenland and Antarctic ice sheets are extensive, see for instance Pritchard et al. [2009]. Even if these processes are poorly understood, they have the potential to accalerate the sea level rise predicted by the last IPCC report. With 10 % of the world population (634 million people) living in coastal regions at elevations lower than 10 m, the projections predicted by IPCC will have a dramatical impact on the human population [FitzGerald et al., 2008]. Rising sea level lead to flooding and salt intrusion of low-laying costal areas, but also increase the damage of storm surges, tsunamis and extream tides. The largest impact may be in developing countries with dense and poor populations in coastal areas, and for some countries (e.g. Vietnam, Egypt and The Bahams) the consequences are even potentially catastrophic [Dasgupta et al., 2007].

1.3 The energy budget of the ocean

Quantifying the energetics of the general ocean circulation is a key component in the understanding of the current climate system. The mechanically forcing by the wind and the tides are transformed into both kinetic, but mainly, potential energy in the large scale ocean circulation.

The meridional overturning circulation describes a major part of the ocean circulation. Heated warm surface water at low latitudes are transported towards higher latitudes. The water is mixed on the way and cooled in sub-polar regions, where dense cold bottom water is formed through convection [Marshall and Schott, 1999] in the North Atlantic and near Antarctica. The dense bottom water spreads towards lower latitudes and forms the deep water in the abyssal ocean.

The meridional overturning circulation has been a controversy in the scientific community. Early studies refer to this circulation as the thermohaline circulation or Sandström's theorem [Kuhlbrodt, 2008]. Later the thermohaline forcing of this circulation has been strongly criticised due to energy budget concerns [Munk, 1966, Munk and Wunsch, 1998, Wunsch and Ferrari, 2004]. These authors claim that the large amount of energy needed to drive the circulation outrank thermohaline forcing.

It was stated by Munk and Wunsch [1998] that: ”Without deep mixing, the ocean would turn...into a stagnant pool of cold salty water”. Assuming a constant climate state, the ocean stratification and circulation can be seen as a statistically steady state. The meridional overturning circulation and large eddy generation ”steal” energy from the large scale ocean circulation. The potential energy stored in the stratified ocean is decreased. Energy is needed to mix deep dense water through the pycnocline, in the interior of the ocean, to keep the steady state stratification. The main agent, or maybe the only one, for mixing in the interior of the ocean is the internal wave field [Wunsch and Ferrari, 2004].

Internal waves can be generated directly from wind forcing at the ocean surface or indirect via fluctuations of the stratification and nonlinear interactions of surface gravity waves. Also flow in the interior, currents, tidal or eddy generated, can generate internal waves when interacting with either bottom topography in the deep oceans or the rough topography in shelf seas. A lot of work has been done since the statement in Munk and Wunsch [1998], in describing and quantifying different sources of energy supply for internal mixing. Still today there is a debate on the importance of thermohaline circulation [Huang et al., 2006, Kuhlbrodt, 2008, Hughes and Griffiths, 2008] and the importance of different sources for mixing [Ivey et al., 2008]. The transportation of energy from large scale barotropic eddies, via the internal wave field, to irreversible mixing, is suggested to be an important source for diapycnal mixing in the interior of the ocean, even if this pathway of energy is still not fully understood [Ferrari and Wunsch, 2009]. There are also questions concerning the importance of processes in the interior of the ocean for the global heat transport [Boccaletti et al., 2005].

The lack of understanding of the generation mechanisms and the mixing processes of the internal wave field is a disadvantage when the parametrisation of mixing in global ocean models and coupled climate models is formulated. Running these models for a past or future climate scenario, the sources of internal mixing in the ocean may be different than today, and the parametrisation a source of error. The last IPPC report [Bindoff et al., 2007] conclude that there are large variations in the circulation but no clear indication that the meridional overturning circulation in the Atlantic has changed over the modern instrumental record. However, there are indications that the wind energy input to the world oceans has significantly increased during the last two centuries [Huang et al., 2006]. The kinetic energy transformation from large geostrophic scales towards smaller scales and the possibilities of either feedback to larger scales or the transformation into irreversible mixing need to be further investigated [Ferrari and Wunsch, 2009].

1.4 Strategies in ocean science

Observing and measuring the ocean for a long time has revealed different methods to analyse physical processes in the ocean. Most of these methods can be described as models, a mathematical description of the dynamics of the ocean. Generally, models can be divided into theoretical and data-based models [Green, 2004]. The theoretical models are based on the principal physical processes in the ocean and can be divided into process-based and hydrodynamic models. The process-based models focus on a limited amount of physical processes and result in a robust and controlled model. In the hydrodynamic models, the equations of motion have to be solved and we need relevant simplifications to either solve the equations analytically or numerically. Data-based models use statistics to model a specific phenomena or area and may be connected to a theoretical model. In the present thesis, a numerical ocean model that solves the equations of motion is the main method applied for different problems. The results from the numerical model will be compared to theory, measurements, and results from other numerical models.

1.5 Towards higher resolution

The available computer capacity together with the numerical models efficiency limit the possible resolution in a numerical study of the ocean.

Both within science studies and engineer related investigations there is a need to use numerical ocean models in high resolution studies. Following the discussion above, about the importance of diapycnal mixing in the overturning circulation, high resolution modelling of these phenomena is necessary. First to understand the processes involved and second to be able to parametrise these effects in a model with a less dense grid resolution. Also with increasing demand in the management of the coastal areas, high resolution studies of shelf seas, estuaries and fjord systems are necessary in for instance environmental, construction and pollution studies.

Most numerical ocean models that are used today were originally constructed for studies with a horizontal grid scale of tens of kilometres. When applying these models to studies with a grid scale of about 1 km and less, smaller scale physical phenomena and high frequency motions may be filtered out. Increasing computer capacity in the near future will make it possible for ocean models to be applied with higher resolution than today [Fringer, 2009]. Though, several aspects concerning the simplifications done in the numerical ocean models have to be reconsidered when applying these models in high resolution studies. Some of these aspects are addressed in the present work.

Chapter 2

Numerical ocean models

In this Chapter the general background for ocean models are presented. To formulate useful hydrodynamic equations that are applicable to ocean situations, it is common to apply different approximations, like the Boussinesq (including incompressibility), hydrostatic, shallow ocean or rigid lid approximation [Gill, 1982, Kundu, 1990, Haidvogel and Beckmann, 1999, Griffies, 2004, Griffies and Adcroft, 2008]. The applied approximations will limit the models capacity to resolve physical processes and phenomena. The purpose of the model, for instance climate, ocean circulation or coastal studies, set the range for the spatial and temporal resolutions that have to be resolved, see Fig. 2.1. The focus in this work is on the hydrostatic approximation and the introduction of nonhydrostatic correction methods, and will be presented in Chapter 3.

2.1 Navier Stokes Equation

A short description of the Navier Stokes Equation will be given here, for a more comprehensive derivation of the Navier Stokes equation see for instance Kundu [1990]. The law of conservation of mass requires that an increase of mass within a fixed volume equals the rate of inflow through the boundaries. This result in the continuity equation that may be written

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla \cdot \mathbf{u} = 0\,. \tag{2.1}$$

where ρ is the density, D/Dt is the material derivative, and $\mathbf{u} = u_i$ is the velocity vector where the index $i = 1,2,3$ indicate the components in the three Cartesian coordinate directions. If the density within the fixed volume may be regarded as constant even if there are changes in the pressure, the fluid is called incompressible and the continuity equation simplifies to

$$\nabla \cdot \mathbf{u} = 0\,. \tag{2.2}$$

The Navier Stokes equations describe the motion of a fluid and is the most common starting point for any type of approach when studying physical processes in fluids. With the assumption of a Newtonian fluid, incompressibility and small temperature differences, the Navier Stokes equation of motion may be written in vector form as

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u}\,, \tag{2.3}$$

where p is the pressure, $\mathbf{g}$ is the gravitational acceleration, μ is the dynamic viscosity, and $\nabla^2\mathbf{u}$ is the Laplacian of $\mathbf{u}$. Together with Eq. 2.3, the continuity equation (Eq. 2.2), conservation criteria, and information about boundary conditions form a solvable set of equations.

2.2 Boussinesq approximation

The Boussinesq approximation is valid in many type of fluid studies, and may be summarised in two statements. First that the density changes in the fluid may be neglected except when they are coupled to the gravitational acceleration. Second that the properties of the fluid, such as the dynamic viscosity, thermal conductivity and specific heat, may be regarded as constants. These approximations are described for an ocean by Gill [1982], Kundu [1990], Griffies [2004] and more thoroughly for an atmospheric condition by Spiegel and Veronis [1960].

Assuming a static reference state with a constant density ρ_0, the density and the pressure may be written in the form of a reference and a deviation from the reference state as $\rho = \rho_0 + \rho'$ respectively $p = p_0 + p'$. When using this definition in Eq. 2.3, removing the static reference state, $-\nabla p_0 = \rho_0 g$, and divide by ρ_0, the term ρ'/ρ_0 will appear in the equation. This term may be regarded as small except in the gravitational term. Further omitting the influence from the Earth's rotation (the Coriolis effect), the momentum equations under the Boussinesq approximation may be written in compressed form as

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho_0}\nabla \cdot \mathbf{p} + \frac{\rho}{\rho_0}\mathbf{g}\hat{k} + \nu\nabla^2\mathbf{u}\,, \tag{2.4}$$

where $\hat{k}$ is the unit vector in the vertical direction and $\nu = \mu/\rho_0$ is the kinematic viscosity. The Boussinesq approximation filter out sound and shock waves and does not allow for large vertical scales of the flow that may create large changes in density. A fluid parcel under the Boussinesq approximation maintains the same volume, and without volume sources the volume of a Boussinesq ocean remains constant. This may be regarded as a good quality in ocean models in general. Though, for instance, the Boussinesq approximation fail to predict an increased mean temperature effect on the sea surface height (steric effect), that may be an important issue in climate studies [Griffies, 2004].

2.3 Earth Rotation

A traditional way to study the effect of the Earth's rotation on a fluid, is to assume a local Cartesian system (x, y, z) on a tangent plane at the surface of the Earth, with the coordinates x eastward, y northward and z upward [Kundu, 1990]. The related velocity components are (u, v, w). The influence of the angular velocity of the Earth, Ω, in the local Cartesian system is called the Coriolis force and may be written as

$$2\Omega \times \mathbf{u} = 2\Omega\left(\hat{i}\,(w\cos\theta - v\sin\theta) + \hat{j}\,(u\sin\theta) + \hat{k}\,(u\cos\theta)\right) \tag{2.5}$$

where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors in the x, y, z directions. The cos-term in the $\hat{k}$ direction may be regarded as small compared to other terms in the vertical momentum equation and with the assumption that $w \ll v$, the cos-term in the $\hat{i}$ direction may also be neglected. The Coriolis parameter may then be defined as $f = 2\Omega \sin\theta$ and only act in the horizontal plane, $-vf$ in the $\hat{i}$ direction respectively fu in the $\hat{j}$ direction. Though, for instance on large scale studies close to the equator, the cos-terms may not be regarded as small and the assumption fails. The assumption discussed above is related to the hydrostatic approximation, where the horizontal scales of motion are regarded as much larger then the vertical scales of motion, see Chapter 3. This is further discussed in Marshall et al. [1997b], where a quasi-hydrostatic ocean model is defined to include the cos-terms. This problem will not be further discussed in the present work, even though it may be an important aspect when formulating a nonhydrostatic ocean model.

2.4 Model types

The hydrodynamic equations and the approximations described above, will be used to categorise different numerical models of fluid dynamics, see Fig. 2.1.

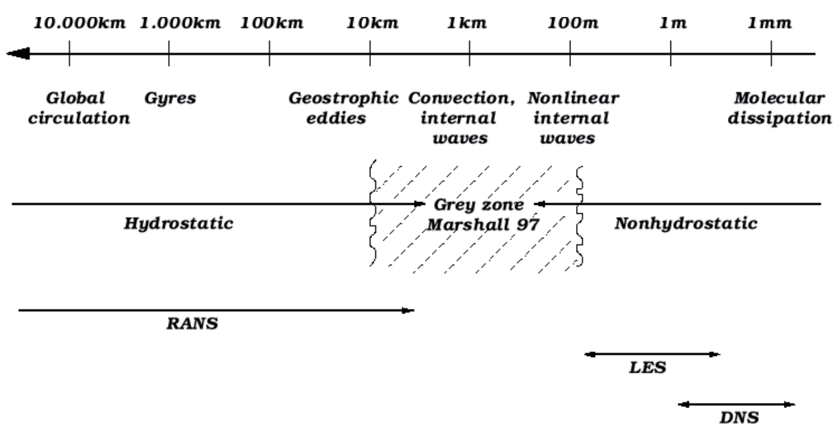

Figure 2.1: Schematic figure describing the length scales of some physical phenomena, hydrostatic and nonhydrostatic physics, and the different types of numerical models. The grey zone describe the transition from hydrostatic length scales to nonhydrostatic length scales Marshall et al. [1997b].

Within engineering studies of dynamical fluid systems, it is common to use the Navier Stokes equation in Direct Numerical Simulation (DNS) models. When using a DNS model for a turbulent fluid, as the ocean or the atmosphere, all turbulent length scales have to be modelled down to the Kolmogorov length scale. At the Kolmogorov microscale the turbulent energy is dissipated into heat using molecular values of viscosity ν and thermal diffusivity κ. The Kolmogorov length scale may be estimated to be in the order of a millimetre in the ocean [Kundu, 1990]. With the limiting factor of the computer capacity, it is only possible to study very small spatial domains with a DNS model, in the order of $1\,\mathrm{m}^3$ in a three dimensional model.

The formulation of Large Eddy Simulation (LES) models is based on the self similarity theory [Lesieur, 1997]. In a LES model, the large eddies in the turbulent flow are solved for, while smaller eddies are assumed to be self-similar and a turbulent model is applied to calculate the influence from the small eddies on the large eddies. Deriving the LES from the Navier Stokes equation, see for instance [Lesieur, 1997], new terms appear in the stress term (last term on the right hand side of equation Eqn. 2.3). The new terms may be described by a subgrid scale tensor such as

$$T_{ij} = \overline{u_i u_j} - \overline{\overline{u_i u_j}} \left(\overline{u_i} u_j^{'} + \overline{u_j} u_i^{'} \right) - \overline{u_i^{'} u_j^{'}}, \tag{2.6}$$

where $\overline{u}$ describe the resolved velocities and $u^{'}$ describe the unresolved, subgrid scale fluctuations. The first term on the right hand side of Eqn. 2.6 may be taken from the resolved velocity field, while the other terms have to be calculated in a subgrid scale turbulent model. To make the assumption of self similarity, a uniform grid scale of about 1 m has to be applied in ocean conditions. A less dense grid is needed in a LES model, compared to a DNS model, though the formulation of a valid subgrid scale turbulent model is not trivial and complex turbulent models may be demanding in computer capacity. An advantage with the LES models is that the model reverts to DNS with high enough resolution [Fringer, 2009].

For numerical studies on ocean scale, it is common to use the Reynolds Averaged Navier Stokes equations (RANS). Based on the Reynolds averaged methods, the flow variables are divided into a mean part and a deviation from the mean part, for the velocity $\mathbf{u} = \overline{\mathbf{u}} + \acute{\mathbf{u}}$, for the density $\rho = \overline{\rho} + \acute{\rho}$, and for the pressure $p = \overline{p} + \acute{p}$. Averaging over all terms in the Navier Stokes equation, the Reynolds stress, $\rho\overline{\acute{u}_i\acute{u}_j}$, appears in the averaged momentum equation. This is the rate of mean momentum transfer by turbulent flow and may be interpreted as the influence from the local fluctuation on the mean motion. Compared to the subgrid scale tensor in LES, Eqn. 2.6, only the last Reynolds stress term is included in the RANS equations [Lesieur, 1997]. In studies on ocean scale and in numerical ocean models, it is common to assume that the molecular stress term is much smaller than the turbulent stress term, given by the Reynolds stresses. Though, studying systems on small scales or over a long time, the molecular stress term may be of importance and should be considered. In the two dimensional Cartesian coordinates (x,z,t), where x is in the horizontal direction, z is in the positive upwards vertical direction, and t is the time, the Reynolds averaged momentum equations using the Boussinesq approximation and assuming no rotational effects may be written

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z}$$

$$= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(A_M\frac{\partial U}{\partial x}) + \frac{\partial}{\partial z}(K_M\frac{\partial U}{\partial z}), \quad (2.7)$$

$$\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial x} + W\frac{\partial W}{\partial z}$$

$$= -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - \frac{g\rho}{\rho_0} + \frac{\partial}{\partial x}(A_M\frac{\partial W}{\partial x}) + \frac{\partial}{\partial z}(K_M\frac{\partial W}{\partial z}). \quad (2.8)$$

In the equations above U and W are the horizontal and vertical velocity components respectively. Together with the continuity equation, Eqn. (2.2), density conservation equation, and values/models for the horizontal and vertical eddy viscosity parameters, A_M and K_M respectively, and eddy diffusivity parameters, A_H and K_H respectively, the set of equations form a solvable system. In the ocean it is common to exchange the density conservation equation with conservation equation for salt and temperature together with an equation relating the density with the pressure, salt and temperature (the equation of state).

2.5 Discretisation

To solve the hydrodynamic equations numerically, the equations have to be discretised on a grid system. The grid have to be specified in the three dimensional space and the most intuitive way to describe the grid is in Cartesian coordinates (x,z,y). Spatial descretisation methods are used to solve the equations within the grid and temporal descretisation methods are applied to step the solution forward in time. Together with the descretisation methods, the applied grid size and time steps further limit the possible temporal and spatial scales that can be resolved in the model.

One way to characterise numerical ocean models, is by the choice of the vertical coordinate. The most common vertical coordinates are shortly described below, together with some advantages and disadvantages. Description of different vertical and horizontal coordinates may be found in Haidvogel and Beckmann [1999], Kantha and Clayson [2000], Griffies [2004]. During the last years also hybrid or generalised vertical coordinates are introduced, combining the advantages with the different coordinates into varying coordinates in space [Griffies, 2004].

The $z-$coordinate models have a vertical coordinate that is fixed in space and time. Often evenly spaced, but may also be focused to the vertical layer of interest, for instance the surface layer. Non-Boussinesq and nonhydrostatic terms can be included together with the full equation of state. The regular grid make the $z-$coordinate simple to implement and to control for instance stability and conservation. A disadvantage is the step wise representation of the bottom. Some of the z-coordinate ocean models today try to solve this problem, by for instance introducing "shaved cells", that may highly improve the representation of the bottom [Adcroft et al., 1997].

In a $\sigma-$ or $s-$coordinate model a terrain following coordinate is introduced. With a constant number of vertical coordinates, stretching from the static depth H to the free surface η, the transformation from Cartesian-coordinates (x,y,z) to the new bottom following $\sigma-$coordinate may be described by

$$\sigma = \frac{z-\eta}{H+\eta}, \tag{2.9}$$

where the total depth $D \equiv H+\eta$ gives a σ-coordinate ranging from $\sigma = 0$ at $z = \eta$ to $\sigma = -1$ at $z = -H(x,y)$. The terrain following $\sigma-$coordinate is popular in coastal ocean models, with its smooth representation of the bottom topography and the advantage of increased resolution in shallow areas. It is also possible to focus the grid at the boundary layers, in both the surface and the bottom. The main disadvantage with a terrain following coordinate is the representation of the internal pressure gradients. In areas with steep topography, the skewness in the grid system may introduce pressure gradient errors. It is accordingly important to choose accurate methods for the estimation of the internal pressure gradients in such models, see for instance [Berntsen and Oey, 2009].

The isopycnal coordinate has favourably been used in large scale ocean studies as climate studies. Under adiabatic and statically stable conditions, the density will be a monotonic function in the vertical and a natural choice of coordinate. The main advantage with the isopycnal coordinate is the Lagrangian treatment of the vertical motion. Advection only have to be calculated for in the horizontal and numerical diffusion in the vertical is avoided. Moreover, the isopycnal models have a good representation of the bottom and conserves volume of density classes. Though, it is not possible to include nonhydrostatic dynamics or the full equation of state and isopycnal models have problems to represent mixing processes in boundary layers.

2.6 Subgrid scale closure

Physical processes that are not resolved within this grid system, subgrid scale processes, have to be parametrised. The ocean fluid is in general turbulent [Thorpe, 2005], and this have to be accounted for when formulating the parametrisation. All numerical models describe a mean value of the state of the fluid [Griffies, 2004], except fully resolving DNS. Statistical methods are then used to formulate a turbulent subgrid scale closure. The subgrid scale parametrisation is a fundamental problem in numerical ocean models [Griffies, 2004] and have to be treated carefully. Even if a physical phenomenon, such as eddies or internal waves, are resolved, all dynamical processes of the phenomenon may not be resolved and the unresolved physical processes have to be parametrised. This may be problematic when trying to formulate general parametrisation models that are case independent.

In ocean modelling, local closure schemes are prevalent. Depending on the numerical model, the special case study, and the resolution, the closure schemes vary from first order gradient theory models (as eddy coefficients) to more sophisticated higher order schemes based on turbulent theory (as the Smagorinsky formulation or $K-\varepsilon$ theory). From an energetic point of view, the subgrid scale parametrisation of the Reynolds stresses in a RANS model, appear as a sink term in the kinetic energy budget equation, energy is dissipated to subgrid scale turbulence. This energy cascade is one way directed, from resolved physical processes in the model to parametrised unresolved subgrid scale physical processes, and prohibit feedback processes transporting energy from smaller scales back to larger scales. In LES there may be interactions between resolved and unresolved scales, and ”back scattering” from the subgrid scale model may be allowed [Davidson, 2009].

2.7 Mode splitting

The models capacity to capture a signal in the model, will be limited by the applied spatial, Δx, and temporal, Δt, resolution.

To be able to resolve fast travelling surface gravity waves at the same time as small scale processes in the interior of the fluid, it is common in numerical ocean models to apply mode splitting. The calculation is splitted into an external mode and an internal mode. The depth averaged momentum equation are propagated with a fast time step in the external mode and the full equations are calculated with longer time steps capturing the internal mode. The external mode basically solves the shallow water equations, where the depth averaged horizontal velocities and the free surface are propagated in time. This is described for a $\sigma-$coordinate model in Blumberg and Mellor [1987]. With the mode splitting scheme, the model is able to solve for a wider range of scales with the same cost in computer capacity.

Even if the idea behind the mode splitting is intuitive, the detailed implementation may differ substantially between models. The specific choice of algorithm may affect the quality of the numerical results, see Ezer et al. [2002]. However, from the numerical evidence so far it is difficult to identify a best approach. For a more detailed description of mode splitting, see Kowalik and Murty [1993].

2.8 Boundary conditions

In numerical ocean models it is common to assume a no-flow condition at the closed boundaries. At the free surface, wind effects may be introduced using drag laws. The effects of bottom friction may also be parametrised through drag laws, see for instance Blumberg and Mellor [1987].

The boundary conditions at open lateral boundaries require very careful treatment. The user has to consider both incoming and outgoing signals and barotropic and baroclinic processes may have to be treated differently. Numerical artifacts may easily appear in the solution near open boundaries and there is accordingly a large literature on the topic, suggesting practical solution techniques, see for instance Engquist and Majda [1977], Garrett and Greenberg [1977], Røed and Smedstad [1984], Martinsen and Engedahl [1987], Gartling [1990], Johnsen and Lynch [1994], Guo and Zeng [1995].

Chapter 3

Nonhydrostatic pressure

Most numerical ocean models today are hydrostatic. With increasing computer power and with the introduction of multiprocessor capacity, today's hydrostatic models are applied with higher and higher resolution. Important questions to ask is then, when can we expect the hydrostatic approximation to fail, how do we implement nonhydrostatic pressure effects, and how may nonhydrostatic pressure effects influence the model results?

3.1 What is nonhydrostatic pressure

At a point in a fluid at rest, the pressure is the weight of the fluid above that point. This is the hydrostatic pressure and form the base for the hydrostatic approximation. When the hydrostatic approximation is applied to the vertical momentum equation, Eqn. 2.7, the acceleration and the eddy viscosity terms for the vertical velocity component are assumed to be much smaller than the gravitational acceleration term, and can thus be neglected. The hydrostatic approximation is a common simplification in numerical ocean models, and makes the set of equations considerably more efficient to solve. In the hydrostatic equations the vertical velocity will be a diagnostic parameter and is calculated directly from the continuity equation, Eqn. 2.2. When a fluid is in motion, the vertical pressure gradient is also influenced by the vertical acceleration and friction, and in the following this will be described as nonhydrostatic pressure effects.

To understand the physical background of the nonhydrostatic pressure, one may go back to Bernoulli's principle. Bernoulli's principle relate the velocity and the pressure in a fluid, and may be derived from the principle of energy conservation. In a barotropic and inviscid fluid Bernoulli stated that

$$p + \frac{1}{2}\rho u^2 + \rho g h = \text{constant} \tag{3.1}$$

along a streamline in a steady flow, where h is the elevation. With the assumption that the streamline follows a constant gravitational potential ($\rho g h = \text{constant}$) and defining p as the "static pressure" and $q = 1/2\rho u^2$ as the "dynamical pressure", then $p + q = \text{constant}$ or

$$\text{"static pressure"} \quad + \quad \text{"dynamical pressure"} \quad = \quad \text{constant.}$$

If q increase along that streamline, then p will decrease accordingly. In other words, if the velocity increase along the streamline, then the pressure in the fluid will decrease due to the "dynamical pressure". The "dynamical pressure" can not be described as a real pressure. The "dynamical pressure" may rather be described as the decrease in the pressure due to velocities in the fluid. In a hydrostatic numerical ocean model ”dynamical pressure” effects will be excluded and velocities and advection of water have to be compensated for in the hydrostatic pressure.

To justify the hydrostatic approximation, one may perform a scale analysis on the momentum equations [Gill, 1982, Marshall et al., 1997b]. The scale analysis show that the validity of the hydrostatic approximation is directly proportional to the ratio of the vertical (H) to the horizontal (L) length scales of the motion. In large scale ocean studies, the horizontal scales of motion is much larger than the vertical scales and the hydrostatic approximation is valid. In ocean studies with a horizontal resolution less than $1-10$ km, the hydrostatic approximation may not be valid and the influence of nonhydrostatic pressure effects have to be considered [Marshall et al., 1997b], see Fig. 2.1. In a stratified water mass, the vertical scale of motion will further be dependent on the stability in the density stratification. This stability may be described by the buoyancy period and in Marshall et al. [1997b] it was stated that ”If the advective timescale is short relative to the buoyancy period, the nonhydrostatic effects can not be neglected”.

There are many phenomena in the ocean where the nonhydrostatic pressure effects are of importance. One of these phenomena, that is well studied during the last years, is nonlinear internal waves. Internal waves are believed to play an important role in the energy budget of the world oceans, and the nonlinear form of internal waves may be more common than earlier believed. Nonlinear internal waves have been found in many places in the world oceans [Apel et al., 2006] and there are several numerical ocean studies that try to model these waves, see for instance Legg and Adcroft [2003], Moum and Smyth [2006], Moum and Nash [2008], Moum et al. [2008]. Negative buoyancy production in the surface, due to cooling at high latitudes or evaporation at low latitudes, may cause deep convection and vertical accelerations [Marshall and Schott, 1999]. Large scale convection at high latitudes are believed to be important for the production of deep water and to play a key role in the meridional over turning circulation. Other phenomena that are connected to strong buoyancy forcing and nonlinear physical processes are bottom density currents. Also bottom density currents are important in the formation and transportation of dense bottom water at high latitudes, in both the southern and northern hemisphere. Bottom density currents may also be essential for the renewal of the stagnant bottom water in semi-enclosed oceans and fjord systems [Stigebrandt, 1987, Arneborg et al., 2007]. There are several numerical investigations of bottom density currents, see for instance Özgökmen et al. [2006] and references herein. One may also find density driven flows at the surface, in the form of gravity plumes generated by freshwater discharge from large rivers. To capture the rotating head in the plume front and the generation of nonlinear waves, nonhydrostatic pressure effects are essential, see for instance Nash and Moum [2005], Orton and Jay [2005], Stashchuk and Vlasenko [2009], Pan and Jay [2009]. Front instabilities in the open ocean may be important for the mixing of water masses and are studied in Mahadevan [2006], Mahadevan and Tandon [2006]. There are many situations of super critical flows over topography in the ocean, and these flows are connected to strong nonlinearities. This may be due to strong forcing in sill regions in fjords and inlets [Farmer and Armi, 1999a, Moum et al., 2003, Xing and Davies, 2006a,b, 2007, Berntsen et al., 2008], but may also be due to strong tidal forcing over steep topography in the deep oceans [Legg and Klymak, 2008].

To capture all the important features in the phenomena mentioned above, nonhydrostatic pressure effects are essential.

3.2 Nonhydrostatic pressure correction methods

When solving for the total pressure in incompressible fluids, in for instance DNS or LES models, it is common to apply fractional-step algorithms to implicitly solve for a divergence free velocity field. In a first step advancing the momentum equation without the non-divergence constraint and in a second step the divergent part is removed to retain a divergence free fluid. The divergent part take the form of a Poisson equation that has to be solved. Some of the methods to solve the Poisson equation are presented and analysed in Armfield and Street [2002], and classified as; projection methods, correction methods, and iterative methods. The iterative methods are the only ones that may, in theory, give exact solutions to the descretised Navier Stokes equation [Armfield and Street, 2002]. Though, the iterative methods tend to be computational demanding and a level of accuracy is often introduced to limit the number of iterations.

Most of the numerical ocean models used today are RANS type of models built on a hydrostatic platform. Some of these models are well established and have large user groups. When formulating a nonhydrostatic ocean model, it may be beneficial to introduce nonhydrostatic capacities in these well established and well tested hydrostatic models. For ocean scale applications, one may expect that even if the fluid is influenced by nonhydrostatic pressure effects, the major part of the total pressure field will still be hydrostatic. Taking advantage of this relation, the pressure field may be decomposed into a hydrostatic, P_h, and a nonhydrostatic, P_{nh}, part. The hydrostatic part includes the pressure from the surface elevation, P_η and the internal pressure, P_{int}, so that $P_h = P_\eta + P_{int}$. The total pressure may be described as

$$P(x,z,t) = P_\eta(x,t) + P_{int}(x,z,t) + P_{nh}(x,z,t), \tag{3.2}$$

where $P_\eta = g\rho_0\eta(x,t)$ and $P_{int} = g\int_z^0 \rho(x,\acute{z},t)d\acute{z}$.

When introducing nonhydrostatic pressure into these models, the concept of the fractional-step methods, described above, are used to correct for nonhydrostatic pressure. The hydrostatic part of the models are used in a first step to calculate a provisional hydrostatic velocity field ($\tilde{U}$, $\tilde{V}$, $\tilde{W}$) and correction methods are used in a second step to achieve a non-divergent nonhydrostatic velocity field (U, V, W). In the second step, momentum equations for the nonhydrostatic pressure, P_{nh}, have to be solved together with the continuity equation, and may be expressed in two dimensional Cartesian coordinates, (x, z, t), as

$$\begin{aligned} \frac{\partial(U-\tilde{U})}{\partial t} &= -\frac{1}{\rho_0}\frac{\partial P_{nh}}{\partial x}, \\ \frac{\partial(W-\tilde{W})}{\partial t} &= -\frac{1}{\rho_0}\frac{\partial P_{nh}}{\partial z}, \\ \frac{\partial U}{\partial x}+\frac{\partial W}{\partial z} &= 0. \end{aligned} \tag{3.3}$$

In RANS models applied for ocean scale problems, the pressure split method described above is much more efficient than to directly solve for the full three dimensional pressure field [Kanarska and Maderich, 2003]. Several nonhydrostatic models are listed and categorised in Kanarska and Maderich [2003]. Some of these models divide the pressure into a hydrostatic and a nonhydrostatic part, and other solve for the full pressure. For ocean application, the numerical ocean model should allow for a free surface and for baroclinicity. There would further be an advantage to perform a mode split, described in Section 2.7, to be able to calculate for a wider range of horizontal scales. Nonhydrostatic pressure effects have been included in the MITgcm model [Marshall et al., 1997b,a]. The MITgcm use z-coordinates in the vertical and spherical coordinates in the horizontal. The Princeton Ocean Model (POM) has been applied with nonhydrostatic pressure capacity [Kanarska and Maderich, 2003]. Nonhydrostatic pressure effects have also been introduced in the ROMS model [Kanarska et al., 2007]. The ROMS model apply terrain following coordinates in the vertical and curve linear coordinates in the horizontal. In recent years several numerical models with a unstructured grid system have been developed. An unstructured grid model, designed for coastal oceans and with nonhydrostatic pressure capacity is described in Fringer et al. [2006].

In the present work the Bergen Ocean Model (BOM) with the capacity to calculate nonhydrostatic pressure have been used. The BOM have a similar platform as the POM, with terrain following coordinates in the vertical and a Cartesian regular grid in the horizontal [Mellor, 2003]. The implementation of nonhydrostatic pressure in BOM follows the ideas with pressure correction methods presented in Casulli [1999], Marshall et al. [1997b], Kanarska and Maderich

[2003]. Nonhydrostatic pressure was first introduced in BOM [Heggelund et al., 2004] in a similar manner as in Kanarska and Maderich [2003], where the nonhydrostatic pressure equations in Eqn. 3.4 are transformed into a $\sigma-$coordinate system. This transformation complicate the equations, and it was suggested in Berntsen and Furnes [2005] to calculate the nonhydrostatic pressure directly in the $\sigma-$coordinate system. The solution methods in Berntsen and Furnes [2005] have been compared to the solution methods in Kanarska and Maderich [2003] in a recent work by Keilegavlen and Berntsen [2009].

3.3 Elliptic solver and boundary condition

Combining the momentum equations and the continuity equation for the nonhydrostatic pressure in Eqn. 3.4 above, an elliptic Poisson equation may be formulated in Cartesian coordinates as

$$\frac{\partial^2 P_{nh}}{\partial x^2} + \frac{\partial^2 P_{nh}}{\partial z^2} = \frac{\rho_0}{\Delta t}\left(\frac{\partial \tilde{U}}{\partial x} + \frac{\partial \tilde{W}}{\partial z}\right), \tag{3.4}$$

where the nonhydrostatic pressure is driven by the velocity gradients in the provisional velocity field. Different methods may be applied to solve the elliptic equation. In the present model iterative methods are applied, starting from the nonhydrostatic pressure field from the last time step. A Successive Over-Relaxation method (SOR) is applied to solve the elliptic equation [Wachspress, 1966]. In most of the calculations in the present work a constant number of iterations are performed. It has been tested that the numerical results are robust to an increase in the number of iterations.

To solve the elliptic equation for the nonhydrostatic pressure, proper boundary conditions are needed. Since no flow are allowed through the bottom and through closed lateral boundaries, a zero pressure gradient condition towards the boundary have to be applied here giving a homogeneous Neumann condition ($\partial P_{nh}/\partial n = 0$). At open lateral boundaries, different boundary conditions may be applied, both Neumann and Dirichlet condition, and will be dependent on the forcing and relaxation methods used at the open boundaries. At the free surface, two different boundary conditions may be found in the literature. One may argue for a Dirichlet condition, $P_{nh} = 0\, at\, z = \eta$, that will make sure that the total pressure becomes equal to the atmospheric pressure at the free surface, since the hydrostatic pressure will be zero at the free surface. In the present work we argue for a Neumann condition for the nonhydrostatic pressure at the free surface. With a homogeneous Neumann condition at the surface the nonhydrostatic pressure field will be divergent free, and will not influence the free surface elevation. This is consistent with the mode splitting idea, see Section 2.7, and is further discussed in Paper B.

Chapter 4

Internal waves

In this Chapter nonlinear internal waves will be presented, with the purpose to give a background for a discussion on the importance of nonhydrostatic pressure for this type of waves. A simple, weakly nonlinear theory for internal waves is presented to show the differences between a hydrostatic and a nonhydrostatic numerical ocean model.

4.1 General aspects of internal waves

Internal waves are common in all stratified fluids, oceans, lakes, and in coastal brackish waters. Mechanical energy from an external source, as the wind or the tide, is needed to start disturbances of the stratified water mass. There are many types of internal waves, and there is a diverse literature on general wave and internal wave theory, see for instance Thorpe [1975], Garrett and Munk [1979], Gill [1982], Kundu [1990], Pedlosky [2003], Helfrich and Melville [2006], Apel et al. [2006].

In summertime the heating from the sun creates a well established warm surface layer and a sharp thermocline is often found in oceans and in lakes. This creates a system with a shallow surface layer and a deep bottom layer. In winter time, enhanced mixing due to strong winds and storms can create deep mixing, and in coastal areas and in shelf seas a pycnocline may be generated close to the bottom. This creates a system with a thick surface layer and a thin bottom layer [Moum et al., 2008]. Also in restricted systems, as fjords and inlets, it is common to find a pycnocline that separates the well mixed surface layer from the stagnant bottom layer. When a large freshwater discharge enters the ocean, a halocline will separate the fresh surface water from the salt ocean water below.

To cover all aspects of internal waves, a theory of internal waves that stretch from the large scale forcing mechanism, as tidal waves and weather systems with length and time scales of hundreds of kilometres and days, down to the turbulent dissipation scale, with length and time scales of millimetre and seconds, would be necessary. This is not possible, so, experiments, measurements, and theories usually cover parts of these length and time scales. Some of these aspects on internal waves are discussed in Thorpe [1975].

4.2 Nonlinear internal waves

The theory of linear internal waves is well known and well studied. With modern in situ and remote sensors a more detailed picture of the interior and the surface of the ocean has revealed new knowledge of internal waves, and that nonlinear internal waves may be a more pronounced phenomenon than was earlier believed

[Jackson and Apel, 2002]. Nonlinear wave theory from the end of the 19th century, in the 1870s by Boussinesq and Releigh and in 1895 by Korteweg and De Wries, was reborn during the 1960s and 1970s and the interest and the knowledge of nonlinear internal waves, and more specifically nonlinear internal waves of soliton character, have been growing since, see for instance Helfrich and Melville [2006], Apel et al. [2006], Grue [2006], Morozov [2006].

Nonlinear waves at the Origon Shelf are well studied during the last years, in summertime as surface trapped waves and in winter time as bottom trapped waves, see Moum et al. [2003], Klymak and Moum [2003], Moum and Smyth [2006], Moum and Nash [2008], Moum et al. [2008]. Nonlinear internal waves may also be generated at the sill in fjord systems as in the Knights Inlet [Farmer and Armi, 1999a,b, Cummins et al., 2003] or in the Loch Etive [Inall et al., 2004]. There are also many places in the open ocean where strong tidal motions create large amplitude solitary waves when passing over topography, for instance in the South China Sea [Zhao et al., 2004, Lien et al., 2005, Klymak et al., 2006, Shaw et al., 2009]. Most of the areas in the ocean where internal solitary waves are found regularly are collected in Jackson and Apel [2002].

Lakes form a controlled volume of water, and may be easily accessible for measurements. Several measurements and theoretical studies of internal waves in lakes have been performed. Some aspects of the nonlinearity of these waves may be found in Farmer [1978], Horn et al. [1986], Boegman et al. [2003], Hodges et al. [2000].

There are several studies of internal waves in laboratory tank experiments. In the laboratory one may repeat the experiments in a controlled environment, and the influence from different initial conditions and strength of the forcing may be detected. The tilted tank experiments presented in Horn et al. [1999, 2001, 2002] are well known and has inspired several similar studies. The energetics for different wave types in this system is calculated and discussed in Boegman et al. [2005]. A different approach with a shoaling single soliton wave is studied in Michallet and Ivey [1999].

Numerical experiments of internal waves on laboratory scale have been performed by for instance [Wadzuk and Hodges, 2004, Berntsen et al., 2006, Kanarska et al., 2007, Botelho et al., 2009, Keilegavlen and Berntsen, 2009]. Nonlinear effects of the internal tide at the Hebride shelf are studied in a numerical ocean model in [Xing and Davies, 2001]. Several aspects of wind-induced internal waves are modelled in Davies and Xing [2004]. The aspects of energy and mixing of shoaling internal waves are the subject in several numerical studies [Vlasenko and Hutter, 2002a,b, Bourgault and Kelley, 2003, Legg and Adcroft, 2003, Vlasenko and Stashchuk, 2007, Bourgault and Kelley, 2007, Bourgault et al., 2007, Shroyer et al., 2008, Thiem and Berntsen, 2009]. There are also several numerical studies of the generation of nonlinear internal waves at the sill in fjords [Vlasenko and Hutter, 2001, Cummins et al., 2003, Lamb, 2004, Xing and Davies, 2006a,b, 2007, Davies and Xing, 2007, Berntsen et al., 2008, 2009].

4.3 Weakly nonlinear theory

With the assumption of a two-layer system, theories may be developed that describe the generation, propagation and transformation of internal gravity waves. With a depth and density in the upper layer, h_1 and ρ_1, and in the lower layer, h_2 and ρ_2, the phase speed of linear internal waves under the long wave approximation may be described by $c_0 = \sqrt{g'H}$, where $H = h_1 + h_2$ is the total depth, and $g' = g(\rho_2 - \rho_1)/\rho_2$ is the reduced gravity.

A widely used theory for weakly nonlinear internal waves is the KdV (Korteweg-de Vries) theory, describing the interface displacement ζ of a unidirectional progressive wave. In the two-layer system described above, the KdV equation may be written as

$$\frac{\partial \zeta}{\partial t} + c_0 \frac{\partial \zeta}{\partial x} + \alpha \zeta \frac{\partial \zeta}{\partial x} + \beta \frac{\partial^3 \zeta}{\partial x^3} = 0 , \tag{4.1}$$

where $\alpha = 3/2c_0(h_1 - h_2)/h_1h_2$ is the nonlinear parameter and $\beta = c_0h_1h_2/6$ is the nonhydrostatic dispersion parameter. The weakly nonlinearity assumption restrict α and β to be comparable and small. Without the two-layer assumption, the parameters α and β take more general forms [Apel et al., 2006]. The KdV equation may be derived from the hydrodynamic equations under incompressibility and the Boussinesq approximation or from the Boussinesq equations [Apel et al., 2006]. A particular solution to Eqn. 4.1 is the solitary wave

$$\zeta(x - c_0t) = a \,\mathrm{sech}^2 \left(\frac{x - c_0t}{\lambda} \right) , \tag{4.2}$$

where a is the wave amplitude, and the phase speed c and the horizontal length scale λ of the wave are given by

$$c = c_0 + \frac{1}{3}\alpha \quad , \quad \lambda^2 = 12 \frac{\beta}{a\alpha} . \tag{4.3}$$

4.4 Basin scale waves

Performing laboratory experiments in a closed tank, analytical wave regimes partly based on the KdV-theory were developed in Horn et al. [2001]. The regimes describe the degeneration of an initial basin scale standing wave into different wave forms. The different regimes, ”damped linear waves”, ”solitary waves”, ”Kelvin-Helmholtz billows”, ”bores and billows”, and ”super critical flow”, are restricted by the initial conditions.

For weak initial disturbances, the viscous damping is relatively large, and the wave take the form of a damped linear wave. With moderate initial disturbances, the nonlinearities are stronger and the wave initially take the form of an internal surge and may later degenerate into a package of solitary waves. Initially the dispersion effects are small, and the wave may be described by the nonlinear and non-dispersive wave equation, excluding the last term on the right hand side of Eqn. 4.1. The strong nonlinearities force the wave to steepen under the balance between the unsteady term $\partial\zeta/\partial t$ and the nonlinear term $\alpha\zeta(\partial\zeta/\partial x)$. A timescale may be calculated for the initial disturbance of the interface to reach to a vertical position, a steepening timescale $T_s = L/\alpha\zeta_0$ [Farmer, 1978, Horn et al., 1999, 2001, Boegman et al., 2005]. The wave dispersion becomes stronger closer to T_s, and the full Eqn. 4.1 is now needed to describe the wave, and finally, the dispersion will partly balance the nonlinear steepening and the wave will degenerate into a package of solitary waves.

Under the hydrostatic approximation, the wave dispersion represented by the last term on the right hand side of Eqn. 4.1, will be absent [Horn et al., 1999, Wadzuk and Hodges, 2004]. A hydrostatic numerical ocean model is not able to represent the solitary waves, and the nonlinear steepening of the internal wave must then be balanced by numerical or parametrised mixing [Hodges et al., 2006]. In a nonhydrostatic numerical model, the dispersion effects may be present. However, the strength of the nonhydrostatic pressure will also depend on the grid resolution, the numerical viscosity, and the subgrid scale parametrisation. Some of these issues are described and discussed in Papers A and C.

4.5 Extended KdV theories

The KdV theory describes the development of a single nonlinear internal wave. In a real situation, the wave motion is a combination of several wave signals on top of each other. Trying to describe this, theories have been developed to extend the KdV theory. One may include an extra cubic nonlinear term to the KdV equation, Eqn. 4.1, to form the eKdV equation [Helfrich and Melville, 2006]. The eKdV equation is able to give more wave solutions than the KdV equation, and a broadening of the wave is possible. The broadened soliton wave form is common in the ocean. More developed theories of the KdV type are found in the literature, see for instance Grue et al. [1997, 2000], Apel et al. [2006], Helfrich and Melville [2006]. These types of theories all describe a wave motion around the position of the undisturbed interface.

4.6 Solitary wave packages

In the ocean and in lakes it is more common to find packages of soliton waves on top of a propagating front, with the depth of the interface being different before and after the passage of the wave package. These packages have the largest wave amplitude in front and decreasing amplitude towards the end of the propagating wave package. These wave packages may for instance be generated due to a strong and short wind pulse in the ocean or in a lake. Advection of the surface water generate downwelling or upwelling at the coast and large vertical displacements of the stratified water masses. These large scale depressions or elevations, respectively, may leave the coast in the form of an internal surge. If the nonlinearities are strong, the surge may develop into a package of solitary waves on top of the surge. This type of solitary wave packages may be described as ”solibores” [Apel et al., 2006].

Extending the KdV-theory with an Jacobian elliptic function $cn_s(x)$, a theory to describe the solibores are developed, and these waves are called ”cnoidal” waves. Further including a periodic elliptic function, $dn_s(x)$, a theory to describe the situation of solibores due to weakly nonlinear internal tide are described in Apel [2003] and Apel et al. [2006], and these waves are accordingly named ”dnoidal” waves.

4.7 Rotational effects

For mesoscale processes in the ocean, the influence of the Earth’s rotation becomes important. With the restriction of the wave motion to have a frequency in between, but not too close to, the Coriolis frequency, f, and the Buoyancy frequency, N, both the high and the low dispersion effects are assumed to be small and a KdV type of theory including the Coriolis effects may be developed. This theory may be called rotational modified KdV theory, or rKdV theory, and is shortly described in Apel et al. [2006].

Chapter 5

Summary and future work

5.1 Summary

Motions in the ocean stretch from large scale ocean circulation down to molecular scale, and imply a range of scales from thousands of kilometres and years down to millimetres and seconds. It is not possible to resolve all these scales in one numerical model. In one end of the range of scales are climate and general ocean circulation models, typically RANS models with the Boussinesq and the hydrostatic approximation. These models have been used successfully in ocean studies with grid scales down to a few kilometres. In the other end of the range of scales are DNS models solving the Navier Stokes equations directly and need a grid resolution on millimetre scale. The LES models are in between, but with the assumption of self similarity a grid resolution on metre scale is needed for these models. There are many important physical phenomena on kilometre scale, and trying to study these phenomena in a numerical model one end up in the gap between traditional ocean circulation models and engineer type of LES and DNS models. When applying a traditional ocean circulation model for these problems, a grid scale less than a kilometre is needed, and some of the basic simplifications in the model may no longer be valid. There are several of the traditional simplifications that are doubtful, but in the present work the focus is on the hydrostatic approximation. By implementing nonhydrostatic pressure correction methods in a RANS type of ocean model, the gap between the RANS models and the LES models may be reduced, see Fig. 2.1.

5.2 Introduction to articles

The strength of the nonhydrostatic pressure and the related pressure gradients will depend on the resolution in the numerical model. In Paper A the influence of the nonhydrostatic pressure on the generation and propagation of wind induced steepening internal waves are studied with a range of grid scales, from 1 km down to 62.5 m. A simple form of the KdV equation is also presented in the paper to describe the differences for a steepening internal wave in a hydrostatic and a nonhydrostatic model. The numerical model results show that with high enough resolution nonhydrostatic dispersion effects become important, as indicated by the KdV theory, and the wave degenerates into a solitary wave package.

When introducing nonhydrostatic pressure into hydrostatic numerical ocean models it is common to use pressure correction methods, that require the solution of an elliptic equation for the nonhydrostatic pressure. Two different boundary conditions at the free surface for the nonhydrostatic pressure are found in the literature, a Neumann condition and a Dirichlet condition. In Paper B a mode split $\sigma-$coordinate ocean model with pressure correction methods is used, and the two boundary conditions described above are applied in the model and tested in three different experiments. With a Dirichlet condition at the free surface, the velocity field will not be divergence free and the free surface elevation have to be adjusted accordingly. In a mode split model, this may influence the solution of the depth averaged momentum equation in the external time step. The influence of the two boundary conditions on the results, will depend on the vertical resolution, the difference between the internal and external time step, and the importance of nonhydrostatic pressure in the surface layer. Some of these aspects are addressed in Paper B, and it is argued that the appropriate surface boundary condition for mode split ocean models is the Neumann condition.

With a horizontal resolution of 62.5 m in Paper A the nonhydrostatic pressure effects were too small to influence the internal wave during the generation phase. One may expect that the strength of the nonhydrostatic pressure increase further with higher resolution. In Paper C the same wind forcing is applied as in Paper A. By introducing an open boundary at the western side, the size of the model domain may be reduced, and the influence of nonhydrostatic pressure during the generation phase is studied with higher resolution. The results show that the nonhydrostatic pressure becomes strong enough with a horizontal grid size of 12.5 m to influence both the density and velocity fields. With this grid size, the solutions are affected by the flow of energy towards the scales of Kelvin-Helmholtz instabilities and wave like patterns appear near the lateral boundary.

5.3 Future work

In the present work the main focus has been on the influence of nonhydrostatic pressure on internal waves, even though, there are still many aspects concerning internal waves and nonhydrostatic pressure that need to be addressed. There are also many other physical phenomena for which nonhydrostatic pressure effects are essential. Some of these phenomena, like tidal flow over sills and constrictions, have been studied with cross sectional models. Other phenomena, especially fully three dimensional phenomena like vorticies, need more attention.

To be able to fully resolve the nonhydrostatic pressure, a grid resolution of about 1 m is needed, where the vertical and horizontal length scales of motion are the same. It would be an advantage in the future, when designing new experiments, to restrict the model domain so that an overall grid resolution of 1 m is possible. This approach, will off course limit the largest scales that can be resolved in the model, and hence not useful when studying some types of physical phenomena.

One of the major challenges in numerical ocean modelling is to represent unresolved physical processes through subgrid scale parametrisation. The subgrid scale parametrisation used in RANS type of ocean models are not designed for grid resolutions down to a metre.

The validity for these types of parametrisations has to be reconsidered in high resolution nonhydrostatic studies. With higher resolution the difference between the vertical and the horizontal scales of motion will be smaller. In hydrostatic ocean models, it is common to separate the vertical and the horizontal subgrid scale parametrisation. This may be problematic in a high resolution nonhydrostatic study. The formulation of subgrid scale parametrisation for these types of studies need to be further investigated, and for this, ideas and influence from the subgrid scale parametrisation in LES models may be used. In general, more high resolution studies of nonhydrostatic small scale physical phenomena, may also form a platform for further developments of subgrid scale parametrisations used in larger scale models.

The parametrisation used in large scale RANS models are based on the assumptions that are questionable and the parameters involved are typically uncertain. The model outputs are accordingly often sensitive to the choice of subgrid scale scheme and/or the parameters involved. A way forward towards better parametrisations may be to perform high resolution nonhydrostatic simulations of a range of physical phenomena, and also to record the time mean model fields. The exercise may be repeated with courser resolution hydrostatic RANS models to investigate the quality of present parametrisations. Possible mismatches may hopefully lead to improved parametrisations.

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Part II

Papers and Reports