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Investigating the Asymmetric Behavior of Oil Price Volatility Using Support Vector Regression

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Abstract

This paper investigates the asymmetric behavior of oil price volatility using different types of Asymmetric Power ARCH (APARCH) model. We compare the estimation and forecasting performance of the models estimated from the maximum likelihood estimation (MLE) method and support vector machine (SVM) based regressions. Combining nonparametric SVM method with parametric APARCH model not only enables to keep interpretations of the parametric models but also leads to more precise estimation and forecasting results. Daily or weekly oil price volatility is investigated from March 8, 1991 to September 13, 2019. This whole sample period is split into four sub-periods based on the occurrence of certain economic events, and we examine whether the asymmetric behavior of the volatility exists in each sub-period. Our results indicate that SVM regression generally outperforms the other method with lower estimation and forecasting errors, and it is more robust to the choice of different APARCH models than the MLE counterparts are. Besides, the estimation results of the SVM based regressions in each sub-period show that the ARCH models with asymmetric power generally perform better than the models with symmetric power when the data sub-period includes large swings in oil price. The asymmetric behavior of oil price volatility, however, is not detected when the analysis is done using the whole sample period. This result underscores the importance of identifying the dynamics of the dataset in different periods to improve estimation and forecasting performance in modelling oil price volatility. This paper, therefore, examines volatility behavior of oil price with both methodological and economic underpinnings.

Keywords Conditional volatility · Oil price · SVR · Asymmetry · APARCH

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1 Introduction

Financial time series estimation and forecasting is of great importance in financial decision-making. Extensive studies in time series analysis have, thus, developed different methods in estimation and forecasting financial returns and volatility. In particular, effects of oil price volatility is far-reaching in the economy as a whole. Sharp increases in oil price volatility have shown close links with asset prices over the past decades (Sadorsky, 1999; El-Sharif et al., 2005; Chiou & Lee, 2009; Fratzscher et al., 2014; Miller & Ratti, 2009). The impacts of oil price fluctuations on the macroeconomy, including on economic growth and inflation, are also well-documented in a considerable body of literature (Hamilton, 1996, 2007; Ferderer, 1996; Barsky & Kilian, 2004; Blanchard & Gail, 2008; Kilian, 2008; Chen, 2009). Hence, estimating and forecasting oil price volatility is very critical. Generalized autoregressive conditional heteroscedasticity (GARCH)-type model was developed by Bollerslev, and it is a standard method in volatility modelling, especially for modelling the volatility clustering in financial data (Kang et al., 2009; Klein & Walther, 2016). Beyond the two stylised characteristics' in the financial series, namely, volatility clustering and fat-tail leptokurtosis, there are also asymmetric effects in financial (Black, 1976; Christie, 1982; French et al., 1987; Bekaert & Wu, 2000)[1] and oil markets (Ramos & Weiga, 2013; Ewing & Malik, 2013). This means that the volatility changes asymmetrically under different market situations—for example, they become more volatile during financial crises or unexpected events and less volatile during periods of relative steady economic growth. Ding et al. (1993) suggested what is known as the Asymmetric Power ARCH (APARCH) model, which has been rapidly used widely in empirical studies on volatility in finance since it can capture asymmetric volatility responding to positive and negative news. The APARCH types of model include correlation structures in odd power forms, thus the leverage effect between asset returns and volatility can also be analysed. Moreover, compared with the assumption on a linear relationship between return and volatility in GARCH models, the APARCH types of model, which also include the GARCH model as one type, allow a more flexible autoregressive structure of returns.

There are a few studies that employ GARCH-type models in the analysis of oil price volatility (Cheong, 2009; Kang et al., 2009; Lux et al., 2016; Wei et al., 2010), which mostly compare modelling performance based on maximum likelihood estimation (MLE). However, the efficiency of MLE depends on the distribution of data and

[1] Two existing hypotheses explain the asymmetric volatility-return in the stock market. The first is the leverage hypothesis (Black, 1976; Christie, 1982), which postulates that the a negative stock return causes the financial leverage of a firm to increase, which subsequently increases the volatility. The second hypothesis is the volatility feedback hypothesis (French et al., 1987), which states that an increase in the volatility raises the required return on equities which leads to an immediate stock price decline (by assuming a constant dividend). The first hypothesis concerns why an increase in the volatility causes a negative return, while the latter hypothesis explains why a negative return results in an increase in the volatility (İnkaya and Okur, 2014). Bekaert and Wu (2000) proposed that these two effects may be interacting and suggested the use of asymmetric GARCH-in-mean models.

innovations in the dataset. If the distribution of innovations is not consistent with the prior assumptions, the MLE can be inefficient (Bollerslev & Wooldridge, 1992; Engle & González-Rivera, 1991). As is described in the following section, studies on oil price return volatility are based on logarithmic prices, which is taken as a random walk process with white noise. This implies that the oil price returns are normally distributed, which lacks clear supporting evidence. Furthermore, the MLE method is purely parametric, where the parameters vary according to the choice of APARCH model. If the assumption of distribution or model structure deviates from the real data, the MLE will be inefficient. Thus, a distribution-free or data-driven estimation method is desired.

There exist certain data-driven nonparametric methods in estimating volatility of time series such as the "realized volatility" forecasting method (Barndorff & Shephard, 2002; Andersson et al. 2003). In those studies, volatility is estimated based on the cumulative intraday squared returns and the estimation is constructed directly from the observed data. This nonparametric realized volatility forecasting method is utilized in the area of energy market in Chan et al. (2008) and Haugom et al. (2010) among others. Some other nonparametric based methods including Buhlamann and McNeil algorithm (Buhlamanna & McNeil, 2000), Chung algorithm (Chung, 2014) are nonparametric regressions which are combined with GARCH models to estimate and forecast volatility. Bildirici and Ersin (2009) combines nonparametric Artificial Neural Network (ANN) with large class of GARCH family models in volatility modeling. Among the above-mentioned methods, the "realized volatility" method is a pure nonparametric method and it has no clear structure of the volatility models. The Buhlamann and McNeil algorithm, Chung algorithm and the model from Bildirici and Ersin (2009) can be viewed as semi-parametric methods and they are more interpretable than pure data driven methods due to their utilization of parametric GARCH models. However, both Buhlamann and McNeil algorithm and Chung algorithms use the assumptions that innovations in data are identical and independently distributed, and that the transformed distribution of innovations is normal (Cassim, 2018). As regards the ANN + GARCH method in Bildirici and Ersin (2009), the optimization solution obtained by ANN are sensitive to the places of initial randomization, therefore, a local solution can be mistakenly achieved instead of a global solution.

Support vector machine (SVM) based regression, called support vector regression (SVR), is also a data-driven technique. It does not require to have a priori assumption regarding the distribution of the data at hand (Chen & Karl, 2010; Li, 2014; Ou & Wang, 2010; Peréz-Cruz et al., 2003). Compared with ANN, however, a unique and global solution can be found in SVR by solving a quadratic optimizing problem with a linear constraint (Peréz-Cruz et al. (2003), Ou and Wang (2010)). Furthermore, SVR aims at minimizing the structure risk, instead of empirical risk, so that the regression function has a well-performed generalization ability (Vapnik, 1995). Thus, instead of using solely either MLE method to estimate the parameters in ARARCH model or other nonparametric methods to estimate the volatility, this paper estimates volatility by combining APARCH model with SVR, and construct a semi-parametric technique to estimate and forecast oil price volatility. This semi-parametric technique not only maintains advantages of the parametric methods, such as being flexible to the data structure, but also can provide better explanations of the works going on inside the black box. As is presented in later sections, the input and output variables used in SVR are decided by the structure of the APARCH model, and the result have the same interpretability as those of the original APARCH model regarding asymmetric effects. Thus, this semi-parametric method is more precise and more transparent compared with pure data-driven nonparametric methods. Some previous studies have used an SVR and GARCH combination, called the SVR-GARCH model, to estimate volatility. For example, Peréz-Cruz et al. (2003) document a higher predictive ability of the estimated GARCH models by using the SVR than those found from using MLE for predicting the conditional volatility of stock market returns. Chen and Karl (2010) also show that SVM-GARCH models tend to perform better than other competing models in one-period-ahead volatility forecasting. Ou and Wang (2010) combine the SVM method with the classical GARCH (1,1), EGARCH (1,1) and GJR (1,1) models to forecast the financial volatility of ASEAN stock markets, and show that the hybrid models are resistant and robust to the highly volatile situation of the financial market. Wang et al. (2013) propose SVM based Markov-Switching Multifractal (MSM) model to forecast volatility in financial time series and applied the approach by using two stock indexes in the Chinese A-share market. Fu et al. (2019) employee SVM models to forecast foreign exchange rates. More recently, Sun and Yu (2020) and Aras (2021) document that hybrid models that are built in GARCH model, which is combined with SVR method, outperforms the standard GARCH models volatility forecasting. Sun and Yu (2020)'s hybrid model contains two-steps process where the first-step is maximum likelihood estimation for the conditional variance. Thus, their method still needs parametric assumptions of distribution of innovations. More specifically, Gaussian and student's t distribution are the pre-assumption of the distribution in Sun and Yu (2020).

Nevertheless, the SVR, does not need any prior information on the data distribution and better approximates the nonlinear characteristics of the data, such as volatility clustering, leptokurtosis and leverage effects, as well as a prediction improvement (Bildirici & Ersin, 2009; Ou & Wang, 2010) for data with any distribution. Furthermore, using a specified kernel function such as Gaussian kernel to achieve nonlinear mapping and using just support vectors in predicting can resolve the problem of the computational complexity of the large amount of data in the APARCH type of model. The SVR model has been used for crude oil price forecasting in some related previous literature (as is listed in Table 3 in Zhang et al., 2015). However, most of the previous literature use the SVR as a pure data-driven nonparametric method and is comparable with some typical machine learning methods such as the artificial neural network (ANN) or the Ensemble EMD (EEMD) methods. Although Zhang et al. (2015) combine SVR with GARCH models to forecast oil price, no studies that combine the SVR and APARCH models to investigate estimate and evaluate oil price volatility including the asymmetric behaviors of oil price volatility have previously been conducted.

Based on these aforementioned previous studies, this paper takes account of the importance of capturing the asymmetric effect in oil price volatility forecasting using different GARCH type models and combine those models with SVR. Additionally, we look at the asymmetric behavior of volatility in different time periods that are split into four sub-periods to examine whether there is a model that suits the best in those different sub-periods or it varies across those periods. To the best of our knowledge, this is the first study that deals with asymmetry using support vector regression model for oil price volatility estimation in different sub-sample periods. The contribution of this study is two-fold. First, we estimate different types of APARCH models containing the GARCH, GJR, TSGARCH and TGARCH models, among which GJR and TGARCH models can capture the asymmetric structures. We evaluate both estimating and forecasting performance based on the results from the MLE and the SVR methods. Second, in the empirical investigation, the sample period is split into four sub-periods, based on the economic events, in order to examine the asymmetric behavior of volatility and to evaluate performance of different hybrid models (that is, APARCH models combined with SVR) in those different sub-periods. Our results indicate that the SVR method outperforms the MLE method with lower estimation and forecasting errors, and it is more robust to the choice of APARCH model than the MLE counterparts are. The findings of this study, therefore, makes a modest contribution to the empirical literature since it compares estimation and forecasting performance of different volatility models with asymmetries using the SVR in different sub-sample periods associated with economic events.

The structure of the paper is as follows. In Sect. 2, we present a brief introduction to APARCH model, SVR method and motivations for using an hybrid APARCH-SVR model to estimate oil price volatility. Section 3, which comprises five sub-sections, is a comprehensive study on estimation and forecasting of oil price volatility based on empirical data from early 1991. This section shows the workflow that was done in steps to perform the empirical study and interpretations at those different stages. The final section, Sect. 4, contains conclusions and scopes for further study.

2 APARCH Model, SVR and Motivations of APARCH-SVR Estimation

Let p_t denote the oil price at time t and define continuously compounded returns as $r_t = 100 * (\ln(p_t) - \ln(p_{t-1}))$, and then centered return is $u_t = r_t - E_{t-1}r_t$ where $E_{t-1}r_t$ is the expectation of r_t in the time periods based on the information at time *t-1*, and we use the average in empirical data as an approximation for this expectation. Let I_{t-1} denote the available information at time $t-1$, and $E(u_t|\mathrm{I}_{t-1}) = 0$. The following APARCH structure can be used to model volatility:

$$\begin{aligned} u_t &= \eta_t\sqrt{h_t};\ \eta_t \sim i.i.d.D(0,1) \\ h_t^{\delta/2} &= w + \alpha(\left|u_{t-1}\right| - \gamma u_{t-1})^{\delta} + \beta h_{t-1}^{\delta/2}, \end{aligned} \tag{1}$$

where $\delta > 0$, $-1 < \gamma < 1$, $w > 0$, $\alpha > 0$, $\beta > 0$, and the conventional stationary condition is $\alpha(1+\gamma^2)+\beta < 1$. Let h_t denotes the conditional variance of u_t, conditional on I_{t-1}, then $u_t|\mathrm{I}_{t-1} \sim D(0, h_t)$, where D is the assumed distribution u_t. In Eq. (1), the power coefficient δ allows different power orders in the data transformation, and the leverage coefficient γ can capture the asymmetric impact on volatility in response to positive and negative returns if $\gamma \neq 0$. The APARCH type of model, therefore, covers different types of models such as: (I) the GARCH model with $\delta = 2$ and $\gamma = 0$; (II) the TS-GARCH model of Taylor (1986) and Schwert (1989) with $\delta = 1$

and $\gamma = 0$; (III) the GJR model by Glosten et al. (1993) with $\delta = 2$ and 4; (IV) the TARCH model of Zakoian (1994) with $\delta = 1$. For GJR and TARCH models, the sign of u_{t-1} will affect the value of h_t and, therefore, these two models can capture an asymmetric relation between returns and changes in variance. This paper includes all four of the above-mentioned types. To estimate the parameters $\theta = (w, \alpha, \beta, \delta, \gamma)$ in the APARCH model under the assumption of normal distribution, the maximum likelihood estimation (MLE) method can be used. Let $\hat{\theta} = (\hat{w}, \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\gamma})$ denote estimation of θ, then for given u_{t-1} and current estimated $\hat{h}_{t-1}$, we can generate predictions of $\hat{h}_t$ based on the Eq. (1).

On the other hand, for a given dataset $\{(\mathrm{x}_1, y_1), ..., (\mathrm{x}_N, y_N)\} \subset \Re^d \times \Re$ with $\mathrm{x}_i \in \Re^d$ being the input vector and $y_i \in \Re$ being the output scalar, $i = 1, 2, ..., N$; the ε-insensitive SVR intends to find a function, $f(\mathrm{x}_t)$, that has a deviation from the observed y_i less than size ε and achieves optimal smoothness, simultaneously. To attain this aim, nonlinear maps are made from the input space into a higher dimension feature space $\Re^k$, where $k > d$. Define $f(\mathrm{x}) = w^T \varphi(\mathrm{x}) + b$, where ε is the nonlinear mapping function. Then, a smaller Euclidean norm of the regression coefficients $\|w\|^2$ indicate a flatter $f(\mathrm{x})$ and corresponds to a better generalization ability (Smola and Scholkopf, 1998a; 1998b). Under the optimization operation, the structure risk function is controlled under the ε-insensitive band constrain condition, which is as follows:

$$\begin{aligned} &\text{Minimize } \frac{1}{2}\|w\|^2 + \frac{C}{N}\sum_{i=1}^{N} L(f(\mathrm{x}_i), y_i); \\ &\text{where } L(f(\mathrm{x}_i), y_i) = \begin{cases} |y_i - f(\mathrm{x}_i)| - \varepsilon \text{ for } |y_i - f(\mathrm{x}_i)| > \varepsilon \\ 0 \text{ otherwise} \end{cases}. \end{aligned} \tag{2}$$

Function $L(f(\mathrm{x}_i), y_i)$ is the ε-insensitive loss function defined by Vapnik (1995) in which training data with an empirical error lower than ε is not penalised and they do not provide information for decisions. Both C and ε are hyperparameters which are predetermined empirically by cross-validation (Vapnik, 1995; Vapnik & Chervonenkis, 1974; Vapnik & Lerner, 1963). As a hard boundary of ε-band is not flexible to outliers and it can result in over-fitting, slack variables ξ_i, ξ_i^*, are introduced to allow certain errors outside the ε- band. After further introducing Lagrange multipliers $\alpha_i, \alpha_i^*, \eta_i, \eta_i^*$, Eq. (2) is transformed into the following dual optimisation:

$$\begin{aligned} L = {} & \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) - \sum_{i=1}^{l}(\eta_i\xi_i + \eta_i^*\xi_i^*) - \sum_{i=1}^{l}\alpha_i(\varepsilon + \xi_i - y_i + \langle w, \varphi(\mathrm{x}_i)\rangle + b) \\ & - \sum_{i=1}^{l}\alpha_i^*(\varepsilon + \xi_i^* + y_i - \langle w, \varphi(\mathrm{x}_i)\rangle - b) \text{ subjects to } \alpha_i, \alpha_i^*, \eta_i, \eta_i^* > 0 \end{aligned} \tag{3}$$

Furthermore, the Karush–Kuhn–Tucker (KKT) conditions (Kuhn & Tucker, 1951) must be satisfied, and it leads to that for $|f(\mathrm{x}_i) - y_i| < \varepsilon$, α_i and α_i^* should be zero. Thus, only a $L < N$ sample points associated with non-zero Lagrange multipliers are kept and referred to as support vectors used in further prediction. Denote the estimated non-zero Lagrange multipliers as $\{(\hat{\alpha}_i, \hat{\alpha}_i^*)\}_{i=1}^{L}$. For given input vector $\mathrm{x}_i \in \Re^d$, the prediction of output y can then be calculated as $\hat{y} = \sum_{i=1}^{L} (\hat{\alpha}_i - \hat{\alpha}_i^*) K(\mathrm{x}_i, \mathrm{x}) + \hat{b}$, with kernel function $K(\mathrm{x}, \mathrm{y}) = \langle \varphi(\mathrm{x}), \varphi(\mathrm{y}) \rangle$ satisfying Mercer's theorem (Mercer, 1909). The current paper applies Gaussian kernel, which is the most commonly used kernel when we have no special prior information of the data structure (Peréz-Cruz et al. (2003), Chen and Karl (2010), Ou and Wang (2010)).

When applying the SVR to estimate different types of APARCH model, identifying the output and input variables based on the model type and the expansion formula of the APARCH structure in Eq. (1) is of great importance. This APARCH-SVR combination can be viewed as a semi-parametric method, as it combines the nonparametric data-driven SVR method with the parametric APARCH model structure. At the same time, the APARCH-SVR method is more flexible compared with using MLE to estimate the APARCH model because the calculation of Lagrange multipliers does not need any prior assumptions of the data distribution. The objective of the current paper is to estimate and forecast conditional volatility, thus, the output variable in the APARCH-SVR model is naturally chosen to be h_t for GARCH and GJR, while it is $h_t^{1/2}$ for TS-GARCH and T-ARCH. The input vectors x_t also differ according to the model types: $\mathrm{x}_t = [u_{t-1}^2,\ h_{t-1}]$ for the GARCH model, $\mathrm{x}_t = [u_{t-1}^2,\ |u_{t-1}|u_{t-1},\ h_{t-1}]$ for the GJR model, $\mathrm{x}_t = [|u_{t-1}|,\ h_{t-1}^{1/2}]$ for the TS-GARCH model and $\mathrm{x}_t = [|u_{t-1}|,\ u_{t-1},\ h_{t-1}^{1/2}]$ for the T-ARCH model. After the APARCH-SVR model is trained by the training data, we only need to import the input vectors x_t and get an estimation for the conditional variance $\hat{h}_t$ by using equation:

$$\hat{y} = \sum_{i=1}^{L} (\hat{\alpha}_i - \hat{\alpha}_i^*) K(\mathrm{x}_i, \mathrm{x}) + \hat{b} \tag{4}$$

where $\hat{h}_t$ corresponds to the estimated output. No specified parameters, such as $\gamma,\ w,\ \alpha,\ \beta$, are estimated and no information on data distribution is required. As for actual dataset, the conditional volatility h_t is unobservable and thus cannot be used directly. Perez-Gruz et al. (2003) suggest a practicable resolution by setting $h_t' = \frac{1}{5} \sum_{k=0}^{4} u_{t-k}^2$ as the measurement of h_t. Li (2014) shows that $h_t' = \frac{1}{5} \sum_{k=0}^{4} u_{t-k}^2$ will lead to an over-smoothing of the volatility and suggests using $h_t' = \frac{1}{3} \sum_{k=0}^{3} u_{t-k}^2$ as an approximation for h_t. This paper, therefore, uses $h_t' = \frac{1}{3} \sum_{k=0}^{3} u_{t-k}^2$ as an approximation for h_t.

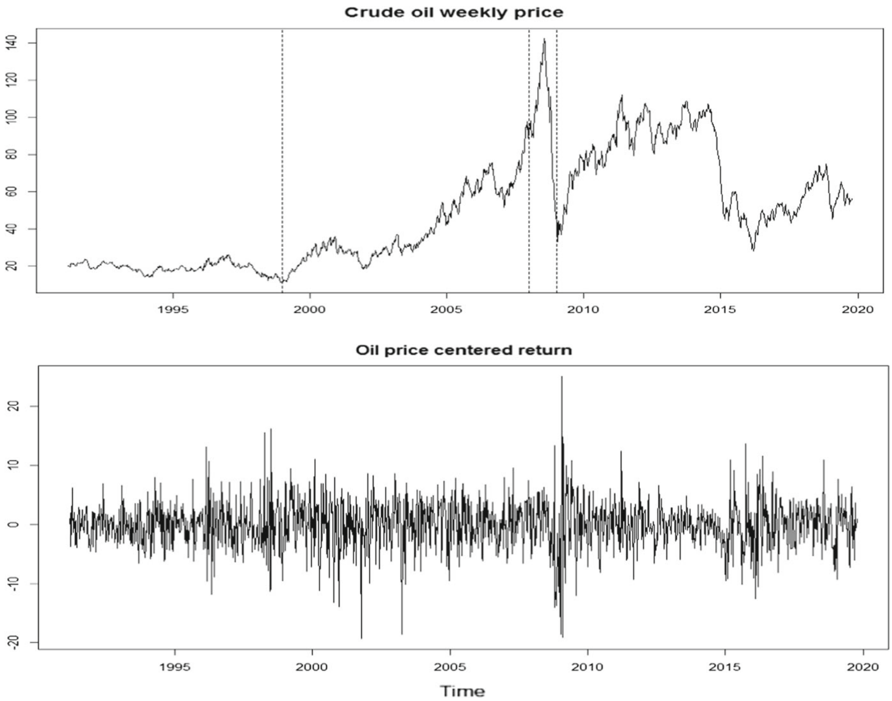

Fig. 1 Crude oil weekly price ($/barrel) and oil price centered return

3 Estimation and Forecasting of the Volatility for the Oil Price

3.1 Overview of Previous Studies and Data for the Empirical Analysis

This paper uses mainly weekly oil (FOB) spot prices (p_t) of West Texas Intermediate ($/barrel) from March 8, 1991 to September 13, 2019. The data retrieval was done from the official website of the US Energy Information Administration (EIA).[2] Then, the centered oil price returns, which are defined as u_t in the beginning of Sect. 2, were calculated, following the analysis in most previous literature (Peréz-Cruz et al., 2003; Chen & Karl, 2010; Qu & Wang, 2010; Mohammadi & Su, 2010; Lux et al., 2016). Figure 1 shows the weekly oil price p_t and the centered return u_t.

The crude oil price per barrel in dollars in Fig. 1 shows that crude oil prices were influenced by economic and geopolitical events that influenced the global economy. For example, the price falls in 1997–8 were due to the Asian financial crisis. Weakening global demand for oil following US recessions and increased uncertainties associated with the 9/11 attack led to decreases in prices in 2001. The general trend in oil prices since 1999 (after the first dashed vertical line), however, shows a steady increase, with growing demand for oil from emerging markets. One of the most noticeable price falls is observable between 2008 and 2009 (between the second and third dashed vertical

[2] Retrieved from https://www.eia.gov/dnav/pet/hist/rwtcW.htm (Accessed September 13, 2019).

lines). This abrupt fall is due to the effects of the global financial crisis, which caused a severe collapse in demand from middle 2008 to the end of 2008, uncertainties in the global economy and liquidity constraints. The most recent price falls were between June 2014 and January 2015, which was the second-largest price drop after the price collapse in 2008. A growing supply glut was a main driver of this huge drop in oil prices. High prices, prior to this oil price collapse, led oil operators to start for the drilling of new wells, yet demand for oil decreased in major oil-importing markets with their economic contraction.

After checking the price and volatility patterns over the whole sample period, we split the sample period into four different sub-periods. It is based on the one of the previously mentioned studies, Ou and Wang (2010), which report differing performance of a (series of) model in different financial market situations. The sub-period split is, therefore, based on major economic events that were behind the movements, which caused the oil market situation to change. The four sub-periods are as below.

Period 1: Weekly data from March 8, 1991 to February 19, 1999. This period includes declines in prices due to the Asian financial crisis in 1997–8.

Period 2: Weekly data from February 26, 1999 to July 4, 2008. This period shows a long and generally increasing trend in prices, due to the strong growth of emerging markets. A price decline, however, is observed in late 2000.

Period 3: Daily data[3] from July 04, 2008 to December 26, 2008. This sub-period includes a sharp decline in oil prices in connection to the financial crisis in 2008.

Period 4: Weekly data from December 26, 2008 to September 13, 2019. This sub-period includes the oil price recovery after the 2008 crash and subsequent price reactions from the world's producers.

We do not merge the dataset in period 3 with other sub-periods due to that the data structure in period 3 is quite distinct with a sharp decrease in price in a very short span of time. Moreover, the precision of parameter estimations in MLE and Lagrange multipliers $\{(\alpha_i, \alpha_i^*)\}_{i=1}^N$ in SVR increase with the number of dataset, yet it is not dependent on data frequency. Thus, we use daily data in period 3 instead of using weekly data, which has a problem of few observations in the period. Figure 2 shows the oil price ($/barrel) in the four different sub-periods and Fig. 3 illustrates oil price centered return of the sub-periods.

Figure 2 shows that period 1 and 4 have more variation in the original price, which includes both increasing and decreasing trends, while periods 2 and 3 have either an increasing or a decreasing trend (rather than including both trends). Figure 3 of the centered return does not show any clear pattern across the different sub-periods; however, it seems that, albeit with some obscurity, the volatility for the dataset in periods 2 and 3 are relatively more similar than those of periods 1 and 4. Period 1 and 4 include dramatic episodes during the 1998–9 (the Asian financial crisis) and 2014–6 (OPEC cuts production targets), respectively. Compared with those two periods (period 1 and 4), period 2 shows a general price increase, although there were price falls shortly after the 9/11 attacks. In comparison to the other sub-periods, period 3 is distinct in terms of the magnitudes and speed of the price falls. This nose-dive in oil price was, however, relatively short-lived and did not stretch over a longer period.

[3] We use daily data in this sub-period since it is a relatively short (only half year). Weekly data is too sparse.

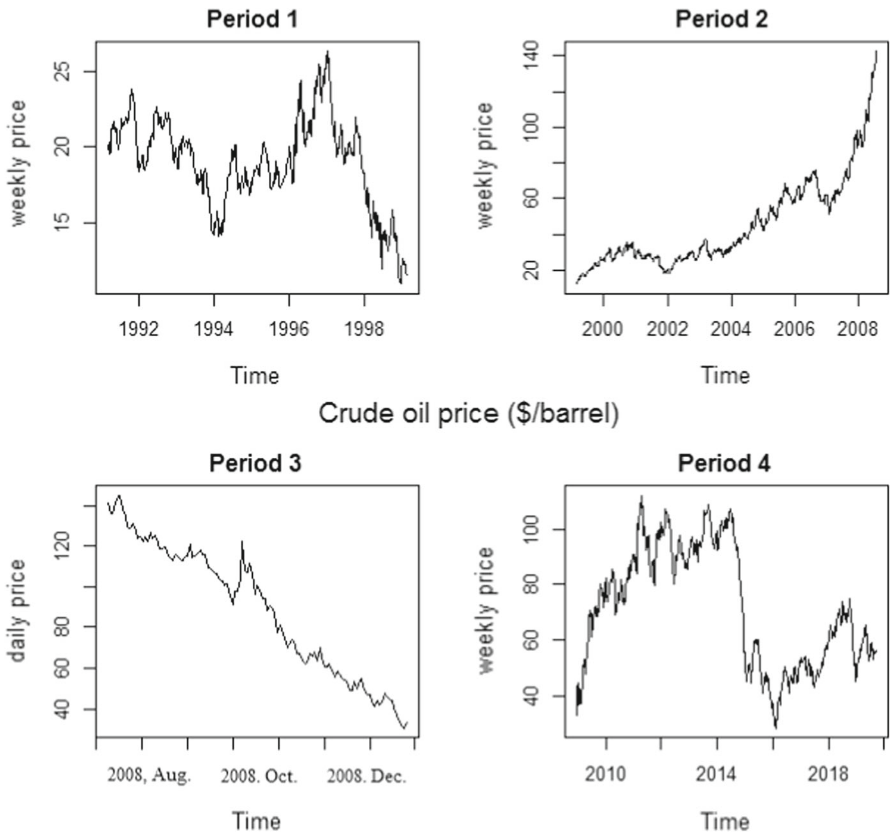

Fig. 2 Crude oil price ($/barrel) in each sub-period

In terms of the magnitudes of fluctuations, the two different periods, period 2 (with a general price increase) and period 3 (with a general price decrease), do not seem to show distinct differences. As Figs. 2 and 3 cannot visually reflect the difference of homogeneity clearly, rigorous statistical tests as well as modelling are needed to reveal the underlying data behavior.

3.2 Pre-Processing Prior to Model Comparison: Statistical Test and Evaluation Criteria

As preliminaries, we check characteristics related to the centered return series u_t. The data length of each sub-period is denoted as N. The Shapiro–Wilk test (Shapiro & Wilk, 1965) is used to test for normality of the centered return, u_t, in each period, with the null hypothesis that the dataset is normally distributed. We also perform a standard Box-Ljung test (Ljung & Box, 1978) to test the correlations of u_t^2 with lag 1 for all the sub-periods, lag 4 for weekly data and lag 7 for daily data, corresponding

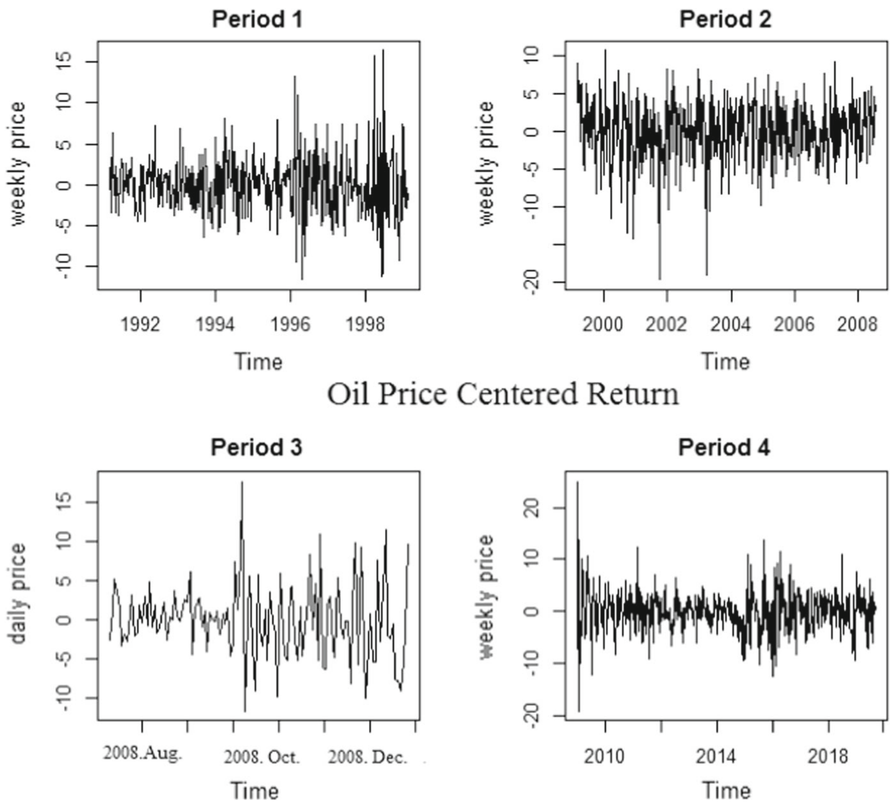

Fig. 3 Oil price centered return in each sub-period

separate correlations in one month for periods 1, 2 and 4 and one week for period 3. The p-values of the tests for four periods are in Table 1.

Results in Table 1 shows that at the 5% significance level, the null hypothesis of a normal distribution of the centered return is rejected for periods 1, 2 and 4, indicating that the MLE with the prior normal assumption is not a suitable estimation method for those datasets. In addition, the Box-Ljung test results of correlations of u_t^2 show that there are significant and persistent correlations for periods 1, 3 and 4, while no correlation is found for period 2. We continue with tests for ARCH effects in the sub-series by applying the Lagrange Multiplier (LM) test and the Rank-based test for conditional heteroscedasticity with the null hypothesis of no ARCH effects. Lag length of 4 for monthly data (periods 1, 2 and 4) and 7 for daily data (period 3) were chosen for the tests. The results are presented in Table 2.

Based on the results from Table 2, at a significant level of 5%, we reject the null hypothesis of no ARCH effects in the series for periods 1, 3 and 4. Both the correlation test of u_t^2 and the conditional heteroscedasticity test indicate that an ARCH type of model can be used for data in those periods while it is not suitable for period 2. (G)ARCH models often work as a remedy for two elements of financial data:

Table 1 [!]P-values for Normality test for u_t and Box-Ljung test for u_t^2

		Period			
		1	2	3	4
N		416	489	121	560
Shapiro – Wilk test		1.100e-07***	3.20e-08***	0.149	1.11e-10***
Box-Ljung test	With lag 1	1.780e-05***	0.235	0.004**	0.008**
	[!!]With lag 4 or 7	6.08e-09***	0.683	0.026*	1.110e-16***

[!!]Lag 4 applies for period 1, 2 and 4 in which weekly data is used. Lag 7 applies for period 3 in which daily data is used
[!]Signif. codes: 0 "***" 0.001; "**" 0.01; "*" 0.05; "." 0.1; "" 1

Table 2 Conditional heteroscedasticity Test statistics and p-values

	Test statistics				[!]P-values			
	Period				Period			
	1	2	3	4	1	2	3	4
LM test	44.112	2.290	15.887	80.236	6.080e-09***	0.682	0.026*	1.110e-16***
Rank test	22.803	3.110	28.156	34.550	0.0001***	0.539	0.0002***	5.746e-07***

[!]Signif. codes: 0 "***" 0.001; "**" 0.01; "*" 0.05; "." 0.1; "" 1

leptokurtosis and volatility clustering. Although it is not particularly clear from Fig. 2, period 2 does not seem to have volatility clustering or pooling. Period 2 shows a general price increase, and the tendency for large returns (of either sign) that are expected to follow large returns, and small returns (of either sign) to follow small returns are not very clearly observed.

Table 1 and 2 show that it is necessary to use not only MLE, but also SVR method to estimate the four types of APARCH model for the four sub-periods. To evaluate the performance of both estimated and forecasted conditional variance $\hat{h}_t$ from both methods, the mean square error (MSE) $MSE = \frac{1}{n}\sum_{t=1}^{n}(\hat{h}_t - h_t^{'})^2$ and the mean absolute deviation (MAD) $MAD = \frac{1}{n}\sum_{t=1}^{n}\left|\hat{h}_t - h_t^{'}\right|$ are utilized as criteria, where $h_t^{'} = \frac{1}{3}\sum_{k=0}^{4} u_{t-k}^2$ is an approximation for h_t. The MSE and MAD are calculated on both in-sample and out-of-sample data.[4] The output and input of in-sample data, also called the training data, are used to estimate the Lagrange multipliers in the SVR and parameters

[4] MSE is a scaled version of the criterion R^2 in Peréz-Cruz et al. (2003). MAD is also calculated as it is less sensitive to large errors, compared with MSE.

$\theta = (w, \alpha, \beta, \delta, \gamma)$ in APARCH models. The out-of-sample data, also called the test data, is not used to train the value of Lagrange multipliers $\{(\alpha_i, \alpha_i^*)\}_{i=1}^N$ or to estimate the APARCH parameters. After having trained $\{(\hat{\alpha}_i, \hat{\alpha}_i^*)\}_{i=1}^L$ for the support vectors and estimated $\hat{\theta} = (\hat{w}, \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\gamma})$ from MLE, we can calculate $\hat{h}_t$ based on the input vectors. If the input vectors x_t come from the in-sample data, $\hat{h}_t$ is called the estimated value for h_t. If input vectors x_t come from the out-of-sample data, the predicted $\hat{h}_t$ will be called forecasting of h_t. The MSE and MAD calculated from the out-of-sample can be used to evaluate the forecasting performance. Compared with the in-sample performance, the out-of-sample forecasting performance is less sensitive to outliers and data mining, and it also can better reflect the available information to the forecaster in 'real-time' (Stock & Watson, 2007a, 2007b).

3.3 Evaluation of the Estimation Performance for the 4 Sub-Periods

In this sub-section, we first present the results of using both MLE and SVR to estimate the conditional variance for all the four sub-periods that were split from the whole sample period as shown in Sect. 3.1. A comprehensive comparison of the performance of the two methods for the four types of APARCH models will follow. As regards SVR, it should be noted that the hyperparameters C and ε-, as well as the bandwidth hyperparameter in Gaussian kernel are tuned by fivefold cross-validation error.[5]

We first use all the dataset in each sub-period as "training data" to estimate volatility of different types of APARCH model based on both MLE and SVR methods. The estimation graphs by using MLE and SVR for the four sub-periods and four types of APARCH models are presented in Figs. 4, 5, 6, and 7. The solid black lines in Figs. 4, 5, 6, and 7 are conditional variance $h_t^{'} = \frac{1}{3}\sum_{k=0}^{3} u_{t-k}^2$ calculated from the centered returns u_t^2. The green dashed lines represent the SVR estimation results for conditional variance $\hat{h}_t$ and the red dashed lines show MLE estimation results for $\hat{h}_t$, respectively. Thus, by comparing the black lines, the green and red dashes, we can see the differences between $h_t^{'}$ and $\hat{h}_t$ from SVR and MLE:

Figures 4, 5, 6, and 7 show that the MLE-based estimation generally fails to catch the peak-points of the volatility, while SVR based estimation can capture all the peaks. Moreover, when it comes to the TARCH and TSGARCH models, the MLE method gives extreme initial estimations, especially for period 2 and 3, while the SVR method does not suffer from this problem. In general, it shows that the SVR can capture volatility clustering better in the data in all four types of models than the MLE method. Figure 5 also shows that the MLE estimation fails to estimate the conditional volatility in period 2, which was not unexpected since Table 1 and 2 already indicate that the ARCH model is not suitable for the dataset in period 2. Besides, all the MLE estimated parameters are around zero and not significant, which results in a constant $\hat{h}_t$. However,

[5] K-fold cross-validation means that dataset is split into K folds (subsets). Models are trained using k − 1 folds as training data and validated on the remaining one fold. The remaining fold can be viewed as test set, and it is used compute a performance measure such as MSE. The tuned hyper-parameter value is the one, which produces lowest average of test error, for example, MSE obtained from test folds computed in the loop.

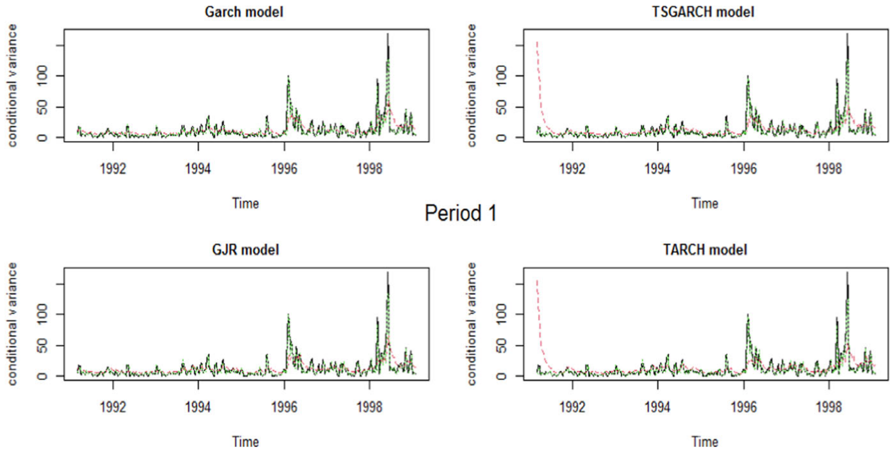

Fig. 4 MLE and SVR modelling results using the four types of models for sub-period 1

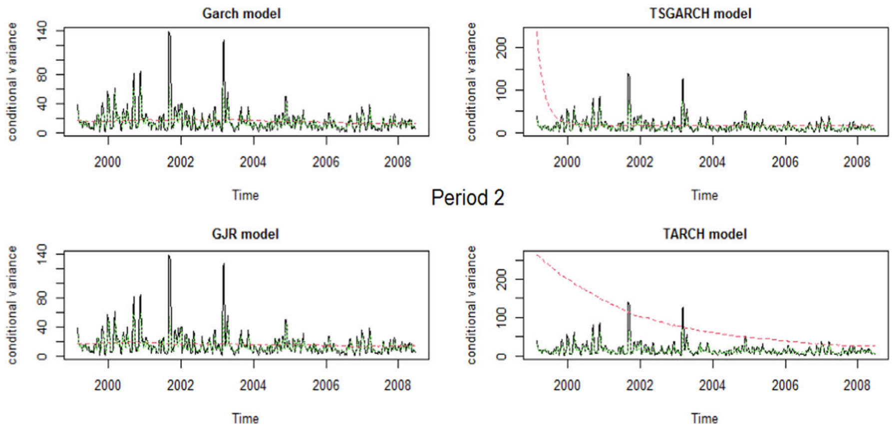

Fig. 5 MLE and SVR modelling results using the four types of models for period 2

the SVR estimate $\hat{h}_t$ from a pure data-driven method, by setting input vectors in to the kernel function $K(\mathrm{x}_i, \mathrm{x})$ in Eq. (4), we can calculate the corresponding $\hat{h}_t$.

To evaluate the estimation performance of the different models in each sub-period, MSE and MAD of those models were compared. As shown in Table 3 and 4, the SVR-based estimation gives much lower MSE and MAD compared with those of the MLE method.

More specifically, Table 3 shows that the MLE method gives very high MSE for TARCH and TSGARCH models, due to the large deviation between $\hat{h}_t$ and $h_t^{'}$ among the initial points. SVR method, however, is more flexible to the model types and it does not give extreme MSE values for TSGARCH and TARCH models. The MSE and MAD values of the SVR method further show that the GJR model, which takes the asymmetric effects into the model, performs the best in periods 1 and 4. Looking

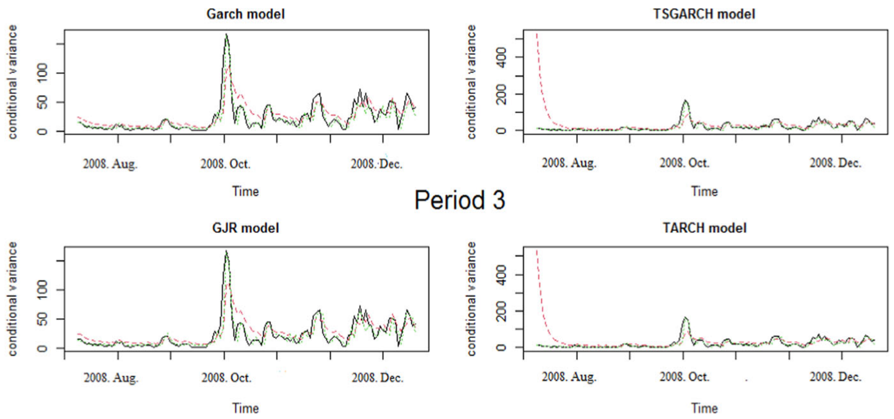

Fig. 6 MLE and SVR modelling results using the four types of models for period 3

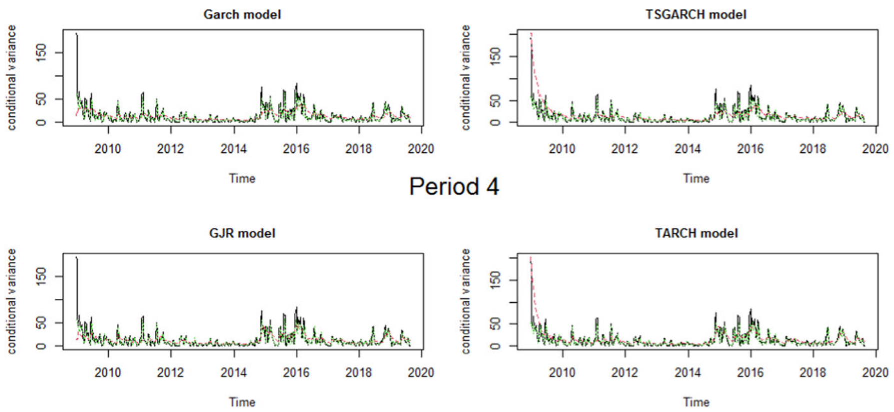

Fig. 7 MLE and SVR modelling results using the four types of models for period 4

at the results of the SVR estimation, in these two periods 1 and 4, GJR model gives significantly lower MSE values, compared with other types of APARCH model. In period 3, the symmetric GARCH model performs the best based on both the MSE and MAD values, while it differs for period 2: GARCH model fits the best in period 2 based on the MSE values and TARCH model fits the best based on MAD values, respectively. This result was expected from Fig. 1 that shows that the original data generally show a general upward or a downward trajectory in period 2 and 3, which leads to relatively more homogenously centered returns, compared with those in period 1 and 4. In short, in period 2 and 3, the symmetric GARCH and TSGARCH models perform better than the other two asymmetric models while in period 1 and 4, the asymmetric GJR model performs the best.

Although the return series in Figs. 4, 5, 6, and 7 cannot reflect the difference of homogeneity clearly, the estimation results based on the SVR method suggest that the GARCH and TSGARCH are suitable for data with a monotonic trend, while the

Table 3 MSE for each sub-period

	MLE-based estimation				SVR-based estimation			
	Period				Period			
	1	2	3	4	1	2	3	4
GARCH	172.794	301.256	310.360	246.359	60.244	**187.815**	**87.922**	135.616
TSGARCH	322.542	897.962	4069.748	281.908	137.513	192.009	150.850	77.333
GJR	176.703	302.817	304.641	234.396	**53.626**	191.284	99.453	**45.126**
TARCH	327.632	9196.208	4163.916	213.456	111.475	152.127	517.485	68.980

Table 4 MAD for each sub-period

	MLE-based estimation				SVR-based estimation			
	Period				Period			
	1	2	3	4	1	2	3	4
GARCH	7.250	9.956	12.073	8.976	3.712	6.238	**5.258**	4.720
TSGARCH	9.066	15.006	23.839	10.015	4.835	6.190	7.021	4.706
GJR	7.364	9.946	12.044	8.187	**3.278**	6.213	5.303	**3.425**
TARCH	9.220	72.674	24.143	8.530	4.723	**5.624**	10.836	4.548

Table 5 Significance of γ for GJR and TARCH models

Periods	Tests	Estimate	Std. Error	t value	! P-value
1	GJR	0.274	0.137	2.010	0.044*
	TARCH	0.350	0.167	2.096	0.036*
2	GJR	1.037e-01	3.610e + 00	0.029	0.977
	TARCH	1.154e-01	8.606e-01	0.134	0.893
3	GJR	− 0.020	0.154	− 0.130	0.896
	TARCH	− 0.041	0.220	− 0.188	0.851
4	GJR	0.344	0.095	3.607	0.000***
	TARCH	0.485	0.123	3.960	7.500e-05***

! Signif. codes: 0 “***” 0.001; “**” 0.01; “*” 0.05; “.” 0.1; “” 1

asymmetric GJR and TARCH models are suitable for the data with larger price swings. More specifically, in period 4, the GJR and TARCH models, respectively, show the lowest and second to the lowest MSE and MAD values among the four models using the SVR method. To confirm that the asymmetric GJR and TARCH models are more suitable for periods 1 and 4 than the other models, we examine the significance of γ in Eq. (1), based on the dataset of centered returns in each sub-period. The estimation results from MLE are included in Table 5.

Table 5 shows that in period 1 and 4, the asymmetric coefficients are statistically significant in both GJR and TARCH models, which yet again indicates that the ARCH models with asymmetric effects are more suitable for modelling volatility for period 1 and 4. Besides, the value of γ is positive in period 1 and 4, which means that a leverage effect is associated in such a way that negative shocks reinforce the volatility by more than positive shocks[6] in those periods. However, the asymmetric effects in the dataset in periods 2 and 3 are not significant. As pointed out in Wu (2001), this asymmetric behavior can be explained by traders’ expectation of volatility in a market.

[6] Hasan et al. (2013), however, report that asymmetry in energy return volatility is observed to the opposite direction of the typical asymmetry of a leverage effect of return volatility.

In a circumstance that increases traders' expectation of volatility in the market, the anticipation augments volatility to a larger degree. In those two periods (period 1 and 4), an economic event caused disruptions in the market, which were showing an increasing trend in price prior to the event, that raises expectation of an increase in volatility among traders. This anticipation of volatility increase leads to an immediate price fall. The downside of the situation continues with a further increase in volatility that can be explained by leverage effect hypothesis (Wu, 2001). For this reason, in both period 1 and 4, this asymmetry in oil price volatility is present, while this mechanism does not work in other periods, such as period 2, with a general upward trend in oil price. Period 3 includes a general downward trend in price, and asymmetry is not observed even though the oil price nudged up in a very short while within that period.

3.4 Evaluation of the Forecasting Performance for the Four Sub-Periods

After detecting the asymmetric behavior of oil price volatility in period 1 and 4, we move to evaluate the forecasting performance of the different models using MLE and SVR methods in those two periods.[7] In doing so, we divide the dataset in period 1 and 4 into in-sample training data and out-of-sample test data. The last 104 dataset in those two periods, which corresponds to a two-year time horizon, is kept as a test dataset. For the in-sample training dataset, the estimation result is quite consistent with the results presented in Figs. 4, 5, 6, and 7, Table 3 and 4: the SVR method combined with GJR, which gives the best estimation in both period 1 and 4. The corresponding figures and tables are not presented here to save the space,[8] yet, we present the forecasting results based on the out-of sample test data for period 1 and 4 in Figs. 8 and 9.

Both Figs. 8 and 9 show that the SVR-based method outperforms the MLE-based method when it comes to forecasting of the oil price return volatility. These figures also shows that the SVR captures the bumps better than the MLE method. However, the test MSE are quite similar among various types of models when using the SVR method in both period 1 and 4, and there is no single type of APARCH model that has significantly lower or larger test MSE value than the others.[9] Thus, when it comes to the out-of-sample forecasting, the result is not very sensitive to the type of the volatility models when the SVR method is used.

3.5 Evaluation of the Estimation Performance for Whole Sample Period and Discussions

Finally, we perform an empirical investigation on oil price volatility by using weekly data of the whole sample period, and examine if the asymmetric behavior is also

[7] It should also be noted here that Fig. 3 shows already that MLE performs not very well for dataset in period 2. In addition, the sample size in period 3 is too small for further split.

[8] The figure and table of the estimation estimation results of the in-sample training dataset are available from the authors upon request.

[9] We do not present the corresponding table with MSE here to save the space, but it is available from the authors upon request.

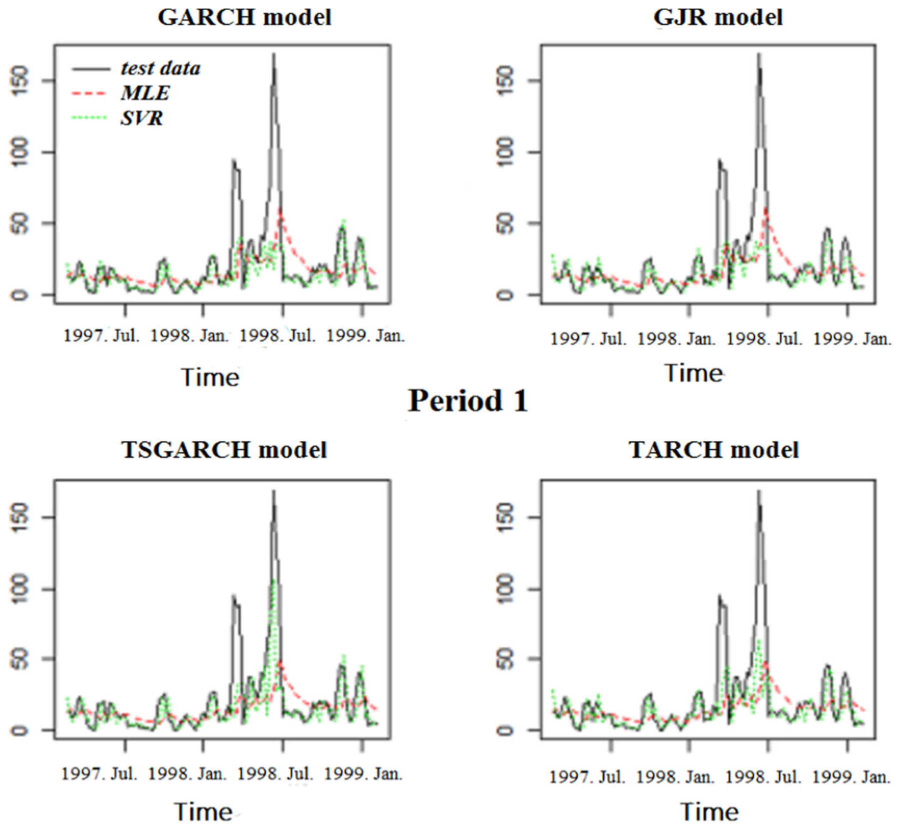

Fig. 8 MLE and SVR forecasting results of the four models for period 1

detected in the whole sample period. The estimation results of the oil price for period from March 8, 1991 to September 13, 2019 are shown in Fig. 10 and Table 6.

Table 6 shows that all the four types of models perform to a quite similar degree to each other when the same estimation method is used. Yet, the SVR method still gives generally better results than the MLE counterparts do, while the differences are not significant. This estimation result of the whole sample is different to those results of each sub-sample presented in Table 3 and 4. In Tables 3 and 4, the GJR asymmetric model performs the best in period 1 and 4. The differing result of Table 6 (to those in Table 3 and 4) can be explained by the fact that when dataset is large and contains different sub-periods with various dynamics in prices and volatility, those dynamics cancel each other out to some extent when being treated all together in one whole sample period. Therefore, we cannot distinguish different volatility patterns even though we compare different models or estimation methods. As was also mentioned along with the results of testing asymmetry in different sub-samples (in Tables 3 and 4), price patterns vary to a large degree in different sub-samples which are associated with different economic events behind the price and volatility mechanisms. Thus, it is of great importance to uncover the different data properties of different sub-periods,

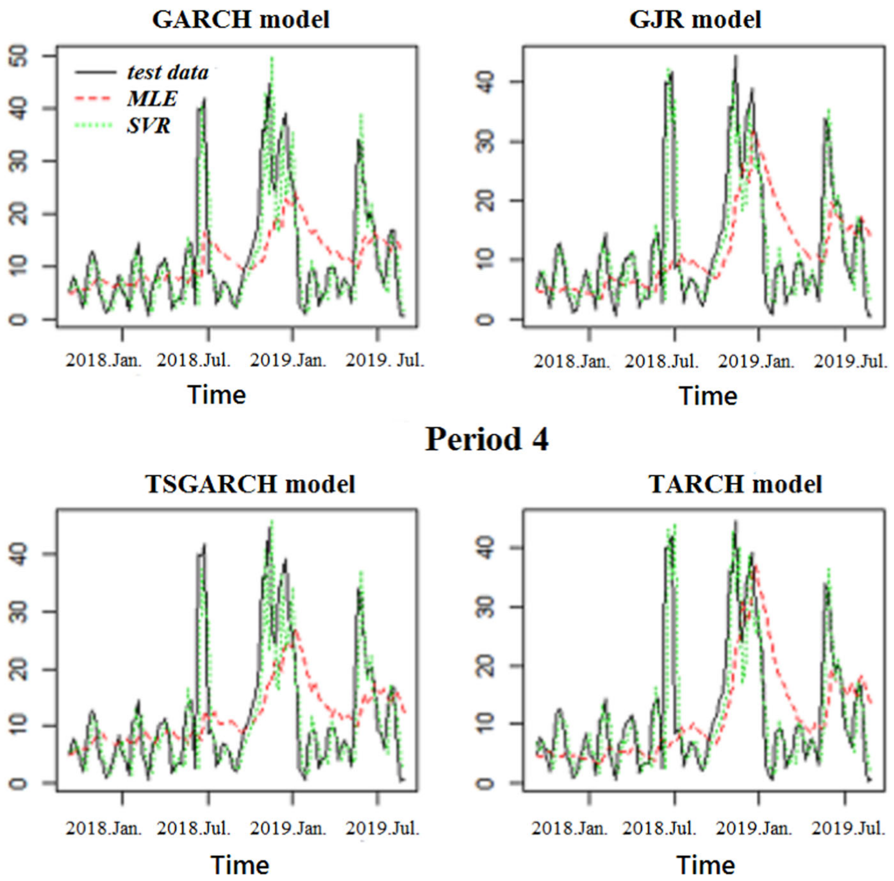

Fig. 9 MLE and SVR forecasting results of the four models for period 4

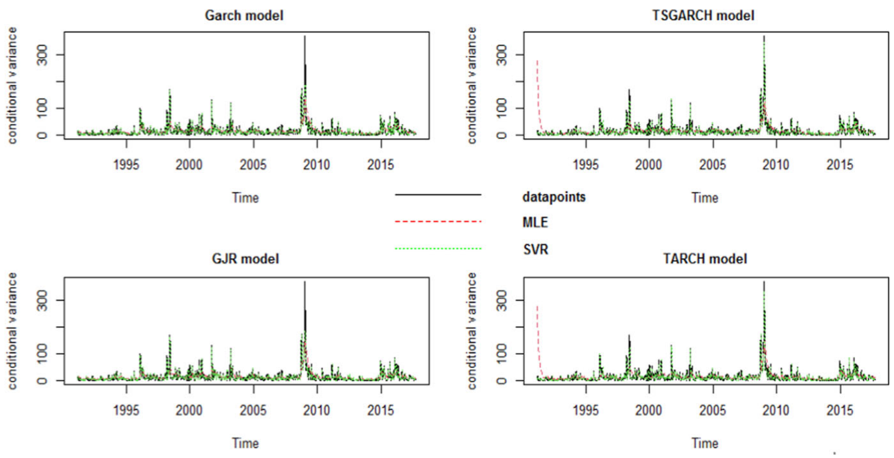

Fig. 10 MLE and SVR modelling results of the four models for the whole sample period

Table 6 MSE and MAD using the whole sample period

	MLE estimation		SVR estimation	
	MSE	MAD	MSE	MAD
GARCH	385.035	10.108	142.554	5.164
TSGARCH	592.011	11.464	**115.780**	5.540
GJR	**361.123**	**9.876**	141.608	**5.050**
TARCH	573.430	11.265	127.415	5.471

which can be masked in the whole sample period. A similar situation is encountered when analyzing the dataset containing structural breaks. Then, we need to split the data into sub-samples and run separate regressions. If we build a regression model based on the whole dataset without splitting the dataset according to locations of the structural breaks, the coefficient estimates suffer from having a high bias.

The empirical example in Sect. 3 can be summarized with the three main findings as below:

1) When estimating the APARCH model, the SVR algorithm, in general, gives better estimation and forecasting results with lower MSE and MAD compared with those from MLE.
2) The asymmetric effect are observed only in sub-periods 1 and 4 where GJR and TARCH models give lower MSE and MAD by SVR estimation. This finding is also supported by the significant γ parameter in GJR and TARCH models in Table 5 for sub-periods 1 and 4.
3) When the whole period is investigated, various dynamics in each sub-period cancel each other out thereby leading to a failure in distinguishing different volatility patterns of different time periods. This finding reinforces the importance of analyzing volatility behaviors which are steered by different economic events in different periods. Putting the different time periods together in one long sample period may mask those varying volatility behaviors.

When it comes to the second finding, the most closely related study using the APARCH model where $\gamma \neq 0$ in oil price forecasting is Mohammadi and Su (2010). By using GARCH, EGARCH and APARCH and FIGARCH, they conduct a forecasting of the conditional mean and volatility of crude oil spot prices from 11 international markets with weekly dataset from 1/2/1997 to 10/3/2009, and show that, in most cases, the APARCH model with asymmetric power outperforms the others. Thus, our findings share both similarities and differences with some previous empirical works on oil volatility. As was also documented by Narayan and Narayan (2007), the evidence of asymmetric behavior of volatility in oil price is inconsistent in the current paper. That is, our results show that asymmetric effects are not present in all sub-sample periods. To be more specific, asymmetric effects are not present in those periods when the original data generally show a general upward or a downward trend in price.[10]

[10] It is also noticeable that inconsistent asymmetric volatility were not only documented in the oil market but also in equity markets (Bekert and Wu, 2000).

This finding of mixed evidence on the presence of asymmetric volatility is closely associated with the question of choosing a better model for forecasting.

For future research works, other types of hybrid models which combine nonparametric data-driven methods with various parametric volatility models, can be compared with our SVR- APARCH model. These hybrid models can also further be applied to investigate volatility behavior of other commodity prices to model volatility behavior with asymmetry. One further possible research scope is to perform an empirical analysis for volatility estimation using the SVR-APARCH model, while utilizing several different types of kernels, including Gaussian kernel, wavelet kernel and polynomial kernel, in the SVR algorithm. This comparison would provide comprehensive results by examining whether the results are sensitive to different kernels or not.

4 Conclusion

This paper primarily compares MLE and SVR methods to estimate and forecast the volatility of weekly or daily oil price returns based on the APARCH type of models. The SVR—APARCH model applied in our paper can be viewed as a hybrid or semi-parametric model which can keep the advantages of both nonparametric SVR estimation algorithm and the interpretability of the parametric APARCH model. In this SVR-APARCH model, the input variables of the nonparametric SVR algorithm are decided by the structure of the parametric APARCH model.

As different types of APARCH model vary in capturing asymmetric behaviors, we can identify whether asymmetric behaviours of volatility exist or not in those subperiods of sharp price falls during when negative shocks may reinforce the volatility by more than positive shocks.

Prior to the model comparison, pre-processing procedures are carried out to investigate the characteristics of the data in different sub-sample periods which are split based on the occurrence of economic events. The results show that in both period 1 and 4, asymmetry in oil price volatility is present, while this mechanism is not present in other periods, such as period 2, with a general upward trend in oil price. Period 3 includes a general downward trend in price, and asymmetry is not observed even though the oil price nudged up in a very short while within that period. This inconsistent behavior of asymmetric in oil price is inconsistent was also documented in a previous study by Narayan and Narayan (2007).

Estimation results of using SVR-APARCH method, overall, suggest that the SVR method can capture volatility clustering better in all the four types of APARCH model than the MLE method. This outperformance of SVR than using MLE method is in line with the previous studies such as Perez-Cruz et al. (2003) and Sun and Yu (2020). SVM regression generally outperforms the other method with lower estimation and forecasting errors, and it is also more robust to the choice of different APARCH models than the MLE counterparts. More precisely, our results show that the MLE method produces extreme initial estimations of conditional variance for TSGARCH and TARCH models in certain cases, while SVR does not seem to be affected by this problem. It should also be noted that, based on the estimation results of the SVR method, the GARCH and TSGARCH models are, in general, more suitable for data with a general monotonic trend, while the asymmetric GJR and TARCH models are more suitable for data with larger swings in price. Unlike previous studies which focus on GARCH models in combination of SVR for a sample period, this paper includes a broader APARCH model and show that performance of different models vary in different sub-sample periods. The estimation results of the paper using SVM based regressions in each sub-period show that the ARCH models with asymmetric power generally perform better than the models with symmetric power when the data sub-period includes large swings in oil price. The asymmetric behavior of oil price volatility, however, is not detected when the analysis is done using the whole sample period.

In sum, our test results do not allow us to conclude that there is a single model that shows better performance in forecasting across all sub-sample periods. Our findings suggest that even though the SVR appears to fit the crude oil data well, different models within the SVR should be carefully chosen based on the characteristics of the data. This result, therefore, underscores the importance of identifying the dynamics of the dataset in different periods to improve estimation and forecasting performance in modelling oil price volatility. The machine learning techniques, in general, start to be increasingly used not only in financial sectors but also in government sectors in policy evaluations and forecasting. However, applying data-driven method itself does not mean that the importance of identifying dynamics of the dataset in different periods for modelling oil price volatility is mitigated. Our study reinforces the importance that employing more effective machine learning technique for volatility forecasting still requires uncovering the different data properties of different periods, during which volatility is influenced by different economic factors. Besides, it is of great importance to examine the different data properties of different sub-periods, which can be masked in the whole sample period.

Author Contributions YL contribute to the methodology and computational part including statistical analysis of data, the construction and estimation of APARCH-SVR model and model comparison. HKK contribute to the discussion of economic motivation and the interpretation of empirical analysis.

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Data availability Available from authors upon request.

Code availability Available from authors upon request.

Declarations

Conflict of interest Not applicable.

[11] Parts of this work have been done in the context of CEDAS (Center for Data Science, University of Bergen, Norway).

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