

Multiplicity and Modifiers

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Abstract

A sentence with an adverbial modifier under negation like *Mike didn't wash the window with soap* gives rise to an inference that Mike did wash the window. A sentence with a plural noun like *Mike washed windows* gives rise to a so-called 'multiplicity' inference that Mike washed multiple windows. In this note, we focus on the interaction between these two inferences in sentences containing both an adverbial modifier and a plural noun under negation, like *Mike didn't wash windows with soap*. We observe that this sentence has a reading conveying that Mike didn't wash any window with soap but that he did wash multiple windows (albeit not with soap). As we discuss, this reading is not predicted by any version of the implicature approach to the multiplicity inference, in combination with the implicature treatment of the inference of adverbial modifiers. We sketch two solutions for this problem. The first keeps the implicature approach to adverbial modifiers but adopts a non-implicature approach to multiplicity based on homogeneity. The second solution holds on to the implicature approach to the multiplicity inference but accounts for the inference of adverbial modifiers as a presupposition. In addition, it adopts the idea that presuppositions can be strengthened via implicatures, as proposed recently in the literature. Either way, the interaction between multiplicity and the inference of adverbial modifiers suggests that we cannot treat both as implicatures: if we want to treat either one as an implicature, we need to do something different for the other. We end by comparing the case above to analogous cases involving different scalar inferences and showing that the ambiguity approach to the multiplicity inference does not provide a solution to our problem.

1 INTRODUCTION

A sentence with a plural noun like (1a) gives rise to the so-called 'multiplicity inference' in (1b).

- (1) a. This morning, Mike washed windows.
b. \rightsquigarrow This morning, Mike washed multiple windows Multiplicity Inference

As is well known, the multiplicity inference arises in positive cases but disappears under negation, as shown in (2). For this reason, a prominent approach in the literature treats

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it as an implicature (Ivlieva, 2013; Mayr, 2015; Sauerland, 2003; Sauerland *et al.*, 2005b; Spector, 2007; Zweig, 2009).

- (2) a. This morning, Mike didn't wash windows.
 b. \neq This morning, Mike didn't wash multiple windows
 c. \rightsquigarrow This morning, Mike didn't wash any window

Similarly, the inference of sentences with an adverbial modifier under negation, as in (3), has been analysed as an implicature (Katzir, 2007; Schlenker, 2008; Simons, 2001).^{1,2}

- (3) a. This morning, Mike didn't wash the window with soap.
 b. \rightsquigarrow This morning, Mike did wash the window

In this squib, we focus on the interaction between these two inferences in sentences containing both an adverbial modifier and a plural noun under negation, as in (4a). We observe that (4a) has a reading which conveys that this morning Mike didn't wash any window with soap but, at the same time, he did wash multiple windows (albeit not with soap) — a reading in which the multiplicity inference appears 'on top' of the inference of adverbial modifiers.^{3,4}

- (4) a. This morning, Mike didn't wash windows with soap.
 b. \rightsquigarrow This morning, Mike did wash multiple windows

As we discuss, this reading is not predicted by any version of the implicature approach to the multiplicity inference combined with the implicature treatment of the inference of adverbial modifiers.

We sketch two solutions for this problem. The first keeps the implicature approach to adverbial modifiers but adopts a non-implicature approach to multiplicity based on homogeneity (Križ, 2017). The second solution keeps the implicature approach to the multiplicity inference but treats the inference of adverbial modifiers as a presupposition (Schlenker, 2008; Simons, 2001); a presupposition which can itself be strengthened via implicatures (Gajewski & Sharvit, 2011; Spector & Sudo, 2017). All in all, the interaction between the multiplicity inference and the inference of adverbial modifiers suggests that we cannot treat both as implicatures: if one is analysed as an implicature, we need to propose a different analysis for the other.

- 1 See also Chemla 2009a and Cremers *et al.* 2018 for experimental evidence of this inference.
- 2 Since this inference is focus-sensitive (Simons, 2001), we use sentence final adverbs and all examples are intended to be read with default intonation.
- 3 The judgments are admittedly subtle, because, the inference under investigation entails the other predicted readings. For this reason, one has to make sure the intuitions about the reading we focus on here are not simply intuitions about compatibility. We examined the intended reading with multiple native speakers using different versions of the main example. The majority of them contributed the intuition that the sentence suggested the relevant inference above. We are therefore confident that this reading exists. Eventually, a full experimental investigation is needed to understand the empirical picture more in detail.
- 4 Note that (4a) has also a pragmatically implausible reading paraphrasable as 'By using soap, Mike didn't wash windows' in which the adverb scopes above negation. Here, we focus on the reading in which the adverb scopes below negation.

The rest of the paper is organised as follows. Section 2 sketches the implicature account of the inference of adverbial modifiers and the main theories within the implicature approach to the multiplicity inference. Section 3 discusses the problem coming from the combination of these two inferences and Section 4 sketches the two solutions to the problem. Some open and related issues are discussed in Section 5, where we also show that the ambiguity approach to the multiplicity inference (Farkas & de Swart, 2010; Grimm, 2013; Martí, 2017) doesn't provide a solution to our problem.

2 MULTIPLICITY, MODIFIERS, AND IMPLICATURES

2.1 *The implicature theory of the inference of adverbial modifiers*

In previous literature, the inference in (3b) has been analysed as a scalar implicature (Katzir, 2007). This account arises from a general theory of alternatives and is not an isolated claim about the inference of adverbial modifiers. Under this approach, (3a) is predicted to have as alternative the corresponding sentence without the adverbial modifier, (5):

(5) This morning, Mike didn't wash the window.

On the basis of this, given a theory of scalar implicatures according to which a hearer tends to conclude that any relevant and more informative alternative sentence that the speaker might have said and didn't say is false (Grice 1975 and much subsequent work), the inference in (3b) is straightforwardly derived.⁵ That is, since (5) is a stronger alternative, upon hearing (3a) the hearer would conclude that (5) is false, giving rise to the inference in (3b).

2.2 *The implicature approach to the multiplicity inference*

The main issue that any theory of the multiplicity inference has to explain is why it arises in positive sentences, such as (1a), but disappears in negative ones, such as (2a), repeated below in (6) and (7). That is, why (7) tends to convey the meaning that Mike didn't wash any window, rather than the negation of (6) that he didn't wash multiple windows.⁶

(6) This morning, Mike washed windows.

(7) This morning, Mike didn't wash windows.

The general consensus in the literature is that the positive sentence in (6) has a reading compatible with Mike washing just one window, paraphrased in (8):

(8) Mike washed one or more windows.

This immediately accounts for the interpretation of (7) as this is simply the negation of (8). What is left unexplained, however, is how the multiplicity inference arises in positive cases. As mentioned, a prominent approach is to analyse this inference as a scalar implicature. The

5 We keep the assumptions about the underlying theory of implicatures as minimal as possible.

6 The reading that Mike didn't wash more than one window can be forced by marked intonation, as in (i). We put this reading aside, as it is not relevant here and can be accounted for by any of the approaches discussed below; see Tieu & Romoli (2019) for discussion.

(i) Mike didn't wash any winDOWS, he washed only one!

main idea is that the literal meaning of a plural sentence in (8) is compared to its singular counterpart, conveying that Mike washed (only) one window. How this competition is implemented differs across the three main types of implicature accounts, which we sketch in the following.

The first account is based on enriching the sentence locally, at the predicate level, where the plural and the singular nouns are assumed to differ (Ivlieva 2013; Mayr 2015; Zweig 2009). While a singular noun in (9) has only atomic elements in its denotation (assuming all the relevant windows in the context are a, b and c), its corresponding plural form in (10) denotes both atoms and sums thereof (e.g., Schwarzschild, 1996).

$$(9) \llbracket \text{windows} \rrbracket = \{a, b, c\}$$

$$(10) \llbracket \text{windows} \rrbracket = \{a, b, c, a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c\}$$

When the plural in (10) combines with the rest of the sentence, it gives rise to the meaning in (8) that Mike washed one or more windows. However, since the singular predicate is ‘stronger’ than the plural one, that is (9) is a subset of (10), the latter can be strengthened with a local implicature at the predicate level (provided a theory of implicature which can apply locally).⁷ That is, the plural predicate in (10) can be compared to the singular predicate in (9) and strengthened by being reduced to the complement of the former, becoming equivalent to (11):

$$(11) \llbracket \text{windows} \rrbracket = \{a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c\}$$

When combined with the rest of the sentence, (11) gives rise to the multiplicity inference that Mike washed multiple windows. Importantly, when plural predicates appear in downward entailing environments, such as in the scope of negation, the singular competitor is not stronger anymore and no implicature is computed.⁸ Therefore this approach correctly captures the contrast between positive and negative sentences.

A second type of implicature account is based on higher order implicatures: implicatures arising on alternatives and the implicatures that such alternatives would have had, had they been asserted (Spector 2007). The starting point is that a singular and a plural sentence are equivalent and both mean (8). The different interpretations they give rise to — and in particular the multiplicity inference of plural sentences — follow from the assumption that they are compared to different alternatives. In particular, the singular sentence in (12) has as alternative the sentence in (13), which is stronger than (12). (12) then can be enriched with the negation of (13) and ends up conveying the meaning which can be paraphrased as in (14).

(12) Mike washed a window.

(13) Mike washed at least two windows.

(14) Mike washed exactly one window.

Sg sentence enriched by its implicature

7 Zweig (2009) and Ivlieva (2013) actually make use of an event semantics and have the strengthening apply at the VP-level. This detail is not important for our purposes so we put it aside here.

8 The sensitivity to monotonicity exhibited by scalar implicatures is generally derived by appealing to a principle preventing them from arising if they weaken the literal meaning of the sentence. This captures the difference between the contribution of implicatures in upward versus downward entailing environments; see Chierchia *et al.* 2012, among many others, for discussion.

The plural sentence then is compared to the corresponding singular sentence already enriched by its implicature, that is to (14). Since (14) is stronger than (8), it will be negated and it will give rise to the multiplicity inference. This is, if it is true that Mike washed one or more windows, but it's not true that he washed exactly one window, then it must be that he washed multiple windows. Again, given that implicatures tend not to appear in the scope of downward entailing operators, this approach too can account for the contrast between upward and downward entailing environments.

Finally, the third account by Sauerland (2003) and Sauerland *et al.* (2005b), is based on the comparison between the singular and the plural predicates at the level of presuppositions, via Maximise Presupposition (e.g., Chemla, 2008a; Heim, 1991; Marty, 2017; Percus, 1998; Singh, 2008). The assumption is that (12) and its corresponding plural counterpart in (15) are equivalent at the assertion level: both are true when Mike washed one or more windows. They differ, however, at the presuppositional level: (12), but not (15), presupposes that the window was exactly one.

(15) Mike washed windows.

On the basis of this, the use of the plural leads to the inference, via Maximise Presupposition, that the exactly-one presupposition of the singular is false.⁹ As a result, the hearer concludes that Mike washed one or more window but not exactly one, i.e., that Mike washed multiple windows. The sensitivity to monotonicity, the contrast between (6) vs. (7), is derived by the assumption that Maximise Presupposition can only apply if it strengthens the entire sentence. This immediately predicts that it will not apply under negation, as it would make the sentence weaker than its literal meaning.

In sum, there are three main types of theories within the implicature approach and they can account for the sensitivity to monotonicity and a variety of complex data in relation to the multiplicity inference which we didn't review here (see Križ 2017; Spector 2007; Zweig 2009 and Ivlieva 2013 for discussion). We turn now to show however that none of these theories can account for the problematic case discussed above, if combined with an implicature approach to the inference of adverbial modifiers.

3 THE PROBLEM

Let us now go back to the inference of the adverbial modifiers and the multiplicity inference, repeated from above:

- (16) a. This morning, Mike didn't wash the window with soap.
 b. \rightsquigarrow This morning, Mike did wash the window

9 The way this is implemented is by having the singular, but not the plural, to be associated with a presupposition that the argument it combines with is an atomic individual. In particular, the assumption is that both (12) and its plural counterpart in (15) are equivalent, as the presupposition of the former would project through the existential quantification associated with the bare plural and becomes part of the assertion. However, at the predicate level the singular has a stronger presupposition than the plural. Maximise Presupposition then is assumed to apply in the scope of the existential quantifier, giving rise to the meaning of plural that Mike washed one or more windows but not exactly one, i.e., that he washed multiple windows.

- (17) a. This morning, Mike washed windows.
 b. \rightsquigarrow This morning, Mike washed multiple windows

When we combine the two, we observe that (18) has a reading in which the multiplicity inference appears on top of the inference of the adverbial modifier:

- (18) This morning, Mike didn't wash windows with soap.
 \rightsquigarrow This morning, Mike did wash multiple windows (albeit not with soap)

Let us now consider the predictions of the implicature approach to adverbial modifiers combined with different versions of the implicature approach to the multiplicity inference, starting from the local-strengthening theories (Ivlieva, 2013; Mayr, 2015; Zweig, 2009). Under this approach, the plural noun can be strengthened locally, or it can be interpreted literally.¹⁰ Either way the reading we are after is not predicted. To illustrate, consider first the case in which the noun is not strengthened, paraphrased in (19). While this may be a possible reading of the sentence, it doesn't account for the reading we are after: it is compatible with Mike washing only one window.

- (19) Mike didn't wash any window with soap but he did wash one or more windows.

If, on the other hand, the predicate is strengthened, the predicted reading is in (20). The multiplicity inference arising from the inference of the adverbial modifier is now correctly captured, but the literal meaning of the sentence (the first conjunct in (20)) appears too weak: it is compatible with Mike washing one window with soap, *contra* intuitions.

- (20) Mike didn't wash multiple windows with soap but he did wash multiple windows.

The situation is even worse for the global approach based on higher-order implicatures by Spector (2007): not only does this approach fail to capture for the described reading, but it also predicts the intuitively unattested reading paraphrased in (21).¹¹

- (21) Mike didn't wash any window with soap but he did wash exactly one window.

Finally, the problem for the presupposed implicature approach (Sauerland, 2003; Sauerland *et al.*, 2005b) is that Maximise Presupposition is not predicted to apply in the case of (18), as it would weaken the meaning of the sentence. Therefore, the predicted reading is again

10 Some of the local-strengthening theories assume that the strengthening of plural is obligatory. It can however happen globally, rather than locally, which in the relevant cases discussed here would be vacuous and therefore equivalent to not strengthening the predicate at all; see Mayr 2015 and Ivlieva 2013 for discussion.

11 We refer the reader to the Appendix for details. As a sketch of why the reading above is predicted, consider that (18) has (i) as its alternative, i.e., the singular alternative enriched with its implicature, which in turn is derived from its alternative in (ii). The negation of (i) then gives rise to the unattested reading in (21).

(i) Mike didn't wash a window with soap but he did wash multiple windows.

(ii) Mike didn't wash at least two windows.

(19). If we lift the restriction and nonetheless apply Maximise Presupposition, the predicted reading is (20), which is also intuitively incorrect.¹²

In sum, none of the implicature theories of the multiplicity inference in combination with an implicature-based analysis of the inference of adverbial modifiers can account for the problematic reading above. In the next section, we sketch two solutions to the problem: the homogeneity approach to the multiplicity inference by Križ (2017) combined with the implicature approach to the inference of adverbial modifiers and a presuppositional theory of the latter combined with the implicature approach to the multiplicity inference.

4 TWO SOLUTIONS

4.1 *The first solution: the multiplicity inference is not an implicature*

Križ (2017) proposes an alternative approach to the multiplicity inference in terms of homogeneity. The main idea is that most predicates are undefined under certain conditions when they apply to pluralities. For what is relevant here, when a predicate like *windows* appears in an episodic sentence such as (1a), repeated below in (22), it gives rise to the following trivalent truth-conditions: it is true when both (22a) and (22b) are true, false when both are false, and undefined otherwise. This predicts the intuitively correct reading of (22), i.e., that it is true if and only if Mike washed more than one window.

- (22) Mike washed windows.
- a. Mike washed one or more windows.
 - b. Mike washed multiple windows.

When (22) is negated, as in (23), negation leaves undefinedness untouched, giving rise to the following truth-conditions: (23) is true when both (23a) and (23b) are true, false when both are false, and undefined otherwise. These truth-conditions capture the intuition that (23) is true if and only if Mike didn't wash any window.¹³ Hence, the homogeneity approach can account for the alternation between positive and negative cases.

- (23) Mike didn't wash windows.
- a. Mike didn't wash one or more windows.
 - b. Mike didn't wash multiple windows.

Let us now go back to the problematic case in (24) and its meaning strengthened with the implicature of the adverbial modifier *à la* Katzir (2007) in (25):

- (24) Mike didn't wash windows with soap.
 (25) Mike didn't wash windows with soap and he did wash windows.

The homogeneity approach predicts that the first conjunct of (25) is true whenever both (22a) and (22b) are true, false when both are false and undefined otherwise. The second

12 Note that under this approach Maximise Presupposition is assumed to apply in the scope of the existential quantification associated with the bare plural. At that level, the inference of the adverbial modifier is not yet derived and hence it is not taken into account by the Maximise Presupposition strengthening mechanism. This is because, under the implicature approach sketched above, the derivation of that inference applies globally on the negated sentence against its negated alternative.

13 The negation assumed here is standard negation, making a true sentence false, a false sentence true and keeping the undefinedness untouched.

conjunct, on the other hand, is true when both (23a) and (23b) are true, false if both are false and undefined otherwise. Putting these together, it follows that the conjunction in (25) is true if and only if all of (22a), (22b), (23a) and (23b) are true.¹⁴ This reading is paraphrased in (26) and this is precisely the reading we are after:

(26) Mike didn't wash any window with soap but he did wash multiple windows.

In sum, adopting the homogeneity approach allows us to account of the problematic case by keeping the implicature approach to the inference of adverbial modifiers.¹⁵

4.2 *The second solution: The inference of adverbial modifiers is not an implicature*

Another solution to the puzzle is to abandon the scalar implicatures theory of the inference of adverbial modifiers and analyse them as presuppositions instead (Schlenker, 2008; Simons, 2001).¹⁶ Under this approach, the sentence in (27) not only entails (27a), but also presupposes it. This idea is supported by the projection data in (28a)–(28d) which suggest that Mike washed windows, i.e., this inference project out of the scope of the operators below and in that it resembles closely the behaviour of presuppositions.¹⁷

(27) Mike washed windows with soap.

a. \sim Mike washed (one or more) windows Presupposition

(28) a. Mike didn't wash windows with soap.

b. Did Mike wash windows with soap?

c. Perhaps Mike washed windows with soap.

d. If Mike washed windows with soap, he will probably do it again.

e. \sim Mike washed (one or more) windows

Given the assumptions that the inference of the adverbial modifiers is a presupposition and that presuppositions project through negation, our problematic case repeated in (29), would also presuppose (27a), repeated below in (29a). This alone however does not solve

14 Note that the implicit assumption here is that the negation of the alternatives is also the standard negation defined above in Footnote 13; see Spector & Sudo 2017 for discussion.

15 An anonymous reviewer suggested us a similar account linked to the homogeneity presupposition associated with distributive predicates. While we think this is a possible alternative route, we do not pursue it here for two reasons. First, as Kríž (2017) argues, homogeneity isn't limited to the distributive predicates only. Therefore linking homogeneity to distributivity seems at best not general enough. Second, for the account to work we would need the homogeneity presupposition to project universally through the existential quantification associated with the episodic bare plural. Though this doesn't seem to be what happens with presuppositions and existential quantification in general (see Chemla 2009b, among others).

16 A third possibility would be to analyse the inference of the adverbial modifier neither as an implicature nor as a presupposition, but as an inference of a different kind altogether. Pursuing this direction would have to minimally include an account of how this inference arises and allow it, under the second solution below, to be strengthened by implicatures. We do not develop this option here.

17 While the projection facts constitute a suggestive piece of evidence in favor of the presuppositional analysis, we do not think they are per se conclusive. For discussion about projection behavior and status of the inference projecting see Chemla 2008b; Romoli 2012; Tonhauser *et al.* 2019; see also Chemla 2009a for an experimental comparison of the inference of adverbial modifiers and presuppositions, on one side, and implicatures on the other.

the puzzle, as (29a) means that Mike washed one or more windows, not that he washed more than one window. What we need is the multiplicity inference arising on top of (29a).

- (29) Mike didn't wash windows with soap.
 a. \sim Mike did wash (one or more) windows Presupposition

The remaining ingredient is adopting the idea that presuppositions can be in itself strengthened by implicatures (Spector & Sudo 2017; see also Gajewski & Sharvit 2011; Magri 2010; Marty 2017; Sudo & Romoli 2017). The gist of the idea is that if a sentence ϕ has an alternative ψ with a stronger presupposition, the negation of that stronger presupposition is added to the presupposition(s) of ϕ . To illustrate, and putting aside for now the details of the different versions of the implicature approach to the multiplicity inference (to which we return in the next subsection), the alternative of (29) is (30).

- (30) Mike didn't wash exactly one window with soap.
 a. \sim Mike did wash exactly one window Presupposition

(30) presupposes (30a), which is stronger than the presupposition of (29) in (29a). Therefore the negation of (30a) is added to the presupposition of (29) and we obtain the right reading that Mike didn't wash any window with soap but he did wash multiple windows, as illustrated in (31).

- (31) Mike didn't wash windows with soap.
 a. \sim Mike didn't wash any window with soap Assertion
 b. \sim Mike did wash one or more windows Presupposition
 c. \sim Mike didn't wash exactly one window
 Added negation of the presupposition of the alternative

In sum, this approach can account for the reading we are after. In fact, it can be seen as an argument for a presuppositional treatment of the inference of adverbial modifiers, an implicature approach to multiplicity inferences, and the idea that presuppositions can be strengthened via implicatures.

4.3 Issues for the presuppositional solution

The presuppositional solution integrates a presuppositional account of the inference of adverbial modifiers with an implicature treatment of the multiplicity inference. However, not all implicature accounts of the multiplicity inference fare equally well in this approach. To illustrate the issue, let us consider first how the three implicature approaches outlined above (the local approach by Mayr (2015), Ivlieva (2013) and Zweig (2009), the global approach by Spector (2007), and the one by Sauerland *et al.* (2005b)) can be applied at the presuppositional level to strengthen the presupposition of the adverbial modifiers in order to obtain the correct inference, e.g. that Mike washed multiple windows. To ease the exposition of the different approaches, let us introduce the following notation in (32): 'EXH' here is an operator which has the effect of adding to a sentence its implicatures. In other words, EXH combines with a sentence and returns its strengthened meaning.¹⁸

- (32) EXH[Mike washed windows] = Mike washed multiple windows

18 See Chierchia *et al.* 2012; Fox 2007; Magri 2010 among many others.

Crucially, under the approach introduced in the previous section, if the sentence has a presupposition, EXH will generally have an indirect effect on the presupposition as well. The idea of treating the inference of adverbial modifiers as presuppositions is that EXH, by being able to strengthen presuppositions, will generate a multiplicity inference on top of it. Let us see whether and how this idea works when we look at the details of each account, starting with the local one. First, the account by Mayr (2015) appears ‘too local’ for the approach developed in the previous section. This is because the multiplicity strengthening happens within the DP containing the plural noun and no presupposition has yet been introduced at that level.¹⁹

The implementation of the local approach by Ivlieva (2013) and Zweig (2009) could be made work, provided we allow ourselves some extra (ad hoc) assumptions about EXH and its distribution. In particular, we would need a version of EXH, call it EXH’, which strengthens the presuppositional level only, when applied locally at the level of the VP. We don’t know though of any independent motivation for such a strengthening operator.²⁰

The situation is even more problematic if one adopts Spector’s (2007) approach to the multiplicity inference. As noted above, this approach gives rise to the unattested reading in (21). This is not only unattested but also in contradiction with the strengthened presupposition that Mike washed multiple windows. The question for Spector 2007 is therefore how to block (21). We discuss this in more detail in the Appendix.

Finally, it is not entirely clear how to combine the presuppositional approach to the inference of adverbial modifiers with Sauerland *et al.*’s (2005a) approach to the multiplicity inference. In particular, as discussed above, Maximise Presupposition is assumed not to apply in this case, so it is unclear how the presuppositional treatment of the adverbial modifier would help.

In sum, none of the implicature accounts are straightforwardly compatible with the presuppositional solution sketched above.

5 DISCUSSION AND CONCLUSION

In this section, we enlarge the focus from the multiplicity inference to scalar terms more generally and their interaction with modifiers under negation. Before that, let us briefly note that the other main non-implicature approach to the multiplicity inference, the ambiguity approach (Farkas & de Swart, 2010; Grimm, 2013; Martí, 2017), does not solve our problem. To illustrate, the main idea of this approach is that a plural noun like *windows* is ambiguous between an inclusive interpretation which includes atomic elements and

19 In other words, the LF predicted by Mayr (2015) is (i) (where EXH₂ is the exhaustivity operator generalised to the predicative level). Therefore the strengthened restrictor will combine with both the assertion and the presupposition of the predicate giving rise to the reading in (ii), which is too weak in the assertion part. So in this sense, Mayr’s account is ‘too local’ to work.

(i) not[\exists [EXH₂ windows]] [λ_2 [Mike washed t_2 with soap]]

(ii) a. Assertion: There is no window plurality which Mike washed with soap.

b. Presupposition: There is a window plurality which Mike washed.

20 We discuss the details in the Appendix.

sums thereof and an exclusive interpretation which only includes sums, giving rise to the interpretations in (33) and (34), respectively. The choice between the two meanings is regulated by the Strongest Meaning Hypothesis which favors the strongest reading—the one that asymmetrically entails the other—i.e., the exclusive interpretation in positive cases and the inclusive one in negative ones.

- (33) Mike washed one or more windows. inclusive
 (34) Mike washed more than one window. exclusive

The ambiguity approach however cannot account for our problematic data, regardless of whether it is combined with the implicature or the presuppositional account of adverbial modifiers. The problem is that in order to obtain the intended reading, we would need to somehow select the inclusive meaning of *windows* in the asserted component of the sentence and the exclusive one in the presupposed/implicated part and it is unclear how to do it compositionally.²¹ Thus the ambiguity approach cannot account for the interpretation we are after and we do not see any obvious amendment to this theory which would allow it to.²²

The discussion on the interaction of multiplicity inference with modifiers under negation extends in interesting ways to other scalar terms, such as scalar adjectives and disjunction.²³ As a start, consider the sentence in (35) with a scalar adjective *good* and an adverbial modifier *gladly* under negation. Intuitively, (35) — if anything — suggests (35a). While this inference is a subtle one, it is clear that the sentence doesn't give rise to the opposite inference, namely that Patricia gave Peter an excellent grade. This however is precisely the inference predicted by the implicature approach.²⁴

- 21 One could consider allowing different denotations of the plural to be used in the assertion and presuppositional levels. This would however predict that in case of lexical ambiguity, different meanings could be selected at these different levels. For instance, (i) should be able to obtain the reading paraphrased in (ii), in which *bank* obtains a different interpretation in the assertion and the presuppositional levels, contrary to facts.
- (i) Patricia knows that the fisherman went to the bank.
 (ii) Patricia believes that the fisherman went to the bank₁ and the fisherman did go to the bank₂.
- 22 As discussed by Farkas & de Swart (2010); Ivlieva (2013); Spector (2007) and Križ (2017), the ambiguity approach also has problems with accounting for the multiplicity inference in sentences with non-monotonic quantifiers, as in (i):
- (i) Exactly two students solved problems.
 ~→ Two students solved more than one problem and all of the others didn't solve any problems
- 23 Many thanks to Benjamin Spector (p.c.) for helpful discussion on the issues discussed in this section.
- 24 This is because among the alternatives of (35) we have (i), which is not entailed by the assertion and therefore ends up being negated, giving rise to the inference that Patricia gave Peter an excellent grade.
- (i) Patricia didn't give Peter an excellent grade.

Note, however, that a sentence like (36) is challenging for the implicature approach to the inference of adverbial modifiers only to the extent that one can show that the alternative in (i) is relevant in the given context. Thanks to an anonymous reviewer for discussion on this point.

- (35) Patricia didn't give Peter a good grade gladly.
 a. \sim Patricia gave Peter a good grade (but not an excellent one)

The presuppositional approach to the inference of adverbial modifiers, on the other hand, predicts the inference in (35a), as it is the (strengthened) presupposition of (36), which projects through negation.

- (36) Patricia gave Peter a good grade gladly.

This could be taken as an argument for the presuppositional view of the inference of adverbial modifiers. However, when we look at other scalar terms, like disjunction, the implicature approach to adverbial modifiers appears to fare better.

Consider first the case of the free choice inference, triggered by a disjunction embedded in the scope of a possibility modal, analysed by many as an implicature.²⁵ (37) gives rise to the inference that each of the disjunct is possible: Miriam can play with her computer and she can play with her i-Pad (though possibly not with both).

- (37) Miriam can play with her computer or her i-Pad.
 a. \sim Miriam can play with her computer and she can play with her i-Pad

Now consider (38): the analogous case to the one we focused on above, involving a free choice inference and the inference of an adverbial modifier under negation.²⁶ It seems to have a reading that Miriam can't play with her computer and that she can't play with her i-Pad after dinner, but before dinner, she has free choice between the two. That is, the free choice inference seems to arise on top of the inference of the adverbial modifiers, similarly to the multiplicity inference discussed above.

- (38) Miriam can't play with her computer or her i-Pad after dinner.
 a. \sim Miriam can't play with her computer and she can't play with her i-Pad after dinner, but before dinner she can play with one and she can play with the other

If this is true, then combining the implicature approach to the inference of the adverbial modifier with an implicature account of the free choice inference straightforwardly predicts the reading in (38).

More surprisingly, the implicature approach to the inference of adverbial modifiers also predicts a conjunctive inference with a simple disjunction like (39). Namely, it predicts the reading in (39a).

- (39) Miriam didn't go to Paris or Berlin by train.
 a. \sim Miriam did go to Paris and she did go to Berlin (albeit not by train)

The presuppositional view, on the other hand, predicts the inference in (38a) but it doesn't predict the inference in (39a). In fact, it predicts (39) to have the inference in (40). While the judgments are subtle and this should be investigated further, we think that if anything (39) suggests (39a) rather than (40).

25 See Bar-lev & Fox 2017; Chemla 2008b; Fox 2007; Franke 2011; Klinedinst 2007; Santorio & Romoli 2017, among others.

26 Again, as in the case of the multiplicity inference and the inference of the adverbial modifiers, the intended reading is one in which *after dinner* is in the scope of negation.

(40) Miriam went to Paris or Berlin but not both

Thus the cases involving disjunction appears to favour the implicature approach to the inference of adverbial modifiers. On the other hand, the case of adjectives like *good* seems to push in the opposite direction, suggesting that we should adopt the presuppositional view. The general interaction between modifiers under negation and scalar terms requires therefore further exploration; an exploration which however goes beyond the scope of this squib.

To summarise, we have focused on the interaction between adverbial modifiers and multiplicity inferences in sentences containing a plural noun and an adverb in the scope of negation like *This morning, Mike didn't wash windows with soap*. We have shown that a reading of this sentence – entailing that Mike didn't wash any window with soap but that he did wash multiple windows (albeit not with soap) – is problematic for the implicature approach to the multiplicity inference in combination with the implicature treatment of the inference of adverbial modifiers. We have sketched two solutions for this problem. The first was to keep the implicature approach to adverbial modifiers but adopting a non-implicature approach to multiplicity, based on homogeneity. The second solution was to keep instead the implicature approach to the multiplicity inference but treat the inference of adverbial modifiers as a presupposition, in combination with the idea that presuppositions can be strengthened via implicatures. Either way, the interaction between multiplicity and the inference of adverbial modifiers suggests that we cannot treat both as implicatures: if we want to treat either one as an implicature, we need to do something different for the other.

Appendix: the formal details

A.1. Background: Magri's approach

We adopt Magri's proposal on the interaction between exhaustification and presuppositions (see Gajewski and Sharvit 2009 for a very similar proposal).²⁷ This proposal is based on the following two components. First, it assumes a two-dimensional theory of meaning (cf. Karttunen & Peters, 1979; see also Mandelkern 2016 for a more recent approach): a sentence φ denotes two propositions, its presupposition $\llbracket \varphi \rrbracket^{prs}$ and its assertion $\llbracket \varphi \rrbracket^{asr}$, which, following Magri's notation, we indicate as $\llbracket \varphi \rrbracket = \llbracket \varphi \rrbracket^{prs}, \llbracket \varphi \rrbracket^{asr}$. Second, when a sentence is parsed with an *exh*, the meaning of *exh* affects both dimensions of meaning. In a nutshell, the assertion and presuppositional levels of a sentence are exhausted.

(1) a. $\llbracket \text{exh } \varphi \rrbracket = (\llbracket \text{exh } \varphi \rrbracket^{prs}, \llbracket \text{exh } \varphi \rrbracket^{asr})$

Magri, (2010 p.54) (see also Marty 2017, p.20-21) defines *exh* as follows: the strengthened presupposition $\langle \text{exh } \varphi \rangle^{prs}$ and the strengthened assertion $\langle \text{exh } \varphi \rangle^{asr}$ as in (2): $\langle \text{exh } \varphi \rangle^{prs}$ is the presupposition of φ conjoined with the negation of all the excludable presuppositions and $\langle \text{exh } \varphi \rangle^{asr}$ is the assertion of φ conjoined with the negation of all the excludable

27 Magri's approach comes with the assumption of bidimensionality of meaning and with some empirical issues pointed out in Marty 2017. For our purposes, however, it will suffice. But everything that is done below could be extended to the unidimensional approach by Marty 2017.

assertions. Here we follow Magri and Marty in assuming that the excludable alternatives are those alternatives (the presuppositions of which) are non-weaker than the (presupposition of the) prejacent.²⁸

(2) a. **Strengthened presuppositions:**

- (i) $\text{excl}_{pres}(\varphi) = \{\psi \in \text{alt}(\varphi) : \varphi_{pres} \not\Rightarrow \psi_{pres}\}$
 (ii) $\llbracket \text{exh}(\varphi) \rrbracket^{pres} = \llbracket \varphi \rrbracket^{pres} \wedge \forall \psi [\psi \in \text{excl}_{pres}(\varphi) \rightarrow \neg \llbracket \psi \rrbracket^{pres}]$

b. **Strengthened assertions:**

- (i) $\text{excl}_{asr}(\varphi) = \{\psi \in \text{alt}(\varphi) : \varphi_{asr} \not\Rightarrow \psi_{asr}\}$
 (ii) $\llbracket \text{exh}(\varphi) \rrbracket^{asr} = \llbracket \varphi \rrbracket^{asr} \wedge \forall \psi [\psi \in \text{excl}_{asr}(\varphi) \rightarrow \neg \llbracket \psi \rrbracket^{asr}]$

At the assertion level, *exh* operates in the same way as before. But now *exh* also has an effect at the presuppositional level: it excludes the presuppositions of the alternatives that are non-weaker than the presuppositions of the prejacent.

A.2. *Back to multiplicity*

Now let's go back to our main case in (3a), presupposing (3b) by assumption in the presuppositional solution we are considering here.

- (3) a. Mike didn't wash windows with soap.
 b. Mike washed windows. presupposition

Using Magri's notation, we have the presupposition and the assertion dimensions as in (4) which is equivalent to (5). (Where *pl* = Mike washed windows (one-or-more), *ws* = with soap, *sg* = Mike washed a window (one-or-more), *two* = Mike washed at least two windows; to avoid clutter we use those symbols to represent both a sentence and its meaning, keeping the assumptions outlined above about what *exh* takes as an argument and operates on).

- (4) $\langle \llbracket \text{Mike didn't wash windows with soap} \rrbracket^{pres}, \llbracket \text{Mike didn't wash windows with soap} \rrbracket^{asr} \rangle$
 (5) $\langle \text{pl}, \neg[\text{pl ws}] \rangle$

A.3. *Spector 2007*

We can now combine the above with Spector's approach to account for the relevant reading of (3a). To illustrate, assume we parse it as in (6), schematised as in (7).

- (6) $\text{exh}[\text{exh}[\text{not}[\text{Mike washed windows with soap}]]]$
 (7) $\text{exh}[\text{exh}[\neg[\text{pl ws}]]]$

The inner most *exh* applies to its prejacent $\neg[\text{pl ws}]$ and exhaustifies its assertion component with respect to its alternatives and its presupposition component with respect to the presupposition of its alternatives. The outermost *exh* does the same: it exhaustifies the assertion component of its prejacent $\text{exh}[\neg[\text{pl ws}]]$ with respect to its (already exhaustified)

28 A more accurate rendition would need to assume innocent exclusion and possibly innocent inclusion; Fox 2007, Bar-lev and Fox 2017 among others.

alternatives and its presupposition component with respect to the (already exhausted) presuppositions of its alternatives.

$$(8) \llbracket \text{exh}[\text{exh}[\neg[\text{pl ws}]]] \rrbracket = \langle \llbracket \text{exh}[\text{exh}[\neg[\text{pl ws}]]] \rrbracket^{\text{prs}}, \llbracket \text{exh}[\text{exh}[\neg[\text{pl ws}]]] \rrbracket^{\text{asr}} \rangle$$

Let us illustrate this step by step focusing on the outcome of the presuppositional level and then moving on the assertion level.

A.3.1. *The presuppositional level* Consider the innermost *exh* first: the alternatives on which it operates are the alternatives of the prejacent $\neg[\text{pl ws}]$. At the presuppositional level, following the definition in (2), *exh* excludes the presuppositions of those alternatives, which are excludable in the sense above (i.e. non-weaker than the presuppositions of the prejacent). At this stage, none of the presupposition is excludable (they are all entailed by the presupposition of the prejacent).

$$(9) \left\{ \begin{array}{l} \neg[\text{pl ws}] = \langle \text{pl}, \neg[\text{plws}] \rangle, \\ \neg[\text{sg ws}] = \langle \text{sg}, \neg[\text{sgws}] \rangle, \\ \neg[\text{sg}] = \langle \emptyset, \neg[\text{sg}] \rangle, \\ \neg[\text{pl}] = \langle \emptyset, \neg[\text{pl}] \rangle \end{array} \right\}$$

Moving on to the second *exh*, leads to the following outcome: the alternatives on which this operates on are the exhausted alternatives of the prejacent and again at the presuppositional level, the meaning of *exh* will negate the presuppositions of those alternatives, which are excludable (ignore for now the assertion level, we will go back to it below).

$$(10) \left\{ \begin{array}{l} \text{exh}[\neg[\text{pl ws}]] = \langle \text{pl}, \neg[\text{plws}] \wedge \text{pl} \rangle, \\ \text{exh}[\neg[\text{sg ws}]] = \langle \text{sg}, \wedge \neg \text{two}, \neg[\text{sgws}] \wedge \text{two} \rangle, \\ \text{exh}[\neg[\text{sg}]] = \langle \emptyset, \neg[\text{sg}] \rangle, \\ \text{exh}[\neg[\text{pl}]] = \langle \emptyset, \neg[\text{pl}] \rangle \end{array} \right\}$$

Note-and this is the crucial step-the presupposition of the alternatives above have already been exhausted by the meaning of the first *exh* at the presuppositional level, with respect to the presupposition of their alternatives. In particular, crucially, the exhausted singular alternative repeated below in (11) is itself exhausted with respect to the alternatives in (12) and at the presuppositional level therefore becomes $\langle \text{sg} \wedge \neg \text{two}, \neg[\text{sgws}] \wedge \text{two} \rangle$.

$$(11) \text{exh}[\neg[\text{sg ws}]]$$

$$(12) \left\{ \begin{array}{l} \neg[\text{sg ws}] = \langle \text{sg}, \neg[\text{sgws}] \rangle, \\ \neg[\text{two ws}] = \langle \text{two}, \neg[\text{two ws}] \rangle, \\ \neg[\text{sg}] = \langle \emptyset, \neg[\text{sg}] \rangle, \\ \neg[\text{two}] = \langle \emptyset, \neg[\text{two}] \rangle \end{array} \right\}$$

The outcome is that the presupposition is indirectly strengthened recursively by *exh* in Magri's and Spector's ways combined, giving rise to the strengthened presuppositional inference that Mike washed at least two windows.

$$(13) \llbracket \text{exh}[\text{exh}[\neg[\text{pl ws}]]] \rrbracket^{\text{prs}} = \text{pl} \wedge \neg(\text{sg} \wedge \neg \text{two}) = \text{pl} \wedge \text{two}$$

(= Mike washed at least two windows)

In sum, the combination of Magri's and Spector's accounts derives the right strengthened reading at the presuppositional level. We note that at each step *exh* only applies to a sentence and its alternatives. However, given Magri's bidimensionality of meaning assumption, it now has an effect both on the assertion and the presuppositional dimensions. On the latter, exhaustification is with respect to the presuppositions of the alternatives. The only novel part in combining Magri's system with Spector's is the fact that we have to deal with more than one *exh*. There is an issue, however, with what Spector's approach predicts *at the assertion level*, independently from the presuppositional solution.

A.3.2. *The assertion level* To illustrate the issue again here: consider now the assertion part of the derivation above repeated in (14).

$$(14) \llbracket \text{exh} [\text{exh} [\neg \text{pl ws}]] \rrbracket^{asr}$$

The alternatives over which the first *exh* operates are in (15), focusing on the assertion part now. At this stage, both *pl* and *sg* (which are equivalent) are excludable, giving rise to the weak inference that Mike did wash one or more windows.

$$(15) \left\{ \begin{array}{l} \neg \text{pl ws} = \langle \text{pl}, \neg [\text{plws}] \rangle, \\ \neg \text{sg ws} = \langle \text{sg}, \neg [\text{sgws}] \rangle, \\ \neg \text{sg} = \langle \emptyset, \neg [\text{sg}] \rangle, \\ \neg \text{pl} = \langle \emptyset, \neg [\text{pl}] \rangle \end{array} \right\}$$

The alternatives the second *exh* operates on are in (16), again focusing on the assertion part on the right. The problem arises in particular given the exhaustified singular alternative in (16).

$$(16) \left\{ \begin{array}{l} \text{exh}[\neg \text{pl ws}] = \langle \text{pl}, \neg [\text{plws}] \wedge \text{pl} \rangle, \\ \text{exh}[\neg \text{sg ws}] = \langle \text{sg} \wedge \neg \text{two}, [\text{sgws}] \wedge \text{two} \rangle, \\ \text{exh}[\neg \text{sg}] = \langle \emptyset, \neg [\text{sg}] \rangle, \\ \text{exh}[\neg \text{pl}] = \langle \emptyset, \neg [\text{pl}] \rangle \end{array} \right\}$$

This is because that alternative is enriched with respect to the alternatives in (18) and at the assertion level therefore becomes $\neg [\text{sgws}] \wedge \text{two}$.

$$(17) \text{exh}[\neg \text{sg ws}]$$

$$(18) \left\{ \begin{array}{l} \neg \text{sg ws} = \langle \text{sg}, \neg [\text{sgws}] \rangle, \\ \neg \text{two ws} = \langle \text{two}, \neg [\text{two ws}] \rangle, \\ \neg \text{sg} = \langle \emptyset, \neg [\text{sg}] \rangle, \\ \neg \text{two} = \langle \emptyset, \neg [\text{two}] \rangle \end{array} \right\}$$

The problem is that this alternative is now excludable therefore giving rise to the unattested inference that Mike didn't wash windows with soap but he did wash exactly one window.

$$(19) \llbracket \text{exh} [\text{exh} [\neg [\text{pl ws}]]] \rrbracket^{asr} = \neg \text{pl ws} \wedge \neg [[\text{sg ws}] \wedge \text{two}] = \neg \text{pl ws} \wedge \neg \text{two}$$

In sum, by combining Magri's and Spector's approach one can derive the right multiplicity inference at the presuppositional level. However, following the definition in (2), each *exh* applies to its prejacent and strengthen both its assertion and presuppositions and this gives rise to the outcome above. As we have seen, there is still an issue about the predictions at the assertion level. So ultimately at this level where Spector's approach might fail to account

for our data. However, one could imagine a separate solution for that; one that maintains the right result at the presuppositional level. For instance, one could imagine a system like Magri's where if the outcome at the presuppositional and assertion level are in conflict, the former wins out to the latter.

A.4. *Zweig 2009 and Ivlieva 2013*

It is possible to make the account work also in Zweig's and Ivlieva's approach, if we make some extra assumptions about *exh* and its distribution, as discussed below. Let's move to an event semantics and let's use the notation in Zweig 2009 for ease of reference, where **window* is the number neutral denotation. Let's also assume that the meaning of the adverb is combined with the rest as follows, introducing the presupposition as in follows (using Heim and Kratzer's 1998 notation for presupposition and assertion here, we will go back to Magri's notation below):

$$(20) \llbracket \text{with soap} \rrbracket = \lambda P \lambda e : P(e). P(e) \wedge \text{with-soap}(e)$$

The meaning of a sentence like *Mike washed windows with soap* before existential closure over events is as in (21):

$$(21) \llbracket \text{Mike washed windows with soap} \rrbracket = \\ \lambda e : \exists X [* \text{window}(X) \wedge * \text{wash}(e) \wedge * \text{ag}(e, m) \wedge * \text{th}(e, X)] \\ \cdot \exists X [* \text{window}(X) \wedge * \text{wash}(e) \wedge * \text{ag}(e, m) \wedge * \text{th}(e, X) \wedge \text{with-soap}(e)]$$

After existential closure and negation the meaning will end up being as in (22a), with the presupposition in (22b), which has projected through negation: there is no event of Mike washing one or more windows with soap and there is an event of Mike washing one or more windows.

$$(22) \llbracket \text{not}[\text{exh}[\text{Mike washed windows with soap}]] \rrbracket = \\ \text{a. } \neg \exists e \exists X [* \text{window}(X) \wedge * \text{wash}(e) \wedge * \text{ag}(e, m) \wedge * \text{th}(e, X) \wedge \text{with-soap}(e)] \\ \text{b. } \exists e \exists X [* \text{window}(X) \wedge * \text{wash}(e) \wedge * \text{ag}(e, m) \wedge * \text{th}(e, X)]$$

This doesn't give us the right reading yet as it is too weak in the presupposition. However, we can now combine Magri and Zweig and compute the multiplicity implicature below the existential closure over events on the presuppositional level.

First we need to generalise *exh* to take predicates of events, so we generalise Magri's definitions in (2a) above as follows, where *P* and *Q* are predicates of events and their presuppositions are also predicates of events, as above in (21).

$$(23) \text{ a. Strengthened presuppositions at predicate level:} \\ \text{(i) } \text{excl}_{pres}(P) = \{ Q \in \text{alt}(P) : P_{pres} \not\subseteq Q_{pres} \} \\ \text{(ii) } \llbracket \text{exh}(P) \rrbracket^{pres} = \llbracket P \rrbracket^{pres} \wedge \forall Q [Q \in \text{excl}_{pres}(P)] \rightarrow \llbracket Q \rrbracket^{pres} \\ \text{b. Strengthened assertions at predicate level:} \\ \text{(i) } \text{excl}_{asr}(P) = \{ Q \in \text{alt}(P) : P_{asr} \not\subseteq Q_{asr} \} \\ \text{(ii) } \llbracket \text{exh}(P) \rrbracket^{asr} = \llbracket P \rrbracket^{asr} \wedge \forall Q [Q \in \text{excl}_{asr}(P)] \rightarrow \llbracket Q \rrbracket^{asr}$$

Now, we apply *exh* locally below existential closure, on the presupposition. In particular, we negate the presupposition of the singular alternative at the predicate level, that is (24),

where lower case variables range over atoms. As a result, we obtain (25).

- (24) $\llbracket \text{Mike washed a window with soap} \rrbracket^{prs} =$
 $\lambda e \exists x [*window(x) \wedge *wash(e) \wedge *ag(e, m) \wedge *th(e, x)]$
- (25) $\llbracket \text{exh}[\text{Mike washed windows with soap}]^{prs} =$
 $\lambda e \exists X [*window(X) \wedge *wash(e) \wedge *ag(e, m) \wedge *th(e, X)] \wedge \neg \exists x [*window(x) \wedge$
 $*wash(e) \wedge *ag(e, m) \wedge *th(e, x)]$

This gives us the right reading at the presuppositional level: a predicate of washing events of more than one windows by Mike. When this is existentially quantified over it gives us the right presupposition that there is an event of washing multiple windows by Mike.

Note, however, that to get the right reading we also need a way not to strengthen at the assertion level or we would weaken the meaning of the assertion under negation. One can imagine different ways of doing that, the choice of which also interacts with the standard assumption of an economy condition that generally bans exhaustification if it weakens the meaning of a sentence. The easiest way to do it is defining another exh, call it exh_3 which applies to a sentence in Magri's way but it only has an effect on its presupposition. We could imagine that this is invoked only if the regular exh would weaken the meaning at the assertion level.

- (26) a. $\llbracket \text{exh}_3 \varphi \rrbracket = \langle \llbracket \text{exh} \varphi \rrbracket^{prs}, \llbracket \varphi \rrbracket^{asr} \rangle$

Putting it altogether, by applying (26) before existential closure and then applying existential closure, we obtain the correct reading as follows: there is no event of Mike washing one or more windows with soap and there is an event of Mike washing more than one window.

- (27) $\llbracket \text{not}[\text{exh}_3[\text{Mike washed windows with soap}]] \rrbracket =$
 a. $\neg \exists e \exists X [*window(X) \wedge *wash(e) \wedge *ag(e, m) \wedge *th(e, X) \wedge \text{with-soap}(e)]$
 b. $\exists e \exists X [*window(X) \wedge *wash(e) \wedge *ag(e, m) \wedge *th(e, X) \wedge \neg \exists x [*window(x)$
 $\wedge *wash(e) \wedge *ag(e, m) \wedge *th(e, x)]]$

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