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Implications of symmetries in the scalar sector

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Abstract. Symmetries play a very important rôle in Particle Physics. In extended scalar sectors, the existence of symmetries may permit the models to comply with the experimental constraints in a natural way, and at the same time reduce the number of free parameters. There is a strong interplay among internal symmetries of the scalar potential, its CP properties and mass degeneracies of the physical scalars. Some of these aspects were discussed in this talk.

1. Introduction

There are strong motivations to extend the scalar sector of the Standard Model of Particle Physics by introducing more than one Higgs doublet. Models with two Higgs doublets have a very rich phenomenology and can give rise to spontaneous CP violation [1], thus allowing to put electroweak symmetry breaking and CP violation on an equal footing. Two Higgs doublet models can also provide good dark matter candidates and can lead to new phenomena through Higgs mediated flavour changing neutral currents (HFCNC). Such models have been extensively studied in the literature. For reviews and references see for example [2, 3].

There are stringent experimental limits on HFCNC. One way of complying with these limits is by imposing natural flavour conservation (NFC) through a \mathbb{Z}_2 symmetry in such a way that all right handed isosinglet fermions of a given charge couple to no more than one Higgs doublet [4]. In the case of NFC there are no tree level HCFNCs. Imposing a discrete symmetry in the scalar sector of two Higgs doublet models eliminates the possibility of having spontaneous CP violation. However, CP can still be spontaneously violated if one allows for the discrete symmetry to be softly broken [5]. An alternative way of suppressing HFCNC was proposed by Branco, Grimus and Lavoura [6], and are the so-called BGL-type models [6, 7], where it was shown that it is possible to have tree level FCNCs completely fixed by the V_{CKM} matrix as a result of an abelian symmetry. In these models, the suppression is given by the small entries of the V_{CKM} matrix with no new flavour structure. The same type of symmetry has also been extended to the leptonic sector [8].

Three Higgs doublets are also well motivated. In particular they allow for the presence of unbroken discrete symmetries and spontaneous CP violation. As the number of Higgs doublets grows so does the number of free parameters [9]. Symmetries play a crucial rôle in reducing



Table 1. Possible symmetries of the 2HDM scalar potential that are respected by the $SU(2) \times U(1)_Y$ gauge kinetic term of the scalar fields. The corresponding symmetry transformation laws are given in a basis where the symmetry is manifest. Table taken from ref. [15]

symmetry	transformation law	
\mathbb{Z}_2	$\Phi_1 \rightarrow \Phi_1$	$\Phi_2 \rightarrow -\Phi_2$
U(1)	$\Phi_1 \rightarrow \Phi_1$	$\Phi_2 \rightarrow e^{2i\theta} \Phi_2$
SO(3)	$\Phi_a \rightarrow U_{ab} \Phi_b$	$U \in U(2)/U(1)_Y$ (for $a, b = 1, 2$)
GCP1	$\Phi_1 \rightarrow \Phi_1^*$	$\Phi_2 \rightarrow \Phi_2^*$
GCP2	$\Phi_1 \rightarrow \Phi_2^*$	$\Phi_2 \rightarrow -\Phi_1^*$
GCP3	$\Phi_1 \rightarrow \Phi_1^* \cos \theta + \Phi_2^* \sin \theta$	$\Phi_2 \rightarrow -\Phi_1^* \sin \theta + \Phi_2^* \cos \theta$ (for $0 < \theta < \frac{1}{2}\pi$)
Π_2	$\Phi_1 \rightarrow \Phi_2$	$\Phi_2 \rightarrow \Phi_1$

the number of free parameters, thus increasing predictability. Finding whether or not there is spontaneous CP violation in multi-Higgs models in the presence of symmetries may, in some cases, prove intricate. In ref. [10] a simple relation is provided, allowing one to answer this question. However, in some cases it may not be straightforward to identify a matrix U corresponding to a symmetry of the Lagrangian that verifies the necessary relation. A simple alternative method based on the existence of scalar bases where only one doublet acquires a non-zero vacuum expectation value (vev) [11, 12] was proposed in ref. [13]. This method may prove particularly useful as the complexity grows with the number of doublets.

This talk is based on several works by the present authors with special emphasis on refs. [14, 15]. In section 2 we summarise the discussion on the interplay between symmetries and mass degeneracy in the case of two Higgs doublet models, as is presented in ref. [15]. In section 3 we discuss specific examples of three Higgs doublet models. In the latter case there is not yet a full study of all possible symmetries.

2. Symmetries and mass degeneracy in two Higgs doublet models

In the two Higgs doublet model (2HDM) the most general gauge-invariant renormalizable scalar potential is:

$$\begin{aligned} \mathcal{V} = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) \\ & + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}. \end{aligned} \quad (1)$$

The complete list of all possible symmetries of the 2HDM potential is known [16, 17, 18, 19, 20, 21]. Table 1 gives the classification presented in ref. [18]. If the scalar potential respects one of the symmetries of Table 1, the coefficients of the scalar potential are constrained according to Table 2. Two different complementary approaches were followed in ref. [15] in order to find out under what conditions there is exact degeneracy of scalar masses without an artificial fine-tuning of the parameters of the scalar potential. The first one relies on deriving directly from the potential the conditions that yield mass degeneracies and checking afterwards whether these conditions correspond to a symmetry. The second one takes as starting point the set of all possible symmetries together with the constraints that these symmetries impose on the coefficients of the potential in the basis where the symmetry is imposed. Both approaches obviously lead to the same conclusions. In the 2HDM a natural mass degeneracy can only arise

Table 2. Impact of the symmetries defined in Table 1 on the coefficients of the 2HDM scalar potential in a basis where the symmetry is manifest. A short dash indicates the absence of a constraint. Table taken from ref. [15]

symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	$\text{Re}\lambda_5$	$\text{Im}\lambda_5$	λ_6	λ_7
\mathbb{Z}_2	-	-	0	-	-	-	-	-	-	0	0
U(1)	-	-	0	-	-	-	-	0	0	0	0
SO(3)	-	m_{11}^2	0	-	λ_1	-	$\lambda_1 - \lambda_3$	0	0	0	0
GCP1	-	-	real	-	-	-	-	-	0	real	real
GCP2	-	m_{11}^2	0	-	λ_1	-	-	-	-	-	$-\lambda_6$
GCP3	-	m_{11}^2	0	-	λ_1	-	-	$\lambda_1 - \lambda_3 - \lambda_4$	0	0	0
Π_2	-	m_{11}^2	real	-	λ_1	-	-	-	0	-	λ_6^*
$\mathbb{Z}_2 \oplus \Pi_2$	-	m_{11}^2	0	-	λ_1	-	-	-	0	0	0
U(1) $\oplus\Pi_2$	-	m_{11}^2	0	-	λ_1	-	-	0	0	0	0

in the case of the inert 2HDM, in which there is an exact discrete \mathbb{Z}_2 symmetry left unbroken by the vacuum. In this case, the two neutral scalars H and A that reside in the inert doublet are mass degenerate provided that λ_5 is equal to zero. Note that we are assuming that the field that is odd under \mathbb{Z}_2 acquires zero vacuum expectation value. This corresponds to the second row of Tables 1 and 2 with an unbroken U(1) symmetry. Furthermore, the global U(1) symmetry responsible for the degeneracy of the masses of H and A is preserved by the interactions of the scalars with the vector bosons. The SO(3) symmetric case (third row of Tables 1 and 2), again with the symmetry imposed in the basis where one of the doublets has zero vev, is even more constrained than the U(1) symmetric one and, in this case, the degeneracy is preserved with both scalars H and A being massless. The presence of two massless states is the consequence of the spontaneous breaking of the SO(3) symmetry. In the next three rows three different cases of CP symmetries are listed, GCP1, GCP2, GCP3. The necessary conditions in each of these cases have been expressed in terms masses and physical couplings in ref. [22], published in these Proceedings.

3. The case of three Higgs doublet models. Particular examples

As mentioned before, in the case of three Higgs doublets there is not yet a full classification of all possible symmetries of the scalar potential. In this section we discuss two particular cases. In the first case we analyse the possibility of having spontaneous CP violation in a three Higgs doublet model with real coefficients where an S_3 symmetry is imposed. This discussion is based on ref. [14]. In the second case we discuss some properties of a three Higgs doublet model proposed by Ivanov and Silva [23] with the striking property of having no real scalar basis and still preserving CP.

3.1. Spontaneous CP violation in the S_3 symmetric scalar sector

The S_3 symmetric three Higgs doublet scalar potential has been studied by several authors both in terms of its irreducible doublet-singlet representation, h_1 , h_2 and h_S representation [24, 25, 26, 27] as well as in terms of a reducible triplet, ϕ_1 , ϕ_2 and ϕ_3 [28, 29]. In ref. [14] a complete list of all possible vacua, starting from a scalar potential with real coefficients was given together with the set of constraints on the parameters of the potential that must be obeyed in each case.

In terms of the reducible triplet fields the potential has the form [28]:

$$V_2 = -\lambda \sum_i \phi_i^\dagger \phi_i + \frac{1}{2} \gamma \sum_{i < j} [\phi_i^\dagger \phi_j + \text{h.c.}], \quad (2)$$

$$\begin{aligned} V_4 = & A \sum_i (\phi_i^\dagger \phi_i)^2 + \sum_{i < j} \{C(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \bar{C}(\phi_i^\dagger \phi_j)(\phi_j^\dagger \phi_i) + \frac{1}{2} D[(\phi_i^\dagger \phi_j)^2 + \text{h.c.}]\} \\ & + \frac{1}{2} E_1 \sum_{i \neq j} [(\phi_i^\dagger \phi_i)(\phi_j^\dagger \phi_j) + \text{h.c.}] + \sum_{i \neq j \neq k, j < k} \{ \frac{1}{2} E_2 [(\phi_i^\dagger \phi_j)(\phi_k^\dagger \phi_i) + \text{h.c.}] \\ & + \frac{1}{2} E_3 [(\phi_i^\dagger \phi_i)(\phi_k^\dagger \phi_j) + \text{h.c.}] + \frac{1}{2} E_4 [(\phi_i^\dagger \phi_j)(\phi_i^\dagger \phi_k) + \text{h.c.}] \}. \end{aligned} \quad (3)$$

Whereas in terms of the S_3 singlet and doublet fields it can be written as [25, 26, 27]:

$$V_2 = \mu_0^2 h_S^\dagger h_S + \mu_1^2 (h_1^\dagger h_1 + h_2^\dagger h_2), \quad (4)$$

$$\begin{aligned} V_4 = & \lambda_1 (h_1^\dagger h_1 + h_2^\dagger h_2)^2 + \lambda_2 (h_1^\dagger h_2 - h_2^\dagger h_1)^2 + \lambda_3 [(h_1^\dagger h_1 - h_2^\dagger h_2)^2 + (h_1^\dagger h_2 + h_2^\dagger h_1)^2] \\ & + \lambda_4 [(h_S^\dagger h_1)(h_1^\dagger h_2 + h_2^\dagger h_1) + (h_S^\dagger h_2)(h_1^\dagger h_1 - h_2^\dagger h_2) + \text{h.c.}] + \lambda_5 (h_S^\dagger h_S)(h_1^\dagger h_1 + h_2^\dagger h_2) \\ & + \lambda_6 [(h_S^\dagger h_1)(h_1^\dagger h_S) + (h_S^\dagger h_2)(h_2^\dagger h_S)] + \lambda_7 [(h_S^\dagger h_1)(h_S^\dagger h_1) + (h_S^\dagger h_2)(h_S^\dagger h_2) + \text{h.c.}] \\ & + \lambda_8 (h_S^\dagger h_S)^2. \end{aligned} \quad (5)$$

Table 3 lists all possible complex vacua. In ref. [14] the set of constraints required for each solution is given. In the discussion of whether or not there is spontaneous CP violation (SCPV) for each of the solutions, it was shown that the parameter λ_4 plays a very important rôle. In fact for λ_4 equal to zero there is an additional $SO(2)$ symmetry. The results are summarised in Table 4 indicating whether or not λ_4 is required to be zero (an X means that it can be different from zero). There is no spontaneous CP violation in any of the cases with λ_4 equal to zero. The simple method described in [13] proved very useful in this discussion. In this framework there are also mass degeneracies among scalar fields, however, the identification of such degeneracies for the different vacuum solutions was not done in our previous work.

3.2. A CP-conserving multi-Higgs model with irremovable complex coefficients

Ivanov and Silva (IS) proposed a three Higgs doublet scalar potential [23] with irremovable complex coefficients that conserves CP. The IS potential has the form:

$$V = V_0 + V_1 \quad (6)$$

where:

$$\begin{aligned} V_0 = & -m_{11}^2 (\phi_1^\dagger \phi_1) - m_{22}^2 (\phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3) + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 [(\phi_2^\dagger \phi_2)^2 + (\phi_3^\dagger \phi_3)^2] + \lambda_3' (\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) \\ & + \lambda_3 (\phi_1^\dagger \phi_1)[(\phi_2^\dagger \phi_2) + (\phi_3^\dagger \phi_3)] + \lambda_4' (\phi_2^\dagger \phi_3)(\phi_3^\dagger \phi_2) + \lambda_4 [(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + (\phi_1^\dagger \phi_3)(\phi_3^\dagger \phi_1)], \end{aligned} \quad (7)$$

with all parameters real, due to hermiticity and

$$\begin{aligned} V_1 = & \lambda_5 (\phi_3^\dagger \phi_1)(\phi_2^\dagger \phi_1) + \frac{1}{2} \lambda_6 [(\phi_2^\dagger \phi_1)^2 - (\phi_1^\dagger \phi_3)^2] + \lambda_8 (\phi_2^\dagger \phi_3)^2 \\ & + \lambda_9 (\phi_2^\dagger \phi_3)[(\phi_2^\dagger \phi_2) - (\phi_3^\dagger \phi_3)] + \text{h.c.} \end{aligned} \quad (8)$$

Table 3. Complex vacua. Notation: $\epsilon = 1$ and -1 for C-III-d and C-III-e, respectively; $\xi = \sqrt{-3 \sin 2\rho_1 / \sin 2\rho_2}$, $\psi = \sqrt{[3 + 3 \cos(\rho_2 - 2\rho_1)] / (2 \cos \rho_2)}$. With the necessary constraints the vacua labelled with an asterisk (*) are in fact real.

	IRF (Irreducible Rep.)	RRF (Reducible Rep.)
	w_1, w_2, w_S	ρ_1, ρ_2, ρ_3
C-I-a	$\hat{w}_1, \pm i\hat{w}_1, 0$	$x, xe^{\pm \frac{2\pi i}{3}}, xe^{\mp \frac{2\pi i}{3}}$
C-III-a	$0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$y, y, xe^{i\tau}$
C-III-b	$\pm i\hat{w}_1, 0, \hat{w}_S$	$x + iy, x - iy, x$
C-III-c	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$	$xe^{i\rho} - \frac{y}{2}, -xe^{i\rho} - \frac{y}{2}, y$
C-III-d,e	$\pm i\hat{w}_1, \epsilon \hat{w}_2, \hat{w}_S$	$xe^{i\tau}, xe^{-i\tau}, y$
C-III-f	$\pm i\hat{w}_1, i\hat{w}_2, \hat{w}_S$	$re^{i\rho} \pm ix, re^{i\rho} \mp ix, \frac{3}{2}re^{-i\rho} - \frac{1}{2}re^{i\rho}$
C-III-g	$\pm i\hat{w}_1, -i\hat{w}_2, \hat{w}_S$	$re^{-i\rho} \pm ix, re^{-i\rho} \mp ix, \frac{3}{2}re^{i\rho} - \frac{1}{2}re^{-i\rho}$
C-III-h	$\sqrt{3}\hat{w}_2 e^{i\sigma_2}, \pm \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau}, y, y$ $y, xe^{i\tau}, y$
C-III-i	$\sqrt{\frac{3(1+\tan^2 \sigma_1)}{1+9\tan^2 \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\pm \hat{w}_2 e^{-i \arctan(3 \tan \sigma_1)}, \hat{w}_S$	$x, ye^{i\tau}, ye^{-i\tau}$ $ye^{i\tau}, x, ye^{-i\tau}$
C-IV-a*	$\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$	$re^{i\rho} + x, -re^{i\rho} + x, x$
C-IV-b	$\hat{w}_1, \pm i\hat{w}_2, \hat{w}_S$	$re^{i\rho} + x, -re^{-i\rho} + x, -re^{i\rho} + re^{-i\rho} + x$
C-IV-c	$\sqrt{1 + 2 \cos^2 \sigma_2} \hat{w}_2,$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho} + r\sqrt{3(1 + 2 \cos^2 \rho)} + x,$ $re^{i\rho} - r\sqrt{3(1 + 2 \cos^2 \rho)} + x, -2re^{i\rho} + x$
C-IV-d*	$\hat{w}_1 e^{i\sigma_1}, \pm \hat{w}_2 e^{i\sigma_1}, \hat{w}_S$	$r_1 e^{i\rho} + x, (r_2 - r_1)e^{i\rho} + x, -r_2 e^{i\rho} + x$
C-IV-e	$\sqrt{-\frac{\sin 2\sigma_2}{\sin 2\sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_2} + re^{i\rho_1} \xi + x, re^{i\rho_2} - re^{i\rho_1} \xi + x,$ $-2re^{i\rho_2} + x$
C-IV-f	$\sqrt{2 + \frac{\cos(\sigma_1 - 2\sigma_2)}{\cos \sigma_1}} \hat{w}_2 e^{i\sigma_1},$ $\hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$re^{i\rho_1} + re^{i\rho_2} \psi + x,$ $re^{i\rho_1} - re^{i\rho_2} \psi + x, -2re^{i\rho_1} + x$
C-V*	$\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S$	$xe^{i\tau_1}, ye^{i\tau_2}, z$

Table 4. Spontaneous CP violation (SCPV)

Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV	Vacuum	λ_4	SCPV
C-I-a	X	no	C-III-f,g	0	no	C-IV-c	X	yes
C-III-a	X	yes	C-III-h	X	yes	C-IV-d	0	no
C-III-b	0	no	C-III-i	X	no	C-IV-e	0	no
C-III-c	0	no	C-IV-a	0	no	C-IV-f	X	yes
C-III-d,e	X	no	C-IV-b	0	no	C-V	0	no

with λ_5, λ_6 real and λ_8, λ_9 complex. This potential is fixed by the following CP symmetry:

$$\phi_i \rightarrow W_{ij} \phi_j^*, \quad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \quad (9)$$

Notice that this symmetry requires λ_5 to be real. On the other hand λ_6 can be made real by an appropriate rephasing of the fields ϕ_2 and ϕ_3 . There exists a range of parameters of the scalar potential corresponding to a vacuum that conserves the above symmetry, i.e.,

$$(v_1, v_2, v_3) = (v, 0, 0). \quad (10)$$

As pointed out by Ivanov and Silva [23] the transformation given above is of order 4 since only after applying it four times are we back to the identity transformation and therefore was called a CP4 transformation. Notice that

$$WW^* = \text{diag}(1, -1, -1) \quad (11)$$

is different from the identity. This may seem paradoxical since one expects that two consecutive CP transformations would correspond to the identity. However, it should be pointed out that this happens because this CP transformation combines the usual CP transformation for a single scalar doublet with an internal symmetry of the potential. From the point of view of spacetime we can only interpret this transformation as a CP transformation if we assume that each time it is applied \vec{x} goes into $-\vec{x}$, so that after applying it twice we are simply left with an internal symmetry transformation of the scalar potential.

It was pointed out in [15] that there is a simpler form for the IS potential with no λ_5 term, with the term in λ_6 changed into $\frac{1}{2}\lambda'_6[(\phi_2^\dagger \phi_1)^2 + (\phi_1^\dagger \phi_3)^2]$, with λ'_6 still real, and all other terms keeping the same form, though with different values for the coefficients. Furthermore, it is still possible to make either λ'_8 or λ'_9 real. This means that the λ_5 term is made redundant and only one coefficient remains complex. This is achieved by a series of unitary transformations under which ϕ_1 does not mix with the other two doublets. As a result the new doublets with indices two and three still have zero vevs. Furthermore, in this basis all fields appearing in ϕ'_2 and ϕ'_3 are already mass eigenstates and do not mix with the Standard Model-like Higgs boson. In addition, the symmetry of the IS model potential is also responsible for the existence of pairwise mass degeneracy among all the fields appearing in ϕ'_2 and ϕ'_3 . Once again, there is a strong interplay between symmetry and mass degeneracies.

The IS model has the curious property that there is no CP transformation of order two (CP2) under which the potential is invariant. It also has the curious property that there is no real basis even though CP is conserved. In ref. [15], we have checked up to three-loop order in the IS model that there is a cancellation of the CP violating form factors of effective ZZZ and ZWW vertices. We have also identified a physical quartic scalar interaction that is consistent with the CP4 symmetry but would vanish for a potential of the same form as the IS potential written in terms of real coefficients.

4. Conclusions

Symmetries play a crucial rôle in Particle Physics. The imposition of symmetries on the scalar potential, with multi-Higgs doublets, leads to degeneracies in the masses of the physical scalars and has implications on its CP properties. A full study of such implications in the case of models with three Higgs doublets has not yet been performed.

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