Optimal funding coverage in a mixed oligopoly with quality competition and price regulation*

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Abstract

We study the optimal design of a public funding scheme in a mixed oligopoly setting (with applications to health care and education) with one welfare-maximising public provider and two profit-maximising private providers, where all providers compete on quality and where providers included in the public funding scheme are subject to price regulation. We find that the first-best solution cannot be implemented without including (at least) one of the private providers in the public funding scheme. However, inclusion of only one of the private providers is sufficient to induce the first-best outcome. Such inclusion allows for the elimination of a negative competition externality between the private providers that, all else equal, yields underprovision of quality.

Keywords: Quality; Competition; Mixed oligopoly; Public funding.

JEL Classification: H44; I11; L13; L33

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1 Introduction

There are many services, among them health and education, which are provided by a mix of public and private providers, but where the relative share of these types of providers varies considerably across different countries. In such mixed markets, where public and private providers coexist, competition typically takes place among providers with different objectives and which are subject to different regulatory schemes. This raises several policy issues. For example, should private providers be included in public funding schemes? And if so, should such providers be allowed to distribute profits? In education markets, for example, many countries do not give public funding to for-profit private schools, while others, including several US states, permit publicly funded charter schools to be operated by for-profit providers (Boeskens, 2016; Lee, 2018). Furthermore, in health and education markets quality is a key concern, and designing policies to ensure a satisfactory provision of quality requires an understanding of how public and private providers strategically interact, and how they respond to different funding schemes.

In this paper we analyse the optimal design of a public funding scheme in a spatial oligopoly market consisting of one public provider and two private providers, and where consumers choose providers based on quality, price and transportation costs. We assume that the providers differ in terms of objectives and regulatory constraints. Whereas the public provider is assumed to maximise social welfare, the two private providers are profit-maximisers. Furthermore, the revenues of each provider depend on whether the provider is included in the public funding scheme. A publicly funded provider receives a regulated price per unit of the good supplied, part of which is paid by the provider's consumers according to a copayment rate set by the regulator. On the other hand, a provider that is not included in the public funding scheme must raise all its revenues from the market by charging a price for its services. In this setting, competition between providers takes place along two different dimensions: quality and price. Whereas all providers choose qualities, only providers without public funding are free to choose prices.

Within this framework, we study the optimal design of the public funding scheme, where a welfarist regulator sets the funding parameters (price and copayment rate) and chooses which providers to include in the funding scheme. Under the basic assumption that quality is not verifiable and thus not contractible, we show that the first-best outcome is not attainable when only the public provider is funded. In this case, the policy that minimises aggregate transportation costs leads to underprovision of quality because of a negative competition externality between the two private providers, whereby each provider has an incentive to reduce its quality in order to induce a price increase from the rival provider. This problem can be solved by including one of the private providers in the public funding scheme, which removes the said competition externality and ensures that the private provider outside the funding scheme always has socially optimal incentives for quality provision. The regulator can then use the two funding instruments (price and copayment rate) to ensure socially optimal incentives also for the included provider, and to ensure an optimal demand allocation across the three providers. Thus, the first-best solution can be implemented by the optimal design of a public funding scheme that includes both public and private providers. Furthermore, inclusion of only one of the private providers is sufficient to induce the first-best outcome. In this sense, our analysis provides a rationale for the co-existence of different types of providers in the same market: public and private providers within the public funding scheme and private providers outside the scheme.

Although our model is not tailor-made to fit one particular industry, our analysis applies in particular to regulated markets such as health care and education. In the health care markets of many European countries, patients can choose between public and private providers within the national health system, where prices and copayments are regulated, or alternatively choose a private provider outside the national health system and pay the expenses either out-of-pocket or via private health insurance.[1] A similar mix of provider options is present in education markets, where tuition fees in publicly funded schools tend to be either absent or regulated, while independent private schools rely on the fees charged to their students. In such markets, publicly funded private schools have become a prominent feature across OECD countries (Boeskens, 2016).[2] Average OECD figures for 2018 show that 13.2% of 15-year-old students attended government-dependent private schools, 81.9% attended public schools, while 4.9% attended independent private schools (OECD, 2020).

Both in health care and education markets, the extent of public funding coverage for private providers is a contentious issue in many countries. In education markets, for example, proponents of extending funding to private providers argue that this stimulates inter-school competition and offers incentives for innovation and quality improvements. On the contrary, opponents argue that

[1] See for example Siciliani et al. (2017) for an overview of the scope for competition between health care providers in five different European countries.

[2] One specific example is Chile, where the educational system is based on three types of schools; municipal, subsidised private and entirely private schools. The first two types are mainly funded by government subsidies per student and may also receive small contributions as school charges. However, the entirely private schools do not receive any public funding and raise their revenues from charging student fees (Chumacero et al., 2011). According to the PISA 2018 results, 34% of students were enrolled in municipal schools, 56.2% in government-funded private schools, and 9.8% in independent private schools (OECD, 2020).

funding private education might lead to public sector resource depletion and ultimately result in a reduction of educational quality (Boeskens, 2016). Our analysis in the present paper can help shed some lights on one particular aspect of this issue, namely how the inclusion of private providers in the public funding scheme can improve incentives for quality provision and thereby reduce inefficiencies in the market.

The rest of the paper is organised as follows. In the next section we present a relatively brief summary and discussion of related literature, before presenting the model in detail in Section 3. In Section 4 we derive the equilibrium quality provision under two different assumptions about public funding coverage, where either no or one private provider is included in the funding scheme, and we analyse how the quality provision depends on the funding parameters (price and copayment rate). In Section 5 we derive the first-best solution and show how the attainability of this solution depends on the funding coverage. In this section we also compare the equilibrium quality provision across the different providers when the copayment rate differs from the first-best level. Finally, some concluding remarks are offered in Section 6.

2 Related literature

Our paper is related to the literature on mixed oligopoly in general and on quality competition between public and private providers in health care and education markets in particular. In the theory of mixed oligopolies, a sizeable literature has grown out of the seminal contributions by De Fraja and Delbono (1989) and Cremer et al. (1989). Later contributions include Cremer et al. (1991), Matsumura (1998), Bennett and La Manna (2012) and Haraguchi and Matsumura (2016). A main message from this literature is that the presence of public firms might yield welfare improving effects in oligopolistic industries, and a key issue has been to determine the optimal degree of public ownership (e.g., Matsumura, 1998).[3] A common assumption in this literature is that firms compete either in prices or quantities, and quality is generally not an issue.

There is however a smaller and more specialised literature dealing with quality competition in mixed oligopolies. Grilo (1994) produced what is probably the earliest contribution in this literature, studying quality and price competition in a vertically differentiated mixed duopoly. A later contribution building on this work is Lutz and Pezzino (2014), who find that a mixed

[3] There is also a recent strand of this literature analysing the policy implications of asymmetries between private firms in mixed oligopolies (e.g., Haraguchi and Matsumura, 2020a, 2020b), which has parallels to our study where there are regulatory asymmetries between the private providers.

duopoly is generally welfare superior to a private duopoly. Laine and Ma (2017) also study quality and price competition in a vertically differentiation framework and show the existence of multiple equilibria that differ with respect to the identity of the high-quality firm (public or private). The latter result has some parallels to the present analysis, where we rank the quality provision across different types of providers when the copayment rate differs from the first-best level. However, one of several important differences between our paper and all of the above mentioned papers on quality competition in mixed oligopolies is that the latter papers apply a vertical differentiation framework, whereas our study is conducted in a setting of horizontal differentiation. Our paper is therefore more closely related to the type of analysis conducted by Ishibashi and Kaneko (2008), who study quality and price competition between a welfare-maximising state-owned firm and a profit-maximising private firm in a Hotelling model. They show that social welfare is maximised if the public firm's objective is a weighted average of welfare and profits, thus indicating that partial privatisation of the state-owned firm would be welfare improving.[4]

Common for all the above mentioned papers is that competition takes place in an unregulated setting, which is another key difference from the present paper, in which a subset of the providers face regulated prices, depending on public funding coverage. In this respect, our paper is more closely related to papers that study quality competition in *regulated* mixed oligopolies, often applied to health care markets. An early study is Barros and Martinez-Giralt (2002) who analyse quality and price competition between a public and a private health care provider under different reimbursement rules. Sanjo (2009) and Herr (2011) also study quality competition between a public and a private health care provider, but under the assumption that prices for both providers are regulated.[5] These studies are all conducted within a horizontal differentiation (Hotelling) framework.[6] More recent studies of mixed duopoly quality competition with fixed prices have addressed issues such as soft budgets (Levaggi and Montefiori, 2013), partial privatization policies (Chang et al., 2018) and location choices (Hehenkamp and Kaarbøe, 2020). A broader review of the merits of mixed markets in health care, presented in a unified framework, is given by Levaggi and Levaggi (2020). A similar type of study, using a Hotelling-type framework, but applied to the education

[4] The studies of quality compeition in mixed oliogopolies can also be seen as being part of a more general literature on quality provision by (private) profit-maximising firms, where a key question is whether an unregulated market is able to provide a socially optimal quality level (e.g., Spence, 1975; Ma and Burgess, 1993; Cellini et al., 2018; Willner and Grönblom, 2021).

[5] On the other hand, Ghandour (2021) studies quality competition in a mixed duopoly where the public provider is subject to price regulation while the private provider is not.

[6] A similar study using instead a vertical differentiation framework is Stenbacka and Tombak (2018).

sector, is Brunello and Rocco (2008), who analyse a mixed duopoly game between a public school choosing quality ('educational standard') and a private school choosing quality and price (tuition fee).

Overall, our paper can be seen as an extension of the above described literature on quality competition in regulated mixed oligopolies, where we investigate a hitherto neglected aspect of optimal regulation in such markets, namely the optimal degree of public funding coverage among the providers in the market.

3 The model

Consider a market for a good (e.g., health care or education) that is supplied by three different providers that are equidistantly located on a circle with circumference equal to 1. Whereas Provider 1 is publicly owned and funded, the two other providers are privately owned, but may or may not be included in the public funding scheme. A publicly funded provider receives a price $\overline{p}$ per unit of the good supplied. A fraction s of this price is paid by the consumers as copayment, whereas the remaining share is paid by a public funder. On the other hand, a private provider that is not publicly funded has to raise revenues in the market by charging its consumers a price p per unit of the good supplied. We also assume that public and private providers differ with respect to their objectives. Here we follow the standard assumption in the mixed oligopoly literature that the public provider maximises social welfare while the private providers are profit maximisers.[7,8]

Consumers are uniformly distributed on the same circle. Each consumer demands one unit of the good from the most preferred provider and the total mass of consumers is normalised to 1. The utility of a consumer located at a distance x_i from Provider i is given by

$$u\left(q_i, r_i, x_i\right) = v + \beta q_i - r_i - t x_i;\ i = 1, 2, 3, \tag{1}$$

[7] Given the significant presence of non-profit private providers in many health care and education markets, our assumption that private providers maximise profits is not trivial. However, non-profit status does not necessarily imply that the provider is not interested in profits, it just implies a restriction on profit distribution. In fact, a common assumption in the literature on non-profit firms is that non-profit status works like a tax on profits for otherwise profit-maximising firms, because profits must be distributed in kind instead of in cash (e.g., Glaeser and Shleifer, 2001; Ghatak and Mueller, 2011). This assumption is validated by empirical evidence showing that for-profit and non-profit providers often tend to respond similarly to financial incentives (see, e.g., Duggan, 2002).

[8] A profit-maximising private provider that is subject to price regulation might have incentives to 'game' the regulator in various ways. For example, in health care markets where funding is based on DRG pricing, a provider might miscode patients in order to obtain a higher price. By the assumptions of single-good providers and a uniform regulated price, the possibility of such gaming behaviour is disregarded in our analysis.

where q_i is the quality offered by Provider i and r_i is the price paid by Provider i's consumers. The parameters $\beta > 0$ and $t > 0$ measure, respectively, the marginal willingness to pay for quality and the marginal transportation cost. The latter can be interpreted either as the marginal cost of travelling in geographical space or the marginal mismatch cost in product space. We also assume that the utility parameter $v > 0$ is sufficiently large to ensure full market coverage for all quality and price configurations.

Suppose that every consumer in the market makes a utility-maximising choice of provider. Let $\widehat{x}_i^{i+1}$ denote the distance between the location of Provider i and the location of the consumer who is indifferent between Provider i and the neighbouring Provider $i+1$. When each consumer maximises utility, this distance is given by

$$\widehat{x}_i^{i+1}\left(q_i, q_{i+1}; r_i, r_{i+1}\right) = \frac{1}{6} + \frac{\beta\left(q_i - q_{i+1}\right) - \left(r_i - r_{i+1}\right)}{2t}. \tag{2}$$

Since each provider has two neighbours, the demand for Provider i is given by

$$D_i\left(q_i, q_{i-1}, q_{i+1}; r_i, r_{i-1}, r_{i+1}\right) = \widehat{x}_i^{i+1}\left(q_i, q_{i+1}; r_i, r_{i+1}\right) + \widehat{x}_i^{i-1}\left(q_i, q_{i-1}; r_i, r_{i-1}\right). \tag{3}$$

Substituting from (2), this yields[9]

$$D_i\left(q_i, q_{i-1}, q_{i+1}; r_i, r_{i-1}, r_{i+1}\right) = \frac{1}{3} + \frac{\beta\left(2q_i - q_{i-1} - q_{i+1}\right) - \left(2r_i - r_{i-1} - r_{i+1}\right)}{2t} \tag{4}$$

The Salop model is generally characterised by localised competition, implying that the demand of each provider only depends on the prices and qualities of that provider and its two neighbours. However, with only three providers, each provider has all the remaining providers in the market as neighbours. Thus, all providers compete directly with each other.

We assume that the cost of provision is separable in quantity and quality, with the cost function of Provider i given by[10]

$$C\left(D_i, q_i\right) = cD_i + \frac{k}{2}q_i^2. \tag{5}$$

[9] Notice the slight abuse of notation, since $i = 1$ implies that $i - 1 = 3$, and $i = 3$ implies that $i + 1 = 1$.

[10] The assumption that the marginal cost of quality provision is independent of output, which is widely used in the theoretical literature on quality competition between health care providers (e.g., Lyon, 1999; Barros and Martinez-Giralt, 2002; Gravelle and Sivey, 2010), implies that quality is a public good for the consumers of a particular provider. For an analysis of quality competition with output-dependent quality costs, see, e.g., Bardey et al. (2012).

The profits of Provider i are thus given by

$$\pi_i = (p_i - c)\, D_i - \frac{k}{2} q_i^2. \tag{6}$$

Whereas the private providers (2 and 3) are assumed to maximise profits, the publicly owned provider is assumed to maximise social welfare, denoted W, which is given by aggregate consumer utility, denoted U, plus total profits, net of public funding:

$$W = U + \sum_{i=1}^{3} \pi_i - (1-s)\, \overline{p} \sum_{j \in J} D_j, \tag{7}$$

where J is the set of publicly funded providers. With a slight abuse of notation, aggregate consumer utility is given by[11]

$$U = \sum_{i=1}^{3} \left(\int_0^{\widehat{x}_i^{i+1}} (v + \beta q_i - r_i - tx)\, dx + \int_0^{\widehat{x}_i^{i-1}} (v + \beta q_i - r_i - tx)\, dx \right). \tag{8}$$

Since total demand is fixed, which implies that social welfare does not depend directly on prices and other monetary transfers, we can more conveniently reformulate the welfare expression as

$$W = v + \beta \overline{q} - T - c - \frac{k}{2} \sum_{i=1}^{3} q_i^2, \tag{9}$$

where

$$\overline{q} := \sum_{i=1}^{3} D_i q_i \tag{10}$$

is average quality (weighted by market shares) and

$$T = \frac{t}{12} + \frac{\sum_{i=1}^{3} r_i (r_i - r_{i+1}) + \beta \left(\beta \sum_{i=1}^{3} q_i (q_i - q_{i+1}) + \sum_{i=1}^{3} q_i (r_{i-1} + r_{i+1}) - 2 \sum_{i=1}^{3} q_i r_i \right)}{2t} \tag{11}$$

is aggregate transportation costs.[12] The last two terms in (9) represent the total cost of provision in the market. It is immediately obvious from (11) that aggregate transportation costs are minimised (at $T = t/12$) for a symmetric outcome, where $r_i = r_j$ and $q_i = q_j$, for all i and j, $i \neq j$.

We consider the following three-stage game:

[11] Notice once more that, if $i = 1$, then $i - 1 = 3$, and if $i = 3$, then $i + 1 = 1$.

[12] Notice that subscripts $i + 1$ and $i - 1$ refer to the two neighours of Provider i located in the clockwise and anticlockwise direction, respectively.

Stage 1 A welfare-maximising regulator chooses its funding scheme.

Stage 2 Each of the three providers chooses its level of quality provision.

Stage 3 Each private provider outside the public funding scheme chooses its price.

The choice of funding scheme involves not only setting the price and copayment, but also deciding which providers to include in the scheme. By placing these decisions at the first stage of the game, we implicitly assume that the regulator is able to precommit to a particular funding policy as a long-term decision. Furthermore, the separation of Stage 2 from Stage 3 is motivated by the implicit assumption that the level of quality provision is more of a long-term decision than the price choice. Finally, in order to ensure equilibrium existence throughout the analysis, we assume that the quality cost parameter k is bounded from below:[13]

$$k \geq \underline{k} := \frac{3\beta^2}{2t}. \tag{12}$$

In order to rule out a negative price-cost margin for any publicly funded private provider, we also assume that $\overline{p} \geq c$.

4 Equilibrium quality provision for a given public funding scheme

In this section we derive and characterise the subgame-perfect Nash equilibrium (SPNE) for a given funding scheme; i.e., we derive the SPNE of a game that starts at Stage 2 of the game outlined in the previous section. More specifically, we derive the equilibrium outcome for given values of $\overline{p}$ and s under two different assumptions about public funding coverage, where either no private provider or one of the private providers is included in the funding scheme.

4.1 No private provider is included in the public funding scheme

Suppose that public funding is not given to either of the two private providers. In this case, both of them have to raise revenues in the market by charging prices p_2 and p_3, respectively. This implies that consumer prices are given by $r_1 = s\overline{p}$, $r_2 = p_2$ and $r_3 = p_3$. Solving the game by backwards induction, we start out by considering the optimal pricing decisions of the private providers at the last stage of the game.

[13] See Appendix A for further details.

The first-order condition for the optimal price set by the private Provider i is given by

$$D_i + (p_i - c)\frac{\partial D_i}{\partial p_i} = 0, \quad i = 2, 3. \tag{13}$$

By simultaneously solving the first-order conditions of the two private providers, we find that the equilibrium prices are given by

$$p_i(q_1, q_i, q_j, r_1) = \frac{2}{3}c + \frac{2}{9}t + \frac{1}{3}r_1 + \frac{\beta}{15}(7q_i - 2q_j - 5q_1); \quad i, j = 2, 3; \quad i \neq j. \tag{14}$$

We see that the optimal price of each of the private providers is decreasing in the quality level of each of the two rival providers. A higher quality by a rival provider leads to a drop in demand, which makes demand more price elastic, all else equal. This reduces in turn the profit-maximising price. Thus, the price of the private provider is a *strategic substitute* to the quality of a rival provider. On the other hand, the optimal price of a private provider is increasing in the provider's own quality. All else equal, a higher quality provision leads to higher demand, which makes demand less price elastic. Consequently, the profit-maximising price increases. In other words, price and quality are *complementary strategies* for each private provider. Finally, notice that each private provider's optimal price is increasing in both the regulated price ($\overline{p}$) and the copayment rate (s). This is due to prices being *strategic complements* for given quality levels. A higher $\overline{p}$ or a higher s implies, all else equal, that the good supplied by the public provider becomes more expensive for consumers (recall that $r_1 = s\overline{p}$). This leads to higher, and thus less price-elastic, demand for each of the private providers, whose optimal response is to increase the price.

Anticipating the price choices of the private providers, all providers simultaneously and independently choose qualities in order to maximise their objective functions. Consider first the problem of the public provider, who chooses q_1 to maximise (9). The first-order condition is given by

$$\beta\left(\frac{\partial \overline{q}}{\partial q_1} + \sum_{i=2}^{3}\frac{\partial \overline{q}}{\partial p_i}\frac{\partial p_i}{\partial q_1}\right) - \left(\frac{\partial T}{\partial q_1} + \sum_{i=2}^{3}\frac{\partial T}{\partial p_i}\frac{\partial p_i}{\partial q_1}\right) - kq_1 = 0, \tag{15}$$

and reflects the public provider's concern for increasing average quality in the market (first term) and reducing aggregate transportation costs (second term). Notice that the quality provision of the public provider has both a direct and an indirect effect (via the pricing decisions of the private providers) on these two variables.

Consider next the problem faced by each of the two profit-maximising private providers. The first order condition for optimal quality provision by the private Provider i is given by

$$\frac{\partial p_i}{\partial q_i} D_i + (p_i - c) \left(\frac{\partial D_i}{\partial p_i} \frac{\partial p_i}{\partial q_i} + \frac{\partial D_i}{\partial p_j} \frac{\partial p_j}{\partial q_i} + \frac{\partial D_i}{\partial q_i} \right) - k q_i = 0, \tag{16}$$

where $i, j = 2, 3$ and $i \neq j$. Using the first-order condition for the optimal price p_i, given by (13), and using the fact that $\partial D_i / \partial q_i = -\beta \left(\partial D_i / \partial p_i \right)$, the first-order condition can be re-written as

$$\beta D_i + (p_i - c) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j}{\partial q_i} - k q_i = 0. \tag{17}$$

The first term captures the direct profit gain of higher quality provision via higher demand, whereas the second term captures a strategic profit effect via the subsequent price decision of the competing private provider. Since $\partial D_i / \partial p_j > 0$ and $\partial p_j / \partial q_i < 0$, this strategic effect is negative. More specifically, each of the private providers has a strategic incentive to lower its quality provision in order to induce a subsequent price increase from the competing private provider.

From the above-stated first-order conditions we derive the best-response functions of the three providers, which fully characterise the strategic interaction at the quality competition stage. These are given by

$$q_1(q_2, q_3) = \frac{\beta \left(6c + 11t - 12\beta \left(q_2 + q_3 \right) - 6r_1 \right)}{27kt - 24\beta^2}, \tag{18}$$

$$q_i(q_1, q_j) = \frac{14\beta \left(10t - 15 \left(c - r_1 \right) - \beta \left(15q_1 + 6q_j \right) \right)}{675kt - 294\beta^2}; \quad i, j = 2, 3; \quad i \neq j. \tag{19}$$

Notice first that qualities are *strategic substitutes*; i.e., higher quality by one provider leads to lower quality by each of the competing providers. For the private providers, this strategic substitutability is caused by the strategic substitutability between own price and rival's quality, as previously described. Consider the private Provider i. If one of the competing providers increases its quality level, the demand for Provider i decreases and therefore becomes more price elastic. Provider i will therefore choose a lower price in the subsequent stage, all else equal, which in turn makes quality provision less profitable because of a lower price-cost margin. Thus, because a private provider correctly anticipates that higher quality by a rival provider will lead to a lower price-cost margin, the provider optimally responds to such a quality increase by reducing its own quality.

For the public provider, the mechanism behind the strategic substitutability is very different. A profit-maximising provider's incentives for quality provision are determined by the marginal utility of the indifferent consumer, which in turn determines the demand response to quality. A welfare-maximising provider, on the other hand, cares about the *average* quality provision in the market, as defined by (10), and the effect of the public provider's quality provision on the average quality depends in turn on the provider's market share. The higher the market share of the public provider, the stronger is the effect of a marginal quality increase (by the public provider) on average quality, and this makes the public provider's quality a strategic substitute to the qualities chosen by private providers. If one of the private providers increases its quality, the market share of the public provider is reduced, all else equal. This implies that a marginal quality increase by the public provider has a smaller effect on average quality, and the optimal response by the public provider is therefore to reduce its quality provision.

The SPNE is given by[14]

$$q_1^* = \beta \frac{t\left(55kt - 42\beta^2\right) + 2\left(14\beta^2 + 15kt\right)(c - r_1)}{27kt\left(5kt - 6\beta^2\right)}, \tag{20}$$

$$q_2^* = q_3^* = 14\beta \frac{t\left(2kt - 3\beta^2\right) + \left(3kt - 2\beta^2\right)(r_1 - c)}{27kt\left(5kt - 6\beta^2\right)}, \tag{21}$$

$$p_2^* = p_3^* = \frac{5t\left(2kt - 3\beta^2\right) + 2c\left(15kt - 22\beta^2\right) + 5r_1\left(3kt - 2\beta^2\right)}{9\left(5kt - 6\beta^2\right)}. \tag{22}$$

The equilibrium quality provision depends on the parameters of the funding scheme, more specifically $\overline{p}$ and s. The relationship between these funding parameters and the equilibrium quality provision are summarised as follows:

Lemma 1 *Suppose that no private provider is included in the public funding scheme. In this case, an increase in the regulated price ($\overline{p}$) and/or the copayment rate (s) leads to lower quality for the public provider and higher quality for each of the private providers.*

Notice first that, when only the welfare-maximising public firm is included in the funding scheme, the equilibrium quality provision can only be affected by changing the price paid by consumers of the public provider. This affects the public provider's incentives for quality provision

[14] Second-order and stability conditions are reported in Appendix A.

directly through a change in relative market shares, which in turn changes the private providers' incentives for quality provision through strategic substitutability. If consumers have to pay a higher copayment to the public provider (because of an increase in $\overline{p}$ and/or s), this provider will have lower demand, which in turn gives the provider an incentive to reduce its quality, as explained above. The private providers will then respond by increasing their qualities.

4.2 Inclusion of one private provider in the public funding scheme

Suppose now that the private Provider 2 is included in the public funding scheme, whereas the remaining private Provider 3 does not receive public funding. Since public funding implies price regulation, this means that $r_1 = r_2 = s\overline{p}$ and $r_3 = p_3$. For notational simplicity, we define $r := r_1 = r_2 = s\overline{p}$. Once more, we solve the game by backwards induction, considering first the optimal price chosen by Provider 3.

At the third stage, the price that maximises Provider 3's profits is given by

$$p_3(q_1, q_2, q_3, r) = \frac{t}{6} + \frac{c+r}{2} + \frac{\beta}{4}(2q_3 - q_1 - q_2). \tag{23}$$

This price depends on qualities in a structurally similar way as the equilibrium prices in the previously considered game, with price competition between the private providers. In the preceding stage of the game, all providers simultaneously and independently choose qualities, anticipating the price choice of Provider 3.

The optimal quality choice of the public provider is given by a first-order condition that is structurally similar to (15), although the inclusion of a private provider in the public funding scheme will affect the magnitudes of $\partial\overline{q}/\partial q_1$ and $\partial T/\partial q_1$.[15] For the private Provider 2, who is now included in the public funding scheme, the first-order condition for profit-maximising quality provision is given by

$$(\overline{p} - c)\left[\frac{\partial D_2}{\partial q_2} + \frac{\partial D_2}{\partial p_3}\frac{\partial p_3}{\partial q_2}\right] - kq_2 = 0, \tag{24}$$

and is structurally similar to (17), in the sense that the provider still has a strategic incentive to reduce quality in order to induce a price increase from the competing private provider (captured by the second term in the square brackets). However, for Provider 3, who remains outside the public

[15] With one included private provider, the quality provision of the public provider has an indirect effect on average quality and aggregate transportation costs only through the pricing decision of the private provider that remains outside the funding scheme.

funding scheme, the first-order condition for profit-maximising quality provision is now qualitatively altered and given by

$$\frac{\partial p_3}{\partial q_3} D_3 + (p_3 - c)\left(\frac{\partial D_3}{\partial p_3}\frac{\partial p_3}{\partial q_3} + \frac{\partial D_3}{\partial q_3}\right) - kq_3 = 0, \tag{25}$$

which, using the first-order condition for the optimal price p_3, can be reformulated as

$$\beta D_3 - kq_3 = 0. \tag{26}$$

Comparing (17) and (26), we see that the second term in (17) is missing from the first-order condition in (26). In other words, Provider 3 has no longer any strategic incentive to affect the competing provider's price decision. The reason is simply that inclusion of Provider 2 in the public funding scheme implies that the price of this provider becomes regulated and thus exogenous ($r = s\overline{p}$), which in turn eliminates the competing Provider 3's ability to strategically influence this price.

The strategic interaction among the three providers are now characterised by the following best-response functions

$$q_1(q_2, q_3) = \beta \frac{2(c-r) + 6t - 3\beta(3q_2 + 2q_3)}{16kt - 15\beta^2}, \tag{27}$$

$$q_2 = \frac{7\beta(\overline{p} - c)}{8kt}, \tag{28}$$

$$q_3(q_1, q_2) = \beta \frac{2t + 6(r-c) - 3\beta(q_1 + q_2)}{6(2kt - \beta^2)}. \tag{29}$$

The best-response functions of the public provider and the private provider without public funding are structurally similar to the corresponding best-response functions in the previously considered game and thus require no further explanation. However, the best-response function of Provider 2 is fundamentally different as a result of the provider being included in the public funding scheme. In fact, Provider 2's best response is now constant, implying *strategic independence*. A quality increase by Provider 2 has a direct and an indirect effect on the provider's demand. The positive direct effect is counteracted by the fact that a quality increase triggers a price reduction by the competing private provider (Provider 3) in the subsequent stage. This indirect effect dampens the incentives for quality provision by Provider 2, all else equal, as shown by (24). However, because of the linearity of the demand function, neither the direct nor the indirect effect of quality on demand depends on the quality levels chosen by the competing providers. Thus, q_2 is strategically independent of the rivals' qualities. Instead, the optimal quality choice by Provider 2 is directly determined by the magnitude of the regulated price $\overline{p}$, which determines the price-cost margin and thus the profitability of attracting more demand through higher quality provision.

The SPNE is given by[16]

$$q_1^{**} = \beta \frac{\left(\left(79kt - 42\beta^2\right) c - 21\left(3kt - 2\beta^2\right)\overline{p}\right)\beta^2 + 16\left(\left(\left(3t + c\right)k - 2\beta^2\right)t - \left(\beta^2 + kt\right)r\right)kt}{8\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)kt}, \quad (30)$$

$$q_2^{**} = \frac{7\beta\left(\overline{p} - c\right)}{8kt}, \quad (31)$$

$$q_3^{**} = \beta \frac{21\left(3\left(2kt - \beta^2\right)c - \left(2kt - 3\beta^2\right)\overline{p}\right)\beta^2 + 4\left(3\left(8kt - 7\beta^2\right)r - 4\left(3\beta^2 + 2\left(3c - t\right)k\right)t\right)kt}{12kt\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)}, \quad (32)$$

$$p_3^{**} = \frac{\left(2kt - 3\beta^2\right)\left(16kt^2 + 3c\left(16kt - \beta^2\right) - 21\beta^2\overline{p}\right) + 12krt\left(8kt - 7\beta^2\right)}{12\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)}. \quad (33)$$

It is once more useful to consider how these equilibrium qualities depend on the funding parameters, and a simple inspection of (30)-(32) is sufficient to reach the following conclusions:

Lemma 2 *Suppose that one of the private providers is included in the public funding scheme. (i) An increase in the regulated price ($\overline{p}$), while keeping the consumer copayment r constant, leads to higher quality for the private provider with public funding and lower qualities for the public provider and the private provider outside the funding scheme. (ii) An increase in the copayment rate (s) leads to higher quality for the private provider without public funding, lower quality for the public provider, and has no effect on the quality provision of the private provider with public funding.*

In contrast to the previously considered game, with no private providers included in the funding scheme, the regulated price has now an influence on the quality provision of all three providers that is independent of the level of consumer copayment. This means in turn that $\overline{p}$ and s can be used as

[16] Second-order and stability conditions are reported in Appendix A.

two separate policy instruments. The independent effect of $\overline{p}$ is caused by the inclusion of a profit-maximising provider in the public funding scheme. Since Provider 2 maximises profits, its incentive for quality provision depends on the profitability of attracting more consumers by increasing the quality, which in turn depends on the unit price $\overline{p}$ it receives from the public funder. Thus, and as previously shown, a higher regulated price leads to higher quality provision by the private provider with public funding. This quality increase leads to lower demand for the two other providers, and both of them respond by reducing their quality provision, though for very different reasons. The public provider reduces its quality because lower demand reduces the effect of a quality increase on the average quality in the market. The private provider without public funding, on the other hand, chooses a lower quality level because the drop in demand makes demand more price-elastic and therefore leads to a lower price, which in turn makes quality provision less profitable.

An increase in the copayment rate s has also different effects for different providers. A higher copayment rate implies that the good supplied by either of the publicly funded providers become more expensive for consumers. However, this has no effect on the quality offered by the publicly funded private provider. Since a higher copayment rate affects neither the profit margin nor the demand responsiveness to quality, it has no effect on the quality choice of a profit-maximsing provider. The incentives are different for the welfare-maximising public provider. Since a higher copayment rate reduces the market share of the public provider, this reduces the effect of the public provider's quality on average quality, which all else equal gives Provider 1 an incentive to reduce its quality provision. Furthermore, since a higher s leads to lower quality for the public provider, the private provider without public funding experiences higher, and thus less price-elastic, demand. This, in turn, gives the private Provider 3 an incentive to increase the price and therefore also leads to higher quality (because price and quality are complementary strategies).

5 Optimal funding policy

We now turn to the question of how a welfarist regulator should optimally design the funding scheme. This question involves setting the optimal values of the funding parameters $\overline{p}$ and s, and deciding which providers to include in the public funding scheme. In order to answer this question, we start out by deriving the first-best solution.

5.1 The first-best solution

Suppose that the regulator is able to control quality and demand directly, and sets quality and demand for each provider in order to maximise social welfare. For a given demand configuration, maximisation of (9) with respect to q_i yields the following first-order condition:

$$\beta D_i - kq_i = 0. \tag{34}$$

Since aggregate transportation costs are minimised if each consumer attends the nearest provider, the welfare-maximising demand configuration has equal market shares for all providers. Thus, due to the symmetry of the model, social welfare is maximised for $D_i = 1/3$, which implies that the first-best quality level – equal for each provider – is given by

$$q_i^{fb} = \frac{\beta}{3k}. \tag{35}$$

Intuitively, the first-best quality level is increasing in the consumers' marginal willingness to pay for quality (β) and decreasing in the marginal cost of quality provision (captured by k).

5.2 Implementation of the first-best solution

Suppose that quality is not verifiable, and therefore not contractible. In other words, the regulator cannot set qualities directly and must use the price instruments of the funding scheme ($\overline{p}$ and s) to induce the desired quality provision. Suppose further that the regulator is able to commit to a particular funding scheme before quality and price decisions are made by the providers in the market. In other words, we let the regulator set both the price and the copayment rate at the first stage of the game. Formally, the regulator maximises (9) with respect to $\overline{p}$ and s. The solution to this problem depends crucially on the public funding coverage; i.e., which providers are included in the funding scheme.

Proposition 1 *(i) If one of the private providers is included in the public funding scheme, the regulator can implement the first-best solution by setting the price*

$$\overline{p}^{fb} = c + \frac{8}{21}t \tag{36}$$

and the copayment rate

$$s^{fb} = \frac{21c + 7t}{21c + 8t}. \quad (37)$$

(ii) If no private provider is included in the public funding scheme, the combination of $\overline{p}$ and s that minimises aggregate transportation costs yields underprovision of quality. The first-best solution is thus not attainable.

A formal proof is given in Appendix B.

Without including a private provider in the public funding scheme, the first-best solution cannot be implemented. This is caused by a negative competition externality between the two private providers that yields underprovision of quality. This is clearly seen by comparing the first-order condition for the social optimal quality, given by (34), with the first-order condition for either of the private providers, given by (17). For a given demand, each of the private providers always chooses a suboptimally low quality level in equilibrium, due to the negative second term in (17). As previously explained, each of the private providers has an incentive to reduce its quality provision in order to induce a price increase from the competing provider, and this leads to underprovision of quality, all else equal.[17] It is worth emphasising that this negative competition externality does not depend on, and thus cannot be corrected by, the parameters of the funding scheme ($\overline{p}$ and s). Thus, the first-best solution is not attainable when none of the private providers are covered by public funding.

This problem can be solved by including one of the private providers in the public funding scheme. As explained in the previous section, for the private provider who remains outside the funding scheme, such inclusion eliminates the strategic incentive to use quality to affect the pricing decision of the competing provider, because the latter's price is 'exogenised' by public funding coverage. Indeed, a comparison of (26) and (34) reveals that, for a given demand level, the private firm without public funding has socially optimal incentives for quality provision, regardless of the values of the funding parameters $\overline{p}$ and s.

Once the socially optimal quality provision of Provider 3 is ensured, implementation of the first-best outcome requires that (i) the other private provider offers quality at the socially optimal level, and that (ii) each consumer's choice of provider is socially optimal (which implies equal

[17] This incentive for underprovision of quality is caused by the assumed sequentiality of quality and price decisions, as shown by Ma and Burgess (1993).

market shares for all providers). Since inclusion of a private provider in the funding scheme implies a separation of $\overline{p}$ and s as policy instruments, the regulator has enough instruments available to achieve the first-best outcome. Socially optimal quality provision by Provider 2 is ensured by setting $\overline{p}$ at the level that implies equality between (24) and (34), whereas optimal consumer choices are ensured by setting s such that $s\overline{p} = p_3$, which minimises transportation costs when all providers have the same quality. If $q_2 = q_3 = q^{fb}$ and $s\overline{p} = p_3$, implying $D_2 = D_3$, it is straightforward to verify that (15) coincides with (34), which implies that the public provider will also set $q_1 = q^{fb}$, which is obvious given that the provider maximises social welfare. Thus, the first-best solution is implemented.[18] Inclusion of one of the private providers in the public funding scheme is therefore a subgame-perfect Nash equilibrium outcome of the three-stage game specified in Section 3.

In sum, the results in Proposition 1 illustrate that the inclusion of private providers in the public funding scheme can be a way to overcome a regulatory problem, caused by a socially undesirable competition externality, and thereby achieve a welfare-superior outcome. However, it is worth stressing that inclusion of only one of the private providers is sufficient to implement the first-best solution. In other words, *partial coverage* of private providers by public funding is enough to ensure a socially efficient outcome.[19] Our analysis therefore provides a potential explanation and rationale for the co-existence of different types of providers in the same market: public and private providers within the public funding scheme and private providers outside the scheme. In Section 6 we provide a brief discussion of the generality of this result.

5.3 Equilibrium quality provision if the copayment rate differs from the first-best level

In the first-best solution, all three providers choose the same quality level and have equal demand. However, in many relevant real-world markets, there are often pronounced differences in both quality and demand across different types of providers. This could of course be explained by real-world asymmetries that are not captured in a symmetric model, but it could also be explained by out-of-model regulatory concerns that preclude the implementation of the first-best solution. In our model, social welfare does not depend directly on the distribution of surplus between consumers

[18] Notice that the first-best solution is implemented for $s < 1$, which implies some degree of cost-sharing between consumers and the public funder.

[19] With provider-specific prices, the first-best solution could also be implemented by including both private providers in the funding scheme, but the key point here is that inclusion of the second provider does not yield additional efficiency gains.

and providers. However, in real-world markets such as health care and education, copayment rates for publicly funded providers are usually set in a way that ensures broad access to the good offered, which in most cases implies relatively low levels of consumer copayment. Thus, it might be relevant to derive the equilibrium outcome under optimal price regulation when the copayment rate does not necessarily coincide with the first-best level.

Plugging the equilibrium outcomes in (30)-(33) into the welfare function and maximising with respect to $\overline{p}$, we derive the optimal regulated price as a function of the copayment rate, given by[20]

$$\overline{p}(s) = \frac{8kst\left(\beta^2 + 16kt\right)\left(kt - \beta^2\right)\left(16kt^2 + 3c\left(16kt - \beta^2\right)\right) + \Lambda}{3\Delta}, \tag{38}$$

where

$$\begin{aligned} \Lambda \ &: = 56t\beta^2\left(12\beta^4\left(17kt - 3\beta^2\right) + k^2t^2\left(144kt - 311\beta^2\right)\right) \\ &+21c\beta^2\left(\beta^4\left(1427kt - 252\beta^2\right) + 16k^2t^2\left(64kt - 137\beta^2\right)\right) \end{aligned} \tag{39}$$

and

$$\begin{aligned} \Delta \ &: = 16kst\left(kt - \beta^2\right)\left(\beta^2 + 16kt\right)\left(7\beta^2 + 4s\left(2kt - \beta^2\right)\right) \\ &+49\beta^2\left(8k^2t^2\left(16kt - 37\beta^2\right) + \beta^4\left(205kt - 36\beta^2\right)\right). \end{aligned} \tag{40}$$

By comparing the equilibrium quality levels across the three providers, when the regulated price is set at the welfare-maximising level, $\overline{p}(s)$, we produce the following ranking:[21]

Proposition 2 *Suppose that Provider 2 is included in the public funding scheme and that the regulated price is set at the welfare-maximising level, $\overline{p}(s)$. In this case,*

(i) if $s < s^{fb}$, the equilibrium quality ranking is given by

$$q_2^{**}(s) > q_1^{**}(s) > q_3^{**}(s);$$

[20] The assumption in (12) ensures that the second-order condition of the welfare-maximising problem is satisfied (see Appendix A for details).

[21] The proof of this proposition relies on a straightforward comparison of equilibrium expressions and is therefore omitted.

(ii) if $s > s^{fb}$, the equilibrium quality ranking is given by

$$q_3^{**}(s) > q_1^{**}(s) > q_2^{**}(s).$$

For any given copayment rate, the equilibrium quality offered by the public provider always lies between the qualities offered by the high-quality and low-quality providers, respectively, and the highest and lowest quality in the market are always offered by a private provider. If the copayment rate is *below* the level that induces the first-best outcome, the highest quality in the market is offered by the private provider that is included in the public funding scheme.

In order to explain this ranking, take the first-best outcome, given by $s = s^{fb}$, as a starting point. If s is reduced below the first-best level, we know from Lemma 2 that this leads to a quality increase for the public provider and a quality reduction for the private provider that is not included in the funding scheme, all else equal. This explains why the latter provider offers the lowest quality in the market for $s < s^{fb}$. However, a lower copayment rate also affects the optimal price. Comparing $\overline{p}(s)$ with the first-best price level, it is possible to verify that $\overline{p}(s) > (<) p^{fb}$ if $s < (>) s^{fb}$. Thus, a reduction of s below s^{fb} implies a higher regulated price, which stimulates the quality provision of the private provider included in the funding scheme, while simultaneously dampening the quality provision of the public provider (cf. Lemma 2). This explains why the private provider with public funding offers the highest quality in the market for $s < s^{fb}$. Obviously, the exact opposite logic applies for copayment rates above the first-best level ($s > s^{fb}$).

6 Concluding remarks

In this paper we have analysed a regulatory problem in mixed oligopoly markets that has, to our knowledge, not previously been addressed in the literature. In many countries, markets for education and health care are characterised by a combination of public and private provision, and a public funding scheme that encompasses a subset of the providers. In such markets, which providers should be included in the funding scheme? What is the optimal public funding coverage? In the present analysis we have addressed this problem in a spatial competition framework with one (welfare-maximising) public and two (profit-maximsing) private providers. In this framework we have shown that the first-best solution can only be implemented by including (at least) one of the private providers in the public funding scheme. Such inclusion allows for an elimination of a negative competition externality between private profit-maximising providers that, all else equal, yields underprovision of quality.

In our three-provider model with one public and two private providers, we have shown that inclusion of only one of the private providers in the public funding scheme is sufficient to implement the first-best outcome. A natural question to ask is how this result generalises to the case of more than three providers, and the underlying mechanisms and intuition behind our main result, as laid out in Section 5, allow us to say something about this.[22] Suppose that there are n public and m private providers equidistantly located on the Salop circle, and that $h \leq m$ of the private providers are included in the public funding scheme. What is the magnitude and locational configuration of h needed to implement the first-best solution? First, the inclusion criteria needs to ensure that the negative competition externality is completely removed. This requires that the $m - h$ private providers outside the funding scheme does compete directly with each other; i.e., none of these firms can be located next to each other. Once this criterion is met, the locational configuration of the h included providers must be such that two regulatory instruments (s and $\overline{p}$) are sufficient to ensure first-best quality provision and demand allocation. For $h \geq 2$, this requires a symmetry among the included private providers in the sense that each of them must compete directly with the same number of non-included providers.[23] This will give each of the h included providers the same incentives for quality provision, all else equal, implying that the first-best quality can be induced from each of these providers by the use of a single price $\overline{p}$.[24] Otherwise, provider-specific prices would be needed to implement the first-best outcome.

Our analysis is obviously not without limitations, and we would here like to mention two of them. Importantly, we have conducted the model in a framework where consumer preferences are heterogeneous only along a horizontal dimension. This means that we are not able to capture effects that might result from vertical preference differentiation, where some consumers have higher willingness to pay for quality than others, for example. However, our model already includes asymmetries along two different dimensions (provider objectives and public funding coverage), and adding asymmetry along a third dimension would simply render the model intractable. Another

[22]We thank an anonymous referee for helping us to pinpoint the exact mechanisms here.

[23]Notice that social welfare is generally non-monotonic in the degree of funding coverage with a uniform regulated price. For example, if $n = 0$ and $m = 4$, the first-best outcome can be implemented for $h = 2$ but not for $h = 3$, since, in the latter case, the three covered providers cannot all compete directly with the uncovered provider. Thus, increasing the extent of funding coverage may worsen social welfare.

[24]If $n = 1$, as in our model, the first-best solution can be implemented for any m if the pattern of funding coverage among the private providers is characterised by alternating funding status, starting with an uncovered provider next to the public firm, and where $n + h = (n + m)/2$ if $n + m$ is even and $n + h = (n + m + 1)/2$ if $n + m$ is odd.

limitation is that we do not allow for any (exogenous or endogenous) differences in cost efficiency across public and private providers. There are several reasons why public versus private ownership might lead to different incentives for cost-efficient provision, for example the presence of soft budgets associated with public ownership. Potential explorations along these lines are left for further research.

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Appendix

A. Equilibrium existence

No private provider is included in the public funding scheme

In the pricing subgame, the second order conditions are satisfied,

$$\frac{\partial^2 \pi_2}{\partial p_2^2} = \frac{\partial^2 \pi_3}{\partial p_3^2} = -\frac{2}{t} < 0, \tag{A1}$$

and equilibrium stability requires that the Jacobian is negative definite, which is easily verified:

$$\frac{\partial^2 \pi_2}{\partial p_2^2}\frac{\partial^2 \pi_3}{\partial p_3^2} - \frac{\partial^2 \pi_2}{\partial p_2 \partial p_3}\frac{\partial^2 \pi_3}{\partial p_2 \partial p_3} = \frac{15}{4t^2} > 0. \tag{A2}$$

In the quality subgame, there are two sets of conditions that do not trivially hold. First, the problem of each profit maximising provider is well-behaved if

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = \frac{\partial^2 \pi_3}{\partial q_3^2} = \frac{1}{225t}\left(98\beta^2 - 225kt\right) < 0, \tag{A3}$$

and the problem of the welfare-maxmising provider is well-behaved if

$$\frac{\partial^2 W}{\partial q_1^2} = \frac{1}{9t}\left(8\beta^2 - 9kt\right) < 0. \tag{A4}$$

Second, the Nash equilibrium is locally stable if the Jacobian is negative definite, which requires

$$\frac{\partial^2 W}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 W}{\partial q_1 \partial q_2} = \frac{kt\left(225kt - 298\beta^2\right) + 56\beta^4}{225t^2} > 0, \tag{A5}$$

and

$$\begin{vmatrix} \frac{\partial^2 W}{\partial q_1^2} & \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_1 \partial q_3} \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_2}{\partial q_2^2} & \frac{\partial^2 \pi_2}{\partial q_2 \partial q_3} \\ \frac{\partial^2 \pi_3}{\partial q_3 \partial q_1} & \frac{\partial^2 \pi_3}{\partial q_3 \partial q_2} & \frac{\partial^2 \pi_3}{\partial q_3^2} \end{vmatrix} = -\frac{k\left(5kt - 6\beta^2\right)\left(25kt - 14\beta^2\right)}{125t^2} < 0, \tag{A6}$$

All the above conditions are satisfied if $k \geq \underline{k}$, where $\underline{k}$ is explicitly given by (12).

Inclusion of one private provider in the public funding scheme

In the quality subgame, there are two conditions that do not trivially hold. First, the problem of the welfare-maxmising public provider is well-behaved if

$$\frac{\partial^2 W}{\partial q_1^2} = -\frac{\left(16kt - 15\beta^2\right)}{16t} < 0, \tag{A7}$$

which requires $k > 15\beta^2/16t$. Second, the Nash equilibrium is locally stable if the Jacobian of the system of first-order conditions is negative definite, which requires

$$\frac{\partial^2 W}{\partial q_1^2}\frac{\partial^2 \pi_2}{\partial q_2^2} - \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1}\frac{\partial^2 W}{\partial q_1 \partial q_2} = \frac{k}{16t}\left(16kt - 15\beta^2\right) > 0 \tag{A8}$$

and

$$\begin{vmatrix} \frac{\partial^2 W}{\partial q_1^2} & \frac{\partial^2 W}{\partial q_1 \partial q_2} & \frac{\partial^2 W}{\partial q_1 \partial q_3} \\ \frac{\partial^2 \pi_2}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_2}{\partial q_2^2} & \frac{\partial^2 \pi_2}{\partial q_2 \partial q_3} \\ \frac{\partial^2 \pi_3}{\partial q_3 \partial q_1} & \frac{\partial^2 \pi_3}{\partial q_3 \partial q_2} & \frac{\partial^2 \pi_3}{\partial q_3^2} \end{vmatrix} = -\frac{k\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)}{16t^2} < 0. \tag{A9}$$

(A8) holds if (A7) holds, while (A9) holds if $kt\left(16kt - 23\beta^2\right) + 6\beta^4 > 0$. Notice that

$$kt\left(16kt - 23\beta^2\right) + 6\beta^4\Big|_{k=\frac{15\beta^2}{16t}} = -\frac{3}{2}\beta^4 < 0 \tag{A10}$$

and

$$\frac{\partial\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)}{\partial k} = t\left(32kt - 23\beta^2\right) > 0 \text{ for } k > \frac{15\beta^2}{16t}, \tag{A11}$$

which implies that the condition in (A9) holds if k is above some threshold value higher than $15\beta^2/16t$, which in turn implies that (A7) and (A8) always hold if (A9) holds.

Furthermore, the regulator's optimal pricing problem (for a given copayment rate) is well-behaved if

$$\frac{\partial^2 W}{\partial \overline{p}^2} = -\frac{\left(2kt - 3\beta^2\right)\Theta}{64kt^2\left(kt\left(16kt - 23\beta^2\right) + 6\beta^4\right)^2} < 0, \tag{A12}$$

where

$$\begin{aligned}\Theta \ :\ &= 16kst\left(\beta^2 + 16kt\right)\left(kt - \beta^2\right)\left(7\beta^2 + 4s\left(2kt - \beta^2\right)\right) \\ &+49\beta^2\left(8k^2t^2\left(16kt - 37\beta^2\right) + \beta^4\left(205kt - 36\beta^2\right)\right).\end{aligned} \tag{A13}$$

Assuming that $\Theta > 0$, the condition in (A12) holds if $k > 3\beta^2/2t$. Evaluating the numerator in (A9) at $k = 3\beta^2/2t$ yields

$$kt\left(16kt - 23\beta^2\right) + 6\beta^4\Big|_{k=\frac{3\beta^2}{2t}} = \frac{15}{2}\beta^4 > 0. \tag{A14}$$

Thus, the condition in (A9) always holds if (A12) holds. It remains to show that $\Theta > 0$. To do so, we derive

$$\frac{\partial^3 \Theta}{\partial k^3} = 768t^3\left(7\beta^2\left(2s + 7\right) + s^2\left(64kt - 23\beta^2\right)\right). \tag{A15}$$

Notice that $\partial^3\Theta/\partial k^3 > 0$ if $k > 3\beta^2/2t$. This implies that $\partial^2\Theta/\partial k^2$ is monotonically increasing in k. Evaluated at the lower bound $k = 3\beta^2/2t$, we derive

$$\left.\frac{\partial^2 \Theta}{\partial k^2}\right|_{k=\frac{3\beta^2}{2t}} = 112t^2\beta^4\left(114s + 272s^2 + 245\right) > 0. \tag{A16}$$

Thus, Θ is strictly convex for $k > 3\beta^2/2t$. Furthermore,

$$\left.\frac{\partial \Theta}{\partial k}\right|_{k=\frac{3\beta^2}{2t}} = t\beta^6\left(6944s + 10\,336s^2 + 8869\right) > 0 \tag{A17}$$

and

$$\Theta|_{k=\frac{3\beta^2}{2t}} = \frac{75}{2}\beta^8\left(56s + 64s^2 + 49\right) > 0. \tag{A18}$$

Since Θ is positive and increasing in k at $k = 3\beta^2/2t$, and since Θ is strictly convex for all $k > 3\beta^2/2t$, it follows that Θ is positive also for all $k > 3\beta^2/2t$. Thus, the second-order condition (A12) is satisfied if

$$k > \underline{k} := \frac{3\beta^2}{2t}, \tag{A19}$$

and this condition ensures that the critical conditions in the quality subgame, (A7)-(A9), are also satisfied.

B. Proof of Proposition 1

(i) Plugging (36) and (37) into (30)-(33), it is easily confirmed that $q_1^{**} = q_2^{**} = q_3^{**} = q^{fb}$ and that $p_3^{**} = s^{fb}\overline{p}^{fb}$, which implies that $D_1 = D_2 = D_3$. (ii) In the equilibrium outcome given by (20)-(22), the two private providers have equal quality and price levels, and in turn demand, while demand for the public provider is generally different. Aggregate transportation costs are minimised when each provider has the same demand. Setting $D_1 = D_2 = D_3$ and solving for r_1, we find that equal demand across the three providers is induced if $\overline{p}$ and s (recall that $r_1 = s\overline{p}$) are set such that

$$r_1 = \frac{\left(5kt - 3\beta^2\right)t + 5c\left(3kt - 2\beta^2\right)}{5\left(3kt - 2\beta^2\right)}. \tag{B1}$$

This yields the following equilibrium qualities:

$$q_1^* = \beta\frac{45kt - 28\beta^2}{45k\left(3kt - 2\beta^2\right)} \tag{B2}$$

and

$$q_2^* = q_3^* = \frac{14\beta}{45k}. \tag{B3}$$

Since $D_1 = D_2 = D_3 = 1/3$, this means that the average quality is given by

$$\overline{q}^* := \frac{q_1^* + q_2^* + q_3^*}{3} = \beta\frac{43kt - 28\beta^2}{45k\left(3kt - 2\beta^2\right)}. \tag{B4}$$

A comparison with the first-best quality yields

$$\overline{q}^* - q^{fb} = -\frac{2\beta\left(kt - \beta^2\right)}{45k\left(3kt - 2\beta^2\right)} < 0. \tag{B5}$$

Thus, when the funding parameters are set such that aggregate mismatch costs are minimised, average quality provision is below the first-best level. *Q.E.D.*