Neutrino-Antineutrino Oscillations

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A thesis presented for the degree of
Master of Physics

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Norway

June 2023

Abstract

This thesis presents the theory describing massive neutrinos. It is shown how neutrino mass and mixing give rise to neutrino oscillations. Having introduced the concept of a Majorana neutrino, several cross sections are calculated for both Majorana and Dirac neutrinos. In particular we confirm that any additional effects due to the Majorana nature of the neutrinos are suppressed by the neutrino mass. Assuming Majorana neutrinos, we derive the neutrino-antineutrino oscillation probability and investigate the link between such oscillations and the formerly derived cross sections.

Acknowledgements

I would like to thank my supervisor Jörn Kersten for his support and knowledge. His willingness to help and the freedom he has given me during the last year have made it a truly enjoyable process. I would also like to thank my parents Silje and Tom for always supporting me and encouraging me to do what I find interesting. Finally, a big thanks to friends, family and colleagues for making each day a bit more fun.

Contents

1 Introduction

In the 1920's nuclear physicists were unable to explain the energy spectrum of the process know as beta decay. The theory of alpha decay was well know, and fitted well together with observations. However, the energy spectrum in beta decay was not well explained by the current theory and it seemed like there was a need for new and radical ideas. In fact, Niels Bohr proposed the violation of energy conservation in order to justify the observed energy spectrum [1]. In a famous letter written in late 1930, the Austrian physicist Wolfgang Pauli suggested a solution, a new electrically neutral particle originally named the neutron, but now known as the neutrino [2]. Originally, the mass of the neutrino was only assumed to be smaller than the electron mass, but subsequent experiments in the coming decades showed no sign of any mass at all. The neutrino, which Pauli feared would never be observed, was detected for the first time in 1956 using the reverse process of beta decay, inverse beta decay. During the 1970's and 1980's the Standard Model (SM) became the accepted theory of the fundamental forces and the neutrino was integrated into the SM as a masless electrically neutral particle coming in three different flavours, each corresponding to a charged lepton.

Meanwhile, experimental progress was also being made. In particular measurements of the neutrinos flux coming from the sun did not match the predictions [3]. This discrepancy became know as the Solar Neutrino Problem and remained unsolved for almost 30 years. This is where neutrino oscillations come into the picture. It turns out that the neutrino has a nonzero chance of changing it's flavour as it travels. That is, a neutrino created as an electron neutrino have a small probability of changing it's flavour into a muon or tau neutrino. This flavour change is what we refer to as neutrino oscillations, and it turns out that it's only possible if neutrinos are massive particles. Even though this was already suggested by Pontecorvo in 1968, strong evidence was not provided until 1998 and 2000 by the Super-Kamiokande Observatory and Sudbury Neutrino Observatory [4], [5]. The observation of neutrino oscillations thus provided a solution to the solar neutrino problem, but at the same time it posed several new questions. If neutrinos are massless in the SM, then the observation of neutrino oscillations requires new physics to explain how the neutrino obtains it's mass. Not even the numerical value of the neutrino mass is known exactly, only certain upper bounds. Massive neutrinos leads to mixing also in the lepton sector, analogous to mixing for quarks. The mixing parameters that governs how much neutrinos mix have to be experimentally measured and are the main goal of several neutrino oscillation experiments. On the other hand neutrinos offers a gateway into new physics. Any new theory of physics should naturally incorporate massive neutrinos and explain how they obtain their mass. Massive neutrinos has also been proposed to make up some of the dark matter[6] and they might help explain the baryon-antibaryon asymmetry in the universe[7]. In any case, neutrino physics has been, and still is, one of the most studied subjects in particle physics and astrophysics.

This thesis aims to provide the necessary theoretical background for performing calculations with massive neutrinos. Section 2 gives a short recap of the Standard Model including mass generation. In Section 3 we investigate several paths to neutrino mass. A Dirac mass is obtained by including the right-handed neutrino fields into our theory. Introducing the concept of a Majorana particle we show that it's possible for the neutrino to also obtain a Majorana mass. Several properties of Majorana particles are discussed and it's shown that Majorana neutrinos behave slightly different than Dirac neutrinos. Section 4 contains the standard derivation of the neutrino oscillation probability as well as transformation properties under CP, CPT and T. We also briefly discuss oscillations with sterile neutrinos. Section 5 gives a short review of neutrino oscillation experiments and provide values for the elements of the mixing matrix. Section 6 investigates differences in the cross sections for Majorana and Dirac neutrinos in both neutral current and charged current Standard Model interactions. In particular we confirm that in the massless limit the two cross sections agree. Having seen this, Section 7 introduces lepton number violating processes. Neutrino-less double beta decay is introduced as a process that can settle the Majorana/Dirac question. We also derive the probability for neutrino-antineutrino oscillations. In Section 8 we investigate a link between the neutrino-antineutrino oscillations and the cross sections derived in Section 6.

2 The Standard Model

The Standard Model is perhaps the most successful scientific theory ever. The theory describes the content and the interaction of all the known fundamental particles to a very high level of accuracy. It predicted the existence of the Higgs boson, quarks and several properties of the W and Z bosons, all of which have been experimentally confirmed. Despite of this, it is known that the Standard Model cannot be completely correct. Several observed phenomena is not explained by the theory and it's believed that the Standard Model must be a low-energy approximation of some higher energy, more fundamental, theory. In any case, the starting point for the work that will be done in this thesis is the Standard Model.

2.1 Gauge Bosons

The Standard Model (SM) describes three of the four fundamental forces in the universe. It combines the different particles, interactions between them and the bosons that mediate the interactions. The main principle behind the SM is the principle of gauge invariance i.e the requirement that the Lagrangian is invariant under local transformations. The set of symmetries that keep the Lagrangian invariant is referred to as the gauge group. For the SM the gauge group is denoted by $SU(3)_c \times SU(2)_L \times U(1)_Y$. The standard model can correspondingly be broken down into three two. The strong interactions corresponding to the $SU(3)_c$ symmetry and the electroweak interactions corresponding to the symmetry $SU(2)_L \times U(1)_Y$. The electroweak sector can be broken down into the weak sector and the quantum electromagnetic sector. Only in the process of symmetry breaking are the two are unified into the electroweak sector.This will be the theory we work with in this thesis. A more thorough treatment of the Standard Model can be found in [8]. We here follow the introduction in [9].

The standard model gauge group has 12 generators which corresponds to 8 gluons and 4 electroweak gauge bosons. For the $SU(2)_L$ symmetry group (weak isospin), there are three gauge bosons W^i_μ, one for each generator T^i and for $U(1)_Y$ (hypercharge) there is only one gauge boson B_μ. To each of these gauge bosons there is a corresponding field strength tensor [9]

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon_{abc} W^b_\mu W^c_\nu \tag{2.1}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2.2}$$

with ϵ denoting the Levi-Cevita tensor and g is the coupling constant. Now, the general form of the gauge transformations for the groups $SU(2)_L$ and $U(1)_Y$ is given by (σ^a denotes the Pauli matrices)

$$W^a_\mu \frac{\sigma^a}{2} \to \mathcal{U}_L W^a_\mu \frac{\sigma^a}{2} \mathcal{U}_L^{-1} + \frac{i}{g} \partial_\mu \mathcal{U}_L \mathcal{U}_L^{-1} \tag{2.3}$$

$$B_\mu \to B_\mu + \frac{i}{g'} \partial_\mu \mathcal{U}_Y \mathcal{U}_Y^{-1}, \tag{2.4}$$

with

$$\mathcal{U}_L = e^{i\alpha^a \frac{\sigma^a}{2}}, \qquad \mathcal{U}_Y = e^{i\alpha_Y}. \tag{2.5}$$

Now, the point is that any Lagrangian in our theory has to be invariant under these transformations. For instance the kinetic Lagrangian for the bosons

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{2.6}$$

is invariant under the gauge transformations described above. The fermion fields, which for the time being are massless, can be separated into two different helicity (chirality) states, which we call left- and right-handed components

$$\psi_L = \frac{1}{2}(1-\gamma_5)\psi, \qquad \psi_R = \frac{1}{2}(1+\gamma_5)\psi. \tag{2.7}$$

Each of these transforms differently under $SU(2)_L$. The left-handed components transforms as doublets, while the right-handed component transforms as singlets. Considering only the first generation of leptons, we order the left handed components into doublets and the right handed components as singlets:

$$L_L = \begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L, \qquad e_R \tag{2.8}$$

In particular we note the absence of the right-handed neutrino fields, which will be important in later sections. The subscript in the symmetry group $SU(2)_L$ now become clear. The elements of the group act trivially on the right-handed components of the fermion fields, that is $Te_R = 0$, but non-trivially on the left handed-component. The relation between the weak isospin and hypercharge is given by the Gell-Mann–Nishijima formula

$$Q = T_3 + \frac{Y}{2}. \tag{2.9}$$

Under the entire symmetry group $SU(2)_L \times U(1)_Y$, the left and right-handed fermion fields transforms as follows

$$\psi_L \to e^{i\alpha^a \frac{\sigma^a}{2}} e^{i\alpha_Y Y} \psi_L \tag{2.10}$$

$$\psi_R \to e^{i\alpha_Y Y} \psi_R. \tag{2.11}$$

To obtain the interaction between the fermions and the gauge bosons we look to the kinetic part of the Lagrangian. In doing so, the normal derivative should be replaced by the covariant derivative

$$\partial_\mu \psi_L \to \mathcal{D}_\mu \psi_L = \left(\partial_\mu + ig\frac{\sigma_a}{2} W^a_\mu + ig' Y B_\mu\right)\psi_L \tag{2.12}$$

$$\partial_\mu \psi_R \to \mathcal{D}_\mu \psi_R = \left(\partial_\mu + ig' Y B_\mu\right)\psi_R, \tag{2.13}$$

so the covariant derivatives of the fields transforms in the same way as the fields themselves. Then we can write the fermion field Lagrangian as

$$\mathcal{L}_F^{kin} = \sum_{doublets} i\overline{\psi}_L\gamma^\mu\mathcal{D}_\mu\psi_L + \sum_{singlets} i\overline{\psi}_R\gamma^\mu\mathcal{D}_\mu\psi_R \tag{2.14}$$

For the most part, we will in this thesis be interested in only the leptons and so a short derivation of the Lagrangian for the interaction between leptons and the gauge bosons is in order. After symmetry breaking (next section) the massless gauge bosons W_μ^a and B_μ will give rise to to the photon A_μ and the massive Z_μ-boson. The relationship between them is given by the Weinberg angle θ_W:

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu, \qquad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu. \tag{2.15}$$

The two remaining gauge bosons are the charged W bosons given by

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}. \tag{2.16}$$

Considering only the first generation of leptons and omitting the kinetic term, we have for the charged current interaction Lagrangian

$$\begin{aligned}\mathcal{L}_{I,L}^{(CC)} &= -\frac{g}{\sqrt{2}}(\overline{\nu_{eL}}W^+e_L + \overline{e_L}W^-\nu_{eL}) \\ &= -\frac{g}{2\sqrt{2}}\overline{\nu_{eL}}\gamma^\mu(1-\gamma^5)eW_\mu + h.c.\end{aligned} \tag{2.17}$$

Using the relations given above and the additional relation between the coupling constants and the Weinberg angle $g\sin\theta_W = g'\cos\theta_W$ we extract the neutral-current Lagrangian

$$\begin{aligned}\mathcal{L}_{I,L}^{(NC)} &= -\frac{g}{2\cos\theta_W}\left(\overline{\nu_{eL}}\not{Z}\nu_{eL} - (1-2\sin^2\theta_W)\overline{e_L}\not{Z}e_L + 2\sin^2\theta_W\overline{e_R}\not{Z}e_R\right) \\ &+ g\sin\theta_W\overline{e}\not{A}e.\end{aligned} \tag{2.18}$$

The last term includes the coupling of the electron with the electromagnetic field and so the Lagrangian includes, as it should, the lepton interactions with photons. Identifying the last part of the above equation with the QED Lagrangian $\mathcal{L}_{I,L}^{(\gamma)}$, we can write the NC-Lagrangian in as

$$\mathcal{L}_{I,L}^{(NC)} = \mathcal{L}_{I,L}^{(Z)} + \mathcal{L}_{I,L}^{(\gamma)}, \tag{2.19}$$

where

$$\mathcal{L}_{I,L}^{(Z)} = -\frac{g}{2\cos\theta_W}j_{Z,L}^\mu Z_\mu \tag{2.20}$$

is the weak neutral-current Lagrangian and the leptonic weak neutral-current is given by

$$j_{Z,L}^\mu = 2g_L^\nu\overline{\nu_{eL}}\gamma^\mu\nu_{eL} + 2g_L^l\overline{e_L}\gamma^\mu e_L + 2g_R^l\overline{e_R}\gamma^\mu e_R. \tag{2.21}$$

The generalization to the rest the leptons is in principle the same procedure as described above for one generation. However, the interactions derived above still solely pertain to massless particles.

2.2 Mass Generation in the Standard Model

In the framework of the SM, the neutrino is a massless particle. Until the discovery of neutrino oscillations, there was no reason to believe that neutrinos were massive. As we will see later, the observation of neutrino masses gives definite evidence for a neutrino mass. The value of the mass and the mechanism responsible for it is still unknown. The other particles in the standard model obtains their mass through the so called Higgs-Mechanism. This section aims to give a short description of this mechanism.

A mass term for the fermions is a term in the Lagrangian on the form $\overline{\psi}\psi = \overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L$. Since the left- and right-handed components transforms differently under the SM gauge group, such a term cannot be invariant under the gauge group. A boson mass term has the form $B^a_\mu B^{a,\mu}$ and is also forbidden since it cannot be invariant under the gauge transformations. In order to give the particles mass we must introduce it in a way that preserves the gauge invariance of the Lagrangian. This is achieved by what is called spontaneous symmetry breaking or the Higgs Mechanism. The idea is that the symmetry is always preserved by the Lagrangian, but that it's no longer a symmetry of the ground state of the system. To implement spontaneous symmetry breaking into the model, we introduce a $SU(2)_L$ scalar doublet with hypercharge $Y = 1$ denoted by

$$\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}. \tag{2.22}$$

This allows us to write the Yukawa Lagrangian for the leptons

$$\mathcal{L}_{Yuk} = -\sum_{\alpha,\beta} {Y'}^l_{\alpha\beta}\overline{L'}_{\alpha L}\Phi l'_{\beta R} + h.c. \tag{2.23}$$

The primed fields are used to distinguish from the massive fields that will be present after the symmetry breaking. The doublet is defined in accordance with equation (2.8) as

$$L'_{\alpha L} = \begin{pmatrix} \nu'_{\alpha L} \\ l'_{\alpha L} \end{pmatrix}, \tag{2.24}$$

where the sum is understood to be over all flavours. The Yukawa Lagrangian in equation (2.23) is evidently invariant under the SM gauge group since the doublet has hypercharge $Y = -1$. The Higgs Lagrangian

$$\mathcal{L}_H = (\mathcal{D}_\mu\Phi)^\dagger(\mathcal{D}^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \tag{2.25}$$

is also invariant under the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group. For $\lambda > 0$ the last part of the Higgs Lagrangian describes a potential bounded from below:

$$V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \tag{2.26}$$

In order for the spontaneous symmetry breaking $SU(2)_L \bigotimes U(1)_Y \to U(1)_Q$ to take place, there is also the requirement that $\mu^2 < 0$. Defining the quantity $v \equiv \sqrt{\frac{-\mu^2}{\lambda}}$, it's clear that the potential has a minimum at $\Phi^\dagger\Phi = \frac{v^2}{2}$. The minimal potential corresponds to the vacuum state and excitations of the fields above the vacuum corresponds to particles. Of course our usual fermion and boson fields must have a zero value in a vacuum. However, a neutral scalar field, like the Higgs field, can have a non-zero value. We call this value VEV or vacuum expectation value. Since we want the vacuum to be electrically neutral, ϕ^0 must be the reason for the VEV of the Higgs field

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \tag{2.27}$$

Now, the symmetries of $SU(2)_L \bigotimes U(1)_Y$ is broken by the VEV:

$$T_a\langle\Phi\rangle \neq 0 \tag{2.28}$$

$$Y\langle\Phi\rangle \neq 0 \tag{2.29}$$

The symmetry of $U(1)_Q$ is on the other hand still unbroken:

$$Q\langle\Phi\rangle = (T_3 + \frac{Y}{2})\langle\Phi\rangle = 0. \tag{2.30}$$

This is the reason for the photon remaining massless even after symmetry breaking. In order to proceed we parameterize the scalar field as

$$\Phi = e^{\frac{i\theta^a(x)\sigma_a}{v}}\begin{pmatrix}0\\ \frac{v+H(x)}{\sqrt{2}}\end{pmatrix} \tag{2.31}$$

where θ^a and $H(x)$ are real fields. The latter will be the field describing the physical Higgs boson. The fields θ^a, can be rotated away by a local $SU(2)_L$ transformation with parameter $\alpha^a = -\frac{2\theta^a}{v}$. This particular choice of gauge is called *unitary gauge*. In this gauge there is only one physical scalar field, the Higgs field. It takes the following form:

$$\Phi(x) \to e^{\frac{-i2\theta^a}{v}\frac{\sigma^a}{2}}\Phi = \begin{pmatrix}0\\ \frac{v+H(x)}{\sqrt{2}}\end{pmatrix}. \tag{2.32}$$

Finally, we can rewrite the Higgs Lagrangian in unitary gauge as

$$
\begin{aligned}
\mathcal{L}_H = \frac{1}{2}(\partial_\mu H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 + \frac{g^2v^2}{4}W_\mu^\dagger W^\mu + \frac{g^2v^2}{8\cos\theta_W^2}Z_\mu Z^\mu \\
+ \frac{g^2 v}{2}W_\mu^\dagger W^\mu H + \frac{g^2 v}{4\cos\theta_W^2}Z_\mu Z^\mu H \\
+ \frac{g^2}{4}W_\mu^\dagger W^\mu H^2 + \frac{g^2}{8\cos\theta_W^2}Z_\mu Z^\mu H^2
\end{aligned} . \tag{2.33}
$$

The mass of the bosons is then given by

$$
m_H = \sqrt{-2\mu^2}, \qquad m_W = \frac{gv}{2}, \qquad m_Z = \frac{gv}{2cos^2\theta_W} \tag{2.34}
$$

2.3 Lepton Masses

Having seen the how the gauge bosons obtain mass via the Higgs mechanism, we move on to the leptons. In particular this section will be important for understanding the mixing of neutrinos. Additionally, the same procedure that is used to obtain the mass of the charged leptons can be used later on to obtain a neutrino mass. The starting point is the Yukawa Lagrangian given in equation (2.23). In unitary gauge the Lagrangian can be written as

$$
\mathcal{L}_{Yuk} = -\left(\frac{v+H}{\sqrt{2}}\right)\sum_{\alpha,\beta} {Y'}^{l}_{\alpha\beta}\overline{l'}_{\alpha L} l'_{\beta R} + h.c. \tag{2.35}
$$

One of the terms in this Lagrangian is then proportional to the the VEV and will become the mass term for the lepton. The term proportional to H will describe the coupling between the Higgs boson and the leptons. Now, the $Y'^{l}_{\alpha\beta}$ are in general non-diagonal matrices known as Yukawa matrices. Since they are non-diagonal, the fields l'_α do not yet have a definite mass. In order to obtain the fields of definite mass, the Yukawa matrices have to be diagonalized. To this extent we define the arrays

$$
l'_L = \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix}, \quad l'_R = \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}. \tag{2.36}
$$

This way we can rewrite the Yukawa Lagrangian as

$$
\mathcal{L}_{Yuk} = -\left(\frac{v+H}{\sqrt{2}}\right)\overline{l'_L}Y'^{l} l'_R + h.c. \tag{2.37}
$$

In order to diagonalize the Yukawa matrices we introduce the unitary matrices ${M^l_L}^\dagger$ and M^l_R that are defined in such a way that

$$
{M^l_L}^\dagger Y'^{l} M^l_R = Y^l, \qquad Y^l_{\alpha\beta} = y^l_\alpha \delta_{\alpha\beta}. \tag{2.38}
$$

In other words, the matrix Y^l is diagonal with the entries given by y^l_α. We define the left- and right-handed components of the fields of definite mass as

$$l_L = {M^l_L}^\dagger l'_L = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}, \quad l_R = {M^l_R}^\dagger l'_R = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}. \tag{2.39}$$

We can then rewrite our Yukawa Lagrangian as

$$\mathcal{L}_{Yuk} = -\sum_\alpha \frac{y^l_\alpha v}{\sqrt{2}} \bar{l}_\alpha l_\alpha - \sum_\alpha \frac{y^l_\alpha}{\sqrt{2}} \bar{l}_\alpha l_\alpha H. \tag{2.40}$$

The fields l_α now denotes the fields of definite masses given by $\frac{y^l_\alpha v}{\sqrt{2}}$. The values of these coefficients cannot be theoretically predicted and must thus be determined experimentally. Meanwhile, the second term in the Yukawa Lagrangian describes the lepton coupling with the Higgs boson. In terms of our fields of definite mass, the leptonic charged current in equation (2.17) can be written as

$$j^\mu_{W,L} = 2\overline{\nu'}_L \gamma^\mu {'}_L = 2\overline{\nu'}_L \gamma^\mu M^l_L l_L. \tag{2.41}$$

Since the neutrino fields are massless we are free to redefine them however we see fit. In particular we can define the field

$$\nu_L = {M^l_L}^\dagger \nu'_L. \tag{2.42}$$

This definition allows us to write the leptonic charged current as

$$j^\mu_{W,L} = 2\sum_\alpha \overline{\nu}_{\alpha L} \gamma^\mu l_{\alpha L}. \tag{2.43}$$

The new fields ν_α are called neutrino flavour fields. Since the neutrino is massless they also correspond to the neutrino mass eigenstates. Each of the fields ν_α couple only to the corresponding flavoured lepton. Once the neutrino obtains mass we will see that the flavour fields will no longer correspond to the mass eigenstates.

3 Extending The Standard Model - Dirac And Majorana Masses

The question of neutrino mass is possibly the most important aspect of neutrino physics. Neither the absolute value of the neutrino mass nor the mechanism that generates it is known. In this section we follow several paths to neutrino mass. First a Dirac mass term is considered, giving the neutrino mass through the same mechanism that gives the rest of the fermions their mass. Next we consider the case of a Majorana mass which for SM particles is a unique possibility for the neutrino mass generation. A parametrization for the mixing matrix is provided in both cases. Finally, we consider the most general mass term, a combined Dirac Majorana mass which is shown to give rise to the so-called seesaw mechanism.

3.1 Dirac Masses

The simplest way to create a system that accommodates neutrino mass is to look to the other leptons in the Standard Model. All of them obtains their mass through the Yukawa Lagrangian (2.23) which requires both left- and right-handed fields. SM neutrinos only have left handed fields and are thus unable to acquire mass. The solution is then simple; introduce the right-handed neutrino fields and use this to write down the mass terms in exactly the same manner as one does for the other fermions. The right handed components are denoted by

$$\nu_{\alpha R}, \qquad \alpha = e, \mu, \tau \tag{3.1}$$

and are evidently singlets of $SU(3)_C \times SU(2)_L$, have hypercharge equal to zero and so are invariant under all the the symmetries of the standard model. For this reason they are often called sterile neutrinos. We will however refer to these as right-handed neutrinos and reserve the term "sterile neutrino" for additional neutrino mass eigenstates beyond the three known light neutrinos. The introduction of the right-handed neutrino fields allows us to add a new term to our Yukawa Lagrangian in equation (2.23)

$$\mathcal{L}_{Yuk} = -\sum_{\alpha,\beta} {Y'}^{l}_{\alpha\beta} \overline{L'}_{\alpha L} \Phi l'_{\beta R} - \sum_{\alpha,\beta} Y'^{\nu}_{\alpha\beta} \overline{L'}_{\alpha L} \tilde{\Phi} \nu'_{\beta R} + h.c, \tag{3.2}$$

where $\tilde{\Phi}$ is a Higgs doublet with hypercharge $Y = -1$ as required in order to make the Lagrangian invariant. In unitary gauge this can be rewritten as

$$\mathcal{L}_{Yuk} = -\left(\frac{v+H}{\sqrt{2}}\right) [\bar{l}'_L Y'^l l'_R + \overline{\nu}'_L Y'^\nu \nu'_R] + h.c, \tag{3.3}$$

with the primed neutrino fields defined analogously to the charged lepton fields in equation (2.36)

$$\nu'_R \equiv \begin{pmatrix} \nu'_{eR} \\ \nu'_{\mu R} \\ \nu'_{\tau R} \end{pmatrix}, \qquad \nu'_L \equiv \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}. \tag{3.4}$$

As before $Y^{'l}$ denotes the Yukawa matrices for the charged leptons and Y'^{ν} the same for the neutrinos. In a similar manner to the charged lepton case, we now diagonalize the neutrino Yukawa matrices by defining unitary matrices $M_L^{\nu\dagger}$ and M_R^{ν} in such a way that the combination

$$Y^{\nu} = M_L^{\nu\dagger} Y_R'^{\nu\nu}, \tag{3.5}$$

is diagonal. The elements of this new diagonal matrix takes the following values

$$Y_{mn}^{\nu} = y_{mn}^{\nu}\delta_{mn}, \qquad y_m^{\nu} \in \mathbb{R}^+, \tag{3.6}$$

with $m, n = 1, 2, 3$. We now define that massive neutrino arrays as

$$N_L = M_L^{\nu\dagger}\nu_L' \equiv \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \qquad N_R = M_L^{\nu\dagger}\nu_R' \equiv \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}. \tag{3.7}$$

Using this we can rewrite the neutrino term in the Yukawa Lagrangian in equation (3.3) as

$$\begin{aligned} \mathcal{L}_{Yuk} &\supset -\left(\frac{v+H}{\sqrt{2}}\right)\overline{N}_L Y^{\nu} N_R + h.c \\ &= -\left(\frac{v+H}{\sqrt{2}}\right)\sum_{m=1}^{3} y_m^{\nu}\overline{\nu}_{mL}\nu_{mR}. \end{aligned} \tag{3.8}$$

Writing this out we have the following form of the neutrino terms in our Yukawa Lagrangian

$$\mathcal{L}_{Yuk} \supset -\sum_{m=1}^{3}\frac{y_m^{\nu} v}{\sqrt{2}}\overline{\nu_m}\nu_m - \sum_{m=1}^{3}\frac{y_m^{\nu}}{\sqrt{2}}\overline{\nu_m}\nu_m H. \tag{3.9}$$

The first term is the mass term and the second represents the neutrinos coupling with the Higgs boson. As for the charged leptons, the neutrino mass is proportional to the Higgs VEV v. Since the neutrino mass is known to be much smaller than for the other leptons, the values of y_m^{ν} would have to be correspondingly small. This method of introducing the neutrino mass gives no explanation for why this is the case. As for all other particles these values have to be measured experimentally.

Let us now investigate the effect of massive neutrinos on the charged leptonic current in equation (2.43). From equation (2.41) and (3.7) we can rewrite the current as

$$j_{W,L}^{\mu} = 2\overline{N}_L U^{\dagger}\gamma^{\mu} l_L, \tag{3.10}$$

with

$$U^{\dagger} = M_L^{\nu\dagger} M_L^{l}. \tag{3.11}$$

In order to have the current on the same form as in equation (2.43) we define the (left-handed) flavour neutrino fields as

$$\nu_L = U N_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}. \tag{3.12}$$

The leptonic charged current then becomes

$$j^{\mu}_{W,L} = 2\sum_{\alpha} \overline{\nu}_{\alpha L}\gamma^{\mu} l_{\alpha L}. \tag{3.13}$$

From equation (3.12) it's clear that U is the mixing matrix for the neutrinos. It describes the relationship between the flavour field and mass eigenstates of the neutrino. In the massless case we could redefine our neutrino mass eigenstates so that they coincided with the flavour field. Once the neutrino is massive this option is no longer available and the consequence is that the flavour fields will now be related to the mass eigenstates by the mixing matrix U.

The weak charged current in equation (3.13) is evidently invariant under the global gauge transformations

$$l_{\alpha L} \to e^{i\phi_{\alpha}} l_{\alpha L}, \qquad \nu_{\alpha L} \to e^{i\phi_{\alpha}} \nu_{\alpha L} \qquad \alpha = e, \mu, \tau \tag{3.14}$$

Finding a transformation that leaves both the right handed neutrino part and the kinetic part of the Lagrangian invariant is not possible. That is, the introduction of the right handed neutrino field has the consequence that flavour lepton number is no longer conserved. In Section 4 we will see that this allows for the possibility of neutrino flavour oscillations. A neutrino created as a electron neutrino has a non-zero chance of turning into a muon neutrino. The total lepton number on the other hand, is still conserved. Consider the global $U(1)$ gauge transformations

$$l_{\alpha L} \to e^{i\phi} l_{\alpha L}, \qquad l_{\alpha R} \to e^{i\phi} l_{\alpha R} \qquad \alpha = e, \mu, \tau \tag{3.15}$$

$$\nu_{mL} \to e^{i\phi} \nu_{mL}, \qquad \nu_{mR} \to e^{i\phi} \nu_{mR} \qquad m = 1, 2, 3. \tag{3.16}$$

Both the Yukawa Lagrangian and the kinetic part of the neutrino Lagrangian is invariant under these transformations[8]. Symmetries lead to conservation laws and so by Noether's Theorem [10] we have a conserved current

$$j^{\mu} = \sum_{m=1}^{3} \overline{\nu_m}\gamma^{\mu}\nu_m + \sum_{\alpha=e,\mu,\tau} \overline{l_{\alpha}}\gamma^{\mu} l_{\alpha}, \tag{3.17}$$

and a corresponding conserved charge which we identify as total lepton number.

$$L = \int d^3x j^0(x) = \int d^3x \left[\sum_{k=1}^{3} {\nu_k}^{\dagger}(x)\nu_k(x) + \sum_{\alpha=e,\mu,\tau} {l_{\alpha}}^{\dagger}(x) l_{\alpha}(x)\right]. \tag{3.18}$$

Upon quantizing, Fourier expanding and normal ordering one obtains in a standard way

$$\begin{aligned} :L: = &\sum_{k=1}^{3}\int \frac{d^3p}{(2\pi^3)2E}\sum_{h=\pm1}\left[{a^{(h)}_{\nu_k}}^{\dagger}(p) a^{(h)}_{\nu_k} - {b^{(h)}_{\nu_k}}^{\dagger}(p) b^{(h)}_{\nu_k}\right] \\ &+ \sum_{\alpha=e,\mu,\tau}\int \frac{d^3p}{(2\pi^3)2E}\sum_{h=\pm1}\left[{a^{(h)}_{l_{\alpha}}}^{\dagger}(p) a^{(h)}_{\nu_k} - {b^{(h)}_{\nu_k}}^{\dagger}(p) b^{(h)}_{l_{\alpha}}\right]. \end{aligned} \tag{3.19}$$

So neutrinos and negatively charged leptons have lepton number $L = +1$ and positively charged leptons and antineutrinos have lepton number $L = -1$. This distinction will be of great importance in later sections. If we give the neutrinos a Dirac mass, the total lepton number is still conserved and this allows us to distinguish between the neutrino and antineutrino through their lepton number. If lepton number is not conserved we will see that it's no longer clear how to distinguish the two. One might worry about breaking the lepton flavour symmetry since after all the Standard Model is a theory of symmetries, but it's important to remember that there is nothing sacred about the lepton symmetries. They are merely accidental symmetries that "fall out" of the theory and are not imposed on our Lagrangian the same way that for instance Lorentz symmetry is.

3.2 Majorana Masses

The other alternative for introducing neutrino masses is to consider neutrinos not as Dirac particles, but Majorana particles. It is well known that a Dirac field, for instance the electron field, is described by four spinors. The right and left handed helicity components which we denote by e_R and e_L and their counterparts $\overline{e}_R$ and $\overline{e}_L$. A Majorana field however only needs two components. To see why, consider the following thought experiment. An electron is travelling in the z-direction with the z-component of it's spin given by $-\frac{1}{2}$. Since helicity is given by projecting the spin onto it's momentum, the electron has negative helicity and is described by the field e_L. An observer moving along the z-axis at a higher speed then the electron would thus see the electron as right handed. However, we have two right handed fields e_R and $\overline{e}_R$. Which one does the observer see? Since charge is Lorentz invariant we conclude that it's indeed e_R and not $\overline{e}_R$ that's observed since the corresponding particle would have opposite charge. In the SM, the neutrino is massless and thus only ν_L and $\overline{\nu}_R$ exists and we have a Weyl particle. We can however also postulate the existence of the fields ν_R and $\overline{\nu}_L$ making the neutrino a Dirac particle. However, there is no charge to distinguish between particle and anti-particle and hence we must turn to lepton number for this distinction. A third option is the Majorana description. The question is if it's possible to describe massive neutrinos without postulating any new fields. This means that the field our observer sees has to be $\overline{\nu}_R$ since this is the only field with the correct helicity. As a result, lepton number is broken and there is no distinction between particle and antiparticle. There are several reasons for suspecting that the neutrino is a Majorana particle. Firstly, the fact that we have only ever observed the left handed neutrino, and the thing we call a right handed antineutrino. It would be economical if it would suffice to use only these two components to describe the field and not the four components that's needed for a Dirac field. Secondly, Majorana neutrinos are a low energy scale predictor of leptogenesis so that CP-violating Majorana neutrinos may help explain the asymmetry between matter and antimatter. Lastly, a small Majorana mass is the first hint me might observe from a higher energy theory than the SM.

3.2.1 Majorana Fields

Writing out the spinor $\psi(x)$ in terms of it's left and right handed components and inserting into the Dirac equation (10.1) we get the coupled equations

$$i\gamma^\mu \partial_\mu \psi_L = m\psi_R \tag{3.20}$$

$$i\gamma^\mu \partial_\mu \psi_R = m\psi_L \tag{3.21}$$

For a masless fermion the equations decouples into the so-called "Weyl" equations

$$i\gamma^\mu \partial_\mu \psi_L = 0 \tag{3.22}$$

$$i\gamma^\mu \partial_\mu \psi_R = 0 \tag{3.23}$$

So that any massless fermion field can be described by a chiral field, ψ_L or ψ_R which are called Weyl spinors. For Dirac field we know that both left- and right-chiral fields are needed to describe the particle. We will see that it does in fact suffice with on Weyl spinor in order to describe a Majorana spinor. Taking the Hermitian conjugate and multiplying by γ^0 on the left in equation (3.20) we have:

$$-i\partial_\mu \overline{\psi_R}\gamma^\mu = m\overline{\psi_L} \tag{3.24}$$

Transposing and multiplying from the left with the charge conjugation matrix C (Appendix C).

$$i\gamma^\mu \partial_\mu C\overline{\psi_R}^T = mC\overline{\psi_L}^T \tag{3.25}$$

The key point is now to realize that the charged conjugated Weyl spinor have opposite chirality of the original field. This is most easily seen by acting on it with the left-chiral projection operator from equation (10.10):

$$P_L C\overline{\psi}_L^T = C(\overline{\psi}_L P_L)^T = C((P_R\psi_L)^\dagger \gamma^0)^T = 0 \tag{3.26}$$

Here we have used the fact that the Charge Conjugation matrix commutes with the fifth gamma matrix (10.33). We are now free to pick our right-chiral field in this manner

$$\psi_R = \xi C\overline{\psi_L}^T, \qquad |\xi|^2 = 1 \tag{3.27}$$

where ξ is some phase factor. Returning to equation (3.19) we replace ψ_R as indicated by (3.26)

$$i\gamma^\mu \partial_\mu \psi_L = m\xi C\overline{\psi}_L^T \tag{3.28}$$

The phase factor can be eliminated by a simple re-phasing of our fields. Upon re-phasing we obtain the Majorana equation for the left chiral field

$$i\gamma^\mu \partial_\mu \psi_L = mC\overline{\psi_L}^T \tag{3.29}$$

Since any fermion field is composed of it's left- and right-chiral components, we can now write up the field

$$\psi = \psi_L + \psi_R = \psi_L + C\overline{\psi_L}^T \tag{3.30}$$

In particular we note that

$$\psi = C\overline{\psi}^T \tag{3.31}$$

is,apart from the phase factor, the same as the action done by the charge conjugation operation. However, the weak interactions maximally violate the symmetries of charge conjugation (Appendix C) and so we are free to choose it however we like, the most convenient choice being unity. Utilizing this, our Majorana condition becomes

$$\psi = \psi^c \tag{3.32}$$

Implying the equality of particle and antiparticle. it's worth noting that neutrinos are the only fermions that can potentially be Majorana particle since all the other fermions have nonzero electric charge. In the massless case, the Majorana and Dirac descriptions are equal. From equation (3.29) it's straightforward to see that in the case $m = 0$ the Majorana equation for the left-chiral field reduce to the Weyl equation and this is of course also the case for the Dirac equation. Starting from the massless Weyl spinors we have constructed a Majorana spinor, consisting of a right- and left chiral Weyl spinor. As we have seen, the presence of both chirality components allows for an environment where mass can develop. The difference for a Majorana spinor as opposed to a Dirac spinor is that we use the charge conjugate of a Weyl spinor in order to give us the opposite chirality component. In particular this is only possible for neutrinos since it gives us the equality of particle and antiparticle. Having created the correct environment for mass generation we now investigate the nature of a Majorana mass.

3.2.2 Majorana Mass Term

In the Dirac case we saw that it was necessary to add the right chiral neutrino fields in order for neutrinos to get a mass term. We will see that it's possible to write down a mass term using only one of the chiral components and the charged conjugated field. Consider the Dirac mass Lagrangian

$$\mathcal{L}^D_{mass} = -m\bar{\nu}\nu = -m\overline{\nu_R}\nu_L + h.c, \tag{3.33}$$

where the last equality follows from the properties of the projection operator (Appendix A). All the terms in the Lagrangian are Lorentz scalars as required by Lorentz invariance, and so the chiral fields transforms as [8]

$$\nu_L(x) \to \nu^{'}_L(x') = S\nu_L(x) \tag{3.34}$$

$$\nu_R(x) \to \nu^{'}_R(x') = S\nu_R(x) \tag{3.35}$$

$$\overline{\nu_L}(x) \to \overline{\nu^{'}_L}(x') = \overline{\nu_L}(x)S^{-1} \tag{3.36}$$

$$\overline{\nu_R}(x) \to \overline{\nu^{'}_R}(x') = \overline{\nu_R}(x)S^{-1} \tag{3.37}$$

From the previous section we know that given a left chiral field, the charge conjugate of that field is right-chiral. Verifying that the charge conjugated field ν_L^c transforms the same way as ν_L is easily done by inserting it in the above equations. Upon finding that this is indeed the case, we then plug this term into the Lagrangian in equation (3.33) to obtain the Majorana mass Lagrangian

$$\mathcal{L}_{mass}^M = -\frac{1}{2} m \overline{\nu_L^c} \nu_L + h.c. \tag{3.38}$$

Including also the kinetic term in the Lagrangian we have

$$\mathcal{L}^M = \frac{1}{2}\left[\overline{\nu_L} i\partial\!\!\!/ \nu_L + \overline{\nu_L^c} i\partial\!\!\!/ \nu_L^c\right] - m(\overline{\nu_L^c}\nu_L + \overline{\nu_L}\nu_L^c). \tag{3.39}$$

The factor $\frac{1}{2}$ is to avoid double counting due to the two fields not being independent. This factor is in fact essential in order to get the correct energy-momentum relation as we will soon see. We precede by defining the neutrino Majorana field

$$\nu = \nu_L + \nu_L^c. \tag{3.40}$$

The field above clearly satisfies the Majorana condition

$$\nu^c = \nu. \tag{3.41}$$

This allows us to rewrite the Lagrangian in a more convenient form

$$\mathcal{L}^M = \overline{\nu_L} i\partial\!\!\!/ \nu_L - \frac{m}{2}(-\nu_L^T C^\dagger \nu_L + \overline{\nu_L} C \overline{\nu_L}^T) \tag{3.42}$$

We note that the kinetic term is of the same for as in the usual SM Lagrangian, and hence in the case of massless Majorana neutrinos the Lagrangian reduces to the usual SM Lagrangian as one would expect.

Since the Majorana mass term in equation (3.39) only contains the the left-handed field ν_L, and this is also present in the SM, it's natural to ask if neutrinos in SM can have Majorana masses. The left-handed neutrino field ν_L has $T_3 = \frac{1}{2}$ and $Y = -1$ which implies that the term $\overline{\nu_L^c}\nu_L$ has $T_3 = 1$ and $Y = -2$. There are no weak isospin triplets with $Y = 2$ in the SM so that any term in the Lagrangian that could potentially generate the Majorana mass of the neutrino would not be renormalizable. In fact, if we consider only one generation, the Lagrangian term that could generate a Majorana mass and still respect the symmetries of the SM is the term [8] [9]:

$$\mathcal{L}_5 = \frac{g}{\mathcal{M}}(L_L^T \tau_2 \phi) C^\dagger (\phi^T \tau_2 L_L) + h.c. \tag{3.43}$$

The electroweak symmetry breaking then leads $\mathcal{L}_5$ to generate the Majorana mass term:

$$\mathcal{L}_{mass}^M = \frac{1}{2}\frac{gv^2}{\mathcal{M}} \nu_L^T C^\dagger \nu_L + h.c, \tag{3.44}$$

with the Majorana mass

$$m = \frac{gv^2}{\mathcal{M}}. \tag{3.45}$$

The Lagrangian $\mathcal{L}_5$ does however contain product of fields with energy dimension 5 and this is not renormalizable[11]. We can thus conclude that it's not possible to give the neutrinos a Majorana mass in the framework of the Standard Model.

3.2.3 Quantized Majorana Field

The quantization of the Majorana field is identical to the quantization of the Dirac field. The general procedure for Dirac fields can be found in for instance [10] and [12]. We follow the notation and conventions in [8] and take the Fourier expanded Dirac field to be

$$\psi^D(x) = \int \frac{d^3p}{(2\pi^3)2E} \sum_{h=\pm 1} [a^{(h)}(p)u^{(h)}(p)e^{-ip\cdot x} + b^{(h)\dagger}(p)v^{(h)}(p)e^{ip\cdot x}]. \quad (3.46)$$

Upon quantization, the coefficients $a^{(h)}$ and $b^{(h)\dagger}(p)$ become lowering- and raising-operators for particles and antiparticles respectively. The only real difference in the Majorana case is the additional constraint that $b^{(h)}(p) = a^{(h)}(p)$ due to the equality of particle and antiparticle. This gives us the following Fourier expansion for a Majorana neutrino field

$$\nu^M = \int \frac{d^3p}{(2\pi^3)2E} \sum_{h=\pm 1} \left[a^{(h)}(p)u^{(h)}(p)e^{-ip\cdot x} + a^{(h)\dagger}(p)v^{(h)}(p)e^{ip\cdot x}\right]. \quad (3.47)$$

From this it's clear that both the Majorana fields ν^M and $\overline{\nu}^M$ contain the same creation and annihilation operators but accompanied by the opposite spinor $u(p)$ and $v(p)$. Both of these observations will be essential for dealing with Majorana neutrinos in later sections. From the field expansion (3.47) one derives conserved currents using Noether's Theorem in the standard manner. In particular the normal ordered Energy-Momentum operator is found to be

$$: P^\mu := \int \frac{d^3p}{(2\pi^3)2E} p^\mu \sum_h a^{(h)\dagger}(p)a^{(h)}(p) \quad (3.48)$$

giving us the standard energy-momentum relation

$$p^0 = E = \sqrt{\vec{p}^2 + m^2}. \quad (3.49)$$

In other words, Dirac and Majorana neutrinos satisfy the same energy-momentum relation and hence they are equal in the kinematic sense. This then justifies the factor $\frac{1}{2}$ in the Majorana Lagrangian (3.39).

Even though Majorana neutrinos and Majorana antineutrinos are identical it still makes sense to distinguish between them. The charged current

$$j^{\mu\,\dagger}_{W,L} = 2\overline{l_L}\gamma^\mu \nu_L \quad (3.50)$$

contains the field ν_L which creates Dirac antineutrinos and annihilates Dirac neutrinos. Considering the field instead to be of Majorana type i.e $\nu_L = \nu_L^M$, it's clear from equation (3.47) that the current creates neutrinos also in the Majorana case:

$$\langle \nu(p,h)|\nu_L(x)|0\rangle = v_L^{(h)}(p)e^{ipx}. \tag{3.51}$$

In the chiral representation of the γ matrices (Appendix B) this can be written explicitly as

$$\langle \nu(p,+)|\nu_L(x)|0\rangle = -\sqrt{2E}\begin{pmatrix} 0 \\ \chi^{(-)}(\vec{p}) \end{pmatrix} e^{ip\cdot x} \tag{3.52}$$

$$\langle \nu(p,-)|\nu_L(x)|0\rangle = -\sqrt{2E}\begin{pmatrix} 0 \\ \frac{m}{2E}\chi^{(+)}(\vec{p}) \end{pmatrix} e^{ip\cdot x}. \tag{3.53}$$

From this we see that the creation of ultra-relativistic neutrinos with negative helicity is suppressed by the factor $\frac{m}{2E}$ which vanishes due to the small neutrino mass. Hence this current mainly creates Majorana neutrinos with positive helicity. Similarly the current $j^{\mu}_{W,L}$ which in the Dirac case creates neutrinos, instead creates Majorana neutrinos with negative helicity. So it makes sense to say that ultrarelativistic Majorana neutrinos with negative helicity interacts as Dirac neutrinos with negative helicity. We will for these reasons call Majorana neutrinos with positive helicity for antineutrinos, and Majorana neutrinos with negative helicity for neutrinos. This distinction becomes essential in later discussions on the interaction of massive neutrinos in Section 6.

We saw earlier that for massive Dirac neutrinos the flavour lepton number is no longer conserved, but the total lepton number is. For massive Majorana neutrinos it turns out that neither is conserved. In particular the global $U(1)$ gauge transformation

$$\nu_L \to e^{i\phi}\nu_L \tag{3.54}$$

does not leave the Lagrangian in equation (3.42) invariant. In the Dirac case we had the conservation due to the lepton numbers having opposite signs for particle-antiparticle, for Majorana particles this is no longer possible due to particle and antiparticle being identical. Alternatively by considering the Dirac lepton number operator in equation (3.18) and imposing the Majorana constraint $b^{(h)}_{\nu_k}(p) = a^{(h)}_{\nu_k}(p)$ it's clear that the neutrino part of (3.18) vanishes and so Lepton number is no longer defined for Majorana neutrinos.There is however nothing sacred about the lepton number symmetries and since the observation of neutrino oscillations proves that flavour lepton number is indeed violated, one might also expect the violation of total lepton number. We will later see that such violations are central for deciding whether neutrinos are a Dirac or Majorana particle.

3.3 Lepton Mixing Matrix

In the SM, the neutrino is massless and so the flavour neutrino fields are also the mass eigenstates. For massive neutrinos this is no longer the case. Similarly to the quark sector, there is also mixing in the lepton sector. That is, to determine the interactions of massive neutrinos within a gauge theory we must do two things. Firstly, the mass matrices that arise from the symmetry breaking must be diagonalized. Secondly we must rewrite the fields in the mass eigenstate basis so that we have physical particles.

For Dirac neutrinos the approach is identical to that for the CKM matrix for quarks. The mixing matrix is a unitary matrix that depends on

$$\frac{N(N-1)}{2} \quad angles \tag{3.55}$$

$$\frac{N(N+1)}{2} \quad phases \tag{3.56}$$

For $N = 3$ generations the mixing matrix thus depend on 3 angles and 6 phases. Using the global phase transformations given in section 3 we can re-phase the neutrino and charged lepton fields such that only one phase remains. Several parametrizations of the mixing matrix is possible but we adopt the conventions of The Particle Data Group [13]

$$\mathrm{U}^D = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{3.57}$$

With $c_{ab} = \cos\theta_{ab}$ and $s_{ab} = \sin\theta_{ab}$. For Majorana neutrinos the Lagrangian is not invariant under the global phase transformation corresponding to conservation of total lepton number in equation. Thus it is only possible to eliminate three out of the six CP-violating phases. Once again following the conventions of The Particle Data Group we take the parametrization of the mixing matrix in case of Majorana neutrinos to be

$$U = U^D U^M = U^D \begin{pmatrix} e^{i\lambda_1} & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.58}$$

The two extra phases in the Majorana case implies that there are additional CP-violations for Majorana neutrinos. In particular we will see that they are present already for $n = 2$ generations, but that they only show up as physical parameters in lepton-number violating processes. The elements of the mixing matrix have to be determined experimentally and has been the main objective of neutrino oscillation experiments. Current best known values are presented in Section 5.

3.4 Dirac-Majorana Mass

We have now seen two possible paths to neutrino mass. A Dirac mass can be obtained in the usual way by adding the neutrino singlets into our theory thereby allowing for mass to be generated through the Higgs Mechanism. The second option was to consider a Majorana mass, where the mass is then generated by some unknown mechanism. In addition to these, we can add a combination of the two. The chiral field ν_L obviously exist as it's included in the Standard Model Lagrangian. The chiral field ν_R may or may not exist, but there is nothing stopping us from adding it to the Lagrangian. If we add the chiral field ν_R we have the Dirac mass term

$$\mathcal{L}^D_{mass} = -m_D \overline{\nu_R} \nu_L + h.c, \tag{3.59}$$

so that neutrinos are Dirac particles. In addition, the Lagrangian can also contain the Majorana mass terms for both of the chiral fields ν_L and ν_R

$$\mathcal{L}^L_{mass} = \frac{1}{2} m_L \nu_L^T \mathcal{C}^\dagger \nu_L + h.c \tag{3.60}$$

$$\mathcal{L}^R_{mass} = \frac{1}{2} m_R \nu_R^T \mathcal{C}^\dagger \nu_R + h.c. \tag{3.61}$$

So that it's possible to have a combination of the three, a Dirac-Majorana mass term

$$\mathcal{L}^{D+M}_{mass} = \mathcal{L}^D_{mass} + \mathcal{L}^L_{mass} + \mathcal{L}^R_{mass} \tag{3.62}$$

As noted before, the Majorana mass term for ν_L is not allowed in The Standard Model. The Majorana mass term for ν_R is however allowed and so the Dirac-Majorana mass term with $m_L = 0$ is allowed in the SM with the added right handed field ν_R. The Majorana mass m_R and the Dirac mass m_D can, by re-phasing the fields ν_R and ν_L, be chosen real and positive. This leaves no additional degrees of freedom for re-phasing to make m_L real and positive and so we treat it as complex. It's convenient to define following column matrix of the left handed fields:

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C} \overline{\nu_R}^T. \end{pmatrix} \tag{3.63}$$

Using that $\overline{\nu_L^C} = -\nu_L^T \mathcal{C}^\dagger$ we rewrite the Dirac-Majorana mass term as

$$\mathcal{L}^{D+M}_{mass} = \frac{1}{2} N_L^T \mathcal{C}^\dagger M N_L + h.c, \tag{3.64}$$

where M is defined as

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}. \tag{3.65}$$

The off-diagonal Dirac masses in M means that the fields ν_L and ν_R do not have definite masses. Once again we have to diagonalize the mass matrix using a unitary transformation

$$N_L = U \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \end{pmatrix}. \tag{3.66}$$

In particular, the matrix U must satisfy the following equation

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \tag{3.67}$$

with real and positive entries. We saw in the section on Majorana mass that this can be done, and so using this transformation we obtain the proper form of our Dirac-Majorana mass term

$$\mathcal{L}^{D+M}_{mass} = \frac{1}{2}\sum_{k=1,2} m_k \nu^T_{kL}\mathcal{C}^\dagger \nu_{kL} + H.c = -\frac{1}{2}\sum_{k=1,2} m_k \overline{\nu_k}\nu_k, \tag{3.68}$$

with

$$\nu_k = \nu_{kL} + \nu^C_{kL} = \nu_{kL} + \mathcal{C}\overline{\nu_{kL}}^T. \tag{3.69}$$

So that it's clear that the Dirac-Majorana mass term means that neutrinos are Majorana particles. In the case where we limit ourselves to one generation and consider both left- and right-handed chiral fields, we get two massive Majorana fields, ν_1 and ν_2 which we refer to as fields in the mass basis. Meanwhile we refer to ν_L and ν^C_R as left handed fields in the flavour basis.

3.5 The Seesaw Mechanism

Having seen the possibility of neutrinos having a Dirac-Majorana mass term, we now consider one of the most popular explanations for the small neutrino mass, the see-saw mechanism. There exist a number of different seesaw models, but the main principle behind them is the same. In the following we investigate a so called Type-1 seesaw model. As we saw in the last section, m_D and m_R can both be chosen real and positive. If we in addition assume that m_L is real, positive or negative, our mass matrix M is real and symmetric. Using the unitary matrix U, the mass matrix can be diagonalized with U given by

$$U = \mathcal{O}\rho = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix}, \tag{3.70}$$

where $\rho_{1,2}$ are phases so that $\rho^2_{1,2} = \pm 1$. The matrix $\mathcal{O}$ is orthogonal and must be chosen so that the mass matrix M becomes diagonal, that is

$$\mathcal{O}^T M \mathcal{O} = \begin{pmatrix} m_1^{'} & 0 \\ 0 & m_2^{'} \end{pmatrix}. \tag{3.71}$$

The eigenvalues of the mass matrix $m_1^{'}$ and $m_2^{'}$ are given by

$$m^{'}_{2,1} = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right]. \tag{3.72}$$

This is indeed the case if we require that

$$\tan 2\theta = \frac{2m_D}{m_R - m_L}. \tag{3.73}$$

There are several interesting cases depending the values of the masses m_D, m_R and m_L. The see-saw mechanism arises as a result of the case where

$$m_D \ll m_R, \qquad m_L = 0 \tag{3.74}$$

Equation (3.72) then gives

$$m_1^{'} \simeq -\frac{m_D^2}{m_R}, \qquad m_2^{'} \simeq m_R. \tag{3.75}$$

The role of the matrix ρ of phases is to flip the sign of $m_1^{'}$ if it's negative. So since $m_1^{'}$ is negative we have $\rho_1^2 = -1$ and so we obtain the masses

$$m_1 \simeq \frac{m_D^2}{m_R} \qquad m_2 \simeq m_R. \tag{3.76}$$

This shows that the light mass of ν_1 is due to the large value of m_R compared to m_D. Similarly the mass of ν_2 is large due to m_R. Therefore this mechanism is given the suitable name, the see-saw mechanism. In this case equation (3.73) reduces to

$$\tan 2\theta = \frac{2m_D}{m_R} \tag{3.77}$$

which is small due to the condition in equation (3.74). This means that the mixing is low so that ν_1 consists for the most part of the active ν_L and ν_2 mainly consists of sterile ν_R.

The see-saw mechanism provides a simple yet plausible way to explain why the neutrino masses are so low compered to the other fermions. The assumptions we made in equation (3.37) are in fact quite natural. As we saw in Section 3.2.2 a Majorana mass for the field ν_L is forbidden in the framework of SM due to the non-renormalizability and so setting it to zero allows us to avoid this problem. The problem we had regard the small values of the eigenvalues y_k^ν in the Yukawa Lagrangian is no longer an issue since we can now expect them to be of the same order as for the other leptons. In fact, since the Dirac mass term arises as a consequence of the symmetry breaking, it has to be proportional to the symmetry-breaking scale, which is of order $10^2 GeV$ [8]. Lastly, ν_R is invariant under all the SM symmetries and so it's expected that the mass m_R is generated through some new high-energy theory which may lie at grand unification scale $\approx 10^{14} - 10^{16} GeV$. Together this implies that the neutrino mass is somewhere in the range of

$$m_1 \simeq \frac{m_D^2}{m_R} \approx 10^{-10} - 10^{-12} GeV. \tag{3.78}$$

This is well withing the bounds of the values suggested by for instance [14] where they suggest an upper bound of $1.1 eV$ for the neutrino mass.

4 Neutrino Oscillations In Vacuum

The observation of neutrino oscillations shows that neutrinos do in fact have a non-zero mass and hence is one of the few observations that shows us the limitations of the Standard Model. In this chapter the theory of neutrino oscillations will be described in detail and a few consequences will be discussed at the end. The idea of neutrino oscillations was first proposed by Pontecorvo in 1967 [15] and has since been a major theme in the study of neutrinos.

As mentioned earlier the neutrino masses are of order $\approx 1eV$ and due to technological limitations, only neutrinos with above $100keV$ of energy can be detected. This means that all neutrinos we can detect are ultra-relativistic. There are two different processes in which neutrinos are detected[8]

1. Neutral or charged current weak scattering processes above a certain energy threshold.

2. Elastic scattering processes of the form $\nu + e^+ \to \nu + e^+$

The charged current interaction Lagrangian

$$\mathcal{L}_{I,L}^{(CC)} = -\frac{g}{2\sqrt{2}} \left(j_{W,L}^{\mu} W_\mu + j_{W,L}^{\mu}{}^{\dagger} W_\mu{}^{\dagger} \right) \tag{4.1}$$

contains the leptonic charged current given by

$$j_{W,L}^{\mu} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}} \gamma^\mu l_{\alpha L} = 2 \sum_{\alpha=e,\mu,\tau} \sum_k U_{\alpha k}^{\star} \overline{\nu_{kL}} \gamma^\mu l_{\alpha L}. \tag{4.2}$$

We have seen that this current applies to both Dirac and Majorana neutrinos. From the Fourier expanded Dirac field it's clear that the field operator $\overline{\nu_{kL}}$ contains creation operators of neutrinos and destruction operators of antineutrinos. Together with the lepton field l_α, which contain destruction operators of l_α^- and creation operators of l_α^+, the leptonic charged current $j_{W,L}^{\mu}$ generates four processes. Two of them are the cases we are interested in, namely the transition $l_\alpha^- \to \nu_k$ and the pair creation of $\nu_k l_\alpha^+$. In the Majorana case we recall that we have $a = b$ and so creation operators are present also in $j_{W,L}^{\mu}{}^{\dagger}$. However, it does not contribute to the processes mentioned as it contains the adjoint form of the charged lepton field. If the energies and momenta of the particles in the production process are not measured accurately enough, it's not possible to determine the flavour of the neutrino created. In this case the leptonic charged current generates a superposition of massive neutrinos. That is, a flavour neutrino is in a superposition of massive neutrinos.

4.1 Neutrino Oscillation Probability

From the previous section it's clear that a flavour neutrino ν_α is created through the leptonic current and so the flavour state can be described as

$$|\nu_\alpha\rangle = \sum_k U^\star_{\alpha k} |\nu_k\rangle . \tag{4.3}$$

If we consider only finite normalization volume, then we have orthonormal massive neutrino states, which together with the fact that the mixing matrix U is unitary implies that also the flavour states are orthonormal

$$\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta}. \tag{4.4}$$

Using that the massive neutrino states are eigenstates of the Hamiltonian

$$\mathcal{H} |\nu_k\rangle = E_k |\nu_k\rangle , \tag{4.5}$$

with eigenvalues given by the usual energy momentum relation. Together with the Schrodinger equation this implies that the massive neutrino states evolve as

$$|\nu_k(t)\rangle = e^{-iE_k t} |\nu_k\rangle . \tag{4.6}$$

Inserting equation (4.3) we obtain

$$|\nu_\alpha(t)\rangle = \sum_k U^\star_{\alpha k} e^{-iE_k t} |\nu_k\rangle , \tag{4.7}$$

so that for $t = 0$ we have no evolution of the flavour state. From the condition that U is unitary we can invert equation (4.3) as

$$|\nu_k\rangle = \sum U_{\alpha k} |\nu_\alpha\rangle . \tag{4.8}$$

Substituting this back into equation (4.7) gives

$$|\nu_\alpha(t)\rangle = \sum_\beta \left(\sum_k U^\star_{\alpha k} e^{-iE_k t} U_{\beta k} \right) |\nu_\beta\rangle . \tag{4.9}$$

So that once again there is no mixing at $t = 0$ but at $t > 0$ the flavour state is a superposition of different flavour states, provided that U is not diagonal. The amplitude of a transition $\nu_\alpha \to \nu_\beta$ is given by

$$A_{\nu_\alpha \to \nu_\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_k U^\star_{\alpha k} U_{\beta k} e^{-iE_k t}. \tag{4.10}$$

The probability is then given by the square of the amplitude

$$P_{\nu_\alpha \to \nu_\beta}(t) = \left| A_{\nu_\alpha \to \nu_\beta}(t) \right|^2 = \sum_{k,j} U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j} e^{-i(E_k - E_j)t}. \tag{4.11}$$

In experiments the time t is not known, but rather the length L which is the distance between the source and detector. For ultra-relativistic neutrinos we can approximate $L = t$ and $E_k \approx E + \frac{m_k^2}{2E}$ which gives

$$P_{\nu_\alpha \to \nu_\beta}(t) = \sum_{k,j} U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j} e^{-i \frac{\Delta m^2_{kj}}{2E} L}, \tag{4.12}$$

where

$$\Delta m^2_{kj} = m^2_k - m^2_j \tag{4.13}$$

is the squared mass difference. In experiments both the values of E and L are known and so neutrino oscillation experiments gives us information about the values of the squared mass difference Δm^2_{kj} and the elements of the mixing matrix U.

Even though the neutrino oscillation amplitude depends on the elements of the mixing matrix which are different in the Majorana and Dirac case, it can not be used to distinguish between them. The product

$$U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j} \tag{4.14}$$

is invariant under the following transformation:

$$U_{\alpha k} \to e^{i\psi_a} U_{\alpha k} e^{i\phi_k} \tag{4.15}$$

As we have seen in Section 3.3 there are additional phases present in the case of Majorana neutrinos. From equation (3.58) it's clear that we can write the elements of the Majorana mixing matrix as

$$U_{\alpha k} = U^D_{\alpha k} e^{i\lambda_k} \tag{4.16}$$

Thus, in writing out the product of the elements from the mixing matrix appearing in the oscillation probability, we obtain

$$\begin{aligned} U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j} &= U^{D\,\star}_{\alpha k} e^{-i\lambda_k} U^D_{\beta k} e^{i\lambda_k} U^D_{\alpha j} e^{i\lambda_j} U^{D\,\star}_{\beta j} e^{-i\lambda_j} \\ &= U^{D\,\star}_{\alpha k} U^D_{\beta k} U^D_{\alpha j} U^{D\,\star}_{\beta j} \end{aligned}. \tag{4.17}$$

With no dependence on the additional Majorana phases. If neutrinos turn out to be Majorana particles, neutrino oscillations would not be able to give us any measurements on the Majorana phases and we would have to look at other processes to probe these values. In Section 7 we take a closer look at such processes.

At times it might be useful to rewrite the oscillation probability in equation (4.12) as

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L,E) = &\sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 \\ &+ 2\mathcal{R}e \sum_{k>j} U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j} exp(-2\pi i \frac{L}{L^{osc}_{kj}}) \end{aligned}. \tag{4.18}$$

The first term is now a constant and the second term describes the oscillation part of the probability. The new variable L^{osc} is given by

$$L^{osc}_{kj} = \frac{4\pi E}{\Delta m^2_{kj}} \tag{4.19}$$

in such a way that the oscillation length L^{osc} is the distance at which the phase generated by the mass difference is equal to 2π. Taking the average over the exponential we get zero and so the average probability is given by

$$< P_{\nu_\alpha \to \nu_\beta} >= \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2. \tag{4.20}$$

Finally, it's worth noting that the oscillation probability satisfies the following useful relations. Summing over all possible final flavours β, the total oscillation probability is equal to unity:

$$\sum_\beta P_{\nu_\alpha \to \nu_\beta}(L, E) = 1. \tag{4.21}$$

Similarly, summing over all possible initial flavours α the total oscillation amplitude is also equal to unity:

$$\sum_\alpha P_{\nu_\alpha \to \nu_\beta}(L, E) = 1. \tag{4.22}$$

4.2 Antineutrino Oscillations and Transformation Properties

A similar treatment to the one above can be given to antineutrinos. The creation of massive neutrinos is done by the weak charged current ${j^{\mu}_{W,L}}^{\dagger}$ and so the creation of antineutrinos is done by the Hermitian conjugated current $j^{\mu}_{W,L}$. In the Dirac case, the Hermitian conjugated charged current creates flavour neutrinos which are in superposition of the massive neutrinos with weights corresponding to the elements of the mass matrix. In other words we have that

$$|\overline{\nu_\alpha}\rangle = \sum_k U_{\alpha k} \, |\overline{\nu_k}\rangle \, . \tag{4.23}$$

In the Majorana case we have the equality of particle and anti-particle and so one could expect the same probability in the antineutrino case as for the neutrino case. However we recall that the Hermitian conjugated charged current mainly creates Majorana neutrinos with positive helicity. Hence the previous equation describes Dirac antineutrinos with positive helicity and Majorana neutrinos with positive helicity. Similarly, the corresponding equation for neutrinos

$$|\nu_\alpha\rangle = \sum_k U^{\star}_{\alpha k} \, |\nu_k\rangle \tag{4.24}$$

describes Dirac neutrinos with negative helicity and Majorana neutrinos with negative helicity. The derivation of the oscillation probability is identical as for the neutrino case, except that we no longer have the complex conjugated mass matrix element. Following the same procedure as in the previous subsection we obtain

$$
\begin{tikzcd}
\nu_\beta \to \nu_\alpha \arrow[r, "T"] \arrow[d, "CP"'] \arrow[dr, "CPT"] & \nu_\alpha \to \nu_\beta \arrow[d, "CP"] \arrow[dl] \\
\overline{\nu_\beta} \to \overline{\nu_\alpha} \arrow[r, "T"] & \overline{\nu_\alpha} \to \overline{\nu_\beta}
\end{tikzcd}
$$

Figure 1: Transformation properties for neutrino oscillations.

$$P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(t) = \sum_{k,j} U_{\alpha k} U^\star_{\beta k} U^\star_{\alpha j} U_{\beta j} e^{-i\frac{\Delta m^2_{kj}}{2E}L}. \tag{4.25}$$

We note that the oscillation length is the same as for neutrino oscillations, reflecting the fact that neutrinos and antineutrinos are kinematically equal. The only difference between the two lies in the elements of the mixing matrix which also makes sense since they are created by different currents.

The transformation properties of the different channels under CP, CPT and T are summarised in Figure 1. Measuring any difference in the channels $\nu_\beta \to \nu_\alpha$ and $\overline{\nu_\beta} \to \overline{\nu_\alpha}$ could thus provide information on CP-violations. In what follows, it will be convenient to rewrite the oscillation probabilities in the following form

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(L, E) = \delta_{\alpha\beta} &- 4 \sum_{k>j} Re[U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j}] \sin^2 (\frac{\Delta m^2_{kj} L}{4E}) \\ &+ 2 \sum_{k>j} Im[U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j}] \sin (\frac{\Delta m^2_{kj} L}{4E}) \end{aligned} \tag{4.26}$$

$$\begin{aligned} P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(L, E) = \delta_{\alpha\beta} &- 4 \sum_{k>j} Re[U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j}] \sin^2 (\frac{\Delta m^2_{kj} L}{4E}) \\ &- 2 \sum_{k>j} Im[U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j}] \sin (\frac{\Delta m^2_{kj} L}{4E}). \end{aligned} \tag{4.27}$$

In this way the CP asymmetry can be quantified the difference between the two probabilities

$$A^{CP}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta} = 4 \sum_{k>j} Im[U^\star_{\alpha k} U_{\beta k} U_{\alpha j} U^\star_{\beta j}] \sin (\frac{\Delta m^2_{kj} L}{4E}). \tag{4.28}$$

Assuming CPT symmetry to hold, it's clear that $A^{CP}_{\alpha\beta} = -A^{CP}_{\beta\alpha}$ which means that CP-violation only occur in the cases with flavour transition. This can also be seen directly from the expression from $A^{CP}_{\alpha\beta}$ by noticing that $|U_{\alpha k}|^2|U_{\alpha j}|^2$ is real. Measuring the CP asymmetry could thus provide information on the Dirac phase present in $A^{CP}_{\alpha\beta}$ as well as the mixing angles [16]. If CPT is assumed to hold, then CP violation also implies T violation. A T transformation simply exchanges α and β so that the difference is

$$A^{T}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}, \tag{4.29}$$

CPT invariance thus implies that

$$A^{T}_{\alpha\beta} = -\overline{A}_{\alpha\beta}{}^{T} = A^{CP}_{\alpha\beta}. \tag{4.30}$$

Finally, in a future theory CPT may not be conserved. If this is the case, then such violations could be verified in neutrino oscillations by measuring a nonzero value of the difference

$$A^{CPT}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\overline{\nu}_\beta \to \overline{\nu}_\alpha} \tag{4.31}$$

. Thus it is clear neutrino oscillations offer several interesting possibilities for probing CP-violation in the lepton sector.

4.3 Active-Sterile Oscillations

In the last two subsections we have only considered oscillations between active neutrinos. One might also wonder if oscillations between active and sterile neutrinos are possible. Recall that we by sterile neutrino refer to additional mass eigenstates beyond the three known ones. The existence of sterile neutrinos is well motivated. They are required for the seesaw mechanism and is also a possible candidate for dark matter particles. As singlets of the SM gauge group, it's not immediately clear how to probe the properties of these particles. Neutrino oscillations are however a mixing phenomena and could thus shed some light on the sterile states. The approach is identical to that of active oscillations, but in addition to the three active neutrinos we assume the existence of n_s sterile neutrinos. A sterile neutrino would then be described by

$$|\nu_s\rangle = \sum_{i=1}^{3+n_s} U^{\star}_{si} |\nu_i\rangle, \qquad s = s_1, ..., s_n, \tag{4.32}$$

where U is now our new mixing matrix i.e a $(3 + n_s) \times (3 + n_s)$ unitary matrix. The total system, that is any active of sterile flavour state is now defined by

$$|\nu_\alpha\rangle = \sum_{i=1}^{3+n_s} U^{\star}_{\alpha i} |\nu_i\rangle, \qquad \alpha = e, \mu, \tau, s_1, .., s_n. \tag{4.33}$$

From the condition that U is unitary we have

$$\langle \nu'_\alpha | \nu_\alpha \rangle = \delta_{\alpha'\alpha}, \qquad \alpha', \alpha = e, \mu, \tau, s_1, .., s_n. \tag{4.34}$$

Now, in an identical manner to the active neutrino case we can derive the amplitudes for active-sterile oscillations using the Schrodinger equation. This gives

$$A(\nu_\alpha \to \nu_{\alpha'}) = \sum_{i=1}^{3+n_s} U_{\alpha' i} \exp\{-iE_i t\} U^\star_{\alpha i}. \tag{4.35}$$

As usual the probability is given by the amplitude squared

$$P(\nu_\alpha \to \nu_{\alpha'}) = \sum_{i,j=1}^{3+n_s} U_{\alpha' i} U^\star_{\alpha' j} \exp\{-i(E_i - E_j)t\} U^\star_{\alpha i} U_{\alpha j}. \tag{4.36}$$

Summing over all possible final states we see that the total probability is normalized as it should be

$$\sum_{\alpha'} P(\nu_\alpha \to \nu_{\alpha'}) = 1 \tag{4.37}$$

Writing out and rearranging the above we obtain

$$\sum_{\alpha'=e,\mu,\tau} P(\nu_\alpha \to \nu_s) = 1 - \sum_{\alpha'=s_1,\ldots s_n} P(\nu_\alpha \to \nu_{\alpha'}). \tag{4.38}$$

In other words, measuring the left hand side of the above equation to be different from unity would imply that active-sterile oscillations do indeed take place. Most of the current oscillation data is well explained by the standard 3-mixing scheme, however there are some anomalies [17] that could potentially be explained by introducing active-sterile oscillations. Several such possibilities are discussed in [18]. For the seesaw mechanism to explain the smallness of the neutrino mass it is however required that the mass of the sterile neutrinos to be of the order $10^2 GeV$. Such a high mass would mean that the probability in equation (4.36) would be suppressed due to the large mass difference between the sterile and active mass states.

4.4 Neutrino Oscillations In the case n=2

Having seen the standard derivation of the neutrino oscillation formulae, we now show how to derive an explicit formula in the case of 2 generations. Deriving explicit formulas for the oscillation probability in the case of 3 generations of neutrinos is often cumbersome and no more instructive than considering only 2 generations. Furthermore, it's often a good approximation to only consider 2 generations. The formula derived here will also prove useful in later sections.

In the case of only two generations of neutrinos, the mixing matrix takes the following form [9]

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{4.39}$$

which is just a clockwise rotation in the plane. The mixing angle θ takes on values in $[0, \frac{\pi}{2}]$. The only mass difference is given by $\Delta m^2 \equiv m_2^2 - m_1^2$ which we define to be positive. From equation (4.12) it's straightforward to derive the formula for transition probability

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \frac{1}{2} sin^2(2\theta)(1 - \cos\frac{\Delta m_{21}^2}{2E}L) \qquad , \alpha \neq \beta. \tag{4.40}$$

The survival probability is obtained by using the fact that the sum of the transition and survival probabilities is equal to unity. For $\alpha = \beta$ we have

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = 1 - \frac{1}{2} sin^2(2\theta)(1 - \cos\frac{\Delta m_{21}^2}{2E}L) \tag{4.41}$$

In the $n = 2$ case we thus see that the oscillation formula take a particular simple form. The approach for three generations is exactly the same, but will involve substantially more algebra.

5 Experimental Status On Mixing Parameters And Absolute Neutrino Mass

Neutrino oscillations cannot give us information on the absolute value of the neutrino masses or the neutrino mass hierarchy. It can however give us information on the squared mass differences and the values of the parameters in the mixing matrix. Over the last decades one has pinpointed almost all the values of the parameters in the neutrino mixing matrix. This section provides a short overview of neutrino oscillation experiments and experimental data on neutrino oscillations in the standard three neutrino framework. There are three sources of neutrinos used in neutrino oscillation experiments. These are atmospheric neutrinos, solar neutrinos and terrestrial neutrinos. In the following, we give a short description of each type of experiment.

Atmospheric neutrinos are produced when atoms in the earths atmosphere collide with cosmic rays thereby producing neutrinos. Such neutrinos typically have an energy range of $0.1GeV$ to $100GeV$ and there are approximately a 2:1 ratio between muon and electron neutrinos [19]. These neutrinos can then again be detected in underground experiments through scattering on nuclei. The fact that the laboratory is underground is essential, it shields the detector from being triggered by other particles. The interactions used to detect these neutrinos are quasi elastic charged current interactions of the form

$$\nu_l + N \to l + N'. \tag{5.1}$$

Thus the flavour of the incoming neutrino is known by the flavour of the outgoing lepton. Knowing also the flux of the atmospheric neutrinos, it's thus possible to calculate the number of outgoing leptons of each flavour. The Kamiokande experiment observed a deficit of such interactions as compared to the theoretically predicted number of interactions. The number of expected μ -like event did not match the theoretical prediction. This discrepancy became known as the atmospheric neutrino anomaly. In 1996 the Super-Kamiokande experiments started up and a few years later able to explain the discrepancy through muon-tau neutrino oscillations [4]. From the oscillation formula (4.12) we have:

$$P_{\nu_\mu \to \nu_\tau}(t) = \sum_{k,j} U^\star_{\mu k} U_{\tau k} U_{\mu j} U^\star_{\tau j} e^{-i\frac{\Delta m^2_{kj}}{2E}L}. \tag{5.2}$$

One is thus able to obtain estimates of the mass differences and mixing angles. For atmospheric neutrinos, it's in particular the mass difference Δm_{31} that is measured [20].

Solar Neutrinos are one of the main areas of research in neutrino physics. The sun mostly produces electron neutrinos which pass for the most part unaffected through the interior of the sun. These neutrinos then eventually reach earth where they are observed in underground experiments. The first experiments done by Homestake [21] with solar neutrinos indicated that the neutrino flux on earth was much lower than what was expected from the theoretical models. Through the process

$$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^- \tag{5.3}$$

it was discovered that the observation of electron neutrinos only was about one-third of what was theoretically expected. Neutrino oscillations in vacuum alone could not explain the discrepancy, but taking into matter effects as well the problem was resolvable through neutrino oscillations. Through global fit's of several solar neutrino experiments such as SAGE and Gallex/GNO, this solution has been confirmed [22][23][24]. Lastly there are also so-called terrestrial neutrino oscillation experiments. Nuclear reactors and accelerators produces neutrinos which are then detected through various interactions. The advantage of these experiments is that one is able to somewhat control parameters that are not possible to control in the case of solar and atmospheric neutrinos. For instance one is not able to adjust the energy of solar neutrinos since, but one have some freedom in altering it for reactor neutrinos [8]. Nuclear fission reactors mainly produce electron antineutrinos through beta decays. These electron neutrinos are then detected through the process of inverse beta decay:

$$\overline{\nu}_e + p \rightarrow e^+ + n. \tag{5.4}$$

Depending on the source-detector distance L on divides such experiments into two kinds: Short baseline (SBL) and Long baseline (LBL). SBL typically have $L \approx 10 - 100m$ and LBL typically has $L \approx 1km$. Additionally there is the KamLAND experiment with $L \approx 200km$ which is classified as a very long baseline (VLBL)[8].

In the following we use the parametrization of the PMNS matrix given in Section 3.3:

$$\text{U}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\lambda_1} & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.5}$$

Of course only the first matrix matter in neutrino oscillations, irrespective of the Majorana or Dirac nature. From solar neutrino experiments one has determined the value of the mass difference $\Delta m_{12}^2 \approx 7.41 \cdot 10^{-5} eV^2$. For the atmospheric mass splittings the current best known value is $\left|\Delta m_{31}^2\right| \approx 2.5 \cdot 10^{-3} eV^2$. In this case the sign of the mass splitting is not clear. This leads to two different scenarios; Normal Ordering (NO) where $\Delta m_{31}^2 > 0$ and Inverted Ordering (IO) where $\Delta m_{31}^2 < 0$. Neutrino oscillation experiments also provide measurements of the elements in the PMNS matrix through global fit's of several results. The current best known values are given in the table below with values obtained from [25]. with Δm_{3l}^2 denoting Δm_{31}^2 for normal ordering and Δm_{32}^2 for inverted ordering. Future neutrino oscillations experiments with reactor and accelerator neutrinos will hopefully be able to provide enough data to decide whether the neutrino mass spectrum is inverted or normal ordered. The value of the CP-violating phase δ_{CP} is not known to a high degree of accuracy, and will also be investigated in future experiments.

Parameter	Normal Ordering	Inverted Ordering
θ_{12}	$33.41^{+0.75}_{-0.72}$	$33.41^{+0.75}_{-0.72}$
θ_{23}	$42.2^{+1.1}_{-0.9}$	$49^{+1.0}_{-1.2}$
θ_{13}	$8.58^{+0.11}_{-0.11}$	$8.57^{+0.11}_{-0.11}$
δ_{CP}	232^{+36}_{-26}	276^{+22}_{-29}
Δm^2_{21}	$7.41^{+0.21}_{-0.20}$	$7.41^{+0.21}_{-0.20}$
Δm^2_{3l}	$2.507^{+0.026}_{-0.027}$	$-2.486^{+0.025}_{-0.028}$

Table 1: The angles are given in degrees and the mass splittings are in units $10^{-3}eV^2$

6 Distinguishing Majorana/Dirac through SM Interactions

Having introduced the formalism for massive neutrinos both in the case of Majorana and Dirac neutrinos the next question should be ”Well which is it?”. We have already seen that neutrino oscillations are not able to distinguish between the two, but how about other SM processes? After all one might expect that the violation of total lepton number in the Majorana case would lead to a number of new processes that could easily distinguish between the two. While the first statement is true, the second is not. Majorana neutrinos does indeed lead to more possibilities since our fields can both annihilate and create. On the other hand, the fact that neutrinos are ultra-relativistic makes it extremely difficult to observe such differences. In the following section we consider two well known standard neutrino interactions, namely Inverse Beta Decay and neutral current neutrino-electron scattering. It is shown that even though there are additional diagrams for Majorana neutrinos, any difference is not observable.

6.1 Inverse β-Decay

In the following section, we consider neutrino interactions with a nucleus via the exchange of a W boson. We will look at the process of inverse beta decay: $\overline{\nu} + p \to e^+ + n$. It has been an historically important process in the study of neutrinos and still is used in several experiments. The charged current Lagrangian for the process is given by

$$\mathcal{L}_{CC} = \frac{-g}{\sqrt{2}}(\overline{u}\gamma^\mu P_L V_{ij} d + \overline{\nu}\gamma^\mu P_L e)W^+_\mu + \frac{-g}{\sqrt{2}}(\overline{d}\gamma^\mu P_L V_{ij} u + \overline{e}\gamma^\mu P_L \nu)W^-_\mu, \quad (6.1)$$

where only the first term is relevant for our study since this is the only term that can create the final state positron. The goal of this section is to compute the cross section for the IVB process both for Majorana neutrinos and Dirac antineutrinos in order to illustrate why this process can not be used to distinguish between the two. For Dirac antineutrinos the process has the Feynman diagram in Figure 2.

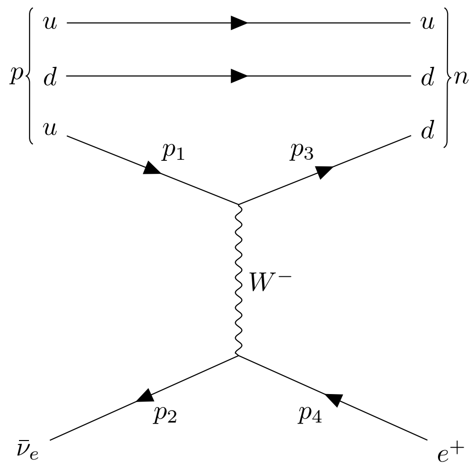

Figure 2: Feynman diagram for IVB process

The corresponding amplitude is obtained directly from the Feynman rules as

$$\mathcal{M}^D = \frac{-iG_F V_{ud}}{\sqrt{2}}[\overline{v}(p_2)\gamma^\alpha(1-\gamma^5)v(p_4)][\overline{u}(p_3)\gamma_\alpha(1-\gamma^5)u(p_1)]. \tag{6.2}$$

In order to calculate the amplitude squared we average over initial states, sum over final states and apply the energy-projection operators. This gives us the following amplitude squared

$$\left|\mathcal{M}^D\right|^2 = \frac{G_F^2 |V_{ud}|^2}{8} Q_{\alpha\beta} N^{\alpha\beta}. \tag{6.3}$$

The quark part of the amplitude is denoted by $Q_{\alpha\beta}$ and is given by

$$Q_{\alpha\beta} = \mathrm{Tr}\big\{\gamma_\alpha(1-\gamma^5)(\not{p}_1 + m_1)\gamma_\beta(1-\gamma^5)(\not{p}_3 + m_2)\big\}. \tag{6.4}$$

Multiplying this out, we find that

$$Q_{\alpha\beta} = 2{p_1}_\mu {p_3}_\nu \,\mathrm{Tr}\big\{\gamma_\alpha\gamma^\mu\gamma_\beta\gamma^\nu + \gamma_\alpha\gamma^\mu\gamma_\beta\gamma^\nu\gamma^5\big\}. \tag{6.5}$$

Employing the well-know trace identities [12], this can be written as

$$\begin{aligned} Q_{\alpha\beta} &= 8{p_1}^\mu {p_3}^\nu (g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu} + g_{\alpha\nu}g_{\mu\beta} - i\epsilon_{\alpha\mu\beta\nu} \\ &= 8({p_1}_\alpha {p_3}_\beta - (p_1\cdot p_3)g_{\alpha\beta} + {p_1}_\beta {p_3}_\alpha - i{p_1}^\mu {p_3}^\nu \epsilon_{\alpha\mu\beta\nu}). \end{aligned} \tag{6.6}$$

Meanwhile, the neutrino part $N^{\alpha\beta}$ is given by

$$N^{\alpha\beta} = \mathrm{Tr}\big\{\gamma^\alpha(1-\gamma^5)(\not{p}_4 - m_4)\gamma^\beta(1-\gamma^5)(\not{p}_2 - m_2)\big\}. \tag{6.7}$$

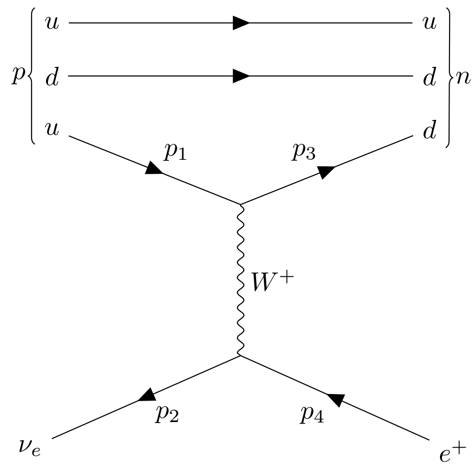

Figure 3: Additional Majorana contribution to IVB

Multiplying out and applying the same trace identities as for the quark part we find

$$N^{\alpha\beta} = 8({p_4}^{\alpha}{p_2}^{\beta} - (p_4 \cdot p_2)g^{\alpha\beta} + {p_4}^{\beta}{p_2}^{\alpha} - ip_{4\sigma}p_{2\kappa}\epsilon^{\alpha\sigma\beta\kappa}. \tag{6.8}$$

The contraction of the quark part $Q_{\alpha\beta}$ and neutrino part $N^{\alpha\beta}$ can now be done:

$$N^{\alpha\beta}Q_{\alpha\beta} = 64(2(p_4 \cdot p_1)(p_2 \cdot p_3) + 2(p_4 \cdot p_3)(p_2 \cdot p_1) - p_{4\sigma}p_{2\kappa}{p_1}^{\mu}{p_3}^{\nu}\epsilon^{\alpha\sigma\beta\kappa}\epsilon_{\alpha\mu\beta\nu}). \tag{6.9}$$

Upon evaluating the last term we find that

$$N^{\alpha\beta}Q_{\alpha\beta} = 256(p_4 \cdot p_1)(p_2 \cdot p_3), \tag{6.10}$$

so the total amplitude squared is

$$\left|\mathcal{M}^D\right|^2 = 32G_F^2|V_{ud}|^2(p_4 \cdot p_1)(p_2 \cdot p_3). \tag{6.11}$$

The corresponding cross section is calculated in Appendix E and is plotted in Figure 4. Having calculated the cross section for IVB in the Dirac case, we now move on to the Majorana cross section. In the Dirac case only antineutrinos will contribute in the process. The process $\nu_e + p \to n + e^+$ is forbidden for Dirac neutrinos since it does not conserve lepton number. A Majorana neutrino created by the current

$$j^{\mu}_{W,L} = 2\sum \overline{\nu}_{\alpha L}\gamma^{\mu}l_{\alpha L} \tag{6.12}$$

can however contribute to the process. In the Majorana case, the field $\overline{\nu}$ contains an annihilation operator of such a particle as can be seen from the Majorana field expansion. For Majorana neutrinos we thus have the additional Feynman diagram in Figure 3. In accordance with the discussion in Section 3.2.3 we find from the Majorana field expansion that

$$\langle\nu(p,h)|\overline{\nu}_L(x)|0\rangle = \overline{u}_L^{(h)}e^{ipx}. \tag{6.13}$$

Neutrinos created by this current are mainly in a negative helicity state which can be seen in the Chiral representation:

$$\langle \nu(p,-)|\overline{\nu}_L(x)|0\rangle \approx \sqrt{2E}\begin{pmatrix} \frac{m}{2E}\chi^{(+)\dagger}(p) & 0 \end{pmatrix} e^{ipx} \tag{6.14}$$

$$\langle \nu(p,+)|\overline{\nu}_L(x)|0\rangle \approx -\sqrt{2E}\begin{pmatrix} \chi^{(-)\dagger}(p) & 0 \end{pmatrix} e^{ipx}. \tag{6.15}$$

The creation of positive helicity neutrinos is suppressed by the ratio $\frac{m}{E}$ while there is no such suppression for the creation of negative helicity neutrinos. The incoming neutrino in Figure 3 can thus be described to be in the following state

$$|\nu(p,h)\rangle = |\nu(p,-)\rangle + \frac{m}{E}\,|\nu(p,+)\rangle\,. \tag{6.16}$$

The incoming neutrino is then annihilated by the field $\overline{\nu}$. We examine the effect of annihilating the neutrino by this field. From the Majorana field expansion we have

$$\langle 0|\overline{\nu}_L(x)|\nu(p,h)\rangle = \overline{\nu}_L^{(h)} e^{ipx}. \tag{6.17}$$

In the Chiral representation we find

$$\langle 0|\overline{\nu}_L(x)|\nu(p,-)\rangle = \sqrt{2E}\begin{pmatrix} 0 & \frac{m}{2E}\chi^{(+)}(p) \end{pmatrix} \tag{6.18}$$

$$\langle 0|\overline{\nu}_L(x)|\nu(p,+)\rangle = -\sqrt{2E}\begin{pmatrix} 0 & \chi^{(-)}(p) \end{pmatrix}. \tag{6.19}$$

In other words, the field $\overline{\nu}$ annihilates the negative helicity states suppressed by a factor $\frac{m}{E}$, while the positive helicity states experience no such suppression from the annihilation. Taking into account both the suppression from the creation of the neutrino and its annihilation we can describe our incoming neutrino state as

$$|\nu(p,h)\rangle = \frac{m}{E}\,|\nu(p,-)\rangle + \frac{m}{E}\,|\nu(p,+)\rangle = \frac{m}{E}\,|\nu(p,h)\rangle\,. \tag{6.20}$$

Since both helicity states are suppressed by the same factor we can now perform the unpolarized spin sum. From the Majorana field expansion and the usual Feynman rules we obtain the Feynman amplitude corresponding to Figure 3 as

$$\mathcal{M}^M = \frac{-iG_F V_{ud}}{\sqrt{2}}[\overline{v}(p_2)\gamma^\alpha(1-\gamma^5)v(p_4)][\overline{u}(p_3)\gamma_\alpha(1-\gamma^5)u(p_1)]. \tag{6.21}$$

The amplitude squared is then given by

$$\left|\mathcal{M}^{\mathcal{M}}\right|^2 = \frac{m_2^2}{E_2^2}\frac{1}{4}\sum_{spins}\mathcal{M}^M\mathcal{M}^{M\dagger}, \tag{6.22}$$

where we have renamed the neutrino mass and energy according to the Feynman diagrams in Figure 2 and 3. Since the amplitude $\mathcal{M}^M$ is equal to the Dirac amplitude $\mathcal{M}^D$ we simply get the result from the corresponding spin sum in the Dirac case:

$$\left|\mathcal{M}^{\mathcal{M}}\right|^2 = 32G_F^2|V_{ud}|^2\frac{m_2^2}{E_2^2}(p_4\cdot p_1)(p_2\cdot p_3) \tag{6.23}$$

Summing up the two contributions we have that the total amplitude squared for Majorana neutrinos is given by

$$\left|\mathcal{M}_{tot}^{M}\right| = \left|\mathcal{M}^{\mathcal{M}}\right|^2 + \left|\mathcal{M}^{D}\right|^2 = 32G_F^2|V_{ud}|^2(p_4 \cdot p_1)(p_2 \cdot p_3)(1 + \frac{m_2^2}{E_2^2}) \qquad (6.24)$$

The Majorana amplitude squared thus contain an additional term as compared to the Dirac case. The term is however suppressed by the small ratio $\frac{m_\nu^2}{E_\nu^2}$. As common wisdom dictates, the Dirac and Majorana descriptions become equal in the massless case where they both reduce to the Weyl description of the neutrino. Assuming the neutrino mass to be $m_\nu \approx 1eV$ and the threshold energy for the IVB process to be approximately $E_\nu \approx 1.8MeV$, we find that the additional term is suppressed by $\mathcal{O}\big(10^{-13}\big)$. Considering that weak interaction cross sections are usually of order $\mathcal{O}\big(10^{-44}cm^2\big)$, the additional term in the cross section will be of order $\mathcal{O}\Big(10^{-57^2}\Big)$, well outside the reach of current detectors. The calculation of the corresponding cross sections has been relegated to Appendix E. The plots can be found in Figures 4,5,6. As expected, the Majorana and Dirac cross sections for IVB look identical. The additional Majorana term in Figure 6 is far to small for having any effect on the Majorana cross section. It's also worth noting that the additional Majorana term decreases with increasing energy. Weak interaction cross sections are usually proportional to $G_F^2 s$, thus lowering the neutrino energy in order to enhance the contribution from the additional Majorana term will reduce the overall cross section.

As a final remark we note that the unpolarized spin sum for the process $\overline{\nu} + p \to e^+ + n$ for Majorana neutrinos is not completely accurate. The positive and negative helicity components have different contributions. In this case it is the "right" field $\overline{\nu_L}$ that annihilates the incoming state and so in analogy with equation (6.20) we can show that

$$|\overline{\nu}(p,h)\rangle = |\overline{\nu}(p,+)\rangle + \frac{m^2}{E^2}\,|\overline{\nu}(p,-)\rangle\,. \qquad (6.25)$$

This means that the above cross section have corrections of order order $(\frac{m^2}{E^2})^2$ which are so small that we will neglect them.

6.2 Neutral Current - Unpolarized Majorana Neutrinos

At first sight, the situation for neutral scattering looks more promising. We will see that the vector part of the current vanishes for Majorana neutrinos and additionally the axial part is twice the Dirac case. In order to see this, we start of by deriving a few useful properties for Majorana particles.

Consider any fermion bi-linear $\overline{\psi}F\psi$ where F is any 4×4 matrix. For Dirac neutrinos we have the following [26]

$$\langle p', s'|\overline{\psi}F\psi|p, s\rangle = \overline{u}_{s'}(p')Fu_s(p). \qquad (6.26)$$

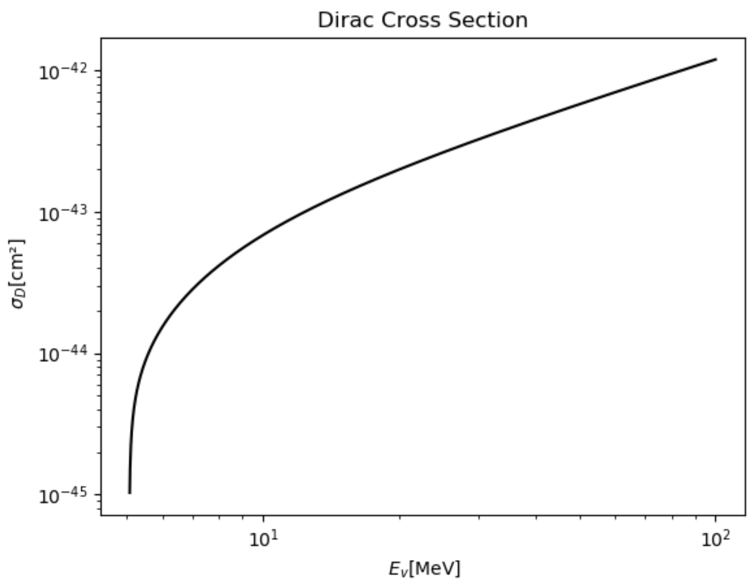

Figure 4: IVB Cross Section Dirac

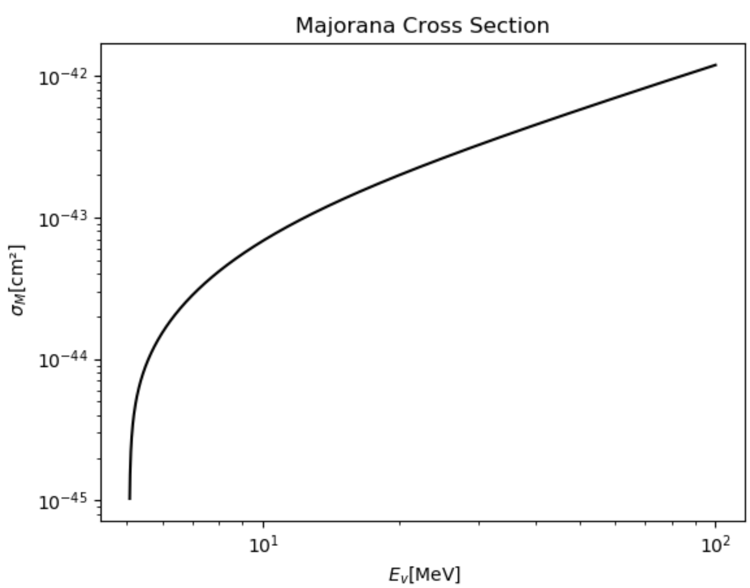

Figure 5: IVB Cross Section Majorana

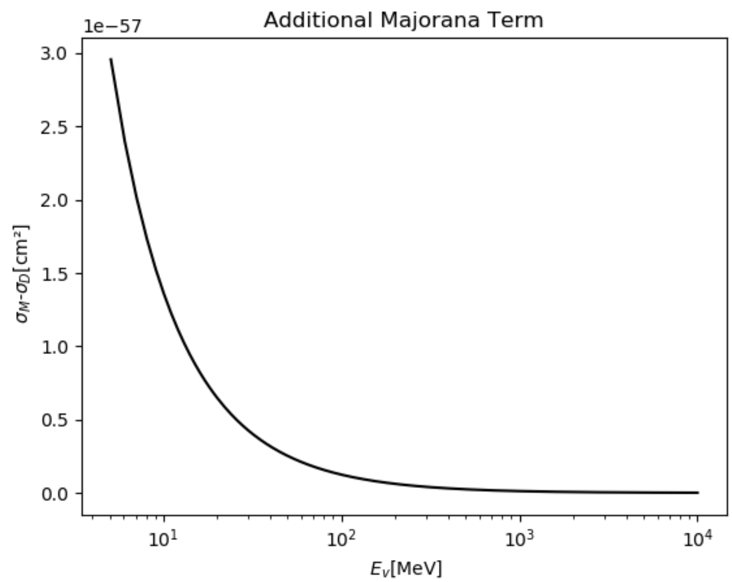

Figure 6: Additional Majorana Term IVB

For Majorana neutrinos there are two possibilities of creating and annihilating the initial and final states and so we have an additional term:

$$\langle p', s'|\overline{\psi} F \psi|p, s\rangle = \overline{u}_{s'}(p') F u_s(p) - \overline{v}_{s'}(p') F v_s(p). \tag{6.27}$$

The Majorana condition $\psi = \psi^c$ implies that

$$u(p, s) = -\overline{v}^T C \tag{6.28}$$

$$v(p, s) = C\overline{u}^T. \tag{6.29}$$

Using the properties for the charge conjugation matrix C that can be found in Appendix C, we have the following useful relations

$$u^T = -\overline{v}C, \quad v^T = -\overline{u}C, \quad \overline{u}^T = C^{-1}v, \quad \overline{v}^T = C^{-1}u. \tag{6.30}$$

Transposing the last term in equation (6.27) and using the properties just derived for the u and v spinors, we have

$$\overline{v}_{s'}(p') F v_s(p) = -\overline{u}_{s'}(p') C F^T C^{-1} u_s(p). \tag{6.31}$$

Plugin back in to equation (6.27)

$$\langle p', s'|\overline{\psi} F \psi|p, s\rangle = \overline{u}_{s'}(p') \left(F + C F^T C^{-1}\right) u_s(p). \tag{6.32}$$

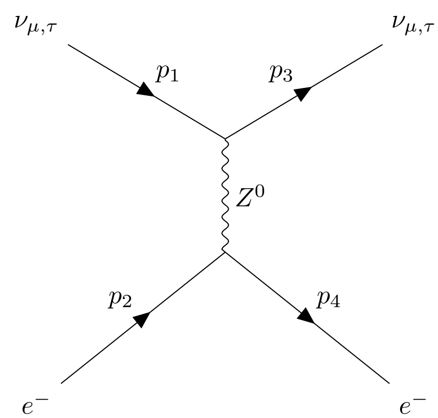

Figure 7: Feynman diagram for neutral current scattering

In particular we note that for $F = \gamma_\mu$

$$\langle p', s'|\overline{\psi}F\psi|p, s\rangle = \overline{u}_{s'}(p')\,(\gamma_\mu - \gamma_\mu)\,u_s(p) = 0, \tag{6.33}$$

where we have used the relation (10.30) in Appendix C. The matrix product vanishes identically. Similarly in the case $F = \gamma_\mu\gamma_5$ we have

$$\langle p', s'|\overline{\psi}F\psi|p, s\rangle = \overline{u}_{s'}(p')\,(\gamma_\mu\gamma_5 - \gamma_5\gamma_\mu)\,u_s(p) = 2\overline{u}_{s'}(p')\gamma_\mu\gamma_5 u_s(p). \tag{6.34}$$

So that in the weak neutral current interactions, the vector part vanishes for Majorana neutrinos and the axial part is twice as much as in the Dirac case as claimed. Thus, we might expect that the neutral currents are well-suited for distinguishing between Dirac and Majorana neutrinos. In order to investigate this, we now consider the process of neutrino-electron scattering. Let us first investigate the unpolarized case. Assume we have a beam of incoming Majorana neutrinos and that they are in an equal mix of positive and negative helicity states. According to our previous discussion, Majorana neutrinos with positive helicity behaves as Dirac antineutrinos, and Majorana neutrinos with negative helicity behave as Dirac neutrinos. We therefore expect that the unpolarized cross section for the process in question should be comparable to the sum of both the corresponding processes for Dirac neutrinos and Dirac antineutrinos. In order to confirm this, we calculate the cross sections Dirac neutrino and antineutrino scattering with an electron. The scattering process $\nu_\alpha + e^- \to \nu_\alpha + e^-$ is mediated by both the neutral current Z^0 and the charged current W^+. Limiting ourselves to the cases where $\alpha = \mu, \tau$ there is only a neutral current contribution with corresponding Feynman diagram given in Figure 7 For low energy neutrinos, the effects of the Z propagator can be neglected. Applying the usual Feynman rules for electroweak theory we have

$$\mathcal{M}^D = \frac{-iG_F}{\sqrt{2}}[\overline{u}(p_4)\gamma^\alpha(g_V - g_A\gamma_5)u(p_2)][\overline{u}(p_3)\gamma_\alpha(1-\gamma_5)u(p_1)] \tag{6.35}$$

for the Dirac amplitude.The coefficients g_A and g_V denotes the axial and vector couplings for the electron. Their values can be found in for instance [8]. We precede by calculating the corresponding cross section for Dirac neutrinos. Averaging over initial and summing over final spins we have

$$\left|\mathcal{M}^D(\nu)\right|^2 = \frac{1}{4}\sum_{spins}\mathcal{M}^D\mathcal{M}^{D\dagger} = \frac{G_F^2}{8}A^{\alpha\beta}B_{\alpha\beta}, \tag{6.36}$$

with

$$A^{\alpha\beta} = \sum_{spins}[\overline{u}(p_4)\gamma^\alpha(g_V - g_A\gamma_5)u(p_2)][\overline{u}(p_4)\gamma^\beta(g_V - g_A\gamma_5)u(p_2)]^\dagger \tag{6.37}$$

$$B_{\alpha\beta} = \sum_{spins}[\overline{u}(p_3)\gamma_\alpha(1-\gamma_5)u(p_1)][\overline{u}(p_3)\gamma_\beta(1-\gamma_5)u(p_1)]^\dagger. \tag{6.38}$$

By applying Casimirs trick or the energy projection operators, [27] the two can be evaluated using the trace techniques, giving

$$\begin{aligned}A^{\alpha\beta} &= 4(g_V^2+g_A^2)[p_2^\alpha p_4^\beta - (p_2p_4)g^{\alpha\beta} + p_2^\beta p_4^\alpha] - 8ig_Vg_Ap_{2\sigma}p_{4\rho}\epsilon^{\alpha\sigma\beta\rho}\\ &+ 4m_e^2(g_V^2-g_A^2)g^{\alpha\beta}\end{aligned} \tag{6.39}$$

$$B_{\alpha\beta} = 8(p_{1\alpha}p_{3\beta} - (p_1p_3)g_{\alpha\beta} + p_{1\beta}p_{3\alpha} - ip_{1\mu}p_{3\nu}\epsilon^{\mu\nu}_{\alpha\beta}). \tag{6.40}$$

The contraction of the two gives us the Feynman amplitude squared as

$$\begin{aligned}\left|\mathcal{M}^D(\nu)\right|^2 &= 8G_F^2((g_V+g_A)^2(p_1p_2)(p_3p_4) + (g_V-g_A)^2(p_1p_4)(p_3p_2)\\ &+ m_e^2(g_A^2-g_V^2)(p_1p_3).\end{aligned} \tag{6.41}$$

In order to obtain the amplitude squared for the corresponding process with antineutrinos, one replace the spinors for the neutrinos in the original amplitude with antineutrino spinors and preform a similar calculation. The result is simply stated as

$$\begin{aligned}\left|\mathcal{M}^D(\overline{\nu})\right|^2 &= 8G_F^2((g_V+g_A)^2(p_2p_3)(p_1p_4) + (g_V-g_A)^2(p_1p_2)(p_3p_4)\\ &+ m_e^2(g_A^2-g_V^2)(p_1p_3).\end{aligned} \tag{6.42}$$

As discussed in the introduction to this section, we add the two amplitudes. This gives the total amplitude squared for Dirac neutrinos and antineutrinos

$$\begin{aligned}\left|\mathcal{M}^D\right|^2 &= 16G_F^2[(g_V^2+g_A^2)((p_2p_3)(p_1p_4) + (p_1p_2)(p_3p_4))\\ &+ m_e^2(g_A^2-g_V^2)(p_1p_3)].\end{aligned} \tag{6.43}$$

The amplitude for Majorana neutrinos must be treated with more care since there are several possibilities of creating or destroying initial and final states. According to the properties derived in the beginning of this subsection, the vector part vanishes and the axial part is twice the Dirac case

$$\mathcal{M}^M = \frac{2iG_F}{\sqrt{2}}[\overline{u}(p_4)\gamma^\alpha(g_V - g_A\gamma_5 u(p_2)][\overline{u}(p_3)\gamma_\alpha\gamma_5)u(p_1)]. \tag{6.44}$$

Then, the Feynman amplitude is given by averaging over initial and summing over final spins

$$\left|\mathcal{M}^M\right|^2 = \frac{1}{4}\sum_{spins}\mathcal{M}^M\mathcal{M}^{M\dagger} = \frac{1}{4}\frac{4G_F^2}{2}A^{\alpha\beta}C_{\alpha\beta}, \tag{6.45}$$

with $A^{\alpha\beta}$ as before and $C_{\alpha\beta}$ is given by

$$C_{\alpha\beta} = \sum_{spins}[\overline{u}(p_3)\gamma_\alpha\gamma_5)u(p_1)][\overline{u}(p_3)\gamma_\alpha\gamma_5)u(p_1)]^\dagger. \tag{6.46}$$

Once again this is evaluated using Casimirs trick and the trace identities, giving

$$C_{\alpha\beta} = 4(p_{1\alpha}p_{3\beta} - (p_1p_3)g_{\alpha\beta} + p_{1\beta}p_{3\alpha} - m_\nu^2 g_{\alpha\beta}). \tag{6.47}$$

Performing the contraction we obtain the Feynman amplitude squared in the Majorana case

$$\begin{aligned}\langle\left|\mathcal{M}^M\right|\rangle = 16G_F^2[&(g_V^2 + g_A^2)((p_2p_3)(p_1p_4) + (p_1p_2)(p_3p_4) + m_\nu^2(p_2p_4)\\ &+ m_e^2(g_A^2 - g_V^2)((p_1p_3) + 2m_\nu^2)].\end{aligned} \tag{6.48}$$

Comparing the above with equation (6.43), we see explicitly that the only terms that differs between the two are proportional to the neutrino mass. The calculations of the corresponding total cross sections can be found in Appendix E. The results are

$$\begin{aligned}\sigma_D = \frac{G_F^2 s}{2\pi}&((g_V^2 + g_A^2)(\frac{1}{4} - \frac{(m_e^2 - m_\nu^2)^2}{4s^2}\\ &+ \frac{1}{2s^2}(m_e^2 - m_\nu^2 + s)^2) + m_e^2(g_A^2 - g_V^2)\frac{(m_\nu^2 - m_e^2 + s)^2}{2s^3})\end{aligned} \tag{6.49}$$

$$\begin{aligned}\sigma_M = \frac{G_F^2 s}{2\pi}&((g_V^2 + g_A^2)(\frac{1}{4} - \frac{(m_e^2 - m_\nu^2)^2}{4s^2} + \frac{(m_e^2 - m_\nu^2 + s)^2}{2s^3}m_\nu^2\\ &+ \frac{1}{2s^2}(m_e^2 - m_\nu^2 + s)^2)\\ &+ m_e^2(g_A^2 - g_V^2)(\frac{(m_\nu^2 - m_e^2 + s)^2}{2s^3} + \frac{4}{s^2}m_\nu^2)).\end{aligned} \tag{6.50}$$

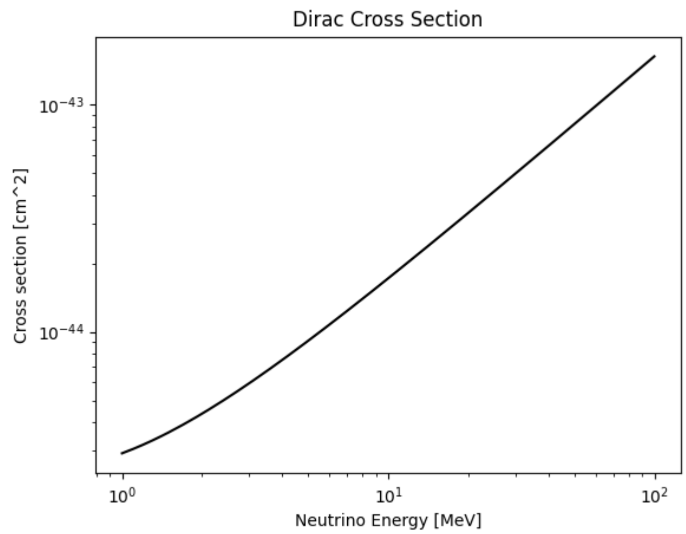

Figure 8: Neutral Current Electron- Dirac Neutrino Scattering Cross Section

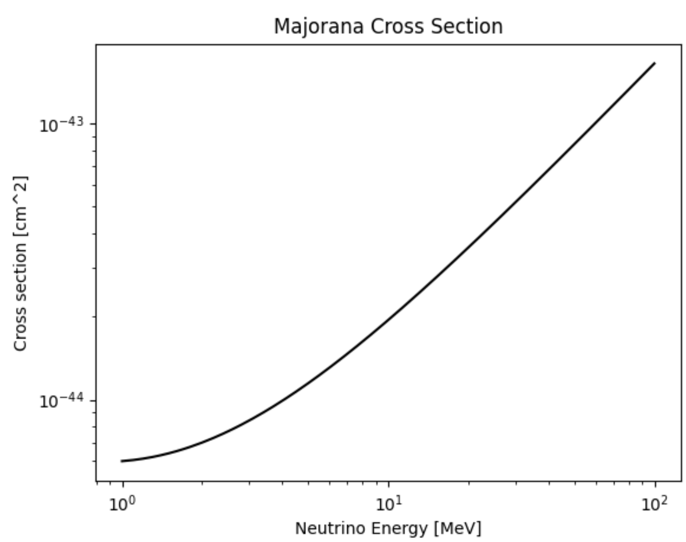

Figure 9: Neutral Current Electron- Majorana Neutrino Scattering Cross Section

The overall cross sections are as expected proportional to $G_F^2 s$, with s denoting the usual Mandelstam variable (Appendix E). The difference between the two cross sections is however suppressed by some power of m_ν/s and hence it's extremely difficult to actually detect any differences. Changing to the LAB-frame with the electron at rest, we obtain plots of the cross sections as a function of the incoming neutrino energy. The cross sections for the Dirac and Majorana case can be found in Figure 8 and Figure 9. As expected, the two plots looks identical. The effect of the additional terms in the Majorana cross section are minimal. The difference between the two cross sections can be quantified by the following:

$$\sigma_{diff} = |\sigma_M - \sigma_D| \tag{6.51}$$

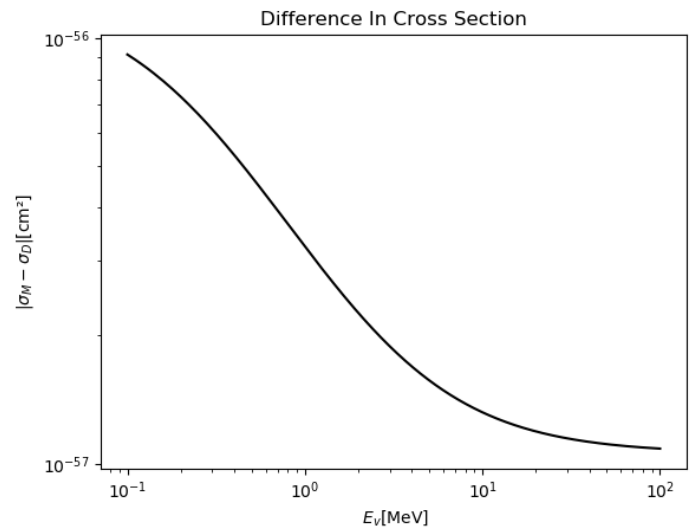

Figure 10: Difference between Majorana and Dirac cross sections

6.3 Neutral Current - Polarized Case

The previous discussion made the assumption that the incoming Majorana neutrino were equally probable of being in a positive and negative helicity state. It is also possible to consider Majorana neutrinos in a specific helicity state. For Majorana neutrinos, both the Feynman diagram in Figure 7 and the corresponding diagram with the initial neutrino replaced by an antineutrino contributes. However, the former will be highly suppressed due to helicity conditions. By applying the helicity projector in the amplitude (6.35) one can compute the polarized cross section with the replacement [10]

$$u(p_1) \rightarrow \frac{1 - \gamma^5 \not{s}}{2} u(p_1). \tag{6.52}$$

One may then compute the polarized amplitudes for neutrino-electron scattering as:

$$\left|\mathcal{M}^D(\nu, h)\right| = \frac{G_F^2}{4} A^{\alpha\beta} C_{\alpha\beta}(h), \tag{6.53}$$

where we have applied the spin sum and averaged over only the spin of the incoming electron. The electron part $A^{\alpha\beta}$ is the same as before, while the neutrino part $C_{\alpha\beta}(h)$, which now depends on the helicity of the incoming neutrino, is

$$C_{\alpha\beta}(h) = \frac{1}{4}\,\mathrm{Tr}\big\{\gamma_\alpha(1-\gamma_5)(1+\gamma_5\not{s})(\not{p}_1 + m_1)(1+\gamma_5\not{s})\gamma_\beta(1-\gamma_5)\big\}(\not{p}_3 + m_3). \tag{6.54}$$

After a substantial amount of algebra it can be shown that the neutrino part can be re-written in the convenient from

$$C_{\alpha\beta}(h) = \frac{1}{4}\,\mathrm{Tr}\big\{\gamma_\alpha(1-\gamma_5)(\not{p}_1 - \not{s}m_1)\gamma_\beta(1-\gamma_5)\big\}(\not{p}_3 - \not{s}m_3). \tag{6.55}$$

The advantage being that we can now define two new four vectors

$$L^\mu = p_1^\mu - s^\mu m_1, \qquad K^\mu = p_3^\mu - s^\mu m_3 \tag{6.56}$$

so that we can use directly our results from the Dirac amplitude squared in equation (6.41). Upon doing this we find

$$\begin{aligned}\left|\mathcal{M}^D(\nu, h)\right|^2 = 4G_F^2((g_V + g_A)^2(L\cdot p_2)(K\cdot p_4) + (g_V - g_A)^2(L\cdot p_4)(K\cdot p_2)\\ + m_e^2(g_A^2 - g_V^2)(L\cdot K)),\end{aligned} \tag{6.57}$$

In a similar way we can compute the polarized amplitude squared for Dirac antineutrinos as

$$\left|\mathcal{M}^D(\overline{\nu}, h)\right| = \frac{G_F^2}{4} A^{\alpha\beta} E_{\alpha\beta}(h), \tag{6.58}$$

where $A^{\alpha\beta}$ is as before and the neutrino part $E_{\alpha\beta}(h)$ is

$$E_{\alpha\beta}(h) = \frac{1}{4}\,\mathrm{Tr}\big\{(1+\gamma_5\not{s})\gamma_\alpha(1-\gamma_5)(\not{p}_3 - m3)\gamma_\beta(1-\gamma_5)(1+\gamma_5\not{s})(\not{p}_1 - m_1)\big\}. \tag{6.59}$$

Analogously to the Dirac neutrino case we can rewrite this as

$$E_{\alpha\beta}(h) = \frac{1}{4}\,\mathrm{Tr}\big\{\gamma_\alpha(1-\gamma_5)(\not{p}_3 + \not{s}m3)\gamma_\beta(1-\gamma_5)(\not{p}_1 + \not{s}m_1)\big\}. \tag{6.60}$$

Defining the four-vectors

$$N^\mu = p_1^\mu + s^\mu m_1, \qquad M^\mu = p_3^\mu + s^\mu m_3, \tag{6.61}$$

and preceding along the same lines as in the Dirac neutrino case, we find

$$\begin{aligned}\left|\mathcal{M}^D(\overline{\nu}, h)\right|^2 = 4G_F^2((g_V + g_A)^2(p_2\cdot M)(N\cdot p_4) + (g_V - g_A)^2(M\cdot p_2)(M\cdot p_4)\\ + m_e^2(g_A^2 - g_V^2)(N\cdot M).\end{aligned} \tag{6.62}$$

We have seen several times that negative helicity Majorana neutrinos behave up to order $\frac{m_\nu}{E_\nu}$ exactly the same as Dirac neutrinos. In order to confirm this, we compute the polarized amplitude squared for negative helicity Majorana neutrinos and compare to negative helicity Dirac neutrinos. To this extent, it's instructive to look at the four-vectors L and N in component form. The definition of the spin polarization four-vector s can be found in Appendix C.

$$L^\mu = \begin{pmatrix} E_1 \\ \vec{p_1} \end{pmatrix} - h \begin{pmatrix} |\vec{p_1}| \\ \frac{E_1 \vec{p_1}}{|\vec{p_1}|} \end{pmatrix} = (E_1 - h|\vec{p_1}|) \begin{pmatrix} 1 \\ \frac{-h\vec{p_1}}{|\vec{p_1}|} \end{pmatrix} \tag{6.63}$$

$$N^\mu = \begin{pmatrix} E_1 \\ \vec{p_1} \end{pmatrix} + h \begin{pmatrix} |\vec{p_1}| \\ \frac{E_1 \vec{p_1}}{|\vec{p_1}|} \end{pmatrix} = (E_1 + h|\vec{p_1}|) \begin{pmatrix} 1 \\ \frac{h\vec{p_1}}{|\vec{p_1}|} \end{pmatrix}. \tag{6.64}$$

Since neutrinos are ultra-relativistic, we can make the following approximations

$$|\vec{p_1}| \approx E_1 - \frac{m_1^2}{2E_1}, \qquad \frac{E_1}{|\vec{p_1}|} \approx 1 + \frac{m_1^2}{2E_1^2}. \tag{6.65}$$

Using these approximations we find that

$$L(h=-1) \approx 2 \begin{pmatrix} E_1 \\ \vec{p_1}, \end{pmatrix}, \quad L(h=+1) \approx \frac{m_1^2}{2E_1^2} \begin{pmatrix} E_1 \\ -\vec{p_1} \end{pmatrix} \tag{6.66}$$

and

$$N(h=-1) \approx \frac{m_1^2}{2E_1^2} \begin{pmatrix} E_1 \\ -\vec{p_1}. \end{pmatrix}, \quad N(h=+1) \approx 2 \begin{pmatrix} E_1 \\ \vec{p_1}, \end{pmatrix}. \tag{6.67}$$

Using this, the polarized amplitude squared for negative helicity Dirac neutrinos becomes

$$\begin{aligned} \left|\mathcal{M}^D(\nu, h=-1)\right|^2 &= 16G_F^2((g_V+g_A)^2(p_1\cdot p_2)(p_3\cdot p_4) \\ &\quad + (g_V-g_A)^2(p_1\cdot p_4)(p_3\cdot p_2) + m_e^2(g_A^2-g_V^2)(p_1\cdot p_3)). \end{aligned} \tag{6.68}$$

For Majorana neutrinos, the above amplitude contributes, but there is also a second contribution corresponding to the antineutrino amplitude. That is,

$$\left|\mathcal{M}^M(h=-1)\right|^2 = \left|\mathcal{M}^D(\nu, h=-1)\right| + \left|\mathcal{M}^D(\overline{\nu}, h=-1)\right| \tag{6.69}$$

From equation (6.62) and (6.67), the last term in the above will be highly suppressed due to helicity conditions. In fact, as can be seen from equations (6.67) and (6.62), it will be suppressed by a factor $\frac{m_\nu^4}{4E_\nu^4}$. We can thus conclude that

$$\left|\mathcal{M}^M(h=-1)\right|^2 = \left|\mathcal{M}^D(\nu, h=-1)\right| + \frac{m_\nu^4}{4E_\nu^4}\left|\mathcal{M}^D(\overline{\nu})\right|. \tag{6.70}$$

Considering that weak interactions normally have cross sections at around $\mathcal{O}(10^{-44}cm^2)$ and the ratio $\frac{m_\nu^4}{4E_\nu^4}$ is approximately of order $\mathcal{O}(10^{-24})$, the additional term due to the Majorana nature of the neutrino will be extremely hard to detect. A completely analogous calculation holds for positive helicity Majorana neutrinos and Dirac antineutrinos.

6.4 Discussion

It seems to be a well known result that the difference between Majorana and Dirac cross sections are proportional to the ratio $\frac{m_\nu}{E_\nu}$ Despite of this, explicit calculations are hard to come by. We have provided these calculations for both neutral and charged currents, and commented on how to deal with the polarization of the neutrinos. In particular we have seen that if one takes the unpolarized cross section, the neutrino helicity is not measured and hence there is an equal contribution from both helicity states. Since helicity is the only property distinguishing Majorana neutrinos from Majorana antineutrinos we thus need to compare the unpolarized cross section for Majorana neutrinos with the the sum corresponding processes for Dirac neutrinos and antineutrinos. On the other hand, if the helicity is measured, a left helical Majorana neutrino is compared to a Dirac neutrino. Similarly a right helical Majorana neutrino is compared to a Dirac antineutrino in accordance with the discussion in Section 3.2.3. In all instances, we find that the difference between the Majorana and Dirac cross sections are proportional to $\frac{m_\nu}{E_\nu}$, and in the massless limit, the difference vanishes. In general, we find the appropriate comparison from the table below. The top

	LH	RH	Unpolarized
Majorana	LH Dirac Neutrino	RH Dirac Antineutrino	Dirac Neutrino + Dirac Antineutrino
Dirac Neutrino	LH Majorana		LH Majorana
Dirac Antineutrino		RH Majorana	RH Majorana

Table 2: Schematic overview for comparison of polarized and unpolarized Majorana and Dirac (anti)neutrinos

row denotes the helicity and the left column indicates the type of particle one considers. The calculations in Section 6.1 showed that a Dirac antineutrinos behaves approximately as a positive helicity Majorana neutrino. In Section 6.2 we were in the unpolarized Majorana case and thus considered both Dirac neutrinos and antineutrinos, in agreement with the table. In Section 6.3 we saw that left negative helicity Majorana neutrino behave as left handed Dirac neutrinos, also in agreement with the table. In all cases, given a particle and polarization, the table gives the correct combination in order to have the only differences in their cross sections proportional to the neutrino mass. The two blank entries also offers interesting information. Considering for instance a left helical Dirac antineutrino scattering with a nucleus. It will only produce a positively charged lepton due to lepton number conservation and the rate at which it does will be highly suppressed due to the helicity. A left helical Majorana neutrino will however also produce negatively charged leptons with no suppression. Thus if one was able to obtain a left helical Dirac antineutrino, there would be no problem distinguishing between Dirac and Majorana. Unfortunately, flipping the helicity of a neutrino is not a straightforward process. The amplitude for a helicity flip through the weak interaction is proportional to the well known ratio m_ν/E_ν [28] The other possibilities, overtaking the neutrino and flipping it with the help of a strong magnetic field is considered in for instance [29] In any case, coming across a left helical antineutrino is extremely difficult.

7 Lepton Number Violating Processes

The calculations in the previous sections shows that distinguishing between Dirac and Majorana neutrinos through standard SM interactions is extremely difficult. Unless one is able to find some way of finding non-relativistic neutrinos, the sensitivity of current experiments is not able to measure the small difference in the cross sections. For this reasons one must for other processes outside of the SM. In the following section we take a closer look at lepton number violating processes. Any observation of such a process would indeed imply that neutrinos are of Majorana type.

7.1 Neutrino-less Double β-Decay

One of the most promising processes for deciding whether neutrinos are Dirac or Majorana particles is the process of neutrino-less double-beta decay. They are processes of the type

$$\mathcal{N}(A,Z) \to \mathcal{N}(A,Z\pm 2) + 2e^{\mp}, \tag{7.1}$$

which we will denote as $2\beta_{0\nu}^{\mp}$. They are of course not allowed in the SM since they violate total lepton number by $\Delta L = \pm 2$. However as we have seen in previous sections Majorana neutrinos violate total lepton number by the same amount. Lets us start of by considering the process of double beta decay

$$(A,Z) \to (A,Z+2) + 2e^{-} + 2\overline{\nu_e}. \tag{7.2}$$

The process is rare, but know to happen for several different nuclei [30]. It's generated at second order in the perturbative expansion of the weak interaction. The Feynman diagram for the decay is drawn in Figure 11

In other words it's two simultaneous instances of the reverse process considered in Section 6.1. The $2\beta_{0\nu}^{\mp}$ process is similar. The starting point is the same, but the final state does not contain any antineutrinos. How can we achieve this? Consider the diagram in Figure 12 where we join the antineutrino lines so that they form a virtual neutrino line. The above process is not possible in the SM for two reasons. Firstly, the lower leptonic vertex is only capable of absorbing a fermion, not an anti-fermion. Secondly, the lower vertex can only absorb a particle with negative helicity, while the emitted antineutrino has definite positive helicity in the SM. Hence for the process to take place, we require two things [8]:

1. The equality of particle and antiparticle i.e $\overline{\nu_e} = \nu_e$ so that it is indeed possible for the lower vertex to absorb the emitted (anti)neutrino.

2. $m_{\nu_e} \neq 0$ in which case the (anti)neutrino emitted from the upper vertex has the possibility of having negative helicity with relative amplitude $\frac{m_{\nu_e}}{E_{\nu_e}}$.

And so it's clear that the process is possible in the case that the neutrino is a massive Majorana particle. Introducing neutrino mixing to the process,

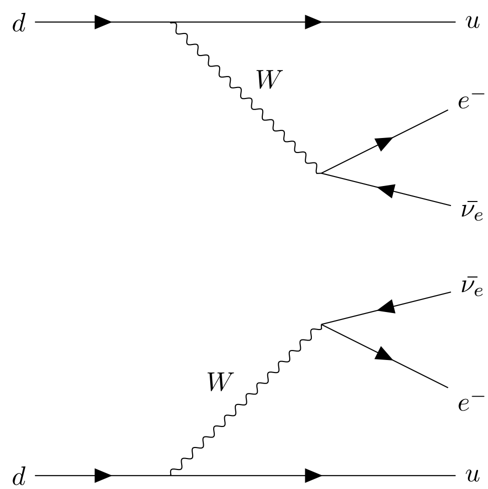

Figure 11: Feynman diagram for double beta decay

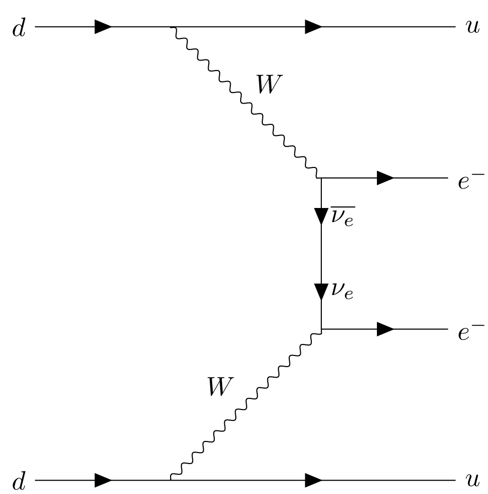

Figure 12: Feynman diagram neutrino-less double beta decay

each leptonic vertex in the Feynman diagram above is described by the charged leptonic current in equation (4.2) and it's hermitian conjugate. In order to have the two electrons in the final state it's in fact the hermitian conjugate we look at:

$$j_W^{\rho\,\dagger} = 2\sum_\alpha\sum_k U_{\alpha k}\bar{l}_{\alpha L}\gamma^\rho\nu_{kL}. \tag{7.3}$$

There are two vertices so this leads to a factor U_{ek}^2 and a factor m_k due to the helicity condition for the upper vertex antineutrino. Since we are considering mixing, we also have to sum over the massive neutrinos which then gives us the effective Majorana mass

$$m_{2\beta} = \sum_{k=1}^{3} U_{ek}^2 m_k, \tag{7.4}$$

and we claim that the amplitude of the process is proportional to the effective Majorana mass. To show this we note that the amplitude for the process will be proportional to the Majorana neutrino propagator. By definition the Feynman propagator [8] is given by

$$M(x_1 - x_2) = \langle 0|T[\nu_{eL}(x_1)\nu_{eL}(x_2)]|0\rangle\,, \tag{7.5}$$

where T denotes the time ordering. Writing everything out in terms of mixing and projection operators we have

$$M(x_1 - x_2) = \frac{1-\gamma_5}{2}\sum_{k=1}^{3} U_{ek}^2 \langle 0|T[\nu_k(x_1)\frac{1-\gamma_5}{2}\nu_k(x_2)]|0\rangle\,. \tag{7.6}$$

Noting that we can safely transpose the last part of the above

$$\frac{1-\gamma_5}{2}\nu_k(x_2) = \left(\frac{1-\gamma_5}{2}\nu_k(x_2)\right)^T \tag{7.7}$$

we employ the Majorana condition

$$\nu_k = C\overline{\nu_k}^T \implies \nu_k^T = \overline{\nu_k}C^T, \tag{7.8}$$

so that our propagator can be written in more convenient form

$$M(x_1 - x_2) = -\frac{1-\gamma_5}{2}\sum_{k=1}^{3} U_{ek}^2 \langle 0|T[\nu_k(x_1)\overline{\nu_k}(x_2)]|0\rangle\, C\frac{1-\gamma_5}{2}. \tag{7.9}$$

Here we have also used the property of the charge conjugation matrix (10.32). The advantage of this expression is that the quantity within the bracket is well-know. It's just the usual Feynman propagator [8]

$$i\int \frac{d^4p}{(2\pi)^4}\frac{\not{p} + m_k}{p^2 + m_k^2}e^-ip\cdot(x_1 - x_2). \tag{7.10}$$

Employing this in equation (7.9) one finds the Majorana neutrino propagator to be

$$M(x_1 - x_2) = -\frac{1-\gamma_5}{2}\sum_{k=1}^{3} U_{ek}^2 i \int \frac{d^4p}{(2\pi)^4}\frac{\not{p} + m_k}{p^2 + m_k^2} e^{-ip\cdot(x_1-x_2)}\frac{1-\gamma_5}{2}C. \quad (7.11)$$

This can be simplified even further noting that the $\not{p}$ terms cancel. Additionally, since the mass term in the denominator is negligible in comparison to the momentum squared, the Majorana propagator can be approximated as

$$M(x_1 - x_2) = -im_{2\beta}\int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x_1-x_2)}}{p^2 + m_k^2}\frac{1-\gamma_5}{2}C. \quad (7.12)$$

Thus the relationship between the effective Majorana mass and the Majorana neutrino propagator is made clear. Since the neutrino-less double beta decay process is proportional to the Majorana propagator the claim is established through transitivity. An appropriate parametrization for the effective Majorana mass can be given in terms of the the elements of the mixing matrix introduced in Section 3.3:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} diag\left(e^{i\lambda_1}, e^{i\lambda_2}, e^{i\lambda_3}\right) \quad (7.13)$$

with $\lambda_1 = 0$ instead of $\lambda_3 = 0$ which is just convention i.e the choices are physically equivalent. In this manner we may rewrite the effective Majorana mass as:

$$m_{2\beta} = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{2i\lambda_2} m_2 + s_{13}^2 e^{2i(\lambda_3 - \delta_{13})} m_3. \quad (7.14)$$

The amplitude of the process is thus proportional to the neutrino masses and depends upon both the mixing angles and the two unknown Majorana phases λ_2 and λ_3.

We have seen that double-beta decay does indeed take place through the standard SM interactions with amplitude proportional to the effective Majorana mass $m_{2\beta}$. But there are of course new particles and interactions in beyond SM (BSM) physics that could also be responsible for the process [9]. For instance, the helicity mismatching could be overcome by some new V+A interaction. The question is then if the observation of neutrino-less double beta decay would actually give us any information on the Majorana nature of the neutrino. Consider the following "black box" diagram

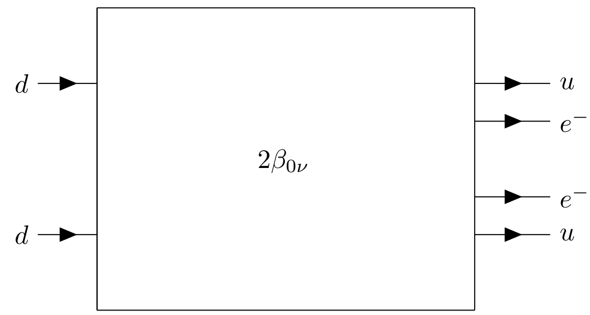

Figure 13: Black Box Diagram

The box contains all possible mechanisms responsible for $2\beta_{0\nu}$ decay at all orders of perturbation theory. This includes the SM interaction we have seen, but also all potential BSM interactions. Considering only standard weak SM interactions, we can join the outgoing fermions to the neutrino-lines in order to obtain the Feynman diagram in Figure 14 [31]. This gives us a diagram for $\overline{\nu}_e \to \nu_e$ transitions and hence a contribution to the Majorana mass at some level of perturbation theory. In effect this guarantees that the observation of $2\beta_\nu$ would mean that the neutrino is a Majorana particle, no matter which mechanism responsible for the process. On the other hand, there is no way of telling which mechanism that is dominating the decay and so the Majorana mass contribution resulting from the SM interaction might be very small. In this case, translating the observed decay rates into the neutrino masses would be misleading. Similarly, if we were to find out that the Majorana contribution to the neutrino mass is large, then we would know that the SM interaction is the leading mechanism for $2\beta_{0\nu}$ decay [9].

Of course the values of the Majorana phases and the masses are not known. Regardless of this it is possible to calculate upper bounds on the decay rates. Expanding the formula in equation (7.14) in terms of the elements of the mixing matrix, we have:

$$|m_{2\beta}| = \left|c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13} m_2 e^{i\lambda_2} + s_{13}^2 m_3 e^{i\lambda_3}\right|, \tag{7.15}$$

with a slight abuse of notation

$$\lambda_2 = 2\lambda_2, \qquad \lambda_3 = 2(\lambda_3 - \delta_{13}). \tag{7.16}$$

The effective Majorana mass can be expanded further using the definition of the modulus and several trigonometric identities

$$\begin{aligned} |m_{2\beta}| =& (((c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13} m_2 \cos\lambda_2 + s_{13}^2 m_3 \cos\lambda_3)^2 \\ &+ (s_{12}^2 c_{13}^2 m_2 \sin\lambda_1 + s_{13}^2 m_3 \sin\lambda_3)^2)^{\frac{1}{2}}. \end{aligned} \tag{7.17}$$

The masses can all be expressed in terms of the lightest mass eigenstate, which in the Normal Ordering regime takes the following form

$$m_0 \equiv m_1, \qquad m_2 = \sqrt{m_0^2 + \Delta m_{21}^2}, \qquad m_3 = \sqrt{m_0^2 + \Delta m_{21}^2 + \Delta m_{32}^2}. \tag{7.18}$$

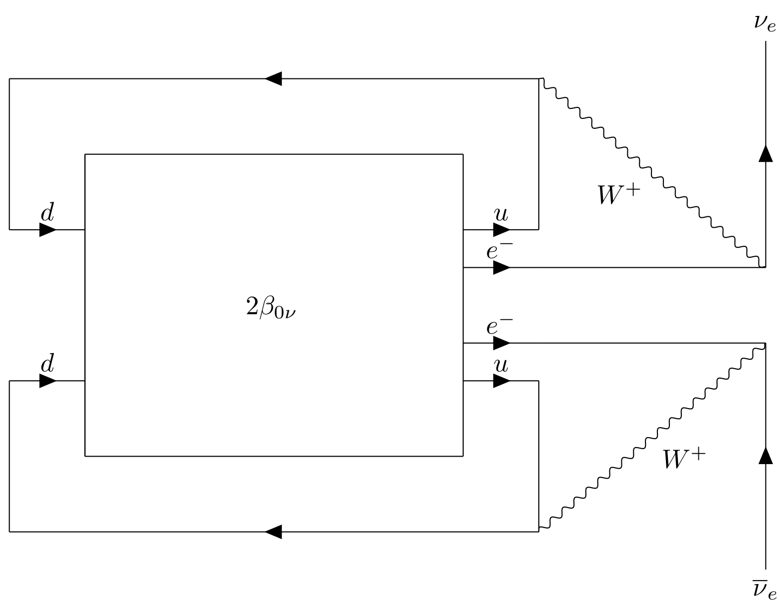

Figure 14: Black Box and SM Interaction

The possible values of the effective Majorana mass are plotted in Figure 15 as a function of the lightest neutrino mass. The values used for the mixing parameters are the ones given in Table 1. For a summary on which values for the effective Majorana mass has been ruled out by experiment see [32]. Similarly, we can calculate the possible spectrum for the inverted hierarchy, in which the mass squared differences are

$$m_3 \equiv m_0, \qquad m_2 = \sqrt{m_0^2 - \Delta m_{32}^2}, \qquad m_1 = \sqrt{m_0^2 - \Delta m_{21}^2 - \Delta m_{32}^2}. \quad (7.19)$$

The corresponding values of the effective Majorana mass can be found in Figure 16

7.2 Neutrino-Antineutrino Oscillations

In this section we derive the probability for neutrino anti-neutrino oscillations. Assuming that neutrinos are Majorana particles, lepton number is violated and oscillations on the form $\nu_\alpha \to \overline{\nu}_\beta$ are possible. We will see that such oscillations are highly suppressed and that they depend on the undetermined Majorana phases. A complete formula for the case of 2 generations is given and the relationship between the effective Majorana mass $m_{2\beta}$ in neutrino-less beta decay and the general effective Majorana mass in neutrino-antineutrino oscillations is given.

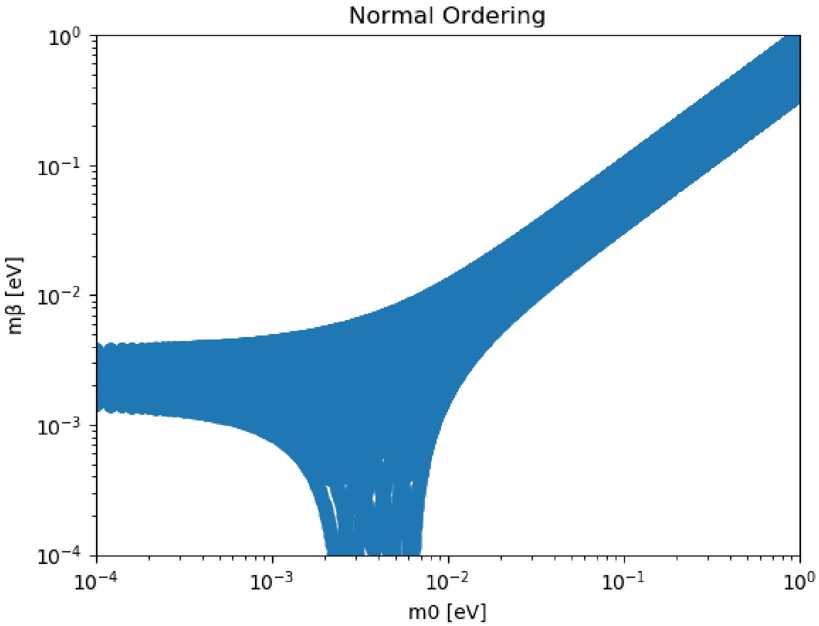

Figure 15: Effective Majorana Mass as a function of the lightest neutrino mass

We follow the derivation given in [33] with some slight change in notation. Consider a theory with n generations of neutrinos denoted by α and with no neutrino singlets. We also assume n neutrino mass eigenstates denoted by k. In the following we will always work in a representation where γ_5 is diagonal. The charged leptonic weak interaction was given in Section 4 as

$$\mathcal{L}_{I,L}^{(CC)} = -\frac{g}{2\sqrt{2}}\left(j^{\rho}_{W,L}W_{\rho} + h.c\right) = \frac{-g}{\sqrt{2}}\sum_{\alpha,=1}^{n}\overline{l_{\alpha L}}\gamma^{\rho}U_{\alpha k}\nu_{kL} + h.c. \tag{7.20}$$

To calculate the oscillation probability we consider the following lepton number violating process:

1. $l^{+}_{\alpha} + n \rightarrow \nu_{\alpha} + p$
2. The produced neutrino travels time t
3. $\nu_{\alpha} + n \rightarrow l^{-}_{\beta} + p.$

The amplitude for the entire process will then be the product of the three amplitudes. Using the interaction Lagrangian above and the Majorana Fourier expansion we can derive the amplitudes. For process 1 we have

$$U_{\alpha k}S^{\rho}\overline{v}\gamma^{\rho}\frac{1+\gamma_5}{2}v^{r}(\vec{p}), \tag{7.21}$$

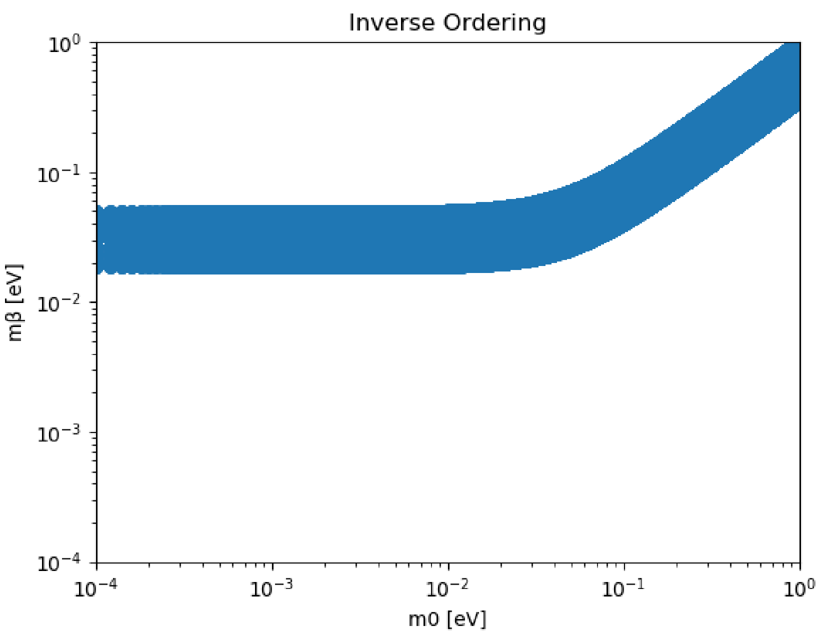

Figure 16: Effective Majorana Mass as a function of the lightest neutrino mass

with S^ρ denoting the kinematic factors involved. The initial positively charged lepton is annihilated by the current in equation (7.20) giving the the spinor u. At the same time, what we would call an antineutrino is created, giving the spinor $v^r(\vec{p}$ according to equation (3.46). The neutrino mass eigenstate then travels for time t, giving the amplitude $e^{-iE_k t}$. For the third process, the annihilation of the (antineutrino) is now done by the same field that created it thus giving the spinor $u^r(\vec{p})$ according to equation (7.20). The final state negatively charged lepton is created by the same field as before, but now inducing a $\overline{u}$ spinor

$$U_{\beta k} T^\rho \overline{u} \gamma^\rho \frac{1+\gamma_5}{2} u^r(\vec{p}), \tag{7.22}$$

with T^ρ denoting another kinematic factor. Combining the three amplitudes we must sum over the spin label r and we encounter the term

$$\sum_r \frac{1+\gamma_5}{2} v^r(\vec{p}) \frac{1+\gamma_5}{2} u^r(\vec{p}) \tag{7.23}$$

Utilizing the Majorana condition $u^r(\vec{p}) = C\overline{v}^T$ and that we work with a diagonal representation of γ_5 we can safely transpose the last part of the equation above to give

$$\sum_r \frac{1+\gamma_5}{2} v^r(\vec{p}) \left(\frac{1+\gamma_5}{2} u^r(\vec{p}) \right)^T = \frac{1+\gamma_5}{2} \sum_r v^r(\vec{p}) \overline{v}^r(\vec{p}) C^T \frac{1+\gamma_5^T}{2}. \tag{7.24}$$

Multiplying everything out and utilizing equations (10.32) and (10.33) we find that all terms involving $\not{p}$ cancel and we are left with

$$m_k \frac{1+\gamma_5}{2} C. \tag{7.25}$$

Combining the three amplitudes we also have to take the sum over the intermediate mass states and divide by E in order for the amplitude to be dimensionless

$$\frac{1}{E} \sum_{\alpha=1}^{n} U_{a\alpha} S^\rho U_{b\alpha} T^\rho e^{-iE_\alpha t} \overline{v} \gamma^\rho \overline{u} \gamma^\rho m_\alpha \frac{1+\gamma_5}{2} C. \tag{7.26}$$

Assuming that the neutrino mass is small enough so that there are no kinematic effects, we factor out the terms depending on k to give

$$A_{\overline{\alpha}\beta}(t) = \frac{1}{E} \sum_{k=1}^{n} U_{\alpha k} U_{\beta k} e^{-iE_k t} m_k, \tag{7.27}$$

with corresponding oscillation probability given by the amplitude squared

$$P(\overline{\nu}_\alpha \to \nu_\beta) = \frac{1}{E^2} \left| \sum_{k=1}^{n} U_{\alpha k} U_{\beta k} e^{-iE_k t} m_k \right|^2. \tag{7.28}$$

We note the suppression of the probability by a factor $\frac{m^2}{E^2}$ which is not present for normal neutrino oscillations (4.11) and that in the limit $m \to 0$ the probability vanishes as expected. In fact, this shows that in the massless limit, such a lepton violating interaction is not possible and thus lepton number is conserved just as in the SM. it's also stressed that this is in fact the antineutrino-neutrino oscillation probability. The corresponding neutrino-antineutrino probability is derived in the same manner, the only difference being that the elements of the mixing matrix appears in the complex conjugated form

$$P(\nu_\alpha \to \overline{\nu}_\beta) = \frac{1}{E^2}\left|\sum_{k=1}^{n} U^\star_{\alpha k} U^\star_{\beta k} e^{-iE_k t} m_k\right|^2 . \tag{7.29}$$

In general, both probabilities (7.28) and (7.29) are exceedingly small owing to the suppression m_ν/E. Additionally, these probabilities will, unlike the flavour oscillation case, also depend on the unknown Majorana phases. To see this, it suffices to look at the case with $n = 2$ generations. Writing out the equation above we have:

$$P(\overline{\nu}_\alpha \to \nu_\beta) = \frac{1}{E^2}\sum_{k,j}^{2} U_{\alpha k} U_{\beta k} U^\star_{\alpha j} U^\star_{\beta j} e^{-i(E_k - E_j)t} m_k m_j . \tag{7.30}$$

For ultra-relativistic neutrinos we can take $E_k - E_j \approx \frac{\Delta m^2_{kj}}{2E}$ and $t \approx L$. The mixing matrix in the case of two generations is [9]

$$U = \begin{pmatrix} \cos\theta & e^{i\theta_{12}}\sin\theta \\ -e^{-i\theta_{12}}\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} c_{12} & e^{i\theta_{12}} s_{12} \\ -e^{-i\theta_{12}} s_{12} & c_{12} \end{pmatrix} . \tag{7.31}$$

As discussed before, for lepton number conserving processes, the Majorana phases θ_{12} vanishes and so processes like normal neutrino oscillations cannot be used to distinguish between Majorana and Dirac neutrinos. Using the parametrization of the mixing matrix above, we obtain an explicit form of the antineutrino-neutrino oscillation probability

$$\begin{aligned} P(\overline{\nu}_\alpha \to \nu_\beta) = \frac{1}{E^2}((m_1^2 + m_2^2)c_{12}^2 s_{12}^2 \\ - 2c_{12}^2 s_{12}^2 m_1 m_2 \cos(\frac{\Delta m^2_{21} L}{2E} - 2\theta_{12})) \end{aligned} \tag{7.32}$$

where the presence of the Majorana phases θ_{12} is evident and so is the suppression $\frac{m^2}{E^2}$. A similar calculation for Majorana (or Dirac) antineutrino flavour oscillations with $n = 2$ generations gives the probability

$$P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = 2c_{12}^2 s_{12}^2 \left(1 - 2\cos(\frac{\Delta m^2_{21} L}{2E})\right), \tag{7.33}$$

with no dependence on the Majorana phases and no suppression $\frac{m^2}{E^2}$. If it turns out that neutrinos are indeed Majorana particles, the question of the values of the Majorana phases is inevitable. To probe these phases it is then necessary to look at lepton number violating processes such as the one we have just described. The key point in the derivation is that for Majorana neutrinos the same field that creates neutrinos may also be used to annihilate them. For a Dirac neutrino the process described above would not be possible since there is no way of annihilating the antineutrino and at the same time create a negatively charged lepton or in other words: Lepton number is conserved.

Similarly to the neutrino-less double beta decay, one can define a ”effective neutrino mass” in the case of neutrino-antineutrino oscillations and it’s CPT conjugate. Firstly, we can use the same approximation as we did for flavour oscillations $E_k - E_j \approx \frac{\Delta m_{kj}^2}{2E}$ and $t \approx L$ in order to obtain

$$A(\overline{\nu}_\alpha \to \nu_\beta) = \frac{1}{E} \sum_k U_{\alpha k} U_{\beta k} m_k exp(-i\frac{m_k^2}{2E}L). \tag{7.34}$$

The effective Majorana mass is then defined to be

$$m_{\alpha\beta}^L = \left| \sum_k U_{\alpha k} U_{\beta k} m_k exp(-i\frac{m_k^2}{2E}L) \right| \tag{7.35}$$

and for the conjugate process we have

$$\overline{m}_{\alpha\beta}^L = \left| \sum_k U_{\alpha k}^\star U_{\beta k}^\star m_k exp(-i\frac{m_k^2}{2E}L) \right|. \tag{7.36}$$

In the case of neutrino-less double beta decay, we have $\alpha = \beta = e$ and L=0, in which case equation (7.33) reduces to

$$m_{ee}^L = \left| \sum_k U_{ek} U_{ek} m_k \right| = m_{2\beta}, \tag{7.37}$$

which is just the effective Majorana mass in the case of neutrino-less double beta decay given in equation (7.4). Thus, the connection between the neutrino-less double beta decay and neutrino-antineutrino oscillations is made clear once again.

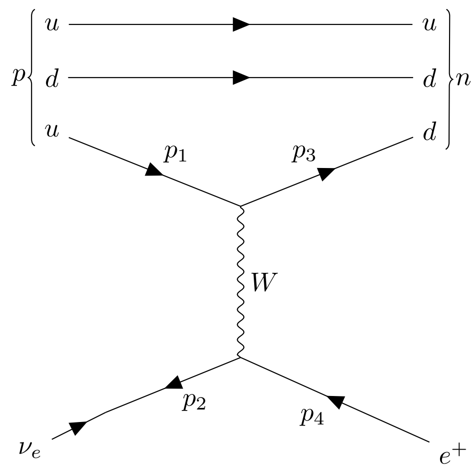

Figure 17: Lepton number violating Feynman Diagram with neutrino-antineutrino oscillations

8 Application Of Neutrino-antineutrino Oscillations

Having seen the formalism for neutrino-antineutrino oscillations one might notice the resemblance between the additional Terms in the Majorana amplitude and the formula for neutrino-antineutrino oscillations. They both involve exchanging particle spinors $u(p)$ to anti-particle spinors $v(p)$ and they both come with a suppression m_ν^2/E_ν^2. In this section we investigate the link between the two and show that they give equal results. In order to do this we once again consider the process of IVB for Majorana neutrinos. In Section 6.1 we saw that there were two Feynman diagrams that contributed to the process. The first one, given in Figure 2, coincides with the Dirac case. The second one, given in Figure 3 arise as a result of the particle-antiparticle duality of Majorana neutrinos. In this section, we instead consider the process where the incoming Majorana neutrino in Figure 3 first undergo a neutrino-antineutrino oscillation and is then annihilated by the "right" field $\overline{\nu}_L$ as in Figure 2. The situation is illustrated in Figure 17, where the clashing arrows indicate a neutrino-antineutrino oscillation.

The amplitude for the Feynman diagram in Figure 17 will be the product of the two amplitudes for the oscillation and decay process:

$$\mathcal{M} = \mathcal{A}_{osc}\mathcal{M}^D, \tag{8.1}$$

where $\mathcal{A}_{osc}$ is given by equation (7.27) (upon conjugating the mixing matrices) and $\mathcal{M}^D$ is defined in equation (6.2). Using the known expressions, the amplitude can be written as

$$\begin{aligned}\mathcal{M} = &\frac{1}{E}\sum_{k=1}^{3} U_{ek}^{\star}U_{ek}^{\star}e^{-iE_k t}m_k \\ &\times \frac{-iG_F V_{ud}}{\sqrt{2}}[\overline{v}(p_2)\gamma^{\alpha}(1-\gamma^5)v(p_4)][\overline{u}(p_3)\gamma_{\alpha}(1-\gamma^5)u(p_1)].\end{aligned} \tag{8.2}$$

The corresponding amplitude squared is readily obtained from equations (6.11) and (7.29) as

$$|\mathcal{M}|^2 = \frac{32G_F^2|V_{ud}|^2}{E_\nu^2}(p_4\cdot p_1)(p_2\cdot p_3)\sum_{k,j=1}^{3} U_{ek}^{\star}U_{ek}^{\star}U_{ek}U_{ek}e^{-i(E_k-E_j)t}m_k m_j. \tag{8.3}$$

If we ignore mixing consider no propagation time i.e $t=0$ this reduce to

$$|\mathcal{M}|^2 = 32G_F^2|V_{ud}|^2\frac{m_\nu^2}{E_\nu^2}(p_4\cdot p_1)(p_2\cdot p_3), \tag{8.4}$$

in agreement with equation (6.23). If we instead wish to consider mixing the derivation of the amplitude in equation (6.21) requires a slight modification. The initial Majorana neutrino in equation (6.21) is created by the current $j^{\mu}_{W,L}$ with weights according to equation (4.2)

$$\mathcal{M}^M \supset U_{ek}^{\star}. \tag{8.5}$$

The neutrino then propagates time t giving a factor

$$\mathcal{M}^M \supset U_{ek}^{\star}e^{-iE_k t} \tag{8.6}$$

according to equation (4.6). It is then annihilated by the same field that created it giving another weight $U_{ek}^{\star}$. Finally, since the flavour neutrino is in a superposition of the mass eigenstates, this also induces a sum over k. Putting all of this together, the amplitude in equation (6.21) including neutrino mass effects become

$$\begin{aligned}\mathcal{M}^M = &\frac{-iG_F V_{ud}}{\sqrt{2}E_\nu}\sum_{k} U_{ek}^{\star}U_{ek}^{\star}e^{-iE_k t}m_k \\ &\times [\overline{v}(p_2)\gamma^{\alpha}(1-\gamma^5)v(p_4)][\overline{u}(p_3)\gamma_{\alpha}(1-\gamma^5)u(p_1)].\end{aligned} \tag{8.7}$$

Upon squaring the amplitude and using the results of equation (6.11) we find that

$$\left|\mathcal{M}^M\right|^2 = \frac{32G_F^2|V_{ud}|^2}{E_\nu^2}(p_4\cdot p_1)(p_2\cdot p_3)\sum_{k,j=1}^{3} U_{ek}^{\star}U_{ek}^{\star}U_{ek}U_{ek}e^{-i(E_k-E_j)t}m_k m_j, \tag{8.8}$$

in perfect agreement with equation (8.3). The two approaches, therefore, appear to be equivalent. This is perhaps no surprise considering that the derivation done in Section 6.1 is essentially a the same as done in Section 6.2. The main difference between the two being that we in Section 6.1 make some approximations regarding the helicity which is not needed in the Neutrino-Antineutrino oscillation derivation.

9 Conclusions

Throughout this thesis we have provided the necessary theoretical background for working with massive neutrinos. Starting from the Standard Model we have introduced several paths to neutrino mass. The usual Dirac mass, Majorana masses and a combined Dirac-Majorana mass are all considered. It is shown that mixed Majorana-Dirac mass term naturally gives rise to a seesaw model that explains the smallness of the neutrino mass in a simple way. The standard derivation of the neutrino oscillation probability has been given. It has been shown that such oscillations cannot distinguish between Majorana and Dirac neutrinos. Additionally we have considered antineutrino oscillations and several transformation properties. In particular we have looked at the possibilities of observing CP-violation in neutrino oscillations. We have discussed some of the experimental aspects of neutrino oscillations and indicate the role of future neutrino oscillation experiments.

Special emphasis has been put on the Majorana/Dirac distinction. It is explained how Majorana neutrinos give rise to several new processes owing to the Majorana properties of the field. Several cross sections has been calculated for various SM processes both for Majorana and Dirac neutrinos. Through these calculations we have confirmed the well know result that such processes differs only by terms proportional to m_ν^2/E_ν^2. As such we have made it clear that none of the SM processes considered can distinguish between Majorana and Dirac neutrinos. Additionally this also shows that in the massless limit, the Dirac and Majorana description of the neutrino are equivalent. In this case they both reduce to the SM Weyl neutrino.

Having seen the difficulties related to distinguishing Dirac/Majorana neutrinos through SM processes we introduce the process of neutrino-less double beta decay. It is shown that any nonzero observation of this process will indicate that the neutrino is a Majorana neutrino. We have provided plots of the possible values of the effective Majorana mass as a function of the lightest neutrino mass, both in the case of a Normal ordered and Inverse ordered regime. The neutrino-antineutrino propagator is computed and shown to be proportional to the Majorana mass. In a similar manner, we derive the neutrino-antineutrino oscillation probability in full detail. The general formula for these oscillations are obtained and it is shown how this relates to neutrino-less double beta decay.

Having calculated the cross section for inverse beta decay with Majorana (anti)neutrinos we have seen that it is possible to obtain the same result through applying to neutrino-antineutrino oscillations. Combing the results of Section 6.1 and Section 7, the inverse beta decay amplitude is calculated once again but this time using the formalism for neutrino-antineutrino oscillations. We conclude that they both give the same result: The IVB process for Majorana neutrinos is, up to order m_ν^2/E_ν^2 identical to the Dirac case.

For future work it would be interesting to look at other possibilities of distinguishing between Dirac and Majorana neutrinos. There are several other processes that violate lepton number, and in the case that neutrinoless double beta decay does not give a positive answer, one of these will be the new main candidate for giving a final answer to the Majorana/Dirac question. If the neutrino turns out to be a Majorana particle the question of the values of the Majorana phases is also completely open and would have to be investigated further.

10 Appendix

10.1 Appendix A - A Few Gamma Identities

The spin $\frac{1}{2}$ fermions can be described by a 4 component spinor $\psi(x)$ which satisfy the Dirac equation:

$$(i\partial\!\!\!/ - m)\psi(x) = 0, \tag{10.1}$$

where we employ the usual convention $\partial\!\!\!/ = \gamma^\mu \partial_\mu$. The γ^μ is a set of 4×4 matrices that satisfy the Clifford Algebra

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \tag{10.2}$$

together with the condition that

$$\gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu. \tag{10.3}$$

The γ matrices are invariant under Lorentz transformations and from equation (10.2) and (10.3) we can derive the following properties

$$\left(\gamma^0\right)^2 = \mathbb{1}, \qquad \left(\gamma^k\right)^2 = -\mathbb{1} \tag{10.4}$$

$$\left(\gamma^0\right)^\dagger = \gamma^0, \qquad \left(\gamma^k\right)^\dagger = -\gamma^k. \tag{10.5}$$

The fifth gamma matrix also know as the chirality matrix is defined through

$$\gamma^5 = \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{10.6}$$

Using the above equations it can be shown to have the following properties

$$\{\gamma^5, \gamma^\mu\} = 0, \qquad (\gamma^5)^2 = \mathbb{1}, \qquad (\gamma^5)^\dagger = \gamma^5. \tag{10.7}$$

The fifth gamma matrix has eigenvalues $1, -1$, and so we have the following equalities

$$\gamma^5\psi_R = +\psi_R \tag{10.8}$$

$$\gamma^5\psi_L = -\psi_L, \tag{10.9}$$

where the field ψ_R denotes the field which is an eigenfunction of γ^5 with eigenvalue $+1$. We call this field the right handed chiral field. Similarly ψ_L denotes the eigenfuntion with eigenvalue -1 and is called the left handed chiral field. Defining the chirality projection matrices

$$P_R = \frac{1+\gamma^5}{2}\psi, \qquad P_L = \frac{1-\gamma^5}{2}\psi, \tag{10.10}$$

these operators satisfy the usual properties for projection operators. In particular we have

$$P_R + P_L = \mathbb{1} \tag{10.11}$$

and so we can decompose the spinor ψ into it's right and left handed components

$$\psi = P_R\psi + P_L\psi \equiv \psi_R + \psi_L. \tag{10.12}$$

All of the above properties are independent of representation. In the cases where we have worked in a specific representation of the gamma matrices we have used the chiral representation. In this representation the gamma matrices take the form

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{10.13}$$

with σ^i denoting the usual Pauli matrices. The advantage of working in this representation is that the γ^5 matrix can be expressed in the convenient form

$$\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{10.14}$$

In particular the fifth gamma matrix is diagonal so that it's evident that it's equal to it's own transpose. In the chiral representation the chiral projectors take the simple form

$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_L = \frac{1-\gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{10.15}$$

10.2 Appendix B - Basis And Solutions Of The Dirac Equation

The Dirac equation has plane wave solutions given as

$$\psi(x) \propto \begin{pmatrix} u_r(p) \\ v_r(p) \end{pmatrix} e^{ipx}. \tag{10.16}$$

The spinors $u_r(p)$ and $v_r(p)$ satisfies the momentum space Dirac equations

$$(\not{p} - m)u_r(p) = 0, \qquad (\not{p} + m)v_r(p) = 0. \tag{10.17}$$

From this one can derive the following relations between the spinors

$$\overline{u}_r(p)u_s(p) = -\overline{v}_r(p)v_s(p) = \delta_{rs} \tag{10.18}$$

$$\overline{u}_r(p)v_s(p) = \overline{v}_r(p)u_s(p) = 0. \tag{10.19}$$

Another important relation is the following completeness relation

$$\sum_{r=1}^{2}[u_{r\alpha}(p)\overline{u}_{r\beta}(p) - v_{r\alpha}(p)\overline{v}_{r\beta}] = \delta_{\alpha\beta}. \tag{10.20}$$

The Energy projection operators acts by projecting out the positive or negative energy solutions from the spinors. They are given by

$$\Lambda^{\pm} = \frac{\pm\not{p} + m}{2m}. \tag{10.21}$$

Of particular importance when calculating amplitudes is the alternative form

$$\Lambda^{+}_{\alpha\beta}(p) = \sum_{r=0}^{2} u_{r\alpha}(p)\overline{u}_{r\beta}(p), \qquad \Lambda^{-}_{\alpha\beta}(p) = -\sum_{r=0}^{2} v_{r\alpha}(p)\overline{v}_{r\beta}(p). \tag{10.22}$$

In the chiral representation of the gamma matrices, the spinor u and v can be written as

$$u^{(h)}(p) = \begin{pmatrix} -\sqrt{E + h|\vec{p}|}\chi^{(h)}(\vec{p}) \\ \sqrt{E - h|\vec{p}|}\chi^{(h)}(\vec{p}) \end{pmatrix} \tag{10.23}$$

$$v^{(h)}(p) = -h \begin{pmatrix} \sqrt{E - h|\vec{p}|}\chi^{(-h)}(\vec{p}) \\ \sqrt{E + h|\vec{p}|}\chi^{(-h)}(\vec{p}) \end{pmatrix}, \tag{10.24}$$

where h denote the helicity of the particle and $\chi^{(h)}(\vec{p})$ are two-component helicity eigenstate spinors. In many cases, particularly in neutrino physics one usually works in the relativistic limit where $m << E$. In this limit, the above representation of the spinors take the form

$$u^{(+)}(p) \approx -\sqrt{2E} \begin{pmatrix} \chi^{(+)}(\vec{p}) \\ -\frac{m}{2E}\chi^{(+)}(\vec{p}) \end{pmatrix}, \qquad u^{(-)}(p) \approx \sqrt{2E} \begin{pmatrix} -\frac{m}{2E}\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix} \tag{10.25}$$

$$v^{(+)}(p) \approx -\sqrt{2E} \begin{pmatrix} -\frac{m}{2E}\chi^{(-)}(\vec{p}) \\ \chi^{(-)}(\vec{p}) \end{pmatrix}, \qquad v^{(-)}(p) \approx \sqrt{2E} \begin{pmatrix} \chi^{(+)}(\vec{p}) \\ \frac{m}{2E}\chi^{(+)}(\vec{p}) \end{pmatrix}. \tag{10.26}$$

10.3 Appendix C- Charge Conjugation

One important transformation that leaves the Dirac Lagrangian unchanged is the charge conjugation transformation. Under charge conjugation, the fermion fields transforms as

$$\psi(x) \xrightarrow{C} \psi^{c}(x) = \xi_c C\overline{\psi}^{T}(x) \tag{10.27}$$

$$\overline{\psi}(x) \xrightarrow{c} \overline{\psi}^{c}(x) = -\xi^{\star}\psi^{T}(x)C^{\dagger}. \tag{10.28}$$

Since performing the transformation twice must leave our fields invariant i.e

$$\psi \xrightarrow{c} \xi_c C\overline{\psi}^{T} \xrightarrow{c} |\xi_c|^2\psi, \tag{10.29}$$

we require that ξ_c be a phase with modulus equal to unity. The matrix C is the charge conjugation matrix satisfying

$$C\gamma_{\mu}^{T}C^{-1} = -\gamma_{\mu} \tag{10.30}$$

$$C^{\dagger} = C^{-1} \tag{10.31}$$

$$C^T = -C. \tag{10.32}$$

Another useful property is that the charge conjugation matrix commutes with the fifth gamma matrix:

$$[C, \gamma^5] = 0. \tag{10.33}$$

Even though the Dirac Lagrangian is invariant under charge transformation, the weak interactions Lagrangian is not. Consider the charged weak interaction Lagrangian

$$\mathcal{L}_{I,L}^{(CC)} = -\frac{g}{\sqrt{2}}(\overline{\nu_{eL}} W^+ e_L + \overline{e_L} W^- \nu_{eL}), \tag{10.34}$$

rewriting this in terms of the Vector and Axial parts one obtains

$$\mathcal{L}_{I,L}^{(CC)} = -\frac{g}{\sqrt{2}}[(V_{ab}^\mu - A_{ab}^\mu)W_\mu + (V_{ba}^\mu - A_{ba}^\mu)W_\mu^\dagger]. \tag{10.35}$$

Using the above transformation properties of the fermion fields and the fact that vector fields transforms like

$$W_\mu \xrightarrow{c} \xi_c^W W_\mu^\dagger \tag{10.36}$$

under charge conjugation, it's straightforward to see that the charged current Lagrangian transforms as

$$\mathcal{L}_{I,L}^{(CC)} \xrightarrow{c} -\frac{g}{\sqrt{2}}[-\xi_c^{a\star}\xi_c^b\xi_c^W(V_{ba}^\mu + A_{ba}^\mu)W_\mu^\dagger - \xi_c^a\xi_c^{b\star}\xi_c^{W\star}(V_{ab}^\mu + A_{ab}^\mu)W_\mu]. \tag{10.37}$$

Comparing this to equation (9.34) one finds that there is no choice of phases that leaves the Lagrangian invariant under the charge conjugation transformation. For this reason one says that the weak currents maximally violates this transformation and hence the phases can be chosen arbitrarily.

10.4 Appendix D- Chirality And Helicity

A major source of confusion in online forums is the distinction between helicity and chirality. The two properties are, for the most part completely unrelated. Except for in the case of a massless fermion, where they coincide. Helicity is a property of the particle. More precisely it's the projection of the spin onto the direction of the momentum. For a particle that obeys the Dirac equation, the helicity operator can be written as

$$h_p = \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|}, \tag{10.38}$$

where

$$\Sigma^i = \frac{1}{2}\epsilon^{ijk}\sigma_{jk} \tag{10.39}$$

are the spin matrices. The eigenvalues of h_p are either $+1$ or -1 and we say that an eigenstate with $h_p = +1(-1)$ is right(left)-handed. it's important to note that for a free particle, h_p commutes with the Dirac Hamiltonian and is thus conserved. However, it's not Lorentz invariant since under a boost the direction of momentum flips.

We have already seen in Appendix A that any Dirac field may be broken into it's left and right chiral part by using the chiral projectors.Thus chirality is a property of the field. The important point is that these chiral fields are invariant under Lorentz transformations:

$$[\gamma_5, \sigma_{\mu\nu}] = 0 \tag{10.40}$$

On the other hand, chirality is not conserved since the Dirac Hamiltonian anti-commutes with γ_5, in particular it's the mass term that breaks the commutation as it contains only one γ matrix. It's thus clear that helicity and chirality have somewhat opposite characteristics. In the massless limit however, these distinctions vanish. In the helicity case it's no longer possible to boost to a Lorentz frame where the momentum of the particle is flipped and thus helicity is Lorentz invariant in the masless case. For chirality, the problem we had regarding the mass term vanishes and so chirality is conserved.

It's useful to define the helicity projection operator which acts on the u and v spinors and projects out the desired helicity states

$$P_h = \frac{1 + \gamma^5 \not{s}_h}{2}, \tag{10.41}$$

with s_h^μ being the polarization four vector defined by

$$s_h^\mu = h \left(\frac{|\vec{p}|}{m} \quad , \frac{E\vec{p}}{m|\vec{p}|} \right). \tag{10.42}$$

This allows us to project out the relevant helicity components of a spinor when we wish to calculate polarized cross sections.

10.5 Appendix E - Explicit Calculation Of The Cross Sections

In this appendix we compute all the necessary kinematics needed in order to obtain the total cross sections in Section 6.

10.5.1 Neutral Current Scattering

We start of by considering the situation in the case of neutral scattering of neutrinos and electrons. The starting point is the amplitude squared derived in equation (6.59) and (6.54). In the COM-system the situation can be illustrated as in Figure 18.

In particular we have $\vec{p_1} = -\vec{p_2} \equiv \vec{p}$ and $\vec{p_3} = -\vec{p_4} \equiv \vec{p}'$ which is the defining feature of the COM-system. Looking at our amplitudes we need to compute the following products:

$$(p_2 \cdot p_3) = E_2 E_3 - |\vec{p}||\vec{p}'| \cos\theta \tag{10.43}$$

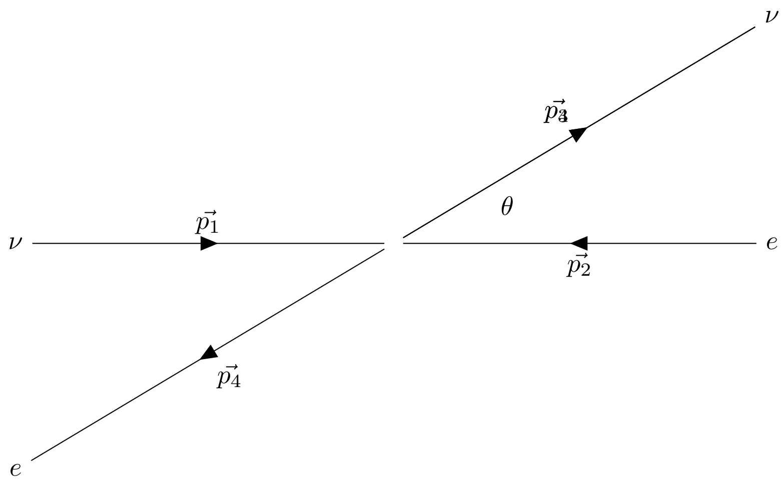

Figure 18: Kinematics for scattering process in COM frame

$$(p_1 \cdot p_4) = E_1 E_4 - |\vec{p}||\vec{p}'| \cos\theta \tag{10.44}$$

$$(p_1 \cdot p_2) = E_1 E_2 + p^2 \tag{10.45}$$

$$(p_3 \cdot p_4) = E_3 E_4 + {p'}^2 \tag{10.46}$$

$$(p_2 \cdot p_4) = E_2 E_4 + |\vec{p}||\vec{p}'| \cos\theta \tag{10.47}$$

$$(p_1 \cdot p_3) = E_1 E_3 + |\vec{p}||\vec{p}'| \cos\theta. \tag{10.48}$$

It is also useful to introduce the Mandelstam variables which are defined by

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \tag{10.49}$$

$$t = (p_1 - p_3)^2 = (p_2^2 - p_4)^2 \tag{10.50}$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \tag{10.51}$$

from which we can derive that

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2. \tag{10.52}$$

In the COM frame we have

$$s = (E_1 + E_2)^2, \tag{10.53}$$

so we can write $|\vec{p}|$ and $|\vec{p}'|$ in the following way

$$|\vec{p}| = \frac{1}{2\sqrt{s}} \sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2} \tag{10.54}$$

$$|\vec{p}'| = \frac{1}{2\sqrt{s}} \sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2}. \tag{10.55}$$

Furthermore, it is possible to express the energies of the involved particles in terms of the variable s as follows

$$E_{1,2} = \frac{1}{2\sqrt{s}}(s + m_{1,2}^2 - m_{2,1}^2) \tag{10.56}$$

$$E_{3,4} = \frac{1}{2\sqrt{s}}(s + m_{3,4}^2 - m_{4,3}^2). \tag{10.57}$$

In the case of elastic scattering, $m_1 = m_3$ and $m_2 = m_4$ and from equation (10.56) and (10.57) we see that $E_1 = E_3 \equiv E_\nu$ and $E_2 = E_4 = E_e$. The differential cross section is obtained from the formula [10]

$$(\frac{d\sigma}{d\Omega})_{CM} = \frac{1}{2E_{\mathcal{A}}2E_{\mathcal{B}}|v_{\mathcal{A}} - v_{\mathcal{B}}|}\frac{|\vec{p_1}|}{(2\pi)^2 4E_{CM}}|\mathcal{M}(p_{\mathcal{A}}, p_{\mathcal{B}} \to p_1, p_2)|^2 \tag{10.58}$$

Since we have a two-particle final state this simplifies further

$$(\frac{d\sigma}{d\Omega})_{CM} = \frac{1}{64\pi^2}\frac{|\vec{p}'|}{|\vec{p}|}\frac{1}{s}|\mathcal{M}|^2. \tag{10.59}$$

In the case of elastic scattering, we have the equality of $|\vec{p}|$ and $\left|\vec{p'}\right|$ and further simplification of the above follows. In order to obtain the total cross section, we preform the integration

$$\sigma = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}. \tag{10.60}$$

Turning our attention to the scattering in question, we can rewrite the scalar products in equations (10.44)-(10.48) as

$$(p_1 \cdot p_2)(p_3 \cdot p_4) = \frac{s^2}{4}(1 - (\frac{m_\nu^2 + m_e^2}{s}))^2 \tag{10.61}$$

$$\begin{aligned}(p_2 \cdot p_3)(p_1 \cdot p_4) =& \frac{s^2}{16}(1 - \cos\theta + \frac{2(m_\nu^2 - m_e^2)}{s}\cos\theta \\ &+ \frac{2m_\nu^2 m_e^2}{s^2}(1+\cos\theta) - \frac{m_\nu^4 + m_e^4}{s^2}(1+\cos\theta))^2\end{aligned} \tag{10.62}$$

$$\begin{aligned}(p_2 \cdot p_4) =& \frac{s}{4}((1+\cos\theta) + \frac{2(m_e^2 - m_\nu^2)}{s} - \frac{2(m_\nu^2 + m_e^2)}{s}\cos\theta \\ &+ \frac{(m_e^4 + m_\nu^4)}{s^2}(1+\cos\theta) - \frac{2m_\nu^2 m_e^2}{s^2}(1+\cos\theta))\end{aligned} \tag{10.63}$$

$$\begin{aligned}(p_1 \cdot p_3) =& \frac{s}{4}((1+\cos\theta) + \frac{2(m_\nu^2 - m_e^2)}{s} - \frac{2(m_\nu^2 + m_e^2)}{s}\cos\theta \\ &- \frac{2m_e^2 m_\nu^2}{s^2}(1+\cos\theta) + \frac{m_e^4 + m_\nu^4}{s^2}(1+\cos\theta)).\end{aligned} \tag{10.64}$$

If one works in the limit where the energy is sufficiently large $\sqrt{s} >> m_e$, these formula simplify considerably. For our purposes however, since we want to illustrate the small differences between the Majorana and Dirac cross sections, we keep terms of all orders and make no such simplification. Inserting into the formula (10.59) and performing the integration indicated in (10.60), we obtain:

$$\begin{aligned}\sigma(D) = &\frac{G_F^2}{2\pi s}((g_V^2 + g_A^2)(\frac{s^2}{4} - \frac{(m_e^2 - m_\nu^2)^2}{4} \\ &+ \frac{1}{2}(m_e^2 - m_\nu^2 + s)^2) + m_e^2(g_A^2 - g_V^2)\frac{(m_\nu^2 - m_e^2 + s)^2}{2s}).\end{aligned} \tag{10.65}$$

The Majorana cross section is similar, but of course contains additional terms:

$$\begin{aligned}\sigma(M) = &\frac{G_F^2}{2\pi s}((g_V^2 + g_A^2)(\frac{s^2}{4} - \frac{(m_e^2 - m_\nu^2)^2}{4} + \frac{(m_e^2 - m_\nu^2 + s)^2}{2s}m_\nu^2 \\ &+ \frac{1}{2}(m_e^2 - m_\nu^2 + s)^2) + m_e^2(g_A^2 - g_V^2)(\frac{(m_\nu^2 - m_e^2 + s)^2}{2s} + 4m_\nu^2)).\end{aligned} \tag{10.66}$$

We may now move to the lab frame where the electron is at rest and thus we have

$$s = m_\nu^2 + m_e^2 + 2m_e E_\nu \tag{10.67}$$

with E_ν denoting the energy of the incoming neutrino. The total cross section is now easily obtained as a function of the neutrino energy.

10.5.2 IVB Decay

The approach is identical to the neutral current case, except from the fact that we are no longer in the case of elastic scattering. The diagram scattering diagram given in the beginning of last section remains the same with the appropriate replacements of the particles. For the Dirac case, we obtain the total cross section from equation (6.11) and (10.59):

$$\sigma_D = \frac{2G_F^2}{\pi s}\frac{\left|\vec{p'}\right|}{|\vec{p}|}(E_4 E_1 E_2 E_3 + \frac{1}{3}\left|\vec{p'}\right|^2 |\vec{p}|^2). \tag{10.68}$$

Then, using equations (10.54)-(10.57) it's a straightforward procedure to express this in terms of s. Finally, we may move to the rest frame of the up-quark by using equation (10.67). This way we can express the cross section in terms of the antineutrino energy. The Majorana cross sections is obtained from the Dirac cross section as

$$\sigma_M = \sigma_D(1 + \frac{m_\nu^2}{E_\nu^2}). \tag{10.69}$$

In the high-energy limit we have

$$E_i \approx |\vec{p}| \approx \left|\vec{p'}\right| \approx \frac{\sqrt{s}}{2} \tag{10.70}$$

for $i = 1, 2, 3, 4$. Employing this approximation, we find that

$$\sigma_M - \sigma_D = \frac{G_F^2}{6\pi} \left(\frac{m_\nu^4}{E_\nu^2} + \frac{m_u^2 m_\nu^2}{E_\nu^2} + \frac{2 m_u m_\nu^2}{E_\nu} \right). \tag{10.71}$$

The resulting plots can be found in Figures 4,5,6.

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