# Monolithic atom interferometry 

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#### Abstract

Atom and, more recently, molecule interferometers are used in fundamental research and industrial applications. Most atom interferometers rely on gratings made from laser beams, which can provide high precision, but cannot reach very short wavelengths and require complex laser systems to function. Contrary to this, simple monolithic interferometers cut from single crystals offer (sub) nanometer wavelengths with an extreme level of stability and robustness. Such devices were conceived and demonstrated several decades ago for neutrons and electrons. Here, we propose a monolithic design for a thermal-beam molecule interferometer based on (quantum) reflection. We show, as an example, how a reflective, monolithic interferometer (Mach-Zehnder type) can be realized for a helium beam using $\operatorname{Si}(111)-\mathrm{H}(1 \times 1)$ surfaces, which have previously been demonstrated to act as very robust and stable diffractive mirrors for neutral helium atoms.


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## I. INTRODUCTION

The field of atom interferometry has expanded enormously over the last few decades. Atom interferometers are used in various applications, from magnetic and gravity sensing [1,2], quantum metrology [3], to atomic clocks [4]. They may even be used as dark matter and gravitational wave detectors [5] also in space [6,7]. Compact, portable atom gravimeters for prospecting, oil survey, and geophysical investigations have recently become commercially available [8]. Atom interferometers will also be useful as accelerometers for subsea navigation in submarines, and more recently, underwater drones $[2,9]$. This, however, will require very compact solutions, which are not presently available.

Atom interferometers use either cold atoms (including Bose-Einstein condensates) [10] or thermal atoms beams [11], and more recently, hot thermal vapors [12]. Most optical interferometers have, by now, been realized as atom interferometers, including Young's double slit, Mach-Zehnder, Talbot-Lau, Ramsey-Bordé, and Sagnac interferometers.

Historically, Young's double slit makes the simplest atom interferometer. The beam is split into two paths by passing through a double slit and the interference pattern is observed on a screen further down the beam path. It was realized for atoms for the first time in 1991 using metastable helium atoms passing through a thin gold foil [13].

The simplest split-path atom interferometer is arguably the Mach-Zehnder interferometer. It exploits the de Broglie wavelength of the atoms in a diffraction grating configuration

[^0]with split beam paths. The first Mach-Zehnder atom interferometer was realized in 1991 [11] using a sodium beam and solid transmission diffraction gratings. Later in 1995, it was developed further by using metastable neon and argon and transmission diffraction gratings made of standing light waves [14,15], in 2002 using ground-state lithium also with light-wave gratings [16] and later again using neutral helium with solid gratings. Results from the last-mentioned instrument were never published, but it was mentioned in a review paper from 2009 [17].

In the Talbot-Lau interferometer, the self-imaging property of a grating is exploited in near-field diffraction. The atom paths are not truly separated; therefore, this type of interferometer has been used extensively for experiments with heavy molecules where the de Broglie wavelength is very small. The first Talbot-Lau atom interferometer was realized in 1994 [18].

Where the Mach-Zehnder interferometer and the TalbotLau interferometers are adapted from light optics, the Ramsey-Bordé interferometer, first realized in 1949 by Ramsey [19], can only be used for atoms: the principle is diffraction by absorption of a single photon on a weakly allowed transition to split the wave package. In the 1980th, this interferometer was further developed by Bordé by using atomic recoil to create a beam splitter [20]. This interferometer type is currently the standard for high-precision measurements, such as atomic clocks.

In light optics, the Sagnac interferometer, also called ring interferometer, relies on a beamsplitter mirror to create two beams that travel equidistant paths in opposite directions through a ring structure guided by reflective mirrors. The two beams meet at the starting point, where they interfere and are made to exit the ring. The first atom interferometer using the Sagnac effect was realized in 1991 using a Ramsey-Bordé


FIG. 1. Sketch of the optical paths within a monolithic, reflection interferometer: the beam is reflected three times between the surfaces of two parallel slabs (gray area) separated by the distance $s$. The incoming beam is reflected at point A with an incidence angle $\alpha$. Two different diffraction orders are selected: Reflection towards point B with diffraction angle $\beta$ and reflection towards $\mathrm{B}^{\prime}$ with diffraction angle $\gamma$. At point B , the incidence angle is the same as the outgoing angle in point $\mathrm{A}: \beta$. Part of this beam is reflected towards point C with diffraction angle $\delta$. At point $\mathrm{B}^{\prime}$, the incidence angle is given by $\gamma$ due to the reflection at point A and reflected at the diffraction angle $\varepsilon$. In point C , the incoming waves with incidence angles $\delta$ and $\varepsilon$ are recombined, leaving the slab with a reflection angle $\zeta$
configuration of a state-labeled atom interferometer based on single-photon transitions, with a beam of atoms traversing two pairs of traveling wave fields. The laser fields within each pair are separated by a distance $D$, while the two pairs are separated by $d$ and are counterpropagating with respect to each other [21]. By rotating the interferometer, the counterpropagating beams collect different phases along their optical paths leading to an interference pattern on the screen. Such a configuration provides an absolute measurement of the rotational speed.

The atomic structure of a single crystal offers a simple periodic diffractive grating. Thus, it could produce many different types of interferometers, where the monolithic construction guarantees extreme stability. Interferometers based on transmission through solid slabs of material have been demonstrated, e.g., x-rays [22,23], neutrons [24], and electrons [25]. Unfortunately, these techniques are inapplicable to atoms, which interact too strongly with any solid material they travel through. Monolithic atom interferometers have been used widely in neutron scattering experiments observing gravitationally induced interference (in transmission) [26] and the quantised states of neutrons in the presence of gravitational fields with perfectly reflecting mirrors [27]. Neutrons are sensitive to external forces, and thus, are suitable candidates for quantum sensing. However, such experiments require an extensive, costly infrastructure to create, control, and detect the neutron beam. This also applies to cold atom interferometers. Thermal atom beams are easier to create and couple more robust to external fields due to the higher mass of the atoms. A further advantage of thermal atom interferometers is that they can operate continuously, dramatically decreasing the temporal resolutions.

Here, we propose an interferometer based on the reflection of atoms on monolithic single-crystal structures. The basic operation principle is depicted in Fig. 1: an incident beam of atoms is reflected by the crystal lattice (A) into two components, which impinge onto a second mirror and recombine on the third reflection.

In the past, atoms had been neglected largely because the atoms most commonly used in interferometry ( Rb [28]; Cs [29]; Ar [30]; Na [11]; K [18]) will stick to surfaces under most conditions. Similarly, metastable atoms, which have also been used for interferometry ( Ar [31]; He [13]), will decay upon impingement. A further practical challenge for a reflection-based interferometer is the contamination of the reflecting surface, which distorts the diffraction. For example, all metal surfaces will be covered in physisorbed molecules within hours, even in an ultrahigh vacuum [32-34].

Noble gasses, including ground-state helium, $\mathrm{H}_{2}, \mathrm{HCl}$, and other molecules are known to scatter from various surfaces over a broad temperature range without sticking to them [35-37]. Over the last years, focusing mirrors for neutral, ground-state helium have been developed for neutral helium microscopes [38]. An important requirement for these mirrors is that they must remain stable in a vacuum for months. One of the solutions implemented was $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$ [39]. Detailed experiments on He and $\mathrm{H}_{2}$ scattering were performed [ 40,41$]$ and the interaction potential between helium and $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$ calculated [41]. This interaction potential was then used to obtain the intensity of the different diffraction peaks for a range of conditions [42].

The advantage of the $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$ surface from an experimental point of view is that it can be prepared chemically by dipping the $\mathrm{Si}(111)$ crystal in an HF solution [43]. This means a monolithic configuration with two reflecting surfaces facing each other can be fabricated at any spacing. The additional advantage of using the $\operatorname{Si}(111)-\mathrm{H}(1 \times 1)$ surface is the small lattice constant of $a_{\mathrm{S}}=3.383 \AA$ [40], which, together with the wavelength of, as an example, helium atoms in a room temperature beam $\lambda_{\mathrm{dB}}=0.55 \AA$, ensures a very big wave-package separation. Recent matter-wave interferometers typically split the wave package over a few milliradian [31,44-46]. In contrast, using the room-temperature helium beam described above, the proposed interferometer splits the matter wave over 0.5 radians.

The atom interferometer we introduce here uses reflective atom-surface diffraction as a beam splitter. Further reflections from a parallel surface yield the recombination of the wave and thus the interference; see Fig. 1. We present a theoretical model determining the expected interference patterns and apply the model to the interference of helium atoms using $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$ surfaces, where we concentrate on describing the general principles by describing an ideal system with a perfectly coherent and monochromatic beam and an experimentally based model for the diffraction probabilities. We chose an experimentally realizable parameter set providing all possible superpositions occurring in such an interferometer: single-path transmission, double-path superposition with vanishing phases, and multipath interference. Finally, we discuss how a reflective interferometer based on quantum reflection
can be addressed. The paper finishes with a conclusion and outlook on future work.

## II. REFLECTIVE INTERFEROMETER

## A. Geometric arrangement

The general arrangement of a monolithic reflection interferometer is depicted in Fig. 1. A slab is cut into a U-shaped monolith to form two parallel planar surfaces with a distance $s$ being sufficiently large to achieve propagating waves inside the interferometer. A particle beam will be diffracted minimally three times at points $\mathrm{A}, \mathrm{B}$, and C . The beam will be split at point A, and each part will be reflected at point B and recombined in point C , where they interfere.

In more detail, a particle beam is sent via an incidence angle $\alpha$ towards one surface. It is reflectively split in point A into a range of diffraction orders determined by the incident beam angle $\alpha$, the periodic surface structure described by the lattice spacing $a_{\mathrm{S}}$, and the beam wavelength $\lambda$ through the well-known reciprocal lattice equation [47]. We pick two orders, the first one with the reflection angle $\beta$

$$
\begin{equation*}
\sin \beta=\sin \alpha+\frac{n_{1} \lambda}{a_{\mathrm{S}}} \tag{1}
\end{equation*}
$$

with an integer $n_{i} \in \mathbb{Z}$ (numerating the diffraction order) and the second one with reflection angle $\gamma$

$$
\begin{equation*}
\sin \gamma=\sin \alpha+\frac{n_{1^{\prime}} \lambda}{a_{\mathrm{S}}} \tag{2}
\end{equation*}
$$

At point A, the two selected diffraction orders propagate towards points $B$ and $B^{\prime}$, where they are reflected towards point C and recombine. Point B denotes the reflection point one diffraction order from point A ; thus, the corresponding incidence angle is $\beta$. To satisfy the recombination of the beam, the reflection angle $\delta$ has to be of a nonzeroth diffraction order expressed as

$$
\begin{equation*}
\sin \delta=\sin \beta+\frac{n_{2} \lambda}{a_{\mathrm{S}}}=\sin \alpha+\frac{\left(n_{1}+n_{2}\right) \lambda}{a_{\mathrm{S}}} \tag{3}
\end{equation*}
$$

Analogously, the reflection at point $\mathrm{B}^{\prime}$ can be determined by

$$
\begin{equation*}
\sin \varepsilon=\sin \gamma+\frac{n_{2^{\prime}} \lambda}{a_{\mathrm{S}}}=\cos \alpha+\frac{\left(n_{1^{\prime}}+n_{2^{\prime}}\right) \lambda}{a_{\mathrm{S}}} \tag{4}
\end{equation*}
$$

To satisfy the recombination of the beam at point C , the diffraction of the incoming beams need to occur under the same diffraction angle, which can be described mathematically by the relation

$$
\begin{equation*}
\sin \zeta=\sin \delta+\frac{n_{3} \lambda}{a_{\mathrm{S}}}=\sin \alpha+\frac{\left(n_{1}+n_{2}+n_{3}\right) \lambda}{a_{\mathrm{S}}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\sin \zeta=\sin \varepsilon+\frac{n_{3^{\prime}} \lambda}{a_{\mathrm{S}}}=\sin \alpha+\frac{\left(n_{1^{\prime}}+n_{2^{\prime}}+n_{3^{\prime}}\right) \lambda}{a_{\mathrm{S}}} \tag{6}
\end{equation*}
$$

These equations yield a constrain for the diffraction orders

$$
\begin{equation*}
n_{3^{\prime}}=n_{1}+n_{2}+n_{3}-n_{1^{\prime}}-n_{2^{\prime}} . \tag{7}
\end{equation*}
$$

In addition to this angular dependence, the distance between points A and C needs to be the same for both paths to satisfy the recombination of the beams. Figure 1 illustrates this
condition: the blue and red beamlines need to recombine in the same point C. Otherwise, they would be reflected without any spatial overlap to interfere directly. If they are reflected into parallel beams from different spots, they will interfere in the far field with a phase shift proportional to the spatial difference between both points. To achieve interference also in the optical near-field regime for the entire interferometer, the condition reads

$$
\begin{equation*}
\tan \beta+\tan \delta=\tan \gamma+\tan \varepsilon \tag{8}
\end{equation*}
$$

Finally, we sum up six parameters characterizing a reflective atom interferometer which have to satisfy the conditions (7) and (8). These conditions can either be used for determining the incidence angle $\alpha$ or by rewriting the equation

$$
\begin{align*}
& \tan \left(c+N_{1}\right)+\tan \left(c+N_{1}+N_{2}\right) \\
& \quad-\tan \left(c+N_{1^{\prime}}\right)-\tan \left(c+N_{1^{\prime}}+N_{2^{\prime}}\right)=0 \tag{9}
\end{align*}
$$

with $c=\cos \alpha$ and $N_{i}=n_{i} \lambda / a_{\mathrm{S}}$; one finds the following conditions leading to an $\alpha$-independent solution:

$$
\begin{equation*}
n_{1}=n_{1^{\prime}}+n_{2^{\prime}} \wedge n_{1^{\prime}}=n_{1}+n_{2} \tag{10}
\end{equation*}
$$

The interference pattern is due to the phase shift along the different optical paths $A B C$ and $A B^{\prime} C$. The path lengths can be determined via these angles for the path along point B

$$
\begin{equation*}
b=s\left(\frac{1}{\cos \beta}+\frac{1}{\cos \delta}\right) \tag{11}
\end{equation*}
$$

and along the point $\mathrm{B}^{\prime}$

$$
\begin{equation*}
b^{\prime}=s\left(\frac{1}{\cos \gamma}+\frac{1}{\cos \varepsilon}\right) \tag{12}
\end{equation*}
$$

The interference occurs via the superposition of two waves with the same wave vector $\boldsymbol{k}$, but are phase shifted with respect to the respective path lengths $b-b^{\prime}$. Hence, the phase shifts between the different paths are given by

$$
\begin{equation*}
\varphi=k\left(b-b^{\prime}\right) \tag{13}
\end{equation*}
$$

It can be observed in Eqs. (11) and (12) that the path lengths are proportional to the slab separation $s$ and, thus, $s$ should be tuned with respect to the wave vector to maximize the phase shift between both interfering beams.

Figure 2 illustrates the positions of the different diffraction for different incidence angles $\alpha$ for a particular interferometer configuration. It can be seen that the diffraction orders are strongly separated. All lines are discontinued due to the finite length of the interferometer, which leads to some beams escaping the interferometer. These particles will likely hit the surface and fall into the interferometer; thus, they will not affect the interference patterns.

## B. Reflection coefficients for the different beam paths inside the interferometer

In the last section, the conditions for interference were obtained. We now consider the intensity distribution in the


FIG. 2. Distribution of the diffraction orders on the screen $\varphi$ depending on the incidence angle $\alpha$ for a monolithic atom interferometer built of silicon with hydrogen-passivized surfaces, which are separated by 5 mm and have an extension of 50 mm . The considered wavelength was $\lambda=0.55 \AA$. The purple area describes the dark regions where no particle will appear. For each incidence angle $\alpha$, the maximum population of the diffraction order is marked in yellow. The remaining peak intensities are plotted relative to the maximum intensity according to the color scale.
interference signal, described via a reflection function. This reflection function depends on the incidence and diffraction angle $\vartheta_{1}$ and $\vartheta_{2}$, respectively. We model the reflected beam via a Gaussian intensity distribution. Consequently, each diffraction order has a Gaussian profile which we normalize to the real-valued probability of each diffraction order $\rho_{n}$

$$
\begin{equation*}
r\left(\vartheta_{1}, \vartheta_{2}\right)=\sum_{n} \rho_{n} e^{-\frac{\left(\vartheta_{2}-\theta_{n}\right)^{2}}{2 \sigma_{n}^{2}}} \tag{14}
\end{equation*}
$$

with the width of the diffracted signal $\sigma_{n}$ and the position of the diffracted beam $\theta_{n}$ determined by Eq. (1). The widths depend on the incidence angle and wavelength $\sigma_{n}=\sigma_{n}\left(\lambda, \vartheta_{1}\right)$. These impacts are negligible for surface diffraction, the paper's content, due to the overall weak reflection signal [41]. The reflection coefficient (14) only includes the inelastic scattering, that the wavelength of the outgoing wave is the same as that of the incoming wave $\lambda_{\text {inc }}=\lambda_{\text {out }}$. Thus, the total reflected signal is smaller than $1, \int d \vartheta_{2} r\left(\vartheta_{1}, \vartheta_{2}\right)<1$.

In general, there are five lengths involved in such an interferometer: the wavelength $\left(\lambda_{\mathrm{dB}}\right)$, the dimensions of the interferometer (length $d$ and slap separation $s$ ), and the freespace propagation lengths (source to interferometer $L_{1}$ and interferometer to detector $L_{2}$ ). Typically, these dimensions are on different length scales $\lambda_{\mathrm{dB}} \ll d, s<L_{1}, L_{2}$. This consideration allows for the separation of length scales. Consequently, each particle will only interfere with itself inside the same optical path in the interferometer. Thus, we can treat each path inside the interferometer separately, and the collected diffraction image will follow from the Gaussian beam envelope. The partial waves will experience a different phase shift due to the optical path (13). Thus, we can describe the reflection properties of the entire interferometer with a single modified

TABLE I. Overview of beams expected from an $83^{\circ}$ incidence angle reflected into the diffraction angle $\varphi$ with the diffraction orders $n_{1}, n_{2}$, and $n_{3}$ with the first diffraction angle $\beta$ in radians. The last columns describe the optical path length $b$ and amplitudes $a$ being the ratio of transmitted atoms into each diffraction channel (\%).

| Angle $\varphi(\mathrm{deg})$ | $n_{1}$ | $\beta\left(n_{1}\right)$ | $n_{2}$ | $n_{3}$ | Path $b(\mathrm{~cm})$ | Ampl. $a(\%)$ |
| :--- | ---: | :---: | ---: | :---: | :---: | :---: |
| 30.32 | 0 | 1.4486 | -1 | -2 | 5.00 | 0.0270 |
| 41.87 | -1 | 0.9791 | 1 | -2 | 5.00 | 0.0135 |
| 41.87 | 0 | 1.4486 | -1 | -1 | 5.00 | 0.0540 |
| 56.10 | -2 | 0.7307 | 2 | -1 | 4.77 | 0.0068 |
| 56.10 | 0 | 1.4486 | -2 | 1 | 4.77 | 0.0270 |
| 56.10 | -1 | 0.9791 | 1 | -1 | 5.00 | 0.0270 |
| 56.10 | 0 | 1.4486 | -1 | 0 | 5.00 | 0.1080 |
| 83.00 | -2 | 0.7307 | 1 | 1 | 1.57 | 0.0135 |
| 83.00 | -2 | 0.7307 | 2 | 0 | 4.77 | 0.0135 |
| 83.00 | 0 | 1.4486 | -2 | 2 | 4.77 | 0.0135 |
| 83.00 | -1 | 0.9791 | 0 | 1 | 1.79 | 0.0540 |
| 83.00 | -1 | 0.9791 | 1 | 0 | 5.00 | 0.0540 |
| 83.00 | 0 | 1.4486 | -1 | 1 | 5.00 | 0.0540 |
| 83.00 | -1 | 0.9791 | -1 | 2 | 1.57 | 0.0135 |

reflection coefficient

$$
\begin{equation*}
r_{\text {inter }}\left(\vartheta_{1}, \vartheta_{2}\right)=\sum_{n_{1} n_{2} n_{3}} \rho_{n_{1}} \rho_{n_{2}} \rho_{n_{3}} f_{n_{1} n_{2} n_{3}} e^{i k b_{n_{1} n_{2}}} e^{-\frac{\left[\vartheta_{2}-\theta_{\left.n_{1} n_{2} n_{3}\left(\vartheta_{1}\right)\right]^{2}}^{2 \sigma^{2}}\right.}{}}, \tag{15}
\end{equation*}
$$

with the wave vector of the matter wave $k=2 \pi / \lambda$ and the indicator function $f_{n_{1} n_{2} n_{3}}$ factoring in the interferometer's geometry (which determines whether the beam can pass through the interferometer or not). The beam spread of all diffraction orders will usually be the same for a monochromatic wave, $\sigma=\sigma_{n}$ for all diffraction orders $n$. Due to the tilted reflective surfaces with respect to the beam incidence, the detected spots will be slightly asymmetric, which we neglect for consideration in this paper. The position of the diffraction order is given by $\theta_{n_{1} n_{2} n_{3}}\left(\vartheta_{1}\right)$, which is the three-times composition of Eq. (1) simplifying to

$$
\begin{equation*}
\theta_{n_{1} n_{2} n_{3}}\left(\vartheta_{1}\right)=\arcsin \left[\sin \vartheta_{1}+\frac{\left(n_{1}+n_{2}+n_{3}\right) \lambda}{a_{\mathrm{S}}}\right] . \tag{16}
\end{equation*}
$$

## C. Monolithic interferometer for He and $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$

Let us consider a helium beam with de Broglie wavelength $\lambda_{\mathrm{dB}}=0.55 \AA$ and a beam spread of 1 mrad at a distance of 1 m from the interferometer (propagation length $L_{1}=$ 1 m ). This corresponds to a $1-\mathrm{mm}$ beam waist ( $w=1 \mathrm{~mm}$ ) as the beam enters the interferometer. The lattice spacing of $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$ is $a_{\mathrm{S}}=3.383 \AA[40]$. We consider an incidence angle of $83^{\circ}$ and the reflection coefficient (14) with the amplitudes $\rho_{0}=0.06, \rho_{ \pm 1}=0.03$, and $\rho_{ \pm 2}=0.015$. These scattering values correspond to the experimentally obtained data for a beam with a wavelength of $0.6 \AA$ and an incidence angle of $52^{\circ}$ reported in Ref. [40]. We changed the incidence angle because the result would be restricted to the zeroth order; see Fig. 2. Table I shows the ratio of transmitted atoms into each diffraction channel. The reflection coefficients influence only the amplitude of the interference patterns, not


FIG. 3. Optical paths in a monolithic reflective atom interferometer: a slab cut into a monolithic crystal of length $d$ ( 50 mm ) and width $s(5 \mathrm{~mm})$. A helium beam with an incidence angle of $83^{\circ}$ (dark blue line) enters the interferometer. It is diffracted at the hydrogen-passivized surfaces with a lattice constant $a_{\mathrm{S}}=3.383 \AA$. The diffracted orders are reflected two more times until they leave the interferometer. It can be seen that the third-order $\left(-3=n_{1}+n_{2}+\right.$ $n_{3}$ ) diffraction beam will not show any interference (blue dashed line at $30.32^{\circ}$ ), see Table I, the second-order beam at $41.87^{\circ}$ (orange lines) will not show any interference due to equal optical path lengths; the diffraction at $56.10^{\circ}$ (green lines), the zeroth order at $83.00^{\circ}$ (red lines) will be measured separately in the near-field regime, whereas they will interfere in the far-field leading to the interference patterns depicted in Fig. 4.
the position of the peaks. Thus, the impact of the correct scattering amplitudes is insignificant. We chose the parameters to demonstrate several effects: the single-beam transmission, the two-path superposition, and the multipath interference. Here we restrict our considerations to the zeroth, first, and second diffraction orders. Furthermore, we consider the reflecting plates to be $50-\mathrm{mm}$ long and $5-\mathrm{mm}$ separated from each other. The optical paths for this scenario are depicted in Fig. 3. It can be seen that the third-order diffraction beam (at $30.32^{\circ}$, blue line) consists of a single beam. It thus will not show any interference; the second-order beam (at $41.87^{\circ}$, orange line) is the superposition of two paths, as described in Sec. II A, but with equal optical paths which again will not interfere; and the first and zeroth order will show two separate signals each that will lead to interference in the far field. Due to the separation of the length scales and the fact that the atoms interfere with themselves and not with each other, we describe each interference pattern via a phase-shifted Gaussian wave in analogy to the Michelson interferometer. Thus, the interference pattern is described by the superposition of phase-shifted Gaussian waves

$$
\begin{equation*}
I(\varphi) \propto\left|\sum_{n} a_{n} e^{i \frac{k \sin \varphi}{2} b_{n}}\right|^{2} e^{-\frac{2 L_{2}^{2} \sin ^{2} \varphi}{w^{2}}}, \tag{17}
\end{equation*}
$$

with the amplitudes $a_{n}=\rho_{n_{1}} \rho_{n_{2}} \rho_{n_{3}}$ and the optical path lengths $b_{n}$, which are given in Table I. The widths of the diffraction orders $\sigma$ are small compared to the width of the Gaussian envelope $L_{2} \sin \sigma \ll w$, and, hence, can be neglected. It can be seen in Eq. (17) that the interference fringes are determined by the wave vector $k=2 \pi / \lambda_{\mathrm{dB}}$. Thus, increasing the wavelength, either by increasing the particle's mass or velocity will reduce the spacing between the interference fringes. The resulting interference patterns are plotted in Fig. 4. One can see that the diffraction at $30.32^{\circ}$ and $41.87^{\circ}$


FIG. 4. Far-field diffraction patterns of each spot at $30.32^{\circ}$ (red lower line), $41.87^{\circ}$ (green line), $56.10^{\circ}$ (orange line), and $83.00^{\circ}$ (blue upper line) for a helium beam with wavelength $\lambda_{\mathrm{dB}}=0.55 \AA$ with an incidence angle of $83^{\circ}$.
will not show any interference features due to the equal optical path lengths of both optical paths. The remaining two spots will show interference effects with a contrast of $48.5 \%$ for the spot at $56.10^{\circ}$ and $84.1 \%$ for the spot at $83.00^{\circ}$. The transmission rates of all channels can be found in Table I: $0.027 \%$ of the atoms will be diffracted under the angle of $30.32^{\circ}, 0.0675 \%$ under $41.87^{\circ}, 0.1688 \%$ under $56.10^{\circ}$, and $0.216 \%$ under $83^{\circ}$. The remaining particles will not leave the interferometer. The intensity of a typical helium beam is so big [48] that a signal fraction of $10^{-4}$ can easily be detected. The velocity spread will be the limiting quantity to measure the interference patterns for the helium atom interferometry configuration depicted in Fig. 4. The velocity spread of a supersonic helium beam depends on the beam temperature, the nozzle diameter, and the reservoir pressure. This has been treated extensively in the literature; see, for example, Ref. [49]. A velocity spread causes two different effects: (i) a broadening of the interference fringes and (ii) a spatial movement of the entire interference pattern, as illustrated in Fig. 5. Finally, to observe interference, the velocity spread has to be sufficiently small to not cause a washing out of the interference fringes. Figure 5 illustrates the positions of the diffraction order for different wavelengths of the incoming beam with a fixed incidence angle of $83^{\circ}$. It can be observed that the zeroth order will stay constant. The remaining orders strongly spread out with increasing wavelength. As in Fig. 2, the lines are not continuous due to the finite size of the interferometer. It can be seen in Table I that the interferometer splits the wave package at the first diffraction point over $\approx 0.71 \mathrm{rad}$.

## D. Quantum reflection interferometer

Quantum reflection occurs on the attractive (outer) part of the atom-surface interaction potential [50] in contrast to surface scattering, where the reflection occurs on the repulsive (inner) part of the interaction potential [51,52]. It is called quantum reflection because, classically, reflection cannot occur with an attractive force interaction potential. Quantum reflection has the very big advantage that an extensive range


FIG. 5. Positions of the diffraction orders [from top to bottom: zeroth (blue); first (orange); second (yellow); third (purple); fourth (green); fifth (light blue); sixth (dark red)] for an incidence angle of $83^{\circ}$ depending on the wavelength of the helium beam. The vertical black dashed line marks the regime for the considered scenario.
of atoms and small, few-atomic molecules that would stick under surface diffraction conditions display quantum reflection. The disadvantage is that quantum reflection requires small perpendicular wave vectors. This means that, for a given wavelength, the spatial extension of a reflective interferometer must be larger than in the surface scattering configuration for the separated beams to recombine.

Quantum reflection is less sensitive than surface scattering to defects and surface contamination because it occurs at larger distances from the surface [53]. Very large specular reflection coefficients of the order of $50 \%$ [54] up to $90 \%$ [55] have been measured. Diffraction via quantum reflection was recently demonstrated experimentally [56] using helium
dimers and trimers with periodically striped surfaces with micron-sized structures. The paper includes a comparison of the experimental result with scattering theory based on the diffraction angle distribution (1), reported in Ref. [57]. There is reasonable agreement between theory and experiment.

## III. CONCLUSION AND FUTURE WORK

This paper presents the first proposal for a reflective interferometer for atoms and molecules. We present calculations for a monolithic configuration based on experimental scattering results for a room-temperature helium beam from $\mathrm{Si}(111)-\mathrm{H}(1 \times 1)$, showing that a beam splitting of more than 0.5 radians is achievable. Furthermore, we argue that quantum reflection diffraction is a viable option for extending the beams and surfaces that can be used and potentially increase the signal intensity. The interference of larger and complex molecules can be achieved by using different interaction potentials, such as evanescent fields [51]. A reflective atom or molecule interferometer, particularly in a monolithic configuration, opens several possibilities for applications, for instance, as an accelerometer, in investigating the coherence of matters near dielectric surfaces, as a continuous velocity selector, and so on. The next obvious first step is to do a demonstration experiment of the interferometer with a helium beam and to do detailed designs of quantum reflection setups. This last will require the calculation of quantum (diffraction) reflection coefficients for a range of realistic system configurations.

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