Smart operations for low-emission maintenance vessels at offshore wind farms

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Abstract

This thesis investigates the efficiency of maintenance operations for offshore wind farms (OWFs) using mathematical programming. The aim is to find cost-effective solutions focusing on implementing hybrid battery technology in maintenance vessels. The developed model optimises short-term maintenance schedules to enhance the profitability and competitiveness of OWFs in the energy market.

The research examines the impact of green maritime technologies on the profitability of maintenance operations. It compares traditional diesel-powered vessels with low-emission hybrid vessels, evaluating their economic impact. A column generation-based algorithm identifies the best routes and schedules for maintenance tasks, considering factors such as technician availability, battery limits, charging schedules, and time constraints.

The results indicate that using low-emission vessels with offshore charging stations can significantly reduce operational costs and increase profitability in maintenance operations. This thesis provides insights into running OWFs and the benefits of low-emission technologies. It supports global efforts to reduce greenhouse gas emissions and shift to green energy sources while addressing economic considerations.

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Chapter 1

Introduction

1.1 Motivation

Global energy consumption has reached unprecedented levels, creating an urgent need for a transition to green, carbon-free energy sources. This dual challenge necessitates innovative solutions to provide more affordable energy while addressing pressing environmental concerns.

Offshore wind farms (OWFs) offer a promising solution. These installations can produce significantly more energy than their onshore counterparts and mitigate issues such as noise pollution, visual disturbances, and local resistance. However, transitioning to offshore environments significantly increases costs, particularly in operation and maintenance (O&M), due to harsh weather conditions and exposure to salty water. Offshore wind turbines are more prone to breakdowns compared to land-based turbines [28], and their remote locations make repairs difficult and expensive. Recent studies indicate that O&M can account for up to a third of the total expenses over an OWF's lifecycle [37].

Floating turbines amplify both the advantages and challenges of OWFs. This technology allows for the exploitation of more consistent winds found further from shore [27], but it also increases the risk of failure and limits accessibility for repairs due to the harsher marine environment. This might results in more downtime which affects the profitability of the farm.

The maritime industry is a significant contributor to climate change and environmental pollution. According to the International Maritime Organization (IMO), the industry aims to reach net-zero greenhouse gas (GHG) emissions from international shipping by around 2050. The revised IMO GHG Strategy includes targets to reduce total GHG emissions by 20-30% by 2030 and 70-80% by 2040, relative to 2008 levels [6]. These goals align with the broader UN climate objectives outlined in the Paris Agreement, which aims to limit global temperature rise to well below 2°C [50].

The motivation for this thesis is to develop a short-term maintenance scheduling optimization model aimed at increasing the profitability of offshore wind farms to achieve competitive market prices. Many of these projects are heavily supported by government funding or innovation funds. However, to accelerate expansion, it is crucial to reduce costs and increase revenues, making offshore wind not only environmentally sustainable but also economically viable. This model will also address environmental concerns by specializing the short-term maintenance scheduling optimization for low-emission vessels with offshore chargers.

1.2 Problem Statement

In the realm of marine wind energy production, maintenance expenditures constitute a substantial fraction of the overall costs. According to [52], the cost of specialized support resources necessary for offshore operations, which include service vessels and personnel, is five to ten times higher compared to onshore resources. Consequently, considerable research is dedicated to finding ways to minimize these expenditures.

This thesis takes a different approach by focusing on the outsourcing model for maintenance work at offshore wind farms. We will assume that the wind farm operators outsource their maintenance tasks to subcontractors. The wind farm operators provide the subcontractors with a set of maintenance tasks that need to be performed, along with a corresponding price they are willing to pay for each task. These prices are typically related to the cost of parts, the risk of breakdowns, and the price of downtime while work is performed.

A subcontractor, specializing in inspections and minor repairs with technicians on site and a fleet of crew transfer vessels, receives this set of tasks and aims to maximize their profit. To achieve this, the subcontractor utilizes a maintenance scheduling model to determine which tasks should be performed and when. This involves several decisions, including:

- Selecting which tasks, among the available tasks, should be performed and when.
- Determining the appropriate vessel to use for each task.
- Assigning specific technicians to each task, matching their skills.
- Planning the routes for the vessels, including decisions on when to use diesel or battery power and when to charge.
- Deciding whether the vessel should remain on turbine while technicians perform maintenance.

Furthermore, the profit-maximizing subcontractor seeks to evaluate the profitability of new green investments in their vessel fleet. This analysis will consider the economic benefits of adapting green maritime technologies and their impact on the overall profitability. This is done by simulating for multiple time horizon and analysing the economic profitability on different sets of vessels.

1.3 Thesis Outline

Chapter 2: Background Theory
This chapter introduces background theory related to the thesis. It covers the fundamentals of mathematical programming, focusing on the concepts relevant to this work. Additionally, it provides an overview of offshore wind farms, focusing on operations and maintenance. Lastly, the chapter gives a introduction of the concept of low-emission vessels, specifically crew transfer vessels, which are central to this thesis.

Chapter 3: Mathematical Formulation and Algorithm
In this chapter, we present the mathematical formulation and the column generation algorithm used to address the problems outlined in the Problem Statement.

Chapter 4: Experiments and Analysis
This chapter describes two experiments conducted using the method detailed in Chapter 3 and provides an analysis of the results.

Chapter 5: Conclusion
Chapter 5 offers concluding remarks, summarizing the key findings of the thesis.

Chapter 6: Transparency and Tools
For full transparency, this chapter includes a brief presentation on how Chat GPT-4 was used as a tool in this thesis to enhance the text, including improvements in spelling and formulations.

Chapter 2

Background Theory

2.1 Mathematical Programming

Mathematical programming, also referred to as mathematical optimization, is a collection of mathematical methods used to determine the optimal values of a quantities, given some conditions [22]. The quantities we want to find the optimal values for are often called decision variables, while the conditions are usually referred to as constraints. The objective of these methods is to turn real-life problems into mathematical models and typically maximize or minimize a function value with respect to some constraints. In these models the function is called the objective function and the corresponding value is called the objective value.

Real-world problems tend to be framed as minimization problems because real-life planners tend to be pessimistic and are interested in minimizing cost. However, in mathematics, we are more optimistic [53], therefore, in this thesis, we will maximise.

Converting a max problem to a min problem and vice versa is often a trivial operation, especially in linear programming, which we will introduce next.

2.1.1 Linear Programming

Linear programming (LP) falls under the larger umbrella of mathematical programming described above. LP is a form of mathematical programming where both the objective function and the constraints are linear. An LP problem can be formulated as

$$Z_{LP} = \max\{c^\top x : Ax \leq b, x \geq 0\},$$

where x is the vector of decision variables. The decision variables can represent different types of quantities depending on the specific problem, such as the number of items sold. The vector c contains coefficients that describe how much each decision variable increases the objective function. For example, c can describe different profits for selling different items. A is a matrix representing the coefficients of the constraints, quantifying how much of each resource is consumed by the different decision variables. This can be seen as how much material is used to create the items sold. Finally, b is the vector of available resources and acts as an upper limit for each constraint. It can describe how much material is available. As b describes the available resources and A describes the amount of resources consumed per decision variable x, it is clear that the

$Ax \leq b$ part of the LP formulation sets limitations on the quantity of each decision variable x. These limitations create an area of allowed solutions, which is usually called the feasible region. The objective of an LP is to find the optimal combination of decision variables, represented as one or sometimes multiple points at the vertices of the feasible region.[53]

The primary advantage of keeping the objective function and constraints linear, as in an LP, is the availability of efficient solution methods and techniques. These methods can solve LP problems with relatively high computational efficiency compared to other types of mathematical programming problems. Among the most commonly used methods are the Simplex Method, which can be described as an iterative algorithm that moves along the edges of the feasible region to find the optimal solution[42].

2.1.2 Dual Problem

Every LP, often called the primal problem, has a related dual problem [53]. The dual problem provides an alternative perspective of the same problem by focusing on the value of the resources. The dual of the primal formulated in the section above can be written as

$$W_{LP} = \min\{b^\top y : A^\top y \geq c, y \geq 0\},$$

where y is a vector of dual variables. The dual variable y, also referred to as the shadow price, represents the value of resources available. It quantifies the rate at which the objective function value changes with respect to changes in the right-hand side of the primal constraints, denoted by the vector b, where each constraint in the primal has a corresponding dual variable. Using the example from the LP section, the value of the dual variable can be interpreted as the additional profit made by increasing the availability of the corresponding resource by no more than one unit.

The dual constraint $A^\top y \geq c$ ensures that the dual variables are bounded by the value that the resources provide to the primal objective. Here, A represents the resource usage by the primal decision variables, and c denotes the contribution of these variables to the primal objective. The dual constraint is important to ensure that the solution to the dual problem provides a valid bound on the primal objective function. This is done by ensuring that the prices of resources are chosen such that the cost of resources is not less than what they contribute to the primal objective function.

The relationship between the primal and dual problems is described through the concepts of strong and weak duality. Weak duality means that, in maximisation problems, the objective function value of any feasible solution to the dual problem is always greater than or equal to the objective function value of any feasible solution to the primal problem. Strong duality means that if the primal problem has an optimal solution, then the dual problem also has one, and their optimal objective function values are equal[53]. This relationship between the primal and dual problems is very useful for many applications in mathematical programming, especially in linear programming, and will be relevant in the remainder of this thesis.

2.1.3 Integer Programming and Mixed-Integer Programming

In addition to linear programming, we encounter Integer Programming (IP) and Mixed-Integer Programming (MIP). These optimisation problems involve decision variables restricted to integer values in the case of IP or a mix of integer and continuous variables in the case of MIP. An IP can be formulated as

$$\max\{c^\top x : Ax \leq b, x \in \mathbb{Z}^n\},$$

while a MIP can be formulated as

$$\max\{c^\top x + h^\top y : Ax + Gy \leq b, x \in \mathbb{Z}^n, y \in \mathbb{R}^m\},$$

where x is the vector of integer decision variables, y is the vector of continuous decision variables, c and h are vectors of coefficients for the objective function, A and G are matrices representing the coefficients of the constraints, and b is the vector of available resources or upper limits for each constraint.[59]

IP and MIP are important because many real-world problems typically involve integer constraints. In real-life problems, fractional solutions often do not make sense. For example, it is irrelevant to know the profit a shop can make if it sells only half of a chair.

Solving IP and MIP problems is complex due to the integer constraints. To make them more solvable, they are often relaxed to LP problems by allowing integer variables to take continuous values[59]. This relaxation enables the use of methods like the Simplex algorithm, which can provide valuable bounds to the IP/MIP.
One effective technique for solving IP and MIP problems is the Branch and Bound method. This method systematically explores branches of the decision tree using the LP relaxation solution, pruning branches based on infeasibility, bounds, or optimality. [59]

2.1.4 VRP

The vehicle routing problem (VRP) is example of a known IP, and in some variants, as a MIP. This problem was first introduced by Dantzig and Ramser [11] in 1959. They examined a scenario where a fleet of trucks delivers gasoline from a central bulk terminal to multiple service stations. The goal is to identify the optimal set of routes for the trucks that satisfy the gasoline demands of the service stations while minimizing the total distance traveled. Various extensions of the problem consider different sets of constraints.

A related extensions is the *VRP with Pickup and Delivery (VRPPD)*, where vehicles are responsible for both picking up and delivering items along their routes. This variant increases complexity as each pickup and delivery pair must be managed such that pickups occur before deliveries and vehicle capacities are not exceeded.[38]

In specific applications, such as the maintenance of offshore wind farms, the problem becomes even more complex. Dai, Stålhane, and Utne [10] formulated this as the *Routing and Scheduling Problem of Maintenance Fleet for Offshore Wind Farms (RSPMFOWF)*. This variant involves scheduling maintenance tasks and routing a fleet of vessels to perform these tasks efficiently. The objective is to minimize the total operational cost while ensuring timely maintenance of the wind turbines. This problem incorporates constraints related to vessel capacities, maintenance schedules, and varying weather conditions, making it a challenging and realistic application of VRP.

In general, the vehicles in VRP can represent various modes of transport, where it the vehicles visits multiple locations and return to the starting point. Depending on the specific problem, several constraints might be involved.

2.2 Column Generation

Column generation is an optimization technique particularly effective for solving large-scale linear programming problems. It was first conceptualized by Ford and Fulkerson in 1958 [17]. While they did not explicitly use the term "column generation," their approach laid the groundwork for its development, particularly in addressing multi-commodity network flow issues. Over the years, this method has been widely adopted in various optimization problems.
In this section, we present the theoretical foundations of column generation, adhering closely to the explanation used by Desrosiers and Lübbecke [13], specifically section 2.1.

2.2.1 Master Problem and Relaxed Problem

We start by defining the *master problem (M)* as the following Integer Linear Program (ILP)

$$Z_M = \max\{c^\top x : Ax \leq b, x \in \mathbb{Z}^n_+\},$$

and the *relaxed problem (LM)* formulated as a the Linear Program (LP)

$$Z_{LM} = \max\{c^\top x : Ax \leq b, x \in \mathbb{R}^n_+\}.$$

When dealing with very large dimensions (n), it becomes impractical to compute all entries in A and c. To manage this, we introduce the *restricted relaxation of the master problem (RLM)*

$$Z_{RLM} = \max\{g^\top \lambda : K\lambda \leq b, \lambda \in \mathbb{R}^p_+\},$$

where λ and g are the first p entries of x and c, respectively, and K is a sub-matrix of A consisting of all rows but only the first p columns. Solving the RLM is assumed to be relatively fast.

2.2.2 Subproblem

The core idea of column generation is to iteratively add variables (columns) to the RLM to improve the solution. This involves solving a *subproblem (SP)* to identify which columns to add. The SP is used to find a column i in A that is not already in K, and the corresponding entry c_i, that, if added to the RLM, would potentially improve the optimal solution. This is done by taking advantage of the RLM's dual problem. The dual RLM is expressed as

$$W_{RLM} = \max\{b^\top \pi : K^\top \pi \geq g,\ \pi \in \mathbb{R}^q_+\},$$

where π is the vector of dual variables, and q is the number of rows in K. We construct a new constraint in the dual RLM corresponding to a column in A, and investigate whether adding this constraint makes the current dual optimal solution π^* infeasible. Let $A_{(:,i)}$ denote column i of A, and c_i denote the i-th entry in c. We want to find the i that violates

$$A^\top_{(:,i)}\pi^* \geq c_i$$

the most. From dual theory, we know that a column in A with a large positive value in $c_i - A^\top_{(:,i)}\pi^*$ indicates that including this column would improve the primal problem's feasible solution. This value is called the reduced cost, and a positive reduced cost tells us how much the objective value can improve if we add the corresponding column to the RLM. We can reformulate it as the optimization problem

$$W_{SP} = \max\{c_i - A^\top_{(:,i)}\pi^* : i \in \{1,2,\dots,n\}\},$$

where maximizing the violation of the added constraint will provide the column with the highest positive reduced cost. We define this as the *Subproblem* W_{SP} in the column generation algorithm. To maintain generality, the only constraint in the SP above is that it is a column in A. However, it is not uncommon to add problem-specific constraints to the SP.

2.2.3 Iterative Process

We solve W_{SP} and denote an optimal solution i^*. If $c_{i^*} - A^\top_{(:,i^*)}\pi^* \leq 0$, we know that no new column in A would provide an increase in the primal RLM objective function. Thus, the current optimal solution to the RLM, λ^*, is also an optimal solution to the LM x^*. This means that the LM is solved to optimality, and the column generation has converged. The stopping criterion is met. On the other hand, if $c_{i^*} - A^\top_{(:,i^*)}\pi^* > 0$, we know that the optimal solution of the RLM may be improved if we add the column $A_{(:,i^*)}$ and the entry c_{i^*} to K and g, respectively. We add the column and repeat the process until no more columns with a positive reduced cost are found.

2.2.4 Results

When the column generation algorithm terminates, it provides the optimal solution x^* of the LM. However, there is no guarantee that x^* is feasible for the Integer Problem (IP) M. The LP relaxation might have some fractional entries in its optimal solution. Therefore, we solve the restricted master problem (RM) with integer constraints, using the same columns as in the RLM that led to the stop condition

$$Z_{RM} = \max\{\hat{c}^\top \hat{x} : \hat{A}\hat{x} \leq b,\ \hat{x} \in \mathbb{Z}^r_+\},$$

where $\hat{c}, \hat{A}, \hat{x}$, and r correspond to g, K, λ, and p. If the optimal value Z_{RM} matches that of the final RLM, the RM solution is also optimal for M. Otherwise, the RM solution may still be a sufficiently good solution for M in practice. In the remainder of this thesis, we will refer to these different solution qualities as LP optimality and IP optimality.

2.3 Offshore Wind Farms

Offshore wind farms (OWFs) are an important part of the transition to renewable energy. OWFs capture stronger and more consistent wind speeds than onshore wind farms, making offshore installations efficient and capable of producing significant amounts of clean energy.[7]

According to a 2022 report by WindEurope [58], Europe installed 19 GW of new wind capacity in 2022, bringing the total wind capacity to 255 GW, with 225 GW onshore and 30 GW offshore. Total capacity additions across Europe are expected to reach 129 GW over the next five years, with 95 GW from onshore wind and 34 GW from offshore wind. Up to 500 MW of this will be floating offshore wind, including projects such as Hywind Tampen [15]. Commitments from EU member states towards wind energy targets for 2030 have increased, with more ambitious goals, particularly for offshore wind, where current pledges stand at 111 GW by 2030.

OWFs usually utilise fixed-bottom structures. These structures are suitable for relatively shallow waters and offer a stable platform for wind turbines. Fixed-bottom turbines are advantageous compared to floating ones, due to their proven technology and established O&M practices, which make maintenance easier because of their proximity to shore and stable platforms. However, they are limited to shallow waters (up to 60 metres), restricting deployment to certain regions [7].

As the demand for renewable energy grows and technological advancements progress, the offshore wind industry has started developing floating offshore wind farms (FOWFs) to overcome the limitations of fixed-bottom designs. FOWFs use floating platforms anchored to the seabed with mooring lines, allowing for the deployment of wind turbines in deeper waters. This innovation expands the potential areas for offshore wind energy generation. Floating wind turbines take advantage of the stronger and more consistent wind speeds found farther from the shore while also minimising visual impact [27]. Nevertheless, FOWFs face higher initial costs and complex installation processes, along with greater O&M challenges due to dynamic marine environments.

2.3.1 Operations and Maintenance (O&M)

The floating wind turbine industry is still in its early stages, and thus, there is limited accumulated experience. However, insights into maintaining floating structures can be drawn from the extensive experience of the offshore oil and gas industry. For the floating turbines, O&M it assumed to be similar to fixed offshore wind turbines[7].

The study by Faulstich et al[16], found that, for onshore wind turbines, minor failures constituting 75% of all failures account for only 5% of the total downtime. In contrast, major failures, which make up 25% of failures, are responsible for 95% of the downtime. Major failures were defined as those causing more than one day of downtime, while minor failures caused less than one day of downtime. The study suggests that the impact of minor failures on downtime may become more significant for offshore wind turbines, where longer waiting, travel, and work times can amplify the impact of minor failures. Preliminary results from existing offshore wind farms confirm that annual downtimes due to minor failures are likely to increase. However, current experience is limited to offshore wind farms located no more than 12 km from shore. For future wind farms planned at distances of 50 km or more from shore, typically for FOWFs, availability may decline even further.

O&M activities are crucial for the longevity and efficiency of wind farms. These include regular inspections, minor and major repairs, and the use of various vessels and equipment. Inspection and minor Repair tasks involve routine checks, maintenance activities, and small-scale repairs, often requiring quick access to the turbines. These tasks are more frequent but less disruptive than major repairs. In OWFs, these activities is often facilitated by the use of Crew Transfer Vessels (CTVs) for easy access, with minor repairs typically completed within hours to days, minimizing downtime.

Logistics play a critical role in the efficiency of maintenance activities, especially for FOWFs located far offshore. Accessibility is a key concern as greater distances from shore increase travel time and dependency on suitable weather conditions. Efficient resource management, including the transport of personnel and equipment, is crucial to minimize downtime and costs. The use of CTVs, SOVs, and helicopters is essential for various repair tasks, with the choice of vessel depending on the repair type, distance, and weather conditions.[3, 1]

2.4 Low emmision vessels

The Norwegian government is committed to leading the green transition by focusing on a greener, smarter, and more innovative workforce. Key to this is renewing Norway's maritime industry. Under the 2019 Granavolden platform, the government aims to cut emissions from domestic shipping and fishing by 50% by 2030. This will involve promoting low-emission technologies across all vessel types. [20]

Norway, a pioneer in CO_2 taxation since 1991, covers over 80% of its greenhouse gas emissions with carbon taxes or the EU quota system. To aid the transition, the government will raise the carbon tax by 5% annually until 2025, using proceeds to reduce taxes for affected groups. Enova has invested over NOK 1.6 billion in ship projects since 2015, including NOK 1.5 billion for battery-powered vessels. These investments, along with NOK 500 million for shore power in ports, strengthen the battery technology value chain, essential for a low-emission future.[20].

The Norwegian government follows DNV GL's definitions of what constitutes a low- and zero-emission ship. A low-emission ship is defined as a ship that has reduced its greenhouse gas emissions by at least 40% compared to conventional technology, with hybrid propulsion and energy-efficient hull design among the main solutions. A zero-emission ship is defined as a ship that has reduced its greenhouse gas emissions by at least 95% compared to conventional technology, with examples including batteries and hydrogen in fuel cells [20].

In an ideal world, all ships would be zero-emissions. Yet, according to a report from Nofima [26], while technology for zero-emission vessels is advancing, it is not fully matured for high-speed vessels that traverse long distances, crucial for offshore wind farm (OWF) service vessels. This report, initially focusing on the Norwegian maritime aquaculture industry, provides insights applicable to the OWF sector. It indicates that while full electrification is on the horizon, the present reality leans favourably towards low-emission-hybrid-solutions. These hybrid vessels are praised for their operational flexibility and lower emissions compared to traditional ships. The adoption of hybrid technologies is accelerating due to their ability to offer substantial emission reductions and improved operational efficiency without limiting the vessels operations. With ongoing advancements and increasing interest in hybrid operations, coupled with a push from crews for better working conditions and the maturing of relevant technologies, hybrid vessels are emerging as a practical intermediary solution. Nevertheless, the development of adequate charging infrastructure remains a critical bottleneck that needs addressing to realise broader adoption [26].

As mentioned, Enova has invested NOK 500 million for shore power in ports. The government has also shown interest in offshore charging infrastructure by investing NOK 38 million in the award-winning Ocean Charger Project through the Green Platform Initiative. The Ocean Charger is a research project aimed at enabling offshore battery-powered ships to charge using the power grid in wind farms and harbours. Led by Vard, this innovation seeks to reduce the environmental footprint of maritime operations by providing a reliable energy source for offshore vessels. The project involves multiple industry and research partners, including the University of Bergen's Institute of Informatics, and has received significant recognition, such as the Vessel Charging Innovation of the Year award [43, 54, 51, 41].

To better optimise vessel logistics for low-emission vessels, it is beneficial to understand the fundamental principles, technology, and engineering involved. Above, we introduced the concept of low-emission ships and incentives to further develop them. For the remaining part of this section, we will briefly introduce the reader to some basic vessel engineering, explain how a low-emission vessel operates, and then focus on low-emission crew transfer vessels (CTVs), the main vessel type in this thesis.

2.4.1 Propulsion

Ship emissions primarily result from propulsion [32], making it logical to begin here. Currently, marine diesel oil paired with mechanical propulsion is preferred due to its low initial cost and reliability. Nevertheless, the industry has increasingly adopted alternative solutions in recent years. This section will briefly review typical propulsion types, before focusing on the low emission alternative hybrid propulsion.

Mechanical Propulsion

Since the steam engine's development during the Industrial Revolution, mechanical propulsion has been the traditional method. A standard mechanical system includes a large engine, called the prime mover or main engine. This prime mover, either a gas turbine or diesel engine, drives the propulsor. The propulsor, typically a propeller or water jet, delivers thrust and is connected to the main engine directly or through a gearbox. [18].
A separate AC electrical network powers auxiliary loads. Diesel, steam-turbine, or gas-turbine generators provide energy for the network, which supports auxiliary systems like variable speed drives, heating, ventilation, air conditioning, and mission-critical systems [18].

Electrical Propulsion

Electrical propulsion (EP), dating back to the early 1900s, is a widely used alternative. A typical diesel-electric propulsion system consists of multiple diesel engines driving generators that produce electrical power. This power is supplied to a high-voltage electrical bus. This bus then delivers electricity to the electric propulsion motors and the hotel load, often through a transformer. The propulsion motors drive the ship's propellers, and the system uses a power electronic converter to regulate the shaft line speed, thereby controlling the ships speed[18].

Hybrid Propulsion

Hybrid propulsion is an emerging concept expected to gain popularity in coming years [20]. In this thesis, "hybrid propulsion" refers to a system that combines a conventional combustion engine with a rechargeable Energy Storage System (ESS), usually a battery [48]. The combustion engine may either generate power similar to electrical propulsion system or operate as a prime mover like in mechanical propulsion. Here, both are referred to as engines.

Peak shaving is a benefit of hybrid propulsion. In simple terms, it means the battery discharges at high load levels and charges with excess energy at low demand, allowing engines to maintain stable load levels [36]. This keeps the main engine within the standard rating without unnecessary spikes. In hybrid ships under full operation, if sudden power demand arises from acceleration or weather conditions, the battery compensates, ensuring the main engine operates within the optimal rating [46].

Similarly, the start-stop method allows the engine to charge the battery at low loads. When sufficiently charged, the engine stops, and the battery supplies power until it runs low. The engine is then reactivated, repeating the process [36]. The battery can manage total vessel load in specific situations like harbour manoeuvring, dynamic positioning, standby, and deck operations. When the battery charge is sufficient, the main engine shuts down, with the battery fully replacing it [46].

The battery also provides redundancy. Ships sometimes need significant backup energy, called spinning reserve, to maintain power in emergencies. In dynamic positioning, redundancy ensures enough power to avoid dangerous situations in case of generator failure or sudden high demand. For non-hybrids, redundancy requires running multiple engines at low load. With batteries, this is unnecessary because they provide instant energy in the event of lost generating capacity [46] [2] [33].
Dynamic Positioning System is a computer-controlled system designed to automatically maintain a vessels heading and position without relying on mooring lines or anchors[25]. For OWFs, positioning system design often integrates compensated walk-to-work gangways to increase efficiency and safety. A pushing-type gangway pushes towards the structure, allowing personnel and equipment to board the turbine platform. The control system compensates for the pushing force to reduce any deviation caused by the gangways connection to a structure. This can be complex, especially in turbulent weather conditions [24].

A combustion engine's efficiency depends on its workload. Most vessels are designed to match the engine profile to their activities, ensuring the engine often operates efficiently. However, exposure to waves, wind, and changing schedules prevents the main propulsion engines from consistently operating at their optimum. This impacts specific fuel consumption (SFC).
The preferred load is where SFC (g/kWh) is low while maintaining engine health, typically between 75-90% of the Maximum Continuous Rating (MCR) [60, 12]. Peak shaving, start-stop systems, and electric redundancy are effective ways to reduce costs and emissions by maintaining optimal engine loads. They also reduce fuel consumption by minimising engine hours required for continued operation [33].

Plug-in Hybrid

Plug-in hybrid systems represent a significant evolution from traditional hybrid propulsion systems, as detailed previously. Unlike hybrids that combine a combustion engine with an Energy Storage System (ESS) for supplementary power and efficiency [46], plug-in hybrids have larger battery capacities. This allows them to power the vessel entirely on electricity for a defined duration without the use of combustion engine entirely. [35].

Both plug-in hybrids and traditional hybrids use batteries for peak shaving and start-stop systems. Batteries discharge during high load demands and charge during low demands, maintaining stable engine loads and ensuring optimal efficiency [36].

The main distinction of plug-in hybrids is their ability to operate independently of the combustion engine, enabled by larger battery packs that store enough energy to power the vessel for limited periods.

Benefits of plug-in hybrids include more than environmental compliance. Operating fully on electricity reduces noise and vibration, enhancing crew comfort and decreasing mechanical wear and tear. Additionally, the ability to plug into shore-side or off-shore chargers allows battery recharging without using the ship's engines, saving fuel and reducing emissions [20].

In emergencies, the larger battery of a plug-in hybrid provides a more substantial spinning reserve than traditional hybrids. This ensures that even if the combustion engine fails, the ship can maintain critical operations and safety protocols using electric power.

Thus, while plug-in hybrid systems share some operational methodologies with standard hybrids, their enhanced battery capacity and ability to connect to external power sources for recharging introduce significant flexibility, cost efficiency, and environmental benefits. The extended electric-only operation and traditional hybrid functionalities define the unique profile of plug-in hybrid vessels, marking them as a forward-looking, intermediate solution towards zero-emission maritime technology [31].

2.4.2 Hull Design

Energy-efficient hull design is crucial for developing low-emission vessels. DNV encourages shipbuilders to consider designs that both accommodate for emerging fuel types, and also innovative hull designs with improved hydrodynamics[14].
Propulsion produces thrust, which can be seen as the force that pushes a ship forward, while resistance opposes this force. An efficient hull has low resistance, requiring less propulsion and resulting in lower emissions. This section introduces a simplified concept of resistance and it's effect on fuel consummation.

Resistance

The amount of propulsion required to reach or maintain a given speed depends heavily on resistance the hull encounters. When a vessel moves through water, it encounters resistance, the ship's propulsion must generate thrust greater than this. The total resistance of a vessel consists of three main components [60]:

Frictional or viscous resistance is the force from tangential forces acting on the vessel's hull due to the boundary layer along the hull [60]. In simpler terms, this resistance comes from friction between the water and the hull. As the ship moves, a thin layer of water clings to the hull (the boundary layer), creating drag.

Form resistance, also known as pressure resistance results from the normal forces on the hull due to the pressure difference in front (bow) and behind (stern) of the moving ship. Pressure losses become considerable when the boundary layer detaches from the hull at the stern [60]. As the ship moves forward, it pushes water aside, creating higher pressure at the bow. Behind the ship, the pressure is lower because the water has been displaced and does not flow smoothly back to its original position. This pressure difference creates resistance.

Wave resistance occurs because the moving ship generates waves. As the ship displaces water, it creates waves that travel away from the ship. Generating these waves requires energy, which increases the overall resistance encountered by the ship[60].

Additionally, the air resistance acting on the portion of the ship above sea level can be significant. The total hull resistance of a vessel is the sum of these resistances. It is generally acceptable to assume that a ship's resistance is roughly proportional to the square of its speed at relatively low speeds. However, at higher speeds, the resistance curve becomes significantly steeper [60].

The total fuel power required can be calculated by multiplying the resistance by the speed and dividing by certain vessel-specific parameters, which we will not detail here[45]. The key takeaway is that fuel consumption is proportional to the resistance properties of the hull and increases approximately with the cube of the vessel's speed. This means that a linear increase in speed, such as doubling (2x), results in a non-linear cubic or greater increase in power demand, for example, eight times (8x) or more[60]. This underscores the challenges in developing fast battery-powered ships, as well as the potential savings from minimizing the ship's resistance.

2.5 Low Emission Crew Transfer Vessels in Offshore Wind Farms

As briefly introduced, Crew Transfer Vessels (CTVs) are specialized ships designed for transporting service teams to OWFs. Traditionally, CTVs are aluminum catamarans that accommodate up to 12 passengers, operating at speeds of 15-25 knots. As wind farms are located further offshore, accessibility becomes challenging, leading to the use of larger Service Operation Vessels (SOVs) and helicopters[1]. However, CTV builders are now constructing larger, faster, and more efficient vessels to enhance rapid transfers and accessibility, essential for minimizing downtime and maximizing operational efficiency at OWFs [31].

CTVs have a dynamic power demand, requiring high-speed transfers from shore to the farm, and significant operational time in standby mode or using positioning systems, while the maintenance work is performed. This underscores the potential for hybrid propulsion systems, as discussed in Section 2.5.

For economic reasons, it is important that CTVs have the ability to maintain high speed. This is economically favorable as it allows technicians to perform more work and reduces the time spent without productive activity. However, it is also important to maintain low consumption for both economic and environmental benefits.
As described in Section 2.4.2, to achieve this balance of high speed and low consumption, it is crucial design the ship's hull for low resistance. Surface Effect Ships (SES), such as WAVECRAFT™[56] and CWind[23], utilize an air cushion system to reduce hull resistance, increase speeds, and enhance fuel efficiency and stability in rough seas. The air cushion, defined as the enclosed volume between the hull, seals, and water plane, is pressurized by centrifugal lift fans, lifting up to 90% of the vessel's weight and raising it higher in the water. Although lift power is needed to maintain the air cushion, the resulting reduction in resistance at high speed decreases the required propulsion power. This makes air-cushion catamarans more efficient than equivalent monohulls [47, 55]. This can be further optimized by powering the lifting fan by the use of on board batteries[44].

Lebkowski and Koznowski [31] conducted a study on the potential of using Plug-In hybrid CTV for OWFs, with offshore chargers. The study focused on evaluating the performance of hybrid diesel-battery propulsion systems. Various battery sizes and configurations were analyzed using mathematical models developed in the Modelica simulation environment, based on hydrodynamic resistance data from real operational vessels. The results demonstrated that the diesel-electric drive system significantly reduced energy consumption, greenhouse gas emissions, and operating costs compared to conventional diesel propulsion. There was a clear correlation between the size of the battery and the benefits provided. The tests were conducted on battery sizes ranging from 276 kWh to 1106 kWh, resulting in reductions in diesel consumption from 46% to 69% and energy cost reductions from 41% to 62%. It was also shown that with the largest battery pack, the vessel has the potential to operate fully electrically at a speed of 20 knots for 34.4 nautical miles (62 km), which opens the door for fully electric CTVs for certain small, close-to-shore wind farms.

The shift towards greener shipping solutions is driven by environmental concerns, regulatory, and market trends. New contract models from charterers like Equinor require emissions reduction for long-term chartering, a trend expected to extend to the offshore wind sector, making hybrid CTVs a potential commercially strategic choice [20].

Chapter 3

Mathematical Formulation and Algorithm

In this chapter, we will present a Master Problem and a Sub-Problem, which together form the Column Generation-based short-term maintenance scheduling optimization model introduced in Section 1.2.
Parts of this chapter are significantly influenced by the work of Laugaland [30], who utilized column generation to optimize the routing and scheduling of maintenance tasks for offshore wind farms. Laugalands model focused on maximizing expected profit by considering various constraints such as technician availability and weather conditions. In contrast, this model modernizes and electrifies this approach by incorporating new concepts such as plug-in hybrid and electric power, charging infrastructure, and dynamic positioning systems. To maintain transparency and continuity, the notations from [30] are preserved whenever possible.

3.1 Graph Representation

We represent our problem using a directed graph. Consider a graph G = (N, A), where N is a set of nodes, and A is a set of arcs consisting of ordered pairs of nodes.

3.1.1 Nodes

We approach our problem as a variant of the *VRPPD*, introduced in Section 2.1.4. We represent each turbine by two distinct nodes in N: one *delivery node* and one *pick-up node*. We adopt a similar representation for each of the k offshore chargers, and one additional pair for the origin/destination node.
For simplicity, the *start charging* nodes are referred to as *delivery nodes*, and the *end charging* nodes as *pick-up nodes*. All pairs of nodes correspond to the same physical locations. We represent the nodes by integers, where m denotes the number of turbines, and k the number of chargers, leading to $n = m + k$, representing the total number of turbines and chargers.
The sets representing the nodes are defined as follows:

$$W = \{w_1, w_2, \dots, w_m\}$$
$$N^D = \{1, 2, \dots, n\}$$
$$N^P = \{n+1, n+2, \dots, 2n\}$$

$$N^{OD} = \{0, 2n+1\}$$
$$N^{DC} = \{m+1, m+2, \ldots, m+k = n\}$$
$$N^{PC} = \{n+m+1, n+m+2, \ldots, n+m+k = 2n\}$$

where

- N^D is the set of delivery nodes.
- N^P is the set of pick-up nodes.
- N^{OD} consists of the origin and destination nodes.
- N^C is the set of charger nodes.

and

$$N^{PC} \subseteq N^P$$
$$N^{DC} \subseteq N^D$$
$$N^C = N^{PC} \cup N^{DC}$$
$$N = N^{OD} \cup N^D \cup N^P \cup N^C$$

3.1.2 Arcs

In constructing the graph, we aim to exclude infeasible arcs. By omitting clearly infeasible arcs, we reduce the number of constraints in our model, potentially decreasing computation time. The sets A consist of following arcs:

- From the origin node to all delivery nodes:

$$(0, j), \quad \forall j \in N^D.$$

- From turbine delivery nodes to other delivery nodes:

$$(i, j), \quad \forall i \in N^D \setminus N^{DC}, \quad j \in N^D.$$

- From turbine delivery nodes to turbine pick-up nodes:

$$(i, j), \quad \forall i \in N^D \setminus N^{DC}, \quad j \in N^P \setminus N^{PC}.$$

- From charger delivery nodes to corresponding pick-up nodes (indicating start and end of charging):

$$(i, i+n), \quad \forall i \in N^{DC}.$$

- From pick-up nodes to the destination node:

$$(i, 2n+1), \quad \forall i \in N^P.$$

- From pick-up nodes to all delivery nodes, excluding the corresponding delivery node:
$$(i,j), \quad \forall i \in N^P, \quad j \in N^D \setminus \{i-n\}.$$
- From pick-up nodes to other pick-up nodes:
$$(i,j), \quad \forall i \in N^P \setminus N^{PC}, \quad j \in N^P \setminus \{i\}.$$

Note that the only permissible move from a *charger delivery node* $i \in N^{DC}$ is to proceed to the corresponding *charging pick-up node* $i+n$. This restriction ensures the prevention of an illogical scenario where a vessel moves while charging. The vessel must complete its current charging session before it can continue on its route. However, for non-charger nodes, arcs from *delivery nodes* to other *delivery nodes* exist, allowing parallel job execution, where the initiation of a second job before the first is completed. Practically, this arrangement means that some technicians remain at a turbine and are picked up after the vessel has visited other turbines or chargers.

3.2 Master problem: Route selection

In this section, we define our Master Problem (MP), which is identical to the one used in [30].

3.2.1 Objective

In the master problem, the objective is to select a combination of routes that maximizes economic profit. A route represents a sequence of operations performed by a single vessel, dictating the vessel's specific path or trajectory. Each maintenance task yields a known revenue when performed. In contrast, charging the vessel does not generate direct revenue but provides battery power, thus reducing costs and increasing profit. Costs vary depending on factors such as the specific path, the source of energy used and time spent in dynamic position. The computation of profit per route, P_r, will be further described in a later section. It is important to clarify that only routes yielding positive profit will be considered in the MP, this logic will be clarified in later sections.

Given a set of vessels V and corresponding time windows T, we define R^{All} as the set containing all feasible routes. From this collection, we select a subset of arbitrary size, denoted R. This subset is subsequently divided into smaller subsets R_{vt}, where each subset represents the routes that a specific vessel $v \in V$ can perform during a specific time window $t \in T$.

$$R_{vt} \subseteq R \subseteq R^{\text{All}}, \tag{3.1}$$
$$R = \bigcup_{v \in V, t \in T} R_{vt}, \tag{3.2}$$
$$R_{vt} \cap R_{v't'} = \emptyset, \forall (v,t),(v',t') \in V \times T, (v,t) \neq (v',t'). \tag{3.3}$$

In Equation (3.3), $R_{vt} \cap R_{v't'} = \emptyset$ states that the sets of routes for different vessel-time window combinations do not overlap. This ensures that each route in the set R is uniquely assigned to a specific vessel v and time window t. It does not mean that different vessels cannot have identical routes in the same time window, rather, the subsets R_{vt} and $R_{v't'}$ remain distinct, maintaining the uniqueness of each route allocation.

As mentioned above, each route $r \in R$, has an economic profit P_r. We maximize the profit by selecting the most profitable combination of routes.

$$\text{maximize} \quad Z = \sum_{v \in V} \sum_{t \in T} \sum_{r \in R_{vt}} P_r x_r, \tag{3.4}$$

Where:

$$x_r = \begin{cases} 1 & \text{if the route is selected,} \\ 0 & \text{otherwise.} \end{cases}$$

3.2.2 Constraints

The first constraint limits each vessel v to undertake at most one route r per time window t,

$$\sum_{r \in R_{vt}} x_r \leq 1 \quad \forall\, v \in V, t \in T. \tag{3.5}$$

Every route r has a corresponding demand D_{br}^T for technicians of type b. We define B as the set of different types of technicians, G_{bt} as the number of available technicians of type b at time t, which yields,

$$\sum_{v \in V} \sum_{r \in R_{vt}} D_{br}^{\mathrm{T}} x_r \leq G_{bt} \quad \forall\, b \in B, t \in T. \tag{3.6}$$

This ensures that the combination of selected routes does not require more technicians than what is available.

Lastly, we need to ensure that each job is performed no more than once. To prevent any task from being done by multiple routes, we introduce a new binary parameter I_{wr}, which indicates whether job w is performed in route r or not. We assume that each turbine $w \in W$ requires exactly one operation, and we define

$$I_{wr} = \begin{cases} 1 & \text{if the operation at turbine } w \text{ is performed in route } r, \\ 0 & \text{otherwise.} \end{cases}$$

We enforce the restriction

$$\sum_{v \in V} \sum_{t \in T} \sum_{r \in R_{vt}} I_{wr} x_r \leq 1 \quad \forall\, w \in W. \tag{3.7}$$

This makes it impossible for two routes to perform the same task.

3.2.3 MP

The complete Master problem can be summarized as following IP:

$$\begin{aligned} \text{maximize} \quad & Z = \sum_{v \in V} \sum_{t \in T} \sum_{r \in R_{vt}} P_r x_r, \\ \text{subject to} \quad & \\ & \sum_{r \in R_{vt}} x_r \leq 1, \forall\, v \in V, t \in T, \\ & \sum_{v \in V} \sum_{r \in R_{vt}} D_{br}^{\mathrm{T}} x_r \leq G_{bt}, \forall\, b \in B, t \in T, \\ & \sum_{v \in V} \sum_{t \in T} \sum_{r \in R_{vt}} I_{wr} x_r \leq 1, \forall\, w \in W, \\ & x_r \in \{0,1\} \forall\, r \in R. \end{aligned} \tag{3.8}$$

3.2.4 RMP

As discussed in Chapter Two, we relax our integer program (IP) to the following linear program (LP):

$$\begin{aligned} \text{maximize} \quad & Z = \sum_{v \in V} \sum_{t \in T} \sum_{r \in R_{vt}} P_r x_r, \\ \text{subject to} \quad & \\ & \sum_{r \in R_{vt}} x_r \leq 1, \forall\, v \in V, t \in T, \\ & \sum_{v \in V} \sum_{r \in R_{vt}} D_{br}^{\mathrm{T}} x_r \leq G_{bt}, \forall\, b \in B, t \in T, \\ & \sum_{v \in V} \sum_{t \in T} \sum_{r \in R_{vt}} I_{wr} x_r \leq 1, \forall\, w \in W, \\ & x_r \geq 0 \forall\, r \in R. \end{aligned} \tag{3.9}$$

Note that the continuous relaxation of the RMP is identical to our MP, except for the last line:

$$x_r \geq 0 \forall\, r \in R,$$

where $x_r \leq 1$ is already bounded by condition (3.7).
As G_{bt} is non-negative, a trivial feasible solution exists where no routes or jobs are performed, resulting in an objective value of zero. Conversely, given that only routes with positive profit are considered, selecting all routes provides a logical upper bound. With the existence of a feasible point and an upper bound, we can guarantee that an optimal solution to (3.9) x_r^* exists.

3.3 Sub Problem

In the MP discussed above, the objective is to select the most profitable combination of routes from a set of feasible routes. In this section, we will further explore what constitutes a feasible route and how to generate them.
The routes will be generated by solving a Sub Problem (SP). The SP is a profit-maximising MIP formulated as a *VRPPD*, as described in Section 3.1.
As stated by Equation (3.3), the routes are divided into different sub sets R_{vt}, where v represents a vessel and t represents a time window. The SP is solved independently for each v-t combination, where each iteration of the SP provides one solution in the form of a route r_{vt} for the respective vessel v and time window t. Therefore, in this section, we simplify the notation by excluding specific indexing for v and t.
In a column generation fashion, each route represents a column in the MP. We want to solve the SP with the dual variables from the RMP as input to find the route that increases the objective value the most. This process will be further described in Section 3.4.
For now, we concentrate on the SP by first introducing some simple logical constraints, then how a routes profit is determine, followed by more problem-specific constraints to ensure the solution is a feasible route.

We start by introduce a new binary variable y_{ij} and present some constraints ensuring logical transfer. Let

$$y_{ij} = \begin{cases} 1 & \text{if the vessel travels directly from node } i \text{ to node } j, \\ 0 & \text{otherwise.} \end{cases} \qquad \forall (i,j) \in A.$$

The vessel is required to leave the origin exactly once by

$$\sum_{j \in N^D} y_{0j} = 1, \tag{3.10}$$

and return to the destination exactly once by

$$\sum_{i \in N^P} y_{i(2n+1)} = 1. \tag{3.11}$$

The vessel cannot visit a node without leaving it,

$$\sum_{j:(j,i) \in A} y_{ji} - \sum_{j:(i,j) \in A} y_{ij} = 0 \quad \forall i \in N \setminus N^{OD}. \tag{3.12}$$

A pick-up node can only be visited if the corresponding delivery node has been visited previously,

$$\sum_{i:(i,j) \in A} y_{ij} - \sum_{i:(i,j) \in A} y_{i(j+n)} = 0, \quad \forall j \in N^D. \tag{3.13}$$

The next constraint ensures that non charger nodes is visited no more than once

$$\sum_{i:(i,j)\in A} y_{ij} \leq 1, \quad \forall j \in N \setminus N^C. \tag{3.14}$$

This constraint has no impact on the optimal solution. However, this constraint has been shown during testing to significantly reduce the overall computational complexity of the problem, this will be shown in 4.3.

3.3.1 Profit

In the MP, P_r to denotes the profit associated with selecting a particular route r. This profit is calculated using the function

$$p_r = \sum_{(i,j)\in A,\, j\in N^D} R_j y_{ij} - \sum_{(i,j)\in A} (EC^T_{ij} E_{ij} + DC^T_{ij} D_{ij}) - \sum_{i\in N^D \setminus N^{DC}} Tdp_i DPC. \tag{3.15}$$

The revenue for each route r is calculated by summing the revenues from all maintenance tasks performed along that specific route. From the triangle inequality, we conclude that in an optimal solution, y_{ij} equals 1 only if operation j is performed, thereby realizing the revenue R_j. It is important to note that R_j equals zero for charger nodes, as charging does not provide direct revenue.

Similarly, costs associated with travelling from node i to node j are incurred. These costs are derived from the travel distances and depend on the energy source used. There is a significant cost difference between the two sources energy. Although the vessel may always operate on electric propulsion (EP), it uses either stored energy from chargers or energy generated by onboard generators. For simplicity, we refer to distances covered using battery-stored energy as 'electric' and those using generator power as 'diesel'.

The specific costs of travelling from node i to node j using electric and diesel power, respectively, are denoted EC^T_{ij} and DC^T_{ij}, while the variables E_{ij} and D_{ij} quantify the fraction of the distance travelled using the respective energy source. These variables are continuous and range from 0 to 1. The last sum in (3.15) relates to the time spent in Dynamic Positioning (DP) where the variable Tdp_i quantifies the time spent in DP at node i, and DPC represents the cost of staying in DP per unit of time. Although DPC is a constant vessel parameter and does not provide extensive scope for optimization, it is greatly influenced by the propulsion attributes of the vessel v, as discussed in Chapter 2.

The difference in costs for the two means of propulsion implies that a profit-maximizing model would prefer routes powered by electricity. However, due to battery capacity limitations, this is not always feasible. Thus, to ensure a realistic model, constraints related to battery capacity must be incorporated.

It is also worth noting that in this thesis, E_{ij} and D_{ij} refer to the fraction of distance travelled. There is no issue applying the same logic to a vessel with non-binary energy usage, meaning both sources can act together at the same time. Here, E_{ij} and D_{ij} simply quantify the proportion of each energy source used.

3.3.2 Battery Limitations

First and foremost, the vessel must use some type of fuel to travel any distance. The following constraint ensures that if the vessel travels a distance, it either consumes power from the battery, generates power using diesel, or it combine the two modes

$$E_{ij} + D_{ij} = y_{ij} \quad \forall (i,j) \in A. \tag{3.16}$$

We introduce new variables to describe the battery level and capacity. The variable θ_i represents the battery level when leaving node i, while ε denotes the vessel's maximum battery capacity. The cost matrix δ_{ij} contains the battery cost for traveling from i to j, while $\bar{\delta}$ denotes the vessel's battery cost for DP operations per unit of time.
The following constraints ensure that the vessel is fully charged when departing from the origin, and the battery level should never exceed the maximum capacity:

$$\theta_0 = \varepsilon, \tag{3.17}$$

$$\theta_i \leq \varepsilon \quad \forall i \in N. \tag{3.18}$$

The next constraint updates the vessel's battery level while also ensuring logical electric usage:

$$\theta_j \leq \theta_i - \delta_{ij} E_{ij} - T dp_i \bar{\delta} + \varepsilon(1 - y_{ij}) \quad \forall (i,j) \in A \, i \notin N^{DC}. \tag{3.19}$$

This constraint updates the battery level at node j as the level at node i minus the battery cost of travelling from i to j and the battery cost used in DP operations at node i. Furthermore, the non-negativity of θ, combined with (3.19), ensures that electric propulsion is feasible only when adequate battery charge is available. The constraint is also non-restrictive for untraveled links ($y_{ij} = 0$),by maintaining the initial maximum battery capacity at all nodes until updated for each traversed distance in A.

The following constraint updates the battery level during charging

$$\theta_i + L(q_{i+n} - q_i - \kappa) = \theta_{i+n} \quad \forall i \in N^C. \tag{3.20}$$

Where L represents the charging rate in battery units per unit of time, $q_j - q_i$ denotes the time spent at the charging node, and κ describes time spent at the charger node that does not contribute to efficient charging, such as positioning the vessel, connecting, and disconnecting the charger. For solutions where $\kappa \geq q_{i+n} - q_i$, this calculation will not hold, but in this profit-maximizing model, such solutions are not optimal.

Due to environmental considerations, it is preferable for the vessel to operate without combustion engines when close to the harbour, origin, and destination node. By avoiding diesel use in population-dense areas, we reduce public hazards related to vibration, pollution, and noise.

The following constraints limit the use of diesel for all traversed distances to and from the base nodes. The parameter π denotes the allowable diesel cost for these distances. Let

$$\sum_{j\in N^D} D_{0j}DC^T_{0j} \leq \pi, \tag{3.21}$$

and

$$\sum_{i\in N^P} D_{i(2n+1)}DC^T_{i(2n+1)} \leq \pi. \tag{3.22}$$

These constraints effectively prohibit engines from operating during in-sailing and other harbour operations. They ensure that this cost is sufficiently low, inversely enforcing all electric propulsion. Note that this is a simplification. The constraints do not specify which part of the distance must run without engines, but (3.21, 3.22) at least guarantee it is done, and consequently that battery power is available.

3.3.3 Time Considerations

To manage the vessel's time windows, we introduce a constraint that restricts the vessel's availability during a specific period t to T^P. Here, T^P represents the latest time by which the vessel must return to the destination node, and q_i, briefly introduced in the context of the charging constraint (3.20), indicates the time the vessel departs from node i.

$$q_{2n+1} \leq T^P, \tag{3.23}$$

This ensures that the vessel returns to the destination in time.

Furthermore, we introduce the matrix T_{ij}, which describes the travel time from node i to node j. The following constraint updates the time variables in a logical manner

$$q_i + T_{ij} \leq (T^P + T_{ij})(1 - y_{ij}) + q_j \quad \forall (i,j) \in A. \tag{3.24}$$

The left-hand side of this inequality combines the departure time from node i with the travel time to node j, establishing the earliest possible arrival time at node j. The right-hand side corresponds to the arrival time at node j when the vessel traverses from i to j ($y_{ij} = 1$), effectively updating q_j to state that the vessel cannot depart a node before it has the time to reach it. If the vessel does not traverse the arc $y_{ij} = 0$, the factor $(T^P + T_{ij})$ is large enough to make the inequality non-binding.

All maintenance tasks require a certain amount of time for the technicians to complete the work. We introduce a new input parameter, $\bar{T}_i$, to denote the minimum time required to perform the task at node i. This time parameter helps ensure that the tasks are performed in a feasible manner. The constraint

$$q_i + \bar{T}_i \leq q_{i+n}, \quad \forall i \in N^D \tag{3.25}$$

guarantees that technicians have adequate time to complete their tasks before being picked up. It also prevents the corresponding pick-up node from being visited before the delivery node, thus enforcing the logical sequence of service delivery and pick-up.

Recalling Tdp_i from the profit Section 3.3.1, which is time spent in Dynamic Positioning (DP) at turbine i, the following constraint updates this time variable:

$$q_{i+n} - q_i + (y_{i,i+n} - 1)T^P \leq Tdp_i, \quad \forall i \in N^D \tag{3.26}$$

This constraint is designed to update the time spent in DP. $q_{i+n} - q_i$ is the time spent between a delivery and corresponding pickup for one turbine. However, since it is possible to perform parallel jobs, where the vessel is not stationary at the turbine while the technicians work, we limit the constraint to only apply if $y_{i,i+n} = 1$, meaning the vessel "travels" straight from delivery to corresponding pickup, meaning in its turn that it stays at the turbine during the whole duration of the operation. If this is not the case, T^P is large enough to make the constraint non-binding.

3.3.4 Technicians

Constraint (3.6) in the Master Problem (MP) ensures that the model does not select a combination of routes that requires more technicians than are available, G_{bt}. To prevent creating unnecessary infeasible routes, we also restrict individual routes to respect technician availability. We introduce a new non-negative variable z_{bi}, representing the count of type b technicians on board the vessel upon leaving node i.

$$z_{b0} \leq G_{bt}, \quad \forall b \in B \tag{3.27}$$

This constraint restricts the number of technicians leaving the origin node to not exceed the available technicians.
Due to space limitations on the vessels, there is a maximum capacity K_v for the number of technicians on board the vessel.

$$\sum_{b \in B} z_{b0} \leq K_v, \tag{3.28}$$

This enforces the vessel's space limitation for technicians.

To represent the number of required technicians of type b to perform a job at node i, we introduce the parameter F_{bi} representing the demand for technicians at node i. We set the demand to be positive at delivery nodes and negative at pick-up nodes. Every node, except the origin, destination, and charging nodes, has a demand for technicians.

$$z_{bi} - F_{bj} \geq z_{bj} + K_v(1 - y_{ij}), \quad \forall (i,j) \in A, b \in B \tag{3.29}$$

and

$$z_{bi} - F_{bj} \leq z_{bj} - K_v(1 - y_{ij}), \quad \forall (i,j) \in A, b \in B \tag{3.30}$$

These constraints update the variable z_{bi} by the technician demand when a node is visited, ensuring the number of technicians on board the vessel is adjusted accordingly.

3.4 Column generation

As discussed in introduction of this Chapter, we use column generation to solve our problem. We start by connecting our sub problem to the RMP. This is done by taking advantages of similarity's in the two problems. We revisit the master problem discussed in Section 3.3, and further focus on the dual of the relaxed master problem. In section 3.3.4, we outlined both a lower bound and an upper bound, and subsequently guaranteeing an optimal solution x_r^* . Let λ_{vt}^*, ω_{vt}^*, and ρ_{vt}^* denote the optimal solution, to the related dual problem:

$$\min \quad \sum_{v\in V}\sum_{t\in T}\lambda_{vt} + \sum_{b\in B}\sum_{t\in T} G_{bt}\omega_{bt} + \sum_{w\in W}\rho_w \tag{3.31}$$

$$\text{s.t.} \quad \lambda_{vt} + \sum_{b\in B} D_{br}^T\omega_{bt} + \sum_{w\in W} I_{wr}\rho_w \geq P_r, \quad \forall v\in V, t\in T, r\in R_{vt} \tag{3.32}$$

$$\lambda_{vt} \geq 0, \quad \forall v\in V, t\in T \tag{3.33}$$

$$\omega_{bt} \geq 0, \quad \forall b\in B, t\in T \tag{3.34}$$

$$\rho_w \geq 0, \quad \forall w\in W, \tag{3.35}$$

where $\lambda_{vt}, \omega_{bt}$ *and* ρ_w correspond to the constraints (3.5),(3.6) and (3.7) in the RMP. Due to the fact that (λ_{vt}^*, ω_{vt}^*, ρ_w^*) is an optimum, and consequently a feasible solution, we can safely assume the inequality (3.32) holds at this solution.

We further substitute P_r in the dual constraint (3.32) with (3.15) formulated in Section 3.3.1. However, it is important to note that the SP will be solved for all vessels v and time windows t, resulting in

$$P_r = \sum_{(i,j)\in A,\, j\in N^D} R_{jt}y_{ij} - \sum_{(i,j)\in A}(EC_{ijvt}^T E_{ij} + DC_{ijvt}^T D_{ij}) - \sum_{i\in N^D\setminus N^{DC}} Tdp_{iv}DPC_v, \quad \forall v\in V, t\in T, r\in R_{vt}. \tag{3.36}$$

In the dual optimum,

$$\lambda_{vt}^* + \sum_{b\in B} D_{br}^T\omega_{bt}^* + \sum_{w\in W} I_{wr}\rho_w^* \geq \sum_{(i,j)\in A:\, j\in N^D} R_{jt}y_{ij} - \sum_{(i,j)\in A}(EC_{ijvt}^T E_{ij} + DC_{ijvt}^T D_{ij}) - \sum_{i\in N^D\setminus N^{DC}} Tdp_{iv}DPC$$

$\forall \quad v \in V,\ t \in T,\ r \in R_{vt}$.(3.37)

Similarly, we substitute other MP parameters with route-specific variables from the SP to connect our SP to the MP. The MP parameter D_{br}^T, representing the demand for technicians, corresponds to the SP's number of technicians z_0 initially on board, which, as we know from (3.27), essentially represents amount of technicians occupied while perform route r.
The MP binary parameter I_{wr}, which describes whether the job at turbine w is performed in route r, is substituted by the SP variable y_{ij}, where $j \in N^D \setminus N^{DC}$. This is feasible because there is a one-to-one relationship between the number of m turbines and delivery nodes, excluding charger nodes $(N^D \setminus N^C)$. As discussed in Section 3.3.1, we know that if a delivery node is visited, the corresponding job is performed.
Hence in (3.37),

$$\sum_{w\in W} I_{wr}\rho_w^* \quad \text{is replaced by:} \quad \sum_{(i,j)\in A:\, j\in N^D\setminus N^{DC}} y_{ij}\rho_j^*,$$

and

$$D_{br}^T \quad \text{by:} \quad z_{b0}$$

Since the dual variable ρ_w is related to the primal constraint (3.7) and represents the shadow price of restricting a job to be performed by at most route, and the chargers can be used in all routes, this transition is logical.
After the substitution, we relocate and contract, and (3.37) becomes,

$$\sum_{(i,j)\in A: j\in N^D\setminus N^{DC}} y_{ij}(R_{jt}-\rho_j^*)-\lambda_{vt}^*-\sum_{(i,j)\in A}(EC_{ijvt}^T E_{ij}+DC_{ijvt}^T D_{ij})-\sum_{i\in N^D\setminus N^{DC}} Tdp_{iv}DPC_v-\sum_{b\in B} z_{bt0}\omega_{bt}^*$$

$\leq 0, \quad \forall v \in V, t \in T.$ (3.38)

We have transitioned from representing a route $r \in R$ with D_{br}^T and I_{wr}, to using the variables y_{ij} and z_{b0}. It is established that a specific route $r \in R$ corresponds to particular values of the variables. We also know by definition, that all $r \in R$ satisfies inequality (3.37) when v and t match the route's designated vessel and period. Recall from Section 3.2.1, where we selected the most profitable combination of routes from the set $R \subset R^{ALL}$. We now aim to discover, or more precisely generate new and potentially superior routes $r' \in \bar{R}$. If a route $r' \in \bar{R}$ is found to be feasible under all constraints in Section 3.3 while also violating (3.37), we consider r' as a likely profitable route and as a strong candidate for inclusion in R. Consequently, we construct a model that maximizes the violations of (3.38) while also ensuring route feasibility, by not breaching the constraints from SP (3.3). This is done for every vessel v within their corresponding time window t, for all $v \in V$ and all periods $t \in T$. [30]

Resulting in the following MIP:

$$
\begin{aligned}
\max \quad & -\lambda_{vt}^{*} + \sum_{(i,j)\in A: j\in N^D} y_{ij}(R_{jt} - \rho_j^{*}) - \sum_{(i,j)\in A} \left(EC_{ijvt}^{T} E_{ij} + DC_{ijvt}^{T} D_{ij} \right) - \sum_{i\in N^D \setminus N^{DC}} Tdp_i DPC - \\
& \sum_{b\in B} z_{b0}\omega_{bt}^{*} \\
\text{s.t.} \quad & \sum_{j:(i,j)\in A} y_{ji} - \sum_{j:(j,i)\in A} y_{ij} = 0, \quad \forall i \in N \backslash N^{OD} \\
& \sum_{j\in N^D} y_{0j} = 1 \\
& \sum_{i\in N^P} y_{i(2n+1)} = 1 \\
& \sum_{i:(i,j)\in A} y_{ij} - \sum_{i:(i,j)\in A} y_{i(j+n)} = 0, \quad \forall j \in N^D \\
& \sum_{i:(i,j)\in A} y_{ij} \le 1, \quad \forall j \in N \setminus N^C. \\
& \theta_0 = \varepsilon \\
& \theta_i \le \varepsilon, \quad \forall i \in N \\
& \theta_j \le \theta_i - \delta_{ij}E_{ij} + \varepsilon(1 - y_{ij}), \quad \forall (i,j) \in A, i \notin N^{DC} \\
& \theta_i + L(q_{i+n} - q_i - \kappa) = \theta_{i+n}, \quad \forall i \in N^C \\
& \sum_{j\in N^D} D_{0j} DC_{0j}^{T} \le \pi \\
& q_i + T_{ij} \le (T^P + T_{ij})(1 - y_{ij}) + q_j, \quad \forall (i,j) \in A \\
& q_i + \tilde{T}_i \le q_{(n+i)}, \forall i \in N^D \\
& z_{b0} \le G_{bt}, \quad \forall b \in B \\
& \sum_{b\in B} z_{b0} \le K_v \\
& z_{bi} - F_{bj} \le z_{bj} + K_v(1 - y_{ij}), \quad \forall (i,j) \in A, b \in B \\
& q_{i+n} - q_i + (y_{i,i+n} - 1)T^P \le Tdp_i, \quad \forall i \in N^D \setminus N^{DC} \\
& z_{bi} - F_{bj} \ge z_{bj} - K_v(1 - y_{ij}), \quad \forall (i,j) \in A, b \in B \\
& y_{ij} \in \{0,1\}, \quad \forall (i,j) \in A \\
& q_i, Tdp_i \in \mathbb{R}_0^{+}, \quad \forall i \in N \\
& z_{bi} \in \mathbb{Z}_0^{+}, \quad \forall b \in B, i \in N \\
& \theta_i \in [0,100], \quad \forall i \in N \\
& E_{ij}, D_{ij} \in [0,1], \quad \forall (i,j) \in A
\end{aligned}
$$

3.5 Algorithm

The Algorithm 1 shown below solves the column generation problem through an iterative process similar to that described in Section 2.2.3.

Initially, the algorithm starts with an empty set of routes R and two time limits, $TimeLimit1$ and $TimeLimit2$, which will be discussed further in Section 4.2.

The process begins by solving the LP relaxation of the MP with no routes, thus reflecting a restricted relaxed MP (RRMP). Subsequently, the SP is solved for each respective time window and vessel separately to find new routes. From Section 3.4, we know that if the SP has a positive objective value, the route is not already in R and it is profitable.

In each iteration, the SP identifies the route that improves the objective function value the most by finding the column, represented as a route, with the highest positive reduced cost. If this route has a positive objective value, the corresponding route is added to the set R, otherwise, the route is not included. After solving the SP for all time windows t and all vessels v, the dual variables are updated by solving the RRMP, ensuring that the next iteration of SP uses the most current dual prices.

The algorithm continues to iterate, adding new routes and resolving the master problem until no further improvements are found, indicated by the absence of new routes with positive reduced costs. When no new routes are found, the column generation terminates, the RRMP is equal to the relaxed MP, and the solution is considered optimal. However, this relaxed solution might have fractional values, meaning that while it provides a very good solution, it may not be an optimal integer solution for the integer program.
Finally, once the iterative process converges, the master problem with integer constraints is solved to obtain the integer solution.

Algorithm 1 Column Generation

```
procedure RunCG(R, TimeLimit, TimeLimit2)
    improvementFound ← True
    while improvementFound do
        improvementFound ← False
        ImprovetCount ← 0
        DualPrices ← SolveMasterProblemLP(R)
        for all v ∈ V) do
            for all t ∈ T do
                (r, subObjValue) ← Solve Sub problem(v, t, DualPrices, TimeLimit)
                if subObjValue > 0 then
                    R ← R ∪ {r}
                    improvementFound ← True
                    ImprovetCount ++
                end if
            end for
        end for
        if not improvementFound and Sub not optimal and ImprovetCount < 1 then
            improvementFound ← CheckWithNewTimeLimit(R, TimeLimit2, DualPrices)
        end if
    end while
    BestIntegerSolution ← Solve Master problem IP (R)
end procedure
```

Chapter 4

Experiments

In this chapter, we conduct two distinct experiments to explore the applications and efficiencies of column generation within our specific context.

The first experiment focuses on the methodology of using column generation for optimizing maintenance tasks at offshore wind farms using low-emission vessels. The primary objective is to determine whether column generation is an efficient approach for solving this problem. We will evaluate the performance of the column generation algorithm and test if minor modifications could enhance its overall efficiency.

The second experiment explores an alternative use of column generation. We will investigate the possibility of utilizing the column generation algorithm as an analytical tool for examining the impacts of innovation in vessel technology. This involves testing different vessel parameters and examining the results, with the goal of gaining insights into the impact of various vessel technologies.

The algorithm is implemented in Python 3.10, with both the MP and SP solved using Gurobi.py. The MP is solved with a default tolerance of 1×10^{-6}, while the SP uses a tolerance of 1×10^{-2}. The algorithm begins with the set of routes R being empty, denoted by $R = \emptyset$. The computational experiments are conducted on a system with an Intel(R) Core(TM) i5-10600K CPU @ 4.10GHz and 16 GB of RAM. The python implementation and instance generating script is available on GitHub [49].

We start this chapter by introducing the specific instances on which the experiments will be conducted. Due to the lack of suitable problem instances in the literature, we will generate test data as realistically as possible to simulate operational conditions. To ensure robustness in our experiments, each instance size will be generated 5 times and solved accordingly.

4.1 Instance generations

4.1.1 Layout

The grid layout is designed according to the available data describing the Utsira Nord project. For turbine positioning, we use coordinates from one of the three designated areas of the Utsira Nord project, specifically Area 1 [39]. Turbines are randomly placed within this grid, constrained by a minimum separation of 1 km as proposed by [34]. A similar approach is used for the placement of the offshore chargers. While the location of the onshore base has not yet been announced, this study assumes the Haugesund-Subsea- Offshorebase-Kyllingøy, as the operational base and uses its coordinates. Distance calculations, and therefore travel times are calculated from the grid layout. Our model only considers those turbines that correspond to a maintenance task, so turbines without associated maintenance are not considered.

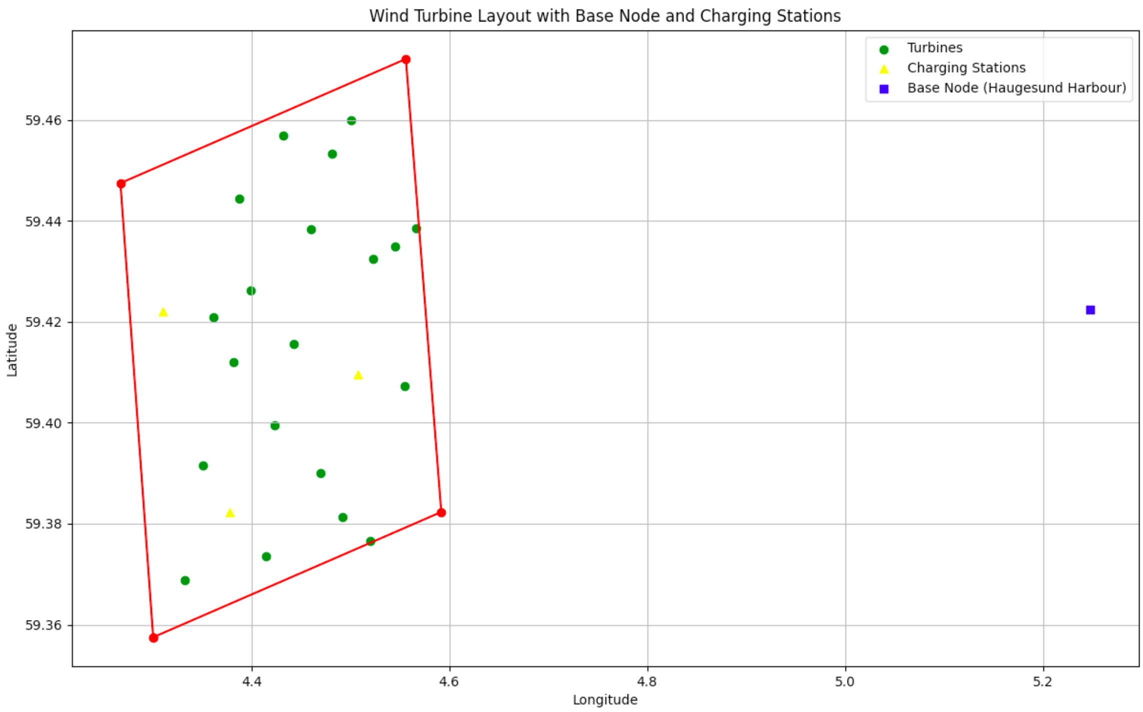

Figure 4.1: Grid layout for 20 turbine and 3 chargers

4.1.2 Revenue and Cost

After multiple unsuccessful attempts to find concrete data on maintenance task revenues, we use the following reasoning:
It is assumed that there are economic incentives to maintain operational turbines, implying that the revenue is significantly higher than the associated costs. Considering that our model excludes costs related to vessel chartering and technician expenses, the revenue is set to an arbitrary figure related to travel costs. This figure lies between covering travel costs and being highly profitable, ranging randomly between \$1000 and \$4000.
Revenue per turbine is subject to daily fluctuations influenced by variables such as weather conditions, wind direction, and part availability. Consequently, revenue for each time window is considered independent and random.
Travel costs will be calculated based on the distance traveled and the specific fuel consumption of each vessel.

4.1.3 Maintenance Tasks

As previously discussed, Crew Transfer Vessel (CTV) operations typically involve inspections and minor repairs. Therefore, we have determined the duration of these tasks to be relatively low, ranging randomly from 1 to 6 hours. For technician demand, safety considerations for offshore operations in adverse weather have been factored in, resulting in a minimum requirement of at least 4 technicians. To avoid overcrowded turbines, a maximum limit of 8 technicians has been set. The quantity of each specific type of technician is also randomized within these bounds.
The availability of technicians has been set to equal the sum of vessel capacities, with the types of technicians being equally distributed.

4.1.4 Vessels

The description of the vessel attributes is further described in the vessel technology analysis below. In experiment 1, we use the parameters associated with Fleet 3 to incorporate the full complexity of the model, including battery management and charging.

4.2 Experiment 1: Column Generation

In this experiment, we will analyze the performance of the algorithm described in Section 3.5. We will investigate the possibility of enhancing performance by adjusting model parameters. Finally, we will discuss the results and the effectiveness of the method for our specific problem.

Form the nature of the two models, the complexity of the sub problem considerably exceeds that of the master problem. This effectively makes the sub problem a computational bottleneck, particularly for larger instances. This was also observed during testing.

To potentially increase the algorithm's ability to handle larger instances, different combinations of time limits were tested. To perform this experiment, we will evaluate the performance of the algorithm on identical instances with various time limit combinations. To ensure the experiment can be conducted within a reasonable timeframe, a total cut-off limit of 6 hours will be enforced. If the algorithm does not terminate within 6 hours, the operation will be stopped and labeled as Did Not Finish (DNF).

The first strategy employs an infinitely large time limit for the SP, ensuring that it is solved optimally in each iteration without the need for a secondary check. This approach is identical to the explanation of the algorithm in Section 3.5. We hypothesize that this approach will be effective for smaller instances where the SP can be resolved quickly. For further references, we will name this approach, the traditional approach.

A more dynamic approach involve to first solve the sub problem with a strict time limit. When no new solutions are found, but the optimum is not yet reached, the process is repeated without time limit for one iteration for all vessels v and time windows t, ensuring optimality. If new profitable routes are found, the routes are added to R, and process reverts back to the strict time limit. This process is repeated until the sub problem is solved to optimality for all vessels and time windows without finding new profitable routes, and we can conclude with LP optimality.

The dynamic approach attempts to shift the computational load towards the MP, which is less complex and executes more quickly. We speculate that this strategy may help to manage computational demands effectively by alleviating the intensity of the SP, especially in the initial iterations.
Furthermore, since the SP is a MIP, and as discussed in Section 2.1.3, MIPs are typically solved using methods such as branch and bound, an optimal solution for the SP may be found early in the process. However, significant computational time is often expended in certifying optimality. This is done by pushing the dual bound towards the objective value of the incumbent solution [57].
Since strict optimality is only required in the final set of iterations to claim LP optimality, we speculate that computational time could be saved by not requiring the program to search all leaves in the branch and bound tree for every iteration. In terms of column generation, this means that if a column with a positive reduced cost is found, we

do not necessarily need to spend time proving it is the column with the highest positive reduced cost. However, if no column with a positive reduced cost is found during all iterations with the tight time limit, we repeat without the time limit to either find one or prove it does not exist, thereby claiming LP optimality.
Moreover, the dynamic time limit approach tends to generate more routes before terminating. This may reduce the gap between the linear programming and integer programming solutions of the master problem, thereby enhancing the algorithms overall quality. However, a trade-off exists. If too many routes are generated, the computational benefit of using the column generation (CG) method might vanish.
We speculate that this approach will handle medium-sized instances better compared to the first strategy, while maintaining the same quality of LP solutions and potentially better for IP.

For the third and final strategy, we tailor our approach to handle larger instances. As the computational complexity and consequently run time increase exponentially with instance size, it becomes practically impossible to solve larger instances to optimality. However, a trade-off is possible, where the quality of the solution is balanced against the ability to handle larger instances.
In this strategy, we follow a similar tactic to the dynamic approach described above, but we no longer guarantee that the SP is solved to optimality. Instead, we impose a relatively generous time limit, allowing us to manage the complexity while still addressing larger scale problems efficiently. The biggest drawback to this approach is, of course, the quality of the solution. We can no longer guarantee LP optimality, and it is also very hard to quantify the quality of the solution, as each route will have an individual gap as well as the MP IP gap.

Table 4.1 shows the different instances sizes used in this experiment.

Table 4.1: Instance sizes

	Turbines	Chargers	Vessels	Periods	Technician Types
Instance size 1	4	3	2	2	2
Instance size 2	6	3	2	2	2
Instance size 3	10	3	2	3	2
Instance size 4	15	3	2	4	2
Instance size 5	25	3	2	5	2

4.3 Results

The results of Experiment 1 are presented in Table 4.3. As outlined in Section 4.2, we tested the performance of our algorithm across different time limits and for various instances and instance sizes. In this section, we will present the results and mentioned some key findings.

In the experiment, we introduced three different approaches, each defined by different combinations of time limits. In this section, we will present results for only the traditional and dynamic approaches. The reason for this is that the dynamic and large-instance approaches performed essentially identically. During the experiments, the SP consistently found routes with a positive reduced cost within the strict time limit without needing the secondary check. This means that, for this specific experiment, the iterative process of finding good routes (not necessarily the best) with the tight time limit, and updating the dual variables was sufficient to achieve LP optimality for all instances of all sizes, making the size of the second time limit irrelevant. Therefore, for clarity and to avoid redundancy, we focus on comparing the traditional and dynamic approaches.

Table 4.2: Performance Results Across Different Time Limits for All Runs

	T_1 (inf s, 0 s)	T_2 (20 s, inf s)
Instance Size	Runtime	Runtime
	Instance 1	
Size 1	8 s	8 s
Size 2	54 s	56 s
Size 3	1495 s	647 s
Size 4	DNF	35 min
Size 5	DNF	132 min
	Instance 2	
Size 1	9 s	10 s
Size 2	49 s	36 s
Size 3	DNF	534 s
Size 4	DNF	26 min
Size 5	DNF	128 min
	Instance 3	
Size 1	13 s	12 s
Size 2	DNF	640 s
Size 3	DNF	889 s
Size 4	DNF	41 min
Size 5	DNF	120 min
	Instance 4	
Size 1	15 s	15 s
Size 2	5.1 Hr	58 s
Size 3	DNF	566 s
Size 4	DNF	21 min
Size 5	DNF	107 min
	Instance 5	
Size 1	7 s	7 s
Size 2	151 s	131 s
Size 3	DNF	563 s
Size 4	DNF	25 min
Size 5	DNF	104 min

Another key finding relates to constraint (3.14) from Section 3.3. This constraint ensures that a node, excluding charger nodes, is not visited more than once. Due to the construction of the model, this is not a logically necessary constraint and does not impact the optimal solution. However, it has significantly reduced the computational time necessary for solving the problem, as shown in the table below.

Table 4.3: Performance Results for Instances 2 and 5 Across Different Time Limits with and without Constraint 3.14

	T_2 (With-Out 3.14)	T_2 (With 3.14)
Instance Size	Runtime	Runtime
Instance 2		
Size 1	451 s	10 s
Size 2	1113 s	36 s
Size 3	2799 s	534 s
Size 4	59 min	26 min
Size 5	112 min	128 min
Instance 5		
Size 1	594 s	7 s
Size 2	1053 s	131 s
Size 3	2446 s	563 s
Size 4	103 in	25 min
Size 5	352 min	104 min

4.4 discussion

The results show that the dynamic approach significantly outperforms the traditional approach.

For the smallest instance size (Size 1), there was no difference between them, as the SP was solved to optimality within the strict time limit, making the approaches effectively identical.

In larger instances, the dynamic approach consistently solved problems to optimality, whereas the traditional method had varied results, often reaching the 6-hour limit. As detailed in Section 4.3, the initial iterations are the main challenge for the traditional approach, which was confirmed in this experiment. For example, in Instance 3, Size 2, the traditional method did not generate a single route within the 6-hour limit, while the dynamic approach solved the problem to optimality in 640 seconds. This supports the notion that generating initial routes is the bottleneck for the traditional method.

Moreover, the dynamic approach's ability to solve larger instances to optimality in a reasonable time demonstrates its robustness and efficiency. This is particularly relevant for practical applications where computational resources and time are limited.

However, some limitations must be acknowledged. The input parameters for this experiment were adjusted to fit both this experiment and Experiment 2, described in next Section. This adjustment may have simplified the model some what. For instance, the parameter π was set infinitely large, simplifying constraints (3.21 and 3.22) related to the vessel running on electricity near the harbor. Additionally, high availability of technicians meant that technician capacity was only a limiting factor in one optimal solution, this will be further discussed in Section 4.9.2. These adjustments may have led to artificially favorable results and may not reflect the most challenging real-world conditions.

For the results related to constraint (3.14). This was not thoroughly investigated, and further insight is needed before drawing any conclusions. However, we speculate that the inclusion of this constraint reduces the solution space, thereby reducing the branches in the solution tree that the branch-and-bound algorithm needs to prune in the search for an optimal solution.

In summary, the dynamic approach demonstrated strong performance, consistently solving instances to optimality in a reasonable time. It showed a significant reduction in runtime compared to the traditional method, indicating improved efficiency. These findings suggest that the Column Generation method could be a valuable tool for optimizing maintenance scheduling at Offshore Wind Farms (OWFs) with low-emission vessels. However, continued experimentation and validation are recommended to ensure broader applicability and reliability.

4.5 Experiment 2: Vessels

This section aims to analyse different types of CTVs and their technology. We will examine the financial implications of using different types of CTVs, aswell as using the value of the dual variables to gain further insight. We will look at three different fleets of vessels and compare their economic performance. Each fleet represents a different generation of maritime technology.

It is important to note that this section is heavily influenced by a lack of data. It is an attempt to create 'realistic' data, combined with some major simplifications. The data and results are in no way presented as real, but rather a display of method.

4.6 Fleets

There will be 3 different sets of vessels, with each set representing a different fleet. The first fleet will consist of traditional CTVs. These have been around for a while, but are still very much in use. The second fleet will include some of the newest and most innovative CTVs available today. Finally, in the third fleet, we will try to guess what CTVs might look like in the coming years. This will be based on current trends and some educated guesses.

4.6.1 Fleet 1

The first vessel in the traditional fleet is the CRC Sentinel. The Sentinel has a length overall (LOA) of 19 metres and a beam of 7.4 metres. It is powered by two diesel engines of 1200 hp each, which supply two waterjets, allowing a maximum speed of 29 knots and a preferred cruising speed of 22 knots. This CTV/Dive Support vessel can transfer 12 technicians, and has the capacity to carry an additional payload of up to 1,500 kg.

The second vessel in our analysis is the CRC Vulcan, a multi-role catamaran. This vessel has a LOA of 16.48 metres and a beam of 6.3 metres. It is powered by two diesel engines powering two waterjets, giving a top speed of 25 knots and a cruising speed of 21 knots. The Vulcan can transfer 12 technicians while maintaining a substantial payload capacity of 4,000 kg.

The last vessel in Fleet 1 is the CRC Gladiator, a multi-role catamaran. This vessel has an overall length of 13 metres and a beam of 5.4 metres. Powered by two 368kW diesel engines supplying waterjets, it has a top speed of 30 knots and a cruising speed of 27 knots. The Gladiator can accommodate 12 technicians and has a payload capacity of 1,500 kg.

As shown in Table 4.4, all fuel consumption data is given at 21 knots. This is particularly suitable for the Vulcan and Sentinel as it is close to their service speed. For the Gladiator, on the other hand, there is a noticeable deviation from its service speed. For the sake of simplicity, we will continue to use the 21 knot speed along with the fuel

Table 4.4: Parameters of Fleet 1, note that fuel burn is at 21 knots

Vessel Name	Service Speed (knots)	Fuel Burn (L/hr)	Technicians	DP (L/hr)
CRC Sentinel	22	400 at 21 kts	12	240
CRC Vulcan	21	270 at 21 kts	12	162
CRC Gladiator	27	160 at 21 kts	12	96

consumption data provided. The consumption in DP is not provided, but for this experiment we will assume it is 60% of consumption at service speed for all diesel-driven vessels .

All information on these boats is taken from the Commercial Rib Charter website and catalogue. [5] [4]

4.6.2 Fleet 2

Fleet 2 includes two modern CTVs, CWind Pioneer and Wavecraft Sprinter 26. Both vessels are surface-effect ships (SES) specifically designed for OWF operations.

The first vessel, CWind Pioneer, has an LOA of 22 metres and a beam of 8.9 metres. The Pioneer can carry up to 5,000 kg of cargo and 24 technicians at a service speed of 27 knots. It utilises a hybrid system, consisting of two 809 kW diesel engines and a 75 kWh electric power bank, providing a peak power of up to 1,800 kW to its waterjets. The Pioneer reaches a top speed of 33 knots on engine power alone and 38 knots with battery support for up to 30 minutes [8, 9].

The second vessel, Sprinter 26, has an LOA of 25.6 metres and a beam of 10.4 metres. It can transfer 24 technicians and 1,000 kg of cargo. The Sprinter is equipped with two main diesel engines, each producing 1,081 kW, enabling a maximum speed of 40 knots and a cruising speed of 35 knots.

Both vessels share similarities, particularly being SES. The key differences lie in their propulsion technologies. The Pioneers hybrid propulsion system enhances efficiency and reduces emissions, whereas the Sprinter employs traditional propulsion methods. This is reflected in their fuel consumption: the Pioneer has a fuel burn of 180 L/hr at a service speed of 27 knots, while the Sprinter consumes 557.5 L/hr at an economic speed of 35 knots. Adjusting for speed, this translates to 6.7 L/nm for the Pioneer and 15.9 L/nm for the Sprinter. However, due to the non-linear relationship between speed and consumption discussed in Section 2.5.2, this comparison is complex. For the Sprinter, consumption in dynamic positioning (DP) mode is set to 60% of service speed, similar to Fleet 1. For the Pioneer, considering the battery usage, we assume a 28% reduction in DP consumption [29], resulting in a DP consumption of 44% of service speed.

Note: While some older sources describe the CWind Pioneer as a plug-in hybrid, recent documentation does not confirm this, and there is no evidence in the final datasheet.

Therefore, we assume it is a non-plug-in or self-charging hybrid and use the provided fuel consumption data.

Table 4.5: Parameters of Fleet 2

Vessel Name	Service Speed (knots)	Fuel Burn (L/hr)	Technicians	DP (L/hr)
CWind Pioneer	27	180	24	78
Sprinter 26	35	557.5	24	347

4.6.3 Fleet 3

To the best of the our knowledge, there are currently no commercially available plug-in hybrid CTVs. Consequently, we will create a hypothetical future CTV based on some rough simplifications and predicted advancements in battery technology.

Fleet 3 will feature two conceptually enhanced Pioneer vessels. There are several reasons for choosing the Pioneer as the basis for this speculative design. According to predictions from [19], the inclusion of SES CTVs is overall economically favorable for maintenance work at OWF. This study considered personnel costs, vessel costs, spare part costs, and lost income due to downtime. The results predicted that the optimal fleet compositions are either one traditional CTV and one SES, or two SES vessels, depending on weather data, where the SES was favorable in terms of rough weather, which we assume will be the case for Utsira Nord which is the site our experiment is conducted on.

The existing battery installation facilitates conceptual expansion and enhancement of this system. In addition, the Pioneer's cargo capacity of 5,000 kg suggests that it could accommodate a larger battery system without the need for extreme retrofitting.

This conceptual update mostly retains the existing configuration of the CWind Pioneer, with the exception of halving the 14,000 L diesel tank, which frees up about 5,950 kg while also incorporating additional electric motors and batteries. This upgrade will allow the vessel to run on both electric and diesel power and to be recharged using offshore chargers.

Current specifications indicate that the Pioneer's twin diesel engines, each delivering 809 kW, allow a top speed of 33 knots. We hypothesize that the addition of two electric motors of equivalent power would maintain a similar level of performance. To determine power and energy requirements, we note the following:

- Power requirement for top speed (33 knots): 1618 kW.
- Energy required for 1 hour at 33 knots: 1618 kWh.
- Distance traveled in 1 hour at 33 knots: 33 nautical miles.

Thus, a battery capacity of at least 1618 kWh is needed to travel 33 nautical miles in one hour. In the orginal data sheet fuel consumption increases by a factor of 1.83 from

27 to 33 knots[9]. Assuming a similar ratio for battery consumption, the vessel can travel approximately 60 nautical miles on electric power at 27 knots. In diesel mode, the fuel consumption remains at 180 litres per hour. For DP mode, energy consumption is 44% of cruising power, in this scenario we expect this to be powered solely on battery power, resulting in a battery reduction of approximately 388 kW, to maintain consistency, energy consumption is converted to an equivalent distance, normalized to NM at 27 knots resulting in a DP consumption 7.9 NM/hr.

For charging, the battery can be recharged in one hour, giving a charging rate of 1618 kW, approximately 60 NM/Hr. It is worth noting that, for modeling reasons, battery power usage described in Constraint 3.19 in Section 3.3.2 excludes the DP usage during charging. To compensate for this, we reduce the "charging" rate by the power spent to maintain position, resulting in an effective charging rate of approximately 52 NM/Hr.

Table 4.6: Specifications of Fleet 3 Vessels

Vessel	E.Range (NM)	Speed (knots)	Fuel Burn (L/hr)	Techs	DP (NM/hr)	(NM) Chrg/hr
Pioneer 2.0	60	27	180	24	7.9	52
Pioneer 2.1	60	27	180	24	7.9	52

Recent studies indicate that Lithium Iron Phosphate (LFP) battery is the preferred type due to its balance of safety, cost, and energy density [40], currently with a gravimetric energy density of 190 Wh/kg, with a potential to reach 220 Wh/kg in the coming years [21], which we will base this futuristic vessel on. To support 1618 kWh, the battery pack would weigh approximately 7355 kg. This weight, plus the additional weight from electric motors and other components, is manageable given the additional 5950 kg freed up by reducing the diesel tank and the 5000 kg cargo capacity.

4.7 Approach

The approach for this experiment involves using the *Algorithm 1* Section 3.5, to solve the problem for each of the three fleets individually and analyzing the outputs. The main goal is to evaluate the economic benefits of green maritime technologies over multiple time horizons and instance sizes. Each instance size, as outlined in Section 4.1, will be run separately for each fleet while keeping all other parameters identical. The parameters for diesel price are set at $2 per liter, and the price of electricity is set at $0.075 per kWh. Since only Fleet 3 has the capability to operate solely on electric power, we set the value of π to indefinitely large, for all fleets, including Fleet 3, to ensure a level playing field. This results in constraint (3.21 and 3.22) being non-binding, and there is no requirement on electric propulsion in and close to the harbor.

To handle larger instances effectively, we will use the dynamic time limit described in Experiment 1, Section 4.2. In the next section, we will interpret the results by examining the total profit and the final dual variables to gain further insight.

4.8 Results

In Experiment 2, we analyzed the economic performance of three different fleets of CTVs. The primary objective was to assess the profitability and efficiency of each fleet in the context of OWF maintenance operations. The fleets included traditional diesel-powered vessels (Fleet 1), current vessels with advanced technology (Fleet 2), and a hypothetical future fleet with potential green technology (Fleet 3).

The results, summarized in Table 4.8, indicate a significant performance advantage for Fleet 3 across nearly all instances and sizes. The average profit per time window was calculated for each fleet, providing a clear comparison of their performance. Fleet 3 had an average profit of $12,064 per time window, compared to $11,196 for Fleet 2 and $10,621 for Fleet 1. Additionally, Fleet 2 similarly outperformed Fleet 1, demonstrating the economic benefits of advanced technology in current vessels.

Table 4.7: Profit comparison among different fleets across multiple instances

Instance	Instance Size	Profit, Fleet 1	Profit, Fleet 2	Profit, Fleet 3
Instance 1	Size 1	10046	10612	11589
Instance 1	Size 2	15340	14926	16315
Instance 1	Size 3	28876	29821	31423
Instance 1	Size 4	43106	45826	47387
Instance 1	Size 5	78537	85426	89418
Instance 2	Size 1	6716	7291	8552
Instance 2	Size 2	15803	16882	18307
Instance 2	Size 3	27790	31463	32817
Instance 2	Size 4	44937	47134	49784
Instance 2	Size 5	77351	85984	89263
Instance 3	Size 1	11599	12805	13192
Instance 3	Size 2	13054	15151	17167
Instance 3	Size 3	28170	29681	31752
Instance 3	Size 4	44846	48134	50427
Instance 3	Size 5	73493	82536	86782
Instance 4	Size 1	10404	10607	11810
Instance 4	Size 2	16974	17243	18591
Instance 4	Size 3	28113	29750	31729
Instance 4	Size 4	47100	49120	51944
Instance 4	Size 5	65523	59377	85654
Instance 5	Size 1	11635	12386	13727
Instance 5	Size 2	12446	15303	16697
Instance 5	Size 3	28591	30062	32233
Instance 5	Size 4	43951	46510	49646
Instance 5	Size 5	65353	61657	58931

4.9 Discussion

As presented in the section above, the results show a significant overall economic benefit of utilising the new green advanced vessel technology. However, this truth comes with some modifications. Both the model and the input data are based on some major simplifications, making the realism of the results questionable. In this section, we will discuss some weaknesses of this experiment. Thereafter, investigate if it is possible to gain further insight by analysing the solution data and the value of the dual variables. Lastly we will introduce a simple financial method to get a better grasp of the bigger economic picture of.

4.9.1 Weakness

The model and the input has limitations that could affect the results, we will now discuses some of them.

Fleet 3's vessel parameters rely on potential future battery advancements, introducing uncertainty. The conceptual retrofitting of Fleet 3 may be too optimistic, not considering negative aspects such as increased weight, higher fuel consumption when not using electric power, reduced speed, and decreased maneuverability. Additionally, the results do not account for the initial investment required to upgrade the vessels, which will be discussed later in Section 4.9.3. These oversights may favour Fleet 3, resulting in unrealistically high profits.

On the other hand, one potential weakness is related to Dynamic Positioning (DP). Constraint (3.19) requires Fleet 3 vessels to use electric energy exclusively for DP operations. This could affect the optimal solution for Fleet 3, where a profitable job might be avoided due to a lack of available battery power, leading to lower results compared to the diesel-driven Fleets 1 and 2. However, in this experiment, π which by Constraints 3.21 and 3.22, inversely determining the distance of all-electric drive near shore and harbour, is set to infinitely large . With π being infinitely large, there are no other scenarios than DP where the vessel relies solely on battery power. Since the cost benefit of battery power is greater in DP operations compared to normal travel, this should not pose a significant problem in an optimal solution.

The random placement of chargers in the model does not reflect real-world scenarios, as optimal charger placement is an optimisation problem itself. This could negatively affect Fleet 3's profitability, where optimally placed chargers would incentivise more charging, leading to higher profits.

The revenue assumptions for maintenance jobs lack real-world data, making the quantitative results somewhat speculative. However, for comparing different fleets, the results provide useful insights.

4.9.2 Solution data and dual variable analyse

We will now analyse the solution data and the value of the dual variables in the optimal solutions in attempt to gain further insight.

As expected, the sum of the dual variables ω^*_{bt}, associated with the availability of technicians of type b in time window t, corresponding to the primal constraint (3.6) in MP, are zero in all optimal solutions, with one exception of Instance 5, size 5. This is because, in this experiment, the focus is primarily on examining the differences between using various vessels. Therefore, the number of available technicians has been set sufficiently high, ensuring it is not a limiting factor. It is important to note that this does not imply that the technician capacity on each vessel is not a limiting factor, but rather that the total availability of technicians G_{bt} of each type in each time window is adequate.

In all instances in the experiments, all available jobs are completed within the given time frame. Consequently, the dual variables $\rho^*_w > 0 \quad \forall w \in W$. ρ_w is associated with the shadow price of performing a job at turbine w multiple times, from primal Constraint (3.7) in MP. The fact that all these values are positive indicates that all jobs are completed. This is as expected since keeping a wind farm operational should be profitable. However, this does not necessarily mean it is profitable to perform all jobs in all time windows. It depends on the value of the job for the different time windows and the value of other jobs, determining whether a particular job is included in the optimal route for a specific time window.

The solution data indicate that there is a relatively high availability of vessels to perform the jobs. For Fleet 3, the dual variable $\lambda^*_{vt} = 0, \forall v \in V, \forall t \in T$ for most observed instances, with the exception of two. The dual variable λ_{vt} corresponds to the primal constraint (3.5) in the MP and represents the value of allowing a vessel v to perform no more than one additional route in a given time window t. The fact that this value is zero for most optimal solutions indicates that vessel availability for Fleet 3 is not a highly limiting factor. Consequently, most jobs can be done in the most profitable time window. This is not the case for the other fleets. As shown in Table X, the λ^*_{vt} values for Fleet 1 and Fleet 2 are often non-zero.

Two main factors determine the value of λ^*_{vt}. First, there may not be enough vessels to perform all jobs in their respective most profitable time windows. This includes the capacity to transfer technicians to the respective jobs and having time to reach all jobs within the time frame. However, given that Fleet 2 has the same capacity to transfer technicians as Fleet 3 and also has higher speed, resulting in higher vessel availability, this is unlikely the case for Fleet 2. Similarly, for Fleet 1, the λ_{vt^*} is non-zero in many cases while only two of the three vessels in the fleet are utilised.

A second and more relevant reason is that, unlike Fleet 3, the other two fleets are heterogeneous, meaning the vessels have different attributes. One of the biggest internal differences in the two fleets is related to fuel consumption and thereby operational costs. Therefore, it is highly likely that the λ^*_{vt} value indicates the potential cost savings if the most operationally expensive vessel in the fleet is substituted with an extra of the cheapest. While this is not always the case, the solution data indicate this trend.

In the optimal solutions, there are relatively low costs related to DP operations. Several reasons might explain this. First, in an optimal solution, there is no incentive to perform DP operations if they are avoidable. Another reason is that the model does not impose a specific speed for the vessels but rather sets a lower limit on the time required to reach a destination, calculated based on speed and distance parameters. Vessels can take longer if desired, but energy consumption and cost will remain the same. Normally, this has little effect since the cost of traversing a distance is the same, and going slower than allowed is usually not optimal. But in this case, the flexibility allows the vessels to operate at a slower speed to minimise costs associated with DP. This might be why, in the optimal solution, DP costs are generally low. This is advantageous as it represents the real world more accurately without adding significant model complexity with variable speed. However, as the time spent in DP (T_{dp_i}) is updated only if a vessel stays at the turbine while work is performed, as described by Constraint (3.26), it is uncertain how big an impact this effect has on the optimal solution.

The most surprising insight from this analysis is the significant value of λ_{vt^*}, where it is still optimal for Fleet 2 to use only the cheaper vessel. This tells us that it is more optimal to wait for the "cheaper" vessel to be available rather than perform maintenance work in the most profitable time window. Since the revenue from each maintenance work typically relates to the risk of breakdowns and the price of downtime while work is performed, this indirectly indicates that having an inefficient fleet promotes maintenance scheduling that may risk the efficiency of wind farm operations. This might further incentivise the promotion of advancing technology. On the other hand, this also signals that the subcontractor could potentially make large cost reductions by reducing its fleet to only one vessel. Furthermore, in other applications, the consistently low value of ω_{bt}^* suggests potential cost savings by reducing the number of technicians in rotation. While the high value of ρ_w indicates that the OFW operator could reduce its costs related to maintenance operations by reducing the price rate for the given maintenance task they want performed.

Table 4.8: Fleet Information Table across multiple instances

Instance	Size	Fleet 1		Fleet 2		Fleet 3	
		Vessel Used	Sum λ	Vessel Used	Sum λ	Vessel Used	Sum λ
Instance 1	Size 1	1	0	1	1231	1	0
	Size 2	1	0	1	1273	1	0
	Size 3	2	798	1	2746	2	0
	Size 4	2	989	1	2505	2	0
	Size 5	2	3531	1	4417	1	0
Instance 2	Size 1	1	395	1	0	1	0
	Size 2	1	102	1	0	1	0
	Size 3	1	283	1	178	1	0
	Size 4	2	288	1	255	1	0
	Size 5	3	307	2	603	2	0
Instance 3	Size 1	1	0	1	0	2	0
	Size 2	2	3175	1	1277	1	0
	Size 3	2	798	1	2746	2	0
	Size 4	1	1917	1	3080	2	0
	Size 5	2	4428	1	3286	2	0
Instance 4	Size 1	1	0	1	571	1	0
	Size 2	1	705	1	191	1	0
	Size 3	1	1237	1	2124	2	0
	Size 4	1	1749	1	1362	2	0
	Size 5	3	5128	2	5402	2	2286
Instance 5	Size 1	1	1325	1	157	1	0
	Size 2	2	1305	1	1885	1	0
	Size 3	2	1168	1	1606	1	0
	Size 4	2	577	1	2647	2	0
	Size 5	3	4920	2	8161	2	32424

4.9.3 Economic Discussion

The findings from Experiment 2 highlight the economic benefits of using advanced green maritime technology for offshore wind farm (OWF) maintenance. Here, we present a financial method to evaluate the economic impact of upgrading vessel fleets, including the initial retrofitting costs and payback time.

We retrofitted two identical vessels with large battery packs allowing for charging and all-electric drive. According to MAN Energy Solutions 2019, the cost of a future retrofit for a marine battery system is estimated be 250 USD/kWh. Adding indirect costs, we set the total price at 400 USD/kWh. This results in an initial investment of $647,200 per vessel and $1,294,400 in total.

The paper by Hasselwander et al. (2023) estimates that future LiFePO4 batteries will last 5000 cycles [21]. Each time window in our experiment represents 12 hours, giving a battery lifetime of about 6.8 years with 730 time windows per year.

From Section 4.6, Fleet 3 makes \$868 more profit per time window than Fleet 2, resulting in an annual profit difference of \$633,731. Using the payback time formula,

$$\text{SPT} = \frac{\text{Initial Investment}}{\text{Annual Cash Inflow}}$$

we find the investment pays off in about 2 years. While the data for Fleet 3 involves some uncertainty, this calculation showcase a highly profitable investment. Further more, with Fleet 3 achieving higher average profits per time window, with increased profit margins with larger instances. This might indicates that the new green technology is more adaptive, scalable, and capable of efficiently handling larger future wind farms.

Chapter 5

Conclusions and Future Work

5.1 Conclusion

This thesis has explored the potential for optimizing maintenance operations for offshore wind farms (OWFs) by using advanced green maritime technology. By implementing a maintenance scheduling model, various fleet configurations were assessed, including traditional diesel-powered vessels, current vessels with advanced technology, and a hypothetical future fleet incorporating potential green technologies. The results indicate that green technology, more specifically, the use of hybrid Crew Transfer Vessels (CTV), might enhance economic profitability and operational efficiency in OWF maintenance operations.

The introduction of green technologies in vessel fleets, such as large battery packs, allowing for electric drive, suggests a potential economic benefit with a relatively rapid payback period. The advanced fleet generally outperformed traditional and current technology fleets in terms of profit per time window, and results indicating that green technology could be more scalable and adaptable for larger future wind farms. Furthermore, adopting low-emission vessels may reduce and environmental impact, aligning with global goals to reduce greenhouse gas emissions.

To perform this analysis, we constructed a short-term maintenance schedule optimisation algorithm based on the mathematical programming concept of column generation. We further experimented by introducing some minor changes to the traditional solving approach to increase efficiency and reduce computational time. This involved a dynamic solving approach where the sub-problem is not necessarily solved to optimality for each iteration. This experiment produced promising results, potentially reinforcing the relevance of column generation in the specific problem of maintenance scheduling at offshore wind farms utilising low-emission vessels.

5.2 Future Work

Future research could address the limitations identified in this study. For example, integrating more realistic revenue assumptions, vessel parameters, and optimal charger placement could make the model more practical. Incorporating real-world constraints such as varying weather conditions and dynamic positioning requirements would enhance the model's accuracy and applicability.

Comparative studies on optimisation techniques could yield valuable insights. For instance, comparing the dynamic column generation approach with the warm start Column Generation approach. Additionally, enhancing the column generation approach by generating a solution pool per iteration in the sub-problem could potentially improve efficiency and robustness.

Chapter 6

Transparency and Tools

6.1 Writing Tool

In this section, I will show how ChatGPT-4 has been used as a writing tool in this thesis. The process can be described as an iterative sparring with self-written text, proofread and finessed by GPT, then rewritten to further fit the intended purpose, and so on until the final product is made. This is done by using the following prompt:

Requirements

- Respect ALL the following rules:
  - Adhere closely to the original wording.
  - Be Concise.
  - Respect the LaTeX syntax e.g.
  - cite
  - Use the dollar sign for the mathematical environment.
  - Use short sentences.
  - Avoid contractions.
  - Use British English.
  - Use a vocabulary understandable by non-native speakers.
  - Avoid hyperbolic language.
  - Use active voice whenever possible.

Task

Proofread the following paragraph:

6.1.1 example

Now, an example will be shown to better illustrate the effects on the final text

Initial input

6.1.2 Dual Problem

Every LP often called as the primal problem, has a related dual problem [53]. The dual problem provides an alternative perspective of the same problem by foucsing on the value of the resources rather than maximizing profit. The dual of the the primal formulated in the Section above can be formulated as

$$W_{LP} = \min\{b^\top y : A^\top y \geq c, y \geq 0\},$$

where is y a vector of dual variables. The dual variabel y or also referd to as shadow price can be seen as the the value of resources available, and quantifies the rate the objective function value changes with respect to changes in the right-hand side of the of the primal constraints, dentoted by the vecor b, where each constraint in the primal has a corresponding dual variabel. Using the exampel from LP section, the value of the dual variabel can be interpreted as how much more profit there is to be made by increasing the avilibility of corresponding rescourse by no more than a unit of one. While the dual constraint ensures that the solution of the dual provides a bound to the primal. The dual constraint $A^\top y \geq c$ enforces the dual variabel to not subsied the the value the resource provide the primal objective. where A describes how much the of the primal decision variables use the resources and c indicates how much primal objective increas with the respective decision variables.

The dual constraints ensure that the solution to the dual problem provides a valid lower bound on the primal objective function. This is achieved by enforcing that the dual variables (prices of resources) are chosen such that the cost of resources is not less than the contribution each resource provides to the primal objective function. Specifically, the condition $A^\top y \geq c$ must hold, where A describes how the primal decision variables use the resources and c indicates how much the primal objective function increases with the respective decision variables. In essence, the dual variables must be sufficient to cover the costs or contributions represented by the primal objective function coefficients.

The relationonship between the primal and dual problems, is decrbied through the concepts of strong and weak duality. Weak duality states that,in maximizatin problems the value of the objective function for any feasible solution of the dual problem provide upper bound to the value of the objective function for any feasble solution of the primal problem. Strong duality states that if the primal has a optimal solution, then the dual also has an optimal solution, and the optimal objective functions value are equal. The Strong duality hold for all This relationship netween primal and duel problem provide important and valuebal insight for many aplications in MP, and are epsacaly important in LP, as will be shown in the reminde of this theis.

6.1.3 First out-put

Every linear programme (LP), often called the primal problem, has a related dual problem [53]. The dual problem provides an alternative perspective of the same problem by focusing on the value of the resources rather than maximising profit. The dual of the primal formulated in the section above can be written as

$$W_{LP} = \min\{b^\top y : A^\top y \geq c, y \geq 0\},$$

where y is a vector of dual variables. The dual variable y, also referred to as the shadow price, represents the value of resources available. It quantifies the rate at which the objective function value changes with respect to changes in the right-hand side of the primal constraints, denoted by the vector b, where each constraint in the primal has a corresponding dual variable. Using the example from the LP section, the value of the dual variable can be interpreted as the additional profit made by increasing the availability of the corresponding resource by no more than one unit.

The dual constraint ensures that the solution of the dual problem provides a bound to the primal problem. The dual constraint $A^\top y \geq c$ enforces that the dual variable does not underestimate the value the resource provides to the primal objective, where A describes how much of the primal decision variables use the resources and c indicates how much the primal objective increases with the respective decision variables.

The dual constraints ensure that the solution to the dual problem provides a valid lower bound on the primal objective function. This is achieved by enforcing that the dual variables (prices of resources) are chosen such that the cost of resources is not less than the contribution each resource provides to the primal objective function. Specifically, the condition $A^\top y \geq c$ must hold, where A describes how the primal decision variables use the resources and c indicates how much the primal objective function increases with the respective decision variables. In essence, the dual variables must be sufficient to cover the costs or contributions represented by the primal objective function coefficients.

The relationship between the primal and dual problems is described through the concepts of strong and weak duality. Weak duality states that, in maximisation problems, the value of the objective function for any feasible solution of the dual problem provides an upper bound to the value of the objective function for any feasible solution of the primal problem. Strong duality states that if the primal has an optimal solution, then the dual also has an optimal solution, and the optimal objective function values are equal. This relationship between the primal and dual problems provides important and valuable insight for many applications in mathematical programming and is especially important in linear programming, as will be shown in the remainder of this thesis.

6.1.4 Second input

Every LP, often called the primal problem, has a related dual problem [53]. The dual problem provides an alternative perspective of the same problem by focusing on the value of the resources rather than maximising. The dual of the primal formulated in the section above can be written as

$$W_{LP} = \min\{b^\top y : A^\top y \geq c, y \geq 0\},$$

where y is a vector of dual variables. The dual variable y, also referred to as the shadow price, represents the value of resources available. It quantifies the rate at which the objective function value changes with respect to changes in the right-hand side of the primal constraints, denoted by the vector b, where each constraint in the primal has a corresponding dual variable. Using the example from the LP section, the value of the dual variable can be interpreted as the additional profit made by increasing the availability of the corresponding resource by no more than one unit.

The dual constraint $A^\top y \geq c$ ensures that the dual variables is bounded to value that the resources provide to the primal objective. Here, A represents the resource usage by the primal decision variables, and c denotes the contribution of these variables to the primal objective. The dual constraints is important to ensure that the solution to the dual problem provides a valid bound on the primal objective function. This is done by ensuring that the prices of resources are chosen such that the cost of resources is not less than what they contribute to the primal objective function.

The relationship between the primal and dual problems is described through the concepts of strong and weak duality. Weak duality means that, in maximization problems, the objective function value of any feasible solution to the dual problm is always greater than or equal to the objctive function value of any fesible solution to the primal problem. Strong duality means that if the primal problem has a optimal solution, then the dual problem also has one, and their optimal objective function values are equal. This relationship between the primal and dual problems is very useful for many applications in mathematical programming, especially in linear programming, as will be shown in the remainder of this thesis.

6.1.5 final out put

Every LP, often called the primal problem, has a related dual problem [53]. The dual problem provides an alternative perspective of the same problem by focusing on the value of the resources rather than maximising profit. The dual of the primal formulated in the section above can be written as

$$W_{LP} = \min\{b^\top y : A^\top y \geq c, y \geq 0\},$$

where y is a vector of dual variables. The dual variable y, also referred to as the shadow price, represents the value of resources available. It quantifies the rate at which the objective function value changes with respect to changes in the right-hand side of the primal constraints, denoted by the vector b, where each constraint in the primal has a corresponding dual variable. Using the example from the LP section, the value of the dual variable can be interpreted as the additional profit made by increasing the availability of the corresponding resource by no more than one unit.

The dual constraint $A^\top y \geq c$ ensures that the dual variables are bounded by the value that the resources provide to the primal objective. Here, A represents the resource usage by the primal decision variables, and c denotes the contribution of these variables to the primal objective. The dual constraint is important to ensure that the solution to the dual problem provides a valid bound on the primal objective function. This is done by ensuring that the prices of resources are chosen such that the cost of resources is not less than what they contribute to the primal objective function.

The relationship between the primal and dual problems is described through the concepts of strong and weak duality. Weak duality means that, in maximisation problems, the objective function value of any feasible solution to the dual problem is always greater than or equal to the objective function value of any feasible solution to the primal problem. Strong duality means that if the primal problem has an optimal solution, then the dual problem also has one, and their optimal objective function values are equal. This relationship between the primal and dual problems is very useful for many applications in mathematical programming, especially in linear programming, as will be shown in the remainder of this thesis.

6.1.6 Final product

Every LP, often called the primal problem, has a related dual problem [53]. The dual problem provides an alternative perspective of the same problem by focusing on the value of the resources. The dual of the primal formulated in the section above can be written as

$$W_{LP} = \min\{b^T y : A^T y \geq c, y \geq 0\},$$

where y is a vector of dual variables. The dual variable y, also referred to as the shadow price, represents the value of resources available. It quantifies the rate at which the objective function value changes with respect to changes in the right-hand side of the primal constraints, denoted by the vector b, where each constraint in the primal has a corresponding dual variable. Using the example from the LP section, the value of the dual variable can be interpreted as the additional profit made by increasing the availability of the corresponding resource by no more than one unit.

The dual constraint $A^T y \geq c$ ensures that the dual variables are bounded by the value that the resources provide to the primal objective. Here, A represents the resource usage by the primal decision variables, and c denotes the contribution of these variables to the primal objective. The dual constraint is important to ensure that the solution to the dual problem provides a valid bound on the primal objective function. This is done by ensuring that the prices of resources are chosen such that the cost of resources is not less than what they contribute to the primal objective function.

The relationship between the primal and dual problems is described through the concepts of strong and weak duality. Weak duality means that, in maximization problems, the objective function value of any feasible solution to the dual problem is always greater than or equal to the objective function value of any feasible solution to the primal problem. Strong duality means that if the primal problem has an optimal solution, then the dual problem also has one, and their optimal objective function values are equal [53]. This relationship between the primal and dual problems is very useful for many applications in mathematical programming, especially in linear programming, and will be relevant in the remainder of this thesis.

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