

NOTES AND INSIGHTS

Measuring the change in behavior of a system with a single metricWilliam Schoenberg,^{a,b,*}  Robert Eberlein^b and Pål Davidsen^a*Abstract*

Loops that Matter (LTM) provides a practical and comprehensive way to understand which feedback loops are driving model behavior at different points in time. LTM describes from which loops the observed change in behavior across all stocks in the model originate. In this paper we present a method to measure the magnitude of the change in behavior of all stocks in the model based on net flow values, relative to the magnitude of change taking place across an entire observation (simulation) period. We call this new metric the “system change”. We then demonstrate how our system change metric can be visualized using loop scores to highlight those loops that are predominantly responsible for the changes in behavior exhibited by the model. This helps analysts focus in on the feedback loops that are prime candidates for interventions to change the model behavior.

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The problem

Loops that Matter (LTM) is an algorithmic method for performing a “formal assessment of dominant structure and behavior” (Duggan and Oliva, 2013) applicable to models of any size and complexity (Schoenberg, 2020; Schoenberg *et al.*, 2020, 2023). LTM is the first of the algorithmic methods,ⁱ developed to perform loop dominance analysis, that has been integrated within commercially available system dynamics software (Stella Architect v2.0 and later) and is, therefore, currently applied by practitioners as well as researchers in the field, including Aboah and Enahoro (2022), Aboah and Setsoafia (2022); Hovmand *et al.* (2022), Mumtaz *et al.* (2022), and Kliem *et al.* (2021). LTM is unique among the behavior domain algorithmic

ⁱOther algorithmic methods are Eigenvalue-based methods—that is, Eigenvalue Elasticity Analysis (Graham, 1977; Forrester, 1982; Eberlein, 1984; Saleh, 2002; Güneralp, 2006; Gonçalves, 2009; Saleh *et al.*, 2010; Kampmann, 2012; Moxnes and Davidsen, 2016; Oliva, 2016; Naumov and Oliva, 2018; Oliva, 2020) and Pathway Participation Metric-based methods (Mojtahedzadeh, 1996; Mojtahedzadeh, 1997; Mojtahedzadeh *et al.*, 2004; Mojtahedzadeh, 2008; Mojtahedzadeh, 2011), including the Loop Impact method (Hayward and Boswell, 2014; Sato, 2016; Hayward and Roach, 2017; Hayward and Roach, 2022).

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methods applied to formal loop dominance in that it defines and measures loop dominance on a model-wide basis as opposed to within the context of a single stock. Schoenberg *et al.*, define loop dominance in their 2020 article in the quote below:

[In LTM] We define loop dominance as a concept which relates to the entirety of a model, as opposed to loop dominance being something that affects a single stock. For loop dominance to apply to the entire model, we require that all stocks are connected to each other by the network of feedback loops in the model. For models where there are stocks that do not share feedback loops, we consider each subcomponent of interrelated feedback loops individually, and we refer to each model substructure as having a separate loop dominance profile. [In LTM] Our measurement of loop dominance is specific to the particular time period selected for analysis. We say that a loop (or set of loops) is dominant if the loop(s) describe at least 50% of the observed change in behavior across all stocks in the model over the selected time period.

Analysts using LTM often struggle with the lack of a formal definition for the concept of “observed change in behavior across all stocks in the model”. For clarity we will refer to this concept as the aggregate state change. The literature published to date has not specifically addressed the aggregate state change and, in this paper, we will provide a computational definition along with a description of visualization techniques for displaying this information. We will then show how this can be combined with the results of LTM dominance measures to help narrow the focus of the analysis to key time ranges.

Background on LTM

LTM computes three metrics which are used to measure loop dominance. These three metrics were first elaborated in Schoenberg *et al.* (2020) and were subsequently updated in Schoenberg *et al.* (2023) (the update first appearing in Stella Architect version 2.1). The three metrics are the link score, the loop score and the relative loop score.

The link score is a measure of the contribution and polarity of any causal link from an independent to a dependent variable regardless of whether the link contains an integration process or not. The link score is illustrated below for the case where there are two inputs (x and y) to the dependent variable z characterized by the equation $z=f(x,y)$. The link score for the link $x\rightarrow z$ is written in a discontinuous form to reflect the implementation of the calculation. Schoenberg *et al.* (2023) shows how to measure the link score between a flow and a stock using the same process.

Equation 1: The link score for the link x to z for the equation $z=f(x,y)$ is given by

$$LS(x \rightarrow z) = \begin{cases} \left[\left| \frac{\Delta_{xz}}{\Delta z} \right| \cdot \text{sign} \left(\frac{\Delta_{xz}}{\Delta x} \right) \right], \\ 0, \Delta z = 0 \text{ or } \Delta x = 0 \end{cases} \quad (1)$$

where Δz is the change in z from the previous to the current time, Δx is the change in x over that same time step, and Δ_{xz} is the change in z with respect to x , which is often called the partial change in z with respect to x . In plain English, it is the amount that z would have changed, had x changed the amount it did, yet y had not changed. The first major term in Eq. 1 represents the magnitude of the link score; the second is the link score polarity.

The loop score is computed by multiplying the link scores for all links in a loop. This is a demonstrably unique measure which has a well-defined relationship to the Loop Impact metric of Hayward and Boswell (2014). For further explanation of the relationships between LTM, Pathway Participation Metric and PPM, see Schoenberg *et al.* (2023).

Equation 2: The definition of loop score for loop x , which contains n links through n variables (v), is given by

$$\text{Loop score}(L_x) = [LS(v_1 \rightarrow v_2) \cdot LS(v_2 \rightarrow v_3) \dots \cdot LS(sv_{n-1} \rightarrow v_n) \cdot LS(sv_n \rightarrow v_1)] \quad (2)$$

The third and final key metric is the relative loop score (Eq. 3), which compares the contribution of feedback loops to determine which loops are dominant at any point in time. The relative loop score requires that there is no independence across the loops it compares and, when possible, uses the exhaustive set of feedback loops as the basis for comparison.

Equation 3: The definition of the relative loop score for loop x normalized over all loops m analyzed in the chosen loop set is given by

$$\text{Relative loop score}(L_x) = \left(\frac{\text{Loop score}(L_x)}{\sum_{y=0}^m |\text{Loop score}(L_y)|} \right) \quad (3)$$

The sign of a relative loop score represents the polarity of the feedback loop. The relative loop score is a normalized measure taking on a value between -1 and 1 . It reports the polarity and instantaneous fractional contribution of a feedback loop to the change in value of all connected stocks. By comparing loop scores, it can easily be determined which loop (or set of loops of the same polarity) are dominant—that is, contribute the most to the behavior of all stocks in the feedback loop set under study at any point in time (indicated by the fact that their loop scores sum to over 50% of the total

score). This normalization is critical to maintaining scores that are easy to work with.

LTM, with the loop score, thus measures dominance as a contribution to the aggregate state change at each point in time. Since both loop scores and the aggregate state change vary over the course of a simulation, it is natural for analysts to ask at which times should they pay the most attention to the loop dominance profile to understand simulated behavior. This can be done, for example, by partitioning a simulation into different segments (e.g., growth, saturation, collapse) and then reviewing the loop scores within each segment of the partitioned loop dominance profile. This partitioning will vary between model scenarios and adds an additional analysis step. By automatically measuring the magnitude of the aggregate state change, it is possible to decrease the burden on the analysts. From the perspective of an LTM analysis, it is most important to look at times when the aggregate state change is large. That is typically around the inflection points of one or more of the stocks in the model. The feedback loops that contribute the most (i.e., have large loop scores) during these time periods are those that, when modified, can cause a the most significant change in the overall behavior of the model at that time.

A simple demonstration of the problem

Using the Bass diffusion model (Bass, 1969), the problem of having no formal definition for the aggregate state change becomes clearer. In Figure 1, the structure and behavior of a Bass diffusion model is portrayed. In Figure 2, the result of an associated LTM analysis is presented.

By studying the feedback loop dominance profile in Figure 2 in isolation, analysts may mistakenly overemphasize the significance of the loop R1, which is completely dominant at the beginning of the run, or B1, which is completely dominant at the end of the run. While these conclusions regarding dominance are a correct interpretation of the relative loop scores in Figure 2, it is clear from Figure 1 that the most significant dynamics (the largest changes in stock values as well as pattern of behavior) are those taking place in the middle of the run, not at the beginning or the end. LTM is doing exactly what it was designed to do—reporting the causes of the aggregate state change, no matter how small that change may be, effectively ignoring the magnitude of the aggregate state change at each point in time. We argue that information about that magnitude is critical to analysts when contextualizing the significance of a loop dominance profile. During those periods, when the aggregate state change is small, then none of the model variables are changing very much (“nothing is happening”), whereas, during periods when the aggregate state change is large, some variables are changing significantly (“something is happening”). The model behavior is made up

Fig. 1. Stock and flow diagram of Bass Diffusion model along with a plot of Potential Adopters (Blue Solid) and Adopters (Red Dashed)

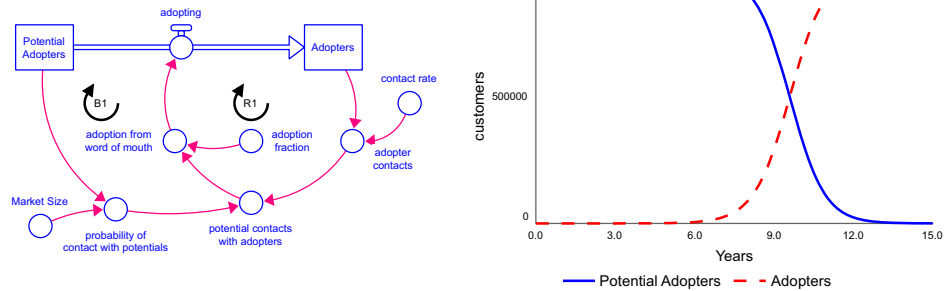
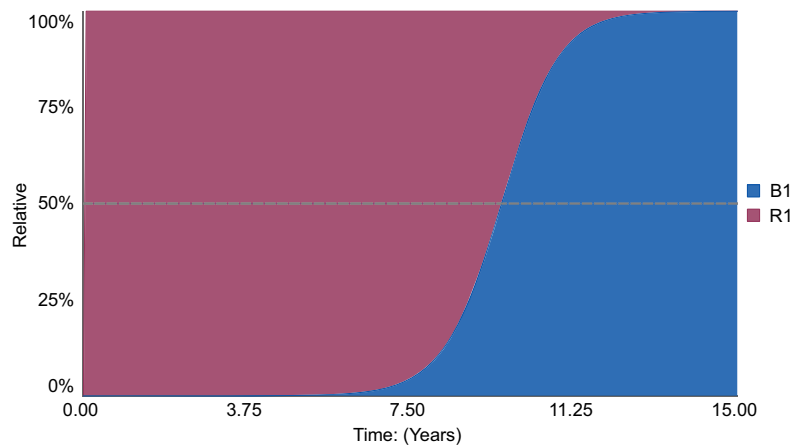


Fig. 2. The loop dominance profile for the Bass Diffusion model—A plot of the Relative Loop Scores for both feedback loops in the model.



of the accumulation of the net flows. Therefore, if the analysts wish to change the behavior of a model, they may well do so by influencing the feedback loops that are actively contributing to significant changes in behavior—that is, when the change in the aggregate state is relatively large.ⁱⁱ

Our goal then, is to superimpose the loop dominance profile from LTM onto a measure of the aggregate state change so that analysts may focus their attention on loops active during times of change. Without such a metric, analysts must take advantage of their intuitive understanding of when the stocks in the model are most in flux and combine that understanding with the loop dominance profile to develop a more complete understanding of how to influence model behavior. This may be fairly easy to do for a simple diffusion model but can be very difficult in models with several stocks.

ⁱⁱIt is important to note that the intervention does not need to happen when the behavior is being generated. Oftentimes it may be beneficial to intervene long before an undesirable change in behavior materializes and is recognized.

Solving the problem

Conceptually, a system as a whole is changing in direct proportion to the net flows of the stocks within it. Therefore, the net flows for each stock will be the key input to any metric quantifying the aggregate state change. The challenge is to combine those net flows in a metric that work well for models of different complexity, formulated in different ways. Though we ultimately settled on an additive measure, it is helpful to understand two alternatives we considered and why we ended up rejecting them.

Taking the product of all net flows, just as we take the product of link scores to compute a loop score was our first inclination. It solved the scaling problem discussed below, but unfortunately has a significant flaw. Consider a model that oscillates, where one stock stops changing (at the turnover point) while the other stocks are changing the fastest (at a point of inflection). The turnover point would give a net change of 0, which when multiplied by anything is still 0. But when one stock is constant, it does not mean that all the other stocks in the model are not changing—in fact they may be changing very rapidly. Multiplying would, when a single net flow is zero, report zero also for the aggregate state change at any point in time. That would be incorrect. Multiplying effectively makes the smallest change (net flow) dominate the measure of the aggregate state change, which is the opposite of what we are striving for.

Another approach we considered was to make the largest change dominate the measure of aggregate state change by selecting the largest net flow at each point in time. There is an obvious scaling issue here, which we will address below. In that case, there is also another conceptual problem. The largest change approach would treat the case of one stock changing rapidly, while another does not change, the same as the case where the second stock is changing—just not as quickly as the first. Clearly, a good measure of the aggregate state change would result in the latter case being larger.

That leaves us with an additive approach. The adding needs to take place in such a way as to satisfy two conditions.

First, each net flow contribution must be dimensionless so that it is at least possible to combine changes in stocks of different units. For instance, it does not make sense to add the change in the number of apples in a warehouse to the change in the number of dollars in a bank account. Any metric with dimensioned components would necessarily be uninterpretable. That is not to say that by making a dimensionless metric we are guaranteed to have one which is safe to add across stocks with values that differ in magnitude. Even after being made dimensionless, we can think of residual “types” as having been left behind in the process of making the stock values dimensionless. This brings us to the second condition.

A good measure of the aggregate state change must be independent of scale and scaling; it must be able, for example, to combine flows for variables ranging from 0 to 1 with flows ranging from 1 billion to 2 billion. To ensure that the dimensionless net flow contribution is safe to combine, the relative system change metric cannot vary across runs if the loop dominance profile does not also vary across those same runs. This means that even if the modeler changes the units (and therefore the values) of a stock (and all of its associated parameters) from grams to kilograms, the computed system change metric must be invariant. Ensuring this means that we are not improperly adding types, which would cause the introduction of an indirect units-dependency into the aggregate state change metric.

A sensible metric is therefore a weighted average of net flows; it is the selection of those weights that turn out to be the most problematic. Our first inclination was to come up with a measure that could be computed at each point in time, using only information available at that point in time (as we do with loop scores). We tried out various derivations of such a measure. In retrospect, we do not believe such an approach can succeed because the comparison we want to make is from time to time, not loop to loop. Thus, the normalization needs to be across time, not across loops, and we cannot conceive of a metric that normalizes across time without first simulating across time.

With all of that in mind, we developed a “system change” metric, which is a weighted average of the absolute value of the net flow for each stock at each point in time. The weights used are defined the same way for all times and are equal to the reciprocal of the sum of the absolute value of the net flows across time. Thus, we divide the amount that each stock is currently changing by the total amount it changes during the entire simulation period, and we then add the result up across all stocks. Finally, we then divide the result by the largest value it takes over all times during the simulation period so as to obtain a normalized score, ranging from 0 to 1 for each time. The details of this computation are outlined in the pseudo code in Figure 3, as well as in the Eqs 4–6. As long as the net flows are not changing dramatically between computational intervals (the model is well behaved) the results will not be sensitive to dt .

Equation 4: The definition of accumulated stock change for stock S is given by

$$A_s = \sum_{\substack{\text{Time=Start time} \\ \text{by } \Delta T}}^{\text{Stop time}} \left| \frac{dS}{dt} \right| \quad (4)$$

Equation 5: The definition of raw system change over time for all stocks in the model is given by

```

let system_change_overtime = []
let raw_system_change_overtime = []
let max_raw_system_change = 0;

for (let stock in model.stocks)
{
  stock.accumulated_stock_change = 0;

  for (let time in model.timeAxis)
  {
    stock.accumulated_stock_change +=
      ABS(stock.net_flow_at(time));
  }
}

for (let time in model.timeAxis)
{
  raw_system_change_overtime[time] = 0;
  for (let stock in model.stocks)
  {
    raw_system_change_overtime[time] +=
      ABS(stock.net_flow_at(time)
        / stock.accumulated_stock_change);
  }

  if (raw_system_change_overtime[time] >
      max_raw_system_change)
  {
    max_raw_system_change = raw_system_change_overtime[time];
  }
}

for (let time in model.timeAxis)
{
  system_change_overtime[time] =
    raw_system_change_overtime[time]
    / max_raw_system_change;
}

```

Fig. 3. Pseudo code used to calculate the system change metric which is executed after the entire simulation is complete.

$$R = \sum_{\text{All stocks } S} \frac{\left| \frac{dS}{dt} \right|}{A_s} \quad (5)$$

Equation 6: The definition of system change over time is given by

$$\text{System change over time} = \frac{R}{\max(R)} \quad (6)$$

As can be seen in Eq. 5, we compare the net flow for a stock $\left| \frac{dS}{dt} \right|$ at a given time with its accumulation over the entire time period. This means that if a stock changes a significant amount of its total change (A_s) over a single time period, that stock will not contribute significantly to the system change metric. This is a beneficial property of the system change metric that will ensure we do not distract the analysts with relatively insignificant changes in stock values and, therefore, inappropriately flag feedback loops as significant when loops are active merely during times which stocks remain predominantly unchanged.

In Eq. 6 astute readers will notice that we are adding dimensionless values across stocks that originally had different units. Because we have made the values dimensionless via scaling by the accumulated stock change for each stock S (A_s), there is no residual scaling effect. For example, changing units from grams to kilograms will change both the numerator and denominator proportionally. Still, the raw system change in any instant (R) is the fraction that was observed at one time T of the total change across all stocks over all time periods—an abstract and unintuitive concept at best. Equation 6 serves to make the system change metric more easily interpreted, and rescales the values to be between 0 and 1 for each point in time. This makes it easier to reason about relative system change values, where a value of, say, 0.5 means the aggregate state change at that instant is half of the maximum observed aggregate state change.

Continuing with the Bass diffusion example, Figure 4 shows the calculated system change over the simulation period. In this plot, we clearly see the inflection point at the peak of adopting. Figure 4 (which is identical to the single net flow, adopting, scaled (to percentages) by its peak value) can now be used by analysts to identify the period of largest change, to help them contextualize the loop dominance profile and focus on times where the model is most in flux.

The obvious visualization extension is to scale (multiply) the relative loop scores in the loop dominance profile by the calculated system change metric, which helps emphasize the loops that matter when the system is changing, and thus show their significance to the overall model behavior. We have created the resulting plot as shown in Figure 5. It is important to note that analysts cannot choose which feedback loops to plot. Instead, all feedback loops

Fig. 4. Calculated system change metric for Bass Diffusion example

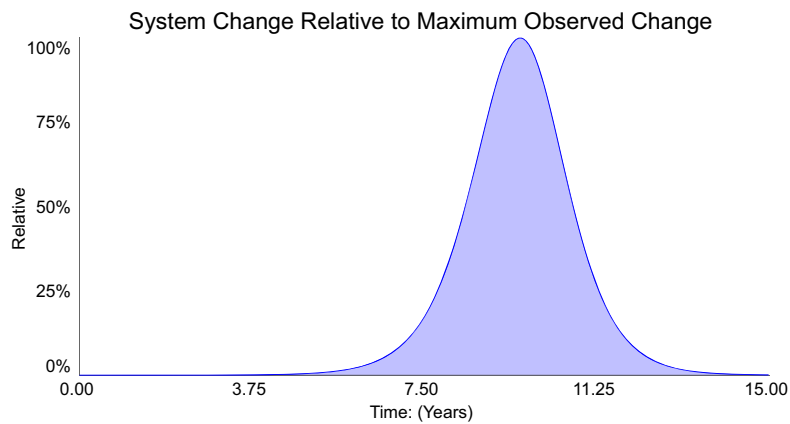
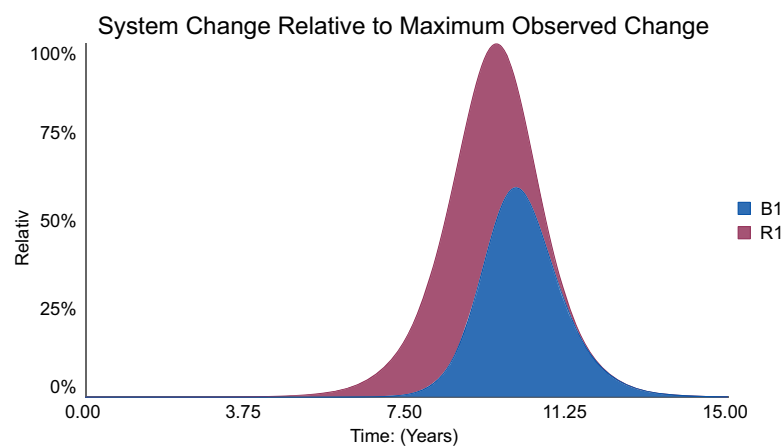


Fig. 5. Contribution to system change plot by feedback loop for the Bass Diffusion example model.



in the model must be plotted so that the total system change is maintained. This mixed visualization method does not work well when the number of explanatory loops (loops of significance) at any one point in time is large—realistically, more than five. In those cases, analysts must fall back to loop dominance profiles (like the one in Figure 2) and use a plot of the system change (like the one in Figure 4) to identify periods of interest. Subsequently, they may narrow the time range over which the loop dominance profile is defined in order to identify important feedback loops that should be made, subject to further studies.

Figure 5 helps reinforce the understanding that the key time period to study is around the inflection point in Adopters. During that period, first R1 is more important, then B1 is. Figure 5 does not have sufficient fidelity to replace Figure 2 for questions about dominance, but is rather meant to be

used in conjunction with it, to focus the analysts' attention to the periods of time with the largest aggregate state change, governed by the loops that contribute most significantly to model behavior. The loops identified through this process are the most likely candidates for interventions aimed at changing the behavior exhibited by the model, since they are the ones predominantly responsible for creating that behavior. The displayed relative loop score values are of limited use numerically. Rather, they are primarily useful pictorially to help analysts focus on the key time periods for further studies into the loop dominance profile.

Applying the system change metric to overshoot and collapse

The yeast alcohol model is a common test model for analysis in the loop dominance literature. In recent literature it has been analyzed by Schoenberg *et al.* (2020), who compared its analysis with other automated loop dominance analysis techniques. The model and behavior shown in Figure 6 are directly reproduced from that paper. This model serves as a good test case for demonstrating the utility of the system change metric because the model contains two stocks measured in non-commensurate units (C = yeast cells; A = mL of alcohol). The loop dominance profile for this model is shown in Figure 7, and the contribution of each of the loops to system change—that is, their significance—is shown in Figure 8.

In this analysis, U1 (a loop of unknown—i.e., changing—polarity) is the “births” loop representing the growth of the yeast cells, and after time 70 additional deaths from alcohol.ⁱⁱⁱ B1 is the natural deaths loops, B2 facilitates the reduction in births from alcohol production, and B3 causes the increase in deaths from alcohol production.

Analysis of this model (Table 1) shows that its dynamics are initially dominated by the reinforcing component of U1 (U1+), which leads to the growth of yeast cells and alcohol. As U1+ loses strength, B2 gains strength with leads to the accumulation of alcohol, reducing births of new yeast cells. This, then, leads the model into a period where the impact of the accumulation of alcohol on deaths (B3) is the dominant feedback loop that finally concludes with the dominance of the natural deaths of yeast cells B1. These phases of dominance are shown in Table 1.

Table 2 reports the total loop score, which is defined in the Stella Architect software as the average magnitude of the relative loop score over the given time period of the simulation (Schoenberg, 2020). In Table 2 this is over the entire simulation time period from 0 to 100. This measure

ⁱⁱⁱThe reason this loop has an unknown polarity is that, under conditions of high alcohol (which are encountered at time 70), the births variable with the equation $B = C*(1.1-0.1*A)/b1$ computes negative values. This flaw has been left uncorrected to maintain the consistency of the model across analyses in the literature.

Fig. 6. Yeast Alcohol model reproduced from Schoenberg *et al.*, 2020 with plot of stock behavior on the left

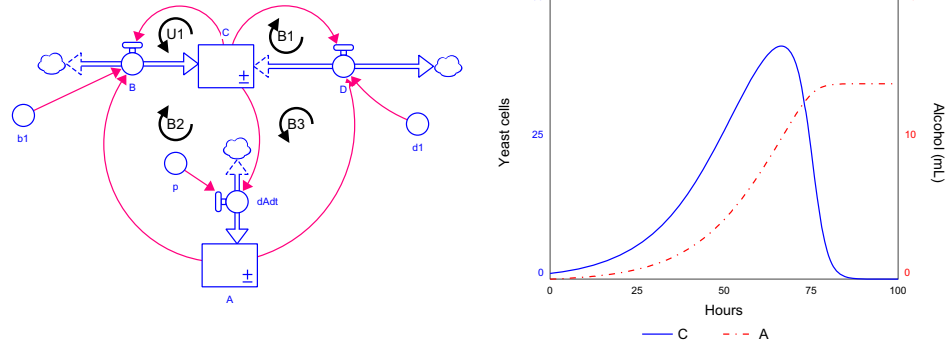
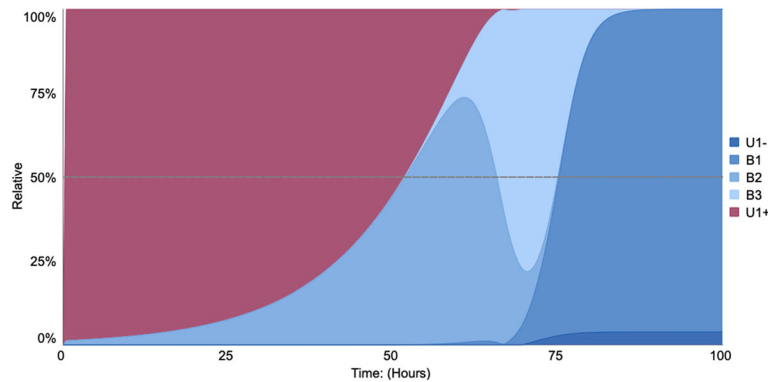


Fig. 7. Loop dominance profile for Yeast Alcohol model. Results are in line with Schoenberg *et al.*, 2020 but uses the improvement in LTM from Schoenberg *et al.*, 2023 which yields small differences in the loop dominance profile.



is referred to as the total loop score because it reports the contribution of a loop to the aggregate stock change over the given time period. We chose to report this metric because it quantifies for analysts the relative importance of each loop in the loop dominance profile over the selected time period. It is the percentage of the area covered by each loop in the loop dominance profile (Figure 7), and it is prominently featured in the Stella Architect software.

By studying only the loop dominance profile and the total loop score over the entire simulation period (Table 2), analysts may place more emphasis on the dominance of U1 because the calculation reports that it describes 49.17% of the aggregate state change over the simulated time period. Analysts may mistakenly assume that B3 is relatively unimportant, as it only describes 9.46% of the aggregate state change over the entire simulation period. Whereas, when the analysts consider the loop dominance profile, contextualized using the system change metric (Figure 8), it becomes apparent that U1+ is not doing all that much before time 25 (because the number of yeast cells is low) and that B1 does not really do much of anything after time 80 (because the yeast cells are dead).

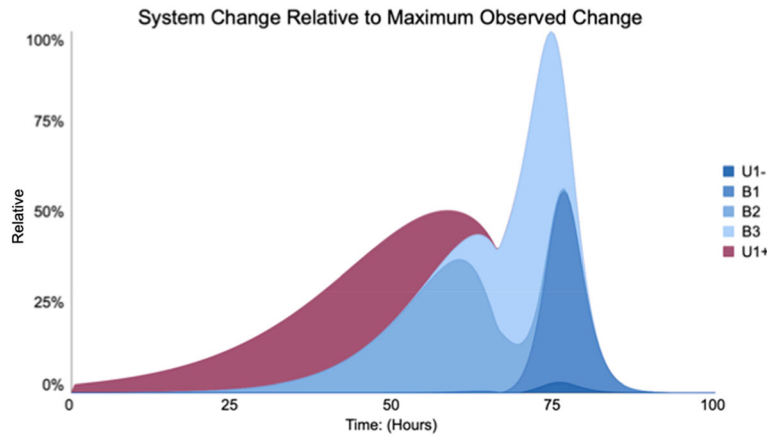


Fig. 8. Contribution to system change for the yeast alcohol model. This shows the analysts that Phases 2 and 3 (Table 1), which are the periods changing the most are the most important areas for study to understand the overshoot and collapse of the system. Looking at just the system change metric (the area under the curve, not the individual loops) the analysts can see that the system change metric is a combination of the two net flows in this model, but not the same as either one of them.

Table 1. Phases of dominance in the yeast alcohol model.

Time range	Phase 1: 0–51.5	Phase 2: 52–66	Phase 3: 66.5–75	Phase 4: 75.5–100
Dominant loop	U1+	B2	B3	B1

Table 2. Loop dominance over the simulation period for all loops in the Yeast Alcohol model

Loop	Total loop score time = [0, 100]
U1	±49.17%
B1	–23.72%
B2	–17.66%
B3	–9.46%

What Figure 8 emphasizes is that the analysts cannot ignore B2 and B3 when making judgments about which loops are important to the behavior generated in this model run and, therefore, about what loops to modulate to change model behavior. Just because these two loops have the least explanatory power over all times (Table 2) does not mean that these loops are unimportant to the behavior exhibited by the model. Figure 8 shows analysts that B2 and B3 are important during periods of major change in the system (the overshoot and the collapse). If analysts wish to intervene in the system in order to change this mode of behavior, then the fact that those feedback loops are directly responsible for the change in system behavior during the

critical overshoot and collapse is information of major importance that guides analysts to the prime targets, the leverage points (feedback loops) to be modulated in order to change the observed behavior.

Figure 8 reinforces the understanding of this model as a shift from the exponential growth in the number of yeast cells from time ~ 25 to ~ 55 driven by (U1+), which leads to the corresponding rise in the level of alcohol. It is recognized that, then, the alcohol governs the behavior of the system, first dominating through the reduction in the birth of yeast cells, and later through the death of yeast cells—ultimately pushing the yeast cells to collapse. After the onset of that collapse, the natural deaths process then takes over, driving the number of yeast cells to 0. By time ~ 80 there is not that much activity left in the system because all the yeast cells are either dead or, thanks to B1, very soon will be. Figure 8 has ensured that analysts do not ignore what they may at first perceive to be two minor, unimportant balancing loops B2 and B3 that turn out to be clearly critical to the development of the behavior of this model, contrary to U1 and B1, which are dominant for far longer periods of “less interesting” time—that is, when there are relatively few changes that take place.

Conclusions

We have developed a new metric, “system change”, by which we can capture in a single time series the aggregate state change, providing a relative measure of the amount of change across all stocks in the model over an entire simulation period. This system change metric is constructed to measure the same change that LTM uses to report its loop dominance metrics. The metric, when plotted with the relative loop scores of all loops in a model, makes clear to analysts which are the most important loops over time, contextualized by the amount the system is changing overall. This guides the analysts towards loops that are dominant when the system is in flux and, therefore, potentially constitute its most effective leverage points. This stands as a significant addition to the usefulness of LTM.

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