Two-Photon Decay of Non-Standard Higgs Bosons

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Chapter 1

Introduction

One of the crucial issues in modern particle physics is to discover the Higgs boson, and understand the origin of particle masses. In the Standard Model [1] there is only one Higgs particle, whereas in more general theories there are typically several, including also charged Higgs particles.

The existence of the Standard Model Higgs boson is not confirmed experimentally yet, but those gauge theorists never give up. Different 'theories' and models have been proposed as extensions beyond the Standard Model. In models with two Higgs doublet fields, there are three electrically neutral and a pair of charged Higgs particles. In the Minimal Supersymmetric Standard Model (MSSM) the neutral Higgs particles are eigenstates of charge-parity symmetry, CP. Two of them (h^0 and H^0) are even, whereas one (A^0) is odd under CP.

In more general theories, the Higgs particles need not be eigenstates of CP. In the present thesis, we consider the decay to two photons of a Higgs particle which is not a CP eigenstate. Thus, we allow it to have an additional coupling to fermions, involving γ_5, which distinguishes between left- and right-handed fields. Our aim is to determine whether such a coupling contributes to the $\gamma\gamma$ decay rate at the one-loop level, and how this contribution depends on the Higgs mass.

Chapter 2

Theoretical framework

2.1 Gauge invariance

By a gauge field theory we mean a field theory which is invariant under a group of local gauge transformations. Current theories of particle interactions are based on local symmetries: the theory is invariant under phase transformations of certain fields, provided there is a coupling to a vector (gauge) field, and provided the vector field also is transformed in a certain way. A familiar example of a gauge theory is Quantum Electrodynamics (QED), where the free Dirac 'matter field' is transformed as

$$\begin{aligned} \psi(x) &\longrightarrow \psi'(x) = e^{-iq\theta(x)}\psi(x) \\ \overline{\psi}(x) &\longrightarrow \overline{\psi}'(x) = e^{iq\theta(x)}\overline{\psi}(x) \end{aligned} \tag{2.1}$$

The Lagrangian for the free Dirac field

$$\mathcal{L}_0 = \overline{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x) \tag{2.2}$$

is *not* invariant under the local gauge transformations (2.1) since it picks up an extra term from the derivative of $\theta(x)$. To deal with this extra term, a so-called covariant derivative of the form

$$D_\mu = \partial_\mu + iqA_\mu(x) \tag{2.3}$$

is introduced. The covariant derivative involves the 'gauge field' $A_\mu(x)$ which transforms as

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x) \tag{2.4}$$

By replacing the ordinary derivative $\partial_\mu\psi(x)$ in the Lagrangian density $\mathcal{L}_0$ by $D_\mu\psi(x)$ we get the transformed Lagrangian density

$$\mathcal{L} = \overline{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) \tag{2.5}$$

$$= \mathcal{L}_0 + q\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu(x) \tag{2.6}$$

which is invariant under the gauge transformation since the covariant derivative undergoes the transformation

$$D_\mu\psi(x) \longrightarrow e^{-iq\theta(x)}D_\mu\psi(x) \tag{2.7}$$

the same as the field $\psi(x)$ itself.

To summarize: if we demand the invariance of a theory —which is described by a Lagrangian that does not contain derivatives higher than the first— with respect to local phase transformations of the matter field, we should introduce a gauge field coupled to the matter field through the replacement of the ordinary derivative $\partial_\mu\psi(x)$ by the covariant derivative $D_\mu\psi(x)$. This structure is imitated in other, more complex theories, like Quantum Chromodynamics and the $SU(2) \times U(1)$ Electroweak Theory.

2.2 Spontaneous symmetry breaking

Not all the symmetries in nature that we believe in are manifest, some are *hidden*. These symmetries are hidden due to so-called *spontaneous symmetry breaking*. Physically speaking, in this spontaneous symmetry breaking, the Lagrangian respects the symmetry, but the vacuum state (lowest energy state) of the system does not. To treat this case we will consider a Lagrangian which has a particular symmetry. If the vacuum state would respect the symmetry of the Lagrangian, then the vacuum should be degenerate, i.e., there is no unique solution for the Lagrangian equations of motion with the same energy.

According to Richard Feynman: *In theoretical physics we discover that all our laws can be written in mathematical form; and that this has a certain simplicity and beauty about it. So, ultimately, in order to understand nature it may be necessary to have a deeper understanding of mathematical relationships.* Seeking that simplicity and beauty which Feynman was talking about, let us — in order to understand the idea of spontaneous symmetry breaking— flavor it with some mathematical expressions in the next two sections. In section 2.3 we will consider spontaneous symmetry breaking for a Lagrangian which is invariant under *global* gauge transformations [the Goldstone Model], while in section 2.4 we will consider spontaneous symmetry breaking for a Lagrangian which is invariant under *local* gauge transformations [the Higgs Model].

2.3 Goldstone model

Let us consider the complex scalar field $\phi(x)$ of the form

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + i\phi_2(x)] \tag{2.8}$$

with mass m and a Lagrangian density given by

$$\mathcal{L}_s(\phi(x)) = \partial_\mu\phi^*(x)\partial^\mu\phi(x) - m^2[\phi^*(x)\phi(x)] - \lambda[\phi^*(x)\phi(x)]^2 \tag{2.9}$$

where the positive real constant λ is the self-interaction coupling strength and the subscript s denotes 'scalar'. The complex field $\phi(x)$ has no direct physical interpretation.

The above Lagrangian is invariant under global $U(1)$ phase transformations of the form $U_\theta = e^{-i\theta}$ with

$$\begin{aligned} \phi(x) &\longrightarrow U_\theta \phi(x) \\ \phi^*(x) &\longrightarrow U_\theta^* \phi^*(x) \end{aligned} \tag{2.10}$$

The Hamiltonian density corresponding to this Lagrangian is

$$\mathcal{H}(\phi(x)) = [\partial^0 \phi^*(x)][\partial_0 \phi(x)] + [\nabla \phi^*(x)] \cdot [\nabla \phi(x)] + V(\phi(x)) \tag{2.11}$$
$$= \mathcal{H}_0 + V(\phi(x)) \tag{2.12}$$

where

$$V(\phi(x)) = m^2 |\phi(x)|^2 + \lambda |\phi(x)|^4 \tag{2.13}$$
$$= m^2[\phi_1^2(x) + \phi_2^2(x)] + \lambda[\phi_1^2(x) + \phi_2^2(x)]^2 \tag{2.14}$$

We are interested in the case when $m^2 < 0$. The potential in this case has the shape of a symmetric Mexican sombrero, with a local maximum at $\phi(x) = 0$ and a whole circle of absolute minima satisfying the relation

$$\phi_{10}^2(x) + \phi_{20}^2(x) = \frac{-m^2}{2\lambda} \tag{2.15}$$

or

$$|\phi(x)| = |\phi_0(x)| = \left(\frac{-m^2}{2\lambda}\right)^{1/2} \tag{2.16}$$

which is pure real and positive. The vacuum state ϕ_0 is not invariant under the transformation $U_\theta = e^{-i\theta}$ since the transformed state is

$$\phi_0' = \phi_0 e^{-i\theta} \tag{2.17}$$

The energy of ϕ_0' is the same as the energy of ϕ_0, moreover, since θ can take on any value between 0 and 2π, the vacuum is infinitely degenerate.

In order to get a physical interpretation of the Lagrangian (2.9) in the quantized theory we make the change of variables

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x) + i\eta(x)] \tag{2.18}$$

with v a constant given by

$$v = \sqrt{2}\phi_0 = \left(\frac{-m^2}{\lambda}\right)^{1/2} \tag{2.19}$$

Thus,

$$-m^2 = \lambda v^2 \tag{2.20}$$

Then, substituting (2.18) into (2.9), and using (2.20), we obtain

$$\begin{aligned} \mathcal{L}_s(\sigma(x), \eta(x)) &= \tfrac{1}{2}[\partial^\mu \sigma(x)][\partial_\mu \sigma(x)] - \tfrac{1}{2}(2\lambda v^2)\sigma^2(x) \\ &+ \tfrac{1}{2}[\partial^\mu \eta(x)][\partial_\mu \eta(x)] \\ &- \lambda v \sigma(x)[\sigma^2(x) + \eta^2(x)] - \tfrac{1}{4}\lambda[\sigma^2(x) + \eta^2(x)]^2 \end{aligned} \tag{2.21}$$

where we have dropped the constant $\lambda v^4/4$.

We see that the $\mathcal{L}$ of eq. (2.9) is the Lagrangian for two real scalar fields $\sigma(x)$ and $\eta(x)$ including interactions. Furthermore, on quantization, both fields lead to neutral spin 0 particles: the field $\sigma(x)$ has mass $m_\sigma^2 = 2\lambda v$, while the field $\eta(x)$ has massless quanta. Thus, although the complex field $\phi(x)$ seems to have no obvious physical interpretation (it appears to correspond to particles of imaginary mass), the real fields $\sigma(x)$ and $\eta(x)$ can be interpreted physically: the first has mass and the other is massless. From eqs. (2.18) and (2.19) we find

$$< 0|\phi(x)|0 >= \phi_0 \tag{2.22}$$

which is the condition for the spontaneous symmetry breaking in the quantized theory, and it is fulfilled in our case, so the symmetry of $\mathcal{L}$ is spontaneously broken. We see that the spontaneous symmetry breaking gives rise to a field $\eta(x)$ whose quanta are massless scalar particles. These particles are known as *Goldstone bosons* (after Jeffrey Goldstone).

2.4 Higgs Model

The Lagrangian given in eq. (2.9) is not invariant under *local* gauge transformations given as

$$U_\theta = e^{-i\theta(x)} \tag{2.23}$$

since it contains derivatives with respect to the fields. We need to find a way to overcome that by replacing the derivative ∂_μ by a *covariant* derivative D_μ defined as

$$D_\mu \phi(x) = [\partial_\mu + iqA_\mu(x)]\phi(x) \tag{2.24}$$

This covariant derivative in eq. (2.24) involves a new vector gauge field $A_\mu(x)$, whose Lagrangian density is given as

$$\mathcal{L}_v = -\tfrac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) \tag{2.25}$$

where $F_{\mu\nu}(x)$ is defined by

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) \tag{2.26}$$

Now we can obtain the Lagrangian for a complex scalar field $\phi(x)$ and a massless vector field $A(x)$ [eq. (13.15) in [2]]

$$\begin{aligned}\mathcal{L}(x) &= [D^\mu\phi(x)]^*[D_\mu\phi(x)] - m^2[\phi^*(x)\phi(x)] - \lambda[\phi^*(x)\phi(x)]^2 \\ &\quad -\tfrac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)\end{aligned} \tag{2.27}$$

which is invariant under the $U(1)$ gauge transformations

$$\begin{aligned}\phi(x) &\longrightarrow \phi'(x) = e^{-iq\theta(x)}\phi(x) \\ \phi^*(x) &\longrightarrow \phi^{*'}(x) = e^{iq\theta(x)}\phi^*(x) \\ A_\mu(x) &\longrightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)\end{aligned} \tag{2.28}$$

Using eq. (2.20) and substituting eq. (2.18) into (2.27) we get

$$\begin{aligned}\mathcal{L}(x) &= \tfrac{1}{2}[\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] - \tfrac{1}{2}(2\lambda v^2)\sigma^2(x) \\ &\quad -\tfrac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \tfrac{1}{2}(qv)^2A_\mu(x)A^\mu(x) \\ &\quad +\tfrac{1}{2}[\partial^\mu\eta(x)][\partial_\mu\eta(x)] \\ &\quad +qvA^\mu(x)\partial_\mu\eta(x) + \mathcal{L}_I\end{aligned} \tag{2.29}$$

with $\mathcal{L}_I$ representing terms which are cubic and quartic in the fields and some 'insignificant' constants.

A direct physical interpretation of the Lagrangian in eq. (2.29) leads to difficulties, furthermore if we count the degrees of freedom in eq. (2.27) and eq. (2.29) we find that the Lagrangian density in (2.27) has four degrees of freedom, while the transformed Lagrangian in (2.29) appears to have five degrees of freedom. This leads to the conclusion that (2.29) contains an unphysical field $\eta(x)$ which corresponds to 'ghost' particles. We can eliminate this field by finding a gauge transformation of the form (2.28) to transform the $\phi(x)$ into a real field as

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x)] \tag{2.30}$$

Substituting eq. (2.30) into eq. (2.27) we find

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_I(x) \tag{2.31}$$

where the quadratic terms are represented by

$$\begin{aligned}\mathcal{L}_0(x) &= \tfrac{1}{2}[\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] - \tfrac{1}{2}(2\lambda v^2)\sigma^2(x) \\ &\quad -\tfrac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \tfrac{1}{2}(qv)^2A_\mu(x)A^\mu(x)\end{aligned} \tag{2.32}$$

while the higher-order interaction terms are expressed in $\mathcal{L}_I(x)$ which we are not interested in at this point.

A physical interpretation can be given now to the Lagrangian density in eq. (2.32) as: the first two terms are the free Lagrangian for a real scalar field $\sigma(x)$ with mass $\sqrt{2\lambda v^2}$

(which we will refer to as Higgs field). The next two terms are the free Lagrangian for a neutral vector field with mass $|qv|$.

We started from the Lagrangian (2.27) with four independent degrees of freedom, two for the *complex* scalar field $\phi(x)$ and two for the massless vector field $A_\mu(x)$. We ended up with a *massive* vector field $A_\mu(x)$, which has three degrees of freedom, and a massive *real* scalar field $\sigma(x)$ which has one degree of freedom. The number of degrees of freedom is four in both cases. This phenomenon by which a vector boson acquires mass without destroying the gauge invariance of the Lagrangian density is known as the *Higgs mechanism* (after Peter Higgs), and the massive spin 0 boson associated with the field $\sigma(x)$ is called a Higgs boson.

When we considered a global gauge transformation, in section 2.3, we found a Goldstone field $\eta(x)$ associated with a massless boson in addition to the massive scalar field $\sigma(x)$. Here the Goldstone boson has disappeared, but its degree of freedom has been transferred to the vector field $A_\mu(x)$ to give it a mass. So by the *Higgs mechanism* the *massless* vector field is transformed into a *massive* one.

The last two terms in eq. (2.32) are identical with the Lagrangian density of a massive neutral vector boson field which leads to the propagator

$$iD_F^{\alpha\beta}(k,m) = \frac{i(-g^{\alpha\beta}+k^\alpha k^\beta/m^2)}{k^2-m^2+i\epsilon} \tag{2.33}$$

where $m=|qv|$. The term $k^\alpha k^\beta/m^2$ in the vector propagator (2.33) does not look nice; it makes the theory non-renormalizable. A way should be found to get rid of it, by choosing to work in another gauge, which was proposed by 't Hooft in a way like QED. 't Hooft imposed the gauge condition

$$\partial_\mu A^\mu(x) - m\eta(x) = 0 \tag{2.34}$$

and if it holds, a 'gauge-fixing' term of the form

$$-\tfrac{1}{2}[\partial_\mu A^\mu(x) - m\eta(x)]^2 \tag{2.35}$$

can be added to the Lagrangian density (2.29) to get the modified Lagrangian density

$$\begin{aligned}\mathcal{L}(x) = {} & \tfrac{1}{2}[\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] - \tfrac{1}{2}(2\lambda v^2)\sigma^2(x) \\ & -\tfrac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \tfrac{1}{2}m^2A_\mu(x)A^\mu(x) - \tfrac{1}{2}[\partial_\mu A^\mu(x)]^2 \\ & +\tfrac{1}{2}[\partial^\mu\eta(x)][\partial_\mu\eta(x)] - \tfrac{1}{2}m^2\eta^2(x) \\ & +\mathcal{L}_I\end{aligned} \tag{2.36}$$

We see from the first and the third lines in eq. (2.36) that $\sigma(x)$ and $\eta(x)$ are real Klein-Gordon fields which on quantization give the usual equations of motion and propagators for such fields. Also from eq. (2.36) one obtains

$$(\Box + m^2)A^\mu(x) = 0 \tag{2.37}$$

which is like the Klein-Gordon equation for a scalar field and leads to the propagator

$$iD_F^{\alpha\beta}(k,m) = \frac{-ig^{\alpha\beta}}{k^2-m^2+i\epsilon} \tag{2.38}$$

The undesirable term $k^\alpha k^\beta/m^2$ disappeared from eq. (2.33). The propagator in (2.38) is like the photon propagator in QED in the sense that it behaves like $1/k^2$ for large k^2.

It is worth to be mentioned here that working in the 't Hooft gauge reintroduces the $\eta(x)$ field as is obvious from eq. (2.35). As we pointed out previously, it does not correspond to physical particles, but the Feynman propagator for this field can be interpreted as representing the exchange of 'ghost' scalar bosons, which are analogous to those of the longitudinal and scalar photons in QED. The detailed properties of these ghost are pretty complicated and moreover, gauge-dependent.

A Few Words About Ghost Particles

As is implied by the name, the 'ghost' particles are not real physical states. The ghost fields are originally introduced as a mathematical device to overcome some of the troublesome terms appearing in the Lagrangian when working in particular gauges. In our calculation we choose the Feynman-'t Hooft gauge in which there are two types of ghost fields interacting with the Higgs, $W^\pm$ and photon fields; the charged Higgs ghosts (G^+, G^-) and the charged Faddeev-Popov ghosts (η^+, η^-) which are associated with the two vector bosons (W^+, W^-) of the weak interactions. At the one-loop level, these ghosts are included as internal lines in the corresponding Feynman diagrams (see as an example fig. 4.7).

2.5 The Standard Model Higgs Boson

Since the time of Fermi's theory of weak interactions, it was known that a charged weak current exists, to include this fact in the framework of a gauge theory, the charged current should be coupled to charged vector (gauge) bosons. Let us introduce the projection operators

$$\begin{aligned} P_R &= \tfrac{1}{2}(1+\gamma_5) \\ P_L &= \tfrac{1}{2}(1-\gamma_5) \end{aligned} \tag{2.39}$$

In the Glashaw-Weinberg-Salam (GWS) model [1], the electroweak interactions are described by a gauge theory based on the group $SU(2)\times U(1)$ with left-handed fermions in doublet representations of $SU(2)$ and right-handed fermions in singlet representations. The Lagrangian density for the standard electroweak theory for purely leptonic processes can be given as [2]

$$\mathcal{L} = \mathcal{L}^L + \mathcal{L}^B + \mathcal{L}^H + \mathcal{L}^{LH} \tag{2.40}$$

where $\mathcal{L}^L$ is the leptonic Lagrangian density given by

$$\mathcal{L}^L = i\left[\overline{\Psi}_l^L(x) \not{D} \Psi_l^L(x) + \overline{\psi}_l^R(x) \not{D} \psi_l^R(x) + \overline{\psi}_{\nu_l}^R(x) \not{D} \psi_{\nu_l}^R(x)\right] \tag{2.41}$$

$\mathcal{L}^B$ is the gauge bosons Lagrangian density

$$\mathcal{L}^B = -\tfrac{1}{4}B_{\mu\nu}(x)B^{\mu\nu}(x) - \tfrac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x) \quad (2.42)$$

Here $B^{\mu\nu}(x)$ and $G_i^{\mu\nu}(x)$ are defined as

$$\begin{aligned} B^{\mu\nu}(x) &\equiv \partial^\nu B^\mu(x) - \partial^\mu B^\nu(x) \\ G_i^{\mu\nu}(x) &\equiv F_i^{\mu\nu}(x) + g\epsilon_{ijk}W_j^\mu(x)W_k^\nu(x) \end{aligned} \quad (2.43)$$

with

$$F_i^{\mu\nu}(x) \equiv \partial^\nu W_i^\mu(x) - \partial^\mu W_i^\nu(x) \quad (2.44)$$

The Higgs part, which is similar to the lepton part, involves the isospinor scalar field

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ v + \sigma(x) + i\eta_3(x) \end{pmatrix} \quad (2.45)$$

and can be written as

$$\mathcal{L}^H = [D^\mu\Phi(x)]^\dagger[D_\mu\Phi(x)] - m^2\Phi^\dagger(x)\Phi(x) - \lambda[\Phi^\dagger(x)\Phi(x)]^2 \quad (2.46)$$

The covariant derivatives are defined as

$$D^\mu\Psi_l^L(x) = [\partial^\mu + ig\tau_j W_j^\mu(x)/2 - ig'B^\mu(x)/2]\Psi_l^L(x) \quad (2.47)$$

$$D^\mu\psi_l^R(x) = [\partial^\mu - ig'B^\mu(x)]\psi_l^R(x) \quad (2.48)$$

$$D^\mu\psi_{\nu_l}^R(x) = \partial^\mu\psi_{\nu_l}^R(x) \quad (2.49)$$

$$D^\mu\Phi(x) = [\partial^\mu + ig\tau_j W_j^\mu(x)/2 + ig'B^\mu(x)/2]\Phi(x) \quad (2.50)$$

The ratio between the two couplings g and g' is given by the weak mixing angle (Weinberg angle) as $\tan\theta_W = g'/g$.

The strategy of Higgs mechanism is applied to the electroweak sector of the Standard Model, where, at least *one* complex scalar doublet is required to give masses to the gauge bosons as well as the fermions.

Again, for $m^2 < 0$, the $SU(2) \times U(1)$ gauge symmetry is spontaneously broken. The vacuum expectation value for the Higgs field is

$$< 0|\Phi(x)|0 > = |\Phi_0| = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad (2.51)$$

with v as defined in (2.19)

The upper component of $\Phi(x)$, which is a charged field, leads to the Higgs ghosts $G^\pm$.

In the unitary gauge one can parametrize $\Phi(x)$ around its vacuum value as:

$$< 0|\Phi(x)|0 > = |\Phi_0| = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + \sigma(x) \end{pmatrix} \quad (2.52)$$

This field can be plugged into the Lagrangian (2.40), which can be simplified by straightforward but tedious transformation of the Lagrangian density (see [2]), and the masses of the weak vector bosons can be read off:

$$m_W = \tfrac{1}{2} v g, \quad m_Z = m_W / \cos\theta_W, \quad m_H = (2\lambda v^2)^{1/2} \tag{2.53}$$

The masses m_W and m_Z are predicted by the theory and their values are in good agreement with the experimental values. The Higgs mass is arbitrary since the self-interaction constant λ is unknown. If $\lambda \gg 1$, perturbation theory would break down; this will lead to an upper limit on the Higgs mass. If $\lambda \ll 1$, radiative corrections become important; these will lead to a lower bound on the Higgs mass [3, 4].

Two-Higgs-doublet models (2HDM)

The 2HDM is one of the simplest extensions of the Standard Model, which is obtained, simply, by the addition of a new scalar $SU(2)$ doublet, which implies that we need at least *two* Higgs doublet fields in order to break the symmetry spontaneously instead of *one* Higgs doublet field as in the SM. There appears different versions of the 2HDM which are referred to as: Model I, Model II, Model III,...etc., depending on whether up and down quarks couple to the same or different scalar doublets. In model I the up and down quarks acquire mass via vacuum expectation value (VEV) of only one Higgs field. In model II, which is consistent with the Higgs sector of the MSSM, all up-type quarks couple to the one doublet, while the down-type quarks couple to the other.

2.6 MSSM

In supersymmetric theories one can transform bosonic states into fermionic ones and vice versa, which means there should be a bosonic (fermionic) superpartner for each fermion (boson). These theories suggest the doubling of the spectrum of the fundamental particles, which contradicts the observed spectrum of the particles. To overcome this difficulty, the supersymmetry should be broken.

What we are interested in is the Higgs sector of these theories. In the minimal supersymmetric extension of the standard model which is referred to as the (MSSM), two complex Higgs doublets are required

$$\Phi_1(x) = \begin{pmatrix} \chi_1^+(x) \\ \phi_1(x) + i\chi_1(x) \end{pmatrix}, \quad \Phi_2(x) = \begin{pmatrix} \chi_2^+(x) \\ \phi_2(x) + i\chi_2(x) \end{pmatrix} \tag{2.54}$$

to break the gauge symmetry spontaneously. The two complex doublets above (2.54) contain eight real scalar fields $\chi_1^{\pm}(x)$, $\chi_2^{\pm}(x)$, $\phi_1(x)$, $\phi_2(x)$, $\chi_1(x)$ and $\chi_2(x)$.

There are five physical Higgs bosons; two neutral CP-even scalars (h^0, H^0—by convention, h^0 is the lightest), one CP-odd neutral pseudoscalar (A^0), and a pair of charged scalars (H^+, H^-). At tree level, the mass of the lightest scalar Higgs should be lighter than the Z boson mass, and the charged Higgs boson mass has to be heavier than the $W^{\pm}$ boson mass. There are radiative corrections to these relations. Besides the four masses, two additional parameters are needed to describe the Higgs sector at tree level: the ratio of the two vacuum expectation values $\tan\beta = v_2/v_1$ and a mixing angle α between the neutral h^0 and H^0. Only two of these parameters are independent; choosing them to be the mass of the pseudoscalar A^0 and $\tan\beta$, the structure of the MSSM Higgs sector can be determined entirely.

Chapter 3

Decay Rate of $H \to \gamma\gamma$

General results for decay rates are given by Mandl and Shaw [2], p. 246. Consider a Higgs boson, with four-momentum $p = (m_H, \mathbf{0})$, decaying into two photons with four-momenta $k_1 = (\omega_1, \mathbf{k}_1)$ and $k_2 = (\omega_2, \mathbf{k}_2)$. The differential decay rate $d\Gamma$ for such a process in terms of the corresponding Feynman amplitude, M, can be expressed as:

$$d\Gamma = \frac{1}{2m_H}(2\pi)^4\delta^{(4)}(p - k_1 - k_2)|M|^2\frac{d^3\mathbf{k}_1}{(2\pi)^3 2\omega_1}\frac{d^3\mathbf{k}_2}{(2\pi)^3 2\omega_2} \tag{3.1}$$

Here, $M = \sum M_j$, where M_j is the Feynman amplitude of each Feynman diagram contributing to the $H \to \gamma\gamma$ decay mode.

We integrate over $\mathbf{k}_2$ to get

$$d\Gamma = \frac{1}{2m_H}\frac{1}{(2\pi)^2}\delta(m_H - \omega_1 - \omega_2)|M|^2\frac{d^3\mathbf{k}_1}{2\omega_1 2\omega_2} \tag{3.2}$$

In the center-of-mass frame the total three-momentum is zero, i.e.

$$\mathbf{k}_1 + \mathbf{k}_2 = 0 \quad \Longrightarrow \quad \mathbf{k}_1 = -\mathbf{k}_2 \tag{3.3}$$

while

$$\omega_1 = \omega_2 = \omega \tag{3.4}$$

and the differential three-momentum can be expressed as

$$d^3\mathbf{k}_1 = |\mathbf{k}_1|^2 d|\mathbf{k}_1| d\Omega = \omega^2 d\omega d\Omega \tag{3.5}$$

Thus,

$$d\Gamma = \frac{1}{2m_H}\frac{1}{(2\pi)^2}\frac{1}{4}\delta(m_H - 2\omega)|M|^2 d\omega d\Omega \tag{3.6}$$

For $|M|^2$ independent of orientation (angular independent), we can integrate over Ω to get a factor of 4π. Thus,

$$d\Gamma = \frac{1}{2m_H}\frac{\pi}{(2\pi)^2}|M|^2\delta(m_H - 2\omega)d\omega$$

and

$$\Gamma = \frac{1}{16\pi m_H}|M|^2 \tag{3.7}$$

By dimensional analysis, since Γ has dimension of [mass], the amplitude M must have dimension of [mass].

We shall extract some factors from the amplitude, and define M_0 by the relation

$$M = \frac{1}{(2\pi)^4}e^2\frac{g}{m_W}(i\pi^2)m_H^2 M_0 \tag{3.8}$$

Thus,

$$|M|^2 = \frac{\pi^4}{(2\pi)^8}e^4\frac{g^2}{m_W^2}m_H^4|M_0|^2 \tag{3.9}$$

Substituting the Fermi coupling constant $G_F = \sqrt{2}g^2/8m_W^2$ and $e^2 = 4\pi\alpha$, with α the fine-structure constant in QED, we find

$$\begin{aligned} |M|^2 &= \frac{\pi^4}{(2\pi)^8}(4\pi\alpha)^2\frac{8G_F}{\sqrt{2}}m_H^4|M_0|^2 \\ &= \frac{\alpha^2}{(2\pi)^2}\frac{2G_F}{\sqrt{2}}m_H^4|M_0|^2 \end{aligned} \tag{3.10}$$

and, from (3.6)

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 G_F}{4\sqrt{2}(2\pi)^3}m_H^3|M_0|^2 \tag{3.11}$$

Here, G_F has dimension of $[\text{mass}]^{-2}$, so M_0 should be dimensionless.

Expressions for the decay rate of $H \to \gamma\gamma$ are given in other articles, for example, see [5, 6, 7]. In comparison to the expression given in [5], our result in eq. (3.11) differs from it in the definition of the amplitude factor as:

$$I \equiv \pm\tfrac{1}{2}M_0 \tag{3.12}$$

while the expression given in [6] will be equivalent to ours in eq. (3.11) if we take

$$F \equiv \pm 2M_0 \tag{3.13}$$

Chapter 4

Decay of the SM Higgs boson (H) into two photons

The Standard Model Higgs boson does not couple directly to the photon; this is in accordance with the fact that the physical Higgs boson couples to other particles proportionally to their masses, while the physical photon is massless. Moreover, the photon couples directly to electrically charged particles while it is known that the SM Higgs boson is a neutral particle, consequently we conclude that there will be no tree-level Feynman diagrams associated with the decaying mode of the SM Higgs boson into two photons $H \to \gamma\gamma$.

An induced coupling is possible starting at the one-loop level Feynman diagrams. These loops may contain all the electrically charged particles whose masses are generated in the Standard Model by the interaction with the Higgs field. First we will consider the $H \to \gamma\gamma$ decay mode through fermion loops, specifically the top quark loops. Next, we will study the $H\gamma\gamma$ coupling through the charged gauge bosons $W^{\pm}$ and related ghost fields.

In order to calculate the total decay rate of $H \to \gamma\gamma$, which is proportional to the square of the modulus of the decay amplitude as given in eq. (3.11), we should take into consideration all the contributions from these diagrams; the fermion-loop diagrams and the W-loop diagrams.

4.1 Fermion loop contributions

The standard model Higgs boson couples to the fermions proportional to their masses, which leads to the conclusion that the contribution of light fermions to the decay amplitude is 'negligible'. A considerable contribution can be received from those 'heavy' fermions, consequently we will only consider the top-quark, which is the 'heaviest' fermion in the standard model, as an effective mediator of the $H \to \gamma\gamma$ decay mode.

For a Higgs boson decaying via a fermion loop, there are two diagrams, as shown in fig. 4.1. We shall label them $(0a)$ and $(0b)$[1]. The analytical expression for the amplitude

[1]This kind of labeling for the diagrams is just chosen for our convention, since it has to do with the notation used in our `REDUCE` program, e.g. the contribution to the total amplitude M_0 from the diagram

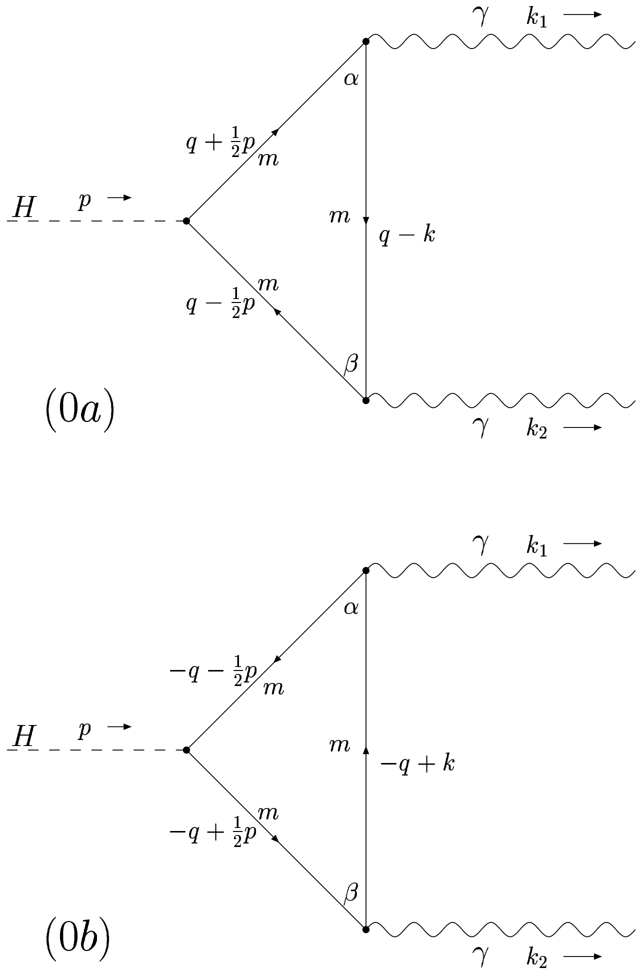


Figure 4.1: Feynman diagrams for Higgs boson decaying into two photons through a fermion loop, m denotes the mass of the fermion (charge carrier) in the loop.

corresponding to diagram $(0a)$ is given by

$$
\begin{aligned}
M_1^{\alpha\beta} &= -\mathrm{Tr}\int \frac{d^4q}{(2\pi)^4}(\alpha m_t)\frac{i(\not{q}-\frac{1}{2}\not{p}+m_t)}{(q-\frac{1}{2}p)^2-m_t^2}(ieQ_t\gamma^\beta) \\
&\quad \times\frac{i(\not{q}-\not{k}+m_t)}{(q-k)^2-m_t^2}(ieQ_t\gamma^\alpha)\frac{i(\not{q}+\frac{1}{2}\not{p}+m_t)}{(q+\frac{1}{2}p)^2-m_t^2}
\end{aligned} \tag{4.1}
$$

where the minus sign appearing in the expression above is characteristic of closed fermion loops. We used the subscript t to denote the top quark (with charge Q_t) as a charge carrier in the intermediate fermion loop, $\alpha = -ig/(2m_W)$, and $-m_t^2$ represents $-m_t^2 + i\epsilon$, with

labeled as $(0a)$ is equivalent to 'term0a' in the `REDUCE` program.

$\epsilon \to 0+$. Thus, the Higgs-quark coupling is

$$-\frac{ig}{2m_W}\, m_q \tag{4.2}$$

where g is the weak gauge coupling.

In our REDUCE program — which is used to calculate all the contributions from different Feynman diagrams to the $H \to \gamma\gamma$ mode— $M_1^{\alpha\beta}$ is translated into that REDUCE code as:

```
term0a:=-qqh(l1,mt)*prq(l1,q-p/2,mt)*qqga(l1,Qt,mu2)
        *prq(l1,q-k,mt)*qqga(l1,Qt,mu1)*prq(l1,q+p/2,mt)$
```

where we used the definitions for the operators appearing in 'term0a' above, as given in Appendix C.

For the crossed diagram $(0b)$, also shown in fig. 4.1, the amplitude is

$$\begin{aligned} M_2^{\alpha\beta} \;=\; & -\mathrm{Tr}\int \frac{d^4q}{(2\pi)^4}(\alpha m_t)\frac{i(-\not{q}-\frac{1}{2}\not{p}+m_t)}{(-q-\frac{1}{2}p)^2-m_t^2}(ieQ_t\gamma^\alpha) \\ & \times\frac{i(-\not{q}+\not{k}+m_t)}{(-q+k)^2-m_t^2}(ieQ_t\gamma^\beta)\frac{i(-\not{q}+\frac{1}{2}\not{p}+m_t)}{(-q+\frac{1}{2}p)^2-m_t^2} \end{aligned} \tag{4.3}$$

The corresponding REDUCE code is

```
term0b:=-qqh(l1,mt)*prq(l1,-q-p/2,mt)*qqga(l1,Qt,mu1)
        *prq(l1,-q+k,mt)*qqga(l1,Qt,mu2)*prq(l1,-q+p/2,mt)$
```

Adding $M_1^{\alpha\beta}$ and $M_2^{\alpha\beta}$ to get the total contribution to $H \to \gamma\gamma$ from the top quark loop, we have

$$M^{\alpha\beta} = -\frac{e^2Q_t^2 g}{2}\,\frac{m_t}{m_W}\int\frac{d^4q}{(2\pi)^4}\frac{N^{\alpha\beta}}{d_1d_2d_3} \tag{4.4}$$

with the denominator factors defined as

$$\begin{aligned} d_1 &= (q-\tfrac{1}{2}p)^2 - m^2 \\ d_2 &= (q+\tfrac{1}{2}p)^2 - m^2 \\ d_3 &= (q-k)^2 - m^2 \end{aligned} \tag{4.5}$$

while the numerator is

$$N^{\alpha\beta} \;=\; N_1^{\alpha\beta} + N_2^{\alpha\beta} \tag{4.6}$$

with, (from eqs. (4.1) and (4.3), and employing the cyclic property of the trace)

$$N_1^{\alpha\beta} \;=\; \mathrm{Tr}\{\gamma^\alpha(\not{q}+\tfrac{1}{2}\not{p}+m_t)(\not{q}-\tfrac{1}{2}\not{p}+m_t)\gamma^\beta(\not{q}-\not{k}+m_t)\} \tag{4.7}$$

$$N_2^{\alpha\beta} \;=\; \mathrm{Tr}\{\gamma^\beta(-\not{q}+\tfrac{1}{2}\not{p}+m_t)(-\not{q}-\tfrac{1}{2}\not{p}+m_t)\gamma^\alpha(-\not{q}+\not{k}+m_t)\} \tag{4.8}$$

In what follows we will show our result for the traces in $N_1^{\alpha\beta}$ in detail. But, first we should list some of the algebraic identities about γ-matrices and their traces (see [2] appendix A), which we will need in order to simplify the traces:

1. For a product of any *odd* number of γ-matrices:

$$\mathrm{Tr}(\gamma^\alpha\gamma^\beta...\gamma^\mu) = 0 \tag{4.9}$$

2. For a product of two or four γ-matrices

$$\mathrm{Tr}(\gamma^\alpha\gamma^\beta) = 4g^{\alpha\beta} \tag{4.10}$$

$$\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\rho\gamma^\delta) = 4(g^{\alpha\beta}g^{\rho\delta} - g^{\alpha\rho}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\rho}) \tag{4.11}$$

A first look at eq. (4.7) shows that we have a trace of 5 γ-matrices at most, which is trivial, according to the identity about the trace of an odd number of γ-matrices given in eq. (4.9). Thus we have at most a non-vanishing trace of 4 γ-matrices. First, by pulling out m from the first, the second and then the third factor inside the trace we get, at most, a trace of 4 γ-matrices, as follows

$$\begin{aligned} N_1^{\alpha\beta} = m_t\mathrm{Tr}\Big\{&\gamma^\alpha(\not{q} - \tfrac{1}{2}\not{p} + m_t)\gamma^\beta(\not{q} - \not{k} + m_t) \\ &+\gamma^\alpha(\not{q} + \tfrac{1}{2}\not{p})\gamma^\beta(\not{q} - \not{k} + m_t) \\ &+\gamma^\alpha(\not{q} + \tfrac{1}{2}\not{p})(\not{q} - \tfrac{1}{2}\not{p})\gamma^\beta\Big\} \end{aligned} \tag{4.12}$$

The third term in eq. (4.12) represents a trace of 4 γ-matrices, so we will keep it as it is. But the first two terms in eq. (4.12) need more simplification, since we still have the chance to get rid of the vanishing traces when 3 γ-matrices are involved:

$$\begin{aligned} N_1^{\alpha\beta} = m_t\mathrm{Tr}\Big\{&m_t^2\gamma^\alpha\gamma^\beta \\ &+\gamma^\alpha(\not{q} - \tfrac{1}{2}\not{p})\gamma^\beta(\not{q} - \not{k}) \\ &+\gamma^\alpha(\not{q} + \tfrac{1}{2}\not{p})\gamma^\beta(\not{q} - \not{k}) \\ &+\gamma^\alpha(\not{q} + \tfrac{1}{2}\not{p})(\not{q} - \tfrac{1}{2}\not{p})\gamma^\beta\Big\} \end{aligned} \tag{4.13}$$

which will be simplified as

$$\begin{aligned} N_1^{\alpha\beta} = m_t\mathrm{Tr}\Big\{&m_t^2\gamma^\alpha\gamma^\beta + 2\gamma^\alpha\not{q}\gamma^\beta(\not{q} - \not{k}) \\ &+\gamma^\alpha(\not{q} + \tfrac{1}{2}\not{p})(\not{q} - \tfrac{1}{2}\not{p})\gamma^\beta\Big\} \end{aligned} \tag{4.14}$$

Similarly, for $N_2^{\alpha\beta}$ (it differs from $N_1^{\alpha\beta}$ by: $\alpha \leftrightarrow \beta$, $q \leftrightarrow -q$ and $k \leftrightarrow -k$), we get

$$\begin{aligned} N_2^{\alpha\beta} = m_t\mathrm{Tr}\Big\{&m_t^2\gamma^\beta\gamma^\alpha + 2\gamma^\beta\not{q}\gamma^\alpha(\not{q} - \not{k}) \\ &+\gamma^\beta(-\not{q} + \tfrac{1}{2}\not{p})(-\not{q} - \tfrac{1}{2}\not{p})\gamma^\alpha\Big\} \end{aligned} \tag{4.15}$$

Adding eqs. (4.13) and (4.15), we get

$$\begin{aligned} N^{\alpha\beta} &= m_t \mathrm{Tr}\Big\{2(m_t^2 + q^2 - \tfrac{1}{4}p^2)\gamma^\alpha\gamma^\beta \\ &\quad +4\gamma^\alpha \not{q}\gamma^\beta(\not{q} - \not{k}) \\ &\quad +\gamma^\alpha \not{p}\not{q}\gamma^\beta - \gamma^\alpha \not{q}\not{p}\gamma^\beta\Big\} \end{aligned} \tag{4.16}$$

Using eqs. (4.10) and (4.11) to evaluate the traces we get

$$\begin{aligned} N^{\alpha\beta} &= m_t\Big\{8(m_t^2 + q^2 - \tfrac{1}{4}p^2)g^{\alpha\beta} \\ &\quad +16[q^\alpha(q-k)^\beta + q^\beta(q-k)^\alpha - q\cdot(q-k)g^{\alpha\beta}] \\ &\quad +8(p^\alpha q^\beta - q^\alpha p^\beta)\Big\} \end{aligned} \tag{4.17}$$

We combine the similar terms and re-arrange the equation to get

$$\begin{aligned} N^{\alpha\beta} &= 8m_t\Big\{(m_t^2 - q^2 - \tfrac{1}{4}p^2 + 2q\cdot k)g^{\alpha\beta} \\ &\quad +4q^\alpha q^\beta - q^\alpha(p+2k)^\beta + (p-2k)^\alpha q^\beta\Big\} \end{aligned} \tag{4.18}$$

More simplifications yield

$$\begin{aligned} N^{\alpha\beta} &= -8m_t\Big\{[(q-k)^2 - m_t^2 + \tfrac{1}{4}p^2 - k^2]g^{\alpha\beta} \\ &\quad +q^\alpha(p+2k)^\beta - (p-2k)^\alpha q^\beta - 4q^\alpha q^\beta\Big\} \\ &= -8m_t\Big\{(d_3 + \tfrac{1}{4}p^2 - k^2)g^{\alpha\beta} \\ &\quad +q^\alpha(p+2k)^\beta - (p-2k)^\beta q^\alpha - 4q^\alpha q^\beta\Big\} \end{aligned} \tag{4.19}$$

where d_3 is defined in eqs. (4.5).

We can now compare the above result to that obtained from the `REDUCE` program. From the `REDUCE` program, the result of $N^{\mu_1\mu_2}$ for the quark loop is:

```
ampl := (mt**2*qt**2*( - 4*k.k*mu1.mu2 + 8*k.mu1*mu2.q + 8*k.mu2*mu1.q
                     + mu1.mu2*p.p + 4*mu1.mu2*d3 - 4*mu1.p*mu2.q
                     + 4*mu1.q*mu2.p - 16*mu1.q*mu2.q))/mh**2$
```

which in LaTeX form can be written as:

$$\begin{aligned} N^{\mu_1\mu_2} &= \frac{m_t^2 Q_t^2}{m_H^2}\Big(-4k^2 g^{\mu_1\mu_2} + 8k^{\mu_1}q^{\mu_2} + 8k^{\mu_2}q^{\mu_1} \\ &\quad +p^2 g^{\mu_1\mu_2} + 4d_3 g^{\mu_1\mu_2} - 4p^{\mu_1}q^{\mu_2} \\ &\quad +4q^{\mu_1}p^{\mu_2} - 16q^{\mu_1}q^{\mu_2}\Big) \end{aligned} \tag{4.20}$$

Now, if we:

1. Interchange the *dummy* indices $(\mu_1, \mu_2) \longleftrightarrow (\alpha, \beta)$.
2. Multiply eq. (4.19) with the factor $(-1/2)m_t Q_t^2$, which is already implied in the original equation (4.4), and then
3. Compare eqs. (4.19) and (4.20), we find that the two results completely fit each other.

We should remark here that in our REDUCE program —for brevity— we exclude the factor ge^2/m_W from eq. (4.4).

4.2 $W^{\pm}$ loop contributions

In the 't Hooft-Feynman gauge, the contribution to Higgs decay via W loops is represented by W loops, G (Higgs ghost) loops, mixed WG loops, and Faddeev-Popov loops. These will be discussed in the present and following sections. The Feynman rules are taken from Bailin and Love [8], and given in Appendix A.

The diagrams with W loops are given in fig. 4.2.

The Feynman amplitude for the diagram $(1a)$ shown in fig. 4.2 is:

$$
\begin{aligned}
M^{\mu\nu} \;=\; & \int \frac{d^4q}{(2\pi)^4} ie[g^{\rho\beta}(q+\tfrac{1}{2}p+q-k)^{\mu} - g^{\rho\mu}(q+\tfrac{1}{2}p+k_1)^{\beta} + g^{\beta\mu}(k_1-q+k)^{\rho}] \\
& \times \frac{-ig^{\rho\alpha}}{(q+\frac{1}{2}p)^2-m_W^2}\, igm_W g^{\alpha\beta} \frac{-ig^{\beta\rho}}{(q-\frac{1}{2}p)^2-m_W^2} \\
& \times ie[g^{\alpha\rho}(q-\tfrac{1}{2}p+q-k)^{\nu} - g^{\alpha\nu}(q-k+k_2)^{\rho} + g^{\rho\nu}(k_2-q+\tfrac{1}{2}p)^{\alpha}] \\
& \times \frac{-ig^{\alpha\beta}}{(q-k)^2-m_W^2}
\end{aligned}
\tag{4.21}
$$

The corresponding REDUCE code for all the diagrams when only W bosons are involved is

```
term1a:=hWW(alpha,beta)          % W^+ clockwise
      *prW(q+p/2,alpha,alpha1)
      *WWga(q+p/2,-q+k,-k1,alpha1,alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(q-k,-q+p/2,-k2,beta2,beta1,mu2)
      *prW(q-p/2,beta1,beta)$
term1b:=hWW(alpha,beta)          % W^- clockwise
      *prW(q+p/2,alpha,alpha1)
      *WWga(-q+k,q+p/2,-k1,alpha2,alpha1,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(-q+p/2,q-k,-k2,beta1,beta2,mu2)
      *prW(q-p/2,beta1,beta)$
term1c:=hWW(alpha,beta)          % with quartic coupling
```

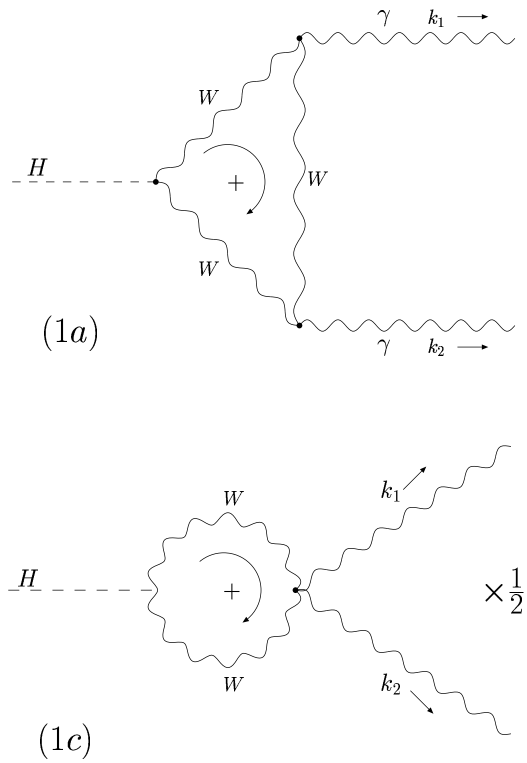


Figure 4.2: Feynman diagrams for $H \to \gamma\gamma$, through loops consisting of W boson lines only. There is also a contribution from the crossed diagram of $(1a)$ which is equivalent to $(1a)$ but with minus charge flowing clockwise in the loop. The diagram with the quartic coupling $WW\gamma\gamma$ is represented in $(1c)$.

```
       *prW(q+p/2,alpha,alpha1)
       *WWgaga(mu1,mu2,alpha1,beta1)
       *d3         % cancel absent propagator of J(1,1,1)
       *prW(q-p/2,beta1,beta)$
term1:=term1a+term1b+term1c$
```

The contributions from `term1a` and `term1b` are identical. The symmetry factor of 1/2 associated with diagram (1c) is compensated for by the fact that there are two such diagrams, one with W^+ and one with W^-.

4.3 G loop contributions

The diagrams with G loops are given in fig. 4.3.

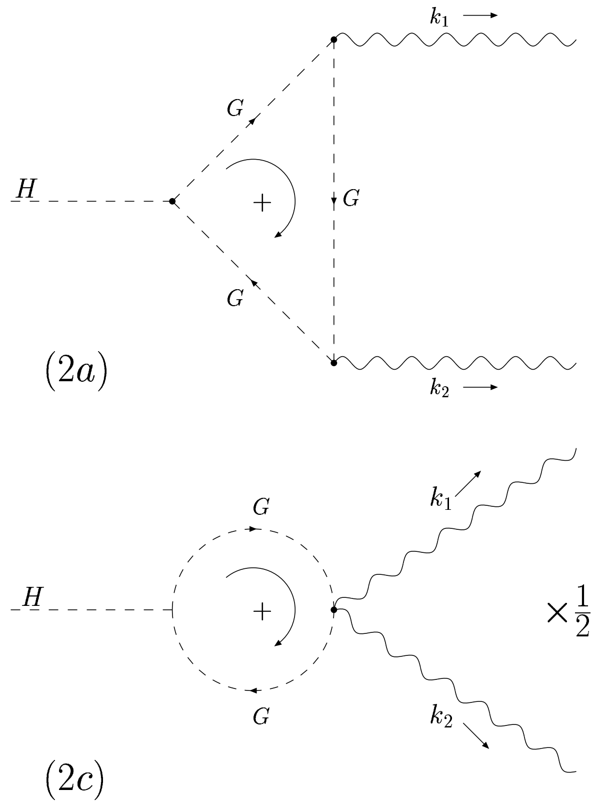


Figure 4.3: Feynman diagrams for $H \to \gamma\gamma$, through loops consisting of Higgs ghost G lines only. There is also a contribution from the crossed diagram of $(2a)$ which is equivalent to $(2a)$ but with minus charge flowing clockwise in the loop. The diagram with the quartic coupling $GG\gamma\gamma$ is represented in $(2c)$.

The corresponding `REDUCE` code is

```
term2a:=hGG       % positive charge clockwise
      *prG(q+p/2)
      *GGga(q+p/2,-q+k,mu1)
      *prG(q-k)
      *GGga(q-k,-q+p/2,mu2)
      *prG(q-p/2)$
term2b:=hGG       % negative charge clockwise
```

```
       *prG(q+p/2)
       *GGga(-q+k,q+p/2,mu1)
       *prG(q-k)
       *GGga(-q+p/2,q-k,mu2)
       *prG(q-p/2)$
term2c:=hGG       % quartic G-G-gamma-gamma vertex
       *prG(q+p/2)
       *GGgaga(mu1,mu2)
       *d3         % cancel absent propagator of J(1,1,1)
       *prG(q-p/2)$
term2:=term2a+term2b+term2c$
```

The contributions from `term2a` and `term2b` are identical. The symmetry factor of 1/2 associated with diagram (2c) is compensated for by the fact that there are two such diagrams, one with G^+ and one with G^-.

4.4 Mixed WWG and GGW loop contributions

The diagrams with mixed WWG and GGW loops are given in figs. 4.4–4.6.

We consider first terms with two W propagators and one G propagator, see Fig. 4.4.

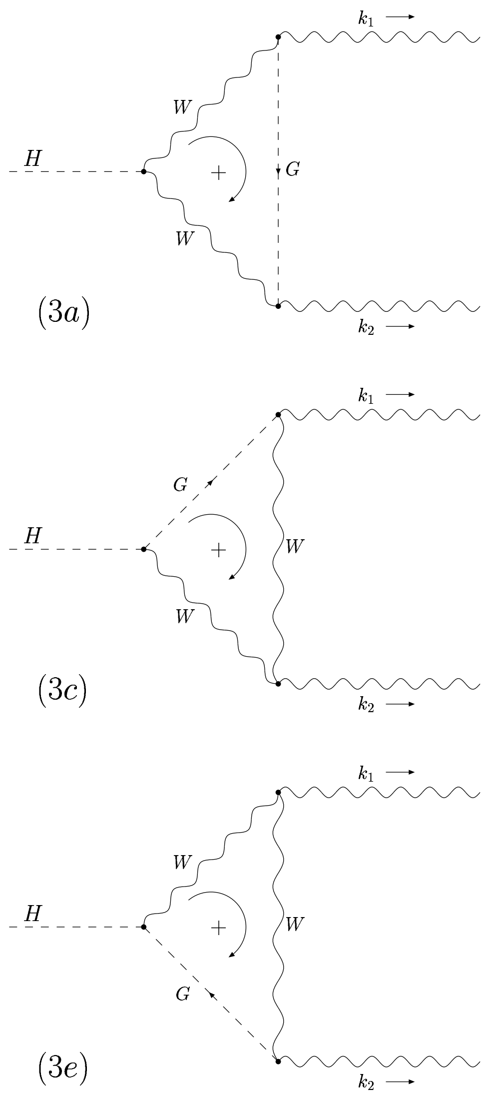


Figure 4.4: Feynman diagrams for $H \to \gamma\gamma$, with two W lines and one G line. There are also contributions from crossed diagrams, (3b), (3d) and (3f).

The corresponding `REDUCE` code is

```
%Clockwise W-W-G (positive charge):
term3a:=hWW(alpha,beta)
      *prW(q+p/2,alpha,alpha1)
      *GWplga(alpha1,mu1)
      *prG(q-k)
      *GWmiga(beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise W-W-G (negative charge) = crossed 3a:
term3b:=hWW(alpha,beta)
      *prW(q+p/2,alpha,alpha1)
      *GWmiga(alpha1,mu1)
      *prG(q-k)
      *GWplga(beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise W-G-W (positive charge):
term3c:=hGWpl(p,-q-p/2,beta)
      *prG(q+p/2)
      *GWmiga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(q-k,-q+p/2,-k2,beta2,beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise G-W-W (negative charge) = crossed 3c:
term3d:=hGWpl(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *WWga(-q+k,q+p/2,-k1,alpha2,alpha1,mu1)
      *prW(q-k,alpha2,beta)
      *GWmiga(beta,mu2)
      *prG(q-p/2)$
%Clockwise G-W-W (positive charge):
term3e:=hGWmi(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *WWga(q+p/2,-q+k,-k1,alpha1,alpha2,mu1)
      *prW(q-k,alpha2,beta)
      *GWplga(beta,mu2)
      *prG(q-p/2)$
%Clockwise W-G-W (negative charge) = crossed 3e:
term3f:=hGWmi(p,-q-p/2,beta)
      *prG(q+p/2)
      *GWplga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(-q+p/2,q-k,-k2,beta1,beta2,mu2)
```

```
      *prW(q-p/2,beta1,beta)$
term3:=term3a+term3b+term3c+term3d+term3e+term3f$
```

The first two terms are identical, since the HWW and $WG\gamma$ vertices are given in terms of the metric tensor only.

We consider next terms with two G propagators and one W propagator, see Fig. 4.5.

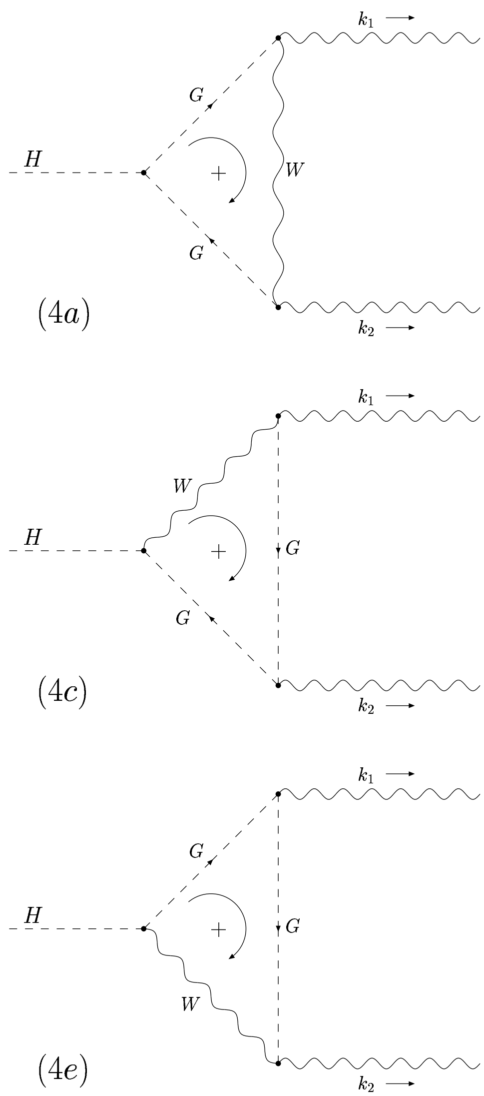


Figure 4.5: Feynman diagrams for $H \to \gamma\gamma$, with two G lines and one W line. There are also contributions from crossed diagrams, (4b), (4d) and (4f).

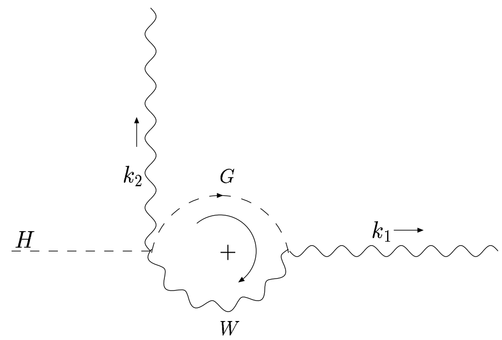
The corresponding `REDUCE` code is

```
% Clockwise: G-G-W (positive charge):
term4a:=hGG
      *prG(q+p/2)
      *GWmiga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *GWplga(beta2,mu2)
      *prG(q-p/2)$
% Clockwise: G-G-W (negative charge) = crossed 4a:
term4b:=hGG
      *prG(q+p/2)
      *GWplga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *GWmiga(beta2,mu2)
      *prG(q-p/2)$
% Clockwise: G-W-G (positive charge):
term4c:=hGWmi(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *GWplga(alpha1,mu1)
      *prG(q-k)
      *GGga(q-k,-q+p/2,mu2)
      *prG(q-p/2)$
% Clockwise: W-G-G (negative charge) = crossed 4c:
term4d:=hGWmi(p,-q-p/2,beta)
      *prG(q+p/2)
      *GGga(-q+k,q+p/2,mu1)
      *prG(q-k)
      *GWplga(beta2,mu2)
      *prW(q-p/2,beta2,beta)$
% Clockwise: W-G-G (positive charge):
term4e:=hGWpl(p,-q-p/2,beta)
      *prG(q+p/2)
      *GGga(q+p/2,-q+k,mu1)
      *prG(q-k)
      *GWmiga(beta2,mu2)
      *prW(q-p/2,beta2,beta)$
% Clockwise: G-W-G (negative charge) = crossed 4e:
term4f:=hGWpl(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *GWmiga(alpha1,mu1)
      *prG(q-k)
      *GGga(-q+p/2,q-k,mu2)
```

```
      *prG(q-p/2)$
term4:=term4a+term4b+term4c+term4d+term4e+term4f$
```

The first two terms, `term4a` and `term4b`, are identical, since the vertices depend only on the metric tensor.

Finally, there are terms with one W and one G propagator, and a four-point vertex. A representative case is given in fig. 4.6. There are four diagrams of this kind. The



(5a)

Figure 4.6: Feynman diagram for $H \to \gamma\gamma$ involving the quartic $HGW\gamma$ vertex. There are two diagrams with the k_2 line attached to the $HGW\gamma$ vertex (G^+ out or G^- out), and two diagrams with k_1 and k_2 interchanged.

corresponding `REDUCE` code is

```
%Clockwise W-G (positive charge, quartic coupling):
term5a:=hGWplga(mu1,beta)
      *d2       % cancel absent propagator of J(1,1,1)
      *prG(q-k)
      *GWmiga(beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise W-G (negative charge, quartic coupling):
term5b:=hGWmiga(mu1,beta)
      *d2       % cancel absent propagator of J(1,1,1)
      *prW(q-k,beta,beta1)
      *GWplga(beta1,mu2)
      *prG(q-p/2)$
%Clockwise W-G (positive charge, quartic coupling) = crossed 5a:
term5c:=hGWplga(mu2,beta)
```

```
      *prG(q+p/2)
      *GWmiga(beta1,mu1)
      *prW(q-k,beta1,beta)
      *d1$        % cancel absent propagator of J(1,1,1)
%Clockwise W-G (negative charge, quartic coupling) = crossed 5b:
term5d:=hGWmiga(mu2,beta)
      *prW(q+p/2,beta,beta1)
      *GWplga(beta1,mu1)
      *prG(q-k)
      *d1$        % cancel absent propagator of J(1,1,1)
term5:=term5a+term5b+term5c+term5d$
```

4.5 Faddeev-Popov loop contributions

For the decay mode $H \to \gamma\gamma$ mediated by Faddeev-Popov ghosts (which are denoted by η^+ and η^-), we should point out that the η^+ field has nothing to do with the η^- field. Consequently, in addition to the crossed diagrams, there will be two η-diagrams; one with an η^+ and one with an η^-. So, in total we have four Feynman diagrams for $H \to \gamma\gamma$ through Faddeev-Popov ghosts; one corresponds to η^+ which is labeled by (6a) in fig. 4.7 and its crossed diagram whose contribution is given in `'term6b'` in the `REDUCE` program, and one corresponds to η^- which is labeled by (6c) in fig. 4.7 and its crossed diagram whose contribution is given in `'term6d'` in the `REDUCE` program. We should also point out that the Faddeev-Popov ghosts behave like fermions in the sense that a direction must be assigned and an extra minus sign should be given for each closed loop of these ghost fields.

The corresponding `REDUCE` code is

```
term6a:=-hghgh      % positive FP ghost clockwise
      *prgh(q+p/2)
      *ghghplga(-q+k,mu1)
      *prgh(q-k)
      *ghghplga(-q+p/2,mu2)
      *prgh(q-p/2)$
term6b:=-hghgh      % crossed diagram  (positive FP ghost anticlockwise)
      *prgh(q-p/2)
      *ghghplga(q-k,mu2)
      *prgh(q-k)
      *ghghplga(q+p/2,mu1)
      *prgh(q+p/2)$
term6c:=-hghgh      % negative FP ghost clockwise
      *prgh(q+p/2)
      *ghghmiga(-q+k,mu1)
      *prgh(q-k)
```

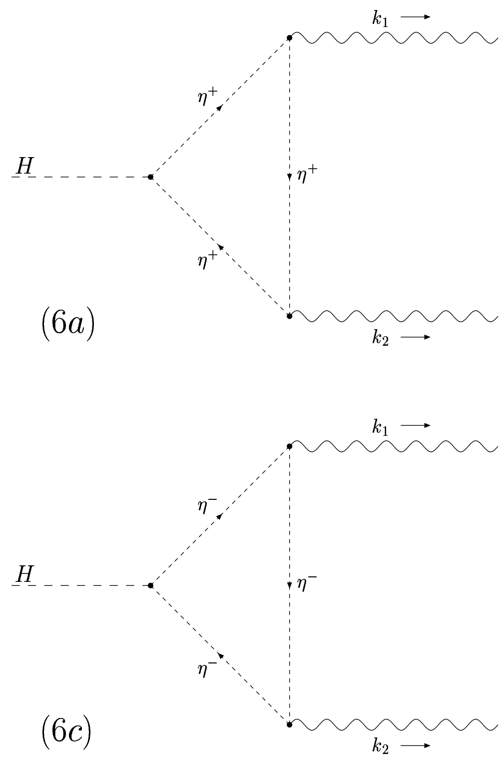


Figure 4.7: Feynman diagrams for $H \to \gamma\gamma$ involving the Faddeev-Popov ghosts η^+ and η^-. There are also crossed diagrams (6b) and (6d).

```
      *ghghmiga(-q+p/2,mu2)
      *prgh(q-p/2)$
term6d:=-hghgh       % crossed diagram  (negative FP ghost anticlockwise)
      *prgh(q-p/2)
      *ghghmiga(q-k,mu2)
      *prgh(q-k)
      *ghghmiga(q+p/2,mu1)
      *prgh(q+p/2)$
term6:=term6a+term6b+term6c+term6d$
```

Chapter 5

The Feynman amplitude

In general, for any process involving external photons, the Feynman amplitude M is of the form

$$M = \epsilon_{1\mu_1}(\mathbf{k_1})\epsilon_{2\mu_2}(\mathbf{k_2})...M^{\mu_1\mu_2\cdots}(\mathbf{k_1},\mathbf{k_2},...) \tag{5.1}$$

There is thus one polarization vector $\epsilon(\mathbf{k})$ for each photon, and the tensor amplitude $M^{\alpha\beta\cdots}(\mathbf{k_1},\mathbf{k_2},...)$ is independent of these polarization vectors.

Since our interest is the $H \to \gamma\gamma$ process, then the Feynman amplitude for this process will take the form

$$M_0 = \epsilon_{1\mu_1}(\mathbf{k_1})\epsilon_{2\mu_2}(\mathbf{k_2})M_0^{\mu_1\mu_2}(\mathbf{k_1},\mathbf{k_2}) \tag{5.2}$$

5.1 The tensor amplitude $M_0^{\mu_1\mu_2}(\mathbf{k_1},\mathbf{k_2})$

As a second rank tensor, the tensor amplitude, $M_0^{\mu_1\mu_2}(k_1,k_2)$, can be decomposed in terms of its arguments, k_1 and k_2, as follows:[1]

$$M_0^{\mu_1\mu_2}(k_1,k_2) = A_0 g^{\mu_1\mu_2} + A_1 k_1^{\mu_1}k_1^{\mu_2} + A_2 k_1^{\mu_1}k_2^{\mu_2} + A_3 k_2^{\mu_1}k_1^{\mu_2} + A_4 k_2^{\mu_1}k_2^{\mu_2} \tag{5.3}$$

We shall here discuss how these scalar coefficient functions A_0, A_1, A_2, A_3, A_4 are extracted. Alternatively, the tensor integrals could be reduced to scalar integrals by the method given in Appendix B.

5.2 The two-photon case

Let us study the tensor amplitude (5.3) when the two external momenta, k_1 and k_2, represent the momenta of two photons, γ_1 and γ_2 respectively. The gauge invariance condition for the real photon case where $k_1^2 = k_2^2 = 0$, requires that:

$$1. \quad k_{1\mu_1}M_0^{\mu_1\mu_2}(k_1,k_2) = 0 \tag{5.4}$$

[1]When a γ_5 coupling is introduced, there will also be a more general term, as discussed in Chapter 8.

$$2. \quad k_{2\mu_2} M_0^{\mu_1\mu_2}(k_1, k_2) = 0 \tag{5.5}$$

The first requirement above can be fulfilled *iff* :

$$(k_1 \cdot k_2) A_4 = 0 \quad \Longrightarrow \quad A_4 = 0 \tag{5.6}$$

and

$$A_0 + (k_1 \cdot k_2) A_3 \;\; = \;\; 0 \tag{5.7}$$

while the second condition requires that:

$$(k_1 \cdot k_2) A_1 = 0, \quad \Longrightarrow \quad A_1 = 0 \tag{5.8}$$

and

$$A_0 + (k_1 \cdot k_2) A_2 \;\; = \;\; 0 \tag{5.9}$$

From eqs. (5.7) and (5.9) we find that

$$A_2 \;\; = \;\; A_3 = \frac{-1}{(k_1 \cdot k_2)} A_0 \tag{5.10}$$

So, only one *independent* coefficient is required which we may take to be A_0.
Then:

$$M_0^{\mu_1\mu_2}(k_1, k_2) \;\; = \;\; A_0 \left\{ g^{\mu_1\mu_2} - \frac{k_1^{\mu_1} k_2^{\mu_2}}{(k_1 \cdot k_2)} - \frac{k_2^{\mu_1} k_1^{\mu_2}}{(k_1 \cdot k_2)} \right\} \tag{5.11}$$

In eq. (5.11), the second term is unphysical, it is orthogonal to the photon polarization vectors. Thus, the form we shall consider, is

$$M_0^{\mu_1\mu_2}(k_1, k_2) \;\; = \;\; A_0 \left\{ g^{\mu_1\mu_2} - \frac{k_2^{\mu_1} k_1^{\mu_2}}{(k_1 \cdot k_2)} \right\}$$

In particular, we note that

$$A_3 = -(k_1 \cdot k_2) A_0 = -\tfrac{1}{2} m_H^2 A_0 \tag{5.12}$$

Thus,

$$M_0^{\mu_1\mu_2} \;\; = \;\; A_0 \left\{ g^{\mu_1\mu_2} - \frac{2 k_2^{\mu_1} k_1^{\mu_2}}{m_H^2} \right\} \tag{5.13}$$

An explicit calculation of A_3 provides a check on gauge invariance.

Photon Polarization sums.

We shall sum over photon polarizations. Using

$$\sum_{\epsilon_1(k_1),\epsilon_2(k_2)} \epsilon_1(k_1)_{\mu_1}\epsilon_2(k_2)_{\mu_2}\epsilon_1(k_1)_{\bar{\mu}_1}\epsilon_2(k_2)_{\bar{\mu}_2} = g_{\mu_1\bar{\mu}_1}g_{\mu_2\bar{\mu}_2} \tag{5.14}$$

the polarization sum for two free photons with external momenta k_1, k_2 is

$$\sum_{\epsilon_1(k_1),\epsilon_2(k_2)} \mid \epsilon_1(k_1)_{\mu_1}\epsilon_2(k_2)_{\mu_2}M_0^{\mu_1\mu_2}(k_1,k_2) \mid^2 = 2 \mid A_0 \mid^2 \tag{5.15}$$

Extracting A_0, A_1, A_2, A_3, A_4

Now we are going to evaluate the coefficients of the tensor decomposition A_0, A_1, A_2, A_3, A_4 in terms of new quantities b_1, b_2, b_3, b_4, b_5, to be defined below.

In order to be able to use the results also for $H \to Z\gamma$, we shall not set k_1^2 and k_2^2 equal to zero.

First, let us make the operations below on eq. (5.3):

I. Multiply by $g_{\mu_1\mu_2}$:

$$g_{\mu_1\mu_2}M_0^{\mu_1\mu_2} = nA_0 + A_1k_1^2 + A_2k_1\cdot k_2 + A_3k_1\cdot k_2 + A_4k_2^2 \tag{5.16}$$

II. Multiply by $k_{1\mu_1}k_{1\mu_2}$:

$$k_{1\mu_1}k_{1\mu_2}M_0^{\mu_1\mu_2} = A_0k_1^2 + A_1(k_1^2)^2 + A_2k_1^2k_1\cdot k_2 + A_3k_1^2k_1\cdot k_2 + A_4(k_1\cdot k_2)^2 \tag{5.17}$$

III. Multiply by $k_{1\mu_1}k_{2\mu_2}$:

$$k_{1\mu_1}k_{2\mu_2}M_0^{\mu_1\mu_2} = A_0(k_1\cdot k_2) + A_1k_1^2k_1\cdot k_2 + A_2k_1^2k_2^2 + A_3(k_1\cdot k_2)^2 + A_4k_2^2k_1\cdot k_2 \tag{5.18}$$

IV. Multiply by $k_{2\mu_1}k_{1\mu_2}$:

$$k_{2\mu_1}k_{1\mu_2}M_0^{\mu_1\mu_2} = A_0(k_1\cdot k_2) + A_1k_1^2k_1\cdot k_2 + A_2(k_1\cdot k_2)^2 + A_3k_1^2k_2^2 + A_4k_2^2k_1\cdot k_2 \tag{5.19}$$

V. Multiply by $k_{2\mu_1}k_{2\mu_2}$:

$$k_{2\mu_1}k_{2\mu_2}M_0^{\mu_1\mu_2} = A_0k_2^2 + A_1(k_1\cdot k_2)^2 + A_2k_2^2k_1\cdot k_2 + A_3k_2^2k_1\cdot k_2 + A_4(k_2^2)^2 \tag{5.20}$$

Let us introduce the new notation

$$x = k_1^2, \quad y = k_2^2, \quad z = k_1\cdot k_2 \tag{5.21}$$

and define

$$\mathcal{K} = k_1^2k_2^2 - (k_1.k_2)^2$$

$$= xy - z^2 \tag{5.22}$$

together with the following abbreviations for the left-hand sides of eqs. (5.16)–(5.20):

$$\begin{aligned}
b_1 &= g_{\mu_1\mu_2} M_0^{\mu_1\mu_2} \\
b_2 &= k_{1\mu_1} k_{1\mu_2} M_0^{\mu_1\mu_2} \\
b_3 &= k_{1\mu_1} k_{2\mu_2} M_0^{\mu_1\mu_2} \\
b_4 &= k_{2\mu_1} k_{1\mu_2} M_0^{\mu_1\mu_2} \\
b_5 &= k_{2\mu_1} k_{2\mu_2} M_0^{\mu_1\mu_2}
\end{aligned} \tag{5.23}$$

Then, eqs. (5.16) through (5.20) can be written as:

$$b_1 = nA_0 + xA_1 + zA_2 + zA_3 + yA_4 \tag{5.24}$$
$$b_2 = xA_0 + x^2A_1 + xzA_2 + xzA_3 + z^2A_4 \tag{5.25}$$
$$b_3 = zA_0 + xzA_1 + xyA_2 + z^2A_3 + yzA_4 \tag{5.26}$$
$$b_4 = zA_0 + xzA_1 + z^2A_2 + xyA_3 + yzA_4 \tag{5.27}$$
$$b_5 = yA_0 + z^2A_1 + yzA_2 + yzA_3 + y^2A_4 \tag{5.28}$$

By solving the simultaneous equations, eqs. (5.24) through (5.28), one gets:

$$A_0 = \frac{1}{\mathcal{K}(n-2)}\{\mathcal{K}b_1 - yb_2 + z(b_3 + b_4) - xb_5\} \tag{5.29}$$
$$A_1 = \frac{1}{\mathcal{K}^2(n-2)}\{-\mathcal{K}yb_1 + y^2(n-1)b_2 - yz(n-1)(b_3 + b_4) + [\mathcal{K} + z^2(n-1)]b_5\} \tag{5.30}$$
$$A_2 = \frac{1}{\mathcal{K}^2(n-2)}\{\mathcal{K}zb_1 - yz(n-1)b_2 - [\mathcal{K} - xy(n-1)]b_3 \tag{5.31}$$
$$+z^2(n-1)b_4 - xz(n-1)b_5\} \tag{5.32}$$
$$A_3 = \frac{1}{\mathcal{K}^2(n-2)}\{\mathcal{K}zb_1 - yz(n-1)b_2 + z^2(n-1)b_3 - [\mathcal{K} - xy(n-1)]b_4 - xz(n-1)b_5\} \tag{5.33}$$
$$A_4 = \frac{1}{\mathcal{K}^2(n-2)}\{-\mathcal{K}xb_1 + [\mathcal{K} + z^2(n-1)]b_2 - xz(n-1)(b_3 + b_4) + x^2(n-1)b_5\} \tag{5.34}$$

If we now take the two photons on the mass-shell, then

$$x = y = 0, \quad \mathcal{K} = -z^2 \tag{5.35}$$

and the expressions simplify:

$$A_0 = \frac{1}{z(n-2)}[zb_1 - b_3 - b_4] \tag{5.36}$$

$$A_1 = \frac{1}{z^2} b_5 \tag{5.37}$$

$$A_2 = \frac{1}{z^2(n-2)}[-zb_1 + b_3 + (n-1)b_4] \tag{5.38}$$

$$A_3 = \frac{1}{z^2(n-2)}[-zb_1 + (n-1)b_3 + b_4] \tag{5.39}$$

$$A_4 = \frac{1}{z^2} b_2 \tag{5.40}$$

Chapter 6

Scalar integrals

In this Chapter we are going to define some of the terms and symbols that frequently appear in our work.

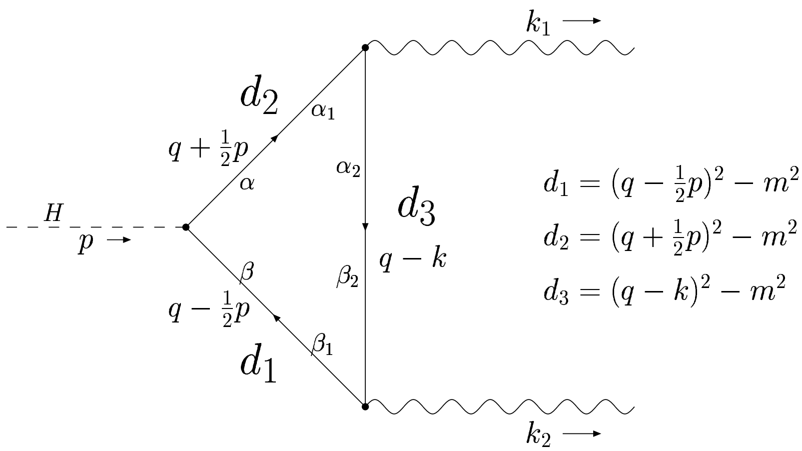


Figure 6.1: Schematic Feynman diagram illustrating $H \to \gamma\gamma$ and $H \to \gamma Z$ at the triangle one-loop level. All of the lines representing the particles involved are labeled by their momenta. Notice that the loop lines represent any of these charged particles: $W^{\pm}$ boson, $G^{\pm}$, $\eta^{\pm}$, top quark,... depending on which case is taken into consideration.

The triangle one loop-level Feynman diagram shown schematically in fig. 6.1 is going to appear frequently in our work. Each line in the diagram is labeled by its four-momentum. In all of the Feynman diagrams included we denote the fermions by solid lines with arrows in the middle, vector bosons by wiggly lines and ghosts by dashed lines (we owe the Higgs boson an apology since we denote it by a dashed line while it is not a ghost, but we have our excuse, since it is not observed yet), we also show the particle symbol next to each line. The symbols used are defined as follows:

- $k_i \equiv$ four momentum of the i-th photon
- $p \equiv k_1 + k_2$; Higgs boson momentum
- $k \equiv \frac{1}{2}(k_1 - k_2)$
- $q \equiv$ loop momentum (integration variable)

Some mathematical operations on the above relations lead to the following new relations

$$q \cdot k = \tfrac{1}{2}(q^2 + k^2 - d_3 - m^2) \tag{6.1}$$
$$q \cdot p = \tfrac{1}{2}(d_2 - d_1) \tag{6.2}$$

where d_1, d_2 and d_3 are defined below in eqs. (6.4).

According to the Feynman rules (see appendix A), each such nice diagram, as we will see, is *rigorously* equivalent to a big messy integral over the loop momentum q. This integral, more or less, will take the general form

$$\mathcal{J}(n_1, n_2, n_3) \equiv \int d^n q \frac{1}{[(q - \frac{1}{2}p)^2 - m^2]^{n_1}[(q + \frac{1}{2}p)^2 - m^2]^{n_2}[(q - k)^2 - m^2]^{n_3}} \tag{6.3}$$

We are going to denote the denominators that will appear in the integrals (6.3) as:

$$\begin{aligned} d_1 &= (q - \tfrac{1}{2}p)^2 - m^2 \\ d_2 &= (q + \tfrac{1}{2}p)^2 - m^2 \\ d_3 &= (q - k)^2 - m^2 \end{aligned} \tag{6.4}$$

where the mass m stands for the top quark mass, m_t, in the fermion-loop case, and for m_W in the others.

In our calculation, since we are dealing with Feynman diagrams at the one-loop level, we will need these integrals with integers $n_i = -1, 0, 1$. Initially, all n_i are 1. However, the numerators of the propagators and the vertex functions are in general dependent on the momenta. Thus, the extraction of the different tensor structures, discussed in Chapter 5, will introduce numerators which cancel some of the denominators. Terms with $n_i < 1$ are generated this way.

We note that in the UV region ($q \to \infty$), the integral—by counting the momenta in the numerator and the denominator—is convergent if $2(n_1 + n_2 + n_3) > n$. For example, in $n = 4$ dimensions, $\mathcal{J}(1,1,1)$ is convergent, whereas $\mathcal{J}(1,1,0)$ is not. Moreover, since the mass is the same in the three factors of the denominator, one can conclude that:

$$\mathcal{J}(1,0,0) = \mathcal{J}(0,1,0) = \mathcal{J}(0,0,1) \tag{6.5}$$

which can be verified easily by shifting the integration variable q.

6.1 One index is −1

If one of the indices is −1 and the other two are non-zeros then this $\mathcal{J}$ can be written in terms of the other $\mathcal{J}$'s of two indices being zero or two indices being one. This may seem like a mysterious statement, but to clarify, let us consider a specific case in details, and see, for example, how we can express $\mathcal{J}(-1,1,1)$ in terms of $\mathcal{J}(0,1,1)$ and $\mathcal{J}(0,0,1)$.

$$\mathcal{J}(-1,1,1) = \int d^n q \frac{d_1}{d_2\, d_3} = \int d^n q \frac{(q-\frac{1}{2}p)^2 - m^2}{[(q+\frac{1}{2}p)^2 - m^2][(q-k)^2 - m^2]} \tag{6.6}$$

Shift the integration variable q as $\tilde{q} \equiv q - \frac{1}{2}p$ (notice that $d^n\tilde{q} \equiv d^n q$ so $\tilde{q}$ and q will be interchanged again):

$$\begin{aligned}\mathcal{J}(-1,1,1) &= \int d^n q \frac{q^2 - m^2}{[q^2 - m^2][(q-\frac{1}{2}p-k)^2 - m^2]} \\ &\quad + p^2 \int d^n q \frac{1}{[q^2 - m^2][(q-\frac{1}{2}p-k)^2 - m^2]} \\ &\quad - 2p_\mu \int d^n q \frac{q^\mu}{[q^2 - m^2][(q-\frac{1}{2}p-k)^2 - m^2]}\end{aligned} \tag{6.7}$$

The first term on the r.h.s. is $\mathcal{J}(0,0,1)$, the second term is $p^2\mathcal{J}(0,1,1)$, while the third term needs further work and simplification, let us work it out.
Define

$$\begin{aligned}I^\mu &= \int d^n q \frac{q^\mu}{[q^2 - m^2][(q-\frac{1}{2}p-k)^2 - m^2]} \\ &= \ell^\mu f(\ell^2, m^2)\end{aligned}$$

where $\ell = \frac{1}{2}p + k$ and

$$f(\ell^2, m^2) = \frac{1}{\ell^2}\int d^n q \frac{q\cdot\ell}{\underbrace{[q^2 - m^2]}_{\tilde{d}_2}\underbrace{[(q-\ell)^2 - m^2]}_{\tilde{d}_3}} \tag{6.8}$$

Here, we have

$$\begin{aligned}\tilde{d}_2 - \tilde{d}_3 &= [q^2 - m^2] - [(q-\ell)^2 - m^2] \\ &= 2q\cdot\ell - \ell^2\end{aligned} \tag{6.9}$$

or

$$q\cdot\ell = \tfrac{1}{2}[\tilde{d}_2 - \tilde{d}_3 + \ell^2] \tag{6.10}$$

Substituting eq. (6.10) into eq. (6.8) we get

$$\begin{aligned} f(\ell^2, m^2) &= \frac{1}{2\ell^2}\int d^n q \frac{\tilde{d}_2 - \tilde{d}_3 + \ell^2}{\tilde{d}_2\tilde{d}_3} \\ &= \frac{1}{2\ell^2}\{\mathcal{J}(0,0,1) - \mathcal{J}(0,1,0) + \ell^2\mathcal{J}(0,1,1)\} \end{aligned} \tag{6.11}$$

Using the fact that $\mathcal{J}(0,0,1) = \mathcal{J}(0,1,0)$ together with $p_\mu \ell^\mu = p_\mu(\frac{1}{2}p + k)^\mu = p \cdot (\frac{1}{2}p + k)$ we get

$$f(\ell^2, m^2) = \tfrac{1}{2}\mathcal{J}(0,1,1) \tag{6.12}$$

Then, after straightforward simplifications we find

$$\mathcal{J}(-1,1,1) = \mathcal{J}(0,0,1) + p \cdot (\tfrac{1}{2}p - k)\mathcal{J}(0,1,1) \tag{6.13}$$

Similar recipe can be followed to find

$$\mathcal{J}(1,-1,1) = \mathcal{J}(0,0,1) + p \cdot (\tfrac{1}{2}p + k)\mathcal{J}(1,0,1) \tag{6.14}$$

and

$$\mathcal{J}(1,1,-1) = \mathcal{J}(0,1,0) + (k^2 - \tfrac{1}{4}p^2)\mathcal{J}(1,1,0) \tag{6.15}$$

6.2 The two-point functions

By two-point functions we mean $\mathcal{J}(1,1,0)$, $\mathcal{J}(0,1,1)$ and $\mathcal{J}(1,0,1)$. The integrals have only two propagators, and the result can be expressed in terms of one momentum squared (and the mass squared). Let us first evaluate the function $\mathcal{J}(1,1,0)$

$$\begin{aligned} \mathcal{J}(1,1,0) &= \int d^n q \frac{1}{d_1 d_2} \\ &= \int d^n q \frac{1}{[(q - \frac{1}{2}p)^2 - m^2][(q + \frac{1}{2}p)^2 - m^2]} \end{aligned} \tag{6.16}$$

First we will work it out in n-dimensional Minkowski space with one time-like dimension and $n-1$ space-like dimensions, then we will approach the limit as $n \to 4$, i.e. 4 dimensions.

Using the one-parameter Feynman formula (see [2], p. 224) for combining the denominators:

$$\frac{1}{ab} = \int_0^1 \frac{dz}{[b + (a-b)z]^2} \tag{6.17}$$

one obtains

$$\mathcal{J}(1,1,0) = \int d^n q \int_0^1 \frac{dz}{[q^2 + \frac{1}{4}p^2 - m^2 + q \cdot p(1-2z)]^2} \tag{6.18}$$

Next, we change the integration variable q into $\tilde{q} \equiv q + \frac{1}{2}p(1-2z)$. This is chosen such that the term in the denominator linear in the integration variable disappears, while $d^n\tilde{q} \equiv d^n q$, so $\tilde{q}$ and q are interchangeable again (q is a dummy variable here):

$$\mathcal{J}(1,1,0) = \int_0^1 dz \int d^n q \frac{1}{[q^2 - m^2 + p^2 z(1-z) + i\epsilon]^2} \tag{6.19}$$

where we have re-introduced $+i\epsilon$. Using eq. (10.23) of [2], we find

$$\mathcal{J}(1,1,0) = i\pi^{n/2}(-1)^2 \frac{\Gamma(2-\frac{n}{2})}{\Gamma(2)} \int_0^1 dz[m^2 - p^2 z(1-z) - i\epsilon]^{\frac{n}{2}-2} \tag{6.20}$$

We are interested in the limit $n \to 4$; and let $n = 4 - 2\varepsilon$ where ε is a small positive parameter ($\varepsilon \to 0$). Using $\Gamma(2) = 1$, eq. (6.20) becomes:

$$\mathcal{J}(1,1,0) \quad = \quad i\pi^2 \Gamma(\varepsilon) \int_0^1 dz[m^2 - p^2 z(1-z) - i\epsilon]^{-\varepsilon} \tag{6.21}$$

With the expansion

$$x^{-\varepsilon} = e^{-\varepsilon \ln x} = 1 - \varepsilon \ln x + \mathcal{O}(\varepsilon^2) \approx 1 - \varepsilon \ln x, \quad \text{for small } \varepsilon \tag{6.22}$$

and the relation

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon), \quad \text{for small } \varepsilon \tag{6.23}$$

where $\gamma = 0.5772...$ is the Euler constant, we can re-express eq. (6.21) in the limit $n \to 4$ as:

$$\mathcal{J}(1,1,0) \quad = \quad i\pi^2 \left(\frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon)\right) \left\{1 - \varepsilon \int_0^1 dz \ \ln[m^2 - p^2 z(1-z) - i\epsilon]\right\} \tag{6.24}$$

where the integral is finite. We shall return to this integral below.

We note that

$$\lim_{n \to 4} (n-4)\mathcal{J}(1,1,0) = -2i\pi^2 \tag{6.25}$$

Next, we consider the integral

$$\mathcal{J}(0,1,1) \quad = \quad \int d^n q \frac{1}{[(q+\frac{1}{2}p)^2 - m^2][(q-k)^2 - m^2]} \tag{6.26}$$

Similar to the Feynman parameterization of $\mathcal{J}(1,1,0)$, we write

$$\mathcal{J}(0,1,1) \quad = \quad \int_0^1 dz \int d^n q \frac{1}{\left\{q^2 + 2q[\frac{1}{2}pz - k(1-z)] + k^2(1-z) + \frac{1}{4}p^2 z - m^2]\right\}^2}$$

(6.27)

Here, we shift the integration variable q into $\tilde{q} = q + \frac{1}{2}pz - k(1-z)$ which implies that $d^n q \equiv d^n \tilde{q}$, so,

$$\mathcal{J}(0,1,1) = \int_0^1 dz \int d^n q \frac{1}{[q^2 + z(1-z)(k+\frac{1}{2}p)^2 - m^2]^2} \tag{6.28}$$

According to our definition of the four-momenta, k and p:

$$\begin{aligned}(k+\tfrac{1}{2}p)^2 &= [\tfrac{1}{2}(k_1 - k_2) + \tfrac{1}{2}(k_1 + k_2)]^2 \\ &= k_1^2 \end{aligned} \tag{6.29}$$

Consequently,

$$\mathcal{J}(0,1,1) = \int_0^1 dz \int d^n q \frac{1}{[q^2 + z(1-z)k_1^2 - m^2 + i\epsilon]^2} \tag{6.30}$$

where we have re-introduced the $i\epsilon$ term. We note the similarity to (6.19). With $k_1^2 = 0$, i.e., the photon on the mass-shell, we get

$$\mathcal{J}(0,1,1) = \int_0^1 dz \int d^n q \frac{1}{[q^2 - m^2]^2} \tag{6.31}$$

$$= i\pi^{n/2}(-1)^2 \frac{\Gamma(2-\frac{n}{2})}{\Gamma(2)} \int_0^1 dz [m^2]^{\frac{n}{2}-2} \tag{6.32}$$

With $n = 4 - 2\varepsilon$, and $\varepsilon \to 0$,

$$\begin{aligned}\mathcal{J}(0,1,1) &= i\pi^2 \left(\frac{1}{\varepsilon} - \gamma + \mathcal{O}(\varepsilon)\right) \left\{1 - \varepsilon \int_0^1 dz\ \ln[m^2]\right\} \\ &= i\pi^2 \left(\frac{1}{\varepsilon} - \gamma\right) \left(1 - \varepsilon \ln m^2\right)\end{aligned} \tag{6.33}$$

In analogy with eq. (6.25), we also have

$$\lim_{n\to 4}(n-4)\mathcal{J}(0,1,1) = -2i\pi^2, \tag{6.34}$$

Finally, we will take a look at:

$$\mathcal{J}(1,0,1) = \int d^n q \frac{1}{[(q-\frac{1}{2}p)^2 - m^2][(q-k)^2 - m^2]} \tag{6.35}$$

Evaluating $\mathcal{J}(1,0,1)$ will not be a big deal; since it is very similar to what we did to $\mathcal{J}(0,1,1)$. The only difference occurs when shifting the integration variable q into $\tilde{q} = q + \frac{1}{2}pz + k(1-z)$, we get

$$\mathcal{J}(1,0,1) = \int_0^1 dz \int d^n q \frac{1}{[q^2 + z(1-z)(\frac{1}{2}p - k)^2 - m^2]^2} \tag{6.36}$$

The factor in the denominator is

$$\begin{aligned}(\tfrac{1}{2}p-k)^2 &= \left[\tfrac{1}{2}(k_1+k_2)-\tfrac{1}{2}(k_1-k_2)\right]^2 \\ &= k_2^2 \end{aligned} \tag{6.37}$$

If we put $k_2^2 = k_1^2$, and with the second photon on the mass-shell, $k_2^2 = 0$, we get the same result as for $\mathcal{J}(0,1,1)$:

$$\mathcal{J}(0,1,1) = \mathcal{J}(1,0,1) = i\pi^2\left(\frac{1}{\varepsilon}-\gamma\right)\left(1-\varepsilon\ln m^2\right) \tag{6.38}$$

Also,

$$\lim_{n\to 4}(n-4)\mathcal{J}(1,0,1) = -2i\pi^2 \tag{6.39}$$

We note that $\mathcal{J}(1,1,0)$, $\mathcal{J}(0,1,1)$ and $\mathcal{J}(1,0,1)$ are all of the same form, given by (6.24), if we for p^2 substitute the squared momentum corresponding to the line *opposite* the propagator that is "missing" (indicated by the zero). We summarize the results for $n \to 4$:

$$\begin{aligned}\lim_{n\to 4}(n-4)\mathcal{J}(1,1,0) &= -2i\pi^2, \\ \lim_{n\to 4}(n-4)\mathcal{J}(1,0,1) &= -2i\pi^2, \\ \lim_{n\to 4}(n-4)\mathcal{J}(0,1,1) &= -2i\pi^2 \end{aligned} \tag{6.40}$$

For the $HZ\gamma$ vertex, the functions $\mathcal{J}(0,1,1)$ and $\mathcal{J}(1,0,1)$ will be different, since then one of the momenta squared will be equal to m_Z^2.

6.3 The integral in eq. (6.24)

Let us now study the integral appearing in eq. (6.24):

$$\begin{aligned}&\int_0^1 dz\;\ln[-p^2 z(1-z)+m^2-i\epsilon] \\ &= \ln m_H^2 + \int_0^1 dz\;\ln\left[-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon\right]\end{aligned} \tag{6.41}$$

where we have used $p^2 = m_H^2$, and defined

$$\mu^2 = \frac{4m^2}{m_H^2} \tag{6.42}$$

Let us now play with the argument of the logarithm

$$-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon = (z-\tfrac{1}{2})^2-\tfrac{1}{4}+\tfrac{1}{4}\mu^2-i\epsilon \tag{6.43}$$

Factorizing, we get

$$-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon=\left((z-\tfrac{1}{2})+\tfrac{1}{2}\sqrt{1-\mu^2+i\epsilon}\right)\left((z-\tfrac{1}{2})-\tfrac{1}{2}\sqrt{1-\mu^2+i\epsilon}\right) \quad (6.44)$$

By looking at the argument of the square root, we see that we have two cases:

$$1.\quad \mu<1 \quad\Longrightarrow\quad \sqrt{1-\mu^2+i\epsilon}=\sqrt{1-\mu^2}+i\epsilon \quad (6.45)$$

$$2.\quad \mu>1 \quad\Longrightarrow\quad \sqrt{1-\mu^2+i\epsilon}=i\sqrt{\mu^2-1} \quad (6.46)$$

Let us first concentrate on the first case, $\mu<1$. Then

$$-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon=\left(z-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}-i\epsilon\right)\left(z-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}+i\epsilon\right) \quad (6.47)$$

Hence,

$$\begin{aligned}
&\int_0^1 dz\ \ln[-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon]\\
=\ &\int_0^1 dz\ln\left\{\left(z-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}-i\epsilon\right)\left(z-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}+i\epsilon\right)\right\}\\
=\ &\int_0^1 dz\ln\left(z-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}-i\epsilon\right)+\int_0^1 dz\ln\left(z-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}+i\epsilon\right)\\
=\ &\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1\right]\\
&-\left(-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}-i\epsilon\right)-1\right]\\
&+\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1\right]\\
&-\left(-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}+i\epsilon\right)-1\right]
\end{aligned} \quad (6.48)$$

Extracting the phases of the arguments of the logarithms, we find

$$\begin{aligned}
&\int_0^1 dz\ \ln[-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon]\\
=\ &\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1\right]\\
&+\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1-i\pi\right]\\
&+\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1\right]\\
&+\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1+i\pi\right]\\
=\ &\left(1-\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1+\frac{i\pi}{2}\right]\\
&+\left(1+\sqrt{1-\mu^2}\right)\left[\ln\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)-1-\frac{i\pi}{2}\right]
\end{aligned}$$

$$
\begin{aligned}
&= \ln\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)+\ln\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)\\
&\quad +\sqrt{1-\mu^2}\left[\ln\left(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-\mu^2}\right)-\ln\left(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-\mu^2}\right)\right]\\
&\quad -2-i\pi\sqrt{1-\mu^2}\\
&= \ln\left[\tfrac{1}{4}-\tfrac{1}{4}(1-\mu^2)\right]+\sqrt{1-\mu^2}\ln\frac{\frac{1}{2}+\frac{1}{2}\sqrt{1-\mu^2}}{\frac{1}{2}-\frac{1}{2}\sqrt{1-\mu^2}}-2-i\pi\sqrt{1-\mu^2}\\
&= \ln\frac{\mu^2}{4}+\sqrt{1-\mu^2}\left[\ln\frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}}-i\pi\right]-2 \qquad (6.49)
\end{aligned}
$$

We consider next the second case, $\mu > 1$. Then

$$
-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon \;=\; \left(z-\tfrac{1}{2}-\tfrac{i}{2}\sqrt{\mu^2-1}\right)\left(z-\tfrac{1}{2}+\tfrac{i}{2}\sqrt{\mu^2-1}\right) \qquad (6.50)
$$

and

$$
\begin{aligned}
&\int_0^1 dz\;\ln[-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon]\\
&= \int_0^1 dz\;\ln[z-\tfrac{1}{2}-\tfrac{i}{2}\sqrt{\mu^2-1}]+\text{c.c.} \qquad (6.51)
\end{aligned}
$$

where c.c. denotes complex conjugation. Performing the integral over z we get

$$
\begin{aligned}
&\int_0^1 dz\;\ln[-z(1-z)+\tfrac{1}{4}\mu^2-i\epsilon]\\
&= \Big\{\left(\tfrac{1}{2}-\tfrac{i}{2}\sqrt{\mu^2-1}\right)\left[\ln\left(\tfrac{1}{2}-\tfrac{i}{2}\sqrt{\mu^2-1}\right)-1\right]\\
&\quad -\left(-\tfrac{1}{2}-\tfrac{i}{2}\sqrt{\mu^2-1}\right)\left[\ln\left(-\tfrac{1}{2}-\tfrac{i}{2}\sqrt{\mu^2-1}\right)-1\right]\Big\}+\text{c.c.} \qquad (6.52)
\end{aligned}
$$

Actually, it turns out that we do not need the explicit result of this integral in our calculations since it is related to $\mathcal{J}(1,1,0)$ and we always meet the $\mathcal{J}(1,1,0)$ multiplied by a factor of $(n-4)$ but, what we need indeed is the result given in eq. (6.25).

6.4 $\mathcal{J}(1,1,1)$

Since we are dealing with Feynman diagrams with triangle one loop and three external momenta; k_1, k_2 and $p = k_1 + k_2$, expressions of the form

$$
\mathcal{J}(1,1,1) \;\equiv\; \int d^nq\frac{1}{[(q-\frac{1}{2}p)^2-m^2+i\epsilon][(q+\frac{1}{2}p)^2-m^2+i\epsilon][(q-k)^2-m^2+i\epsilon]} \qquad (6.53)
$$

will come in the way, where $n = 4 - 2\varepsilon$ is the space-time dimension. Let us first point out that, in four dimensions ($n = 4$) and for large q (the UV region), such an integral behaves as:

$$\int d^4q \, \frac{1}{(q^2)^3}$$

which is — by counting the momenta in the denominator and the numerator— convergent.

In order to evaluate the integral in eq. (6.53), we are going to use some of the 'Feynmanology'; the two-parameter Feynman formula for combining denominators (see [2], p. 224)

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} dy \frac{1}{[a + (b-a)x + (c-a)y]^3} \tag{6.54}$$

or

$$\begin{aligned} \frac{1}{abc} &= 2\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \int_0^1 d\alpha_3 \frac{\delta(1-\alpha_1-\alpha_2-\alpha_3)}{[\alpha_1 a + \alpha_2 b + \alpha_3 c]^3} \\ &= 2\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{1}{[\alpha_1 a + \alpha_2 b + (1-\alpha_1-\alpha_2)c]^3} \end{aligned} \tag{6.55}$$

Define:

$$\mathcal{D} = \alpha_1 a + \alpha_2 b + (1-\alpha_1-\alpha_2)c \tag{6.56}$$

From eq. (6.53), we have

$$\begin{aligned} a &= (q - \tfrac{1}{2}p)^2 - m^2 + i\epsilon \\ b &= (q + \tfrac{1}{2}p)^2 - m^2 + i\epsilon \\ c &= (q-k)^2 - m^2 + i\epsilon \end{aligned} \tag{6.57}$$

Then:

$$\begin{aligned} \mathcal{D}(q,\alpha_1,\alpha_2) &= \alpha_1[(q-\tfrac{1}{2}p)^2 - m^2] + \alpha_2[(q+\tfrac{1}{2}p)^2 - m^2] \\ &\quad +(1-\alpha_1-\alpha_2)[(q-k)^2 - m^2] \\ &= q^2 + q[-\alpha_1 p + \alpha_2 p - 2(1-\alpha_1-\alpha_2)k] \\ &\quad +\tfrac{1}{4}(\alpha_1+\alpha_2)p^2 + (1-\alpha_1-\alpha_2)k^2 - m^2 \end{aligned} \tag{6.58}$$

and

$$\mathcal{J}(1,1,1) = 2\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \int d^n q \frac{1}{[\mathcal{D}(q,\alpha_1,\alpha_2)]^3} \tag{6.59}$$

where the term $i\epsilon$ has been dropped for a while.

Define the new variables:

$$\begin{aligned} \alpha &\equiv \alpha_1+\alpha_2 \\ \beta &\equiv \alpha_1-\alpha_2 \end{aligned} \tag{6.60}$$

$$\tilde{q} \equiv q-[\tfrac{1}{2}\beta p+(1-\alpha)k] \quad \Longrightarrow \quad d^n\tilde{q}\equiv d^n q \tag{6.61}$$

then, in terms of the new variables,

$$\mathcal{D}(\tilde{q},\alpha,\beta) = \tilde{q}^2+\tfrac{1}{4}(\alpha-\beta^2)p^2+\alpha(1-\alpha)k^2-\beta(1-\alpha)p\cdot k-m^2 \tag{6.62}$$

and

$$\mathcal{J}(1,1,1) = \int_0^1 d\alpha\int_{-\alpha}^{\alpha}d\beta\int d^nq\frac{1}{[\mathcal{D}(q,\alpha,\beta)]^3} \tag{6.63}$$

Here, we should remark that when we moved from eq. (6.59) to eq. (6.63), the factor of 2 in eq. (6.59) disappeared, since it has been canceled by the factor of 2 from the Jacobian due to the changing of variables in (6.60).

According to our definition for the four-momenta appearing in eq. (6.62), we have (taking all the momenta on mass-shell, i.e. $k_1^2=k_2^2=0$):

$$p^2 = (k_1+k_2)^2=2k_1\cdot k_2=m_H^2 \tag{6.64}$$
$$k^2 = \tfrac{1}{4}(k_1-k_2)^2=-\tfrac{1}{4}m_H^2 \tag{6.65}$$
$$p\cdot k = \tfrac{1}{2}(k_1^2-k_2^2)=0 \tag{6.66}$$

so,

$$\mathcal{D} = \tilde{q}^2+\tfrac{1}{4}(\alpha^2-\beta^2)m_H^2-m^2 \tag{6.67}$$

and consequently

$$\mathcal{J}(1,1,1) = \int_0^1 d\alpha\int_{-\alpha}^{\alpha}d\beta\int d^nq\frac{1}{[q^2+\frac{1}{4}(\alpha^2-\beta^2)m_H^2-m^2+i\epsilon]^3} \tag{6.68}$$

This integral can be compared to the formula (10.23) in Mandl and Shaw [2]:

$$\mathcal{J}(1,1,1) = i\pi^{n/2}(-1)^3\frac{\Gamma(3-\frac{n}{2})}{\Gamma(3)}\int_0^1 d\alpha\int_{-\alpha}^{\alpha}\frac{d\beta}{[m^2-\frac{1}{4}(\alpha^2-\beta^2)m_H^2-i\epsilon]^{3-\frac{n}{2}}} \tag{6.69}$$

Since it is convergent, we put $n=4$ to get

$$\mathcal{J}(1,1,1) = \frac{i\pi^2}{2}\int_0^1 d\alpha\int_{-\alpha}^{\alpha}\frac{d\beta}{\frac{1}{4}(\alpha^2-\beta^2)m_H^2-m^2+i\epsilon} \tag{6.70}$$

To perform the integral over β, let:

$$\mathcal{I}(\alpha) \equiv \int_{-\alpha}^{\alpha} \frac{d\beta}{\frac{1}{4}(\alpha^2-\beta^2)m_H^2 - m^2 + i\epsilon} \tag{6.71}$$

$$= \frac{4}{m_H^2}\int_{-\alpha}^{\alpha} \frac{d\beta}{\alpha^2-\mu^2-\beta^2+i\epsilon} \tag{6.72}$$

with the constant

$$\mu^2 = \frac{4m^2}{m_H^2} \tag{6.73}$$

Then,

$$\mathcal{J}(1,1,1) = -\frac{i\pi^2}{2}\int_0^1 d\alpha\, \mathcal{I}(\alpha) \tag{6.74}$$

We introduce the new variable,

$$\bar\alpha = \sqrt{\alpha^2-\mu^2} + i\epsilon, \qquad \text{Re}\ \bar\alpha < \alpha \tag{6.75}$$

and perform a partial fractioning

$$\mathcal{I}(\alpha) = \frac{2}{m_H^2\bar\alpha}\int_{-\alpha}^{\alpha} d\beta \left\{\frac{1}{\bar\alpha-\beta} + \frac{1}{\bar\alpha+\beta}\right\} \tag{6.76}$$

but note that we must distinguish two cases.

Case 1. $\mu < \alpha$, **Re** $\bar\alpha < \alpha$

In this case

$$\begin{aligned}\int_{-\alpha}^{\alpha}\frac{d\beta}{\bar\alpha-\beta} &= \lim_{\delta\to 0}\left\{\int_{-\alpha}^{\bar\alpha-\delta}\frac{d\beta}{\bar\alpha-\beta} + \int_{\bar\alpha+\delta}^{\alpha}\frac{d\beta}{\bar\alpha-\beta} + \int_{\bar\alpha-\delta}^{\bar\alpha+\delta}\frac{d\beta}{\bar\alpha-\beta}\right\}\\ &= \lim_{\delta\to 0}\left\{-\ln|\bar\alpha-\beta|\Big|_{-\alpha}^{\bar\alpha-\delta} - \ln|\beta-\bar\alpha|\Big|_{\bar\alpha+\delta}^{\alpha} + I_0\right\}\\ &= \lim_{\delta\to 0}\left\{-\ln\delta + \ln(\bar\alpha+\alpha) - \ln(\alpha-\bar\alpha) + \ln\delta + I_0\right\}\\ &= \ln\frac{\alpha+\bar\alpha}{\alpha-\bar\alpha} + I_0 \end{aligned} \tag{6.77}$$

The term I_0 is non-vanishing, since the pole singularity is off the real axis,

$$\text{Im}\ \bar\alpha = \epsilon > 0 \tag{6.78}$$

We deform the contour into the complex plane, by taking

$$\beta = \bar\alpha + \delta e^{i\phi}, \qquad d\beta = i\delta e^{i\phi} d\phi, \tag{6.79}$$

and find

$$I_0 = \int_\pi^0 \frac{i\delta e^{i\phi} d\phi}{\delta e^{i\phi}} = -i\pi \tag{6.80}$$

Furthermore,

$$\int_{-\alpha}^{\alpha} \frac{d\beta}{\bar{\alpha}+\beta} = \ln|\bar{\alpha}+\beta|\Big|_{-\alpha}^{\alpha} + I_0' = \ln\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}} + I_0' \tag{6.81}$$

where

$$I_0' = \int_{-\bar{\alpha}-\delta}^{-\bar{\alpha}+\delta} \frac{d\beta}{\bar{\alpha}+\beta} \tag{6.82}$$

With $\beta = -\bar{\alpha} + \delta e^{i\phi}$, we find $I_0' = I_0 = -i\pi$.

Hence, for $\mu < \alpha$, we have

$$\mathcal{I}(\alpha) = \frac{4}{m_H^2 \bar{\alpha}} \left[\ln\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}} - i\pi \right] \tag{6.83}$$

In this region, which contributes if $\mu < 1$, or $4m^2 < m_H^2$, $\mathcal{I}(\alpha)$ is complex, and $\mathcal{J}(1,1,1)$ will be complex.

Case 2. $\mu > \alpha$**,** $\bar{\alpha}$ **imaginary,** $\bar{\alpha} = ia$

In this case

$$\begin{aligned} \int_{-\alpha}^{\alpha} \frac{d\beta}{\bar{\alpha}-\beta} &= -\ln(\beta - ia)|_{-\alpha}^{\alpha} \\ &= -\ln(\alpha - ia) + \ln(-\alpha - ia) \end{aligned} \tag{6.84}$$

and

$$\begin{aligned} \int_{-\alpha}^{\alpha} \frac{d\beta}{\bar{\alpha}+\beta} &= \ln(\beta + ia)|_{-\alpha}^{\alpha} \\ &= \ln(\alpha + ia) - \ln(-\alpha + ia) \end{aligned} \tag{6.85}$$

Defining the logarithm with a cut along the negative real axis, we have

$$\begin{aligned} \ln(-\alpha + ia) &= \ln(\alpha - ia) + i\pi \\ \ln(-\alpha - ia) &= \ln(\alpha + ia) - i\pi \end{aligned} \tag{6.86}$$

and

$$\begin{aligned} \int_{-\alpha}^{\alpha} \left(\frac{d\beta}{\bar{\alpha}-\beta} + \frac{d\beta}{\bar{\alpha}+\beta} \right) &= -\ln(\alpha - \bar{\alpha}) + [\ln(\alpha + \bar{\alpha}) - i\pi] \\ &\quad + \ln(\alpha + \bar{\alpha}) - [\ln(\alpha - \bar{\alpha}) + i\pi] \end{aligned}$$

$$= \quad 2\left[\ln\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}} - i\pi\right] \tag{6.87}$$

Hence, for $\mu > \alpha$, we have

$$\mathcal{I}(\alpha) = \frac{4}{m_H^2\bar{\alpha}}\left[\ln\frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}} - i\pi\right] \tag{6.88}$$

We note that the logarithm is pure imaginary, and that $\mathcal{I}(\alpha)$ is real.

Thus, the contribution to $\mathcal{J}(1,1,1)$ for $\alpha < \mu$ will be real. In particular, for $1 < \mu$, $\mathcal{J}(1,1,1)$ will be real.

Substituting back into eq. (6.70), we obtain $\mathcal{J}(1,1,1)$. The remaining integration over α can be performed in terms of dilogarithms [10]:

$$\mathrm{Li}_2(z) = -\int_0^1 \frac{dt}{t}\ln(1-tz) \tag{6.89}$$

Using properties of the dilogarithm, in particular,

$$\mathrm{Li}_2\left(\frac{1}{y}\right) + \mathrm{Li}_2\left(\frac{1}{1-y}\right) = -\tfrac{1}{2}[\ln(-y) - \ln(1-y)]^2 \tag{6.90}$$

one finds the result [11]

$$m^2\mathcal{J}(1,1,1) = -i\pi^2\frac{\mu^2}{2}\begin{cases}\arcsin^2\sqrt{1/\mu^2}, & 1<\mu\\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}} - i\pi\right]^2 & \mu<1\end{cases} \tag{6.91}$$

Details can be found in [12].

It is convenient to define the function

$$f(\mu^2) = \begin{cases}\arcsin^2\sqrt{1/\mu^2}, & 1<\mu\\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}} - i\pi\right]^2 & \mu<1\end{cases} \tag{6.92}$$

We may then write

$$\mathcal{J}(1,1,1)/(i\pi^2) \quad = \quad -\frac{\mu^2}{2m^2}\,f(\mu^2) \tag{6.93}$$

$$= \quad -\frac{2}{m_H^2}\,f(\mu^2) \tag{6.94}$$

6.5 The 't Hooft-Veltman functions

The integrals discussed above, have also been studied by 't Hooft and Veltman [13]. We shall here relate our functions to their functions.

'T Hooft and Veltman define a general two-point function as (they use a different metric)

$$B(k, m_1, m_2) = \int d_n q \, \frac{1}{(q^2 + m_1^2 - i\epsilon)((q + k)^2 + m_2^2 - i\epsilon)} \tag{6.95}$$

Comparing with $\mathcal{J}(0, 1, 1)$, $\mathcal{J}(1, 0, 1)$ and $\mathcal{J}(1, 1, 0)$, we see that

$$\begin{aligned} \mathcal{J}(0,1,1) &= B(k_1, m, m) \\ \mathcal{J}(1,0,1) &= B(k_2, m, m) \\ \mathcal{J}(1,1,0) &= B(p, m, m) \end{aligned} \tag{6.96}$$

Furthermore, they define the three-point function as

$$\begin{aligned} &C(p_1, p_2, m_1, m_2, m_3) \\ &= \int d_n q \, \frac{1}{(q^2 + m_1^2)((q + p_1)^2 + m_2^2)((q + p_1 + p_2)^2 + m_3^2)} \end{aligned} \tag{6.97}$$

where the $i\epsilon$ is omitted. This integral can be brought into the form (6.53) by the following shift of the integration variable:

$$q \to q - \tfrac{1}{2}p \tag{6.98}$$

$$q + p_1 \to q + \tfrac{1}{2}p \qquad \Longrightarrow p_1 = p \tag{6.99}$$

$$q + p_1 + p_2 \to q - k \qquad \Longrightarrow p_2 = -k - \tfrac{1}{2}p = -k_1 \tag{6.100}$$

Thus,

$$\mathcal{J}(1, 1, 1) = -C(p, -k_1, m, m, m) \tag{6.101}$$

6.6 Numerical results

It is convenient to define the dimensionless quantity

$$\tilde{\mathcal{J}}(1, 1, 1) = -\frac{m^2}{i\pi^2} \mathcal{J}(1, 1, 1) \tag{6.102}$$

The sign is chosen such that the real part is positive for 'small' Higgs mass. This function is shown in fig. 6.2, for $m = 175$ GeV, corresponding to a top quark loop.

From fig. 6.2 we can read off that if the SM Higgs mass is below the $t\bar{t}$ pair threshold, then the amplitude is real, above the threshold region the amplitude is complex. Furthermore, $-m_t^2 \mathcal{J}_t(1, 1, 1)/(i\pi^2) \to 1/2$ for light Higgs masses (compared to the quark mass) which is seen to agree with what we get in eq. (6.105), where $-m^2 \mathcal{J}(1, 1, 1)/(i\pi^2) \to 1/2$ as $m_H \to 0$, or, equivalently, $m \to \infty$.

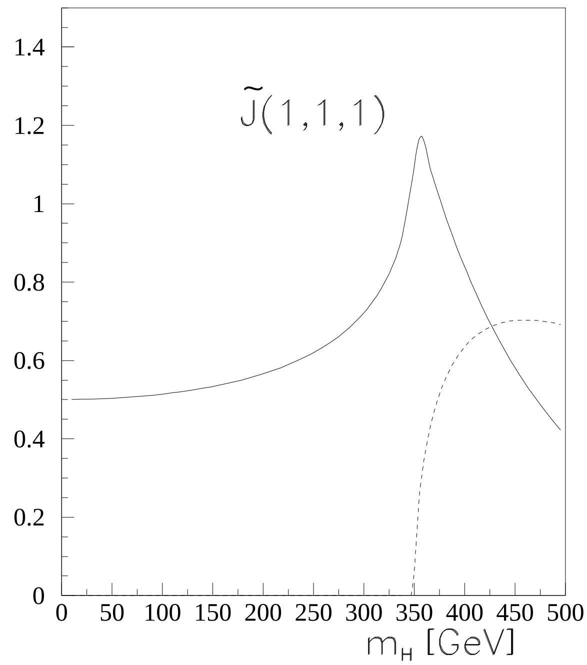


Figure 6.2: Plot of $-m_t^2\mathcal{J}_t(1,1,1)/(i\pi^2)$ vs. m_H, for $m_t = 175$ GeV. The real part is represented by the solid-line curve while the imaginary part is represented by the dashed one.

In fig. 6.3 we show the analogous result for the W mass. The structure has now moved down to lower values of m_H, corresponding to the fact that $m_W \ll m_t$.

6.7 The low-mass limit: $m_H \ll m_W, m_t$

When the Higgs mass is small compared to the other masses, $\mathcal{J}(1,1,1)$ simplifies. From eq. (6.71), we find

$$\mathcal{I}(\alpha) = \int_{-\alpha}^{\alpha} \frac{d\beta}{-m^2 + i\epsilon} \tag{6.103}$$

$$= -\frac{2\alpha}{m^2} \tag{6.104}$$

Thus, from eq. (6.70),

$$\mathcal{J}(1,1,1) = \frac{i\pi^2}{2}\int_0^1 d\alpha \frac{-2\alpha}{m^2}$$

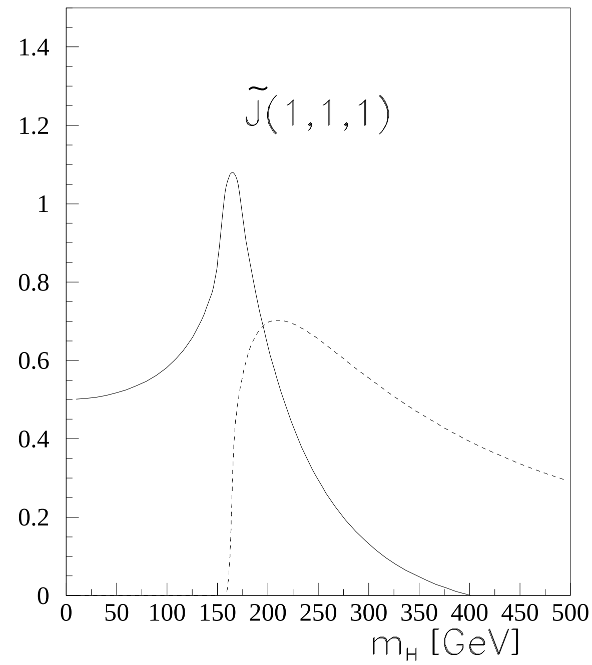


Figure 6.3: Plot of $-m_W^2 \mathcal{J}_W(1,1,1)/(i\pi^2)$ vs. m_H, for $m_W = 80$ GeV. The real part is represented by the solid-line curve while the imaginary part is represented by the dashed one.

$$= -\frac{i\pi^2}{2m^2} \tag{6.105}$$

which is seen to agree with figs. 6.2 and 6.3, where $-m^2\mathcal{J}(1,1,1)/(i\pi^2) \to 1/2$ as $m_H \to 0$, or, equivalently, $m \to \infty$.

We are interested in the approach to this limit. For

$$m_H \ll 2m_t, \qquad \text{or} \qquad \mu_t \gg 1 \tag{6.106}$$

we have

$$f(\mu^2) = \arcsin^2 \sqrt{1/\mu^2} \tag{6.107}$$

For small x,

$$\arcsin(x) = x + \frac{x^3}{3!} + \cdots \tag{6.108}$$

so

$$\arcsin^2(x) = x^2 + \frac{x^4}{3} + \cdots \tag{6.109}$$

and

$$f(\mu^2) \simeq \frac{1}{\mu^2} + \frac{1}{3\mu^4} + \cdots, \qquad \text{for } \mu \gg 1 \tag{6.110}$$

Substituting this into eq. (6.93), we find

$$\mathcal{J}(1,1,1) \simeq -\frac{i\pi^2}{2m^2}\left(1 + \frac{1}{3\mu^2}\right) \tag{6.111}$$

Chapter 7

Results for $H \to \gamma\gamma$

7.1 Quark loops

As an example we will show the result for A_0 in the form as we got it from the REDUCE program. For the quark loop contribution, we find

```
a00 :=(2*mt**2*qt**2*(j(1,1,1)*mh**2*n - 2*j(1,1,1)*mh**2
- 8*j(1,1,1)*mt**2 + 2*j(1,1,0)*n - 8*j(1,1,0)))/(mh**2*(nn - 2))$
```

Provided we multiply from the left by $i\pi^2$ which was extracted as an overall factor, then, in LaTeX form, A_0 can be written as

$$i\pi^2 A_0 = \frac{2m_t^2 Q_t^2}{(n-2)m_H^2}\left\{\left[(n-2)m_H^2 - 8m_t^2\right]\mathcal{J}_t(1,1,1) + 2(n-4)\mathcal{J}_t(1,1,0)\right\} \quad (7.1)$$

$$i\pi^2 A_3 = \frac{-4m_t^2 Q_t^2}{(n-2)m_H^4}\left\{\left[(n-2)m_H^2 - 8m_t^2\right]\mathcal{J}_t(1,1,1) + 2(n-4)\mathcal{J}_t(1,1,0)\right\} \quad (7.2)$$

We note that the result is finite, and that also the gauge-invariance condition, eq. (5.12), is satisfied.

With the abbreviation

$$\mu_t^2 = \frac{4m_t^2}{m_H^2} \quad (7.3)$$

we find

$$i\pi^2 A_0 = \frac{2m_t^2 Q_t^2}{(n-2)}\left\{\left[(n-2) - 2\mu_t^2\right]\mathcal{J}_t(1,1,1) + \frac{2(n-4)}{m_H^2}\mathcal{J}_t(1,1,0)\right\} \quad (7.4)$$

We are interested in the limit $n \to 4$, and use eq. (6.25), together with the abbreviation (6.92) and eq. (6.93):

$$A_0 = Q_t^2\left\{\left[2 - 2\mu_t^2\right]\left(-\frac{1}{2}\mu_t^2\right) f(\mu_t^2) - 4\frac{m_t^2}{m_H^2}\right\}$$

$$= -Q_t^2 \Big\{ (1-\mu_t^2)\mu_t^2 f(\mu_t^2) + \mu_t^2 \Big\} \tag{7.5}$$

Let us study the limit

$$m_H \ll 2m_t, \qquad \text{or} \qquad \mu_t \gg 1 \tag{7.6}$$

Substituting (6.111) into (7.5), we obtain

$$\begin{aligned} A_0 &= -Q_t^2 \left\{ (1-\mu_t^2)\mu_t^2 \left(\frac{1}{\mu_t^2} + \frac{1}{3\mu_t^4} \right) + \mu_t^2 \right\} \\ &= -\frac{2}{3} Q_t^2 \left[1 + \mathcal{O}(1/\mu_t^2) \right] \end{aligned} \tag{7.7}$$

Using eq. (3.12), we see that this result agrees with ref. [5]. This result was first obtained in 1973 [9].

Also, introducing a factor $N_c = 3$ for quark color, we have

$$i\pi^2 A_0 = \frac{2m_t^2 Q_t^2 N_c}{(n-2)} \left\{ \left[(n-2) - 2\mu_t^2 \right] \mathcal{J}_t(1,1,1) + \frac{2(n-4)}{m_H^2} \mathcal{J}_t(1,1,0) \right\} \tag{7.8}$$

or

$$A_0 = -Q_t^2 N_c \mu_t^2 \left\{ 1 + (1-\mu_t^2) f(\mu_t^2) \right\} \tag{7.9}$$

7.2 W (and related) loops

For the W and ghost loop contributions, we find

$$\begin{aligned} i\pi^2 A_0 &= \frac{-1}{(n-2)m_H^2} \{ 4 \left[(2n-5)m_H^2 - 2(n-1)m_W^2 \right] m_W^2 \mathcal{J}_W(1,1,1) \\ &\qquad + \left[(n-4)m_H^2 + 2(n^2-5n+4)m_W^2 \right] \mathcal{J}_W(1,1,0) \} \end{aligned} \tag{7.10}$$

$$\begin{aligned} i\pi^2 A_3 &= \frac{2}{(n-2)m_H^4} \{ 4 \left[(2n-5)m_H^2 - 2(n-1)m_W^2 \right] m_W^2 \mathcal{J}_W(1,1,1) \\ &\qquad + \left[(n-4)m_H^2 + 2(n^2-5n+4)m_W^2 \right] \mathcal{J}_W(1,1,0) \} \end{aligned} \tag{7.11}$$

We note that this result is finite and gauge invariant, compare eq. (5.12).

With $n \to 4$, the result for A_0 can be re-expressed as

$$\begin{aligned} i\pi^2 A_0 &= -\tfrac{1}{2} \{ 2 \left[6 - 3\mu_W^2 \right] m_W^2 \mathcal{J}_W(1,1,1) + (n-4) \left[1 + \tfrac{1}{2}(n-1)\mu_W^2 \right] \mathcal{J}_W(1,1,0) \} \\ &= -\tfrac{1}{2} \{ \tfrac{1}{2}(2+3\mu_W^2)(n-4)\mathcal{J}_W(1,1,0) + 2(6-3\mu_W^2) m_W^2 \mathcal{J}_W(1,1,1) \\ &= -\tfrac{1}{4} \{ (2+3\mu_W^2)(n-4)\mathcal{J}_W(1,1,0) + 12(2-\mu_W^2) m_W^2 \mathcal{J}_W(1,1,1) \} \end{aligned} \tag{7.12}$$

The result can be expressed in terms of the function $f(\mu^2)$ of eq. (6.92) as

$$A_0 = \tfrac{1}{2} \left[2 + 3\mu_W^2 + 3\mu_W^2 (2-\mu_W^2) f(\mu_W^2) \right] \tag{7.13}$$

where we have also used (6.40).

The result given by Spira et al. (eq. (3) of [7]) is:

$$A_W(\tau) = -\left[2\tau^2 + 3\tau + 3(2\tau - 1) f_S(\tau)\right]/\tau^2 \tag{7.14}$$

with

$$\tau = \frac{m_H^2}{4m_W^2} = \frac{1}{\mu_W^2} \tag{7.15}$$

and their $f_S(\tau)$ equal to our $f(\mu^2)$. Up to an over-all convention-dependent factor of -2 (their result is minus twice of ours), this result is in agreement with (7.12).

Limit of $m_H \ll m_W, m_t$

In the limit of a very low Higgs mass, the contributions of individual diagrams are given by Ellis et al [5] in the form

$$igm_W \frac{e^2\pi^2}{(2\pi)^4} \left\{ A\Gamma\left(2 - \frac{n}{2}\right) g_{\mu\nu} + Bg_{\mu\nu}\frac{m_H^2}{2m_W^2} + C\frac{k_{2\mu}k_{1\nu}}{m_W^2} \right\} + \mathcal{O}(n-4) \tag{7.16}$$

and reproduced below. The coefficients A, B and C which are used in [5] are related to our notation as follows

$$A \longleftrightarrow -\tfrac{1}{2}\lim_{n\to 4}(n-4)A_0 \tag{7.17}$$

$$B \longleftrightarrow \left.\frac{2m_W^2}{m_H^2}A_0\right|_{\mathcal{J}(1,1,0)\to 0} \tag{7.18}$$

$$C \longleftrightarrow m_W^2 A_3 \tag{7.19}$$

For comparison, our corresponding results for A are given.

It is seen that, for some individual terms, the dependence on n is different, but our total A has the form

$$\begin{aligned} A &= (n-4)\frac{m_H^2}{m_W^2} + (2n^2 - 10n + 8) \\ &= (n-4)\left(\frac{m_H^2}{m_W^2} + (n-1)\right) \end{aligned} \tag{7.20}$$

and gives a finite result when multiplied by the divergent two-point function $\mathcal{J}(1,1,0)$. Also, in the limit $n \to 4$, they agree. The different n-dependence of individual terms could be related to different ways of approaching the limit $n \to 4$.

Diagrams	A	B	C
a + crossed 1a + 1b	$3(n-1)$ $2(-4n+7)$	$37/6$	$-23/6$
b 1c	$-2(n-1)$ $4(n-1)$	-2	0 0
c + d + crossed 3c + 3d + 3e + 3f	$-(n-1)/2$ $2n-5$	$19/12$	$-23/12$
e + crossed 3a + 3b	0 0	$2m_W^2/m_H^2 + 1/6$ $m_W^2/m_H^2 + 1/12$	0 0
f + crossed 4a + 4b	0 0	1 $1/2 + m_H^2/24m_W^2$	0 0
g + h + crossed 4c + 4d + 4e + 4f	1 -2	$1/6$	$-7/6$
i + crossed 5a + 5c	-2 4	0	0 0
2×j + crossed 6a + 6b + 6c + 6d	$-1/2$ 1	$-1/12$	$-1/12$ $-1/12$
2a + 2b + 2c	$n-4$		

7.3 Total decay amplitude

The full decay amplitude can in the Standard Model be written as [see (7.9) and (7.13)]

$$\begin{aligned} A_0 &= A_0(t) + A_0(W) \\ &= -Q_t^2 N_c \mu_t^2 [1 + (1 - \mu_t^2) f(\mu_t^2)] \\ &\quad + \tfrac{1}{2}\left[2 + 3\mu_W^2 + 3\mu_W^2 (2 - \mu_W^2) f(\mu_W^2)\right] \end{aligned} \tag{7.21}$$

For low values of m_H, it approaches the limit [14]

$$\begin{aligned} A_0 &= -Q_t^2 N_c \mu_t^2 \frac{2}{3\mu_t^2} + \frac{7}{2} \\ &= -\frac{1}{2}\left[Q_t^2 \frac{4N_c}{3} - 7\right] \end{aligned} \tag{7.22}$$

There is destructive interference between the quark-loop and the W-loop contributions.

Chapter 8

Decay of a non-SM Higgs boson into two photons

Let us now consider the effect of an additional qqH coupling involving γ_5. Thus, we take the qqH coupling to be

$$-\frac{ig}{2m_W}\, m_q(1+\beta\gamma_5) \tag{8.1}$$

as would be appropriate for a Higgs particle that is not an eigenstate of CP.

This extra term generates a contribution to the amplitude which is easily evaluated using the `REDUCE` program attached in Appendix D. The additional term is

$$\begin{aligned} M_5^{\mu_1\mu_2}(k_1,k_2) &= \frac{4im_t^2Q_t^2}{m_H^2}\,\beta\,\epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma}\mathcal{J}_t(1,1,1) \\ &\equiv \frac{2\beta}{m_H^2}\,A_5\,\epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma} \end{aligned} \tag{8.2}$$

with

$$i\pi^2 A_5 = 2im_t^2Q_t^2\mathcal{J}_t(1,1,1) \tag{8.3}$$

or, using (6.94),

$$A_5 = -iQ_t^2\mu_t^2 f(\mu_t^2) \tag{8.4}$$

In the low-mass limit, $\mu_t^2 \gg 1$, it takes the form

$$\begin{aligned} A_5 &= -iQ_t^2\mu_t^2\left\{\frac{1}{\mu_t^2}+\frac{1}{3\mu_t^4}\right\} \\ &= -iQ_t^2\left\{1+\frac{1}{3\mu_t^2}\right\} = -iQ_t^2+\mathcal{O}\left(\frac{1}{\mu_t^2}\right) \end{aligned} \tag{8.5}$$

Because of the ϵ tensor, it is automatically gauge invariant,

$$\begin{aligned} &k_{1\mu_1}\,\epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma} = 0, \\ &k_{2\mu_2}\,\epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma} = 0 \end{aligned} \tag{8.6}$$

8.1 Cross section

The unpolarized cross section is proportional to the sum

$$\sum_{\epsilon_1(k_1),\epsilon_2(k_2)} |\epsilon_1(k_1)_{\mu_1}\epsilon_2(k_2)_{\mu_2} M_0^{\mu_1\mu_2}(k_1,k_2)|^2 \tag{8.7}$$

Let us consider the full amplitude, and sum over polarizations

$$M_0^{\mu_1\mu_2}(k_1,k_2) = A_0\left\{g^{\mu_1\mu_2} - \frac{k_2^{\mu_1}k_1^{\mu_2}}{(k_1\cdot k_2)}\right\} + \frac{2\beta}{m_H^2}A_5\,\epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma} \tag{8.8}$$

The interference terms between A_0 and A_5 are seen to vanish, since [compare (5.14) and (5.15)]

$$g^{\mu_1\mu_2}\epsilon^{\bar\mu_1\bar\mu_2\bar\rho\bar\sigma}\,k_{1\bar\rho}k_{2\bar\sigma}\,g_{\mu_1\bar\mu_1}g_{\mu_2\bar\mu_2} = 0 \tag{8.9}$$

and

$$k_2^{\mu_1}k_1^{\mu_2}\epsilon^{\bar\mu_1\bar\mu_2\bar\rho\bar\sigma}\,k_{1\bar\rho}k_{2\bar\sigma}\,g_{\mu_1\bar\mu_1}g_{\mu_2\bar\mu_2} = 0 \tag{8.10}$$

Let us next consider the new term squared:

$$\begin{aligned}&\epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma}\,\epsilon^{\bar\mu_1\bar\mu_2\bar\rho\bar\sigma}\,k_{1\bar\rho}k_{2\bar\sigma}g_{\mu_1\bar\mu_1}g_{\mu_2\bar\mu_2}\\ &= \epsilon^{\mu_1\mu_2\rho\sigma}\,k_{1\rho}k_{2\sigma}\,\epsilon_{\mu_1\mu_2}{}^{\bar\rho\bar\sigma}\,k_{1\bar\rho}k_{2\bar\sigma}\end{aligned} \tag{8.11}$$

We use the fact that (see [2], Appendix A)

$$\epsilon^{\alpha\beta\mu\nu}\,\epsilon_{\alpha\beta}{}^{\sigma\tau} = -2(g^{\mu\sigma}\,g^{\nu\tau} - g^{\mu\tau}\,g^{\nu\sigma}) \tag{8.12}$$

to obtain

$$\begin{aligned}&-2(g^{\rho\bar\rho}g^{\sigma\bar\sigma} - g^{\rho\bar\sigma}g^{\sigma\bar\rho})k_{1\rho}k_{2\sigma}\,k_{1\bar\rho}k_{2\bar\sigma}\\ &= -2\left[k_1^2k_2^2 - (k_1\cdot k_2)^2\right]\end{aligned} \tag{8.13}$$

Thus, for $H\to\gamma\gamma$ (this will be different for the $H\to Z\gamma$ case)

$$\begin{aligned}\sum_{\epsilon_1(k_1),\epsilon_2(k_2)} |\epsilon_1(k_1)_{\mu_1}\epsilon_2(k_2)_{\mu_2} M_0^{\mu_1\mu_2}(k_1,k_2)|^2 &= 2|A_0|^2 + 2\beta^2(k_1\cdot k_2)^2\frac{4}{m_H^4}|A_5|^2\\ &= 2|A_0|^2 + 2\beta^2|A_5|^2\end{aligned} \tag{8.14}$$

The decay mode $H\to\gamma\gamma$ is insignificant far below the W and top thresholds. The width is proportional to m_H^3 as given in eq. (3.11). The decay rate of $H\to\gamma\gamma$ vs. m_H is shown in fig. 8.1, for $m_W = 80.41$ GeV and $m_t = 173.8$ GeV, $\beta = 1$. The dashed curve at the bottom right shows the contribution to the decay rate from the additional term A_5 in the decay amplitude. The additional term is seen to be 'small' in the region $m_H < 2m_t$ and totally negligible in the region $m_H < 2m_W$. For a heavier Higgs particle, though, such an additional interaction could contribute significantly to the $\gamma\gamma$ decay rate.

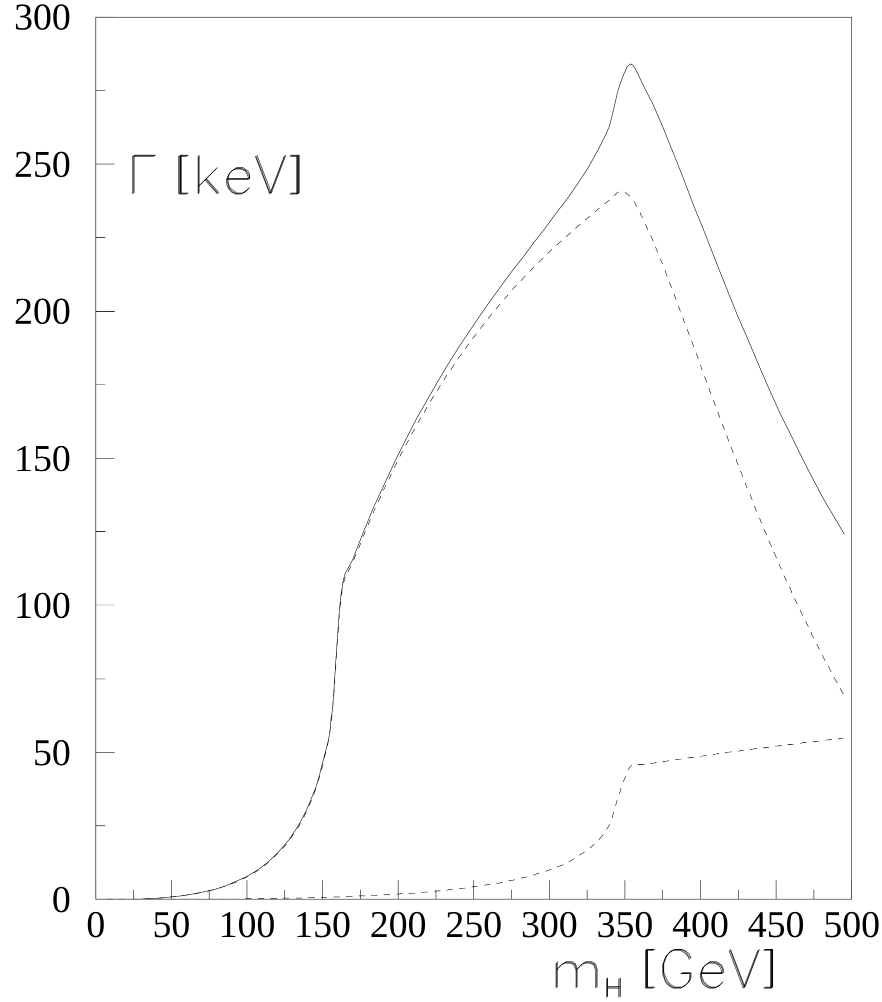


Figure 8.1: Decay rates for $H \to \gamma\gamma$. Dashed: Standard model and contribution of A_5 term. Solid: Total decay rate.

Chapter 9

Conclusions

We have analyzed the contributions from quark, $W^{\pm}$ bosons, $G^{\pm}$ ghosts, mixed WWG and WGG loops, and $\eta^{\pm}$ Faddeev-Popov ghosts, to the coupling of the Higgs boson to two photons ($H \to \gamma\gamma$) in the Standard Model, as well as in a more general theory.

Our results are as follows: The contribution from of the quark loop to the decay amplitude $A_0(t)$ is finite and gauge invariant, for the top quark loop it approaches the finite value $-8/9$ in the low Higgs mass regime. The contributions from the individual diagrams of pure W bosons, mixed WWG, WGG and $\eta^{\pm}$-loops are not gauge invariant, but the over all sum of these contributions gives a finite and gauge invariant result. The contribution from the G loops is finite and gauge invariant by itself. These results, as discussed in the text, are found to agree with results obtained by others who worked on the same topic.

We have also calculated the contribution of the top quark loop to the coupling of a non-standard Higgs to two photons. The decay amplitude A_5 should be similar to that of the CP-odd Higgs particle A^0 of the MSSM.

This detailed investigation of the Standard Higgs coupling to two photons was done in order to provide a solid starting point for studies of the coupling of non-standard Higgs particles to two photons.

In the decay of a Higgs to $Z\gamma$ final states, things might be somewhat different. Since the coupling of the Z to the quark involves a γ_5, an additional coupling of the form A_5 would be generated even without the $\beta\gamma_5$-term in the Higgs-quark coupling. Furthermore, the $\beta\gamma_5$-term could then, with the γ_5 from the Zqq coupling, lead to a contribution to the amplitude A_0. Thus, the contribution to the decay rate would be proportional to β, not β^2 as in the case of $H \to \gamma\gamma$.

To be or not to be: The Higgs Boson

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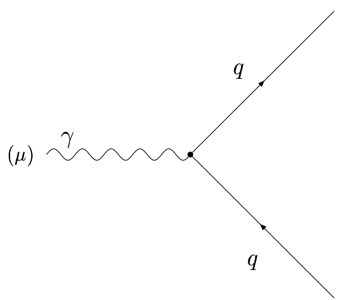
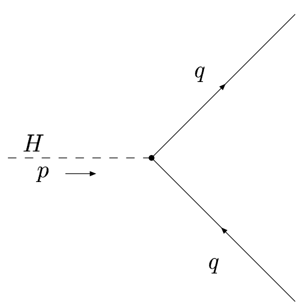
Appendix A: Feynman Rules for selected vertices

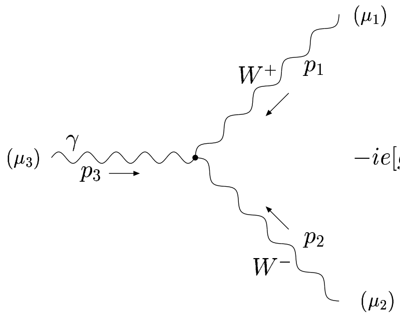
We here give the Feynman rules for Selected Vertices, in the Feynman gauge, according to Bailin and Love [8]. (Some signs are corrected according to T.T. Wu.)



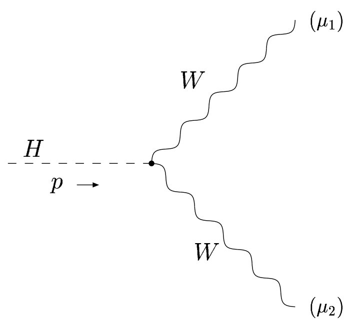
$$iQe\gamma^{\mu}$$



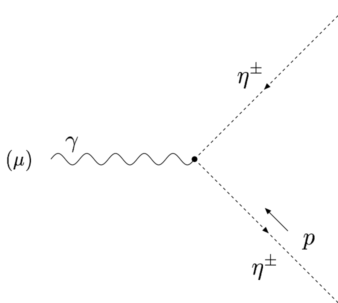
$$\frac{-i}{2} g \frac{m}{m_W}$$



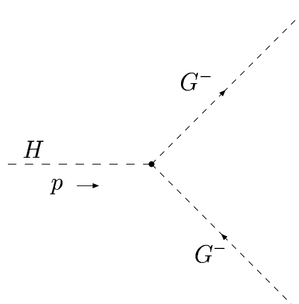
$$-ie[g_{\mu_1\mu_2}(p_1-p_2)_{\mu_3}+g_{\mu_2\mu_3}(p_2-p_3)_{\mu_1}$$
$$+g_{\mu_1\mu_3}(p_3-p_1)_{\mu_2}]$$



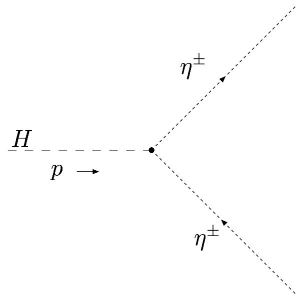
$$igm_W g_{\mu_1\mu_2}$$



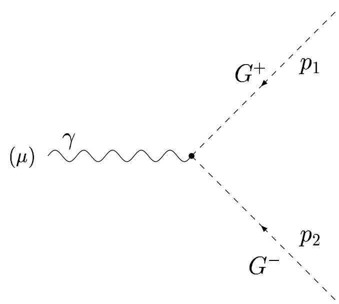
$$\pm iep_\mu$$



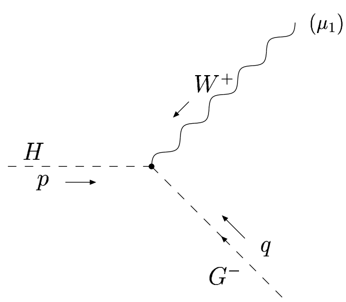
$$\frac{-ig}{2m_W}m_H^2$$



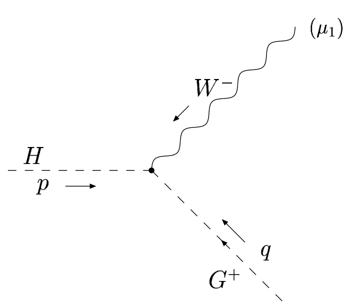
$$\frac{-ig}{2} m_W$$



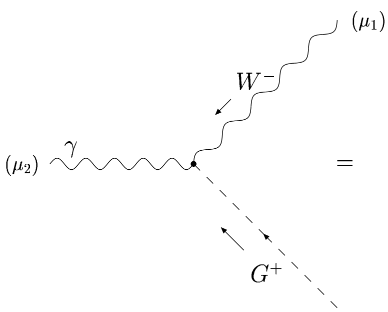
$$ie(p_1 - p_2)_\mu$$



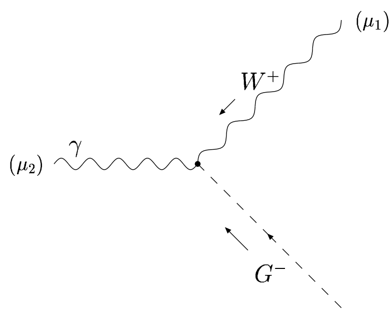
$$+\frac{1}{2} ig(p-q)_{\mu_1}$$



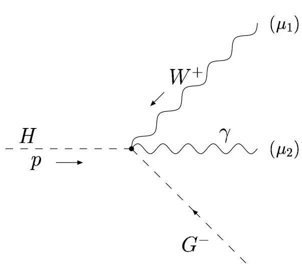
$$-\frac{1}{2} ig(p-q)_{\mu_1}$$



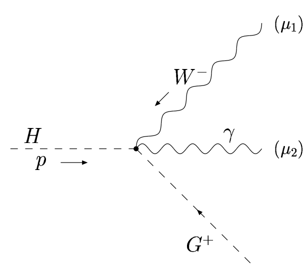
=



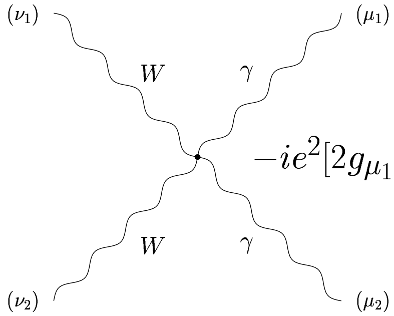
$$iem_W g_{\mu_1\mu_2}$$



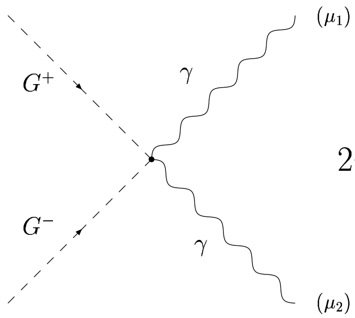
=



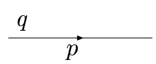
$$\frac{ieg}{2} g_{\mu_1\mu_2}$$



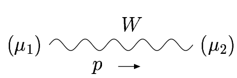
$$-ie^2[2g_{\mu_1\mu_2}g_{\nu_1\nu_2} - g_{\nu_1\mu_1}g_{\nu_2\mu_2} - g_{\nu_1\mu_2}g_{\nu_2\mu_1}]$$



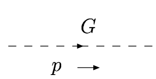
$$2ie^2 g_{\mu_1\mu_2}$$



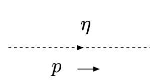
$$\frac{i(\not p + m)}{p^2 - m^2 + i\epsilon}$$



$$\frac{-ig_{\mu_1\mu_2}}{p^2 - m_W^2}$$



$$\frac{i}{p^2 - m_W^2}$$



$$\frac{i}{p^2 - m_W^2}$$

Appendix B: Tensor decomposition of the integral $I^{\mu\nu}$

Let us introduce the tensor integral:

$$\begin{aligned} I^{\mu\nu} &\equiv \int d^n q \frac{q^\mu q^\nu}{[(q-\frac{1}{2}p)^2 - m^2][(q+\frac{1}{2}p)^2 - m^2][(q-k)^2 - m^2]} \\ &= \mathcal{X}_0 g^{\mu\nu} + \mathcal{X}_1 p^\mu p^\nu + \mathcal{X}_2 (p^\mu k^\nu + k^\mu p^\nu) + \mathcal{X}_3 k^\mu k^\nu \end{aligned}$$

which is totally symmetric under the permutation of its indices, μ and ν.

The next step is to express the coefficients ($\mathcal{X}_0$, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$) in terms of the scalar integrals $\mathcal{J}$ of Chapter 6. Let us define the following new parameters:

$$\begin{aligned} \mathcal{C}_1 &= g_{\mu\nu} I^{\mu\nu} \\ &= n\mathcal{X}_0 + p^2 \mathcal{X}_1 + 2p.k\mathcal{X}_2 + k^2 \mathcal{X}_3 \qquad \text{(B.1)} \\ \mathcal{C}_2 &= p_\mu p_\nu I^{\mu\nu} \\ &= p^2 \mathcal{X}_0 + (p^2)^2 \mathcal{X}_1 + 2p^2(p.k)\mathcal{X}_2 + (p.k)^2 \mathcal{X}_3 \qquad \text{(B.2)} \\ \mathcal{C}_3 &= (p_\mu k_\nu + k_\mu p_\nu) I^{\mu\nu} \\ &= 2p.k\mathcal{X}_0 + 2p^2(p.k)\mathcal{X}_1 + 2[p^2 k^2 + (p.k)^2]\mathcal{X}_2 + 2(p.k)k^2 \mathcal{X}_3 \qquad \text{(B.3)} \\ \mathcal{C}_4 &= k_\mu k_\nu I^{\mu\nu} \\ &= k^2 \mathcal{X}_0 + (p.k)^2 \mathcal{X}_1 + 2(p.k)k^2 \mathcal{X}_2 + (k^2)^2 \mathcal{X}_3 \qquad \text{(B.4)} \end{aligned}$$

and, for simplification, let:

$$a \equiv p^2, \qquad b \equiv k^2, \qquad c \equiv p.k \qquad \text{(B.5)}$$

Solving eq. (B.2) through eq. (B.4) simultaneously, we get:

$$\begin{aligned} \mathcal{X}_0 &= \frac{1}{(n-2)(ab-c^2)} \{(ab-c^2)\mathcal{C}_1 - b\mathcal{C}_2 + c\mathcal{C}_3 - a\mathcal{C}_4\} \qquad \text{(B.6)} \\ \mathcal{X}_1 &= \frac{1}{(n-2)(ab-c^2)^2} \{-b(ab-c^2)\mathcal{C}_1 + (n-1)b^2\mathcal{C}_2 - (n-1)bc\mathcal{C}_3 \\ &\qquad + [ab + (n-2)c^2]\mathcal{C}_4\} \qquad \text{(B.7)} \\ \mathcal{X}_2 &= \frac{1}{(n-2)(ab-c^2)^2} \{c(ab-c^2)\mathcal{C}_1 - (n-1)bc\mathcal{C}_2 + \tfrac{1}{2}[(n-2)ab + nc^2]\mathcal{C}_3 \end{aligned}$$

$$-(n-1)ac\mathcal{C}_4\} \tag{B.8}$$

$$\mathcal{X}_3 = \frac{1}{(n-2)(ab-c^2)^2}\{-a(ab-c^2)\mathcal{C}_1 + [ab+(n-2)c^2]\mathcal{C}_2 - (n-1)ac\mathcal{C}_3 + (n-1)a^2\mathcal{C}_4\} \tag{B.9}$$

The $\mathcal{C}$ can be expressed in terms of the $\mathcal{J}$ as

$$\mathcal{C}_1 = \tfrac{1}{2}\mathcal{J}(0,1,1) + \tfrac{1}{2}\mathcal{J}(1,0,1) + (m^2 - \tfrac{1}{4}p^2)\mathcal{J}(1,1,1) \tag{B.10}$$

$$\mathcal{C}_2 = \tfrac{1}{4}[\mathcal{J}(1,-1,1) + \mathcal{J}(-1,1,1) - 2\mathcal{J}(0,0,1)] \tag{B.11}$$

$$\mathcal{C}_3 = \tfrac{1}{2}\{\tfrac{1}{2}[\mathcal{J}(1,-1,1) - \mathcal{J}(-1,1,1)] - \mathcal{J}(1,0,0) + \mathcal{J}(0,1,0) + (k^2 - \tfrac{1}{4}p^2)[\mathcal{J}(1,0,1) - \mathcal{J}(0,1,1)]\} \tag{B.12}$$

$$\mathcal{C}_4 = \tfrac{1}{4}\{\tfrac{1}{4}[\mathcal{J}(-1,1,1) + \mathcal{J}(1,-1,1) + 2\mathcal{J}(0,0,1)] + \mathcal{J}(1,1,-1) + (k^2 - \tfrac{1}{4}p^2)^2\mathcal{J}(1,1,1) - \mathcal{J}(0,1,0) - \mathcal{J}(1,0,0) + (k^2 - \tfrac{1}{4}p^2)[\mathcal{J}(0,1,1) + \mathcal{J}(1,0,1) - 2\mathcal{J}(1,1,0)]\} \tag{B.13}$$

Appendix C: Listing of REDUCE program

Standard Model calculation

```
set_bndstk_size 50000;

% Calculation of one-loop h \gamma\gamma vertex

off nat$
%on gcd$
% Must do this in n dimensions, divergent!
vecdim n$
%vecdim 4$ % Must be used with Gamma5;

vector alpha,beta,alpha1,alpha2,beta1,beta2$

vector p,q,k1,k2$

operator j$

operator prq,qqga,qqh$
operator prG,GGga,GGgaga$
operator hGWpl,hGWmi,GWplga,GWmiga$
operator ghghplga,ghghmiga$
operator prW,prZ,prgh,WWga,WWgaga,hWW$

diagram:=1$

%=================================================================
% FEYNMAN RULES (according to Bailin & Love, signs corrected by Tai Tsun Wu)

% exclude denominators in propagators!

% Quark propagator (massive) January 1998
```

```
for all l1,p01,mq
    let prq(l1,p01,mq)=i*(G(l1,p01)+mq)$

% Quark-Quark-photon vertex:
for all l1,Qf,mu1
    let qqga(l1,Qf,mu1)=i*Qf*G(l1,mu1)$

% Quark-Quark-Higgs vertex:
for all l1,mq
    let qqh(l1,mq)=-(1/2)*i*(mq/mW)$     % (leave out "g")
%    let qqh(l1,mq)=-(1/2)*i*(mq/mW)*(1+bb*G(l1,a))$

%----------------------------------------------------------------------

% Feynman gauge

% W and Z propagators

for all p01,mu1,mu2 let
prW(p01,mu1,mu2)=-i*mu1.mu2,
prZ(p01,mu1,mu2)=-i*mu1.mu2$

%----------------------------------------------------------------------

% Goldstone propagator:
for all p01
let prG(p01)=i$

% Goldstone-Goldstone-Higgs vertex:
% Bailin & Love, p. 228, (14.85) (leave out "g")
let hGG=-(1/2)*i*mh**2/mW$

% Goldstone-Goldstone-photon vertex:
% Bailin & Love, p. 226, (14.74) (leave out "e")
% G^+ in, momentum p01 in
% G^- in, momentum p02 in
for all p01,p02,mu1
let GGga(p01,p02,mu1)=i*(p01-p02).mu1$

% Goldstone-Goldstone-photon-photon vertex:
% Bailin & Love, p. 335, (14.74) (leave out "e^2")
% G^+ in
% G^- in
% photon in, Lorentz index mu1
% photon in, Lorentz index mu2
```

```
for all mu1,mu2
let GGgaga(mu1,mu2)=2*i*mu1.mu2$
%--------------------------------------------------------------------

% (Faddeev-Popov) Ghost propagators
for all p01
let prgh(p01)=i$

% Ghost-Ghost-Higgs vertex:
% Bailin & Love, p. 231, (14.99) (leave out "g")
let hghgh=-(1/2)*i*mW$

% Ghost-Ghost-photon vertex:
% Bailin & Love, p. 230, (14.93) (leave out "e")
% out-ghost has momentum p01 incoming
for all p01,mu1
let ghghplga(p01,mu1)= i*p01.mu1,
    ghghmiga(p01,mu1)=-i*p01.mu1$

%--------------------------------------------------------------------

% Three-vector vertex (all momenta p01,p02,p03 are in-going)

for all p01,p02,p03,mu1,mu2,mu3 let   %  (14.66), (leave out "e")
WWga(p01,p02,p03,mu1,mu2,mu3)         %  order: W^+, W^-, gamma
          =-i*((p01-p02).mu3*mu1.mu2
              +(p02-p03).mu1*mu2.mu3
              +(p03-p01).mu2*mu1.mu3)$

% Four-vector W-W-gamma-gamma vertex: (leave out "e^2")
% W+, W-:           Lorentz indices nu1,nu2
% photon1, photon2: Lorentz indices mu1,mu2
for all mu1,mu2,nu1,nu2
let WWgaga(mu1,mu2,nu1,nu2)
    =-i*(2*mu1.mu2*nu1.nu2-nu1.mu1*nu2.mu2-nu1.mu2*nu2.mu1)$

% WW-higgs vertex:  % (leave out "g")
for all mu1,mu2 let
hWW(mu1,mu2)=i*mW*mu1.mu2$

%--------------------------------------------------------------------

% Higgs-Goldstone-W vertices: % (leave out "g")
% Bailin & Love, (14.78), (14.79)
% Higgs in: momentum p01
```

```
% G^- in:   momentum p02
% W^+ in:   index mu1
for all p01,p02,mu1
let hGWpl(p01,p02,mu1)= (1/2)*i*(p01-p02).mu1,   % Tai's sign

% G^+ in:   momentum p02
% W^- in:   index mu1
    hGWmi(p01,p02,mu1)=-(1/2)*i*(p01-p02).mu1$   % Tai's sign

% Goldstone-W-photon vertices: % (leave out "e")
% G^- in
% W^+ in:    index mu1
% photon in: index mu2
for all mu1,mu2
let GWplga(mu1,mu2)= i*mW*mu1.mu2,
    GWmiga(mu1,mu2)= i*mW*mu1.mu2$ % same sign. Tai, p. 4-6

% Higgs-Goldstone-W-photon vertices: % (leave out "eg")
% G^- in
% W^+ in:    index mu1
% photon in: index mu2
for all mu1,mu2
let hGWplga(mu1,mu2)= i*(1/2)*mu1.mu2,

% G^+ in
% W^- in:    index mu1
% photon in: index mu2
    hGWmiga(mu1,mu2)= i*(1/2)*mu1.mu2$
%===================================================================

% The generic diagram

%                      -------- k1
%                    /|
%             q+p/2 / |                       d1=(q-p/2)^2 - m^2
%             (up) /  |
%                 /   |                       d2=(q+p/2)^2 - m^2
%       p        /    |
%        -------      | q-k (down)            d3=(q-k)^2 - m^2
%                \    |
%                 \   |                       p=k1+k2
%             (up) \  |
%             q-p/2 \ |                       k=(k1-k2)/2
%                    \|
%                      -------- k2
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The ttt diagram (two diagrams)

term0a:=-qqh(ll,mt)*prq(ll,q-p/2,mt)*qqga(ll,Qt,mu2)
        *prq(ll,q-k,mt)*qqga(ll,Qt,mu1)*prq(ll,q+p/2,mt)$
term0b:=-qqh(ll,mt)*prq(ll,-q-p/2,mt)*qqga(ll,Qt,mu1)
        *prq(ll,-q+k,mt)*qqga(ll,Qt,mu2)*prq(ll,-q+p/2,mt)$

% Note: Trace in REDUCE implies division by 4.
% This factor must be multiplied back in.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The WWW diagrams

term1a:=hWW(alpha,beta)          % W^+ clockwise
      *prW(q+p/2,alpha,alpha1)
      *WWga(q+p/2,-q+k,-k1,alpha1,alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(q-k,-q+p/2,-k2,beta2,beta1,mu2)
      *prW(q-p/2,beta1,beta)$
term1b:=hWW(alpha,beta)          % W^- clockwise
      *prW(q+p/2,alpha,alpha1)
      *WWga(-q+k,q+p/2,-k1,alpha2,alpha1,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(-q+p/2,q-k,-k2,beta1,beta2,mu2)
      *prW(q-p/2,beta1,beta)$
term1c:=hWW(alpha,beta)          % with quartic coupling
      *prW(q+p/2,alpha,alpha1)
      *WWgaga(mu1,mu2,alpha1,beta1)
      *d3        % cancel absent propagator of J(1,1,1)
      *prW(q-p/2,beta1,beta)$
term1:=term1a+term1b+term1c$

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The Goldstone (Higgs ghost) diagrams:

term2a:=hGG       % positive charge clockwise
      *prG(q+p/2)
      *GGga(q+p/2,-q+k,mu1)
      *prG(q-k)
      *GGga(q-k,-q+p/2,mu2)
      *prG(q-p/2)$
```

```
term2b:=hGG       % negative charge clockwise
      *prG(q+p/2)
      *GGga(-q+k,q+p/2,mu1)
      *prG(q-k)
      *GGga(-q+p/2,q-k,mu2)
      *prG(q-p/2)$
term2c:=hGG       % quartic G-G-gamma-gamma vertex
      *prG(q+p/2)
      *GGgaga(mu1,mu2)
      *d3         % cancel absent propagator of J(1,1,1)
      *prG(q-p/2)$
term2:=term2a+term2b+term2c$

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Goldstone-W diagrams:

%Clockwise W-W-G (positive charge):
term3a:=hWW(alpha,beta)
      *prW(q+p/2,alpha,alpha1)
      *GWplga(alpha1,mu1)
      *prG(q-k)
      *GWmiga(beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise W-W-G (negative charge) = crossed 3a:
term3b:=hWW(alpha,beta)
      *prW(q+p/2,alpha,alpha1)
      *GWmiga(alpha1,mu1)
      *prG(q-k)
      *GWplga(beta1,mu2)
      *prW(q-p/2,beta1,beta)$

%Clockwise W-G-W (positive charge):
term3c:=hGWpl(p,-q-p/2,beta)
      *prG(q+p/2)
      *GWmiga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(q-k,-q+p/2,-k2,beta2,beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise G-W-W (negative charge) = crossed 3c:
term3d:=hGWpl(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *WWga(-q+k,q+p/2,-k1,alpha2,alpha1,mu1)
      *prW(q-k,alpha2,beta)
      *GWmiga(beta,mu2)
```

```
      *prG(q-p/2)$
%Clockwise G-W-W (positive charge):
term3e:=hGWmi(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *WWga(q+p/2,-q+k,-k1,alpha1,alpha2,mu1)
      *prW(q-k,alpha2,beta)
      *GWplga(beta,mu2)
      *prG(q-p/2)$
%Clockwise W-G-W (negative charge) = crossed 3e:
term3f:=hGWmi(p,-q-p/2,beta)
      *prG(q+p/2)
      *GWplga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *WWga(-q+p/2,q-k,-k2,beta1,beta2,mu2)
      *prW(q-p/2,beta1,beta)$
term3:=term3a+term3b+term3c+term3d+term3e+term3f$

% Clockwise: G-G-W (positive charge):
term4a:=hGG
      *prG(q+p/2)
      *GWmiga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *GWplga(beta2,mu2)
      *prG(q-p/2)$
% Clockwise: G-G-W (negative charge) = crossed 4a:
term4b:=hGG
      *prG(q+p/2)
      *GWplga(alpha2,mu1)
      *prW(q-k,alpha2,beta2)
      *GWmiga(beta2,mu2)
      *prG(q-p/2)$

% Clockwise: G-W-G (positive charge):
term4c:=hGWmi(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *GWplga(alpha1,mu1)
      *prG(q-k)
      *GGga(q-k,-q+p/2,mu2)
      *prG(q-p/2)$
% Clockwise: W-G-G (negative charge) = crossed 4c:
term4d:=hGWmi(p,-q-p/2,beta)
      *prG(q+p/2)
      *GGga(-q+k,q+p/2,mu1)
      *prG(q-k)
      *GWplga(beta2,mu2)
```

```
      *prW(q-p/2,beta2,beta)$
% Clockwise: W-G-G (positive charge):
term4e:=hGWpl(p,-q-p/2,beta)
      *prG(q+p/2)
      *GGga(q+p/2,-q+k,mu1)
      *prG(q-k)
      *GWmiga(beta2,mu2)
      *prW(q-p/2,beta2,beta)$
% Clockwise: G-W-G (negative charge) = crossed 4e:
term4f:=hGWpl(p,q-p/2,alpha)
      *prW(q+p/2,alpha,alpha1)
      *GWmiga(alpha1,mu1)
      *prG(q-k)
      *GGga(-q+p/2,q-k,mu2)
      *prG(q-p/2)$
term4:=term4a+term4b+term4c+term4d+term4e+term4f$


%Clockwise W-G (positive charge, quartic coupling):
term5a:=hGWplga(mu1,beta)
      *d2       % cancel absent propagator of J(1,1,1)
      *prG(q-k)
      *GWmiga(beta1,mu2)
      *prW(q-p/2,beta1,beta)$
%Clockwise W-G (negative charge, quartic coupling):
term5b:=hGWmiga(mu1,beta)
      *d2       % cancel absent propagator of J(1,1,1)
      *prW(q-k,beta,beta1)
      *GWplga(beta1,mu2)
      *prG(q-p/2)$

%Clockwise W-G (positive charge, quartic coupling) = crossed 5a:
term5c:=hGWplga(mu2,beta)
      *prG(q+p/2)
      *GWmiga(beta1,mu1)
      *prW(q-k,beta1,beta)
      *d1$        % cancel absent propagator of J(1,1,1)
%Clockwise W-G (negative charge, quartic coupling) = crossed 5b:
term5d:=hGWmiga(mu2,beta)
      *prW(q+p/2,beta,beta1)
      *GWplga(beta1,mu1)
      *prG(q-k)
      *d1$        % cancel absent propagator of J(1,1,1)
term5:=term5a+term5b+term5c+term5d$
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% The Faddeev-Popov ghost diagrams:

term6a:=-hghgh       % positive FP ghost clockwise
      *prgh(q+p/2)
      *ghghplga(-q+k,mu1)
      *prgh(q-k)
      *ghghplga(-q+p/2,mu2)
      *prgh(q-p/2)$
term6b:=-hghgh       % crossed diagram  (positive FP ghost anticlockwise)
      *prgh(q-p/2)
      *ghghplga(q-k,mu2)
      *prgh(q-k)
      *ghghplga(q+p/2,mu1)
      *prgh(q+p/2)$
term6c:=-hghgh       % negative FP ghost clockwise
      *prgh(q+p/2)
      *ghghmiga(-q+k,mu1)
      *prgh(q-k)
      *ghghmiga(-q+p/2,mu2)
      *prgh(q-p/2)$
term6d:=-hghgh       % crossed diagram  (negative FP ghost anticlockwise)
      *prgh(q-p/2)
      *ghghmiga(q-k,mu2)
      *prgh(q-k)
      *ghghmiga(q+p/2,mu1)
      *prgh(q+p/2)$
term6:=term6a+term6b+term6c+term6d$

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if(diagram=0) then
  let term=4*(term0a+term0b), m=mt$

if(diagram=1) then
  let term=term1+term2+term3+term4+term5+term6, m=mW$
%  let term=term1+term3+term4+term5+term6, m=mW$
%  let term=term1a+term1b, m=mW$
%  let term=term1c, m=mW$
%  let term=term2a+term2b, m=mW$
%  let term=term3a+term3b, m=mW$
%  let term=term3c+term3d+term3e+term3f, m=mW$
%  let term=term4a+term4b, m=mW$
%  let term=term4c+term4d+term4e+term4f, m=mW$
%  let term=term5a+term5b+term5c+term5d, m=mW$
```

```
%  let term=term6, m=mW$
%  let term=term2, m=mW$
%=====================================================================

% Start contracting indices:

index alpha,beta$

term:=term;

index alpha1,alpha2,beta1,beta2$

term:=term;

% Diagram 0,1:
% d1 = (q - p/2)**2 - m**2$
% d2 = (q + p/2)**2 - m**2$
% d3 = (q - k)**2 - m**2$
% d3 = (q - k).(q - k) - m**2$

let q.q = (1/2)*( d1 + d2) - p.p/4 + m**2$
let q.k = (q.q + k.k - d3 - m**2)/2$

term:=term;

%=====================================================================

% Start extracting coefficients:

% M^{mu1,mu2} = A0*g^{mu1,mu2} + A1*k1^{mu1}*k1^{mu2} + A2*k1^{mu1}*k2^{mu2}
%                              + A3*k2^{mu1}*k1^{mu2} + A4*k2^{mu1}*k2^{mu2}

% First, evaluate contractions:

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Test decomposition:
term00:=aa0*mu1.mu2 + aa1*k1.mu1*k1.mu2 + aa2*k1.mu1*k2.mu2
                   + aa3*k2.mu1*k1.mu2 + aa4*k2.mu1*k2.mu2$
% For testing, un-comment next line:
%term:=term00$
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Rescale amplitude:

ampl:=(mW/mh**2)*term*j(1,1,1)$
```

```
b1:=mu1.mu2*ampl$
b2:=k1.mu1*k1.mu2*ampl$
b3:=k1.mu1*k2.mu2*ampl$
b4:=k2.mu1*k1.mu2*ampl$
b5:=k2.mu1*k2.mu2*ampl$

index mu1,mu2$

% Do this tensor algebra in nn dimensions:

a0:=(1/(kkk*(nn-2)))
     *(kkk*b1 - k2.k2*b2
       + k1.k2*(b3+b4) - k1.k1*b5)$

a1:=(1/(kkk**2*(nn-2)))
     *(-kkk*k2.k2*b1 + (nn-1)*k2.k2**2*b2
       -(nn-1)*k2.k2*k1.k2*(b3+b4) + (kkk+(nn-1)*k1.k2**2)*b5)$

a2:=(1/(kkk**2*(nn-2)))
     *(kkk*k1.k2*b1 - k2.k2*k1.k2*(nn-1)*b2 - (kkk - k1.k1*k2.k2*(nn-1))*b3
       + k1.k2**2*(nn-1)*b4 - k1.k1*k1.k2*(nn-1)*b5)$

a3:=(1/(kkk**2*(nn-2)))
     *(kkk*k1.k2*b1 - k2.k2*k1.k2*(nn-1)*b2 + k1.k2**2*(nn-1)*b3
       -(kkk - k1.k1*k2.k2*(nn-1))*b4 - k1.k1*k1.k2*(nn-1)*b5)$

a4:=(1/(kkk**2*(nn-2)))
     *(-kkk*k1.k1*b1 + (kkk + k1.k2**2*(nn-1))*b2
       - k1.k1*k1.k2*(nn-1)*(b3+b4) + k1.k1**2*(nn-1)*b5)$

let k1.q = (1/2)*( (d2 - d1)/2 + (q.q + k.k - d3 - m**2)),
    k2.q = (1/2)*( (d2 - d1)/2 - (q.q + k.k - d3 - m**2))$

let k=(k1-k2)/2$
let p=k1+k2$

for all n1,n2,n3,s
let d1**s*j(n1,n2,n3)=j(n1-s,n2,n3),
    d2**s*j(n1,n2,n3)=j(n1,n2-s,n3),
    d3**s*j(n1,n2,n3)=j(n1,n2,n3-s)$

for all n1,n2,n3
let d1*j(n1,n2,n3)=j(n1-1,n2,n3),
    d2*j(n1,n2,n3)=j(n1,n2-1,n3),
    d3*j(n1,n2,n3)=j(n1,n2,n3-1)$
```

```
let kkk=k1.k1*k2.k2 - k1.k2**2$

let j(1,0,0)=j(0,0,1),
    j(0,1,0)=j(0,0,1)$


let J(1,1,-1) = J(0,1,0) + (k.k - (1/4)*p.p)*J(1,1,0),
    J(1,-1,1) = J(0,0,1) + p.(p/2 + k)*J(1,0,1),
    J(-1,1,1) = J(0,0,1) + p.(p/2 - k)*J(0,1,1)$

a0;
a1;
a2;
a3;
a4;

% Now take photons on mass-shell:

let k1.k1=0, k2.k2=0$

a00:=a0;
a10:=a1;
a20:=a2;
a30:=a3;
a40:=a4;

%out "diagram0.res";
%out "diagram1.res";
diagram;

a00;
a10;
a20;
a30;
a40;

num00:=num(a00);
den00:=den(a00);

num00;
c110:=coeffn(num00,j(1,1,0),1)
     +coeffn(num00,j(1,0,1),1)
     +coeffn(num00,j(0,1,1),1);
```

```
let k1.k2=(mh**2-k1.k1-k2.k2)/2$

a00;
a30;

% Check gauge invariance:

check:=a00+(mH**2/2)*a30;
end;

let nn=n;

% Study coefficients of Ellis et al:

capA:=(mh**2/mW**2)*c110*(n-4)*j(1,1,0)/den00;
capB:=J(1,1,1)*coeffn(a00,j(1,1,1),1);
capC:=mW**2*(mh**2/mW**2)*a30;

% J(1,1,0) etc give -2*i*pi**2/(n-4)
% (i*pi**2 extracted in overall factor)

in "low_mass.red";

%end;

capA;
capB;
capC;

end;

capC+capB;
comp:=(4*mW**2/mh**2)*capA0;

end$
%======================================================================
```

Appendix D: Listing of REDUCE program

Non-Standard Model calculation

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
set_bndstk_size 50000;

% Calculation of one-loop h \gamma\gamma vertex

off nat$
%on gcd$
% Must do this in n dimensions, divergent!
%vecdim n$
vecdim 4$ % Must be used with Gamma5;

vector alpha,beta,alpha1,alpha2,beta1,beta2$

vector p,q,k1,k2$

operator j$

operator prq,qqga,qqh$
operator prG,GGga,GGgaga$
operator hGWpl,hGWmi,GWplga,GWmiga$
operator ghghplga,ghghmiga$
operator prW,prZ,prgh,WWga,WWgaga,hWW$

diagram:=0$

%=====================================================================
% FEYNMAN RULES (according to Bailin & Love, signs corrected by Tai Tsun Wu)

% exclude denominators in propagators!

% Quark propagator (massive) January 1998
```

```
for all l1,p01,mq
    let prq(l1,p01,mq)=i*(G(l1,p01)+mq)$

% Quark-Quark-photon vertex:
for all l1,Qf,mu1
    let qqga(l1,Qf,mu1)=i*Qf*G(l1,mu1)$

% Quark-Quark-Higgs vertex:
for all l1,mq
%    let qqh(l1,mq)=-(1/2)*i*(mq/mW)$     % (leave out "g")
    let qqh(l1,mq)=-(1/2)*i*(mq/mW)*(1+bb*G(l1,a))$

%----------------------------------------------------------------------

% Feynman gauge

% W and Z propagators

for all p01,mu1,mu2 let
prW(p01,mu1,mu2)=-i*mu1.mu2,
prZ(p01,mu1,mu2)=-i*mu1.mu2$

%----------------------------------------------------------------------

% Goldstone propagator:
for all p01
let prG(p01)=i$

% Goldstone-Goldstone-Higgs vertex:
% Bailin & Love, p. 228, (14.85) (leave out "g")
let hGG=-(1/2)*i*mh**2/mW$

% Goldstone-Goldstone-photon vertex:
% Bailin & Love, p. 226, (14.74) (leave out "e")
% G^+ in, momentum p01 in
% G^- in, momentum p02 in
for all p01,p02,mu1
let GGga(p01,p02,mu1)=i*(p01-p02).mu1$

% Goldstone-Goldstone-photon-photon vertex:
% Bailin & Love, p. 335, (14.74) (leave out "e^2")
% G^+ in
% G^- in
% photon in, Lorentz index mu1
% photon in, Lorentz index mu2
```

```
for all mu1,mu2
let GGgaga(mu1,mu2)=2*i*mu1.mu2$
%-----------------------------------------------------------------------

% (FP) Ghost propagators
for all p01
let prgh(p01)=i$

% Ghost-Ghost-Higgs vertex:
% Bailin & Love, p. 231, (14.99) (leave out "g")
let hghgh=-(1/2)*i*mW$

% Ghost-Ghost-photon vertex:
% Bailin & Love, p. 230, (14.93) (leave out "e")
% out-ghost has momentum p01 incoming
for all p01,mu1
let ghghplga(p01,mu1)= i*p01.mu1,
    ghghmiga(p01,mu1)=-i*p01.mu1$

%-----------------------------------------------------------------------

% Three-vector vertex (all momenta p01,p02,p03 are in-going)

for all p01,p02,p03,mu1,mu2,mu3 let   %  (14.66), (leave out "e")
WWga(p01,p02,p03,mu1,mu2,mu3)         %  order: W^+, W^-, gamma
          =-i*((p01-p02).mu3*mu1.mu2
              +(p02-p03).mu1*mu2.mu3
              +(p03-p01).mu2*mu1.mu3)$

% Four-vector W-W-gamma-gamma vertex: (leave out "e^2")
% W+, W-:           Lorentz indices nu1,nu2
% photon1, photon2: Lorentz indices mu1,mu2
for all mu1,mu2,nu1,nu2
let WWgaga(mu1,mu2,nu1,nu2)
    =-i*(2*mu1.mu2*nu1.nu2-nu1.mu1*nu2.mu2-nu1.mu2*nu2.mu1)$

% WW-higgs vertex:  % (leave out "g")
for all mu1,mu2 let
hWW(mu1,mu2)=i*mW*mu1.mu2$

%-----------------------------------------------------------------------

% Higgs-Goldstone-W vertices: % (leave out "g")
% Bailin & Love, (14.78), (14.79)
% Higgs in: momentum p01
```

```
% G^- in:   momentum p02
% W^+ in:   index mu1
for all p01,p02,mu1
let hGWpl(p01,p02,mu1)= (1/2)*i*(p01-p02).mu1,   % Tai's sign

% G^+ in:   momentum p02
% W^- in:   index mu1
    hGWmi(p01,p02,mu1)=-(1/2)*i*(p01-p02).mu1$   % Tai's sign

% Goldstone-W-photon vertices: % (leave out "e")
% G^- in
% W^+ in:    index mu1
% photon in: index mu2
for all mu1,mu2
let GWplga(mu1,mu2)= i*mW*mu1.mu2,
    GWmiga(mu1,mu2)= i*mW*mu1.mu2$ % same sign. Tai, p. 4-6

% Higgs-Goldstone-W-photon vertices: % (leave out "eg")
% G^- in
% W^+ in:    index mu1
% photon in: index mu2
for all mu1,mu2
let hGWplga(mu1,mu2)= i*(1/2)*mu1.mu2,

% G^+ in
% W^- in:    index mu1
% photon in: index mu2
    hGWmiga(mu1,mu2)= i*(1/2)*mu1.mu2$
%====================================================================

% The generic diagram

%                       -------- k1
%                     /|
%              q+p/2 / |                       d1=(q-p/2)^2 - m^2
%              (up) /  |
%                  /   |                       d2=(q+p/2)^2 - m^2
%        p        /    |
%         -------      | q-k (down)            d3=(q-k)^2 - m^2
%                 \    |
%                  \   |                       p=k1+k2
%              (up) \  |
%              q-p/2 \ |                       k=(k1-k2)/2
%                     \|
%                       -------- k2
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% The ttt diagram (two diagrams)

term0a:=-qqh(ll,mt)*prq(ll,q-p/2,mt)*qqga(ll,Qt,mu2)
        *prq(ll,q-k,mt)*qqga(ll,Qt,mu1)*prq(ll,q+p/2,mt)$
term0b:=-qqh(ll,mt)*prq(ll,-q-p/2,mt)*qqga(ll,Qt,mu1)
        *prq(ll,-q+k,mt)*qqga(ll,Qt,mu2)*prq(ll,-q+p/2,mt)$

% Note: Trace in REDUCE implies division by 4.
% This factor must be multiplied back in.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if(diagram=0) then
  let term=4*(term0a+term0b), m=mt$
if(diagram=1) then
  let term=tt1*term1+gg*term2+term3+term4+term5+popov*term6, m=mW$
%=======================================================================

% Start contracting indices:

index alpha,beta$

term:=term;

index alpha1,alpha2,beta1,beta2$

term:=term;


% Diagram 0,1:
% d1 = (q - p/2)**2 - m**2$
% d2 = (q + p/2)**2 - m**2$
% d3 = (q - k)**2 - m**2$

%end

let q.q = (1/2)*( d1 + d2) - p.p/4 + m**2$
let q.k = (q.q + k.k - d3 - m**2)/2$

term:=term;
```

```
%=====================================================================

% First, evaluate contractions:

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Rescale amplitude:
%ampl:=mW*term/mH**2$
ampl:=mW*coeffn(term,bb,1)/mH**2$

index mu1,mu2$

let k1.q = (1/2)*( (d2 - d1)/2 + (q.q + k.k - d3 - m**2)),
    k2.q = (1/2)*( (d2 - d1)/2 - (q.q + k.k - d3 - m**2))$

let k=(k1-k2)/2$
let p=k1+k2$

for all n1,n2,n3,s
let d1**s*j(n1,n2,n3)=j(n1-s,n2,n3),
    d2**s*j(n1,n2,n3)=j(n1,n2-s,n3),
    d3**s*j(n1,n2,n3)=j(n1,n2,n3-s)$

for all n1,n2,n3,s
let d1*j(n1,n2,n3)=j(n1-1,n2,n3),
    d2*j(n1,n2,n3)=j(n1,n2-1,n3),
    d3*j(n1,n2,n3)=j(n1,n2,n3-1)$

let kkk=k1.k1*k2.k2 - k1.k2**2$

let j(1,0,0)=j(0,0,1),
    j(0,1,0)=j(0,0,1)$


let J(1,1,-1) = J(0,1,0) + (k.k - (1/4)*p.p)*J(1,1,0),
    J(1,-1,1) = J(0,0,1) + p.(p/2 + k)*J(1,0,1),
    J(-1,1,1) = J(0,0,1) + p.(p/2 - k)*J(0,1,1)$

ampl;

end$
%=====================================================================
```