## Department of APPLIED MATHEMATICS

Kinetic Theory for Evolution of a Plasma in External Electromagnetic Fields toward a State characterized by Balance of Forces Transverse to the Magnetic Field

by

Alf H. Øien

Report No.40

April 1973



# UNIVERSITY OF BERGEN Bergen, Norway



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Two parameters,  $\varepsilon$  and  $\alpha$ , related to weak inhomogenity and mass ratio between electrons and ions are considered in the kinetic and macroscopic equations for a two component, weakly coupled, weakly inhomogeneous electron - ion plasma in electromagnetic fields. For certain orderings between these parameters ( $\varepsilon << \alpha$ ) the plasma components evolve toward local equilibrium of the same temperature before the typical transport processes set in. In this report we consider an extreme such ordering. Distribution functions and macroscopic functions are expanded in both parameters. Kinetia Theory for Evolution of a Plaama in External Electromagnetic Fields toward, State characterized by Balance of Forces Transverse to the Magnetic Field

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Solutions of equations are discussed and we show in particular that when solutions of kinetic eqations are assumed bounded on short effective collision time scales the plasma evolves so that forces transverse to the magnetic field in the mass velocity equation balance on longer time scales. Bolutions of the enconstances of the second of a first second of the sec

### Introduction.

The transport equations of Chapman and Cowling [1] for ionized gases, we can derive by use of the multiple time scale method of Sandri [2], Frieman [3] and Su [4], starting from the following set of parametrized kinetic and macroscopic equations (cf. Appendix 1).

$$\frac{\partial f_{i}}{\partial t_{o}} + \varepsilon_{1} \left( \underline{c}_{o} \cdot \frac{\partial f_{i}}{\partial \underline{r}} + \underline{C}_{i} \cdot \frac{\partial f_{i}}{\partial \underline{r}} + \underline{F}_{i} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} + \frac{\varepsilon_{i}}{\underline{m}_{i}} \underline{c}_{o} \times \underline{H} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} - \underline{c}_{o} \cdot \frac{\partial \underline{c}_{o}}{\partial \underline{r}} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} - \frac{\partial f_{i}}{\partial \underline{C}_{i}} \cdot \frac{\partial f_{i}}{\partial \underline{c}_{i}} + \underline{F}_{i} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} + \frac{\varepsilon_{i}}{\underline{m}_{i}} \underline{c}_{o} \times \underline{H} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} - \underline{c}_{o} \cdot \frac{\partial \underline{c}_{o}}{\partial \underline{r}} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} - \frac{\partial f_{i}}{\partial \underline{c}_{i}} \cdot \frac{\partial f_{i}}{\partial \underline{c}_{i}} + \frac{\varepsilon_{i}}{\underline{m}_{i}} \underline{c}_{i} \times \underline{H} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} = \sum_{j} c_{ij} [f_{i}f_{j}] \qquad (1)$$

$$\frac{\partial \rho_{i}}{\partial \underline{c}_{i}} + \varepsilon_{1} \frac{\partial}{\partial \underline{r}} \cdot (\rho_{i}\underline{c}_{o}) + \varepsilon_{1} \frac{\partial}{\partial \underline{r}} \cdot (\rho_{i} \ \overline{c}_{i}) = 0 \qquad (2)$$

$$\rho \left( \frac{\partial \underline{c}}{\partial t_{o}} + \varepsilon_{1}\underline{c}_{o} \cdot \frac{\partial \underline{c}}{\partial \underline{r}} \right) = \varepsilon_{1} \sum_{i} \rho_{i}\underline{F}_{i} + \varepsilon_{1}\rho_{e}\underline{c}_{o} \times \underline{H} + \underline{j} \times \underline{H} - \varepsilon_{1} \frac{\partial}{\partial \underline{r}} \cdot \underline{P} \qquad (3)$$

$$\frac{3}{2}nk \left( \frac{\partial T}{\partial t_{o}} + \varepsilon_{1}\underline{c}_{o} \cdot \frac{\partial T}{\partial \underline{r}} \right) = \varepsilon_{1} \frac{3}{2}kT \frac{\partial}{\partial \underline{r}} \cdot \sum_{i} n_{i}\underline{c}_{i} + \varepsilon_{1} \sum_{i} \rho_{i}\underline{c}_{i} \cdot \underline{F}_{i} + \varepsilon_{1} + \varepsilon_{1} \frac{\partial}{\partial \underline{r}} \cdot \underline{F}_{i} + \varepsilon_{1} \frac{\partial}{\partial \underline{r}} \cdot \underline{F}$$

Besides, the following a priori condition has to be fulfilled

$$\sum_{i} \rho_{i} \overline{\underline{C}}_{i} = 0$$
 (5)

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The transport country of Bagmen and Owling [13 for tonized pars to get desive by use of the milalpha time scale method of Sandri [3], Stingar [3] and 34 [4], starping from the following set of presenctived binetic and metroscopic equations (cf. Symendix 1).

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Resident the following a priori condition has to be fulliled

Indexes i and j refer to types of particles. Later we shall be concerned with a fully ionized plasma of electrons and one type of ions only. Instead of Boltzmann collision terms we shall use Fokker - Planck (FP) collision terms in the kinetic equations. F, and  $\frac{e_i}{m_i - o} \times H$  are accellerations due to an electric field and a magnetic field H. The definitions of mass densities  $\rho_{i}$  , charge density  $\rho_{e}$  , mass velocity  $\underline{c}_{0}$  , conduction current j, thermal flow vector q, temperature T and pressure tensor P are the same as in [1].  $\overline{C}_i$  is the average peculiar velocity of particles of type i.  $\epsilon_1$ is a small parameter which partly can be interpreted as  $\lambda/L$  where  $\lambda$  is the effective mean free path of particles and L is the scale of inhomogenities, and partly as the ratio  $|\underline{c}_{0|}|/|\underline{c}_{1}|$  where  $\underline{c}_{0|}$  is the mass velocity perpendicular to the magnetic field. However, this ordering between  $\underline{c}_{i}$  and  $\underline{C}_{i}$  in the equations of Chapman and Cowling is made only in the magnetic force terms. Therefore the parametrization of Chapman and Cowling is only formally consistent and the range of validity of the final equations obtained is not so clear. To clear up this Naze Tjøtta and Øien, [5], [6] and [7] introduced two small parameters  $\varepsilon$  and  $\alpha$  in the kinetic and macroscopic equations. ε relates to weak inhomogenities and weak (electric) fields and  $\alpha = (m_1/m_2)^{\frac{1}{2}}$ , the square root mass ratio between electrons and ions. Relating these two parameters

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to each other a new consistent ordering between all terms in the equations was obtained. The evolution of the gas through its various stages was studied by use of the multiple time scale method. The range of validity of resulting equations for the models studied was clear. These models were:

a:  $\alpha \sim \varepsilon$ ,  $\alpha^2 \sim \varepsilon \alpha \sim \varepsilon^2$ , etc. b:  $\varepsilon = 0$ ,  $\alpha$  finite c:  $\alpha = 0$ ,  $\varepsilon$  finite

Model c discribes the evolution of electrons on a background of immobile ions. The hydrodynamic stage was discribed by two variables, the density and the temperature. The results for model b are easily obtained from the results for model a . For model a the short time scale is  $\tau_{20}$  , the effective time between electron - electron collisions. On the long time scale  $\tau_{22} \sim \tau_{20}/\alpha^2$  macroscopic equations similar, but not identical to those of Chapman and Cowling were obtained. The relaxation of temperature for electrons and ions also takes place on this long time scale. This feature also differs from the theory of Chapman and Cowling since they consider electrons and ions to have the same temperature. If the gas is homogeneous and no electric field is present, on the  $\tau_{22}$  time scale the mass velocity perpendicular to the magnetic field dies away and the temperatures of electrons and ions relax against each other.

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In this report we study a model where the relaxation of temperature for electrons and ions takes place before the typical transport phenomena due to inhomogenities and electric fields set in. We study an extreme model having this feature:

d:  $\varepsilon < \alpha^n$ , all  $n \ge 0$  as  $\alpha \to 0$ .

Distribution functions and macroscopic quantities are sought as two parameter expansions. Also the time derivative is expanded in this way. The sets of equations to zeroth and first order in the parameter e are written down in section 1. In section 2 we study these sets of equations as time goes to infinity on the short time scale. The equations to zeroth order in  $\varepsilon$  are essentially the equations for model b. Using a H - theorem we easily see that on the short time scale electrons and ions evolve toward local equilibrium having then the same temperature to this order, and the perpendicular mass velocity to this order, allowed to be of the same order as the thermal velocity of ions, dies away. As time goes to infinity on the short time scale, from the equations to first order in  $\varepsilon$  we easily obtain continuity equations and equations for the temperature and mass transport vector parallel to the magnetic field of the same form as in [1] (cf. Appendix 1) except that only the parallel mass transport vector is seen in these equations to this order of approximation. Difficulty arises when trying to solve the kinetic equations and the equation for the perpen-

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\*Th this report we study a addpit where the rillaranton of \*Th this report to elitectrons and itons' takes, jizzes of ore the \*Typical transport" is encoded in the to information \*Typical transport" is encoded in the elitect of the terminoli is a strenet model is a strenet

dicular mass transport vector to first order in  $\varepsilon$ . On the short time scale these equations are strongly coupled, and because of this we have not succeeded in deriving their general form as time goes to infinity on the short time scale. To bypass this difficulty we make the plausible assumption in section 2 that the short time derivative of distribution functions to first order in  $\varepsilon$  goes to zero as the short time variable goes to infinity. This is a sufficient condition to avoid too strong growth of the distribution functions to first order in  $\varepsilon$ , but not necessary conditions for the distribution functions to be bounded on the short time scale. Consistent with this assumption we assume that the short time derivative of the perpendicular mass transport vector to first order in  $\varepsilon$  (of the order of  $\varepsilon$  times the thermal velocity of electrons) also goes to zero as the short time variable goes to infinity. This gives a balance of transverse forces at the end of the short time scale. With these assumptions we obtain a set of equations for the distribution functions to first order in  $\varepsilon$  at the end of the short time scale, corresponding to the equations of Chapman and Cowling. The balance of transverse forces determines the perpendicular mass transport vector. In section 3 we partly derive what is assumed in section 2: We derive up to a certain order of approximation that the gas evolves so that time derivatives of bounded distribution functions do vanish as time grows, and in particular that the gas evolves into a state where the transverse forces balance each other. To reach this

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conclusion we expand the sets of equations to zeroth and first order in  $\varepsilon$  in the parameter  $\alpha$ . These equations show the above feature as time grows on the  $\tau_{20}$ ,  $\tau_{21} \sim \tau_{20}/\alpha$  and  $\tau_{22} \sim \tau_{20}/\alpha^2$  time scales.

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#### 1. Basic equations and assumptions

The notations of [1] are used almost throughout. As in [5], [6] and [7] we study a two component inhomogeneous electron - ion plasma in an external electromagnetic field. The gas is assumed to be weakly coupled, [2], [3] and [4] so that collisions between particles give rise to Fokker-Planck (FP) collision terms only. The temperatures  $T_1$  and  $T_2$  of electrons and ions are initially not equal, but of equal. orders of magnitude

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 $\frac{1}{2}m_1\overline{c}_1^2\sim\frac{1}{2}m_2\overline{c}_2^2$ 

 $\frac{\overline{c}_2}{\overline{c}_1} \sim \left(\frac{m_1}{m_2}\right)^{\frac{1}{2}} = \alpha \ll 1$ 

Thus

(Subscripts 1 and 2 refer to electrons and ions respectively).

This is a basic assumption. Besides we have  $n_1 \sim n_2$  and  $e_1 \sim e_2$  and assuming for mean particle velocities

 $|\overline{c}_1| \sim \overline{c}_1$ 

 $|\overline{\underline{c}}_2| \sim \alpha \overline{\underline{c}}_1$ 

for the mass transport vector  $\underline{c}_{0}$  we have

$$|\underline{c}_{0}| = \frac{1}{\rho} |n_{1}m_{1}\overline{\underline{c}}_{1} + n_{2}m_{2}\overline{\underline{c}}_{2}| \sim \alpha \overline{c}_{1}$$

In the same way we estimate peculiar velocities  $\frac{C}{-1}$ , i.e.

$$\frac{|\underline{c}_1| \sim \overline{c}_1}{|\underline{c}_2| \sim \alpha \overline{c}_1}$$

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Besides the small parameter  $\alpha$  we also introduce

$$\varepsilon \sim \frac{c_1 \tau_{20}}{L} \ll 1$$

Here  $\tau_{20}$  is the effective time between electron-electron collisions and L is the characteristic length scale for all (weak) inhomogenities. For the electric and magnetic fields E and H we assume that

$$\frac{\tau_{20}|\mathbf{F}_1|}{|\mathbf{C}_1|} \sim \varepsilon$$

where  $\underline{F}_{i} = e_{i}/m_{i}\underline{E}$  are particle accellerations, and

$$\tau_{20}\Omega_{1} \sim 1$$

where  $\Omega_i = e_i/m_i H$  are gyrofrequencies. Both <u>E</u> and <u>H</u> are assumed to be stationary and uniform on the scales we consider, and we neglect the generation of an electromagnetic field by the evolution of the plasma itself. This assumption may be omitted, cf. Appendix 2. Since the distribution functions  $f_1$  and  $f_2$  are functions of the peculiar velocities <u>C</u><sub>1</sub> and <u>C</u><sub>2</sub>, besides position vector <u>r</u> and time t, we must be aware of the appriori condition, [1],

$$\int d\underline{C}_{1}f_{1}(\underline{r},\underline{C}_{1},t)m_{1}\underline{C}_{1} + \int d\underline{C}_{2}f_{2}(\underline{r},\underline{C}_{2},t)m_{2}\underline{C}_{2} = 0$$

Till now assumptions necessary for parametrizing the kinetic equations have been made. When parametrizing the moment equations we assume velocity moments vary on scales the same as the as the scales for corresponding powers of velocity, i.e. Beaides the small parameter a we also introduce

$$| >> \frac{\partial S^T P^2}{\partial T} - s$$

Here t<sub>eo</sub> is the effective time between electron-sleptron collisions and i, is the characteristic length scale for all (weak) inhomogenities. For the slectric and magnetic fields E and H we assume that

where  $E_1 = o_1/a_1E$  are particle accellerations, and

where  $\Omega_1 = e_1 \ln_2 B$  are gravinequencies. Noth E and E are assumed to be stationary and uniform on the scales, we consider, and we regiont the gravestice of an electromargaphic field by the evolution of the places [teaff, This cosmutifier may be omitted, of, Appendix 8 Since the distribution functions f, and for sectors of the peculiar velocities  $G_1$  and  $G_2$ , besides period vector E and time t, we must be evolved of the electron condition. [3].

 $\int dQ_1 r_1(r, Q_1, r) m_1 Q_1 + \int dQ_2 r_2(r, Q_2, r) m_2 Q_2 = 0$ 

Till now essumptions necessary for parametrizing the kinetic equalitans have been unde. When parametrizing the moment equalitans are assume valoatly moments vary on scales the same as the scales for connerponding powers of velocity, i.e.

$$\begin{aligned} |\underline{\overline{c}}_{1}| &\sim |\underline{c}_{1}| &\sim \overline{c}_{1} \\ |\underline{\overline{c}}_{2}| &\lesssim |\underline{c}_{2}| &\sim \alpha \overline{c}_{1} \\ |\underline{m}_{i} \underline{\overline{c}_{i} \underline{c}_{i}}| &\sim m_{i} \overline{\overline{c}}_{i}^{2} &\sim m_{i} \overline{\overline{c}}_{i}^{2} \\ |\underline{1}_{2}^{2} m_{i} \overline{\overline{c}_{i} \underline{c}_{i}}| &\sim m_{i} \overline{\overline{c}}_{i}^{2} \overline{c}_{i} \end{aligned}$$

Consequently we parametrize the apriori condition in the following way

$$\alpha \int d\underline{C}_{1} f_{1} \underline{m}_{1} \underline{C}_{1} + \int d\underline{C}_{2} f_{2} \underline{m}_{2} \underline{C}_{2} = 0 \qquad (6)$$

which also shows necessary in order to avoid a break down of the multiple time scale method when we consider evolution from an initial state far from equilibrium. Kinetic and macroscopic equations are parametrized as follows(t is a time variable on a scale with unit  $\tau_{20}$ )

$$\frac{\partial f_{1}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial f_{1}}{\partial \underline{r}} + \varepsilon \underline{c}_{1} \cdot \frac{\partial f_{1}}{\partial \underline{r}} + \varepsilon \underline{F}_{1} \cdot \frac{\partial f_{1}}{\partial \underline{c}_{1}} -$$

$$- \alpha \left( \frac{\partial \underline{c}_{0}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial \underline{c}_{0}}{\partial \underline{r}} \right) \cdot \frac{\partial f_{1}}{\partial \underline{c}_{1}} + \alpha \frac{\underline{e}_{1}}{\underline{m}_{1}} \underline{c}_{0} \times \underline{H} \cdot \frac{\partial f_{1}}{\partial \underline{c}_{1}} + \frac{\underline{e}_{1}}{\underline{m}_{1}} \underline{c}_{1} \times \underline{H} \cdot \frac{\partial f_{1}}{\partial \underline{c}_{1}} -$$

$$- \varepsilon \alpha \frac{\partial f_{1}}{\partial \underline{c}_{1}} \cdot \underline{c}_{1} : \frac{\partial \underline{c}_{0}}{\partial \underline{r}} =$$

$$(7)$$

$$= \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{c}_{1}} \cdot \int d\underline{c}_{1}' \underline{\sigma}^{-1} \cdot \left( \underline{c}_{1} - \underline{c}_{1}' \right) \cdot \left( \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{c}_{1}} - \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{c}_{1}'} \right) f_{1}(\underline{c}_{1}) f_{1}(\underline{c}_{1}') +$$

$$+ \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{c}_{1}} \cdot \int d\underline{c}_{2} \underline{\sigma}^{-12} \left( \underline{c}_{1} - \alpha \underline{c}_{2} \right) \cdot \left( \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{c}_{1}} - \frac{\alpha}{\underline{m}_{2}} \frac{\partial}{\partial \underline{c}_{2}} \right) f_{1}(\underline{c}_{1}) f_{2}(\underline{c}_{2}) .$$

Consequently ve parametrize the apriori condition in the following way

$$(33) = \frac{1}{2} \left( \frac{1}{2} \left( \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} \left( \frac{1}{2} \right) - \frac{1}{2} \left( \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} \right) +$$

reside elles abeve secencery in order to evolute threek down of the militiple time scale sector inven we consider evolution from an indifici dowe for from equilibrium. Mirretic and metroscopic equilibrium ane parametrized as rollowed to is a time variable on a scale ratio with antible app

$$\frac{\partial f_2}{\partial t} + \varepsilon \alpha \underline{c}_0 \cdot \frac{\partial f_2}{\partial \underline{r}} + \varepsilon \alpha \underline{c}_2 \cdot \frac{\partial f_2}{\partial \underline{r}} + \varepsilon \alpha \underline{F}_2 \cdot \frac{\partial f_2}{\partial \underline{C}_2} - \left(\frac{\partial \underline{c}_0}{\partial t} + \varepsilon \alpha \underline{c}_0 \cdot \frac{\partial \underline{c}_0}{\partial \underline{r}}\right) \cdot \frac{\partial f_2}{\partial \underline{C}_2} + \alpha^2 \frac{\underline{e}_2}{\underline{m}_2} \underline{c}_0 \times \underline{H} \cdot \frac{\partial f_2}{\partial \underline{C}_2} + \alpha^2 \frac{\underline{e}_2}{\underline{m}_2} \underline{c}_1 + \alpha^2 \frac{\underline{e}_2}{\underline{m}_2} \underline{c}_2 \times \underline{H} \cdot \frac{\partial f_2}{\partial \underline{C}_2} + \alpha^2 \frac{\underline{d}_2}{\underline{d}_2} \underline{c}_2 : \frac{\partial \underline{c}_0}{\partial \underline{r}} = (8)$$

$$= \frac{\alpha}{\underline{m}_2} \frac{\partial}{\partial \underline{C}_2} \cdot \int d\underline{c}_2' \underline{\Phi}^{22} (\underline{c}_2 - \underline{c}_2') \cdot \left(\frac{1}{\underline{m}_2} - \frac{\partial}{\underline{d}\underline{C}_2} - \frac{1}{\underline{m}_2} - \frac{\partial}{\underline{d}\underline{C}_2'}\right) f_2(\underline{c}_2) f_2(\underline{c}_2') + \alpha \frac{\alpha}{\underline{m}_2} \frac{\partial}{\underline{d}\underline{C}_2} \cdot \int d\underline{c}_1 \underline{\Phi}^{21} (\alpha \underline{c}_2 - \underline{c}_1) \cdot \left(\frac{\alpha}{\underline{m}_2} - \frac{\partial}{\underline{d}\underline{C}_2} - \frac{1}{\underline{m}_1} - \frac{\partial}{\underline{d}\underline{C}_1}\right) f_1(\underline{c}_1) f_2(\underline{c}_2) \cdot$$

Here  $\Phi^{ij}(\underline{w})$  are tensors taking into account the weak interaction between particles. For suitable cut-offs they take Landau's form [8]. Note that  $\alpha$  also appears inside some collision terms.

$$\frac{\partial \rho}{\partial t} + \varepsilon \alpha \frac{\partial}{\partial r} \cdot (\rho \underline{c}_{0}) = 0$$
(9)

$$\rho\left(\frac{\partial \underline{c}_{0}}{\partial t} + \varepsilon \alpha \underline{c}_{0}, \frac{\partial \underline{c}_{0}}{\partial \underline{r}}\right) = -\varepsilon \alpha \frac{\partial}{\partial \underline{r}}, \sum_{i=1}^{2} n_{i} \underline{m}_{i} \underline{\overline{c}_{i}} + \alpha^{2} \rho_{e} \underline{c}_{0} \times \underline{H} + \alpha n_{1} e_{1} \underline{\overline{c}_{1}} \times \underline{H} + \alpha^{2} n_{2} e_{2} \underline{\overline{c}_{2}} \times \underline{H} + \varepsilon \alpha \sum_{i=1}^{2} \rho_{i} \underline{F}_{i}$$

$$(10)$$

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$$\frac{3}{2} \operatorname{nk} \left( \frac{\partial T}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial T}{\partial \underline{r}} \right) = \varepsilon \frac{3}{2} \operatorname{kT} \frac{\partial}{\partial \underline{r}} \cdot \left( n_{1} \underline{\overline{c}}_{1} \right) + \\ + \varepsilon \alpha \frac{3}{2} \operatorname{kT} \frac{\partial}{\partial \underline{r}} \cdot \left( n_{2} \underline{\overline{c}}_{2} \right) + \varepsilon \rho_{1} \underline{F}_{1} \cdot \underline{\overline{c}}_{1} + \varepsilon \alpha \rho_{2} \underline{F}_{2} \cdot \underline{\overline{c}}_{2} + \\ + \alpha n_{1} \varepsilon_{1} \underline{\overline{c}}_{1} \cdot \left( \underline{c}_{0} \times \underline{H} \right) + \alpha^{2} n_{2} \varepsilon_{2} \underline{\overline{c}}_{2} \cdot \left( \underline{c}_{0} \times \underline{H} \right) - \varepsilon \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{1} - \\ - \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{2} - \varepsilon \alpha \sum_{i=1}^{2} n_{i} \underline{m}_{i} \underline{\overline{c}}_{1} \underline{\overline{c}}_{1} : \frac{\partial \underline{c}}{\partial \underline{r}}$$

$$(11)$$

We also take into account the following moment equations for each gas component:

$$\frac{\partial \rho_1}{\partial t} + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_1 \underline{c}_0) + \varepsilon \frac{\partial}{\partial \underline{r}} \cdot (\rho_1 \underline{\overline{c}}_1) = 0$$
(12)

$$\frac{\partial \rho_2}{\partial t} + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_2 \underline{c}_0) + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_2 \underline{\overline{c}}_2) = 0$$
(13)

$$\frac{\partial}{\partial t} \left( \rho_{1} \overline{\underline{c}}_{1} \right) + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial}{\partial \underline{r}} \left( \rho_{1} \overline{\underline{c}}_{1} \right) + \varepsilon \alpha \rho_{1} \overline{\underline{c}}_{1} \frac{\partial}{\partial \underline{r}} \cdot \underline{c}_{0} +$$

$$+ \varepsilon \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_{1} \overline{\underline{c}}_{1} \overline{\underline{c}}_{1} \right) - \varepsilon \rho_{1} \overline{\underline{F}}_{1} + \alpha \rho_{1} \left( \frac{\partial \underline{c}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial \underline{c}}{\partial \underline{r}} \right) -$$

$$- \alpha n_{1} \varepsilon_{1} \underline{c}_{0} \times \underline{H} - n_{1} \varepsilon_{1} \overline{\underline{c}}_{1} \times \underline{H} + \varepsilon \alpha \rho_{1} \overline{\underline{c}}_{1} \cdot \frac{\partial \underline{c}}{\partial \underline{r}} =$$

$$= - \int d\underline{c}_{1} d\underline{c}_{2} \underline{\phi}^{12} (\underline{c}_{1} - \alpha \underline{c}_{2}) \cdot \left( \frac{1}{m_{1}} \frac{\partial}{\partial \underline{c}_{1}} - \frac{\alpha}{m_{2}} \frac{\partial}{\partial \underline{c}_{2}} \right) f_{1} (\underline{c}_{1}) f_{2} (\underline{c}_{2})$$

$$(14)$$

(44)

$$\begin{split} \frac{\partial}{\partial t} \left( \rho_{2}\overline{\underline{C}}_{2} \right) &+ \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial}{\partial \underline{r}} \left( \rho_{2}\overline{\underline{C}}_{2} \right) + \varepsilon \alpha \rho_{2}\overline{\underline{C}}_{2}\frac{\partial}{\partial \underline{r}} \cdot \underline{c}_{0} + \\ &+ \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_{2}\overline{\underline{C}}_{2}\overline{\underline{C}}_{2} \right) - \varepsilon \alpha \rho_{2}\underline{F}_{2} + \rho_{2} \left( \frac{\partial \underline{c}_{0}}{\partial t} + \varepsilon \alpha \underline{c}_{0}, \frac{\partial \underline{c}_{0}}{\partial \underline{r}} \right) - \\ &- \alpha^{2}n_{2}e_{2}\underline{c}_{0} \times \underline{H} - \alpha^{2}n_{2}e_{2}\overline{\underline{c}}_{2} \times \underline{H} + \varepsilon \alpha \rho_{2}\overline{\underline{c}}_{2} \cdot \frac{\partial \underline{c}_{0}}{\partial \underline{r}} = (15) \\ &= -\alpha \int d\underline{c}_{1}d\underline{c}_{2} \ \underline{\phi}^{12}(\underline{c}_{1} - \alpha\underline{c}_{2}) \cdot \left( \frac{\alpha}{\underline{m}_{2}} \frac{\partial}{\partial \underline{c}_{2}} - \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{c}_{1}} \right) f_{1}(\underline{c}_{1})f_{2}(\underline{c}_{2}) \ . \end{split}$$

$$\begin{split} \\ \frac{2}{2}n_{1}k \left( \frac{\partial T_{1}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial T_{1}}{\partial \underline{r}} \right) - \frac{2}{2}kT_{1} \ \frac{\partial}{\partial \underline{r}} \cdot (n_{1}\overline{\underline{c}}_{1}) + \varepsilon \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{1} - \\ &- \varepsilon \rho_{1}\overline{\underline{F}}_{1} \cdot \underline{c}_{1} + \alpha \rho_{1}\overline{\underline{c}}_{1} \cdot \left( \frac{\partial \underline{c}_{0}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial \underline{c}_{0}}{\partial \underline{r}} \right) - \\ &- \alpha n_{1}e_{1}\overline{\underline{c}}_{1} \cdot \underline{c}_{0} \times \underline{H} + \varepsilon \alpha \rho_{1}\overline{\underline{c}}_{1}\overline{\underline{c}}_{1} = (16) \\ &= -\alpha \int d\underline{c}_{1}d\underline{c}_{2}\underline{c}_{2} \cdot \frac{\phi^{12}(\underline{c}_{1} - \alpha\underline{c}_{2}) \cdot \left( \frac{1}{\underline{m}_{1}} \frac{\partial}{\underline{c}}_{1} - \frac{\alpha}{\underline{m}_{2}} \frac{\partial}{\underline{c}}_{2} \right) f_{1}(\underline{c}_{1})f_{2}(\underline{c}_{2}) \end{split}$$

$$\frac{3}{2} n_{2} k \left( \frac{\partial T_{2}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial T_{2}}{\partial \underline{r}} \right) - \varepsilon \alpha \frac{3}{2} k T_{2} \frac{\partial}{\partial \underline{r}} \cdot (n_{2} \underline{\underline{c}}_{2}) + \\ + \varepsilon \alpha \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{2} - \varepsilon \alpha \rho_{2} \underline{F}_{2} \cdot \underline{\underline{c}}_{2} + \rho_{2} \underline{\underline{c}}_{2} \cdot \left( \frac{\partial \underline{c}_{0}}{\partial t} + \varepsilon \alpha \underline{c}_{0} \cdot \frac{\partial \underline{c}_{0}}{\partial \underline{r}} \right) - \\ - \alpha^{2} n_{2} e_{2} \underline{\underline{c}}_{2} \cdot (\underline{c}_{0} \times \underline{H}) + \varepsilon \alpha \rho_{2} \underline{\underline{c}}_{2} \underline{\underline{c}}_{2} : \frac{\partial \underline{c}_{0}}{\partial \underline{r}} =$$
(17)
$$= - \alpha \int d\underline{\underline{c}}_{1} d\underline{\underline{c}}_{2} \underline{\underline{c}}_{2} \cdot \underline{\phi}^{12} (\underline{\underline{c}}_{1} - \alpha \underline{\underline{c}}_{2}) \cdot \left( \frac{\alpha}{\underline{m}_{2}} \frac{\partial}{\partial \underline{\underline{c}}_{2}} - \frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{\underline{c}}_{1}} \right) f_{1} (\underline{\underline{c}}_{1}) f_{2} (\underline{\underline{c}}_{2})$$

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The models studied in [5] and [6] were

a)  $\varepsilon \sim \alpha$ ,  $\varepsilon^2 \sim \varepsilon \alpha \sim \alpha^2$  etc. b)  $\varepsilon = 0$ ,  $\alpha \neq 0$  but small c)  $\alpha = 0$ ,  $\varepsilon \neq 0$  but small.

We shall now study a model which we shall characterize by

d) 
$$(0 <) \varepsilon < \alpha^n$$
 all  $n \ge 0$  as  $\alpha \to 0$ .

This is an extreme model having the property that electrons and ions reach the same temperature before the typical transport processes set in. We first expand the distribution functions  $f_1$  and  $f_2$  and macroscopic functions in series in the parameter  $\varepsilon$ , for instance

$$f_{i} = f_{i}^{(0)} + \epsilon f_{i}^{(1)} + \cdots$$

$$\rho = \rho^{(0)} + \varepsilon \rho^{(1)} + \dots$$

 $\rho^{(k)} = \sum_{i=1}^{2} \int f_{i}^{(k)}(\underline{C}) m_{i} d\underline{C}_{i}$ 

where

There is no such connection between expansions for  $f_i$  and the mass transport velocity  $\underline{c}_0$ . However,

The models stadied in [5] and [6] have

We shall now study a model which we shall churdeterise by

1 (0<) E < 0<sup>2</sup> all n 2 0 as 6 4 0 .

This is an artistic hodel having the property that electrons and ions reach the seme targerature bafore the tipical transport processes set in. We first expand the distribution functions of and of anti-antibution functions of and of anti-anti-antiinstance in the parameter of for instance

 $\dots + (n)_{n-2} + (n)_{n-2} + (n)_{n-2}$ 

There is no audo connection haddeen computation four fit and the mean the mass bremmores velocities of ... However,

- (3) -

we seek <u>c</u> as

 $\underline{c}_{0} = \underline{c}_{0}^{(0)} + \varepsilon \underline{c}_{0}^{(1)} + \dots$ 

Here we note that  $|\underline{c}_0|/\overline{c}_1 \sim |\underline{c}_0^{(0)}|/\overline{c}_1 \sim \alpha$ . However  $|\underline{c}_0^{(1)}|/\overline{c}_1 \sim \varepsilon$  so that  $|\underline{c}_0^{(1)}|/|\underline{c}_0^{(0)}| \sim \varepsilon/\alpha$ .

This must be taken into account in the equations to first and higher orders in the parameter  $\epsilon$ .

We introduce time variables 
$$t_i$$
,  $i = 2,3, ...,$  on time  
scales  $\tau_i = \frac{\tau_2}{\epsilon^{i-2}}$ . Therefore the time derivative is  
expanded as follows

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_2} + \varepsilon \frac{\partial}{\partial t_3} + \varepsilon^2 \frac{\partial}{\partial t_4} + \cdots$$

We substitute these expansions into Eqs(6) - (17) and separate in zeroth, first, second order etc in  $\varepsilon$ . In each of these sets of equations we have the parameter  $\alpha$ . We can expand functions in each set in series of the parameter  $\alpha$  and in each set separate in zeroth, first, second order etc in the parameter  $\alpha$ . Thus, in all, we make a double expansion of functions in series of parameters  $\varepsilon$  and  $\alpha$ , for instance

$$f_{i} = f_{i}^{(00)} + \alpha f_{i}^{(01)} + \alpha^{2} f_{i}^{(02)} + \dots + \epsilon \left[ f_{i}^{(10)} + \alpha f_{i}^{(11)} + \dots \right] + \epsilon^{2} \left[ f_{i}^{(20)} + \dots \right] + \epsilon^{2} \left[ f_{i$$

expanded as fall, love

Me estatitute diere expansions into fee (0) = (4) and generate is resto, first, second order of is a in sech of diese sets of equations is define the parameter d we can expand functions in each set is defines of the parameter of and in each set separate is satuth, first, second order set is the parameter of Thus, in All, we make a double organizing of functions in second of parameters a side d's Similarly the time derivative in all is expanded as follows

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_{20}} + \alpha \frac{\partial}{\partial t_{21}} + \alpha^2 \frac{\partial}{\partial t_{22}} + \dots + \epsilon \left[ \frac{\partial}{\partial t_{30}} + \alpha \frac{\partial}{\partial t_{31}} + \dots \right] + \epsilon^2 \left[ \frac{\partial}{\partial t_{40}} + \dots \right] + \dots$$

consistent with introduction of time variables  $t_{ij}$ i = 2,3, ... and j = 0,1,2, ... on time scales  $\tau_{ij} = \frac{\tau_i}{\alpha^j}$ .

The  $\phi^{ij}$  functions appearing in the collision integrals of Eqs.(7) and (8) are expanded in terms of  $\alpha$  by use of Taylors formula, assumed distributional convergent. Each set of equations to each order in  $\varepsilon$  and  $\alpha$  should be solved eliminating all secular terms that arise.

\*

We end this section writing down kinetic and macroscopic equations including the a priori condition to zeroth and first order in the parameter  $\varepsilon$ . Note the differences between this set of equations and the corresponding set of equations of Chapman and Cowling (cf.Appendix 1). All () in superscripts are cmitted from now on. Similarly the third destroition in all de broanded as foilons

constants with introduction of the variables  $i_{ij}$   $\frac{k_i}{2}$ 1 = 2.3, ... and j = 0.112, ... on the solider  $i_{ij} = \frac{k_i}{2}$ 

The of functions supervise in the dufficien integrals of Eqs.(7) and (8) are expended in terms of a by use of Taylors formula, hadustd distributions! obresident and fat of equations to each State in a and a should be solved elthinesting all secular terms that arise.

We and this section writting form Einstle and macroscopic squations including the s priori condition to denote and first order 15 the parameter s. Note the differences between this set of equilibric and the corresponding pot of equations of Chapman and Corresponding for Zeroth order equations in the parameter  $\epsilon$ 

$$a \int d\underline{c}_{1} f_{1}^{0} \underline{n}_{1} \underline{c}_{1} + \int d\underline{c}_{2} f_{2}^{0} \underline{n}_{2} \underline{c}_{2} = 0$$

$$(18)$$

$$\frac{\partial f_{1}^{0}}{\partial t_{2}} - a \frac{\partial \underline{c}_{0}^{0}}{\partial t_{2}} \cdot \frac{\partial f_{1}^{0}}{\partial \underline{c}_{1}} + a \frac{\underline{e}_{1}}{\underline{m}_{1}} \underline{c}_{0} \times \underline{H} \cdot \frac{\partial f_{1}^{0}}{\partial \underline{c}_{1}} + \frac{\underline{e}_{1}}{\underline{m}_{1}} \underline{c}_{1} \times \underline{H} \cdot \frac{\partial f_{1}^{0}}{\partial \underline{c}_{1}} =$$

$$(19)$$

$$= FP_{11} \Big[ f_{1}^{0} (\underline{c}_{1}) f_{1}^{0} (\underline{c}_{1}') \Big] + FP_{12} \Big[ f_{1}^{0} (\underline{c}_{1}) f_{2}^{0} (\underline{c}_{2}) \Big]$$

$$\frac{\partial f_{2}^{0}}{\partial t_{2}} - \frac{\partial \underline{c}_{0}^{0}}{\partial t_{2}} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} + a^{2} \frac{\underline{e}_{2}}{\underline{m}_{2}} \underline{c}_{0}^{0} \times \underline{H} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} + a^{2} \frac{\underline{e}_{2}}{\underline{m}_{2}} \underline{c}_{2} \times \underline{H} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} =$$

$$(20)$$

$$= a FP_{22} \Big[ f_{2}^{0} (\underline{c}_{2}) f_{2}^{0} (\underline{c}_{2}') \Big] + a FP_{21} \Big[ f_{2}^{0} (\underline{c}_{2}) f_{1}^{0} (\underline{c}_{1}) \Big]$$

$$\frac{\partial p^{0}}{\partial t_{2}} = 0$$

$$(21)$$

$$p^{0} \frac{\partial \underline{c}_{0}^{0}}{\partial t_{2}} = a^{2} \rho_{0}^{0} \underline{c}_{0}^{0} \times \underline{H} + an_{1}^{0} \underline{e}_{1} \underline{\overline{c}}_{1}^{0} \times \underline{H} + a^{2} n_{2}^{0} \underline{e}_{2} \underline{\overline{c}}_{2}^{0} \times \underline{H}$$

$$(22)$$

$$\frac{\partial p^{0}}{\partial t_{2}} = an_{1}^{0} \underline{e}_{1} \underline{\overline{c}}_{1}^{0} \cdot (\underline{c}_{0}^{0} \times \underline{H}) + a^{2} n_{2}^{0} \underline{e}_{2} \underline{\overline{c}}_{2}^{0} \cdot (\underline{c}_{0}^{0} \times \underline{H})$$

$$(23)$$

$$\frac{\partial p_{1}^{0}}{\partial t_{2}} = 0$$

$$(24)$$

$$\frac{\partial}{\partial t_{2}} \left( \rho_{1}^{\circ} \underline{\overline{C}}_{1}^{\circ} \right) + \alpha \rho_{1}^{\circ} \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}} - \alpha n_{1}^{\circ} e_{1} \underline{\underline{c}}_{0}^{\circ} \times \underline{H} - n_{1}^{\circ} e_{1} \underline{\overline{C}}_{1}^{\circ} \times \underline{H} =$$

$$= -\int d\underline{\underline{C}}_{1} d\underline{\underline{C}}_{2} \underline{\Phi}^{12} (\underline{\underline{C}}_{1} - \alpha \underline{\underline{C}}_{2}) \cdot (\frac{1}{\underline{m}_{1}} \frac{\partial}{\partial \underline{\underline{C}}_{1}} - \frac{\alpha}{\underline{m}_{2}} \frac{\partial}{\partial \underline{\underline{C}}_{2}}) f_{1}^{\circ} (\underline{\underline{C}}_{1}) f_{2}^{\circ} (\underline{\underline{C}}_{2})$$

$$(26)$$

$$\frac{\partial}{\partial t_2} (\rho_2^{\circ} \overline{\underline{C}}_2^{\circ}) + \rho_2^{\circ} \frac{\partial \underline{c}_2^{\circ}}{\partial t_2} - \alpha^2 n_2 e_2 \underline{c}_2^{\circ} \times \underline{H} - \alpha^2 n_2 e_2 \overline{\underline{C}}_2^{\circ} \times \underline{H} =$$

$$= -\alpha \int d\underline{c}_1 d\underline{c}_2 \Phi^{12} (\underline{c}_1 - \alpha \underline{c}_2) \cdot (\frac{\alpha}{\underline{m}_2} \frac{\partial}{\partial \underline{c}_2} - \frac{1}{\underline{m}_1} \frac{\partial}{\partial \underline{c}_1}) f_1^{\circ} (\underline{c}_1) f_2^{\circ} (\underline{c}_2)$$

$$(27)$$

$$\frac{3}{2}n_{1}^{\circ}k \frac{\partial T_{1}^{\circ}}{\partial t_{2}} + \alpha \rho_{1}^{\circ}\underline{C}_{1}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}} - \alpha n_{1}^{\circ}e_{1}\underline{\overline{C}}_{1}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) =$$

$$= -\alpha \int d\underline{c}_{1} d\underline{c}_{2}\underline{c}_{2} \cdot \underline{\phi}^{12} (\underline{c}_{1} - \alpha \underline{c}_{2}) \cdot (\frac{1}{m_{1}} \frac{\partial}{\partial \underline{c}_{1}} - \frac{\alpha}{m_{2}} \frac{\partial}{\partial \underline{c}_{2}}) f_{1}^{\circ} (\underline{c}_{1}) f_{2}^{\circ} (\underline{c}_{2})$$

$$(28)$$

$$\frac{3}{2}n_{2}^{\circ}k \frac{\partial T_{2}^{\circ}}{\partial t_{2}} + p_{2}^{\circ} \underline{\overline{C}}_{2}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}} - \alpha^{2} n_{2}^{\circ}e_{2}\underline{\overline{C}}_{2}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) =$$

$$= -\alpha \int d\underline{c}_{1} d\underline{c}_{2}\underline{c}_{2} \cdot \underline{\phi}^{12} (\underline{c}_{1} - \alpha \underline{c}_{2}) \cdot (\frac{\alpha}{m_{2}} \frac{\partial}{\partial \underline{c}_{2}} - \frac{1}{m_{1}} \frac{\partial}{\partial \underline{c}_{1}}) f_{1}^{\circ} (\underline{c}_{1}) f_{2}^{\circ} (\underline{c}_{2})$$

$$(29)$$

First order equations in the parameter  $\epsilon$ 

$$\alpha \int d\underline{C}_{1} \underline{f}_{1}^{1} \underline{m}_{1} \underline{C}_{1} + \int d\underline{C}_{2} \underline{f}_{2}^{1} \underline{m}_{2} \underline{C}_{2} = 0$$
(30)

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$$\frac{\partial f_{1}^{1}}{\partial t_{2}} + \frac{\partial f_{1}^{0}}{\partial t_{3}} + \alpha \underline{c}_{0}^{\circ} \cdot \frac{\partial f_{1}^{\circ}}{\partial \underline{r}} + \underline{c}_{1} \cdot \frac{\partial f_{1}^{\circ}}{\partial \underline{r}} + \underline{F}_{1} \cdot \frac{\partial f_{1}^{\circ}}{\partial \underline{c}_{1}} - \alpha \left(\frac{1}{\alpha} \frac{\partial \underline{c}_{0}^{1}}{\partial \underline{t}_{2}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{t}_{2}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{t}_{2}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{t}_{3}} + \alpha \underline{c}_{0}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}} \right) \cdot \frac{\partial f_{1}^{\circ}}{\partial \underline{c}_{1}} - \alpha \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{t}_{2}} \cdot \frac{\partial f_{1}^{1}}{\partial \underline{c}_{1}} + \alpha \underline{f}_{1}^{\circ} \underline{c}_{0}^{\circ} \times \underline{H} \cdot \frac{\partial f_{1}^{1}}{\partial \underline{c}_{1}} + \frac{\partial f_{1}^{1}}{\partial \underline{c}_{1}} + \frac{\partial f_{1}^{1}}{m_{1}} \underline{c}_{1} \times \underline{H} \cdot \frac{\partial f_{1}^{1}}{\partial \underline{c}_{1}} - \alpha \frac{\partial f_{1}^{\circ}}{\partial \underline{c}_{1}} : \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}} = (31)$$

$$= FP_{11} \left[ f_{1}^{\circ} (\underline{c}_{1}) f_{1}^{1} (\underline{c}_{1}') + f_{1}^{1} (\underline{c}_{1}) f_{1}^{\circ} (\underline{c}_{1}') \right] + FP_{12} \left[ f_{1}^{\circ} (\underline{c}_{1}) f_{2}^{\circ} (\underline{c}_{2}) + f_{1}^{1} (\underline{c}_{1}) f_{2}^{\circ} (\underline{c}_{2}) \right]$$

$$\frac{\partial f_{2}^{1}}{\partial t_{2}} + \frac{\partial f_{2}^{0}}{\partial t_{3}} + \alpha \underline{c}_{0}^{0} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{r}} + \alpha \underline{c}_{2} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{r}} + \alpha \underline{F}_{2} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} - \left(\frac{1}{\alpha} \frac{\partial \underline{c}_{0}}{\partial t_{2}} + \frac{\partial \underline{c}_{0}}{\partial t_{2}} + \frac{\partial \underline{c}_{0}}{\partial t_{2}} + \alpha \underline{c}_{0}^{0} \cdot \frac{\partial \underline{c}_{0}^{0}}{\partial \underline{r}}\right) \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} - \frac{\partial \underline{c}_{0}^{0}}{\partial t_{2}} \cdot \frac{\partial f_{2}^{1}}{\partial \underline{c}_{2}} + \alpha^{2} \frac{\underline{e}_{2}}{\underline{m}_{2}} \underline{c}_{0}^{0} \times \underline{H} \cdot \frac{\partial f_{2}^{1}}{\partial \underline{c}_{2}} + \alpha + \alpha \frac{\underline{e}_{2}^{0}}{\underline{c}_{2}} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} + \alpha^{2} \frac{\underline{e}_{2}^{0}}{\underline{c}_{2}} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} + \alpha^{2} \frac{\underline{e}_{2}^{0}}{\underline{c}_{2}} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} + \alpha^{2} \frac{\underline{e}_{2}^{0}}{\underline{c}_{2}} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} - \alpha \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} \cdot \underline{c}_{2} \times \underline{H} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} - \alpha \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} \cdot \underline{c}_{2} \times \underline{H} \cdot \frac{\partial f_{2}^{0}}{\partial \underline{c}_{2}} + \alpha + \alpha + \alpha + \alpha + \mu + \alpha + \mu + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} \cdot \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha^{2} + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha^{2} + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha + \alpha + \mu + \alpha + \mu + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha^{2} + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha^{2} + \frac{\partial f_{2}^{0}}{\underline{c}_{2}} + \alpha + \frac{\partial f_{2}^{0}}{\underline{c}_{2}$$

$$\frac{\partial \rho^{1}}{\partial t_{2}} + \frac{\partial \rho^{\circ}}{\partial t_{3}} + \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho^{\circ} \underline{c}_{0}^{\circ}) = 0$$
(33)

((2))?\*\*(2))?\* + (2))?\*(2)?\* (3)?\* (3)?\*

-- 891-

$$\rho^{\circ}\left(\frac{1}{\alpha}\frac{\partial \underline{c}_{0}^{1}}{\partial \underline{t}_{2}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{t}_{3}} + \alpha \underline{c}_{0}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}}\right) + \rho^{1}\frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{t}_{2}} = -\alpha \frac{\partial}{\partial \underline{r}} \cdot \sum_{i=1}^{2} n_{i}^{\circ} \underline{m}_{i} \underline{\overline{c}_{i}} \underline{\overline{c}_{0}}^{\circ} + \alpha \rho_{i}^{\circ} \underline{c}_{0}^{\circ} \times \underline{H} + \alpha n_{1}^{\circ} \underline{c}_{1} \underline{\overline{c}_{1}}^{1} \times \underline{H} + \alpha n_{1}^{1} \underline{c}_{1} \underline{\overline{c}_{1}}^{\circ} \times \underline{H} + \alpha \rho_{1}^{\circ} \underline{c}_{1} \underline{\overline{c}_{1}}^{\circ} \times \underline{H} + \alpha n_{1}^{\circ} \underline{c}_{1} \underline{\overline{c}_{2}}^{\circ} \times \underline{C} + \alpha n_{1}^{\circ} \underline{c}_{1} \underline{c}_{1} \underline{\overline{c}_{1}}^{\circ} \times \underline{H} + \alpha n_{1}^{\circ} \underline{c}_{1} \underline{c}_{1} \underline{c} + \alpha n_{1}^{\circ} \underline{c}_{1} \underline{c} + \alpha n_{1}^{\circ} \underline{c} + \alpha n$$

$$\begin{split} &\frac{3}{2}n^{\circ}k\left(\frac{\partial \underline{r}^{1}}{\partial \underline{t}_{2}} + \frac{\partial \underline{r}^{\circ}}{\partial \underline{t}_{3}} + \alpha \underline{c}^{\circ}_{\circ} \cdot \frac{\partial \underline{r}^{\circ}}{\partial \underline{r}}\right) + \frac{3}{2}n^{1}k \frac{\partial \underline{r}^{\circ}}{\partial \underline{t}_{2}} = \frac{3}{2}kT^{\circ} \frac{\partial}{\partial \underline{r}} \cdot (n_{1}^{\circ}\underline{C}_{1}^{\circ}) + \\ &+ \alpha \frac{3}{2}kT^{\circ} \frac{\partial}{\partial \underline{r}} \cdot (n_{2}^{\circ}\underline{C}_{2}^{\circ}) + \rho_{1}^{\circ}\underline{F}_{1} \cdot \underline{C}_{1}^{\circ} + \alpha \rho_{2}^{\circ}\underline{F}_{2} \cdot \underline{C}_{2}^{\circ} + n_{1}^{\circ}e_{1}\underline{C}_{1}^{\circ} \cdot (\underline{c}_{0}^{1} \times \underline{H}) + \\ &+ \alpha n_{1}^{\circ}e_{1}\underline{C}_{1}^{-1} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) + \alpha n_{1}^{1}e_{1}\underline{C}_{1}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) + \alpha n_{2}^{\circ}e_{2}\underline{C}_{2}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) + \alpha n_{2}^{\circ}e_{2}\underline{C}_{2}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) + \\ &+ \alpha^{2} n_{2}^{\circ}e_{2}\underline{C}_{2}^{-1} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) + \alpha^{2} n_{2}^{1}\underline{e}_{2}\underline{C}_{2}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) - \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{1}^{\circ} - \alpha \frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{2}^{\circ} - \\ &- \alpha \sum_{i=1}^{2} n_{i}^{\circ}m_{i}\underline{C}_{i}\underline{C}_{i}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}} \end{aligned}$$

$$\frac{\partial \rho_1^1}{\partial t_2} + \frac{\partial \rho_1^0}{\partial t_3} + \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^0 \underline{c}_0^0) + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^0 \underline{\overline{c}}_1^0) = 0$$
(36)

$$\frac{\partial \rho_2^1}{\partial t_2} + \frac{\partial \rho_2^0}{\partial t_3} + \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_2^0 \underline{c}_0^0) + \alpha \frac{\partial}{\partial \underline{r}} \cdot (\rho_2^0 \underline{c}_2^0) = 0$$
(37)

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$$\frac{\partial}{\partial t_{2}} (\rho_{1} \overline{\underline{c}}_{1})^{1} + \frac{\partial}{\partial t_{3}} (\rho_{1} \overline{\underline{c}}_{1})^{\circ} + \alpha \underline{c}_{0}^{\circ} \cdot \frac{\partial}{\partial \underline{r}} (\rho_{1}^{\circ} \overline{\underline{c}}_{1}^{\circ}) + \alpha \rho_{1}^{\circ} \underline{\underline{c}}_{1}^{\circ} \frac{\partial}{\partial \underline{r}} \cdot \underline{c}_{0}^{\circ} + \\ + \frac{\partial}{\partial \underline{r}} \cdot (\rho_{1}^{\circ} \overline{\underline{c}}_{1} \overline{\underline{c}}_{1}^{\circ}) - \rho_{1}^{\circ} \underline{F}_{1} + \alpha \rho_{1}^{\circ} \left(\frac{1}{\alpha} \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{3}} + \alpha \underline{c}_{0}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}}\right) + \\ + \alpha \rho_{1}^{1} \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}} - n_{1}^{\circ} \underline{e}_{1} \underline{c}_{0}^{1} \times \underline{H} - \alpha n_{1}^{1} \underline{e}_{1} \underline{c}_{0}^{\circ} \times \underline{H} - n_{1}^{\circ} \underline{e}_{1} \overline{\underline{c}}_{1}^{1} \times \underline{H} - \\ - n_{1}^{1} \underline{e}_{1} \overline{\underline{c}}_{1}^{\circ} \times \underline{H} + \alpha \rho_{1}^{\circ} \overline{\underline{c}}_{0}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}} =$$
(38)
$$= - \int d\underline{c}_{1} d\underline{c}_{2} \underline{\phi}^{12} (\underline{c}_{1} - \alpha \underline{c}_{2}) \cdot \left(\frac{1}{m_{1}} \frac{\partial}{\partial \underline{c}_{1}} - \frac{\alpha}{m_{2}} \frac{\partial}{\partial \underline{c}_{2}}\right) \\ \left(f_{1}^{\circ} (\underline{c}_{1}) f_{2}^{1} (\underline{c}_{2}) + f_{1}^{1} (\underline{c}_{1}) f_{2}^{\circ} (\underline{c}_{2})\right) \right)$$

$$\frac{\partial}{\partial t_2} (\rho_2 \overline{\underline{c}}_2)^1 + \frac{\partial}{\partial t_3} (\rho_2 \overline{\underline{c}}_2)^\circ + \alpha \underline{c}_0^\circ \cdot \frac{\partial}{\partial \underline{r}} (\rho_2 \overline{\underline{c}}_2^\circ) + \alpha \rho_2 \overline{\underline{c}}_2^\circ \frac{\partial}{\partial \underline{r}} \cdot \underline{c}_0^\circ + \alpha \underline{c}_0^\circ \cdot \frac{\partial \underline{c}}_2^\circ) + \alpha \rho_2 \overline{\underline{c}}_2^\circ \underline{c}_2^\circ + \alpha \underline{c}_0^\circ \cdot \frac{\partial \underline{c}}_2^\circ) + \alpha \rho_2 \overline{\underline{c}}_2^\circ \underline{c}_2^\circ \underline{c}_2^\circ + \alpha \underline{c}_0^\circ \cdot \frac{\partial \underline{c}}_2^\circ) + \alpha \rho_2 \overline{\underline{c}}_2^\circ \underline{c}_2^\circ \underline{c}_$$

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$$\begin{split} \frac{3}{2}n_{1}^{\circ}k\left(\frac{\partial T_{1}^{1}}{\partial t_{2}}+\frac{\partial T_{1}^{\circ}}{\partial t_{3}}+\alpha \underline{c}_{0}^{\circ}\cdot \frac{\partial T_{1}^{\circ}}{\partial \underline{r}}\right)+\frac{3}{2}n_{1}^{1}k \frac{\partial T_{1}^{\circ}}{\partial t_{2}}-\frac{3}{2}kT_{1}^{\circ}\frac{\partial}{\partial \underline{r}}\cdot(n_{1}^{\circ}\underline{c}_{1}^{\circ})+\\ +\frac{\partial}{\partial \underline{r}}\cdot \underline{q}_{1}^{\circ}-\rho_{1}^{\circ}\underline{F}_{1}\cdot \underline{c}_{1}^{\circ}+\alpha(\rho_{1}\underline{c}_{1})^{\circ}\cdot\left(\frac{1}{\alpha}\frac{\partial \underline{c}_{0}^{1}}{\partial t_{2}}+\frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{3}}+\alpha \underline{c}_{0}^{\circ}\cdot\frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}}\right)+\\ +\alpha(\rho_{1}\underline{c}_{1})^{1}\cdot\frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}}-n_{1}^{\circ}e_{1}\underline{\overline{c}}_{1}^{\circ}\cdot\left(\underline{c}_{0}^{1}\times\underline{H}\right)-\alpha n_{1}^{\circ}e_{1}\underline{\overline{c}}_{1}^{1}\cdot\left(\underline{c}_{0}^{\circ}\times\underline{H}\right)-\\ -\alpha n_{1}^{1}e_{1}\underline{\overline{c}}_{1}^{\circ}\cdot\left(\underline{c}_{0}^{\circ}\times\underline{H}\right)+\alpha \rho_{1}^{\circ}\underline{\overline{c}}_{1}\underline{\overline{c}}_{1}^{\circ}\cdot\frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}}= (40)\\ &=-\alpha \int d\underline{c}_{1}d\underline{c}_{2}\underline{c}_{2}\underline{c}_{2}\cdot\underline{\phi}^{12}\left(\underline{c}_{1}-\alpha\underline{c}_{2}\right)\cdot\left(\frac{1}{m_{1}}\frac{\partial}{\partial\underline{c}_{1}}-\frac{\alpha}{m_{2}}\frac{\partial}{\partial\underline{c}_{2}}\right)\\ &\left(f_{1}^{\circ}\left(\underline{c}_{1}\right)f_{2}^{1}\left(\underline{c}_{2}\right)+f_{1}^{1}\left(\underline{c}_{1}\right)f_{2}^{\circ}\left(\underline{c}_{2}\right)\right)\right) \end{split}$$

$$\begin{split} \frac{3}{2}n_{2}^{\circ}k\left(\frac{\partial T_{2}^{1}}{\partial t_{2}}+\frac{\partial T_{2}^{\circ}}{\partial t_{3}}+\alpha \underline{c}_{0}^{\circ}\cdot \frac{\partial T_{2}^{\circ}}{\partial \underline{r}}\right)+\frac{3}{2}n_{2}^{1}k \frac{\partial T_{2}^{\circ}}{\partial t_{2}}-\alpha \frac{3}{2}kT_{2}^{\circ}\frac{\partial}{\partial \underline{r}}\cdot (n_{2}^{\circ}\underline{c}_{2}^{\circ})+\\ &+\alpha \frac{\partial}{\partial \underline{r}}\cdot \underline{q}_{2}^{\circ}-\alpha \rho_{2}^{\circ}\underline{F}_{2}\cdot \underline{c}_{2}^{\circ}+(\rho_{2}\underline{c}_{2})^{\circ}\cdot \left(\frac{1}{\alpha}\frac{\partial \underline{c}_{0}^{1}}{\partial t_{2}}+\frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{3}}+\alpha \underline{c}_{0}^{\circ}\cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}}\right)+\\ &+(\rho_{2}\underline{c}_{2})^{1}\cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{2}}-\alpha n_{2}^{\circ}e_{2}\underline{c}_{2}^{\circ}\cdot (\underline{c}_{0}^{1}\times\underline{H})-\alpha^{2}n_{2}^{\circ}e_{2}\underline{c}_{2}^{\circ}\cdot (\underline{c}_{0}^{\circ}\times\underline{H})-\\ &-\alpha^{2}n_{2}^{1}e_{2}\underline{c}_{2}^{\circ}\cdot (\underline{c}_{0}^{\circ}\times\underline{H})+\alpha \rho_{2}^{\circ}\underline{c}_{2}\underline{c}_{2}^{\circ}\cdot (\frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}})= (41)\\ &=-\alpha\int d\underline{c}_{1}d\underline{c}_{2}\underline{c}_{2}\cdot \underline{\phi}^{12}(\underline{c}_{1}-\alpha \underline{c}_{2})\cdot \left(\frac{\alpha}{\underline{m}_{2}}\cdot\frac{\partial}{\partial \underline{c}_{2}}-\frac{1}{\underline{m}_{1}}\cdot\frac{\partial}{\partial \underline{c}_{1}}\right)\\ &\left(f_{2}^{\circ}(\underline{c}_{2})f_{1}^{1}(\underline{c}_{1})+f_{2}^{1}(\underline{c}_{2})f_{1}^{\circ}(\underline{c}_{1})\right) \end{split}$$

 $((2))^{2}(2)^{2} + (2)^{2} + (2)^{2}(2)^{2})$ 

## 2. Solutions in the limit $t_2 = \infty$ .

In this section we study the equations to zeroth and first order in  $\epsilon$  in the limit  $t_2 = \infty$ , [9]. We do not need then to expand functions and time derivatives in the parameter  $\alpha$ .

The solutions of the zeroth order equations can be discussed by studying the H - function, [1]

$$H = \int f_1^{\circ}(\underline{C}_1) ln f_1^{\circ}(\underline{C}_1) d\underline{C}_1 + \int f_2^{\circ}(\underline{C}_2) ln f_2^{\circ}(\underline{C}_2) d\underline{C}_2$$

H is bounded below and

$$\frac{\partial t^3}{\partial H} \leq 0$$

the sign of equality holding if and only if ([10]for a one component gas)

$$\mathbf{f}_{1}^{O}(\underline{C}_{1}) = \exp\left[-\frac{1}{2}A_{1}C_{1}^{2} + \underline{B}_{1}\cdot\underline{C}_{1} + D_{1}\right]$$
$$\mathbf{f}_{2}^{O}(\underline{C}_{2}) = \exp\left[-\frac{1}{2}A_{2}C_{2}^{2} + \underline{B}_{2}\cdot\underline{C}_{2} + D_{2}\right]$$

and

$$\frac{1}{m_1} \frac{\partial}{\partial \underline{C}_1} \left( -\frac{1}{2} \underline{A}_1 \underline{C}_1^2 + \underline{B}_1 \cdot \underline{C}_1 + \underline{D}_1 \right) - \frac{1}{m_2} \frac{\partial}{\partial \underline{C}_2} \left( -\frac{1}{2} \underline{A}_2 \underline{C}_2^2 + \underline{B}_2 \cdot \underline{C}_2 + \underline{D}_2 \right) = \mu \cdot \left( \underline{C}_1 - \underline{C}_2 \right)$$

## 2. Solutions in the list to an

In this section we study the equations to receptionand direct order in a in the limit to - [2]. We do reconced then to expand functions and the derivatives in the parameter a

The colutions of the servic order coustions can be discussed by studying the H - Cinepicon []]

H = J. 29(2, ) ant? (21) 42, + / 128 (2, ) sais (25) 42.

H is bounded below and

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the sign of equality holding it and only if ([10] ger a.

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Here  $\mu$  (a scalar function)  $A_i$ , <u>B</u> and D<sub>i</sub>, i = 1,2, are independent of <u>C</u> and <u>C</u>.

The last relation is fulfilled if and only if

$$\frac{A_{1}}{m_{1}} = \frac{A_{2}}{m_{2}}$$
(42)  
$$\frac{B_{1}}{m_{1}} = \frac{B_{2}}{m_{2}}$$
(43)

 $A_i$  ,  $\underline{B}_i$  and  $\underline{D}_i$  , i = 1, 2 , can be expressed by the first moments :

$$A_{i} = \frac{\frac{2}{2}m_{i}}{\frac{3}{2}kT_{i}^{\circ} - \frac{1}{2}m_{i}\overline{\underline{C}}_{i}^{\circ}}$$

$$\underline{B}_{i} = \frac{\frac{3}{2}m_{i}\overline{\underline{C}}_{i}^{\circ}}{\frac{3}{2}kT_{i}^{\circ} - \frac{1}{2}m_{i}\overline{\underline{C}}_{i}^{\circ}}$$

$$e^{D_{i}} = n_{i}^{o} \left(\frac{A_{i}}{2\pi}\right)^{3/2} \exp\left[-\frac{B_{i}^{2}}{2A_{i}}\right]$$

The a priori condition to zeroth order, Eq. (18), together with Eqs (42) and (43) give that

$$\overline{\underline{C}}_{1}^{O} = \overline{\underline{C}}_{2}^{O} = 0 \tag{44}$$

Here  $\mu$  (a scalar function)  $A_1$ ,  $A_2$ , and  $D_1$ , 1 = 1, 2, are independent of  $C_1$  and  $S_2$ 

The last relation is fulfilled if and only if

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The a priori condition to zerath order, Eq. (18) together with Eqs (42) and (43) give that.

$$\overline{\mathcal{G}}_{1}^{0} = \overline{\mathcal{G}}_{2}^{0} = 0$$

$$T_1^{\circ} = T_2^{\circ} \quad (= T^{\circ})$$
 (45)

in the limit  $t_2 = \infty$ , and the distribution functions then take Maxwellian forms ,

$$f_{iM}^{o} = n_{i}^{o} \left(\frac{m_{i}}{2\pi kT^{o}}\right)^{3/2} exp\left(-\frac{m_{i}C_{i}^{2}}{2kT^{o}}\right)$$
(46)

(The subscript "M" is used occasionally to denote quantities in "local" equilibrium).

Substituting Eqs. (44) and (46) into Eqs. (23), (28) and (29) show that

$$\frac{\partial T^{0}}{\partial t_{2}}$$
,  $\frac{\partial T^{0}_{i}}{\partial t_{2}}$ ,  $i = 1, 2$ , all  $\rightarrow 0$  as  $t_{2} \rightarrow \infty$  (47)

We now study the kinetic equations to zeroth order in the limit  $t_2 = \infty$  . We assume

$$\frac{\partial \mathbf{f}_{i}^{\circ}}{\partial \underline{\mathbf{C}}_{i}} \rightarrow \frac{\partial \mathbf{f}_{iM}^{\circ}}{\partial \underline{\mathbf{C}}_{i}}$$
$$\frac{\partial \mathbf{f}_{i}^{\circ}}{\partial t_{2}} \rightarrow \frac{\partial \mathbf{f}_{iM}^{\circ}}{\partial t_{2}}$$

as  $t_2 \to \infty$ , and from the last relation it follows, using Eqs. (24) and (25) and Eq. (47), that

$$\frac{\partial f_{i}^{o}}{\partial t_{2}} \rightarrow \frac{\partial T^{o}}{\partial t_{2}} \frac{\partial f_{iM}^{o}}{\partial T^{o}} = 0 \quad \text{as } t_{2} \rightarrow \infty .$$

in the limit to a b , and he distribution difference

$$\left(\frac{1}{2}\right)^{2} = n\left(\frac{1}{2}\right)^{2} = n\left(\frac{1}{2}\right)$$

Substituting Eqs. (45) 266 (46) 1660 Eqs. (23), (28) and (29) show that

(74) 
$$\sim - c^2$$
 as  $a - iis ; cir + i ,  $\frac{276}{276}$  ,  $\frac{276}{276}$$ 

We now study the kinetic equiviliant to zeroth official in the the level of the state of the sta

as  $v_2 \rightarrow w$ , and from the list relation it follows. using Equation (i) that



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Substituting from Eq.(22), Eqs.(19) and (20) in the limit  $t_2 = \infty$  become

$$\left(-\alpha^{2} \frac{\rho_{e}^{\circ}}{\rho^{\circ}} \underline{c}_{o}^{\circ} \times \underline{H} + \frac{e_{1}}{m_{1}} \underline{c}_{o}^{\circ} \times \underline{H}\right) \cdot \underline{C}_{1} = 0$$

$$\left(-\frac{\rho_{e}^{\circ}}{\rho^{\circ}} \underline{c}_{o}^{\circ} \times \underline{H} + \frac{e_{2}}{m_{2}} \underline{c}_{o}^{\circ} \times \underline{H}\right) \cdot \underline{C}_{2} = 0$$

showing that the perpendicular (1) mass transport vector in the limit  $t_2 = \infty$  vanishes,

$$\underline{c}_{0\perp}^{0} = 0 \tag{48}$$

(while the parallel ( $\parallel$ )mass transport vector  $\frac{c_0}{c_0}\parallel$  may be non-vanishing).

Thus the equations to zeroth order in the parameter  $\varepsilon$ , which correspond to the equations in the absence of an external electric field and inhomogenities, describe the evolution into a local equilibrium state in space and time where electrons and ions are of equal temperature and where only a parallel mass transport to zeroth order may exist. Before this local state thermal energy of electrons and ions has grown due to the damping of  $\underline{c}_{ol}^{o}$ , and electrons and ions have exchanged energy. This is in agreement with "fine structure solutions" (expanding in the parameter  $\alpha$ ) obtained in [5], [6], [7] and in section 3 of this report.

showing that the perpendicular (1) mass transport vertor in the the limit  $t_2 = -$  vanishes :

this report.

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From the equations to first order in  $\varepsilon$  we find easily that  $\rho^1$ ,  $\rho_1^1$ ,  $c_{0\parallel}^1$ ,  $T^1$  and  $T_1^1$ , i = 1,2, have transients on the  $\tau_2$  time scale, for instance

$$\rho_{1}^{1}(t_{2}, \ldots) = \rho_{1}^{1}(t_{2} = 0, t_{3}, \ldots) - \int_{0}^{t_{2}} d\tau \left[ \alpha \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_{1}^{\circ} \underline{c}_{0\perp}^{\circ}(\tau) \right) + \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_{1}^{\circ} \underline{c}_{0\perp}^{\circ}(\tau) \right) \right]$$

and as non secularity conditions we obtain

$$\frac{\partial \rho^{\circ}}{\partial t_{3}} + \alpha \frac{\partial}{\partial \underline{r}} \cdot \left( \rho^{\circ} \underline{c}^{\circ}_{\circ} \right) = 0$$
(49)

$$\frac{\partial \rho_{i}^{\circ}}{\partial t_{3}} + \alpha \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_{i}^{\circ} \underline{c}_{0}^{\circ} \right) = 0 , \quad i = 1, 2 . \quad (50)$$

$$\frac{3}{2}n^{\circ}k\left(\frac{\partial T^{\circ}}{\partial t_{3}} + \alpha \underline{c}^{\circ}_{\circ}\|, \frac{\partial T^{\circ}}{\partial \underline{r}}\right) = -\alpha \underline{I} p^{\circ} : \frac{\partial \underline{c}^{\circ}_{\circ}\|}{\partial \underline{r}}$$
(51)

The equation for the parallel mass transport vector becomes

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{\circ}^{\circ}}{\partial t_{3}} + \alpha \underline{c}_{\circ}^{\circ} + \frac{\partial \underline{c}_{\circ}^{\circ}}{\partial \underline{r}}\right) = -\alpha \frac{\partial p^{\circ}}{\partial \underline{r}_{\parallel}} + \alpha \sum_{i=1}^{2} \rho_{i}^{\circ} \underline{F}_{i\parallel}$$
(52)

Equations (49) - (52) are similar to corresponding equations of Chapman and Cowling [1] (cf.Appendix 1, Eqs.A12-A14) except that it is the mass transport vector parallel to the magnetic field that dominates in Eqs. (49)-(52).

We now study the kinetic equations, Eqs.(31) and (32), and the transverse part of Eq.(34). These equations are strongly coupled as to the evolution on the  $\tau_2$  time scale. However, the problem to find the behaviour of  $f_1^1$ ,  $f_2^1$ and  $\underline{c}_{o\perp}^1$  as  $\underline{t}_2 \rightarrow \infty$  we here simplify as follows: We seek bounded solutions of Eqs.(31) and (32) which obey

$$\frac{\partial f_1^1}{\partial t_2} \to 0 , \text{ as } t_2 \to \infty$$

$$\frac{\partial f_2^1}{\partial t_2} \to 0 , \text{ as } t_2 \to \infty$$
(53)

We note that the conditions Eq.(53) alone are neither necessary nor sufficient for  $f_1^1$  and  $f_2^1$  to be bounded as  $t_{20} \rightarrow \infty$ . Chapman and Cowling implicitly make similar assumptions concerning  $f_1^1$  and  $f_2^1$  (cf.Appendix 1, Eq.A15). In the limit  $t_2 = \infty$   $f_1^1$  and  $f_2^1$  from Eqs.(31) and (32) will then be functionals of  $n_1^0$ ,  $n_2^0$ ,  $T^0$ ,  $\underline{c}_{0\parallel}^0$  and  $\underline{c}_{0\perp}^1$ . All except  $\underline{c}_{0\perp}^1$  we have found are either independent of  $t_2$ , Eqs.(24) and (25) and the parallell part of Eq.(22), or relax according to Eq.(47) as  $t_2 \rightarrow \infty$ . To be consistent with the assumptions above we also assume that

$$\frac{\partial \underline{c}_{0\perp}^{1}}{\partial t_{2}} \to 0 \quad \text{as} \quad t_{2} \to \infty \tag{54}$$

This assumption will now give a balance of transverse forces in the limit  $t_2 = \infty$ : From the transverse part

We now study the Kinstle squations, Eqs. (\*1) and (72) and the transverse part of Eq.(3). These squations are strongly coupled as to the Evolution on the sectors of However, the problem to find the behaviour of f. f. as to --- we here similarly as follows: We sector bounded solifions of Eq. (3) and (30) which oney

We note that the conditions  $E_{1}(2)$  and  $E_{2}(2)$  alois are hereast  $E_{0}$  and  $E_{1}$  and  $E_{2}$  to be bounded as  $E_{0}$  and  $E_{1}$  and  $E_{2}$  to be bounded as  $E_{0}$  and  $E_{1}$  and  $E_{2}$  (cf. Appendix 1, Eq. 815). In the concerning  $E_{1}$  and  $E_{2}$  (cf. Appendix 1, Eq. 815). In the then be functionals of  $E_{1}$  (cf. Appendix 1, Eq. 815). In the final to  $E_{2}$  and  $E_{1}$  (cf. Appendix 1, Eq. 815). In the final to  $E_{2}$  and  $E_{1}$  (cf. Appendix 1, Eq. 815). In the final to  $E_{2}$  and  $E_{1}$  (cf. Appendix 1, Eq. (31) and (32) will also the finat to  $E_{2}$  and  $E_{2}$  and E

this assigntion will now give a balance of t

of Eq.(34) using the results from zeroth order equations and the assumption Eq.(54) we get

$$\frac{\partial p^{\circ}}{\partial \underline{r}_{\perp}} + \rho_{\underline{e}}^{\circ} \underbrace{c}_{0}^{1} \times \underline{H} + n_{1}^{\circ} e_{1} \underbrace{\overline{c}}_{1}^{1} \times \underline{H} + \alpha n_{2}^{\circ} \underbrace{e_{2}}_{\underline{c}} \underbrace{\overline{c}}_{2}^{1} \times \underline{H} + \sum_{i=1}^{2} \rho_{i}^{\circ} \underbrace{F_{i}}_{\underline{i}=1} = 0$$
(55)

In this equation we find  $\underline{\overline{c}}_1^1$  and  $\underline{\overline{c}}_2^1$  from the solutions of the kinetic equations to first order in the limit  $t_2 = \infty$ . Eq.(55)then determines  $\underline{c}_{ol}^1$  in this limit. We note here that in the next section we partly derive what is assumed above concerning the time derivatives of  $f_1^1$ ,  $f_2^1$  and  $\underline{c}_{ol}^1$  as  $t_2 \to \infty$ .

Making use of the results to zeroth order and also Eqs.(50), (51) and (52) and the assumptions above  $(f_1^1 \text{ and } f_2^1$ bounded and Eqs.(53) and (54)), the kinetic equations to first order in  $\varepsilon$ , Eqs.(31) and (32) in the limit  $t_2 = \infty$ can be written

$$f_{1M}^{\circ} \left( \frac{m_{1}C_{1}^{2}}{2kT^{\circ}} - \frac{5}{2} \right) \frac{1}{T^{\circ}} \frac{\partial T^{\circ}}{\partial \underline{r}} \cdot \underline{C}_{1} + f_{1M}^{\circ} \left( \frac{\partial}{\partial \underline{r}_{\parallel}} lnp_{1}^{\circ} - \frac{p_{1}^{\circ}}{p_{1}^{\circ}p^{\circ}} (p^{\circ} - \alpha^{2}p_{1}^{\circ}) (\underline{F}_{1\parallel} - \alpha^{2}\underline{F}_{2\parallel}) - \alpha^{2} \frac{p_{1}^{\circ}}{p_{1}^{\circ}p^{\circ}} \frac{\partial p^{\circ}}{\partial \underline{r}_{\parallel}} \right) \cdot \underline{C}_{1} - f_{1M}^{\circ} \frac{e_{1}}{kT^{\circ}} \left( \frac{m_{1}}{e_{1}} \underline{F}_{1\perp} + \underline{c}_{0}^{1} \times \underline{H} - \frac{kT^{\circ}}{p_{1}^{\circ}p^{\circ}} \frac{\partial}{\partial \underline{r}_{\perp}} lnp_{1}^{\circ} \right) \cdot \underline{C}_{1} + \alpha f_{1M}^{\circ} \frac{m_{1}}{kT^{\circ}} \underline{C}_{1}^{\circ} \underline{C}_{1} : \frac{\partial \underline{c}_{0\parallel}^{\circ}}{\partial \underline{r}} =$$

$$= -\frac{e_{1}}{m_{1}} \frac{C}{2} \ln p_{1}^{\circ} \right) \cdot \underline{C}_{1} + \kappa f_{1M}^{\circ} \frac{m_{1}}{kT^{\circ}} \underline{C}_{1}^{\circ} \underline{C}_{1} : \frac{\partial \underline{c}_{0\parallel}^{\circ}}{\partial \underline{r}} =$$

$$= -\frac{e_{1}}{m_{1}} \frac{C}{2} \times \underline{H} \cdot \frac{\partial f_{1M}^{\circ}}{\partial \underline{C}_{1}} + FP_{11} \left[ f_{1M}^{\circ} (\underline{C}_{1}) f_{1M}^{1} (\underline{C}_{1}^{\circ}) + f_{1M}^{1} (\underline{C}_{1}) f_{1M}^{\circ} (\underline{C}_{1}^{\circ}) \right] + FP_{12} \left[ f_{1M}^{\circ} (\underline{C}_{1}) f_{2M}^{1} (\underline{C}_{2}) + f_{1M}^{1} (\underline{C}_{1}) f_{2M}^{\circ} (\underline{C}_{2}) \right]$$

of Eq. (34) using the results from zeroth greet equations and the assumption Eq. (54) we get

In this equation we find  $\underline{C}$  and  $\underline{C}$  from the colutions of the kinetic equations to first order in the limit  $t_2 = \infty$ . Eq. (55) then determines  $\underline{C}_{01}$  in this limit. We note here that in the mart section we partly derive what is assumed shows concerning the time derivatives of  $\underline{C}_{1}$ ,  $\underline{C}_{2}$  and  $\underline{C}_{1}$  as  $\underline{t}_{2} = \overline{c}_{1}$ .

Halding use of the results to schold offer and also Eqs. (30), (51) and (53) and the assumptions above (1, and the bounded and Eqs. (53) and (54)), the sinelic equations to first order in a , Eqs. (31) and (32) in the limit to the can be written

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$$a f_{2M}^{\circ} \left( \frac{m_2 C_2^2}{2kT^{\circ}} - \frac{5}{2} \right) \frac{1}{T^{\circ}} \frac{\partial T^{\circ}}{\partial \underline{r}} \cdot \underline{C}_2 + f_{2M}^{\circ} \left( \alpha \frac{\partial}{\partial \underline{r}} \right) \ln p_2^{\circ} - = \alpha \left( \rho^{\circ} - \rho_2^{\circ} \right) \frac{\rho_2^{\circ}}{p_2^{\circ} \rho^{\circ}} \left( \alpha^2 \underline{F}_2 \right) - \underline{F}_1 \right) - \alpha \frac{\rho_2^{\circ}}{\rho p_2^{\circ}} \frac{\partial p^{\circ}}{\partial \underline{r}_1} \right) \cdot \underline{C}_2 - = \alpha f_{2M}^{\circ} \frac{e_2}{kT^{\circ}} \left( \frac{m_2}{e_2} \underline{F}_{2L} + \underline{c}_0^{\circ} \times \underline{H} - \frac{kT^{\circ}}{e_2} \frac{\partial}{\partial \underline{r}_1} \ln p_2^{\circ} \right) \cdot \underline{C}_2 + + \alpha f_{2M}^{\circ} \frac{m_2}{kT^{\circ}} \underline{C}_2^{\circ} \underline{C}_2 : \frac{\partial \underline{c}_0^{\circ}}{\partial \underline{r}} =$$

$$= -\alpha^2 \frac{e_2}{m_2} \underline{C}_2 \times \underline{H} \cdot \frac{\partial f_{2M}^{\circ}}{\partial \underline{C}_2} + \alpha FP_{22} \left[ f_{2M}^{\circ} (\underline{C}_2) f_{2M}^{\circ} (\underline{C}_2') + + f_{2M}^{\circ} (\underline{C}_2) f_{2M}^{\circ} (\underline{C}_2') \right] + \alpha FP_{21} \left[ f_{2M}^{\circ} (\underline{C}_2) f_{1M}^{\circ} (\underline{C}_1) + f_{2M}^{\circ} (\underline{C}_2) f_{1M}^{\circ} (\underline{C}_1) \right]$$

These equations together with the constraint Eq.(30) in the limit  $t_2 = \infty$  correspond to the "second order kinetic equations" of Chapman and Cowling.

Eqs(30), (56) and (57) can be solved by successive approximations expanding in the parameter  $\alpha$ . Leaving out terms of order  $\alpha$  in Eqs.(30), (56) and (57) we obtain

$$\int d\underline{C}_{2} f_{2M}^{1}(\underline{C}_{2}) m_{2} \underline{C}_{2} = 0$$
(58)

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These equations together with the constraint Rd. (20) in the limit  $t_2 = \infty$  correspond to the "second order idnetic equations" of Chapman and Cowling.

Eqa(30), (a) and (57) can be solved by auccessive approximations expanding in the parameter of a listving out forms of order & in Eqs. (30), (56) and (57) we obtain

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$$f_{1M}^{o} \left[ \left( \frac{m_{1}C_{1}^{2}}{2kT^{o}} - \frac{5}{2} \right) \frac{1}{T^{o}} \frac{\partial T^{o}}{\partial \underline{r}} - \frac{e_{1}}{kT^{o}} \left( \frac{m_{1}}{e_{1}} F_{1} + \underline{c}_{0}^{1} \times \underline{H} - \frac{kT^{o}}{e_{1}} \frac{\partial p_{1}}{\partial \underline{r}} e_{1} \right) \right] \cdot \underline{c}_{1} =$$

$$= -\frac{e_{1}}{m_{1}} \frac{c_{1}}{c_{1}} \times \underline{H} \cdot \frac{\partial f_{1M}^{1}}{\partial \underline{c}_{1}} + F_{11} \left[ f_{1M}^{o} (\underline{c}_{1}) f_{1M}^{1} (\underline{c}_{1}') + f_{1M}^{1} (\underline{c}_{1}) f_{1M}^{o} (\underline{c}_{1}') \right] + D_{1}^{o} \left[ f_{1M}^{1} \right]$$

$$+ D_{1}^{o} \left[ f_{1M}^{1} \right]$$

$$f_{2M}^{\circ}\left(\frac{m_{2}C_{2}^{2}}{2kT^{\circ}} - \frac{5}{2}\right)\frac{1}{T^{\circ}}\frac{\partial T^{\circ}}{\partial \underline{r}}\cdot\underline{C}_{2} + f_{2M}^{\circ}\frac{m_{2}}{kT^{\circ}}\underline{C}_{2}^{\circ}\underline{C}_{2} : \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}} =$$

$$= FP_{22}\left[f_{2M}^{\circ}(\underline{C}_{2})f_{2M}^{1}(\underline{C}_{2}') + f_{2M}^{1}(\underline{C}_{2})f_{2M}^{\circ}(\underline{C}_{2}')\right]$$

$$(60)$$

Here 
$$D_1^{\circ} = \frac{n_2^{\circ}}{m_1^2} \frac{\partial}{\partial \underline{C}_1} \cdot \left( \bigoplus_{i=1}^{n_2} (\underline{C}_1) \cdot \frac{\partial}{\partial \underline{C}_1} \right)$$
 is a diffusion operator.

To obtain Eq.(60) use is also made of Eq.(39) in the limit  $t_2 = \infty$ . Eqs.(59) and (60) are of the same form as Eqs.(2.50) and (2.86) of [5] or Eqs.(10) and (15) of [6], see also [7].

Using the Landau form for the tensor  $\Phi^+{}^{\prime}$  ,

$$\Phi_{\mu}^{ij}(\underline{w}) = \kappa \frac{w^2 \underline{I} - \underline{w} \underline{w}}{w^3} , \quad \kappa = 2\pi e_i^2 e_j^2 \ln \left[ \frac{3\lambda_D k \underline{T}^0}{2e_1^2} \right], \quad \text{where } \underline{I}$$

is the unit tensor and  $\lambda_{\rm D}$  the Debye length, Leversen and Naze Tjøtta [11] have proved existence of solutions of Eqs. (59) and (60) with certain properties. The solution of Eq. (59) contains two arbitrary parameters, and imposing

$$n_1^1 = 0$$
$$T_1^1 = 0$$

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in the limit  $t_2 = \infty$ , these parameters vanish. The solution of Eq.(60) contains 5 arbitrary parameters so that Eq.(58) can be fulfilled too. To see the form of the solution of Eq.(59), which is sufficient in the discussion later, we substitute

$$v_1(f_{1M}^0 - f_1)$$
 (61)

for the collision terms of Eq.(7) when terms of order  $\alpha$ are left out. Here  $v_1$ , assumed constant, is a measure of electron-electron and electron-ion effective collision frequency, with ions at rest. The solution of Eq.(59) thus takes the form, [5]

$$\mathbf{f}_{1M}^{1} = -\mathbf{f}_{1M}^{0} \left[ \frac{1}{\nu_{1}} \underline{C}_{1} \right] + \frac{\frac{1}{\nu_{1}}}{1 + \left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}} \left( \underline{C}_{1\perp} - \frac{\Omega_{1}}{\nu_{1}H} \underline{C}_{1} \times \underline{H} \right) \right] \cdot \underline{\mathbf{h}}_{1}^{1}$$
(62)

where  $f_{1M}^{\circ} \xrightarrow{h_1}^{1} \cdot \underline{C}_1$  is the left hand side of Eq.(59). From Eq.(62) we derive for the homogeneous case

$$n_{1}^{O}\overline{\underline{C}}_{1\perp}^{1} = \frac{\frac{1}{v_{1}}}{1 + \left(\frac{\Omega_{1}}{v_{1}}\right)^{2}} \left[\frac{n_{1}^{O}\underline{e}_{1}}{m_{1}} \left(\frac{\underline{m}_{1}}{\underline{e}_{1}}\underline{F}_{1} + \underline{c}_{0}^{1} \times \underline{H}\right) - \frac{n_{1}\underline{e}_{1}}{1 + \left(\frac{\Omega_{1}}{v_{1}}\right)^{2}} \left[-\frac{n_{1}}{\underline{m}_{1}}\frac{\Omega_{1}}{v_{1}}\frac{\underline{H}}{\underline{H}} \times \left(\frac{\underline{m}_{1}}{\underline{e}_{1}}\underline{F}_{1} + \underline{c}_{0}^{1} \times \underline{H}\right)\right]$$
(63)

in the limit to ever these parameters ramiah. The colution of Eq. (60) contains 5 arbitrary parameters so that Eq. (53) can be fulfilled too. To see the form of the solution of Eq. (53), which is sufficient in the discussion later, we suboting a

$$v_1(z_{131}^2 - z_1)$$
 (61)

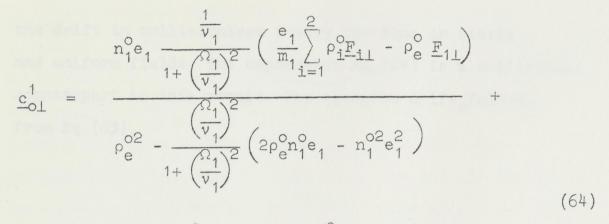
for the collision terms of Eq. (7) when terms of order a are left out. Here  $v_1$ , assumed constant, is a measure of electron-electron and electron-ion effective collision frequency, with ions at rest. The solution of Eq. (59) thus takes the form, [5]

(so) 
$$[\underline{a}, [\underline{a}, \underline{a}] = \underline{a} = \underline$$

where  $\Gamma_{1N}^{0} = \frac{1}{2} \cdot 2_{1}$  is the left hand side of Eq.(59). From Eq. (52) we derive for the homogeneous are

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Leaving out terms of order  $\alpha$  in the balance of transverse forces relation Eq.(55) and substituting from Eq.(63) we obtain the form of  $\underline{c}_{o\perp}^1$  in the limit  $t_2 = \infty$ for the homogeneous case ,



$$+ \frac{\left(\rho_{e}^{\circ} - n_{1}^{\circ}e_{1} - \frac{\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}{1 + \left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}\right)\left(n_{1}^{\circ}m_{1} - \frac{\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}{1 + \left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}} + \frac{H \times F_{1} - H \times \sum_{i=1}^{2} \rho_{i}^{\circ}F_{i}}{H \times F_{1} - H \times \sum_{i=1}^{2} \rho_{i}^{\circ}F_{i}}\right)}{H^{2} \left[\rho_{e}^{\circ 2} - \frac{\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}{1 + \left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}} \left(2\rho_{e}^{\circ}n_{1}^{\circ}e_{1} - n_{1}^{\circ2}e_{1}^{2}\right)\right]} - \frac{1}{1 + \left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}$$

$$\frac{n_{1}^{02}e_{1}m_{1}}{H^{2}\left[1+\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}\frac{H\times F_{1}}{\left[1+\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}\right]^{2}} + \frac{H\times F_{1}}{H\times F_{1}}\right]}{H^{2}\left[\rho_{e}^{e_{2}}-\frac{\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}{1+\left(\frac{\Omega_{1}}{\nu_{1}}\right)^{2}}\left(2\rho_{e}^{o}n_{1}^{o}e_{1}-n_{1}^{o2}e_{1}^{2}\right)\right]}$$

The first part vanishes when  $\underline{F}_{i}$  arises from an electric field.

Leaving out terms of order a in the ballance of transverse forces religition Eq. (55) and substituting from  $E_{1}$  (53) we obtain the form of  $2_{01}$  if the limit  $t_{2} = w$ 



(43)





The first part vanishes when In arises from an electric

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From Eq. (64) we get

$$c_{ol}^{1} \rightarrow \frac{m_{2}F_{2} \times H}{e_{2}H^{2}} \text{ as } \frac{\Omega_{1}}{\nu_{1}} \rightarrow \infty \text{ , i.e.}$$

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the drift in collisionless theory for ions in static and uniform fields. The expression Eq.(64) is a collisional counterpart to this result. The electron drift follows from Eq.(63).

From Eq. (64) we get

the drift in collisionless thoory for loss in static and uniform flaids. The expression Eq. (54) is a collisional counterpart to this result. The electron drift follows from Eq. (53). In this section we expand in  $\alpha$  the sets of equations to zeroth and first order in  $\varepsilon$ . Then we shall in more detail see the evolution of distribution functions and macroscopic quatities. Assuming bounded solutions of all kinetic equations we shall derive to some extent the assumptions Eq. (53) and (54) and show that the gas evolves into a state where forces transverse to the magnetic field balance in the mass transport equation.

We obtain equations to zeroth order in  $\varepsilon$  and zeroth, first, second orders etc. in  $\alpha$  from the equations (18)-(29) or from the equations of [5] putting  $\frac{\partial}{\partial \underline{r}}$  and  $\underline{F}_i$ , i=1,2, all equal to zero. We will discuss some of these equations up to third order in  $\alpha$  here. Since  $\rho^{\circ}$ ,  $\rho_i^{\circ}$  and  $\underline{c}_{\circ\parallel}^{\circ}$  are all independent of  $t_2$  we do not expand them in  $\alpha$  for simplicity here.

The equations to <u>zeroth order in  $\varepsilon$  and zeroth order in  $\alpha$ </u> show that  $f_2^{00}$ ,  $\underline{c}_0^{00}$ ,  $T^{00}$ ,  $T_1^{00}$ , i = 1, 2, and  $\overline{\underline{C}}_2^{00}$  are all independent of  $t_{20}$  while  $\overline{\underline{C}}_1^{00}$  evolves according to the evolution of  $f_1^{00}$  on the  $\tau_{20}$  time scale. A H-theorem can be established which shows that In this section we expand in a the sets of equations to zeroth and first order in a . That we shall in more detail see the evolution of distribution functions the macroscopic quatifies. Assuming bounded solutions of all kinetic equations we shall derive to some extent the assumptions Eq. (53) and (54) and show that the gas evolves into a state where forces transverse to the magnetic field balance in the mass transport equation.

We obtain equations to zeroth order in a and zeroth, first, second orders etc. In a from the equation (18)-(29) or from the equations of [5] mutthe  $\frac{1}{22}$  and  $\frac{1}{21}$ , 1=1,2, all equal to zero. We will discuss some of these equations up to third order in a more. Since  $e^2$ ,  $e_1^2$  and  $2e_1^2$  bre all independent of  $e_2$  we do not expand them in a for simplicity here. The equations to zeroth order in a and zeroth order in show that  $f_2^0$ ,  $f_2^0$ ,  $f^{00}$ ,  $f_1^{00}$ ,  $f_2^0$ , 1=1,2, and  $f_2^0$  are the svolution of  $f_{20}$  while  $f_1^0$  evolves according to the svolution of  $f_{20}^0$  an the  $r_{20}$  time scale. A

$$f_{1}^{00} \rightarrow f_{1M}^{00} = n_{1}^{0} \left(\frac{m_{1}}{2\pi k T_{1}^{00}}\right)^{3/2} exp\left(-\frac{m_{1} C_{1}^{2}}{2k T_{1}^{00}}\right) \text{ as } t_{20} \rightarrow \infty$$
 (65)

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Accordingly  $\underline{\underline{C}}_{1}^{00} \rightarrow 0$  as  $t_{20} \rightarrow \infty$ .

From the equations to zeroth order in  $\varepsilon$  and first in  $\alpha$ we get from the mass transport equation

$$\underline{c}_{0\perp}^{\circ 1}(t_{20}, t_{21}, \ldots) = \underline{c}_{0\perp}^{\circ 1}(t_{20}=0, t_{21}, \ldots) - t_{20} \frac{\partial \underline{c}_{0\perp}^{\circ 0}}{\partial t_{21}} + \frac{n_{1}^{\circ e} \underline{1}}{\rho^{\circ}} \int_{0}^{t_{20}} d\tau \underline{\overline{c}}_{1}^{\circ \circ}(\tau) \times \underline{H}$$

and eliminating the secular term we get that  $\underline{c}_{01}^{00}$  is also independent of  $t_{21}$  and

$$\underline{c}_{0\perp}^{01}(t_{20}, t_{21}, \ldots) = \underline{c}_{0\perp}^{01}(t_{20} = 0, t_{21}, \ldots) + \\ + \frac{n_1^{0} e_1}{\rho^0} \int_{0}^{t_{20}} d\tau \underline{c}_1^{00}(\tau) \times \underline{H} ,$$

i.e.  $\underline{c}_{01}^{01}$  has a transient on the  $\tau_{20}$  time scale. Similarly we find that  $\underline{T}^{00}$ ,  $\underline{T}_{1}^{00}$  and  $\underline{T}_{2}^{00}$  are also independent of  $t_{21}$ ,  $\underline{T}_{2}^{01}$  is independent of  $t_{20}$ , while  $\underline{T}^{01}$ ,  $\underline{T}_{1}^{01}$ ,  $\underline{C}_{2}^{01}$  and  $\underline{f}_{2}^{01}$  all have transients on the  $\tau_{20}$  time scale.

A H-theorem can again be established showing that

$$(63) = 6 \left( \frac{1}{2} + 1 \right)^{2} \left( \frac{1}{2} + \frac{1}{2} \right)^{2} \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right)^{2} \left( \frac{1}{2} + \frac{1}$$

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$$f_2^{00} \to f_{2M}^{00} = n_2^0 \left(\frac{m_2}{2\pi k T_2^{00}}\right)^{3/2} exp\left(-\frac{m_2 C_2^2}{2k T_2^{00}}\right) as t_{21} \to \infty$$
 (66)

Here use has been made of the a priori condition to zeroth order in  $\alpha$ . The electron kinetic equation to first order is of the form

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$$\frac{\partial \mathbf{f}_{1}^{\circ 1}}{\partial \mathbf{t}_{20}} + \frac{\mathbf{e}_{1}}{\mathbf{m}_{1}} \underbrace{\mathbf{C}}_{1} \times \underline{\mathbf{H}} \cdot \frac{\partial \mathbf{f}_{1}^{\circ 1}}{\partial \underline{\mathbf{C}}_{1}} - FP_{11} \left[ \mathbf{f}_{1M}^{\circ \circ}(\underline{\mathbf{C}}_{1}) \mathbf{f}_{1}^{\circ 1}(\underline{\mathbf{C}}_{1}) + \mathbf{f}_{1}^{\circ 1}(\underline{\mathbf{C}}_{1}) \mathbf{f}_{1M}^{\circ \circ}(\underline{\mathbf{C}}_{1}) \right] - D_{1}^{\circ} \left[ \mathbf{f}_{1}^{\circ 1}(\underline{\mathbf{C}}_{1}) \right] = -\frac{\mathbf{e}_{1}}{\mathbf{m}_{1}} \underbrace{\mathbf{c}}_{0}^{\circ \circ} \times \underline{\mathbf{H}} \cdot \frac{\partial \mathbf{f}_{1M}^{\circ \circ}}{\partial \underline{\mathbf{C}}_{1}} + \mathbf{G}^{\circ 1}(\mathbf{t}_{20})$$
(67)

where  $G^{01}(t_{20}) \to 0$  as  $t_{20} \to \infty$ . Some assumptions concerning commutation of differential operations and "limit as  $t_{20} \to \infty$ " operations have been made. In the appendixes of [5], [7] and [12] it is shown that every bounded solution of Eq.(67) evolves as  $t_{20} \to \infty$  toward the solution of

$$-\frac{e_{1}}{m_{1}}\underline{C}_{1} \times \underline{H} \cdot \frac{\partial f_{1M}^{\circ 1}}{\partial \underline{C}_{1}} + FP_{11} \left[ f_{1M}^{\circ 0}(\underline{C}_{1}) f_{1M}^{\circ 1}(\underline{C}_{1}') + f_{1M}^{\circ 1}(\underline{C}_{1}) f_{1M}^{\circ 0}(\underline{C}_{1}') \right] + D_{1}^{\circ} \left[ f_{1M}^{\circ 1}(\underline{C}_{1}) \right] = \frac{e_{1}}{m_{1}} \underline{c}_{0}^{\circ \circ} \times \underline{H} \cdot \frac{\partial f_{1M}^{\circ \circ}}{\partial \underline{C}_{1}}$$
(68)

which has the same form as Eq.(59). Substituting for the collision terms the relaxation term Eq.(61) we get that the solutions of Eq.(67) evolve toward a solution of the form Eq.(62) with  $-e_1/kT_1^{OO}c_0^{OO} \times \underline{H}$  instead of  $\underline{h}_1^1$ .

From the equations to zeroth order in  $\varepsilon$  and second in  $\alpha$ we derive from the mass transport equation, eliminating the secular terms when integrating on the t<sub>20</sub> time scale

$$\rho^{O}\left(\frac{\partial \underline{c}_{01}^{O1}}{\partial \underline{t}_{21}}(\underline{t}_{20} = 0, \underline{t}_{21}, \ldots) + \frac{\partial \underline{c}_{01}^{OO}}{\partial \underline{t}_{22}}\right) = \rho_{e}^{O} \underline{c}_{0}^{OO} \times \underline{H} - n_{1}^{O} e_{1} \overline{\underline{c}}_{1M}^{O1} \times \underline{H}$$

$$\underline{c}_{0\perp}^{02}(t_{20}, t_{21}, ...) = \underline{c}_{0\perp}^{02}(t_{20} = 0, t_{21}, ...) + \\ + \frac{n_{1}^{0} e_{1}}{\rho^{0}} \int_{0}^{t_{20}} d\tau \left( \overline{\underline{c}}_{1}^{01}(\tau) - \overline{\underline{c}}_{1M}^{01} \right) \times \underline{\underline{H}}$$

We have assumed that  $\frac{\partial \underline{c}_{0\perp}^{01}}{\partial t_{21}} = \frac{\partial \underline{c}_{0\perp}^{01}}{\partial t_{21}} (t_{20}, t_{21}, ...) =$ =  $\frac{\partial \underline{c}_{0\perp}^{01}}{\partial t_{21}} (t_{20} = 0, t_{21}, ...)$ . Integrating on the  $\tau_{21}$  time scale we see that  $\underline{c}_{0\perp}^{01} (t_{20} = 0)$  is independent of  $t_{21}$ . For  $\underline{c}_{0\perp}^{00}$  and likewise for  $\underline{T}^{00}$ ,  $\underline{T}_{1}^{00}$  and  $\underline{T}_{2}^{00}$  we get the following equations on the  $\tau_{22}$  time scale

$$\rho^{\circ} \frac{\partial \underline{c}_{\circ}^{\circ \circ}}{\partial \underline{t}_{22}} = \rho_{e}^{\circ} \underline{c}_{\circ}^{\circ \circ} \times \underline{H} + n_{1}^{\circ} e_{1} \overline{\underline{c}}_{1M}^{\circ 1} \times \underline{H}$$
(69)

$$\frac{3}{2}n^{\circ}k\frac{\partial T^{\circ\circ}}{\partial t_{22}} = n_1^{\circ}e_1\underline{\overline{C}}_{1M}^{\circ1}(\underline{c}_0^{\circ\circ} \times \underline{H})$$
(70)

From the equations to teroth order in  $\varepsilon$  and second in  $\tau$ we derive from the mass transport equation, eliminating the secular terms when integrating on the  $\tau_{20}$  time scale

$$\underline{\mathbf{H}} \times \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{N}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}}_{\mathbf{M}} \underbrace{\mathbf{M}} \underbrace$$

$$2^{\circ 0}_{\circ 1}(t_{20}, t_{21}, \dots) = 2^{\circ 1}_{\circ 2}(t_{20}, \dots, t_{21}, \dots)$$

$$\underline{H} \times \begin{pmatrix} 1 & \underline{0} \\ \underline{0} \\ \underline{0} \\ \underline{0} \\ \underline{0} \\ \underline{0} \end{pmatrix} = \begin{pmatrix} \underline{0} & \underline{0} \\ \underline{0}$$

We have assumed that  $\frac{62}{621}$   $\frac{62}{621}$   $\frac{62}{621}$   $(t_{20}, t_{21}, ...)$  =  $\frac{62}{621}$   $(t_{20} = 0, t_{21}, ...)$ . Integrating on the relations scale we see that  $\frac{61}{621}$   $(t_{20} = 0)$  is integrating on the relation relation  $\frac{62}{61}$   $(t_{20} = 0, t_{21}, ...)$ .

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$$\frac{3}{2}n_{1}^{0}k \frac{\partial T_{1}^{00}}{\partial t_{22}} = n_{1}^{0}e_{1}\underline{\vec{C}}_{1M}^{01} \cdot (\underline{c}_{0}^{00} \times \underline{H}) -$$

$$- \frac{n_{1}^{0}n_{2}^{0}}{m_{2}} \left(\frac{m_{1}}{2\pi k}\right)^{3/2} \left(\frac{T_{1}^{00} - T_{2}^{00}}{T_{1}^{00} - 5/2}\right) \int d\underline{c}_{1}\underline{\phi}^{12}(\underline{c}_{1}) : I \exp\left(-\frac{m_{1}c_{1}^{2}}{2kT_{1}^{00}}\right)$$
(71)

$$\frac{3}{2}n_{2}^{\circ}k \frac{\partial T_{2}^{\circ\circ}}{\partial t_{22}} = \frac{n_{1}^{\circ}n_{2}^{\circ}}{m_{2}} \left(\frac{m_{1}}{2\pi k}\right)^{3/2} \left(\frac{T_{1}^{\circ\circ} - T_{2}^{\circ\circ}}{T_{1}^{\circ\circ} - 5/2}\right) \int d\underline{c}_{1} \underline{\phi}^{12}(\underline{c}_{1}) :$$

$$: \underline{I} \exp\left(-\frac{m_{1}c_{1}^{2}}{2kT_{1}^{\circ\circ}}\right) \qquad (72)$$

In general Eqs.(69)-(72) are a coupled set of equations describing the relaxation of  $\underline{c}_{0\perp}^{00}$ ,  $\underline{T}^{00}$ ,  $\underline{T}_{1}^{00}$  and  $\underline{T}_{2}^{00}$  on the  $\tau_{22}$ -time scale. We simplify using for  $\underline{\overline{C}}_{1M}^{01}$  an expression corresponding to Eq.(63). Eq.(69) then can be solved and shows that  $\underline{c}_{0\perp}^{00}$  oscillates and is damped away exponentially on the  $\tau_{22}$  time scale having solutions

proportional to (5) 
$$e^{\pm i\omega_{\pm}t^{2}22}$$
,  $\omega_{\pm} = \eta \pm i\frac{\delta H^{2}}{\rho^{\circ}}$ ,  
 $\delta = \frac{n_{1}^{\circ}e_{1}^{2}}{m_{1}} \frac{\frac{1}{\nu_{1}}}{1+(\frac{\Omega_{1}}{\nu_{1}})^{2}} > 0$ ,  $\eta = \frac{H}{\rho^{\circ}} \left(\rho_{e}^{\circ} - \delta \frac{\Omega_{1}H}{\nu_{1}}\right)$ 

This is in agreement with Eq.(48). Due to this damping Eq.(70) shows how the temperature grows, and Eqs.(71) and (72) describe how  $T_1^{00}$  and  $T_2^{00}$  relax against each other. This should be compared with Eq.(45). Due to the damping of  $\underline{c}_{01}^{00}$  also  $f_{1M}^{01}$  from Eq.(68) damps away as  $t_{22} \rightarrow \infty$  (when  $n_1^{01}$  and  $T_1^{01}$  are set equal to zero in the limit  $t_{20} = \infty$ ).

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$$\frac{\partial f_2^{\circ 1}}{\partial t_{21}} - FP_{22} \left[ f_{2M}^{\circ \circ}(\underline{c}_2) f_2^{\circ 1}(\underline{c}_2') + f_2^{\circ 1}(\underline{c}_2) f_{2M}^{\circ \circ}(\underline{c}_2') \right] = H^{\circ 1}(t_{21})$$
(73)

where  $H^{01}(t_{21}) \rightarrow 0$  as  $t_{21} \rightarrow \infty$ , [5]. Using the appendix of [12] shows that every bounded  $f_2^{01}$  evolves toward the solution of

$$FP_{22}\left[f_{2M}^{00}(\underline{C}_{2})f_{2}^{01}(\underline{C}_{2}') + f_{2}^{01}(\underline{C}_{2})f_{2M}^{00}(\underline{C}_{2}')\right] = 0$$
(74)

as  $t_{21} \rightarrow \infty$ .

Using the a priori condition Eq.(18) to first order in  $\alpha$  in the limits  $t_{20} \rightarrow \infty$  and  $t_{21} \rightarrow \infty$  and also  $n_2^{01} = 0$  and  $T_2^{01} = 0$  in the same limits give

$$f_{2M}^{01} = 0$$
 (75)

From the electron kinetic equation to this order we get, using the appendixes of [5], [7] and [12] and the solution of Eq.(68) and Eq.(75), that every bounded  $f_1^{o2}$  evolves toward the solution of The ion kinetic equation to this order shows that  $f_2^{or}$  has a transient on the  $r_{20}$  time scale. On the  $r_{21}$  time scale the equation is of the form,

 $\frac{\delta r_{2}^{2}}{\delta r_{2}} = r_{22} \left[ \frac{c_{2}}{c_{2}} + \frac{c_{2}}{$ 

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 $\mathbf{r}_{222} \left[ \mathbf{r}_{22}^{2} \left[ \mathbf{r}_{22}^{2} \right] \mathbf{r}_{22}^{2} \left[ \mathbf{r}_{22}^{2} \left[ \mathbf{r}_{2$ 

so to ist as

Using the a priori condition  $\frac{1}{2}$   $\frac{1}{$ 

fo1 M = 0

From the electron kinetic equation to this order we get, using the appendixes of [5]. [7] and [12] and the solution of Eq. (68) and Eq. (75), that every bounded  $f_1^{O2}$  evolves toward the solution of

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 $\frac{\partial f_{1M}^{00}}{\partial t_{22}} + \frac{e_{1}}{m_{1}} \frac{e_{0}^{-0}}{e_{0}} \times \underline{H} \cdot \frac{\partial f_{1M}^{00}}{\partial \underline{c}_{1}} + \frac{e_{1}}{m_{1}} \frac{e_{0}^{-0}}{e_{0}} \times \underline{H} \cdot \frac{\partial f_{1M}^{01}}{\partial \underline{c}_{1}} -$   $= FP_{11} \left[ f_{1M}^{01}(\underline{c}_{1}) f_{1M}^{01}(\underline{c}_{1}') \right] - \frac{1}{m_{1}} \frac{\partial}{\partial \underline{c}_{1}} \cdot \int d\underline{c}_{2} \left( \underline{c}_{2} \cdot \frac{\partial \underline{b}^{12}}{\partial \underline{c}_{1}} (\underline{c}_{1}) \right) \cdot \frac{\partial f_{2M}^{00}}{\partial \underline{c}_{2}} f_{1M}^{00} -$   $= \frac{1}{m_{1}^{2}} \frac{\partial}{\partial \underline{c}_{1}} \cdot \int d\underline{c}_{2} \left( \frac{\underline{c}_{2}\underline{c}_{2}}{2} : \frac{\partial^{2} \underline{b}^{12}(\underline{c}_{1})}{\partial \underline{c}_{1} \partial \underline{c}_{1}} \right) \cdot f_{2M}^{00} \frac{\partial f_{1M}^{00}}{\partial \underline{c}_{1}} =$   $= - \frac{e_{1}}{m_{1}^{2}} \frac{\partial}{\partial \underline{c}_{1}} \times \underline{H} \cdot \frac{\partial f_{1}^{02}}{\partial \underline{c}_{1}} + FP_{11} \left[ f_{1M}^{00}(\underline{c}_{1}) f_{1}^{02}(\underline{c}_{1}') + f_{1}^{02}(\underline{c}_{1}) f_{1M}^{00}(\underline{c}_{1}') \right] +$ 

- 40 --

+  $D_{1}^{0} \left[ f_{1}^{02} \right]$  (76)

as  $t_{20}$  and  $t_{21}$  go to infinity (in the limit  $t_{20} = \infty$ only,  $f_2^{00}$  instead of  $f_{2M}^{00}$ ), or, rewriting terms on the left hand side,  $f_1^{02}$  evolves toward the solution of

$$f_{1M}^{00} \left( \frac{m_{1}C_{1}^{2}}{2\kappa T_{1}^{00}} - \frac{3}{2} \right) \frac{1}{T_{1}^{00}} \frac{\partial T_{1}^{00}}{\partial t_{22}} + \frac{e_{1}}{m_{1}^{0}C_{0}^{0}} \times \underline{H} \cdot \frac{\partial f_{1M}^{00}}{\partial \underline{C}_{1}} + \frac{e_{1}}{m_{1}^{1}C_{0}^{0}} \times \underline{H} \cdot \frac{\partial f_{1M}^{01}}{\partial \underline{C}_{1}} - F_{11}^{0} \left[ f_{1M}^{01}(\underline{C}_{1}) f_{1M}^{01}(\underline{C}_{1}^{\prime}) \right] - \kappa \frac{2m_{2}^{00}}{m_{1}^{m_{2}}} \frac{T_{1}^{00} - T_{2}^{00}}{T_{1}^{00}} \frac{\partial}{\partial \underline{C}_{1}} \cdot \left( \frac{\underline{C}_{1}}{C_{3}^{3}} f_{1M}^{00} \right) = (77)$$

$$= - \frac{e_{1}}{m_{1}^{0}C_{1}} \times \underline{H} \cdot \frac{\partial f_{1}^{02}}{\partial \underline{C}_{1}} + F_{11} \left[ f_{1M}^{00}(\underline{C}_{1}) f_{1}^{02}(\underline{C}_{1}^{\prime}) + f_{1}^{02}(\underline{C}_{1}) f_{1M}^{00}(\underline{C}_{1}^{\prime}) \right] + D_{1}^{0} \left[ f_{1M}^{02} \right] .$$

Here we have used Landau's form for the tensor  $\Phi^{12}$  (cf.p. 30) To prove existence of solutions of Eq.(77) whose left hand side has terms of zeroth, first and higher order anistropy in velocity space, we may follow the lines of [11]. We also need the mass transport equation to zeroth order in  $\varepsilon$  and third in  $\alpha$ ,

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{0}^{\circ 3}}{\partial t_{20}} + \frac{\partial \underline{c}_{0}^{\circ 2}}{\partial t_{21}} + \frac{\partial \underline{c}_{0}^{\circ 1}}{\partial t_{22}} + \frac{\partial \underline{c}_{0}^{\circ \circ}}{\partial t_{23}}\right) = \rho_{e}^{\circ} \underline{c}_{0}^{\circ 1} \times \underline{H} +$$
$$+ n_{1}^{\circ} e_{1} \underline{\overline{c}}_{1}^{\circ 2} \times \underline{H} + n_{2}^{\circ} e_{2} \underline{\overline{c}}_{2}^{\circ 1} \times \underline{H}$$
(78)

Integrating on the  $\tau_{20}$  time scale, eliminating secular terms, using that  $\overline{\underline{C}}_2^{01} \rightarrow 0$  as  $t_{20} \rightarrow \infty$  according to the a priori condition and assuming

$$\frac{\partial \underline{c}_{0}^{02}}{\partial t_{21}}(t_{20} = \infty) = \frac{\partial \underline{c}_{0}^{02}}{\partial t_{21}}(t_{20}, t_{21}, \ldots) = \frac{\partial \underline{c}_{0}^{02}}{\partial t_{21}}(t_{20} = 0, t_{21}, \ldots)$$

we get

$$\frac{c_{0}^{03}(t_{20}, t_{21}, \ldots) = \underline{c}_{0}^{03}(t_{20}=0, t_{21}, \ldots) + \\ + \int_{0}^{t_{20}} d\tau \left[ \frac{\partial}{\partial t_{22}} \left( \underline{c}_{0}^{01}(t_{20}=\infty) - \underline{c}_{0}^{01}(\tau) \right) + \frac{\rho_{e}^{0}}{\rho^{0}} \left( \underline{c}_{0}^{01}(\tau) - \underline{c}_{0}^{01}(\tau) \right) + \frac{\rho_{e}^{0}}{\rho^{0}} \left( \underline{c}_{0}^{01}(\tau) - \underline{c}_{0}^{01}(\tau) - \underline{c}_{0}^{01}(\tau) - \underline{c}_{0}^{02}(\tau) - \underline{c}_{0}^{02}(\tau) - \underline{c}_{1}^{02}(\tau) - \underline{c}_{1}^$$

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Integrating on the T<sub>20</sub> vine 52215, shininging couldr' terms, using that  $\underline{d}_{2}^{(1)} \rightarrow 0$  as  $\bar{v}_{20} \rightarrow v$  a addining to the s priori condition and assumed

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$$\rho^{\circ}\left(\frac{\partial \underline{c}_{0}^{\circ 2}}{\partial t_{21}}(t_{20}=\infty) + \frac{\partial \underline{c}_{0}^{\circ 1}}{\partial t_{22}}(t_{20}=\infty) + \frac{\partial \underline{c}_{0}^{\circ \circ}}{\partial t_{23}}\right) = \\ = \rho_{\underline{c}_{0}}^{\circ} \underline{c}_{0}^{\circ 1}(t_{20}=\infty) \times \underline{H} + n_{1}^{\circ} \underline{c}_{1} \underline{\overline{C}}_{1M}^{\circ 2} \times \underline{H}$$

Integrating here on the  $\tau_{21}$  time scale and eliminating secular terms again show that  $\underline{c}_{0}^{02}(t_{20}=\infty)$  (and from the above assumption  $\underline{c}_{0}^{02}(t_{20}, \ldots)$  and  $\underline{c}_{0}^{02}(t_{20}=0)$ ) is independent of  $t_{21}$  and  $\underline{c}_{0M}^{01}$  obeys

$$\rho^{\circ} \frac{\partial \underline{c}_{\circ M}^{\circ 1}}{\partial t_{22}} = -\rho^{\circ} \frac{\partial \underline{c}_{\circ}^{\circ \circ}}{\partial t_{23}} + \rho_{\underline{e}_{\circ M}}^{\circ} \times \underline{H} + n_{1}^{\circ} e_{1} \overline{\underline{c}}_{1 M}^{\circ \circ} \times \underline{H}$$
(79)

 $\overline{\underline{C}}_{1M}^{002} \text{ we find from the solution of Eq.(77), and the} \\ expression depends on <math>\underline{\underline{c}}_{0}^{01}$  in the same way as  $\overline{\underline{C}}_{1M}^{01}$  depends on  $\underline{\underline{c}}_{0}^{00}$ . Consequently the "homogeneous" equation for  $\underline{\underline{c}}_{0}^{01}$  corresponding to Eq.(79) is the same as the equation for  $\underline{\underline{c}}_{0}^{00}$  and (using for  $\underline{\underline{C}}_{1}^{02}$  an expression corresponding to Eq.(63))therefore has solutions proportional to  $e^{\pm i\omega_{\pm}t_{22}}$ . The inhomogeneous term vary like  $e^{\pm i\omega_{\pm}t_{22}}$  (the term  $\frac{\underline{\partial}\underline{\underline{c}}_{0}^{00}}{\underline{\partial}t_{23}}$ ). To avoid  $\underline{\underline{c}}_{0M}^{01}$  to vary like  $t_{22}e^{\pm i\omega_{\pm}t_{22}}$  (which is secular compared to  $\underline{\underline{c}}_{0M}^{00}$ ) we set  $\frac{\underline{\partial}\underline{\underline{c}}_{0}^{00}}{\underline{\partial}t_{23}} = 0$ 

 $c_{\rm oM}^{\rm o1}$  is now damped away like  $c_{\rm o}^{\rm oo}$  on the  $\tau_{22}$ -time scale.



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We now expand some equations to first order in  $\epsilon$  in terms of the parameter  $\alpha$  .

Equations to first order in  $\epsilon$  and zeroth order in  $\alpha$  are

$$\int d\underline{C}_2 f_2^{10} m_2 \underline{C}_2 = 0 \tag{80}$$

$$\frac{\partial f_{1}^{10}}{\partial t_{20}} + \frac{\partial f_{1}^{00}}{\partial t_{30}} + \underline{C}_{1} \cdot \frac{\partial f_{1}^{00}}{\partial \underline{r}_{1}} + \underline{F}_{1} \cdot \frac{\partial f_{1}^{00}}{\partial \underline{C}_{1}} - \frac{\partial \underline{c}_{0}^{10}}{\partial t_{20}} \cdot \frac{\partial f_{1}^{00}}{\partial \underline{C}_{1}} + \frac{e_{1}}{m_{1}} \underline{c}_{0}^{10} \times \\ \times \underline{H} \cdot \frac{\partial f_{1}^{00}}{\partial \underline{C}_{1}} + \frac{e_{1}}{m_{1}} \underline{C}_{1} \times \underline{H} \cdot \frac{\partial f_{1}^{10}}{\partial \underline{C}_{1}} = FP_{11} \Big[ f_{1}^{00} (\underline{C}_{1}) f_{1}^{10} (\underline{C}_{1}') + \qquad (81) \\ + f_{1}^{10} (\underline{C}_{1}) f_{1}^{00} (\underline{C}_{1}') \Big] + D_{1}^{00} \Big[ f_{1}^{10} \Big] + D_{1}^{10} \Big[ f_{1}^{00} \Big] \\ \text{where} \qquad D_{1}^{\underline{i}\underline{j}} = \frac{n_{\underline{2}}^{\underline{i}\underline{j}}}{m_{\underline{1}}^{2}} \frac{\partial}{\partial \underline{C}_{1}} \cdot \left( \Phi^{12} (\underline{C}_{1}) \cdot \frac{\partial}{\partial \underline{C}_{1}} \right) \quad ,$$

$$\frac{\partial \rho^{10}}{\partial t_{20}} + \frac{\partial \rho^{0}}{\partial t_{30}} = 0$$
(82)

$$\rho^{\circ} \frac{\partial \underline{c}_{\circ}^{1 \circ}}{\partial t_{2 \circ}} = 0 \tag{83}$$

$$\frac{3}{2}n^{\circ}k\left(\frac{\partial T^{1\circ}}{\partial t_{2\circ}} + \frac{\partial T^{\circ\circ}}{\partial t_{3\circ}}\right) = \frac{3}{2}kT^{\circ\circ}\frac{\partial}{\partial \underline{r}} \cdot (n_{1}^{\circ}\underline{\underline{C}}_{1}^{\circ\circ}) + \rho_{1}^{\circ}\underline{\underline{F}}_{1}\cdot\underline{\underline{C}}_{1}^{\circ\circ} + n_{1}^{\circ}\underline{\underline{F}}_{1}\cdot\underline{\underline{C}}_{1}^{\circ\circ} + n_{1}^{\circ}\underline{\underline{C}}_{1}^{\circ\circ}\cdot(\underline{\underline{c}}_{0}^{\circ\circ}\times\underline{\underline{H}}) - \frac{\partial}{\partial \underline{r}}\cdot\underline{q}_{1}^{\circ\circ}$$

$$(84)$$

Ve now expand some equations to filter order fin e:

Equations to first order in a and search order in a are:

$$\int \frac{dg_2}{2} g_2^2 e^{-\alpha_2} g_2 = 0$$
 (85)

$$0 = \frac{906}{900} + \frac{9166}{900}$$

(83) (83)

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$$\frac{\partial \rho_1^{1\circ}}{\partial t_{2\circ}} + \frac{\partial \rho_1^{\circ}}{\partial t_{3\circ}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^{\circ} \underline{\overline{c}}_{1\circ}^{\circ\circ}) = 0$$
(85)

$$\frac{\partial \rho_2^{10}}{\partial t_{20}} + \frac{\partial \rho_2^{0}}{\partial t_{30}} = 0$$
(86)

$$\frac{3}{2}n_{1}^{\circ}k\left(\frac{\partial T_{1}^{1\circ}}{\partial t_{2\circ}} + \frac{\partial T_{1}^{\circ\circ}}{\partial t_{3\circ}}\right) = \frac{3}{2}kT_{1}^{\circ\circ}\frac{\partial}{\partial \underline{r}}\cdot(n_{1}^{\circ}\underline{\overline{c}}_{1}^{\circ\circ}) + \rho_{1}^{\circ}\underline{F}_{1}\cdot\underline{\overline{c}}_{1}^{\circ\circ\circ} - \rho_{1}^{\circ}\underline{\overline{c}}_{1}^{\circ\circ\circ}\cdot\frac{\partial \underline{c}_{2\circ}^{1\circ}}{\partial t_{2\circ}} + n_{1}^{\circ}e_{1}\underline{\overline{c}}_{1}^{\circ\circ\circ}\cdot(\underline{c}_{2\circ}^{1\circ}\times\underline{H}) - \frac{\partial}{\partial \underline{r}}\cdot q_{1}^{\circ\circ\circ}$$

$$(87)$$

Eqs.(82)-(87) show that  $\rho^{10}$ ,  $\underline{c}_0^{10}$  and  $\rho_2^{10}$  are  $t_{20}$ independent while  $\rho_1^{10}$ ,  $T^{10}$  and  $T_1^{10}$  all have transients on the  $\tau_{20}$  time scale.  $\rho^0$ ,  $\rho_1^0$ ,  $\rho_2^0$ ,  $T^{00}$  and  $T_1^{00}$  are all independent of  $t_{30}$ . The electron kinetic equation Eq.(81) is of the form

$$\frac{\partial f_{1}^{10}}{\partial t_{20}} + \frac{e_{1}}{m_{1}} \underbrace{\underline{C}}_{1} \times \underline{\underline{H}} \cdot \frac{\partial f_{1}^{10}}{\partial \underline{C}_{1}} - \underline{FP}_{11} \Big[ f_{1M}^{00}(\underline{C}_{1}) f_{1}^{10}(\underline{C}_{1}^{1}) + f_{1}^{10}(\underline{C}_{1}) f_{1M}^{00}(\underline{C}_{1}^{1}) \Big] - \underline{D}_{1}^{00} \Big[ f_{1}^{10} \Big] = f_{1M}^{00} \Big[ \left( \frac{m_{1}C_{1}^{2}}{2kT_{1}^{00}} - \frac{5}{2} \right) \frac{1}{T_{1}^{00}} \frac{\partial T_{1}^{00}}{\partial \underline{r}} - \frac{e_{1}}{kT_{1}^{00}} \Big( \frac{m_{1}}{e_{1}} \underbrace{\underline{F}}_{1} + \underline{c}_{0}^{10} \times \underline{\underline{H}} - \frac{kT_{1}^{00}}{e_{1}} \frac{\partial}{\partial \underline{r}} \ell n \underline{p}_{1}^{00} \Big] \cdot \underline{\underline{C}}_{1} + \underline{G}^{10}(t_{20})$$
(88)

Here  $G^{10}(t_{20}) \rightarrow 0$  as  $t_{20} \rightarrow \infty$ . Using the appendixes of [5],

 $\frac{662}{662} + \frac{662}{662} = 0$ (660)

[7] and [12] we find that every bounded solution of this equation evolves toward the solution of Eq.(59) as  $t_{20} \rightarrow \infty$  with obvious changes of superscripts and  $T_1$  in place of T.

Equations to first order in  $\varepsilon$  and first in  $\alpha$  are, using results obtained above,

$$\int d\underline{C}_{1} f_{1}^{10} m_{1} \underline{C}_{1} + \int d\underline{C}_{2} f_{2}^{11} m_{2} \underline{C}_{2} = 0$$
(89)

$$\frac{\partial f_2^{10}}{\partial t_{20}} + \frac{\partial f_2^{00}}{\partial t_{30}} - \left(\frac{\partial \underline{c}_0^{11}}{\partial t_{20}} + \frac{\partial \underline{c}_0^{10}}{\partial t_{21}} + \frac{\partial \underline{c}_0^{00}}{\partial t_{30}}\right) \cdot \frac{\partial f_2^{10}}{\partial \underline{c}_2} = 0$$
(90)

$$\frac{\partial \rho^{11}}{\partial t_{20}} + \frac{\partial \rho^{10}}{\partial t_{21}} + \frac{\partial \rho^{0}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho^{0} \underline{c}_{0}^{00}) = 0$$
(91)

$$\rho^{O}\left(\frac{\partial \underline{c}_{0}^{11}}{\partial t_{20}} + \frac{\partial \underline{c}_{0}^{10}}{\partial t_{21}} + \frac{\partial \underline{c}_{0}^{00}}{\partial t_{30}}\right) = 0$$
(92)

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(60)

 $(\hat{s}_{2})) = -\left(\left(\frac{3}{3}, \frac{3}{3}, \frac{3}{3},$ 

$$\frac{\partial \rho_1^{11}}{\partial t_{20}} + \frac{\partial \rho_1^{10}}{\partial t_{21}} + \frac{\partial \rho_1^{0}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^0 \underline{c}_0^{00}) + \frac{\partial}{\partial \underline{r}} \cdot (\rho_1^0 \underline{\overline{c}}_1^{01}) = 0$$
(94)

$$\frac{\partial \rho_2^{11}}{\partial t_{20}} + \frac{\partial \rho_2^{20}}{\partial t_{21}} + \frac{\partial \rho_2^{0}}{\partial t_{31}} + \frac{\partial}{\partial r} \cdot (\rho_2^0 c_0^{00}) = 0$$
(95)

$$\frac{3}{2}n_{1}^{0}k\left(\frac{\partial T_{1}^{11}}{\partial t_{20}} + \frac{\partial T_{1}^{10}}{\partial t_{21}} + \frac{\partial T_{1}^{01}}{\partial t_{30}} + \frac{\partial T_{1}^{00}}{\partial t_{31}} + \frac{\partial T_{1}^{00}}{\partial t_{31}} + \frac{\partial T_{1}^{00}}{\partial t_{31}} + \frac{\partial T_{1}^{00}}{\partial t_{20}}\right) + \\ + \frac{3}{2}n_{1}^{10}k\frac{\partial T_{1}^{01}}{\partial t_{20}} = \frac{3}{2}kT_{1}^{01}\frac{\partial}{\partial t_{21}} \cdot (n_{1}^{0}\overline{c}_{1}^{00}) + \frac{3}{2}kT_{1}^{00}\frac{\partial}{\partial t_{21}} \cdot (n_{1}^{0}\overline{c}_{1}^{01}) + \\ + \rho_{1}^{0}\underline{F}_{1}\cdot\overline{C}_{1}^{01} + n_{1}^{0}e_{1}\overline{C}_{1}^{00}\cdot(\underline{c}_{0}^{11}\times\underline{H}) + n_{1}^{0}e_{1}\overline{C}_{1}^{01}\cdot(\underline{c}_{0}^{10}\times\underline{H}) + \\ + n_{1}^{0}e_{1}\overline{C}_{1}^{10}\cdot(\underline{c}_{0}^{00}\times\underline{H}) + n_{1}^{10}e_{1}\overline{C}_{1}^{00}\cdot(\underline{c}_{0}^{00}\times\underline{H}) - \frac{\partial}{\partial t}\underline{c}_{1}^{01} - \\ - \rho_{1}^{0}\overline{C}_{1}\overline{C}_{1}^{00}\cdot(\underline{c}_{0}^{00}\cdot\underline{H}) + n_{1}^{0}e_{1}\overline{C}_{1}^{00}\cdot(\underline{c}_{0}^{00}\times\underline{H}) - \frac{\partial}{\partial t}\underline{c}_{0}^{00}$$

$$\frac{3}{2}n_{2}^{\circ}k\left(\frac{\partial T_{2}^{1\circ}}{\partial t_{2\circ}} + \frac{\partial T_{2}^{\circ\circ}}{\partial t_{3\circ}}\right) = 0$$
(97)

These equations show that  $f_2^{10}$ ,  $\rho^{11}$ ,  $\rho_2^{11}$ ,  $\underline{c}_0^{11}$  and  $T_2^{10}$  are all independent of  $t_{20}$ , while  $\rho_1^{11}$ ,  $T^{11}$  and  $T_1^{11}$  all have a  $t_{20}$  transient.  $\underline{c}_0^{10}$  is independent of  $t_{21}$  and  $f_2^{00}$ ,  $\underline{c}_0^{00}$  and  $T_2^{00}$  are independent of  $t_{30}$ . The variations on the  $\tau_{21}$  - and  $\tau_{22}$  time scales in Eq.(91) and Eqs. (93) - (96) may be absorbed by

 $\frac{32}{5} = \left( \frac{32}{375} + \frac{32}{575} \right) = 0$ (97)

 $\rho^{10}$ ,  $\rho_1^{10}$ ,  $\rho_2^{10}$ ,  $T_1^{10}$  and  $T^{10}$  (using that  $t_{22} = \alpha t_{21}$ ), for instance

$$\rho_{1}^{1\circ}(t_{20}, t_{21}, \dots) = \rho_{1}^{1\circ}(t_{20}=0, t_{21}=0) - \int_{0}^{t_{20}} \frac{\partial}{\partial \underline{r}} \cdot (\rho_{1}^{0}\underline{\overline{c}}_{1}^{0\circ}(\tau))d\tau - \int_{0}^{t_{21}} \frac{\partial}{\partial \underline{r}} \cdot \left[\rho_{1}^{0}\underline{\underline{c}}_{0}^{0\circ}(\alpha\lambda) + \rho_{1}^{0}\underline{\overline{c}}_{1}^{0\circ1}(\alpha\lambda)\right] d\lambda$$

the first integral comming from the equation to first order in  $\varepsilon$  and zeroth in  $\alpha$ . The term  $\rho_1^{10}(t_{20}=0,t_{21}=0)$  may vary on the  $\tau_{22}$  time scale. In the limits  $t_{20} = \infty$ ,  $t_{21} = \infty$  and  $t_{22} = \infty$  we obtain the equations

$$\frac{\partial \rho^{\circ}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot \left(\rho^{\circ} \underline{c}^{\circ \circ}_{\circ \parallel}\right) = 0$$
(98)

$$\frac{\partial \rho_1^0}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_1^0 \underline{c}_0^{00} \right) = 0$$
(99)

$$\frac{\partial \rho_2^{\circ}}{\partial t_{31}} + \frac{\partial}{\partial \underline{r}} \cdot \left( \rho_2^{\circ} \underline{c}_{\circ}^{\circ \circ} \right) = 0$$
(100)

$$\frac{3}{2}n^{\circ}k\left(\frac{\partial T^{\circ\circ}}{\partial t_{31}} + \underline{c}^{\circ\circ}_{\circ}\right) \cdot \frac{\partial T^{\circ\circ}}{\partial \underline{r}} = - Ip^{\circ\circ} : \frac{\partial \underline{c}^{\circ}_{\circ}}{\partial \underline{r}}$$
(101)

the first integral comming free the coustion to first order. In a sund zeroth in a, The term  $\rho_1^{(0)}(t_{00}^{-1}0, t_{01}^{-1}0)$  may rary on the  $t_{02}$  time totale. In the the life  $t_{00} = 0$ ,  $t_{00} = 0$ ,  $t_{01} = 0$ , and  $t_{02} = 0$  we obtain the the dimetions

 $(99) = \left(\frac{1}{20}, \frac{1}{20}, \frac{1}{$ 

$$\frac{392}{353}$$
 +  $\frac{3}{32}$  ( $9229$ ) = 0

- 48 -

$$\frac{3}{2}n_{1}^{\circ}k\left(\frac{\partial T_{1}^{\circ\circ}}{\partial t_{31}} + \underline{c}_{\circ}^{\circ\circ}\right) \cdot \frac{\partial T_{1}^{\circ\circ}}{\partial \underline{r}} = - Ip_{1}^{\circ\circ} : \frac{\partial \underline{c}_{\circ}^{\circ\circ}}{\partial \underline{r}}$$
(102)

Eqs.(98) - (101) correspond to Eqs.(49) - (51)

Equations to <u>first order in  $\varepsilon$  and second in  $\alpha$ </u> are  $\frac{\partial f_{2}^{11}}{\partial t_{20}} + \frac{\partial f_{2}^{10}}{\partial t_{21}} + \frac{\partial f_{2}^{01}}{\partial t_{30}} + \frac{\partial f_{2}^{00}}{\partial t_{31}} + \frac{c_{00}}{c_{0}} \cdot \frac{\partial f_{2}^{00}}{\partial \underline{r}} + \underline{c}_{2} \cdot \frac{\partial f_{2}^{00}}{\partial \underline{r}} + \underline{F}_{2} \cdot \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial c_{0}^{11}}{\partial \underline{c}_{20}} + \frac{\partial c_{0}^{10}}{\partial \underline{t}_{20}} + \frac{\partial c_{0}^{01}}{\partial \underline{t}_{30}} + \frac{\partial c_{0}^{00}}{\partial \underline{t}_{31}} + \underline{c}_{0}^{00} \cdot \frac{\partial c_{0}^{00}}{\partial \underline{r}} - \frac{\partial c_{0}^{00}}{\partial \underline{r}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} + \frac{\partial c_{0}^{11}}{\partial \underline{c}_{2}} + \frac{\partial c_{0}^{00}}{\partial \underline{t}_{30}} + \frac{\partial c_{0}^{00}}{\partial \underline{t}_{31}} + \underline{c}_{0}^{00} \cdot \frac{\partial c_{0}^{00}}{\partial \underline{r}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} + \frac{\partial c_{0}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} + \frac{\partial f_{2}^{00}}{\partial \underline{c}_{2}} - \frac{\partial f_{2}^{00}}{$ 

$$\rho^{-}\left(\frac{\partial t_{20}}{\partial t_{20}} + \frac{\partial t_{21}}{\partial t_{21}} + \frac{\partial t_{22}}{\partial t_{22}} + \frac{\partial t_{30}}{\partial t_{30}} + \frac{\partial t_{31}}{\partial t_{31}} + \frac{c_{00}}{c_{0}} \cdot \frac{-0}{\partial \underline{r}}\right) +$$

$$+ \rho^{10} \frac{\partial \underline{c}_{0}^{01}}{\partial t_{20}} = - \frac{\partial}{\partial \underline{r}} \cdot \sum_{i=1}^{2} n_{i}^{0} \underline{m}_{i} \overline{\underline{C}_{i}} \overline{\underline{C}_{0}}^{00} + \rho_{\underline{e}}^{0} \underline{c}_{0}^{10} \times \underline{H} + n_{1}^{0} \underline{e}_{1} \overline{\underline{C}_{1}}^{10} \times \underline{H} +$$

$$+ n_{1}^{10} \underline{e}_{1} \overline{\underline{C}_{1}}^{00} \times \underline{H} + \sum_{i=1}^{2} \rho_{i}^{0} \underline{F}_{i}$$

$$(104)$$



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$$\frac{3}{2}n_{2}^{\circ}k\left(\frac{\partial T_{2}^{11}}{\partial t_{20}} + \frac{\partial T_{2}^{10}}{\partial t_{21}} + \frac{\partial T_{2}^{\circ 1}}{\partial t_{30}} + \frac{\partial T_{2}^{\circ \circ}}{\partial t_{31}} + \frac{\partial T_{2}^{\circ \circ}}{\partial t_{31}} + \frac{\partial T_{2}^{\circ \circ}}{\partial \underline{r}}\right) =$$

$$= -\frac{\partial}{\partial \underline{r}} \cdot \underline{q}_{2}^{\circ \circ} - \rho_{2}^{\circ} \underline{c}_{2} \underline{c}_{2}^{\circ \circ} : \frac{\partial \underline{c}_{0}^{\circ \circ}}{\partial \underline{r}} \qquad (105)$$

The parallel part of Eq. (104) is

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{0}^{12}}{\partial t_{20}} + \frac{\partial \underline{c}_{0}^{11}}{\partial t_{21}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{22}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{31}} + \underline{c}_{0}^{\circ\circ} \cdot \frac{\partial}{\partial \underline{r}} \underline{c}_{0}^{\circ}\right) =$$

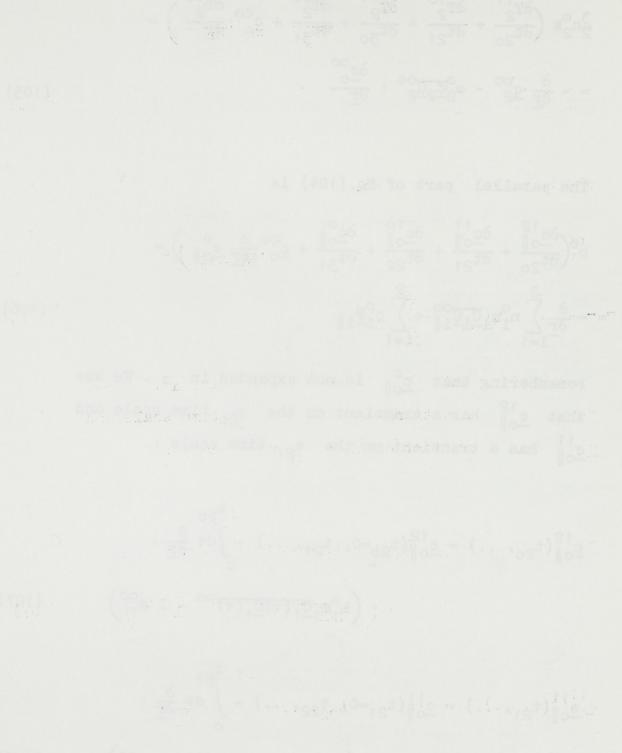
$$= -\frac{\partial}{\partial \underline{r}} \cdot \sum_{i=1}^{2} n_{i}^{\circ} \underline{m}_{i} \cdot \overline{\underline{c}_{0}^{\circ}} + \sum_{i=1}^{2} \rho_{i}^{\circ} \underline{F}_{i} \parallel$$

$$(106)$$

remembering that  $\underline{c}_{0\parallel}^{0}$  is not expanded in  $\alpha$ . We see that  $\underline{c}_{0\parallel}^{12}$  has a transient on the  $\tau_{20}$  time scale and  $\underline{c}_{0\parallel}^{11}$  has a transient on the  $\tau_{21}$  time scale :

$$\underline{c}_{0\parallel}^{12}(t_{20}, \ldots) = \underline{c}_{0\parallel}^{12}(t_{20}=0, t_{21}, \ldots) - \int_{0}^{t_{20}} d\tau \frac{\partial}{\partial \underline{r}} \cdot \left( n_{1}^{0} m_{1} \overline{\underline{c}_{1}(\tau)} \underline{c}_{1}(\tau)^{00} - \mathbf{I} p_{1}^{00} \right)$$
(107)

$$\underline{\mathbf{c}}_{0}^{11}(\mathbf{t}_{21}, \ldots) = \underline{\mathbf{c}}_{0}^{11}(\mathbf{t}_{21}=0, \mathbf{t}_{22}, \ldots) - \int_{0}^{\mathbf{t}_{21}} d\lambda \, \frac{\partial}{\partial \underline{\mathbf{r}}} \cdot \left( n_{2}^{0} m_{2} \overline{\underline{\mathbf{c}}_{2}}(\overline{\lambda}) \underline{\mathbf{c}}_{2}(\overline{\lambda})^{00} - \mathbf{I} \mathbf{p}_{2}^{00} \right)$$
(108)



In the limits  $t_{20} = \infty$  and  $t_{21} = \infty$  Eq.(106) reduces to

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{22}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{31}} + \underline{c}_{0}^{\circ\circ} \cdot \frac{\partial}{\partial \underline{r}} \underline{c}_{0}^{\circ}\right) = -\sum_{i=1}^{2} \frac{\partial p_{i}^{\circ\circ}}{\partial \underline{r}_{\parallel}} + \sum_{i=1}^{2} \rho_{i}^{\circ} \underline{F}_{i\parallel}$$
(109)

As  $t_{22} \rightarrow \infty$ ,  $\underline{c}_{01}^{00} \rightarrow 0$  and  $\underline{T}_{1}^{00} \rightarrow \underline{T}^{00}$ . Due to this  $\underline{c}_{0\parallel}^{10}$  has a transient on the  $\tau_{22}$  time scale and in the limit  $t_{22} = \infty$  Eq.(109) becomes

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{\circ}^{\circ}}{\partial t_{31}} + \underline{c}_{\circ}^{\circ}\|, \frac{\partial}{\partial \underline{r}} \underline{c}_{\circ}^{\circ}\|\right) = -\frac{\partial p^{\circ\circ}}{\partial \underline{r}} + \sum_{i=1}^{2} \rho_{i}^{\circ} \underline{F}_{i}\|$$
(110)

which corresponds to Eq. (52).

The transverse part of Eq.(104) shows that  $c_{0\perp}^{12}$  has a transient on the  $\tau_{20}$  time scale and  $c_{0\perp}^{11}$  a transient on the  $\tau_{21}$  time scale. In the limits  $t_{20} = \infty$  and  $t_{21} = \infty$  the transverse part is

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{01}^{1\circ}}{\partial t_{22}} + \frac{\partial \underline{c}_{01}^{\circ\circ1}}{\partial t_{30}} + \frac{\partial \underline{c}_{01}^{\circ\circ\circ}}{\partial t_{31}} + \underline{c}_{0}^{\circ\circ} \cdot \frac{\partial}{\partial \underline{r}} \underline{c}_{01}^{\circ\circ}\right) = -\sum_{i=1}^{2} \frac{\partial p_{i}^{\circ\circ}}{\partial \underline{r}_{\perp}} + \rho_{\underline{i}}^{\circ\circ} + \rho_{\underline{i}}^{\circ\circ} \underline{c}_{1}^{\circ\circ} + \frac{\partial \underline{c}_{01}^{\circ\circ}}{\partial \underline{r}_{\perp}} + \rho_{\underline{i}}^{\circ\circ} \underline{c}_{1}^{\circ\circ} \times \underline{H} + n_{1}^{\circ} \underline{c}_{1}^{\overline{c}_{1}^{\circ\circ}} \times \underline{H} + \sum_{\underline{i}=1}^{2} \rho_{\underline{i}}^{\circ} \underline{F}_{\underline{i}}$$

$$(111)$$

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 $\overline{C}_{1}^{1\circ}$  we find from Eq.(88) in the limit  $t_{2\circ} = \infty$ . Thus  $\overline{C}_{1}^{1\circ}$  depends on  $\underline{c}_{0}^{1\circ}$  in the same way as  $\overline{C}_{1}^{\circ 1}$  depends on  $\underline{c}_{0}^{\circ \circ}$  or  $\overline{C}_{1}^{\circ 2}$  depends on  $\underline{c}_{0}^{\circ 1}$ , and therefore the "homogeneous" equation corresponding to Eq.(111) for  $\underline{c}_{0}^{1\circ}$ is the same as the equation for  $\underline{c}_{0}^{\circ \circ}$ . The inhomogeneous term of Eq.(111) for  $\underline{c}_{0}^{1\circ}$  tends toward a nonzero limit as  $t_{22} \rightarrow \infty$ . Using for  $\overline{C}_{1}^{1\circ}$  a simplified expression corresponding to Eq.(63) and for  $\underline{c}_{0}^{\circ \circ}$  and  $\underline{c}_{0}^{\circ 1}$ expressions obtained from Eqs.(69) and (79) in the same way, we easily solve Eq.(111) and find that  $\underline{c}_{0}^{1\circ}$  evolves toward a limit as  $t_{22} \rightarrow \infty$  which we can find setting the right hand side of Eq.(111) in the limit  $t_{22} = \infty$ equal to zero, i.e.

$$-\sum_{i=1}^{2} \frac{\partial p_{i}^{\circ\circ}}{\partial \underline{r}_{\perp}} + \rho_{\underline{e}\underline{c}}^{\circ\circ} \times \underline{H} + n_{1}^{\circ} e_{1} \overline{\underline{C}}_{1}^{1\circ} \times \underline{H} + \sum_{i=1}^{2} \rho_{\underline{i}\underline{F}}^{\circ} \underline{F}_{\underline{i}} = 0 \qquad (112)$$

So, if further  $\underline{c}_{01}^{02}$ ,  $\underline{c}_{01}^{03}$  etc. like  $\underline{c}_{01}^{00}$  and  $\underline{c}_{01}^{01}$ all tend to zero as  $t_{22} \rightarrow \infty$  we have shown that for bounded distribution functions,  $\underline{c}_{01}$  up to terms of order  $\varepsilon$  evolves so that balance of transverse forces is established as  $t_{22} \rightarrow \infty$ , using simplified expressions for  $\underline{\overline{c}}_{1}^{01}$ ,  $\underline{\overline{c}}_{1}^{02}$  etc. However, this result is expected to hold using for  $\underline{\overline{c}}_{1}^{01}$ ,  $\underline{\overline{c}}_{1}^{02}$  etc. exact expressions obtained from the solution of Eqs.(68), (77) etc.

From Eq.(105) we see that  $T_2^{11}$  is independent of  $t_{20}$ while  $T_2^{10}$  may absorbe, apart from the variation on the  $\tau_{21}$  time scale, also the variations on the  $\tau_{22}$ time scale. In the limits  $t_{20} = \infty$ ,  $t_{21} = \infty$  and  $t_{22} = \infty$  we obtain

$$\frac{3}{2}n_{2}^{\circ}k\left(\frac{\partial T_{2}^{\circ\circ}}{\partial t_{31}} + \underline{c}_{\circ}^{\circ}\| \cdot \frac{\partial T_{2}^{\circ\circ}}{\partial \underline{r}}\right) = -Ip_{2}^{\circ\circ} : \frac{\partial \underline{c}_{\circ}^{\circ}\|}{\partial \underline{r}}$$
(113)

which when added to Eq.(102) corresponds to Eq.(51) .

Eq.(103) shows that  $f_2^{11}$  has a transient on the  $\tau_{20}$  time scale and substituting from Eq.(39) to second order in  $\alpha$  for the terms in paranteses on the left hand side of Eq.(103) and also from Eqs.(100) and (113) we derive that in the limit  $t_{20} = \infty$  Eq.(103) reduces to

$$\frac{\partial f_{2}^{10}}{\partial t_{21}^{2}} - FP_{22} \left[ f_{2M}^{00}(\underline{c}_{2}) f_{2}^{10}(\underline{c}_{2}^{\prime}) + f_{2}^{10}(\underline{c}_{2}) f_{2M}^{00}(\underline{c}_{2}^{\prime}) \right] =$$

$$= - f_{2M}^{00} \left( \frac{m_{2}c_{2}^{2}}{2kT^{00}} - \frac{5}{2} \right) \frac{1}{T^{00}} \frac{\partial T^{00}}{\partial \underline{r}} \cdot \underline{c}_{2} - f_{2M}^{00} \frac{m_{2}}{kT^{00}} \underline{c}_{2}^{0} \underline{c}_{2} \right] : (114)$$

$$: \frac{\partial \underline{c}_{0}^{0}}{\partial \underline{r}} + H^{10}(t_{21}, t_{22})$$

where  $H^{10}(t_{21}, t_{22}) \rightarrow 0$  as  $t_{21} \rightarrow \infty$  and  $t_{22} \rightarrow \infty$ .

weigh when added to Eq. (402) corresponds to Eq. (51) .

Eq.(4.03) shows that  $f_2^{(1)}$  has a transfort on the  $f_{20}$  time scale and substituting from Eq.(39) to second order is a for the terms in paractases on the left hand alds of Eq.(103) and also from Eq.(100) and (113) we derive that is the limit  $t_{20} = \pi$  Eq.(403) reduces to

Again using the appendix of [12], every bounded solution of Eq.(114) evolves toward the solution of

$$FP_{22} \left[ f_{2M}^{00}(\underline{C}_{2}) f_{2}^{10}(\underline{C}_{2}') + f_{2M}^{10}(\underline{C}_{2}) f_{2M}^{00}(\underline{C}_{2}') \right] =$$

$$= f_{2M}^{00} \left( \frac{m_{2}C_{2}^{2}}{2kT^{00}} - \frac{5}{2} \right) \frac{1}{T^{00}} \frac{\partial T^{00}}{\partial \underline{r}} \cdot \underline{C}_{2} + f_{2M}^{00} \frac{m_{2}}{kT^{00}} \underline{C}_{2}^{0} \underline{C}_{2} : \frac{\partial \underline{C}_{0}^{0}}{\partial \underline{r}} \right]$$

as  $t_{21} \rightarrow \infty$  and  $t_{22} \rightarrow \infty$ , the  $\partial/\partial t_{21}$  of Eq.(114) also taking account of the variation on the  $\tau_{22}$  time scale. This result for the ion distribution function to first order in  $\varepsilon$  and zeroth in  $\alpha$  corresponds to the result of Eq.(60) in the same way as Eq.(88) for the electron distribution function to the same order in the limits  $t_{20} = \infty$ ,  $t_{21} = \infty$  and  $t_{22} = \infty$ , corresponds to Eq.(59).

This ends our partial derivation of assumptions Eqs.(53) and (54).

Again using the appendix of [12], every bounded solution at Eq. (114) evolves toward the solution of

 $\mathbb{P}_{23}\left[ \mathcal{L}_{2M}^{00}(\underline{0}_{2}) \mathcal{L}_{2}^{0}(\underline{0}_{2}) + \mathcal{L}_{2M}^{00}(\underline{0}_{2}) \mathcal{L}_{2M}^{00}(\underline{0}_{2}) \right] =$ 

we be the the and the reaction door of the the time sise testing account of the variation on the Tap time sealed This result for the ion distribution function to signat order in a and seroid in a corresponds to the readit of Eq. (60) in the same may as Eq. (38) for the electron distribution function to the same order in the hints to a corresponds

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### Appendix 1.

We here briefly derive the equations of Chapman and Cowling [1] from Eqs. (1)-(5) by use of the multiple time scale method.

The distribution functions and macroscopic quantities are expanded in terms of the parameter  $\ \epsilon_1$  , for instance

 $f_1 = f_1^{(0)} + \varepsilon_1 f_1^{(1)} + \varepsilon_1^2 f_1^{(2)} + \dots$ 

Later we leave out the paranteses in the superscripts. Also the time derivative is expanded in this way

 $\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon_1 \frac{\partial}{\partial t_1} + \varepsilon_1^2 \frac{\partial}{\partial t_2} + \cdots$ 

consistent with introduction of time variables  $t_i$  on time scales  $\tau_i = \frac{\tau_0}{\epsilon_1^i}$ ,  $i = 0, 1, 2, \dots$ , where  $\tau_0$ , roughly speeking, is the time scale on which the gas evolves toward local thermodynamic equilibrium.

## The equations to zeroth order are

$$\sum_{i} \rho_{i}^{\circ} \overline{\underline{C}}_{i}^{\circ} = 0$$
(A1)

$$\frac{\partial f_{i}^{\circ}}{\partial t_{o}} - \frac{\partial c_{o}^{\circ}}{\partial t_{o}} \cdot \frac{\partial f_{i}^{\circ}}{\partial \underline{C}_{i}} + \frac{e_{i}}{\underline{m}_{i}} \underbrace{\underline{C}}_{i} \times \underline{\underline{H}} \cdot \frac{\partial f_{i}}{\partial \underline{C}_{i}} = \sum_{j} C_{ij} \begin{bmatrix} f_{i}^{\circ} f_{j}^{\circ} \end{bmatrix}$$
(A2)

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$$\frac{\partial \rho_{i}^{\circ}}{\partial t_{\circ}} = 0 \tag{A3}$$

$$\rho^{\circ} \frac{\partial \underline{c}_{\circ}}{\partial t_{\circ}} = \underline{j}^{\circ} \times \underline{H}$$
 (A4)

$$\frac{3}{2}n^{\circ}k \frac{\partial T^{\circ}}{\partial t_{\circ}} = 0$$
 (A5)

A H - theorem can be established showing that

$$f_{i}^{\circ} \rightarrow n_{i}^{\circ} \left(\frac{m_{i}}{2\pi kT^{\circ}}\right)^{3/2} \exp\left(-\frac{m_{i}C_{i}^{2}}{2kT^{\circ}}\right) \text{ as } t_{\circ} \rightarrow \infty$$
 (A6)

Thus  $\underline{c}_{0}^{\circ}$  transverse to  $\underline{H}$ ,  $\underline{c}_{0\perp}^{\circ}$ , obeys  $\frac{\partial \underline{c}_{0\perp}^{\circ}}{\partial t_{0}} \rightarrow 0$  as  $t_{0} \rightarrow \infty$ , or  $\underline{c}_{0\perp}^{\circ}$  has a transient on the  $\tau_{0}$  time scale,

$$\underline{c}_{0}^{0}(t_{0}, \ldots) = \underline{c}_{0}^{0}(t_{0}=0, t_{1}, \ldots) + \frac{1}{\rho_{0}^{0}} \int_{0}^{0} \underline{j}^{0}(\tau) \times \underline{H} d\tau$$

$$\rho_{1}^{0} \text{ and } T^{0} \text{ are independent of } t_{0}.$$

The equations to first order are (they correspond to the second order equations of Chapman and Cowling)

$$\sum_{i} \rho_{i}^{O} \, \overline{\underline{C}}_{i}^{1} + \sum_{i} \rho_{i}^{1} \, \overline{\underline{C}}_{i}^{O} = 0 \qquad (A7)$$

- 55 -

(A3)

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A II - theorem can be establighted showing binkt

$$r_{1}^{\circ} \rightarrow n_{1}^{\circ} \left(\frac{m_{1}}{2\pi \kappa_{1}^{\circ}}\right)^{3/2} \exp\left(-\frac{m_{1}^{\circ}c_{1}^{2}}{2\kappa_{1}^{\circ}}\right) \quad as \quad t_{o} \rightarrow \infty$$
(A6)

$$\sum_{i=1}^{n} \left\{ \mathbf{t}_{0} \cdot \mathbf{t}_{i} \right\} = \sum_{i=1}^{n} \left\{ \mathbf{t}_{0} = 0, \mathbf{t}_{1} \cdot \mathbf{t}_{i} \right\} + \frac{1}{p_{0}} \int_{0}^{n} \sum_{i=1}^{n} \left\{ \mathbf{t}_{i} \right\} \times \mathbf{H} \times \mathbf{t}_{i}$$

The equations to first order are (they converpond to the second order equations of Chapman and Conding)

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$$\frac{\partial f_{i}^{1}}{\partial t_{o}} + \frac{\partial f_{i}^{0}}{\partial t_{1}} + \underline{c}_{o}^{0} \cdot \frac{\partial f_{i}^{0}}{\partial \underline{r}} + \underline{c}_{i} \cdot \frac{\partial f_{i}^{0}}{\partial \underline{r}} + \underline{f}_{i} \cdot \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} + \frac{e_{i}}{\underline{m}_{i}} \underline{c}_{o}^{0} \times \underline{H} \cdot \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} - \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} - \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} - \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} - \frac{\partial c_{o}^{0}}{\partial \underline{c}_{i}} - \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} + \frac{\partial c_{o}^{0}}{\partial \underline{c}_{i}} + \frac{\partial c_{o}^{0}}{\partial \underline{c}_{i}} - \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} + \frac{\partial c_{o}^{0}}{\partial \underline{c}_{i}} + \frac{\partial c_{o}^{0}}{\partial \underline{c}_{i}} + \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} + \frac{e_{i}}{\underline{m}_{i}} \underline{c}_{o}^{0} \times \underline{H} \cdot \frac{\partial f_{i}^{0}}{\partial \underline{c}_{i}} + \frac{e_{i}}{\underline{c}_{i}} + \frac{e_{i}}{\underline{c}} + \frac{e_{i}}{\underline{c}_$$

$$\frac{\partial \rho_{i}^{1}}{\partial t_{0}} + \frac{\partial \rho_{i}^{0}}{\partial t_{1}} + \frac{\partial}{\partial \underline{r}} \cdot (\rho_{i}^{0} \underline{c}^{0}) + \frac{\partial}{\partial \underline{r}} \cdot (\rho_{i}^{0} \overline{\underline{c}}^{0}) = 0$$
(A9)

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{0}^{1}}{\partial t_{0}} + \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{1}} + \underline{c}_{0}^{\circ} \cdot \frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}}\right) + \rho^{1} \frac{\partial \underline{c}_{0}^{\circ}}{\partial t_{0}} = \sum_{i} \rho_{i}^{\circ} \underline{F}_{i} + \rho_{e}^{\circ} \underline{c}_{0}^{\circ} \times \underline{H} + \frac{\underline{j}^{1}}{2} \times \underline{H} - \frac{\partial}{\partial \underline{r}} \cdot \underline{P}^{\circ} \qquad (A10)$$

$$\frac{3}{2}n^{\circ}k\left(\frac{\partial T^{1}}{\partial t_{0}} + \frac{\partial T^{\circ}}{\partial t_{1}} + \underline{c}_{0}^{\circ} \cdot \frac{\partial T^{\circ}}{\partial \underline{r}}\right) = \frac{3}{2}kT^{\circ}\frac{\partial}{\partial \underline{r}} \cdot \sum_{i}n_{i}^{\circ}\underline{\overline{c}}_{0}^{\circ} + \frac{\partial}{\partial \underline{r}} + \sum_{i}\rho_{i}^{\circ}\underline{\overline{c}}_{0}^{\circ} \cdot \underline{F}_{i} + \underline{j}^{\circ} \cdot (\underline{c}_{0}^{\circ} \times \underline{H}) - \underline{P}^{\circ}:\frac{\partial \underline{c}_{0}^{\circ}}{\partial \underline{r}} - \frac{\partial}{\partial \underline{r}} \cdot \underline{q}^{\circ}$$
(A11)

From Eq.(A9) we see that  $\rho_1^1$  has a transient on the  $\tau_0$  time scale due to the evolution of  $\underline{c}_{0\perp}^0$  and  $\overline{\underline{C}}_1^0$ , and in the limit  $t_0 = \infty$  we obtain

$$\frac{\partial \rho_{i}}{\partial t_{1}} + \frac{\partial}{\partial r} \cdot (\rho_{i} \stackrel{\circ}{=} \stackrel{\circ}{OM}) = 0$$
 (A12)

 $(e_{A}) = (22^{e_{A}}) \cdot \frac{e_{A}}{26} + (22^{e_{A}}) \cdot \frac{e_{A}}{26} + \frac{e_{A}}{$ 

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From Eq. (A9) we see that  $p_1$  has a frametent on the  $r_0$ ,  $r_0$  time scale due to the evolution of  $c_{01}^{0}$  and  $c_{11}^{0}$ .

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(A18)

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where M means expressions in the limit  $t_0 = \infty$ . Similarly Eq.(A.11) shows that  $T^1$  has a transient on the  $\tau_0$  time scale due to the transients of  $\underline{c}_{0\perp}^0$ ,  $\overline{\underline{C}}_1^0$ ,  $\underline{j}^0$ ,  $\underline{P}^0$  and  $\underline{q}^0$ . In the limit  $t_0 = \infty$  we get

$$\frac{3}{2}n^{\circ}k\left(\frac{\partial T^{\circ}}{\partial t_{1}} + \frac{c^{\circ}}{\partial M} \cdot \frac{\partial T^{\circ}}{\partial \underline{r}}\right) = -p^{\circ}\frac{\partial}{\partial \underline{r}} \cdot \underline{c^{\circ}}_{OM}$$
(A13)

From the parallel version of Eq.(A.10) we get that  $\underline{c}_{0\parallel}^{1}$  has a transient on the  $\tau_{0}$  time scale due to the transients of  $\underline{c}_{0\perp}^{0}$  and  $\underline{P}^{0}$  and in the limit  $t_{0} = \infty$  we obtain

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{\circ}}{\partial t_{1}} + \underline{c}_{\circ M}^{\circ} \cdot \frac{\partial}{\partial \underline{r}} \underline{c}_{\circ}^{\circ}\|\right) = \sum_{i} \rho_{i}^{\circ}\underline{F}_{i}\| - \frac{\partial p^{\circ}}{\partial \underline{r}}\|$$
(A14)

From Eq.(A8) we seek bounded solutions for  $f_i^1$  which obey  $\frac{\partial f_i^1}{\partial t_0} \to 0$  as  $t_0 \to \infty$  (A15)

Then from Eq.(A.10) we get that  $c_{O\perp}^1$  has a transient on the  $\tau_{O}$  time scale due to the transients of  $c_{O\perp}^O$ ,  $j^1$ and  $P_{O}^O$ , and in the limit  $t_{O} = \infty$  we have

$$\rho^{\circ}\left(\frac{\partial \underline{c}_{OML}}{\partial t_{1}} + \underline{c}_{OM}^{\circ}, \frac{\partial \underline{c}_{OML}}{\partial \underline{r}_{1}}\right) = \sum_{i} \rho_{i}^{\circ}\underline{F}_{i\perp} + \rho_{e}^{\circ}\underline{c}_{OM}^{\circ} \times \underline{H} + \frac{j_{M}^{1} \times \underline{H}}{2} + \frac{j_{M}^{1} \times \underline{H}}{2} - \frac{\partial p^{\circ}}{\partial \underline{r}_{\perp}}$$
(A16)

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12.-.

Here we find  $\underline{j}_{M}^{1}$  from  $f_{\underline{i}}^{1}$  in the limit  $t_{0} = \infty$ . The equation for  $f_{\underline{i}M}^{1}$  is, substituting from Eqs.(A.14) and (A.16)

$$\frac{\partial f_{\underline{i}M}^{\circ}}{\partial t_{1}} + \underline{c}_{\underline{o}M}^{\circ} \cdot \frac{\partial f_{\underline{i}M}^{\circ}}{\partial \underline{r}} + \underline{c}_{\underline{i}} \cdot \frac{\partial f_{\underline{i}M}^{\circ}}{\partial \underline{r}} + \underline{F}_{\underline{i}} \cdot \frac{\partial f_{\underline{i}M}^{\circ}}{\partial \underline{c}_{\underline{i}}} + \frac{e_{\underline{i}}}{m_{\underline{i}}} \underline{c}_{\underline{o}M}^{\circ} \times \underline{H} \cdot \frac{\partial f_{\underline{i}M}^{\circ}}{\partial \underline{c}_{\underline{i}}} -$$

$$- \frac{\partial f_{\underline{i}M}^{\circ}}{\partial \underline{c}_{\underline{i}}} \underline{c}_{\underline{i}} \cdot \frac{\partial \underline{c}_{\underline{o}M}^{\circ}}{\partial \underline{r}} - \frac{1}{\rho^{\circ}} \left( \sum_{j} \rho_{j}^{\circ} \underline{F}_{j} + \rho_{\underline{e}}^{\circ} \underline{c}_{\underline{o}M}^{\circ} \times \underline{H} + \underline{j}_{\underline{M}}^{1} \times \underline{H} - \frac{\partial p^{\circ}}{\partial \underline{r}} \right) \cdot \frac{\partial f_{\underline{i}M}^{\circ}}{\partial \underline{c}_{\underline{i}}} +$$

$$+ \frac{e_{\underline{i}}}{m_{\underline{i}}} \underline{c}_{\underline{i}} \times \underline{H} \cdot \frac{\partial f_{\underline{i}M}^{1}}{\partial \underline{c}_{\underline{i}}} = \sum_{j} c_{\underline{i}j} \left[ f_{\underline{i}M}^{\circ} f_{\underline{j}M}^{1} + f_{\underline{i}M}^{1} f_{\underline{j}M}^{\circ} \right]$$

or, substituting further from Eqs.(A.6), (A.12) and (A.13)

 $f_{iM}^{\circ} \left( \frac{m_{i}C_{i}^{2}}{2kT^{\circ}} - \frac{5}{2} \right) \frac{1}{T^{\circ}} \frac{\partial T^{\circ}}{\partial \underline{r}} \cdot \underline{C}_{i} + f_{iM}^{\circ} \frac{m_{i}}{kT^{\circ}} \left[ \frac{1}{\rho^{\circ}} \left( \sum_{j} \rho_{j}^{\circ} \underline{F}_{j} + \rho_{e}^{\circ} \underline{C}_{oM}^{\circ} \times \underline{H} - \frac{\partial p^{\circ}}{\partial \underline{r}} \right) - \frac{1}{T^{\circ}} \frac{1}{T^{\circ}} \frac{\partial p_{i}}{\partial \underline{r}} + f_{iM}^{\circ} \frac{m_{i}}{kT^{\circ}} \left[ \frac{1}{\rho^{\circ}} \left( \sum_{j} \rho_{j}^{\circ} \underline{F}_{j} + \rho_{e}^{\circ} \underline{C}_{oM}^{\circ} \times \underline{H} - \frac{\partial p^{\circ}}{\partial \underline{r}} \right) - \frac{1}{T^{\circ}} \frac{1}{T^{\circ}} \frac{\partial p_{i}}{\partial \underline{r}} + \frac{1}{T^{\circ}} \frac{1}{T^{\circ}} \frac{\partial p_{i}}{\partial \underline{r}} + f_{iM}^{\circ} \frac{m_{i}}{kT^{\circ}} \left[ \frac{1}{T^{\circ}} \left( \underline{C}_{i} \underline{C}_{i} - \frac{1}{T^{\circ}} \underline{C}_{i} \right) + \frac{1}{T^{\circ}} \frac{\partial p_{i}}{\partial \underline{r}} \right] + \frac{1}{T^{\circ}} \frac{1}{T$ 

Eqs. (A.12), (A.13), (A.14), (A.16) and (A.17) are the "second order" equations of Chapman and Cowling [1]. Eq.(A.7) in the limit  $t_0 = \infty$ , i.e.

$$\sum_{i} \rho_{i}^{\circ} \overline{\underline{C}}_{iM}^{1} = 0 \qquad (A.18)$$

has to be fulfilled when solving Eq.(A.17) for  $f_{iM}^1$ .

Eqs. (A. T2), (A. T3), (A. T4), (A. T6) and (A. T7), are the "second order" equations of Chapman and Comling [1]; Eq. (A. T) to the limit  $t_0 = \infty$ , i. 1. s.

(81.A).,

has to be fulfilled when solving. Eq. (A. )7) for find

### Appendix 2.

Till now the electromagnetic field has been assumed stationary and uniform, imposed on the plasma by external means. However, the generation of a field by the evolution of the plasma itself may also be taken into account with only small modifications of the results obtained in sections 2 and 3. For such a model we study the kinetic and macroscopic equations of the previous sections together with the Maxwell equations for the total electric field E and magnetic field B:

$$\frac{\partial}{\partial \mathbf{r}} \cdot \underline{\mathbf{E}} = 4\pi e^2 \rho_e \tag{A.19}$$

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{\underline{B}} = 0 \tag{A.20}$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$
(A.21)

$$\frac{\partial}{\partial \underline{\mathbf{r}}} \times \underline{\mathbf{B}} = 4\pi (\rho_{\underline{\mathbf{e}}} - \underline{\mathbf{j}}) + \frac{1}{c^2} \frac{\partial \underline{\mathbf{E}}}{\partial t}$$
(A.22)

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As in section 1 we assume that

$$\frac{\overline{c}_{1}\tau_{2}}{\overline{L}} \sim \varepsilon$$

$$\frac{\overline{E}_{0}}{\overline{c}_{1}B_{0}} \sim \varepsilon$$

$$2 \sim \frac{1}{\Omega_{1}} = \frac{m_{1}}{\overline{e_{1}B_{0}}}$$

(A.23)

where  $E_0$  and  $B_0 = H$  are the magnitude of the externally imposed fields. New assumptions on the sources of the fields now have to be made: Since the fields shall not be stronger than before the sources must be weak to a certain extent. Also to solve the Maxwell equations requires additional assumptions. We therefore parametrize the Maxwell equations as we did with the kinetic and macroscopic equations and make the assumptions when we need to make them.

The order of magnitude of the terms in Eq.(A.19) thus become

$$\frac{E_{L}}{L}: 4\pi c^{2} \rho_{e}$$

Here  $E_{L}$  is the part of  $\underline{E}$  that varies in space. We further get, dividing by  $\frac{E_{L}}{L}$ :

$$1: \frac{4\pi c^{2} n_{1} e_{1}^{2}}{m_{1}} \cdot \frac{m_{1} \rho_{e}}{n_{1} e_{1}^{2}} \cdot \frac{L}{E_{L}}$$

As in section i we assume that



White B and B - H are the magnitude of the externally imposed Mields. New assumptions on the sources of the fields now May's to be made: Since the fields shall not be stronger with before the sources must be weak to a certain spice. Aless to shive the Maxwell squations requires additional seconditions. We therefore purchase the sequines additional as we did with the Kinetic and macroscoppic equations and make the assumptions when we need to make them.

The order of mainitude for the therma in Eq. (A. Q). thus become

and using the relations Eq.(A.23) and  $\omega_{p1} = \left(\frac{4\pi c^2 n_1 e_1^2}{m_1}\right)^{\frac{1}{2}}$ , the electron plasma frequency,

$$1 : \left(\frac{\omega_{p1}}{\Omega_1}\right)^2 \frac{1}{\epsilon^2} \left(\frac{E_0}{E_L}\right) \left(\frac{\rho_e}{n_1 e_1}\right)$$

Since for the model we consider  $\omega_{p1} >> \Omega_1$  and we allow  $E_L \sim E_o$ , for a balance of the terms of Eq.(A.19) we must have

$$\sigma_{\rm e} \sim (\Omega_1/\omega_{\rm p1})^2 \cdot \varepsilon^2 n_1 e_1$$

i.e. charge neutrality to a certain extent.

In a similar way we estimate the terms in Eq.(A.21)

$$\frac{E_{L}}{L}:\frac{B_{t}}{\tau_{2}}$$

where  $B_t$  is the part of <u>B</u> that varies in time. We further get

$$\frac{B_{t}}{B_{0}}\frac{1}{\varepsilon^{2}}$$

We have used the relations Eq.(A.23) and  $E_L \sim E_o$ . For the terms in Eq.(A.21) to balance we have  $B_t \sim \epsilon^2 B_o$ .

The order of magnitude of the terms in Eq.(A.22) becomes

$$\frac{B_{L}}{L}: 4\pi n_{1}e_{1}\overline{c}_{1}: \frac{1}{c^{2}}\frac{E_{t}}{\tau_{2}}$$

and value the valations Eq. (A233) and  $\omega_{p1} = \left(\frac{4\pi\sigma^2 n_1 e_1^2}{m_1}\right)^{\frac{1}{2}}$ .

Since For the model we foundider on Stills and the first of Stills and the former of t

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where  $B_L$  and  $E_t$  denote the parts of <u>B</u> and <u>E</u> that vary in space and time. We have set  $|\underline{j}| \sim n_1 e_1 \overline{c_1}$ . We further get

 $1 : \left(\frac{\omega_{p1}}{\Omega_{1}}\right)^{2} \frac{1}{\varepsilon} \frac{B_{o}}{B_{L}} \left(\frac{\overline{c}_{1}}{\varepsilon}\right)^{2} : \left(\frac{\overline{c}_{1}}{-\varepsilon}\right)^{2} \left(\frac{E_{t}}{E_{o}}\right)^{2} \left(\frac{B_{o}}{B_{L}}\right)$ 

We assume that  $\left(\frac{\omega_{p1}}{\Omega_1}\right)^2 \frac{1}{\epsilon} \left(\frac{\overline{c}_1}{c}\right)^2 << 1$ . More specifically, let

 $\left(\frac{\omega_{p1}}{\Omega_{1}}\right)^{2} \frac{1}{\epsilon} \left(\frac{\overline{c}_{1}}{c}\right)^{2} \sim \epsilon^{2}$ . Then  $B_{L} \sim \epsilon^{2}B_{0}$  for the first and second term to be of equal order of magnitude. The third term will always be vanishing small when  $E_{t} \sim E_{0}$ , which is the order of magnitude of  $E_{t}$  that we can permit. We shall therefore neglect the last term (displacement current) of Eq.(A.22), and the approximative Maxwell equations for the gas model are

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{E} = 4\pi c^2 \rho_e \qquad (A.19')$$

$$\frac{\partial}{\partial r} \cdot \underline{B} = 0 \qquad (A.20')$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{E} = - \frac{\partial \underline{B}}{\partial \underline{t}}$$
 (A.21')

$$\frac{\partial}{\partial \underline{r}} \times \underline{B} = 4\pi (\epsilon^2 \alpha \rho_{\underline{e}} \underline{c}_0 + n_1 e_1 \overline{\underline{c}}_1 + \alpha n_2 e_2 \overline{\underline{c}}_2)$$
(A.22')

where E and B are sought as

$$\underline{E}(\underline{r},t) = \underline{E}^{O}(\underline{r},t) + \underline{\varepsilon}\underline{E}^{1}(\underline{r},t) + \dots$$
$$\underline{B}(\underline{r},t) = \underline{H} + \underline{\varepsilon}^{2}\underline{B}^{2}(\underline{r},t) + \dots$$

 $\begin{pmatrix} 2+1\\ 2+1 \end{pmatrix} = \begin{pmatrix} 2\\ -1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 2\\ -1 \end{pmatrix} \end{pmatrix}$  Then B. - e'B. - c'B. - cor the first cand accord term to be of equal carder of menutude The third term will sinks be rentelying amall when B. - Fig. which to the the order of meruluque of S. that we can parmite No anall therefore neglect the last term (disc). According to the second the suproductive second (disc). According to the second the suproductive second (disc).

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(A.21)

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where E and E are sought as

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Making the multiple time scale expansion of  $\frac{\partial}{\partial t}$ , we obtain as the lowest order sets of equations in the parameter  $\epsilon$ :

$$\frac{\partial}{\partial \mathbf{r}} \cdot \underline{\mathbf{E}}^{\circ} = 4\pi c^2 \rho_{e}^{\circ}$$

$$\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{\bar{E}}^{\circ} = - \frac{\partial \mathbf{\bar{E}}^{2}}{\partial \mathbf{\bar{E}}^{2}}$$

$$\frac{\partial}{\partial \underline{r}} \times \underline{B}^2 = 4\pi (n_1^{\circ} e_1 \overline{\underline{C}}_1^{\circ} + \alpha n_2^{\circ} e_2 \overline{\underline{C}}_2^{\circ})$$

and

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{E}^{1} = 4\pi c^{2} \rho_{e}^{1}$$

$$\frac{\partial}{\partial \underline{r}} \cdot \underline{B}^{3} = 0$$
(A.25)
$$\frac{\partial}{\partial \underline{r}} \times \underline{E}^{1} = -\left(\frac{\partial \underline{B}^{3}}{\partial t_{2}} + \frac{\partial \underline{B}^{2}}{\partial t_{3}}\right)$$

 $\frac{\partial}{\partial \underline{r}} \times \underline{B}^{3} = 4\pi (n_{1}^{0} e_{1} \overline{\underline{C}}_{1}^{1} + \alpha \underline{p}_{2}^{0} e_{2} \overline{\underline{C}}_{2}^{1}) + 4\pi (n_{1}^{1} e_{1} \overline{\underline{C}}_{1}^{0} + \alpha n_{2}^{1} e_{2} \overline{\underline{C}}_{2}^{0})$ 

Necessary conditions for existence of solutions are

$$\frac{\partial}{\partial \underline{r}} \cdot (n_1^0 e_1 \overline{\underline{C}}_1^0 + \alpha n_2^0 e_2 \overline{\underline{C}}_2^0) = 0$$

$$(A.26)$$

$$\frac{\partial}{\partial \underline{r}} \cdot (n_1^0 e_1 \overline{\underline{C}}_1^1 + \alpha n_2^0 e_2 \overline{\underline{C}}_2^1) + \frac{\partial}{\partial \underline{r}} \cdot (n_1^1 e_1 \overline{\underline{C}}_1^0 + \alpha n_2^1 e_2 \overline{\underline{C}}_2^0) = 0$$

(A.24)

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which also can be sufficient conditions.

The sets of equations (A.24) and (A.25) now have to be solved together with the zeroth and first order equations of section 1, Eqs.(18)-(29) and Eqs.(30)-(41). The zeroth order equations (18)-(29) are uncoupled with the Maxwell equations, so the evolution of the zeroth order distribution functions and macroscopic quatities are the same as before. The first order equations (30)-(41) are coupled with the zeroth order Maxwell equations (A.24). Due to the zeroth order evolution the electric field  $E^{\circ}$ and magnetic field  $B^2$  show a transient variation on the  $\tau_2$  - time scale. The transient of  $E^{\circ}$  give rise to new transients in the first order kinetic and macroscopic equations in addition to the transients studied before. The transient of  $B^2$  is too small to be seen in these equations. In the limit  $t_2 = \infty$ , however, the first order kinetic and macroscopic equations are the same as before, the electric field  $\underline{E}^{\circ}$  now obeying the Poisson equation.

- 64 -

which also can be sufficient conditions.

the electric field E now obeying the Poinson squation.

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