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Abstract

When we apply the Richtmyer procedure [8] to solve a system of parabolic differential equations which describe the propagation of a finite amplitude sound beam, the initial conditions and the boundary conditions may cause unphysical effects. In this paper we explain why these unwanted effects arise, and we describe how we may approximate the initial and boundary conditions in order to make the Richtmyer procedure applicable. In earlier papers[1,9,6,4] the fully implicit method has been applied to solve the described system of equations. The performances of the two methods are compared in a numerical experiment.

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1 Introduction

In several papers [1,9,6] systems of coupled partial differential equations of the form

$$\frac{\partial a_n}{\partial \sigma} = -c(n,\sigma)a_n - k(n,\sigma)\nabla_x^2 b_n + il_1(n,\sigma,\underline{a},\underline{b})$$
$$n = 1,...,m \qquad (1)$$
$$\frac{\partial b_n}{\partial \sigma} = -c(n,\sigma)b_n + k(n,\sigma)\nabla_x^2 a_n + il_2(n,\sigma,\underline{a},\underline{b})$$

where $\nabla_x^2 = (\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x})$, $\underline{a} = (a_1,...,a_m)$, $\underline{b} = (b_1,...,b_m)$ and m the number of harmonics retained in the numerical solution, have been used to describe the propagation of nonlinear sound beams generated by circular pistons. The first terms on the right hand side are due to absorbtion, the second terms to diffraction and the last terms to nonlinearity.

In the case of moderate or weak nonlinearity, the diffraction terms have to be integrated with greatest care, and therefore we will pay special attention to how these terms should be integrated. Zhileikin [10] applied the Richtmyer procedure [8] to integrate these terms while Aanonsen used a fully implicit method. The source in [10] is Gaussian while the source in [1] is a piston.

In this paper we discuss two problems that may arise when we apply the Richtmyer procedure to solve (1). In sec.2 we discuss the problem that arises when we have a piston source. The problem is explained for in [4], and here we describe a way to get around it. We also discuss the problems that arise because we have to use a finite range of x and therefore to introduce unphysical boundary conditions.

The use of the Richtmyer procedure together with new approximations of the initial conditions in the case of a source piston we believe is a efficient way of integrating the diffraction terms if also the boundary conditions are approximated with care. In sec.3 we compare this method with the method described in [1].

2 The initial and boundary conditions

If we simplify (1) bearing in mind that the diffraction terms are the important terms, we get the problem studied by Richtmyer

$$\frac{\partial v}{\partial t} = -\frac{\partial^2 w}{\partial x^2}$$
$$\frac{\partial w}{\partial t} = \frac{\partial^2 v}{\partial x^2} \qquad 0 \leq x \leq 1, t \geq 0 \tag{2}$$

The initial conditions are given by $v(x,0) = v_0(x)$ and $w(x,0) = w_0(x)$. The boundary conditions are $v(0,t) = f_0(t), v(1,t) = f_1(t), w(0,t) = g_0(t), w(1,t) = g_1(t)$. We introduce some notation used by Fairweather and Gourlay [3]

$$\Omega = \begin{pmatrix} v \\ w \end{pmatrix} \qquad A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{3}$$

Equation (2) may then be rewritten

$$\frac{\partial \Omega}{\partial t} = A\frac{\partial^2 \Omega}{\partial x^2} \tag{4}$$

A rectangular network of points with mesh sizes h and k in the x and t directions respectively, where $Nh = 1$ is superimposed on the region $0 \leq x \leq 1, t \geq 0$. The values of $\Omega(x,t)$ at the mesh points $x=ih, t=jk(i = 0, 1, ..., N; j = 0, 1, ...)$ are given by $\Omega_{i,j}$. The methods we consider may then be written

$$(I - \lambda r A\delta_x^2)\Omega_{i,j+1} = (I + (1-\lambda) r A\delta_x^2)\Omega_{i,j} \quad i = 1, ..., N-1 \tag{5}$$

where $r=k/h^2$, I is the $2 * 2$ unit matrix and δ_x^2 is the usual central difference operator in the x-direction.

For $\lambda = 1/2$ we get the Richtmyer procedure and for $\lambda = 1$ we get the fully implicit method. In [4] we show that the eigenvectors of the solution matrices for both methods are

$$\nu_{\pm s} = \begin{pmatrix} isin(s\pi/N) \\ \pm sin(s\pi/N) \\ \vdots \\ isin((N-1)s\pi/N) \\ \pm sin((N-1)s\pi/N) \end{pmatrix} \qquad s = 1, 2, ...N-1 \tag{6}$$

The eigenvalues of the solution matrix for the fully implicit method are

$$\mu_{\pm s} = \frac{1 \pm 4risin^2(s\pi/2N)}{1 + 16r^2sin^4(s\pi/2N)} \qquad s = 1, 2, ..., N-1 \tag{7}$$

The corresponding eigenvalues for the Richtmyer procedure are

$$\mu_{\pm s} = \frac{1 \mp 2risin^2(s\pi/2N)}{1 \pm 2risin^2(s\pi/2N)} \qquad s = 1, 2, ..., N-1 \tag{8}$$

It is well known, see [2], that when a step function is approximated by a finite Fourier series, Gibbs oscillations do appear. When we have a piston source and a solution matrix with eigenvectors (6), Gibbs oscillations will also appear in the numerical solution.

The eigenvalues of the fully implicit method are all inside the unit circle. The eigenvalues of the higher harmonic eigenvectors are small in magnitude, and the contribution of these eigenvectors to the numerical solution will therefore soon be damped, and after some steps the Gibbs oscillations will disappear from the numerical solution. If the step sizes used in t direction are small, the lower and moderate harmonics are only to a small extent damped, and this explains why the fully implicit method for small step sizes has produced solutions of (1) that have proved to be in good agreement with physical experiments. However, as we shall see in sec. 4, if we increase the step size, much energy is lost and the side lobes in the beam patterns gradually disappear.

The eigenvalues when we apply the Richtmyer procedure, all lie on the unit circle, and therefore as shown in [4] the Gibbs oscillations are maintained in the numerical solution.

Thus both methods considered suffer from severe defects in the case of a piston source. In this paper we try to remove the Gibbs oscillations in the initial data before we start solving the system of differential equations by using a filter. We may then apply the Richtmyer procedure without further loss of energy. The filtering achieved with the fully implicit method has already proved to give satisfactory solutions of (1) for small step sizes, and it was therefore natural to make a filter that simulated the filtering achieved by going some steps with this method.

When we complicate (2) by introducing cylindrical coordinates, we get the equation

$$\begin{aligned} \frac{\partial v}{\partial t} &= -(\frac{\partial^2 w}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x}) \\ \frac{\partial w}{\partial t} &= (\frac{\partial^2 v}{\partial x^2} + \frac{1}{x}\frac{\partial v}{\partial x}) \end{aligned} \qquad 0 \le x \le x_{max}, t \ge 0 \tag{9}$$

To find a general expression for the eigenvectors and corresponding eigenvalues of the solution matrices for this problem is very difficult. It is therefore difficult to predict the exact filtering achieved by using the fully implicit method on problem (9). However, the most important terms on the left hand side of (9) are the second order terms. Therefore, it is reasonable to base a subroutine for simulating the filtering achieved by the fully implicit method, on the expressions (6) and (7). The subroutine is listed in Appendix A.

The test problem used in the numerical experiments to be described is

$$\frac{\partial a_n}{\partial \sigma} = -p_{abs} n^2 a_n - \frac{1}{4n(1+\sigma)^2}\nabla_x^2 b_n +$$

$$p_{non}\frac{n}{2(1+\sigma)}[\sum_{p=1}^{n-1}(a_{n-p}b_p) + \sum_{p=n+1}^{m}(b_p a_{p-n} - a_p b_{p-n})]$$

$$n = 1, ..., 8 \qquad (10)$$

$$\frac{\partial b_n}{\partial \sigma} = -p_{abs} n^2 b_n + \frac{1}{4n(1+\sigma)^2}\nabla_x^2 a_n +$$

$$p_{non}\frac{n}{2(1+\sigma)}[\frac{1}{2}\sum_{p=1}^{n-1}(b_{n-p}b_p - a_{n-p}a_p) - \sum_{p=n+1}^{m}(a_p a_{p-n} + b_p b_{p-n})]$$

The initial conditions when we use the fully implicit method, are given by

$$a_1(x, \sigma = 0) = \begin{cases} sin(x^2) & when\ 0 \le x \le 1 \\ 0 & when\ 1 < x \le 8 \end{cases}$$

$$b_1(x, \sigma = 0) = \begin{cases} cos(x^2) & when\ 0 \le x \le 1 \\ 0 & when\ 1 < x \le 8 \end{cases}$$

$$a_n(x, \sigma = 0) = \quad 0$$

$$0 \le x \le 8, n = 2, ..., 8 \qquad (11)$$

$$b_n(x, \sigma = 0) = \quad 0$$

When we apply the Richtmyer procedure, the initial conditions given by (11) are filtered with the subroutine in Appendix A. (Except where otherwise stated.)

The boundary conditions when we use the fully implicit method are

$$a_n(x = 8, \sigma) = \quad 0$$

$$\sigma \ge 0, n = 1, ..., 8 \qquad (12)$$

$$b_n(x = 8, \sigma) = \quad 0$$

Let the values of $a_n(x,t)$ and $b_n(x,t)$ at the mesh points $x = ih, t = jk(n = 1, ..., 8, i = 0, ..., IMAX, j = 0, 1, ...)$ be given by $a_n^{i,j}$ and $b_n^{i,j}$ respectively. When we apply the Ricthmyer procedure, the boundary conditions are(except for the plot in Figure 4)

$$a_n^{IMAX,j} = 2a_n^{IMAX-1,j} - a_n^{IMAX-2,j}$$

$$n = 1, ..., 8, j = 1, 2, ... \qquad (13)$$

$$b_n^{IMAX,j} = 2b_n^{IMAX-1,j} - b_n^{IMAX-2,j}$$

At x=0 the solution is symmetric, and $\frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} = 2\frac{\partial^2}{\partial x^2}$. $p_{abs} = 1d-6$ and $p_{non} = 1d-1$ in our experiments. When we apply the Richtmyer procedure,

the absorbtion terms are integrated by using the Crank-Nicolson method. (The Crank- Nicolson method applied to (2) is usually called the Richtmyer procedure, see [7].) When we use the fully implicit method on the diffraction terms, the absorbtion terms are integrated by the fully implicit method also. The nonlinear terms are in both cases integrated by an explicit method. $h = 8.0/250$ in all our experiments. $k = 3.5 * 10^{-4} * (1 + \sigma)^2$ in the experiments in this section(except for the plot in Figure 4). We adjust k in order to keep $k/(h^2(1+\sigma)^2)$ constant, see (10).

To illustrate the usefulness of our filter we have applied the Richtmyer procedure on (10). In Figure 1 we plot the amplitude of the initial values of the fundamental (n=1) before and after(the dotted line) we have applied the filter. 1.24 per cent of the energy measured in the 2-norm is lost by using the filter.

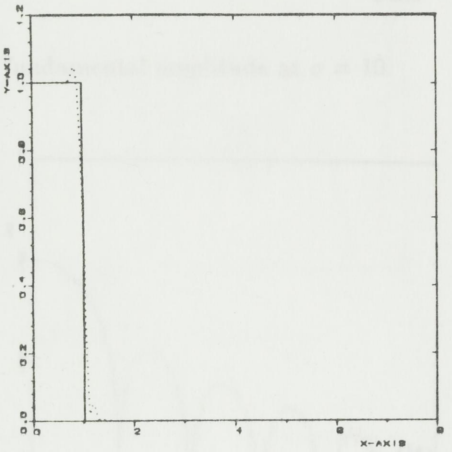

Figure 1. The initial values.

In Figure 2 we show the computed amplitude of the fundamental at $\sigma = 10$. for the initial values in Figure 1 when we apply the Richtmyer procedure. The dotted line shows the results without the filter. We notice that almost all Gibbs oscillations are removed from the numerical solution when we apply our filter. Some unphysical oscillations are left in the side lobes farthest away from the axis. These could to some extent have been removed by using a stronger filter. However, we have to balance between removing the Gibbs oscillations and maintaining the energy in the solution.

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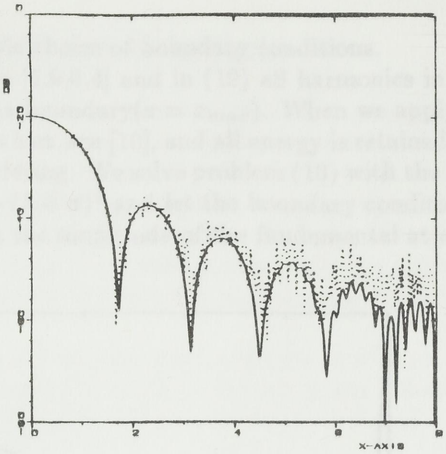

Figure 2. The fundamental amplitude at $\sigma = 10$.

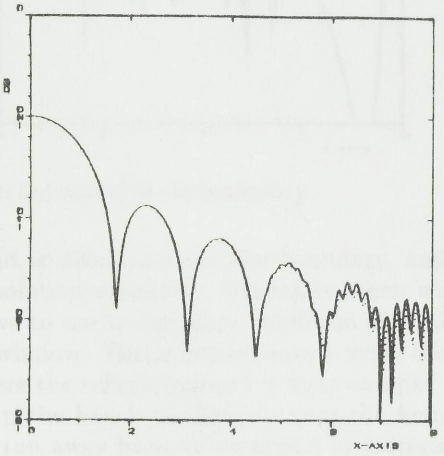

Figure 3. A comparison with the fully implicit method.

In Figure 3 we show the computed amplitude of the fundamental at $\sigma = 10$. when we apply the Richtmyer procedure and the fully implicit method (the dotted line). We see that even though the system of equations (10) is more complicated than (2), we achieve almost the same filtering by using the two techniques.

We end this section with some remarks on the boundary conditions. To restrict the values of x to $0 \leq x \leq 8$, as we do in (11) and (12), is clearly unphysical. However, numerically we have to define a finite range of x and try

to make a reasonable choice of boundary conditions.

In earlier papers [1,9,6,4] and in (12) all harmonics in the sound beam are set equal to 0 at the boundary($x = x_{max}$). When we apply the Richtmyer procedure, no energy is lost, see [10], and all energy is retained inside the window of x that we are considering. We solve problem (10) with the Richtmyer procedure for $k = 2.0 * 10^{-2} * (1 + \sigma)^2$ and let the boundary conditions be given by (12). In Figure 4 we plot the amplitude of the fundamental at $\sigma = 10$.

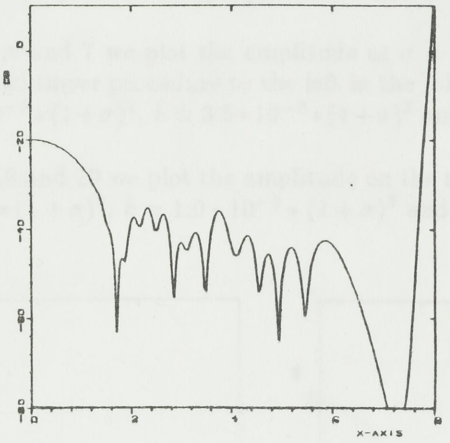

Figure 4. Reflected sound from boundary.

We see that sound is reflected from the boundary, and for smaller values of k the numerical solution overflows. Therefore when we apply the Richtmyer procedure, we have to use a boundary condition that allows energy to escape from the given x-window. This is not necessary when the fully implicit method is used because here the reflected sound is soon damped.

As we see from the heavy oscillations near the boundary in the figures 2 and 3, we do not run away from all problems by introducing (13), but (13) at least allows sound to escape from our x-window. Therefore, when we apply the Richtmyer procedure in sec.3, (13) will be used to approximate the boundary.

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In earlier papers [1,3,6,4] and in (12) all harmonics in the sound beam are set equal to 0 at the boundary($r = r_{max}$). When we apply the Richtmyer procedure, no energy is lost, see [10], and all energy is retained inside the window of x that we are considering. We solve problem (10) with the Richtmyer procedure for $k = 2.0 * 10^{-5} * (1 + x)^2$ and let the boundary conditions be given by (12). In Figure 4 we plot the amplitude of the fundamental at $r = 10$.

Figure 4. Reflected sound from boundary.

We see that sound is reflected from the boundary, and for smaller values of k the numerical solution overflows. Therefore when we apply the Richtmyer procedure, we have to use a boundary condition that allows energy to escape from the given x-window. This is not necessary when the fully implicit method is used because here the reflected sound is soon damped.

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3 Numerical experiments

In sec.2 we showed that if we used the Richtmyer procedure with a filter on the initial values, the results would be almost identical to the results produced by the fully implicit method for $k = 3.5 * 10^{-4} * (1+\sigma)^2$.

In this section we will study to which extent it is possible to increase the step size and still have a satisfactory solution. We use problem (10) as a test problem and study the amplitude of the fundamental only. As a reference solution we use the solution given by the Richtmyer procedure and $k = 3.5 * 10^{-4} * (1+\sigma)^2$(the dotted lines).

In figures 5,6 and 7 we plot the amplitude at $\sigma = 10.$ produced by the two methods(the Richtmyer procedure to the left in the following figures.) when we use $k = 1.0 * 10^{-3} * (1+\sigma)^2$, $k = 3.5 * 10^{-3} * (1+\sigma)^2$ and $k = 1.0 * 10^{-2} * (1+\sigma)^2$ respectively.

In figures 8,9 and 10 we plot the amplitude on the axis (x=0.0) when we use $k = 3.5 * 10^{-3} * (1+\sigma)^2$, $k = 1.0 * 10^{-2} * (1+\sigma)^2$ and $k = 3.5 * 10^{-2} * (1+\sigma)^2$ respectively.

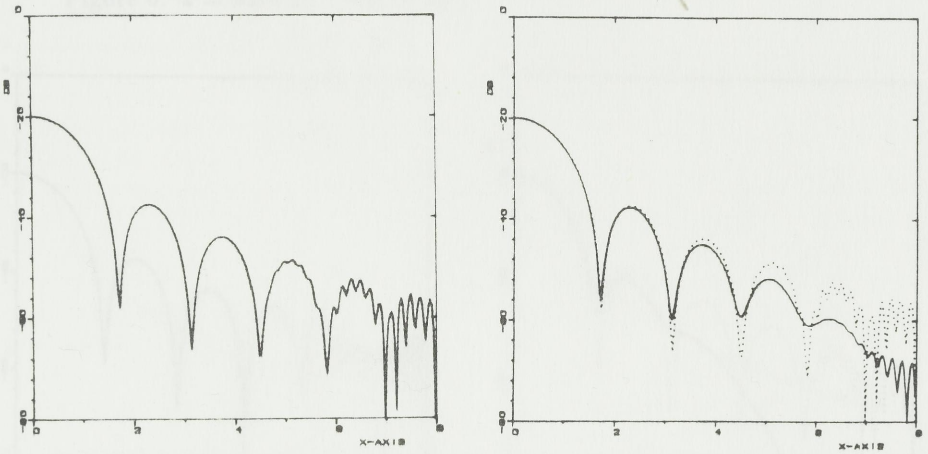

Figure 5. $k = 1.0 * 10^{-3} * (1+\sigma)^2$.

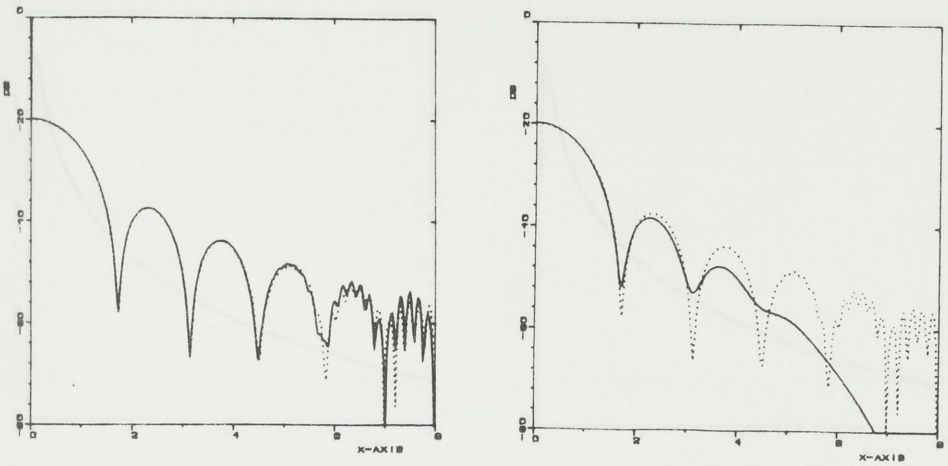

Figure 6. $k = 3.5 * 10^{-3} * (1 + \sigma)^2$.

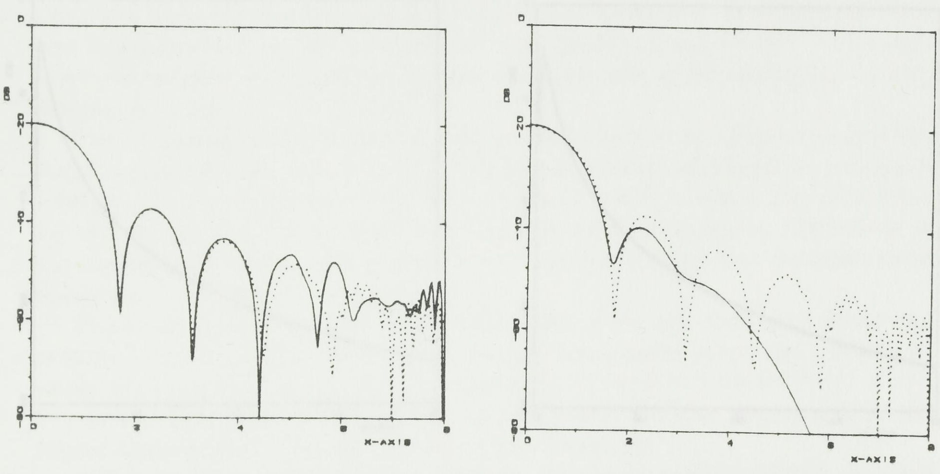

Figure 7. $k = 1.0 * 10^{-2} * (1 + \sigma)^2$.

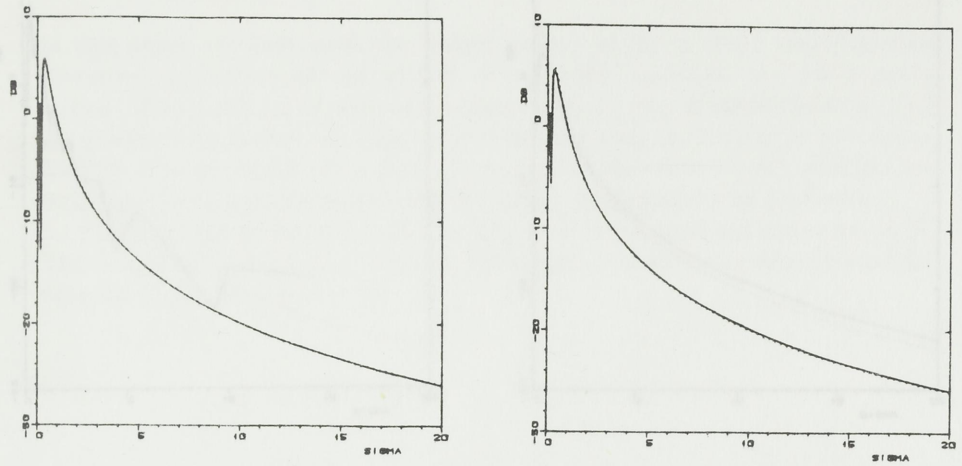

Figure 8. $k = 3.5 * 10^{-3} * (1+\sigma)^2$.

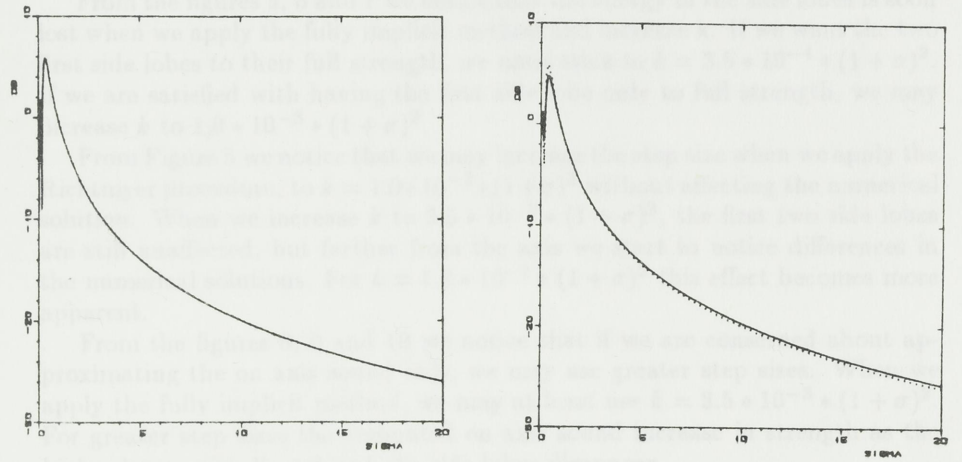

Figure 9. $k = 1.0 * 10^{-2} * (1+\sigma)^2$.

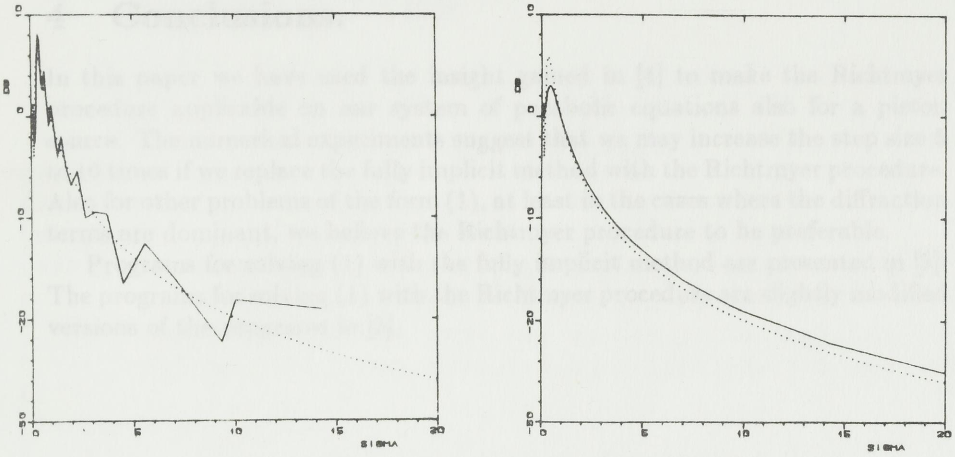

Figure 10. $k = 3.5 * 10^{-2} * (1 + \sigma)^2$.

From the figures 5, 6 and 7 we notice that the energy in the side lobes is soon lost when we apply the fully implicit method and increase k. If we want the two first side lobes to their full strength, we must stick to $k = 3.5 * 10^{-4} * (1 + \sigma)^2$. If we are satisfied with having the first side lobe only to full strength, we may increase k to $1.0 * 10^{-3} * (1 + \sigma)^2$.

From Figure 5 we notice that we may increase the step size when we apply the Richtmyer procedure, to $k = 1.0 * 10^{-3} * (1 + \sigma)^2$ without affecting the numerical solution. When we increase k to $3.5 * 10^{-3} * (1 + \sigma)^2$, the first two side lobes are still unaffected, but farther from the axis we start to notice differences in the numerical solutions. For $k = 1.0 * 10^{-2} * (1 + \sigma)^2$ this effect becomes more apparent.

From the figures 8, 9 and 10 we notice that if we are conserned about approximating the on axis sound only, we may use greater step sizes. When we apply the fully implicit method, we may at least use $k = 3.5 * 10^{-3} * (1 + \sigma)^2$. For greater step sizes the computed on axis sound increase in strength as the higher harmonics die out and the side lobes disappear.

When we use the Richtmyer procedure, all chosen values of k less or equal to $1.0 * 10^{-2} * (1 + \sigma)^2$ give almost the same solution on the axis. However, for k equal or greater than $3.5 * 10^{-2} * (1 + \sigma)^2$ the numerical solution becomes apparently unphysical. We did not notice this dramatic change in the numerical solution when we applied the fully implicit method and increased k.

In the figures 5 to 10 we have studied the amplitude of the fundamental only. However, if we study the amplitude of the second or higher harmonics (or their phases), the relative performances of our two methods are approximately the same.

4 Conclusions.

In this paper we have used the insight gained in [4] to make the Richtmyer procedure applicable on our system of parabolic equations also for a piston source. The numerical experiments suggest that we may increase the step size 5 to 10 times if we replace the fully implicit method with the Richtmyer procedure. Also for other problems of the form (1), at least in the cases where the diffraction terms are dominant, we believe the Richtmyer procedure to be preferable.

Programs for solving (1) with the fully implicit method are presented in [5]. The programs for solving (1) with the Richtmyer procedure are slightly modified versions of the programs in [5].

A Program FILTER

```
      SUBROUTINE FILTER(X,N,EXTEND,WORK)
C***BEGIN PROLOGUE FILTER
C***DATE WRITTEN 870223 (YYMMDD)
C***AUTHOR Jarle Berntsen
C***DESCRIPTION The routine filter the Gibbs oscillations
C       present in the initial values stored in X in
C       approximately the same way as the fully implicit
C       method (100 steps with k=3.5d-4*(1+sigma)**2 and h=8/250)
C
C***INPUT PARAMETERS
C   X        Double precision array of dimension N.
C            Contains the initial values to be filtered.
C   N        Integer.
C            N must not have prime factors greater than 19
C            when we use the given NAG routines for the FFT.
C   EXTEND Double precision array of dimension 4*N.
C            Contains an extension of X.
C   WORK     Double precision array of dimension 4*N.
C            Used as working storage by the NAG-routines.
C***OUTPUT PARAMETER
C   X        Contain on exit the filtered initial values.
C***ROUTINES CALLED C06FAF, C06GBF, C06FBF (From NAG.)
C***END PROLOGUE FILTER
      INTEGER N,IFAIL,I,NX,NDAMP
      DOUBLE PRECISION RSM,LAMBDA
      DOUBLE PRECISION X(N),WORK(4*N),PI,COEFF,EXTEND(4*N)
      NX=4*N
      RSM=16.D0*0.38**2
      NDAMP=100
C
C       Extend X to make the Fourier expansion a pure
C       sin expansion.
C
      EXTEND(1)=0.D0
      DO 5 I=1,N-1
        EXTEND(I+1)=X(N-I+1)
        EXTEND(N+I+1)=X(I+1)
        EXTEND(2*N+I+1)=-EXTEND(I+1)
```

```
         EXTEND(3*N+I+1)=-X(I+1)
5      CONTINUE
       EXTEND(N+1)=X(1)
       EXTEND(2*N+1)=0.D0
       EXTEND(3*N+1)=-X(1)
       PI=X01AAF(1.D0)
C
C      Fourier expand EXTEND.
C
       CALL C06FAF(EXTEND,NX,WORK,IFAIL)
C
C      Filter EXTEND by reducing the Fourier
C      coefficients by the same factor as the
C      fully implicit method with 100 steps and
C      k=3.5d-4*(1+sigma)**2 and h=8/250.
C
       DO 10 I=1,N
         LAMBDA=1.D0+RSM*(SIN(I*PI/DBLE(2*N+1)))**4
         LAMBDA=SQRT(1.D0/LAMBDA)
         LAMBDA=LAMBDA**NDAMP
         EXTEND(NX-2*I+2)=LAMBDA*EXTEND(NX-2*I+2)
10     CONTINUE
C
C      Form the complex conjugates of the discrete Fourier transform.
C
       CALL C06GBF(EXTEND,NX,IFAIL)
C
C      Compute the inverse Fourier transform.
C
       CALL C06FBF(EXTEND,NX,WORK,IFAIL)
C
C      Restrict EXTEND to X.
C
       DO 20 I=1,N
         X(I)=EXTEND(N+I)
20     CONTINUE
       RETURN
       END
```

References

[1] S.I. Aanonsen. Numerical computation of the nearfield of a finite amplitude sound beam. *Dept. of Math., Univ. of Bergen, Norway*, Report no. 73, 1983.

[2] A.Papoulis. *The Fourier Integral and its Applications.* McGraw-Hill, 1962.

[3] G.Fairweather and A.R.Gourlay. Some stable difference approximations to a fourth-order parabolic partial differential equation. *Math.Comp.*, 21:1–11, 1967.

[4] J.Berntsen and E.Vefring. Numerical computation of a finite amplitude sound beam. *Dept. of Math., Univ. of Bergen, Norway*, Report no. 81, 1986.

[5] J.Berntsen and E.Vefring. User documentation. Programs SOLTRI, SOLFIV and SOLSEV. *Dept. of Math., Univ. of Bergen, Norway*, 1986.

[6] M.F.Hamilton,J.Naze Tjøtta and S.Tjøtta. Nonlinear effects in the farfield of a directive sound source. *J.Acoust.Soc.Am.*, 78:202–216, 1985.

[7] M.K.Jain. *Numerical solution of differential equations.* John Wiley, 1984.

[8] R.D.Richtmyer. *Difference methods for initial value problems.* Intersc. Tracts in Pure and Appl. Math.,Tract 4,Interscience, 1957.

[9] S.I.Aanonsen,T.Barkve,J.Naze Tjøtta and S.Tjøtta. Distortion and harmonic generation in the nearfield of a finite amplitude sound beam. *J.Acoust.Soc.Am.*, 75:749–768, 1984.

[10] Ya.M.Zhileikin. Numerical computation of the equation for nonlinear acoustics of confined beams. *U.S.S.R.Comput.Maths.Math.Phys.*, 22:140–156, 1982.

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