# Department of APPLIED MATHEMATICS

Optimal Portfolio management Rules in a Non-Gaussian Market with Durability and intertemporal Substitution

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# OPTIMAL PORTFOLIO MANAGEMENT RULES IN A NON-GAUSSIAN MARKET WITH DURABILITY AND INTERTEMPORAL SUBSTITUTION

# FRED ESPEN BENTH, KENNETH HVISTENDAHL KARLSEN, AND KRISTIN REIKVAM

ABSTRACT. We determine the optimal portfolio management rules for a portfolio selection problem with consumption which incorporates the notions of durability and intertemporal substitution. The logreturns of the uncertain assets are not necessarily normally distributed. The natural models then involve Lévy processes as the driving noise instead of the more frequently used Brownian motion. The optimization problem is a state constrained singular stochastic control problem and the associated Hamilton-Jacobi-Bellman equation is a nonlinear second order degenerate elliptic integro-differential equation subject to gradient and state constraints. For utility functions of HARA type, we calculate the optimal investment and consumption policies together with an explicit expression for the value function. Also for the classical Merton problem, which is a special case of our optimization problem, we provide explicit policies. Instead of relying on a classical verification theorem, we verify our results within a viscosity solution framework. This framework is an adaption of the one used in our companion paper [4], which is devoted to a characterization of the value function as the unique constrained viscosity solution of the associated Hamilton-Jacobi-Bellman equation in the case of general utilities and pure-jump Lévy processes.

# 1. INTRODUCTION

The present paper continues our study in [4] of an optimal portfolio selection problem with consumption. The optimization problem captures the notions of durability and intertemporal substitution, and was first suggested and studied extensively by Hindy and Huang [14] for a market modeled by a geometric Brownian motion. In [4], we extended their model to exponential pure-jump Lévy processes and showed that the value function is the unique constrained viscosity solution of the associated Hamilton-Jacobi-Bellman equation, which is a first-order integro-differential equation subject to a gradient constraint (i.e., a first order integro-differential variational inequality).

The main topic here is to present explicit consumption and portfolio allocation rules in a Lévy market for power utility functions. We shall use a viscosity solution framework to validate our solutions, contrary to [14] who relies on a verification theorem. To this end, we extend the results on viscosity solutions in [4] to also account for Lévy processes having a

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Key words: Portfolio choice, intertemporal substitution, singular stochastic control, dynamic programming method, integro-differential variational inequality, viscosity solution, closed form solution.

JEL classification (1991): G11, C61, D91.

Mathematics Subject Classification (1991): 45K05, 49L20, 49L25, 93E20.

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# ORTIMAL FORTFOLIO MANAGELENT RULES IN A NON-GAUSSIAN MARKET WITH DÜRARMITY AND EVTERTEMPORAL SUBSTITUTION

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Eberlein and Keller [8] and Barndorff-Nielsen [3] propose to model logreturns (i.e., the logarithmic price changes) of stock prices using distributions from the generalized hyperbolic family. Following their perspective, one is lead to an exponential stock price dynamics driven by *pure-jump* Lévy processes having paths of infinite variation. This was the main motivation in [4] for concentrating on Lévy models without a continuous martingale part.

In this paper our basic model for the asset price dynamics will be

(1.1) 
$$S_t = S_0 e^{\sigma W_t + L_t},$$

where  $L_t$  is a pure-jump Lévy process,  $W_t$  is a Wiener process independent of  $L_t$  and  $\sigma$ ,  $S_0$  are constants. There are several reasons for studying such a model. First of all, from the Lévy-Khintchine representation, we know that every Lévy process can be decomposed into a purejump process and a Wiener process where the Wiener process is the continuous martingale part. Hence, from a theoretical point of view, (1.1) is a generalization of the asset price dynamics considered in [4]. However, we can also view (1.1) as a model for the asset price where  $L_t$  is a pure-jump Lévy process accounting for sudden "big" changes in the price. The Brownian motion part, on the other hand, models the "small" or "normal" variations in the price movements. This is the modeling perspective of Honoré [13], although he considers a slightly different price process (see also Section 6). Rydberg [18] discusses an approximation procedure for numerical simulation of the normal inverse Gaussian Lévy process  $L_t$ . She proposes to decompose  $L_t$  into a Brownian motion part and a pure-jump part, i.e.,

$$L_t = \sigma W_t + \tilde{L}_t.$$

For a given  $\varepsilon$ , the jump process  $L_t$  is assumed to be a Lévy process with Lévy measure

$$\tilde{\nu}(dz) = \mathbf{1}_{(-\varepsilon,\varepsilon)} \,\nu(dz),$$

where  $\nu(dz)$  is the Lévy measure of  $L_t$  and

$$\sigma^2 = \int_{-\varepsilon}^{\varepsilon} z^2 \,\nu(dz).$$

Thus,  $L_t$  has paths of finite variation. We remark that this procedure is not restricted to the normal inverse Gaussian process alone. Such an approximation is highly relevant for a numerical treatment of the portfolio optimization problem using a Markov chain discretization (see [9]). In conclusion, generalizing the theory to asset price dynamics of the form (1.1) is of interest from both a practical and theoretical point of view.

Here is an outline of the paper: In Section 2, we formulate the portfolio optimization and consumption problem and state the basic assumptions. The resulting singular stochastic control problem with a state space constraint is analyzed via the dynamic programming method and the theory of viscosity solutions in Section 3. Sections 4 and 5 present explicit rules for portfolio allocation and consumption when the utility function is of HARA type. We consider both portfolio management with intertemporal substitution and durability as well as the more classical Merton problem where utility is derived from present consumption only. Finally, we discuss some related problems in Section 6.

#### BENTH, RAMESER, AND BENEVAN

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# PORTFOLIO MANAGEMENT IN NON-GAUSSIAN MARKETS

# 2. The portfolio optimization problem and basic assumptions

Let  $(\Omega, \mathcal{P}, \mathcal{F})$  be a probability space and  $(\mathcal{F}_t)$  a given filtration satisfying the usual hypotheses. We consider a financial market consisting of a stock and a bond. Assume that the value of the stock follows the stochastic process

$$(2.1) S_t = S_0 e^{L_t},$$

where  $L_t$  is a Lévy process with Lévy-Khintchine decomposition

$$L_t = \mu t + \sigma W_t + \int_0^t \int_{|z| < 1} z \,\tilde{N}(ds, dz) + \int_0^t \int_{|z| \ge 1} z \,N(ds, dz)$$

Here,  $\mu$  and  $\sigma$  are constants,  $W_t$  is a Wiener process, N(dt, dz) is Poisson random measure on  $\mathbb{R}_+ \times \mathbb{R}$  with intensity measure  $dt \times \nu(dz)$ ,  $\nu(dz)$  is a  $\sigma$ -finite Borel measure on  $\mathbb{R} \setminus \{0\}$ with the property

(2.2) 
$$\int_{\mathbb{R}\setminus\{0\}} \min(1, z^2) \,\nu(dz) < \infty,$$

and  $N(dt, dz) = N(dt, dz) - dt \times \nu(dz)$  is the compensated Poisson random measure. We assume  $W_t$  and N(dt, dz) are independent stochastic processes. The measure  $\nu(dz)$  is called the Lévy measure. We choose to work with the unique càdlàg version of  $L_t$  and denote this also by  $L_t$ . Under the additional integrability condition on the Lévy measure

(2.3) 
$$\int_{|z|\geq 1} \left| e^{z} - 1 \right| \nu(dz) < \infty,$$

we can write the differential of the stock price dynamics as (using Itô's Formula [15])

(2.4) 
$$dS_t = \hat{\mu} S_t \, dt + \sigma S_t \, dW_t + S_{t-} \int_{\mathbb{R} \setminus \{0\}} (e^z - 1) \, \tilde{N}(dt, dz).$$

Here we have introduced the short-hand notation

(2.5) 
$$\hat{\mu} = \mu + \frac{1}{2}\sigma^2 + \int_{\mathbb{R}\setminus\{0\}} \left(e^z - 1 - z\mathbf{1}_{|z|<1}\right)\nu(dz).$$

Note that condition (2.3) is effective only when  $z \ge 1$  due to (2.2), and says essentially that  $e^z$  is  $\nu(dz)$  - integrable on  $\{z \ge 1\}$ . Moreover, this condition implies that  $\int_0^t \operatorname{E}[S_s] ds < \infty$  for all  $t \ge 0$ . Observe also that  $e^z - 1 - z \ge 0$  for all  $z \in \mathbb{R}$ .

We let the bond have dynamics

$$dB_t = rB_t \, dt,$$

where r > 0 is the interest rate. Assume furthermore that  $r < \hat{\mu}$ , which means that the expected return from the stock is higher than the return of the bond.

Consider an investor who wants to put her money in the stock and the bond so as to maximize her utility. Let  $\pi_t \in [0, 1]$  be the fraction of her wealth invested in the stock at time t and assume that there are no transaction costs in the market. If we denote the cumulative consumption up to time t by  $C_t$ , we have the wealth process  $X_t^{\pi,C}$  given as

$$\begin{aligned} X_t^{\pi,C} &= x - C_t + \int_0^t \left( r + (\hat{\mu} - r)\pi_s \right) X_s^{\pi,C} \, ds + \int_0^t \sigma \pi_s X_s^{\pi,C} \, dW_s \\ &+ \int_0^t \pi_{s-} X_{s-}^{\pi,C} \int_{I\!\!R \setminus \{0\}} \left( e^z - 1 \right) \tilde{N}(ds, dz), \end{aligned}$$

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where r > 0 is the interest rate. Assume furthermore that,  $r < \mu$ , which means that the expected resurn from the mode is higher than the returned the bond.

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where x is the initial wealth of the investor. To incorporate the idea of intertemporal substitution, Hindy and Huang [14] introduce the process  $Y_t^{\pi,C}$  modeling the average past consumption. The process has dynamics

(2.6) 
$$Y_t^{\pi,C} = y e^{-\beta t} + \beta e^{-\beta t} \int_{[0,t]} e^{\beta s} \, dC_s,$$

where y > 0 and  $\beta$  is a positive weighting factor. We shall frequently use the notation  $Y_t$  for  $Y_t^{\pi,C}$  and  $X_t$  for  $X_t^{\pi,C}$ . The integral is interpreted pathwise in a Lebesgue-Stieltjes sense. The differential form of  $Y_t$  is

$$dY_t = -\beta Y_t \, dt + \beta \, dC_t.$$

The objective of the investor is to find an allocation process  $\pi_t^*$  and a consumption pattern  $C_t^*$  which optimizes the expected discounted utility over an investment horizon. We shall here focus on an investor with an infinite investment horizon. We define the value function as

(2.7) 
$$V(x,y) = \sup_{\pi,C \in \mathcal{A}_{x,y}} \mathbb{E}\Big[\int_0^\infty e^{-\delta t} U(Y_t^{\pi,C}) dt\Big],$$

where  $\delta > 0$  is the discount factor and  $\mathcal{A}_{x,y}$  is a set of admissible controls. Let

$$\mathcal{D} = \Big\{ (x, y) \in \mathbb{R}^2 : x > 0, y > 0 \Big\}.$$

We say that a pair of controls  $(\pi, C)$  is admissible for  $x, y \in \overline{\mathcal{D}}$  and write  $\pi, C \in \mathcal{A}_{x,y}$  if:

- (c<sub>i</sub>)  $C_t$  is an adapted process that is right continuous with left-hand limits (càdlàg), nondecreasing, with initial value  $C_{0-} = 0$  (to allow an initial jump when  $C_0 > 0$ ), and satisfies  $E[C_t] < \infty$  for all  $t \ge 0$ .
- $(c_{ii})$   $\pi_t$  is an adapted càdlàg process with values in [0, 1].
- $(c_{iii}) X_t^{\pi,C}, Y_t^{\pi,C} \ge 0$  almost everywhere for all  $t \ge 0$ .

Note that condition  $(c_{iii})$  introduces a state space constraint into our control problem. The utility function  $U : [0, \infty) \to [0, \infty)$  is assumed to have the following properties:  $(u_i) \ U \in C([0, \infty))$  is nondecreasing and concave.

 $(u_{ii})$  There exist constants K > 0 and  $\gamma \in (0, 1)$  such that  $\delta > k(\gamma)$  and

$$U(z) \le K(1+z)^{\gamma},$$

for all nonnegative z, where

(2.8) 
$$k(\gamma) = \max_{\pi \in [0,1]} \left[ \gamma (r + (\hat{\mu} - r)\pi) - \frac{1}{2} \sigma^2 \pi^2 \gamma (1 - \gamma) \frac{\sigma^2}{2} \pi^2 + \int_{\mathbb{R} \setminus \{0\}} \left( \left( 1 + \pi (e^z - 1) \right)^\gamma - 1 - \gamma \pi (e^z - 1) \right) \nu(dz) \right]$$

By a Taylor expansion we see that the integral term of  $k(\gamma)$  is well-defined in a neighborhood of zero. Condition (2.3) ensures that the integral is finite outside this neighborhood, which shows that (2.8) is finite for  $\gamma \in (0, 1]$ . Note that condition  $(u_{ii})$  guarantees that the value function of the related Merton problem is well-defined, see Section 5. where z is the initial wealth of the investor. To locarporate the idea of intercomporal substitution, Bindy and Huang (14) invoduce the process [7<sup>15]</sup> messeling the average past onesumption. The process has dynamics

$$Y_{1}^{*} = \mu \sigma^{*} + \beta \sigma^{*} \int_{[0,1]} e^{\beta t} dC_{0}$$

where y > 0 and  $\beta$  is a positive weighting factor. We shall furghently use the notation K for  $V_{i}^{(1)}$  and  $X_{i}$  for  $V_{i}^{(2)}$ . The integral is interpreted publicate in a following scale sense. The differential form of  $X_{i}$  is

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where 5 > 0 is the discount factor and A., is a set of admissible controls. Lat

$$\mathcal{D} = \left\{ (a, y) \in \mathcal{M}^{1} : a > a > y = 0 \right\}$$

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(c)  $G_i$  is an adapted process that is right condumous with blockand muits (cliffied), undercreating, with initial value  $G_{n-} = 0$  (to allow an unified jump when  $G_0 > 0$ ), and ratables  $E[G_i] < \infty$  for all  $i \ge 0$ .

c.) n is an adapted carrier process with values in [0.1]

$$(c_{\rm eff}) X_1^{\pi, D}, Y_1^{\pi, D} \geq 0$$
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Note that condition  $\{e_{ni}\}$  introduces a state space constraint into our control opplant. The mility function U (6.00) + [0.00] to assumed to have the following properties: (a.)  $U \in C([0.00])$  is nondervesting and concave.

(v.) There exist constants 
$$K > 0$$
 and  $\gamma \in (0, 1)$  such that  $\delta > k(\gamma)$  and

for all nonnegative a where it

$$(2.2) = k(r) = \lim_{n \to \infty} \left( r(r + (k - r)r) - \frac{1}{2}r^{n+1}r(r - r) + \frac{1}{2}r^{n+1}r(r - r) \right)$$

$$+ \int_{R^{-1}(r)} ((1 + r(r - 1))^{n-1} - rr(r^{n-1})) r(r) dr$$

By a Taylor expansion we see that the integral term of v(y) is well-defined in a mighloritron of zero. Condition (2.3) examines that the integral is finite rataine this neighborhood, which shows that (2.8) is inter for  $v(y) \in (0, 1)$ . If she that condition (u.,) guarantees that the value function of the related Marton pressers is well-defined, see Section 5. In this paper we will assume that the dynamic programming principle holds, that is, for any stopping time  $\tau$  and  $t \ge 0$ ,

(2.9) 
$$V(x,y) = \sup_{\pi,C \in \mathcal{A}_{x,y}} \mathbb{E} \Big[ \int_0^{t \wedge \tau} e^{-\delta s} U(Y_s^{\pi,C}) \, ds + e^{-\delta(t \wedge \tau)} V(X_{t \wedge \tau}^{\pi,C}, Y_{t \wedge \tau}^{\pi,C}) \Big],$$

where  $a \wedge b = \min(a, b)$ .

Straightforward modifications (which we omit) of the proofs of Lemma 3.1 and Theorem 3.1 in [4] (see also Alvarez [1]) yield the next theorem concerning the regularity properties of the value function.

**Theorem 2.1.** The value function defined in (2.7) is non-decreasing, concave and uniformly continuous in  $\overline{\mathcal{D}}$ . Furthermore, V is non-negative and has the same sublinear growth as the utility function, i.e.,  $0 \leq V(x,y) \leq K(1+x+y)^{\gamma}$  for  $x, y \in \overline{\mathcal{D}}$ . If for some  $\alpha \in (0,1]$ , we have  $\delta > k(\alpha)$  and  $U \in C^{0,\alpha}([0,\infty))$ , then  $V \in C^{0,\alpha}(\overline{\mathcal{D}})$ . If  $\delta > k(1+\alpha)$  and  $U \in C^{1,\alpha}([0,\infty))$ , then  $V \in C^{1,\alpha}(\overline{\mathcal{D}})$ .

To our optimization problem we can associate a Hamilton-Jacobi-Bellman equation, which is a degenerate elliptic integro-differential equation subject to a gradient constraint:

(2.10) 
$$\max \left\{ \beta v_y - v_x; U(y) - \delta v - \beta y v_y + \max_{\pi \in [0,1]} \left[ (r + (\hat{\mu} - r)\pi) x v_x + \frac{1}{2} \sigma^2 \pi^2 v_{xx} + \int_{\mathbb{R} \setminus \{0\}} \left( v(x + \pi x(e^z - 1), y) - v(x, y) - \pi x v_x(x, y)(e^z - 1) \right) \nu(dz) \right] \right\} = 0 \text{ in } \mathcal{D}.$$

In other words, the Hamilton-Jacobi-Bellman equation is an integro-differential variational inequality. Note that  $x + \pi x(e^z - 1) \ge 0$  for all  $x \ge 0$  and  $z \in \mathbb{R}$ . If v is  $C^2$  and sublinearly growing, then a straightforward Taylor expansion shows that (2.10) is well-defined (see [4]).

It will be convenient to write the Hamilton-Jacobi-Bellman equation in a more compact and simplified form. To this end, we introduce the following notations:  $X = (x_1, x_2) \in \overline{\mathcal{D}}$ ,  $D_X = (\partial_{x_1}, \partial_{x_2}), D_X^2 = (\partial_{x_i x_j}^2)_{i,j=1,2}$  and  $G(D_X v) = \beta v_{x_2} - v_{x_1}$ . Furthermore, let  $\mathcal{B}^{\pi}$  be the integral operator

(2.11) 
$$\mathcal{B}^{\pi}(X,v) = \int_{\mathbb{R}\setminus\{0\}} \left( v(x_1 + x_1\pi(e^z - 1), x_2) - v(X) - \pi x_1 v_{x_1}(X)(e^z - 1) \right) \nu(dz),$$

and let

$$F(X, v, D_X v, D_X^2 v, \mathcal{B}^{\pi}(X, v)) = U(x_2) - \delta v - \beta x_2 v_{x_2} + \max_{\pi \in [0,1]} \Big[ (r + (\hat{\mu} - r)\pi) x_1 v_{x_1} + \frac{\sigma^2}{2} \pi^2 x_1^2 v_{x_1 x_1} + \mathcal{B}^{\pi}(X, v) \Big].$$

The Hamilton-Jacobi-Bellman equation (2.10) can now be written as

(2.12) 
$$\max\left(G(D_X v); F(X, v, D_X v, D_X^2 v, \mathcal{B}^{\pi}(X, v))\right) = 0 \text{ in } \mathcal{D}.$$

This is the form that we will employ in Section 3. Finally, we define the set

(2.13) 
$$C_{\ell}(\overline{\mathcal{D}}) = \left\{ \phi \in C(\overline{\mathcal{D}}) : \sup_{\overline{\mathcal{D}}} \frac{|\phi(X)|}{(1+x_1+x_2)^{\ell}} < \infty \right\}, \qquad \ell \ge 0.$$

#### PORTFOLIO MANAGEMENTIN' NON-PARISIAN MARKET

in this paper we will assume that the dynamic programming principle holds, that is, for any stopping time r and s > 0.

where  $a \wedge b = \min(a, b)$ .

Straightforward modifications (which we omit) of the proofs of Longon & Land 7 heaten. 3.1 [a [4] (see also Aivares [1]) yourd the next theorem concerning the regularity proporties of the value function.

Theorem 2.1. The value function defined in (2,7) is measured, cohome and the forming cohome 2 and the form 1 is accounted in (2,7) is measured and the second reaches a substance 2. Substance 1 is accounted in (2,7) is a second and the second reaches 2 is (2,7). If a second reaches 2 is (2,7) is a second reaches 2 is (2,7). If a second reaches 2 is (2,7) is a second reaches 2 is (2,7). The second reaches 2 is (2,7) is a second reaches 2 is (2,7). The second reaches 2 is (2,7) is a second reaches 2 is (2,7). The second reaches 2 is (2,7) is (2,7). The second reaches 2 is (2,7) is (2,7). The second reaches 2 is (2,7). The second reaches 2 is (2,7) is (2,7). The second reaches 2 is (2,7) is (2,7). The second reaches 2 is (2,7). The

To eur optimization problem we can associate a Bravilion-Jacobi-Bollakas equation, which is a degenerate eliptic integro-differential equation architect to a eradion constraint:

In other words, the Hamilton-Jacoba Beliman equation is an integradifierential variational inequality. Note that  $r \in r\pi(r^2 + 1) \geq 0$  for all  $r \geq 0$  and  $r \in I^2$  if  $r \geq 0^2$  and solutionally growing, then a straightforward Taylor expansion shows that (210) is well-lefted (3cc [4]). It will be convented to write the Hamilton-Jacobi Balinan equation in a more compact and submate  $D_r = (R_r, R_r) \geq 0$  for all  $r \geq 0$  and  $r \in I^2$  if  $r \geq 0^2$  and solution  $P_r$  and  $P_r$  and  $P_r$  and solution  $P_r$  and  $P_r$  a

$$(2.11) \quad (27'(3', t)) = \int_{23}^{t} (m) \left( 2(2) + m(t) - (1) + (1) - m(t) - m(t) + (1) \right) p(ds)$$

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$$[u, Z_{i}]^{2} \xrightarrow{} \left[ u - \delta - \frac{1}{2} u - \delta - \frac{1}{2} u - \frac{1}{2} \left[ (x + \lambda) + x \right] \frac{1}{2} \left[ (x - \lambda) + x \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) + x \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - \delta - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda - \lambda) - (\lambda - \lambda) \right] \frac{1}{2} \left[ (x - \lambda) - (\lambda -$$

the Hamilton-decobilishiman-equation (2.19) cut now be written as

$$(2.12) \qquad \max\{G(D_{\mathcal{X}}, v), B(X_{\mathcal{X}}, v, D_{\mathcal{Y}}v, D_{\mathcal{Y}}v, B^{*}(X_{\mathcal{X}}, v))\} = 0 \text{ in } D$$

This is the focus ther we will employ to Serden's. Finally, we define bits see

$$(2.13) \quad (245) = \{a \in C(5) : aga (1+a^{-1}+1) \}$$

# BENTH, KARLSEN, AND REIKVAM

# 3. VISCOSITY SOLUTIONS OF THE HAMILTON-JACOBI-BELLMAN EQUATION

We shall rely on a viscosity solution framework to verify the closed form solutions derived in Sections 4 and 5. The (constrained) viscosity solution framework presented below is a straightforward adaption (to the second order case) of the framework developed in [4] for first order integro-differential variational inequalities. Because of the strong similarities with [4], we will be very brief in this section and instead refer to [4] for details not found herein. Also, we refer to [4] for an overview of the existing literature on viscosity solutions of integrodifferential equations. For a general overview of the viscosity solution theory, we refer to the survey paper by Crandall, Ishii, and Lions [7] and the book by Fleming and Soner [10].

A constrained viscosity solution of (2.12) is defined as follows:

**Definition 3.1.** (i) Let  $\mathcal{O} \subset \overline{\mathcal{D}}$ . Any  $v \in C(\overline{\mathcal{D}})$  is a viscosity subsolution (supersolution) of (2.12) in  $\mathcal{O}$  if and only if we have, for every  $X \in \mathcal{O}$  and  $\phi \in C^2(\overline{\mathcal{D}}) \cap C_1(\overline{\mathcal{D}})$  such that X is a global maximum (minimum) relative to  $\mathcal{O}$  of  $v - \phi$ ,

$$\max\left(G(D_X\phi); F(X, v, D_X\phi, D_X^2\phi, \mathcal{B}^{\pi}(X, \phi))\right) \ge 0 \ (\le 0).$$

(ii) Any  $v \in C(\overline{\mathcal{D}})$  is a constrained viscosity solution of (2.12) if and only if v is a supersolution of (2.12) in  $\mathcal{D}$  and v is a subsolution of (2.12) in  $\overline{\mathcal{D}}$ .

Exactly the same argumentation (which we omit) as in the proof of [4, Thm. 4.1] leads to the constrained viscosity property of the value function.

**Theorem 3.1.** The value function V(x, y) defined in (2.7) is a constrained viscosity solution of the integro-differential variational inequality (2.12).

To prove that the value function is the *only* solution of (2.12), we need a comparison principle similar to Theorem 4.2 in [4]. We outline below how we can extend the proof of [4, Thm. 4.2] to the second order integro-differential variational inequality (2.12).

First, note that to distinguish the singularities at zero and infinity it is advantageous to split the integral operator into two parts. For any  $\kappa \in (0,1), X \in \overline{\mathcal{D}}, \phi \in C_1(\overline{\mathcal{D}})$  and  $P = (p_1, p_2) \in \mathbb{R}^2$ , define

$$\mathcal{B}^{\pi,\kappa}(X,\phi,P) = \int_{|z|>\kappa} \Big(\phi(x_1+x_1\pi(e^z-1),x_2) - \phi(X) - \pi x_1 p_1(e^z-1)\Big)\nu(dz),$$
$$\mathcal{B}^{\pi}_{\kappa}(X,\phi) = \int_{|z|\leq\kappa} \Big(\phi(x_1+x_1\pi(e^z-1),x_2) - \phi(X) - \pi x_1\phi_{x_1}(X)(e^z-1)\Big)\nu(dz).$$

Observe that for  $\phi \in C^2(\overline{\mathcal{D}}) \cap C_1(\overline{\mathcal{D}})$ , we can write (see [4])

(3.1) 
$$\mathcal{B}^{\pi}(X,\phi) = \mathcal{B}^{\pi,\kappa}(X,\phi,D_X\phi) + \mathcal{B}^{\pi}_{\kappa}(X,\phi).$$

Equipped with this decomposition, we introduce the (slightly shorter) notation

$$F(X, v, P, A, \mathcal{B}^{\pi,\kappa}(X, v, P), \mathcal{B}^{\pi}_{\kappa}(X, \phi)) := F(X, v, P, A, \mathcal{B}^{\pi,\kappa}(X, v, P) + \mathcal{B}^{\pi}_{\kappa}(X, \phi))$$

for  $v \in C_1(\overline{\mathcal{D}})$  and  $\phi \in C^2(\overline{\mathcal{D}})$ .

When proving comparison results for second order equations, it is more convenient to use a formulation of viscosity solutions based on the notions of subjet and superjet.

6

#### BENTRE KARLEN AND BELKYLED

## 3. VISCORITY SOBULTONS OF THE REMITTOR-LANCED BELLMAN EQUATION

We shall raty on a viscosity solution insurance to verse, the closed form solutions to a subin Sections 4 and 5. The (constrained) viscosity solution framework presented below is a straightforward adaption (to the second order eases) of the framework developed in [4] for first order integro differential versions and managed refer to [4] for floading and rate below is [6], we will be very brief in this second, and integrate refer to [4] for floading on frame below. [6], we will be very brief in this second, and integrate refer to [4] for floading on frame below. [6], we will be very brief in this second, and integrate refer to [4] for floading on frame below. [6], we will be very brief in this second, and the contract below to [4] for floading on frame below. [6], we will be very brief in this second out integrate the first of the floading of frame below of the second address to [4] for an excerct overview of the contract below to [4] for floading of the second below. [6], we refer to [6] for an external overview of the contract below to [4] for floading on the second below of the second address below to [6] for an external description of the below by free the second below to be the second second below to [6] for second below of the second below of the second below of the second below of the below by free the second below of the second below of the second below of the below of the below by free the second below of the second below of the second below of the below of the below of the below by free the below of the second below of the second below of the second below of the second below of the below of the

Definition 3.1. (3.1ec O C T. Any we COD) is a stanif, annalation (anacalution) of (2.12) in O R and only R Madate, for every 5 e C and a SC (7) CO(C) such that N is a stobal measurem (minimum) relative to O of a -io

$$\max\{G(B, y\phi); F(X, y, D, y\phi, D, \phi), F'(X, \phi)\}\} \geq 0 \leq 0$$

(ii) Any  $v \in C(D)$  is a construined viscosity solutions of (2.22) if and only lively a supposed that of (2.22) in D and v is a substitution of (2.22) in  $\tilde{D}$ .

Exactly the same argumentation (which we emit) as in the proof of [4, 45m, 4.1] leads to the constrained viscosity property of the value histicae

Theorem 3.1. The value function 1 (2.7) refined in (2.7) as a construction according belongen of the integra-differential contextant inspective (2.22).

To prove that the fatue function is the only solution of (2.13), we need a comparison principle similar to? become 4.2 in [4]. We outline below boy we rate extend the proof of [4, Thu. 4.2) to the second order integro-differential variational inequility (2.13).

First, note that to distinguish the singularities at sets and infinite it is advantagenerate split the integral operator into two parts. For any  $n \in (0, 1)$ ,  $N \in \overline{D}$ ,  $\phi \in G(D)$ , and  $F = (m, m) \in \overline{D}^*$ , define

$$P^{\alpha}(X, u; P) = \int_{D, u} \left( \phi(x_1 + x_1 u) e^{-1} (x_2) - \phi(X) - \pi u_1 u(e^{-1} u) \right) P(d_1) \\ P(X, u) = \int_{D, u} \left( \phi(x_1 + x_1 u) e^{-1} (x_2 - 1) + u(X) - \pi u_1 u_2 (e^{-1} u) \right) P(d_2) \\ P(X, u) = \int_{D, u} \left( \phi(x_1 + x_1 u) e^{-1} (x_2 - 1) + u(X) - \pi u_1 u_2 (u) \right) P(d_2) \right) P(d_2)$$

Discrete that for S. C. (D) (LG, (D), we can write (see M)

$$(X, A) = B^{-n}(X, A) = B^{-n}(X, A) + B^{-n}(X, A)$$

Equipped with this decomposition, we involve the (slightly monther), northing

## or sie C.(D) and o e C\*(D).

When anyting comparison results for eccord order equalions. It is more correlable to use a formulation of viscosity solutions inseed on the actions of subject and superject.

**Definition 3.2.** Let  $S^N$  denotes the set of  $N \times N$  symmetric matrices,  $\mathcal{O} \subset \overline{\mathcal{D}}$ ,  $v \in C(\mathcal{O})$ , and  $X \in \mathcal{O}$ . The second order superjet (subjet)  $J_{\mathcal{O}}^{2,+(-)}v(X)$  is the set of  $(P, A) \in \mathbb{R}^2 \times S^2$ such that

$$v(Y) \le (\ge 0) v(X) + \langle P, Y - X \rangle + \frac{1}{2} \langle A(Y - X), Y - X \rangle + o(|X - Y|^2) \text{ as } \mathcal{O} \ni Y \to X.$$

The closure  $\overline{J}_{\mathcal{O}}^{2,+(-)}v(X)$  is the set of (P,A) for which there exists a sequence  $(P_n,A_n) \in J_{\mathcal{O}}^{2,+(-)}v(X_n)$  such that  $(X_n,v(X_n),P_n,A_n) \to (X,v(X),P,A)$  as  $n \to \infty$ .

Before we can give a suitable definition of viscosity solutions based on sub- and superjets, we need an equivalent formulation of viscosity solutions in  $C_1(\overline{\mathcal{D}})$  based on test functions (which takes into account the decomposition (3.1)).

**Lemma 3.1.** Let  $v \in C_1(\overline{D})$  and  $\mathcal{O} \subset \overline{D}$ . Then v is a viscosity subsolution (supersolution) of (2.12) in  $\mathcal{O}$  if and only if we have, for every  $\phi \in C^2(\overline{D})$  and  $\kappa > 0$ ,

 $\max\left(G(D_X\phi); F(X, v, D_X\phi, D_X^2\phi, \mathcal{B}^{\pi,\kappa}(X, v, D_X\phi), \mathcal{B}^{\pi}_{\kappa}(X, \phi))\right) \ge 0$ 

whenever  $X \in \mathcal{O}$  is a global maximum (minimum) relative to  $\mathcal{O}$  of  $v - \phi$ .

This lemma is a straightforward extension of [4, Lem. 4.1] and the proof is therefore omitted. Let  $v \in C(\overline{\mathcal{D}})$  and  $\mathcal{O} \subset \overline{\mathcal{D}}$ . Then using the arguments in, e.g., [10] one can easily prove that  $(P, A) \in J_{\mathcal{O}}^{2,+(-)}v(X)$  if and only if there exists  $\phi \in C^2(\overline{\mathcal{D}})$  such that  $\phi(x) = v(x)$ ,  $D_X\phi(X) = P$ ,  $D_X^2\phi(X) = A$ , and  $v - \phi$  has a global maximum (minimum) relative to  $\mathcal{O}$  at X. In view of Lemma 3.1 and continuity of the governing equation, the following formulation of viscosity solutions in  $C_1$  based on sub- and superjets is now immediate.

**Lemma 3.2.** Let  $v \in C_1(\overline{\mathcal{D}})$  be a subsolution (supersolution) of (2.12) in  $\mathcal{O} \subset \overline{\mathcal{D}}$ . Then, for all  $\kappa > 0, X \in \mathcal{O}, (P, A) \in \overline{J}_{\mathcal{O}}^{2,+(-)}v(X)$ , there exists  $\phi \in C^2(\overline{\mathcal{D}})$  such that

 $\max\left(G(P); F(X, v, P, A, \mathcal{B}^{\pi,\kappa}(X, v, P), \mathcal{B}^{\pi}_{\kappa}(X, \phi))\right) \ge 0 \ (\le 0).$ 

The test function  $\phi$  is such that  $v - \phi$  has a global maximum (minimum) relative to  $\mathcal{O}$  at  $X_n$  with  $X_n \to X$  as  $n \to \infty$ .

A similar formulation is also used in Pham [17]. To prove a comparison principle for (2.12), we shall need the following maximum principle for semicontinuous function taken from Crandall, Ishii, and Lions [7]:

**Lemma 3.3** ([7]). Let  $\mathcal{O} \subset \mathbb{R}^N$  be locally compact. Let  $u_1, -u_2$  be upper semicontinuous and  $\varphi$  twice continuously differentiable in a neighborhood of  $\mathcal{O} \times \mathcal{O}$ . Suppose  $(\hat{X}, \hat{Y}) \in \mathcal{O} \times \mathcal{O}$  is a local maximum of  $u_1(X) - u_2(Y) - \varphi(X, Y)$  relative to  $\mathcal{O} \times \mathcal{O}$ . Then for every  $\varsigma > 0$  there exist two matrices  $A, B \in S^N$  such that

$$(D_X\varphi(\hat{X},\hat{Y}),A) \in \overline{J}_{\mathcal{O}}^{2,+}u_1(\hat{X}), \qquad (-D_Y\varphi(\hat{X},\hat{Y}),B) \in \overline{J}_{\mathcal{O}}^{2,-}u_2(\hat{Y}),$$

and

$$(3.2) \qquad -\left(\frac{1}{\varsigma} + \|D^2\varphi(\hat{X},\hat{Y})\|\right)I \le \begin{pmatrix} A & 0\\ 0 & -B \end{pmatrix} \le D^2\varphi(\hat{X},\hat{Y}) + \varsigma\left(D^2\varphi(\hat{X},\hat{Y})\right)^2.$$

Definition 3.2. Let  $S^{n}$  determs the set of  $N \times N$  symmetric matrices  $O \in \mathbb{C}$ ,  $n \in C(O)$ , and  $X \in O$ . The becould order superint (subjet)  $J_{O}^{2,+(-)}v(X)$  is the set of (2.4)  $\in \mathbb{R}^{2} \times S^{4}$  such that

 $a(Y) \le (\ge 0) a(X) + (Y, Y - X) + b(X(Y - X), Y - X) + a(X - Y) + a(X) - b(X - Y) + a(X) - b(X) - b$ 

The domain  $X_0^{(1)}$   $u(X_0)$  is the set of (F, A) for which there easily a requested  $(F_0, A_0) \in V_0^{(1)}$ ,  $u(X_0)$  such that  $(A_0, e(X_0), F_0, A_0) = \{X_0, e(X_0), F(X_0)\}$  is a set of

Before we can give a subable definition of viscosity valuations haved or sub- and viperates we need as equivalent formalation of viscosity solutions is G(P) haved on test functions (which takes into account the decomposition (3.1))

Lemma 3.1. Let  $\sigma \in C_{\delta}(D)$  and  $O \in T$ . Then p is a uncounty subsolution (repotseletion) of (2.12) in O if and only if we blue, for curves  $\in C^{2}(T)$  and  $\kappa > 0$ .

whenever X & O is a divid maximum (minimum) relative to O of a - 161

This lemma is a straightforward extension of [4, 1 and 4 [6 and the proof is therefore control, Let  $v \in C(\overline{D})$  and  $D \in \overline{D}$ . Then using the arguments in, eq. (16) one can easily prove that  $(P, A) \in \overline{D}_{1}^{-1/2}(X)$  if and only if there exists  $v \in C^{1}(\overline{D})$  such that D(v) = v(v)  $D_X \phi(X) = P, D_{1}^{1} \phi(X) = A$ , and v = b has a global maximum (minimum) relative to O at X. In view of Lemma 3.1 and continuity of the governing equation, the following formula tion of vieweity solutions in C. based on sub- and superints is now immediate

**Lemma 3.2.** Let  $v \in C_1(\mathbb{D})$  be a subsolution (approvchiteca) of (2.12) in  $\mathcal{O} \subset \mathbb{D}$ . Then, for all u > 0,  $X \in O$ .  $(P, X) \in \mathbb{C}^{(1+1)}$ , there exists  $v \in C^{(1)}$  such that

hex (0(P) F(X, r, P, 1, 55° X, r, P) 37(X, 6)) > 0(5 m)

The test function  $\phi$  is such that  $v - \phi$  has a global maximum (manuscul relation to  $\Omega$  as  $X_{\mu}$ , with  $X_{\mu} \rightarrow X$  as  $a \rightarrow \infty$ .

A similar formulation is also used in Phane [17]. To prove a comparison principle for (2.12), we shall need the following maximum principle for semicontinuous function takes from Crankall, ishii, and Islans [7].

Lemma 3.3 ([7]). Let  $O \subset \mathbb{R}^{n-1}$  is leasily compare: bet  $n_{1} = n_{2}$  be upper semicontration on the p funce continuously differentiable in a neighborizon of  $O \times O$ . Suppose  $(X, Y) \in O \times O$  is a local maximum of  $n_{1}(X) = n_{2}(Y) = n_{2}(Y, Y)$  relative to  $O \times O$ . Then, for absence > 0 there exist too matrices A,  $G \in S^{n-1}$  such that

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$$(3.2) \qquad -\left(\frac{1}{2} + \frac{1}{2} \ln^2 n + \frac{1}{2} \ln^2 n \le \left(\frac{1}{2} - \frac{n}{2}\right) \le D^2 n + \frac{1}{2} \ln^2 n + \frac{1}{2}$$

Let  $\underline{v} \in C(\overline{\mathcal{D}})$  be a subsolution of (2.12) in  $\overline{\mathcal{D}}$  and  $\overline{v} \in C(\overline{\mathcal{D}})$  a supersolution of (2.12) in  $\mathcal{D}$ . Choosing  $\tilde{K}$  and  $\overline{\gamma}$  properly, one can show (following closely the proof of Lemma 4.3 in [4]) that  $w = \tilde{K} + (1 + x_1 + \frac{x_2}{2\beta})^{\overline{\gamma}}$  and thus

$$\overline{v}^{\theta} = (1 - \theta)\overline{v} + \theta w, \qquad \theta \in (0, 1],$$

are strict supersolutions of (2.12) in any bounded subset of  $\mathcal{D}$ . We claim that

 $\underline{v} \leq \overline{v}^{\theta}$  in  $\overline{\mathcal{D}}$ ,

which immediately implies that the comparison principle holds between v and  $\overline{v}$ .

Except for the treatment of the second order term, which relies in an essential way on Lemma 3.3, the proof of our comparison principle is very similar to the proof of Theorem 4.2 in [4], which the reader is referred to for details not found below.

As in the first-order case [4], we utilize our choice of a strict supersolution  $\overline{v}^{\theta}$  to "localize" the proof to the following bounded domain

(3.3) 
$$\mathcal{K} := \left\{ (x_1, x_2) : 0 < x_1 < R(1 + e^1), 0 < x_2 < R \right\},$$

where R is some positive constant chosen such that  $\underline{v} \leq \overline{v}^{\theta}$  in  $\{x_1, x_2 \geq R\}$ . To prove the comparison result it is now sufficient to show that  $\underline{v} \leq \overline{v}^{\theta}$  in  $\overline{\mathcal{K}}$ .

Assume that the contrary is true, i.e.,

(3.4) 
$$M := \max_{\overline{K}} (\underline{v} - \overline{v}^{\theta})(Z) > 0$$

holds for some  $Z \in \overline{\mathcal{K}}$ . Then either  $Z \in (0, R) \times (0, R)$  or  $Z \in \Gamma_{SC}$ , where

$$\Gamma_{\rm SC} = \Big\{ (x_1, x_2) : x_1 = 0, 0 \le x_2 < R \text{ or } 0 \le x_1 < R, x_2 = 0 \Big\}.$$

Here we consider only the latter case, the case  $Z \in \mathcal{K}$  is treated similarly (consult Case II in the proof of [4, Thm. 4.2]).

Let  $(X_{\alpha}, Y_{\alpha})$  be a maximizer of the function  $\Phi(X, Y) : \overline{\mathcal{K}} \times \overline{\mathcal{K}} \to \mathbb{R}$ , defined for any  $\alpha > 1$ and  $0 < \varepsilon < 1$  as

(3.5) 
$$\Phi(X,Y) = \underline{v}(X) - \overline{v}^{\theta}(Y) - |\alpha(X-Y) + \varepsilon \eta(Z)|^2 - \varepsilon |X-Z|^2.$$

The uniformly continuous function  $\eta: \overline{\mathcal{K}} \to \mathbb{R}^2$  satisfies

$$B(X + t\eta(Z), td) \subset \mathcal{K}$$
 for all  $X \in \overline{\mathcal{K}}$  and  $t \in (0, t_0]$ ,

for positive constants  $d, t_0$  and B(X, r) denotes the open ball in  $\mathbb{R}^2$  centered at X and with radius r. The construction (3.5) is ultimately due to Soner [19].

It is standard to see that the penalized maxima  $(X_{\alpha}, Y_{\alpha})$  satisfy as  $\alpha \to \infty$  (see, e.g., [4]): (i)  $X_{\alpha}, Y_{\alpha} \to Z$ , (ii)  $\alpha(X_{\alpha} - Y_{\alpha}) + \varepsilon \zeta(Z) \to 0$ , (iii)  $(\underline{v}(X_{\alpha}) - \overline{v}^{\theta}(Y_{\alpha})) \to M$ , (iv)  $M_{\alpha} \to M$ . In view of (ii) and (3.4), we conclude that  $Y_{\alpha} \in (0, R) \times (0, R)$  and  $X_{\alpha} \in [0, R) \times [0, R)$ . Using the maximum principle for semicontinuous functions (Lemma 3.3) with

$$\varphi(X,Y) = |\alpha(X-Y) + \varepsilon \eta(Z)|^2 + \varepsilon |X-Z|^2, \quad u_1 = \underline{v}, \quad u_2 = \overline{v}^{\theta}, \quad \mathcal{O} = \overline{\mathcal{K}},$$

we conclude that there exist matrices  $A = (a_{ij})_{i,j=1,2}, B = (b_{ij})_{i,j=1,2} \in S^2$  such that

$$(P, A) \in \overline{J}_{\overline{\mathcal{K}}}^{2,+} \underline{v}(X_{\alpha}), \qquad P = D_X \varphi(X_{\alpha}, Y_{\alpha}) = 2\alpha [\alpha(X_{\alpha} - Y_{\alpha}) + \varepsilon \eta(Z)] + 2\varepsilon (X_{\alpha} - Z),$$
  

$$(Q, B) \in \overline{J}_{\overline{\mathcal{K}}}^{2,-} \overline{v}^{\theta}(Y_{\alpha}), \qquad Q = -D_Y \varphi(X_{\alpha}, Y_{\alpha}) = 2\alpha [\alpha(X_{\alpha} - Y_{\alpha}) + \varepsilon \eta(Z)].$$

Let  $y \in C(D)$  be reactionation of (2,12) in D and  $U \in C(D)$  a superconduction of (2,12) in D. Choosing K and  $\Xi$  property, analogs above (following closely the proof of Lemma 4.3 in [4]) that  $w = K + (1 + x_1 + 45)^2$  and thus

are strict supersatutions of (2.12) in any bounded subset of D. We shall that

which immediately immines that the companisors minciple holds between y and re-

Except for the treatment of the second order true, which relies in an evently way are it. Lemma 3.3, the proof of our comparison protocole is very similar to the proof of Theorem 4.2 in [d], which the resider is referred to for quarks not found below.

As in the first-order case [4], we utilize our choice of a strict approach tion 7° to Toosfize? the proof to the following bounded domain

$$(3.3) = \{(\gamma + 1), (\gamma + 1)\}, (\beta < \beta) > 0, ((\beta + \beta))\} = \{(\gamma + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta)\}, (\beta + \beta) = \{(\beta + \beta), (\beta + \beta)\}, (\beta + \beta)\},$$

where R is some positive constant chosen such that  $g \leq W$  in  $\{x_0, z_0 \geq R\}$ . To prove the comparison result it is now unlichent to spay that  $g \in W$  in  $\hat{L}$ .

Assume that the company is merical

$$(3.4) = (3.4) = 0$$

holds for some Z C C. Then either Z C 10. El X 10. El or Z C Leo, which

$$f_{SC} = \{(x_1, x_2) : x_1 = 0, 0 \le x_2 < R \text{ or } 0 \le x_1 < R, x_2 = 0\}$$

Here we consider only the tablet case, the case  $\mathcal{I} \in \mathcal{K}$  is breaked similarly (crossfit Case II in the proof of [4, Thus, 4.2]).

Let  $(X_n, Y_n)$  be a maximizer of the function  $\Psi(X, Y) : K \times K \mapsto R$ , defined for any  $n \geq 4$  and 0 < c < 1 as

The uniformly continuous function w: X -> R' satisfies

$$[M(X) \mapsto m(X), M(Y) \subset K \text{ for all } X \in K \text{ and } Y \in (0, \infty).$$

for positive constants  $d_i h_i$  and B(X, r) denotes the open ball in  $A^{i}_{i}$  centered at X and with radius r. The construction (3.5) is ultimately due to Same [19].

It is standard to see that the pendiced maxima  $(X_{i,i}, b_i)$  satisfy is  $m \rightarrow \infty$  (see, e.g., (1): (i)  $X_{i,i}, Y_{i,j} \rightarrow Z_{i,j}$  (ii)  $a_i(X_{i,j} \rightarrow Y_{i,j}) \rightarrow 0$ . (iii)  $(g(X_{i,j}) \rightarrow g'(X_{i,j})) \rightarrow g'(X_{i,j})$   $a_i(M_{i,j}) \rightarrow M_{i,j}$ Is view of (ii) and (5.45, we concluse that  $Y_{i,j} \in [0, R] \times (0, R)$  and  $Y_{i,j} \in [0, R) \times [0, R)$ . Using the maximum mindule for settioner functions (Lemma 3.3) with

$$x = 0$$
  $x = e_1(X - X) = e_2(X) + x(x - X) = e_1(X - X) = e_1(X)$ 

we conclude that there exist matrices A = (eq.), e.e. a B = (br.), e.e. c & such they

$$(P, A) \in \mathcal{F}_{2}^{*}(X_{0}), \quad P = O_{X} \varphi(X_{0}, Y_{0}) = 2o(\varphi(X_{0} - Y_{0}) + 2o(Y_{0})) + 2o(Y_{0} - Y_{0})$$

$$(O, B) \in \mathcal{F}_{2}^{*}(Y_{0}), \quad O = -D(-D(-Y_{0}), Y_{0}) = 2o(\varphi(X_{0} - Y_{0}) + 2o(Y_{0}))$$

Following, e.g., [7] it is not difficult to show that (3.2) implies

(3.6) 
$$\lim_{\varepsilon \to 0} \lim_{\alpha \to \infty} \left( \frac{\sigma^2}{2} \pi x_{\alpha 1}^2 a_{11} - \frac{\sigma^2}{2} \pi y_{\alpha 1}^2 b_{11} \right) \le 0.$$

Since  $\overline{v}^{\theta}$  is a strict supersolution of (2.12) in  $\mathcal{D}$  there exists, thanks to Lemma 3.2,  $\psi \in C^2(\overline{\mathcal{D}})$  such that

(3.7) 
$$F(Y_{\alpha}, \overline{v}^{\theta}, Q, B, \mathcal{B}^{\pi, \kappa}(Y_{\alpha}, \overline{v}^{\theta}, Q), \mathcal{B}^{\pi}_{\kappa}(Y_{\alpha}, \psi)) < -\vartheta,$$

for some constant  $\vartheta > 0$ . Similarly, since  $\underline{v}$  is a subsolution of (2.12) in  $\overline{\mathcal{D}}$ , there exists  $\phi \in C^2(\overline{\mathcal{D}})$  such that

(3.8) 
$$F(X_{\alpha}, \underline{v}, P, A, \mathcal{B}^{\pi,\kappa}(X_{\alpha}, \underline{v}, P), \mathcal{B}^{\pi}_{\kappa}(X_{\alpha}, \phi)) \ge 0.$$

Having (3.6) in mind, we now subtract (3.7) from (3.8) and send (in that order)  $\alpha \to \infty$ ,  $\varepsilon \to 0$ , and  $\kappa \to 0$ . These limit operations lead (after some tedious work) to the contradiction  $(\underline{v} - \overline{v}^{\theta})(Z) < 0$  (consult Case I in the proof of [4, Thm. 4.2]).

Summing up, we have proven the following comparison (uniqueness) theorem:

**Theorem 3.2.** Let  $\gamma' > 0$  be such that  $\delta > k(\gamma')$ . Assume  $\underline{v} \in C_{\gamma'}(\overline{D})$  is a subsolution of (2.12) in  $\overline{D}$  and  $\overline{v} \in C_{\gamma'}(\overline{D})$  is a supersolution of (2.12) in D. Then  $\underline{v} \leq \overline{v}$  in  $\overline{D}$ . Consequently, in the class of sublinearly growing solutions, the Hamilton-Jacobi-Bellman equation (2.12) admits at most one constrained viscosity solution.

# 4. EXPLICIT CONSUMPTION AND PORTFOLIO ALLOCATION RULES

In this section we study a case where we can construct an explicit solution to the control problem. The case is taken from Hindy and Huang [14], who construct an explicit solution to the optimization problem when the utility function is of HARA (Hyperbolic Absolute Risk Aversion) type and the price of the stock follows a geometric Brownian motion. We show in this section that a more realistic price model with a Lévy process instead of Brownian motion leads to a similar solution. We consider a general Lévy process which leads to the second-order integro-differential variational inequality (2.10). We are able to solve this equation, and construct optimal consumption and portfolio allocation strategies by closely following the arguments in [14]. Note, however, that our results are not as explicit as those in [14]. For instance, the optimal allocation strategy  $\pi^*$  is the solution of an integral equation involving the Lévy measure of the noise process.

For  $\gamma \in (0, 1)$ , consider the utility function

$$U(z) = \frac{z^{\gamma}}{\gamma}.$$

We recall that  $1 - \gamma$  is the risk aversion coefficient. Motivated by Hindy and Huang [14], we guess that the optimization problem has a constrained viscosity solution of the form

(4.1) 
$$V(x,y) = \begin{cases} k_1 y^{\gamma} + k_2 y^{\gamma} \left[\frac{x}{ky}\right]^{\rho}, & 0 \le x < ky, \\ k_3 \left(\frac{y+\beta x}{1+\beta k}\right)^{\gamma}, & x \ge ky > 0, \end{cases}$$

for some constants  $k_1, k_2, k_3, k$ , and  $\rho > \gamma$ . This solution is constructed from the assumption that we can split the state space into two parts, on which each of the terms in the variational

Following, e.g., [7] It is not difficult to show that (3.2] implies all set

Since V is a shirt supersolution of (2.12) in Il there onists, reader to Lemma 3.2. & C (2): such that

for some constant  $\theta > 0$ . Similarly, since g is a subsolution of (2.12) in  $\overline{P}_{1}$  there exists  $\phi \in C^{2}(\overline{D})$  such that

Having (3.6) in minst, we now antitact (3.7) from (3.9) and good (in that order) for  $\rightarrow \infty$ ,  $z \rightarrow 0$ , and  $z \rightarrow 0$ . These finit operations lead (after some redents work) to the contradiction (z - 7)(Z) < 0 (consult Case 1 in the proof of [4. That, (2.2)].

Summing up, we have proven the following computings (uniquencia) theorem:

Theorem 3.2. Let  $\gamma' > 0$  be such that  $\delta > b(\gamma')$ , standard  $\gamma \in C.(D)$  is a velocitizer of (2.13) in D, C.(D) is a velocitizer of (2.13) in D. Consequently, in D. Consequently, in the class of antineardy property pointion of (2.13) in D. Consequently, and the class of antineardy property pointions, the Banditon-Lacobi-Ballanan equition. (2.13) are a minimum to construct accord y solution.

## EXPLICIT CONSUMPTION AND BOATFOLIO ALLOCATION BEERS

In this section we study a case where we can construct an explicit addition to this control problem. The case is taken from Hindy and Huang [14], who construct an explicit solution to the optimization problem when the utility function is of UARA (it patholic Atashu e Risk Average) type and the proof of the stock follows a geometric Browging motion. We also the section tiest a more realistic price model with a Lévy process instead of Browning motion backs to a conflict solution. We consider a general Lévy process instead of Browning motion coder integro-differential estational inequality (2.13). We are able to solve the secondtion construct ordinal constantion and portfolio allocation strategies by donest following the assumetts in [14]. Note, however, that our remits we get as explicit as those in [14]. For the Lévy researce of the quier process the Lévy researce of the units process.

noiton a filling and isolation (1.0) by not

We recall that  $1 - \gamma$  is the risk aversion coefficient, Monivated by Ricchy and Ready (14), we areas that the annalization problem has a constrained viscourty solution of No term

$$\left[ \left\{ a, b \right\}^{-1} = \left\{ \left\{ a, b \right\}^{-1} = \left\{$$

for some constants  $k_1$ ,  $k_2$ ,  $k_3$ , and  $\mu > \gamma$ . This solution is constructed from the symptron that we can solve the state space into two parts, on which each of the terms in the variational

inequality (2.10) is effective. Hence, for  $0 \le x < ky$ , we construct the solution from the assumption that

(4.2) 
$$\frac{y^{\gamma}}{\gamma} - \delta V - \beta y V_y + \max_{\pi \in [0,1]} \left[ (r + (\hat{\mu} - r)\pi) x V_x + \frac{1}{2} \sigma^2 \pi^2 x^2 V_{xx} + \int_{\mathbb{R} \setminus \{0\}} \left( V(x + \pi x (e^z - 1), y) - V(x, y) - \pi x V_x(x, y) (e^z - 1) \right) \nu(dz) \right] = 0$$

and, when  $x \ge ky > 0$ ,

$$(4.3)\qquad\qquad\qquad\beta V_y - V_x = 0.$$

We see that the integral in (4.2) is well defined by the condition in (2.3). In what follows, all the displayed integrals are convergent by the same condition. In the rest of this section we derive expressions for the different constants in the solution, and find the optimal allocation and consumption processes. Optimize the kernel of (4.2) with respect to  $\pi$  to find the first order condition for an optimum

$$(\hat{\mu} - r)xV_x + \sigma^2 \pi x^2 V_{xx} + \int_{\mathbb{R} \setminus \{0\}} \left( V_x(x + \pi x(e^z - 1), y)x(e^z - 1) - xV_x(x, y)(e^z - 1) \right) \nu(dz) = 0.$$

Inserting the guessed solution (4.1) for x < ky, we get the expression

(4.4) 
$$(\hat{\mu} - r) - (1 - \rho)\sigma^2 \pi + \int_{\mathbb{R}\setminus\{0\}} \left( \left(1 + \pi(e^z - 1)\right)^{\rho - 1}(e^z - 1) - (e^z - 1) \right) \nu(dz) = 0.$$

Assume from now on that  $\pi^*$  is a solution of (4.4). Note that  $\pi^*$  is constant with respect to time which gives that the optimal investment rule is to hold a constant fraction of the wealth in the stock. With this  $\pi^*$ , we can find equations for the unknown constants  $k_1$  and  $\rho$ . Inserting (4.1) into (4.2), we obtain

$$y^{\gamma} \left(\frac{1}{\gamma} - \delta k_{1} - \beta \gamma k_{1}\right) + k_{2} y^{\gamma} \left[\frac{x}{ky}\right]^{\rho} \left\{-\delta - \beta(\gamma - \rho) + (r + (\hat{\mu} - r)\pi^{*})\rho - \frac{1}{2}\sigma^{2}\pi^{2}\rho(1 - \rho) + \int_{\mathbb{R}\setminus\{0\}} \left(\left(1 + \pi^{*}(e^{z} - 1)\right)^{\rho} - 1 - \rho\pi^{*}(e^{z} - 1)\right)\nu(dz)\right\} = 0.$$

The only way the left-hand side can be zero is when

$$\left( r + (\hat{\mu} - r)\pi^* + \beta - \frac{1}{2}\sigma^2\pi^{*2}(1-\rho) \right) \rho$$
  
=  $\delta + \beta\gamma - \int_{\mathbb{R}\setminus\{0\}} \left( \left( 1 + \pi^*(e^z - 1) \right)^\rho - 1 - \rho\pi^*(e^z - 1) \right) \nu(dz)$ 

and

(4.5)

$$k_1 = \frac{1}{\gamma(\delta + \beta\gamma)}.$$

The first equation is an expression for  $\rho$ .

From now on we assume that (4.4) and (4.5) have a solution  $(\pi^*, \rho) \in [0, 1] \times (\gamma, 1)$ . We can find expressions for  $k_2$  and  $k_3$  by imposing a *smooth fit* condition along the boundary x = ky. From continuity we easily get

$$k_1 + k_2 = k_3.$$

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inequality (2.10) is effective. Rende, for  $0 \leq x < ky$ , we construct the solution from the assumption that

and, when a > bus

We see that the integral in (4.2) is well defined by the condition in (2.3) in what follows, all the displayed integrals are convergent by the same capitition. In the cast of this section we derive expressions for the different constration the solution, and find the optical allocation and construction processes. Optimize the level of gl.2] with respect to 7 to find the first order condition for an optimize

$$(a-v)xV_{2} + e^{2vx^{2}}V_{2} + \int_{av(c)} \left(V_{a}(x+vx(a^{2}-v),v)x(a^{2}-v) - a^{2}(x+vx(a^{2}-v))\right) e^{2vx(a^{2}-v)}$$

Inserting the guessed solution (4,1) for x < ky, we get the exprassion

$$(4.4) \quad (k-r) - (1-r)\sigma^2 \pi + \int_{R^2(r)} ((1+r)\sigma^2 (1-r))^{r-1} (r^2 - 1) - (r-4) = 0, \quad (4.4)$$

Assume from now on that s" is a solution of (4.4). Note that s" is constant with mapped to time which gives that the optimal investment role is to hold a constant traction of the wealth in the stock. With this s", we can find equations lot the unknown constants by and a inserting (4.1) into (4.2), we obtain

$$\frac{1}{2} = \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{2} \right] \right] \right] \right] \right] \right]$$

The order servicile left-hand side can be sero a when

$$(a - b^{n} a^{2} - b + bai - b^{1+n})$$

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$$(a - b^{n} a^{2} - b^{2} - b^{2}$$

Diks

The first equation is an expression for a

Frequences on we assume that (4.4) and (4.5) have a solution  $(6.6) \leq (0.1) \times (0.1)$ . We can find expressions for by and by by imposing a smooth fit condition along the boundary z = ky. From continuity we easily set

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Moreover, if the derivatives of V are to be continuous as well, we need to have  $V_x = \beta V_y$  when x = ky for the solution (4.1) (x < ky). But differentiating and equating give

$$k_2 = \frac{\beta k_1 \gamma}{\rho/k - \beta(\gamma - \rho)} = \frac{\beta k}{(\delta + \beta \gamma)(\rho(1 + \beta k) - \beta k \gamma)}.$$

For x < ky, we need to show that  $\beta V_y - V_x \leq 0$ . Direct differentiation gives

$$V_{x} = k_{2}y^{\gamma} \left[\frac{x}{ky}\right]^{\rho-1} \frac{\rho}{ky} = k_{2}\frac{\rho}{k}y^{\gamma-1} \left[\frac{x}{ky}\right]^{\rho-1},$$
  
$$V_{y} = k_{1}\gamma y^{\gamma-1} + k_{2}(\gamma-\rho)y^{\gamma-\rho-1} \left[\frac{x}{k}\right]^{\rho} = k_{1}\gamma y^{\gamma-1} + k_{2}(\gamma-\rho)y^{\gamma-1} \left[\frac{x}{ky}\right]^{\rho}.$$

Hence

$$\beta V_y - V_x = y^{\gamma - 1} \left( k_1 \beta \gamma + \beta k_2 (\gamma - \rho) \left[ \frac{x}{ky} \right]^{\rho} - k_2 \frac{\rho}{k} \left[ \frac{x}{ky} \right]^{\rho - 1} \right).$$

Inserting the expressions for  $k_1$  and  $k_2$  yields

$$\beta V_y - V_x = \frac{\beta y^{\gamma - 1}}{\delta + \beta \gamma} \Big( 1 - (1 - \rho) \Big[ \frac{x}{ky} \Big]^{\rho} - \rho \Big[ \frac{x}{ky} \Big]^{\rho - 1} \Big).$$

We see that  $\beta V_y - V_x \leq 0$  if and only if

$$h(z) := 1 - (1 - \rho)z^{\rho} - \rho z^{\rho - 1} \le 0,$$
 for all  $z \in [0, 1].$ 

But h(1) = 0 and

$$h'(z) = \rho(1-\rho)z^{\rho-2}(1-z) \ge 0.$$

Hence h(z) is an increasing function on [0, 1] with maximum h(1) = 0, which implies  $h(z) \le 0$ . This completes the proof of  $\beta V_y - V_x \le 0$  for x < ky.

For the second case we specify the value of k to be the same as in [14] and show that this gives the desired inequality under an additional condition on the parameters in the problem. Let

$$k = \frac{1-\rho}{\beta(\rho-\gamma)}.$$

This gives

$$k_3 = \frac{\rho(1-\gamma)}{\gamma(\rho-\gamma)(\delta+\beta\gamma)}$$

and thus

$$V(x,y) = c(y+\beta x)^{\gamma}, \quad \text{for } x \ge ky, \quad c = \frac{\rho}{\gamma(\delta+\beta\gamma)} \Big(\frac{1-\gamma}{\rho-\gamma}\Big)^{1-\gamma}$$

We show next that

$$\frac{y^{\gamma}}{\gamma} - \delta V - \beta y V_y + \max_{\pi \in [0,1]} \Big[ (r + (\hat{\mu} - r)\pi) x V_x + \frac{1}{2} \sigma^2 \pi^2 x^2 V_{xx} + \int_{\mathbb{R} \setminus \{0\}} \Big( V(x + \pi x (e^z - 1), y) - V(x, y) - \pi x V_x (e^z - 1) \Big) \nu(dz) \Big] \le 0,$$

whenever  $x \ge ky$ . Inserting the expression for V(x, y) in the left-hand side of the above inequality and using  $\frac{\beta x}{y+\beta x} \in (0, 1)$ , we get

$$\frac{y^{\gamma}}{\gamma} - \delta c(y + \beta x)^{\gamma} - \beta \gamma \frac{y}{y + \beta x} c(y + \beta x)^{\gamma}$$

$$(1,0) = 1$$
 is not  $(0,2)^{1-2} = 2^{1-2} = 1$ 

$$h(z) = p(1 - p)z^{n-1}(1 - z) \ge 0.$$

$$\frac{1}{(10+3)(2-3)^2} = e^{2}$$

$$= \left\{ \frac{1}{2} \left\{ \frac{1}$$

$$+ c(y+\beta x)^{\gamma} \max_{\pi \in [0,1]} \left[ (r+(\hat{\mu}-r)\pi)\gamma \frac{\beta x}{y+\beta x} + \frac{1}{2}\sigma^{2}\pi^{2} \left(\frac{\beta x}{y+\beta x}\right)^{2}\gamma(\gamma-1) \right. \\ \left. + \int_{\mathbb{R}\setminus\{0\}} \left( \left(1 + \frac{\beta x}{y+\beta x}\pi(e^{z}-1)\right)^{\gamma} - 1 - \gamma \frac{\beta x}{y+\beta x}\pi(e^{z}-1)\right)\nu(dz) \right] \\ \left. \leq \frac{y^{\gamma}}{\gamma} - c(y+\beta x)^{\gamma} \left(\delta - k(\gamma)\right). \right.$$

But since  $x \ge ky$  and  $\delta - k(\gamma)$  and c are both positive, we have

$$\frac{y^{\gamma}}{\gamma} - c(\delta - k(\gamma))(y + \beta x)^{\gamma}$$
  
$$\leq \frac{y^{\gamma}}{\gamma} - c(\delta - k(\gamma))(1 + \beta k)^{\gamma}y^{\gamma} = y^{\gamma} \Big(\frac{1}{\gamma} - c(\delta - k(\gamma))(1 + \beta k)^{\gamma}\Big),$$

which is less than or equal to zero if and only if

$$\frac{1}{\gamma} - c(\delta - k(\gamma))(1 + \beta k)^{\gamma} \le 0.$$

But this happens if and only if

(4.6) 
$$\frac{\rho(1-\gamma)}{\rho-\gamma} \ge \frac{\delta+\beta\gamma}{\delta-k(\gamma)}.$$

By construction V is a constrained viscosity solution in  $\{x \ge 0, y > 0\}$ . Note that a subsolution in  $\{x \ge 0, y > 0\}$  is also a subsolution in  $\overline{\mathcal{D}}$ . We refer to the first remark in Section 3 in [1] for a proof of this. Thanks to the Theorems 3.1 and 3.2, V is thus the unique constrained viscosity solution of (2.10) and hence coincides with the value function (2.7). Summing up, we have proven the following theorem:

**Theorem 4.1.** For  $\gamma \in (0, 1)$ , let  $U(y) = \frac{y^{\gamma}}{\gamma}$  and assume (4.6) holds. Then the value function V(x, y) associated with our optimization problem is explicitly given by (4.1), where

$$k_1 = \frac{1}{\gamma(\delta + \beta\gamma)}, \quad k_2 = \frac{1-\rho}{(\rho - \gamma)(\delta + \beta\gamma)}, \quad k_3 = \frac{\rho(1-\gamma)}{\gamma(\rho - \gamma)(\delta + \beta\gamma)}, \quad k = \frac{1-\rho}{\beta(\rho - \gamma)}.$$

The optimal allocation of money in the stock is given by  $\pi^*$  where  $\pi^* \in [0, 1]$  and  $\rho \in (\gamma, 1]$  are solutions (when such exist) to the system of equations

$$\begin{aligned} (\hat{\mu} - r) - (1 - \rho)\sigma^2 \pi + \int_{\mathbb{R}\setminus\{0\}} (1 + \pi(e^z - 1))^{\rho - 1}(e^z - 1) - (e^z - 1)\nu(dz) &= 0\\ \left(r + (\hat{\mu} - r)\pi + \beta - \frac{1}{2}\sigma^2\pi^2(1 - \rho)\right)\rho\\ &= \delta + \beta\gamma - \int_{\mathbb{R}\setminus\{0\}} \left((1 + \pi(e^z - 1))^{\rho} - 1 - \rho\pi(e^z - 1)\right)\nu(dz).\end{aligned}$$

Note that  $k_1, k_2$ , and  $k_3$  are equal to the constants found by Hindy and Huang [14]. However, our expressions for  $\rho$  and  $\pi^*$  are quite different. Furthermore,  $\pi^*$  is independent of time and thus gives a constant fraction of wealth to be invested in the stock. It is easily seen that in the case of geometric Brownian motion, Theorem 4.1 coincides with the results of Hindy and Huang [14].

$$+ \alpha (a + a a)^{2} \max_{a \neq a} \left[ (a + (a - n)n)^{2} + \frac{a^{2}}{a + a} + \frac{a^{2}}{a} e^{2n} \left( \frac{a^{2}}{a + a} \right)^{2} n(n - 1) + \frac{a^{2}}{a + a} \right]^{2} n(n - 1)$$

$$+ \int_{a_{1}(a_{1})} \left( \left( 1 + \frac{a^{2}}{a + a} + \frac{a^{2}}{a} + \frac{a^{2}}{a} + \frac{a^{2}}{a} + \frac{a^{2}}{a} + \frac{a^{2}}{a} + \frac{a^{2}}{a} \right)^{2} n(n - 1) \right)$$

$$+ \int_{a_{1}(a_{1})} \left( \left( 1 + \frac{a^{2}}{a + a} + \frac{a^{2}}{a} \right)^{2} n(n - 1) \right)$$

But since a 2 ky and d - ky and care both positive, we have

$$= -\alpha(\delta - k(\tau))(\tau + \beta \tau)^{2}$$

$$\leq \sum_{i=1}^{2} -\alpha(\delta - k(\tau))(\tau + \beta \tau)^{2} = V^{2} \left( \sum_{i=1}^{2} -\alpha(\tau - k(\tau))(\tau + \beta \tau)^{2} \right)$$

which is less than or equal to zero if and only if

But this happens if and only if

$$\frac{r_{0}^{2}+6}{(r_{0}^{2})^{2}-3} \leq \frac{(r_{0}-1)_{0}}{r_{0}-6}$$
(3.1)

By construction V is a constrained viscosity volution in  $\{x \ge 0, y \ge 0\}$  from that a subsolution  $x \in \{x \ge 0, y \ge 0\}$  is also a subsolution in  $\tilde{N}$ . We refer to the first reports in Section 3 in [1] for a proof of this. Thanks to the Theorems 3.1 and 3.2 V is thus the unique  $\infty$ constrained give cases volution of (2.16) and hence concores with the relation function (2.7). Summing up, we have constant the tablewing theorem.

**Theorem 4.1.** For  $\gamma \in (0, 1)$ , let  $U(y) = \frac{1}{2}$  and assume (4.6) hould? Then the value (approximation) V(x, y) associated with one obtainization problem is amplicitly given by (4.1), where

$$\frac{(n-1)}{(n-q)n} = n - \frac{(n-1)n}{(n+q)(n-q)n} = n^2 - \frac{(n-1)n}{(n+q)(n-q)} = n^2 - \frac{(n-1)n}{(n+q)(n-q)(n-q)(n-q)} = n^2$$

The optimal atlaced introj atomy in the stock is given by  $\pi^*$  where  $\pi^* \in \{0,1\}$  and  $p \in \{2,1\}$  are solutions (when each crief) to the restem of equations

$$(a - a) - (a - a)a^{2} + aa_{a}a^{2} + a(a^{2} - a)^{2} + (a^{2} - a)^{2} + (a^{2} - a) - (a -$$

Note that M<sub>1</sub>, M<sub>2</sub>, and K<sub>2</sub> are equal to the constants found by Hlady and Hours? [14], More ever, our expressions for  $\rho$  and  $\pi^*$  are quite different. Furthermore,  $\pi^*$  is independent of time and thus gives a constant fraction of which to be invested in the stark? To irreadly that in the case of geometric Browning, motion. Theorem 4.1 consider with the results of Biddy and Huang [14]. **Example 4.1.** To include the possibility of a sudden price drop (a "crack") in a stock, a natural model could be a geometric Brownian motion with a Poisson component:

$$S_t = S_0 e^{\mu t + \sigma W_t - \xi N_t}$$

where  $\mu, \sigma, \xi, S_0$  are constants and  $N_t$  is a Poisson process with intensity  $\lambda > 0$ . The Lévy measure is easily seen to be

$$\nu(dz) = \lambda \delta_{-\xi}(dz),$$

where  $\delta_a$  is the Dirac measure located at a. Assume now that  $0 < \xi < 1$ . The expected rate of return for this stock is

$$\hat{\mu} = \mu + \frac{1}{2}\sigma^2 - \lambda(1 - e^{-\xi}).$$

Moreover, the equations for  $\pi^*$  and  $\rho$  become

$$\hat{\mu} - r - \lambda (1 - e^{-\xi}) \left( \left( 1 - \pi (1 - e^{-\xi}) \right)^{\rho - 1} - 1 \right) - \sigma^2 (1 - \gamma) \pi = 0,$$
  
$$\left( r + (\hat{\mu} - r)\pi + \beta - \frac{1}{2} \sigma^2 \pi^2 (1 - \rho) \right) \rho = \delta + \beta \gamma - \lambda \left( \left( 1 - \pi (1 - e^{-\xi}) \right)^{\rho} - 1 + \rho \pi (1 - e^{-\xi}) \right).$$

If  $\sigma = 0$  and the conditions

 $\mu > r$  and  $(\mu - r)e^{-(1-\rho)\xi} < \lambda(1 - e^{-\xi}) < \mu - r$ 

hold, we have the following explicit expression for  $\pi^* \in [0, 1]$  in terms of  $\rho$ :

$$\pi^* = \frac{1}{1 - e^{-\xi}} \left( 1 - \left[ \frac{\lambda(1 - e^{-\xi})}{\mu - r} \right]^{\frac{1}{1 - \rho}} \right).$$

An optimal consumption process is provided by the following theorem:

**Theorem 4.2.** An optimal consumption process  $C_t^*$  is given as

$$C_t^* = \Delta C_0^* + \int_0^t \frac{X_t^*}{1+\beta k} \, dZ_s, \qquad k = \frac{1-\rho}{\beta(\rho-\gamma)},$$

$$\Delta C_0^* = \left[\frac{x - kY_{0-}}{1 + \beta k}\right]^+, \quad Z_t = \sup_{0 \le s \le t} \left[\ln \frac{\hat{X}_t}{\hat{Y}_t} - \ln k\right]^+, \qquad \hat{Y}_t = (Y_0 + \beta \Delta C_0^*)e^{-\beta t},$$

and

$$\hat{X}_{t} = (x - \Delta C_{0}^{*}) + \int_{0}^{t} (r + (\hat{\mu} - r)\pi^{*}) \hat{X}_{s} \, ds + \int_{0}^{t} \sigma \pi^{*} \hat{X}_{s} \, dB_{s} + \int_{0}^{t} \pi^{*} \hat{X}_{s-} \int_{\mathbb{R} \setminus \{0\}} (e^{z} - 1) \, \tilde{N}(ds, dz).$$

The processes  $X^*$  and  $Y^*$  are the state variables associated with  $C^*$ .

*Proof.* This argument follows closely the proof in [14, Prop. 5]. From the results in [14], we need to find a k ratio barrier policy which ensures that  $X_t^*/Y_t^* \leq k$ , P-a.s. at every t. This leads to an initial jump of  $C_t^*$  if  $x/Y_{0-} > k$ , from where we get the expression of  $\Delta C_0^*$ . Now define

$$Z_t = \sup_{0 \le s \le t} \left[ \ln \frac{X_t}{\hat{Y}_t} - \ln k \right]^+$$

Example 4.1. To include the possibility of a sudden prior drop (a "crack") in a stock, a netural model could be a geometric Brownian motion with a Poisson component;

where  $\mu, \sigma, \xi, S_0$  are constants and  $N_1$  is a Poisson process with intensity  $\lambda > 0$ . The Levy measure is easily seen to be

where  $\delta_{i}$  is the Dirac measure located at a. Assants now that  $0 \le i \le 1$ . The expected rule of return for this stock is

$$\hat{\mu} = \mu + \frac{1}{2}\sigma^2 - \lambda(1 - e^{-\xi}).$$

Moreover, the coustions for r and a become

$$\begin{array}{l} & -r - \lambda (1 - e^{-\gamma}) \left( (1 - \pi (1 - e^{-\gamma}))^{-1} - 1 \right) = e^{\gamma} (1 - \gamma) \pi = 0 \\ & \left( r + (k - r) \pi + n - \frac{1}{2} e^{2\pi^{2}} (1 + n) \right) e^{-\alpha} + 4\eta - \lambda \left( (1 - \pi (1 - e^{-\gamma}))^{2} - 1 + \mu \pi (1 - e^{-\gamma}) \right) \end{array}$$

If e = 0 and the conditions

$$n > n > (2^{-}n - 1)k > 2(n - 1) - n(n - 1)k > 2(n - n)$$

hold, we have the following explicit expression for r. . . [0, 1] in terms of a

An optimal consumption process is provided by the failowing theorem:

Theorem 4.2. In onlines concumption process 62 is given as

here

$$\begin{aligned} \hat{x}_{i} &= (x - \Delta C_{3}) + \int_{0}^{1} (x + (x - x)x) \hat{x}_{i} dx + i \int_{0}^{1} dx^{i} dx \\ &+ \int_{0}^{1} dx^{i} \hat{x}_{i} dx \\ &+ \int_{0}^{1} dx^{i} \hat{x}_{i} dx \\ \end{aligned}$$

The processes X", and Y", and the state periodics associated with C".

Proof. This argument follows closely the proof in [14, Prop. 3]. From the results in [14], we need to find a k ratio barrier policy which easilies that  $X_{i}/b_{i}^{n} \leq h_{i}$  Proas, at every to [15] is leads to an initial jump of  $C_{i}$  II  $x/b_{i}$ . Now where we get the expression of  $X_{i}^{n}$ . Now define

and let  $\ln(X_t^*/Y_t^*)$  be the "regulated" process defined by

(4.7) 
$$\ln \frac{X_t^*}{Y_t^*} = \ln \frac{\hat{X}_t}{\hat{Y}_t} - Z_t.$$

Note that the processes  $\hat{X}_t$  and  $\hat{Y}_t$  are unregulated in the sense that we do not apply any consumption process except for the initial jump. The process  $Z_t$  is easily seen to be nondecreasing,  $Z_0(\omega) = 0$ , and increasing only when  $\ln(X_t^*/Y_T^*) = \ln k$ . Applying Itô's formula, we find that

$$d\ln\frac{X_t^*}{Y_t^*} = d\ln X_t^* - d\ln Y_t^* - \left(\frac{1}{X_t^*} + \frac{\beta}{Y_t^*}\right) dC_t^*$$
  
=  $(r + \beta + (\hat{\mu} - r)\pi^* - \frac{1}{2}\sigma^2\pi^{*2}) dt - \left(\frac{1}{X_t^*} + \frac{\beta}{Y_t^*}\right) dC_t^*$   
+  $\int_{\mathbb{R}\setminus\{0\}} \ln\left(1 + \pi^*(e^z - 1)\right) \tilde{N}(dt, dz) + \sigma\pi^* dB_t$   
+  $\int_{\mathbb{R}\setminus\{0\}} \left(\ln\left(1 + \pi^*(e^z - 1)\right) - \pi^*(e^z - 1)\right) \nu(dz)$ 

and

$$d\ln\frac{X_t}{\hat{Y}_t} = d\ln\hat{X}_t - d\ln\hat{Y}_t = (r + \beta + (\hat{\mu} - r)\pi^* - \frac{1}{2}\sigma^2\pi^{*2}) dt + \int_{\mathbb{I\!R}\setminus\{0\}} \ln\left(1 + \pi^*(e^z - 1)\right) \tilde{N}(dt, dz) + \sigma\pi^* dB_t + \int_{\mathbb{I\!R}\setminus\{0\}} \left(\ln\left(1 + \pi^*(e^z - 1)\right) - \pi^*(e^z - 1)\right) \nu(dz)$$

Thus, relation (4.7) is fulfilled exactly when

$$Z_t = \int_0^t \left(\frac{Y_s^* + \beta X_s^*}{X_s^* Y_s^*}\right) \, dC_s^* \quad \text{or} \quad C_t^* = \int_0^t \frac{X_s^* Y_s^*}{Y_s^* + \beta X_s^*} \, dZ_s = \int_0^t \frac{X_s^*}{1 + \beta k} \, dZ_s.$$

Here the relation for  $C_t^*$  follows since  $Z_t$  only increases when  $X_t^*/Y_t^* = k$ . This completes the proof of the theorem.

# 5. MERTON'S PROBLEM WITH CONSUMPTION AND HARA UTILITY

In this section we consider Merton's problem with consumption when the stock price is modeled as (2.1). Merton's problem can be thought of as the limiting case when  $\beta \to \infty$  in the particular model considered in Section 4. In this problem we thus optimize the expected utility of the consumption directly. The consumption process is assumed to be absolute continuous with respect to the Lebesgue measure on the real positive half-line, and can thus be specified on the form  $C_t = \int_0^t c_s ds$ , where  $c_s$  is the consumption rate at time s. The value function will only be dependent on one variable, namely the initial fortune x. We note that this problem has been treated by Framstad *et al.* [11] when the price process  $S_t$  is modeled as the solution of a stochastic differential equation with jumps, see also [12] where they take into account transaction costs. However, they have a more restrictive condition on the Lévy measure in a neighborhood of zero. For example, the normal inverse Gaussian Lévy process of Barndorff-Nielsen [3] does not fit into the framework of [11, 12].

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and let in XTAVT) he the "regulated" process defined by

Note that the processes X, and Y, are surregulated in the sense that we do not apply any constant the process succept for the initial jump. The process Z, is omily each to be nearly creasing  $Z_0(\omega) = 0$ , and increasing only when  $in(X_i^*/Y_i) = ink$ . Applying 15 is formula, we find that

$$= a_{10} \frac{a_{10}}{b_{10}} = a_{10} \frac{a_{10}}{b_{10}} = (a + a + (b - a_{10}) - a_{10}) \frac{a_{10}}{b_{10}} + a_{10} \frac{a_{10}}{b_$$

$$z_{1} = \int_{0}^{\infty} \left( \frac{Y_{1}^{2} + g_{1} \chi_{1}}{\chi_{2} \chi_{2}} \right) dz_{1} = -\zeta_{1} = \int_{0}^{\infty} \frac{\chi_{1}^{2} \chi_{1}}{Y_{2}^{2} + g_{2} \chi_{2}} dz_{1} = \int_{0}^{\infty} \frac{\chi_{1}}{Y_{2} + g_{2} \chi_{2}} dz_{1} = \int_{0}^{\infty} \frac{\chi_{1}}{Y_{2} + g_{2} \chi_{2}} dz_{1} = -\int_{0}^{\infty} \frac{\chi_{1}}{Y_{2} + g_{2} \chi_{2}} dz_{1} = -\frac{\chi_{1}}{Y_{2} + g_{2} \chi_{2}} dz_{1} = -\frac{\chi_{1}}{Y_{2} + g_{2} \chi_{2}} dz_{1} = -\frac{\chi_{1}}{Y_{2} + -\frac{\chi_{1}}{Y_{2} + g_{2} \chi_{2}} dz_{1} = -\frac{\chi_{1}}{Y_{2} + -$$

Here the relation for  $C_1^*$  follows since  $Z_1$  only increases then  $X_1^*/W = 4$ . This completes the proof of the theorem.

## NTRAFON'S PROBLEM WITH CONSUMPTION AND HARAS PROVIDENCY

In this section we consider Marton's problem with consumption when the stack price is modeled as (2.1). Method's problem can be thinght of as the intriting case when 3 --- or it the particular model considered is Section 4. In this problem we thus operance the expected stinty of the quastingtion dirocity. The consumption process is resumed to be abalance continues with respect to the Lebesgue measure on the real particle fail-line, and can thus be specified all the form C<sub>1</sub> = {; o, ds, where c, is the commutation rule of the problem s. We can this problem has the form C<sub>1</sub> = {; o, ds, where c, is the commutation rule of the time, and can thus this problem has the form C<sub>1</sub> = {; o, ds, where c, is the commutation rule of time, and can thus this problem has the form C<sub>1</sub> = {; o, ds, where c, is the commutation rule of time, and can thus this problem has then treated by Framsted et al. (1) when the print proton s. We call this problem has then treated by Framsted et al. (1) when the print proton s. [3] where the states are also the stochastic differential equation with jumps, are also [1] where they price into account trategotion costs. However, they have a more remembered to all the long measure in a mightion house of zero. For example, the quantal inverse Caused in the Long of Baradord Friderer (1) does not it into the treasy or of (1). [2]

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In the present context, the wealth process is given as

$$dX_{t} = (r + (\hat{\mu} - r)\pi_{t})X_{t} dt - c_{t} dt + \sigma X_{t}\pi_{t} dB_{t} + X_{t-}\pi_{t-} \int_{\mathbb{R}\setminus\{0\}} (e^{z} - 1) \tilde{N}(dt, dz)$$

with initial wealth  $X_0 = x$  and  $\hat{\mu}$  as defined in (2.5). We consider the optimal control problem

$$V(x) = \sup_{c,\pi \in \mathcal{A}_x} \mathbb{E}^x \left[ \int_0^\tau e^{-\delta t} \left[ \frac{c_t^{\gamma}}{\gamma} \right] dt \right], \quad \text{for } \gamma \in (0,1)$$

where the set of admissible controls  $\mathcal{A}_x$  is defined as follows:  $\pi, c \in \mathcal{A}_x$  if

 $(cm_i)$   $c_t$  is a positive and adapted process such that  $\int_0^t \mathbb{E}[c_s] ds < \infty$  for all  $t \ge 0$ .

 $(cm_{ii})$   $\pi_t$  is an adapted cádlág process with values in [0, 1].

 $(cm_{iii})$   $c_t$  is such that  $X_t^{\pi,c} \ge 0$  almost everywhere for all  $t \ge 0$ .

Note that condition  $(cm_{iii})$  introduces a state space constraint into our control problem. The Hamilton-Jacobi-Bellman equation for this problem is

(5.1) 
$$\max_{\substack{c \ge 0, \pi \in [0,1]}} \left[ (r + (\hat{\mu} - r)\pi) x v'(x) - cv'(x) - \delta v(x) + \frac{c^{\gamma}}{\gamma} + \frac{1}{2} \sigma^2 \pi^2 x^2 v''(x) \right] \\ \int_{\mathbb{R} \setminus \{0\}} \left( v(x + \pi x (e^z - 1)) - v(x) - \pi x v'(x) (e^z - 1) \right) \nu(dz) = 0 \text{ in } \{x > 0\}.$$

Note that the integral in (5.1) as well as the other integrals displayed in this section are convergent by the condition in (2.3). We now construct an explicit (unique) constrained viscosity solution to this problem. First maximize with respect to c to obtain

$$-V'(x) + c^{\gamma-1} = 0 \implies c = [V'(x)]^{\frac{1}{\gamma-1}}.$$

Maximizing with respect to  $\pi$  gives the expression

$$(\hat{\mu} - r)xV'(x) + \sigma^2 \pi x^2 V''(x) + \int_{\mathbb{R} \setminus \{0\}} \left( V'(x + \pi x(e^z - 1))x(e^z - 1) - xV'(x)(e^z - 1) \right) \nu(dz) = 0.$$

We guess a solution on the form  $V(x) = Kx^{\gamma}$ . Then a straightforward calculation gives the following integral equation for  $\pi$ :

(5.2) 
$$(\hat{\mu} - r) - (1 - \gamma)\sigma^2 \pi + \int_{\mathbb{R} \setminus \{0\}} \left( \left( 1 + \pi (e^z - 1) \right)^{\gamma - 1} - 1 \right) (e^z - 1) \nu(dz) = 0.$$

Note that a  $\pi$  solving this equation will be independent on t. Using the guessed solution, we can obtain an expression for c as well:

(5.3) 
$$c = (K\gamma)^{\frac{1}{\gamma-1}}x.$$

This expression gives us an explicit consumption rule, that is, consume the fraction  $(K\gamma)^{1/\gamma-1}$  of the present wealth. We now set out to find the constant K. Inserting (5.3) into the Hamilton-Jacobi-Bellman equation (5.1), we get

$$\max_{\pi \in [0,1]} \left[ (r + (\hat{\mu} - r)\pi)\gamma - (K\gamma)^{\frac{1}{\gamma-1}}\gamma - \delta + (K\gamma)^{\frac{\gamma}{\gamma-1}-1} - \frac{1}{2}\sigma^2\gamma(1-\gamma)\pi^2 \right]$$

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in the present concept, the weath process is when as

$$dX_{0} = (r + (\beta - r)\pi_{0})X_{0}dt - b_{0}dt + \sigma X_{0}\pi_{0}dB_{0} + X_{0}\pi_{0} - \int_{dM_{0}} [e^{z} - 1]N(dt, dz)$$

with initial wealth  $X_0 = x$  and  $\beta$  as defined in (2.5). We consider the optimum control problem

where the set of admissible controls A, is defined as follows: d.o.8 My iff

(emi) of is a positive and adapted process such that [, Fig. 4s c os for all 1 > 0.

(orag) we is an adapted catifar process with values in [0, 1].

(emp) or is such that  $X^{m} \ge 0$  simost everywhere for all  $t \ge 0$ .

Note that condition (cm<sub>in</sub>) introduces a statespace constraint into our control problem. The list that condition (cm<sub>in</sub>) introduces a statespace constraint into our control problem. The

$$(5.1) = \frac{\max_{x \in [0,1]} \left[ (r + (x - r)\pi) \pm u'(x) - u'(x) - u'(x) - u'(x) + \frac{c}{r} + \frac{1}{r} a^{2} r^{2} r^{2} u'(x) \right]}{\left[ (r(x + \pi x(r^{2} - 1)) - v(x) - \pi x u'(x)(r^{2} - 1)) \right] u(x)} = 0 \text{ in } (h > 0).$$

Note that the integral in (5.1) as well as the orther integrals displayed in this section the convergent by the condition in (2.3). We now construct an equicit (unique) constrained viscosity solution to this problem. First naximize with respect to c to obtain

$$e^{\frac{1}{1-1}}[(a)^{n}A] = 0 \iff 0 = \frac{1}{1-1} + (a)^{n}A = 0$$

Maximizing with respect to r gives the expression

$$(\mu - n) = V'(a) + d' \pi a^{-1} ('(a) + \int_{B_{1}(a)} (V'(a + \pi a(a^{-1})) x(a^{-1}) - \pi V'(a)(a^{-1})) x(a^{-1}) = 0.$$

We guess a solution on the form  $V(x) = Nx^2$ . Then a straightforward calculation gives the following integral equation for x:

$$(5.2) - (1-1)(4-1)(-1)(2+1) \left( (1+1)(-1)(1+1) \right) = (1-1)(4-1)(4-1)(4-1) = 0$$

Note that air contraction this replation will be independent on to lising the greesed solution, we can obtain as electron are

This expression gives us an explicit consumption only, that is, consume the fraction (K 7) VCC of the present weakly. We now set out to find the constant, K. Specting (5.3) into the Barritron-Brank Bellman constitut (5.1), we get

$$+\int_{\mathbb{R}\setminus\{0\}} \left( \left(1 + \pi(e^z - 1)\right)^{\gamma} - 1 - \gamma \pi(e^z - 1) \nu(dz) \right) \right] K x^{\gamma} = 0$$

We thus conclude that

$$K = \frac{1}{\gamma} \left[ \frac{1-\gamma}{\delta - k(\gamma)} \right]^{1-\gamma},$$

where  $k(\gamma)$  is defined in (2.8). Note that the condition  $\delta > k(\gamma)$  imposed in Section 2 implies that K is positive.

We state a condition ensuring the existence of a unique solution  $\pi \in [0, 1]$  to (5.2). To this end, define the function

$$f(\pi) = (\hat{\mu} - r) - (1 - \gamma)\sigma^2 \pi + \int_{\mathbb{R} \setminus \{0\}} \left( \left( 1 + \pi(e^z - 1))^{\gamma - 1}(e^z - 1) - (e^z - 1) \right) \nu(dz) dz \right)$$

Inserting  $\pi = 0$  and  $\pi = 1$ , we obtain

$$f(0) = \hat{\mu} - r > 0$$

and

$$f(1) = (\hat{\mu} - r) - (1 - \gamma)\sigma^2 + \int_{\mathbb{R}\setminus\{0\}} \left( e^{(\gamma - 1)z} (e^z - 1) - (e^z - 1) \right) \nu(dz)$$
$$= (\hat{\mu} - r) - (1 - \gamma)\sigma^2 - \int_{\mathbb{R}\setminus\{0\}} (1 - e^{-(1 - \gamma)z}) (e^z - 1) \nu(dz).$$

In order to have a solution in [0, 1], we need f(1) < 0, i.e.,

(5.4) 
$$\int_{\mathbb{R}\setminus\{0\}} (1 - e^{-(1-\gamma)z})(e^z - 1)\,\nu(dz) > (\hat{\mu} - r) - (1-\gamma)\sigma^2.$$

This solution is unique since

$$f'(\pi) = -(1-\gamma) \left\{ \sigma^2 + \int_{\mathbb{R} \setminus \{0\}} (1+\pi(e^z-1))^{\gamma-2}(e^z-1)^2 \nu(dz) \right\} < 0$$

It is well known that in the case of a geometric Brownian motion,  $S_t = S_0 \exp(\mu t + \sigma B_t)$ , the optimal allocation of money in the portfolio is independent of time; namely,

$$\pi_{\text{GBM}}^* = \frac{\mu + \sigma^2/2 - r}{(1 - \gamma)\sigma^2}.$$

On the other hand, we have seen that  $S_t$  given as in (2.1) also gives a constant fraction, denoted by  $\pi_J^*$ , which solves (5.2). A straightforward calculation shows that

$$f(\pi_{\rm GBM}^*) = \int_{\mathbb{R}\setminus\{0\}} \left(e^z - 1 - z\mathbf{1}_{|z|<1}\right)\nu(dz) + \int_{\mathbb{R}\setminus\{0\}} \left(\left(1 + \pi_{\rm GBM}^*(e^z - 1)\right)^{\gamma-1} - 1\right)\left(e^z - 1\right)\nu(dz).$$

Thus,  $\pi_J^* < \pi_{\text{GBM}}^*$  if  $f(\pi_{\text{GBM}}^*) < 0$  and  $\pi_J^* > \pi_{\text{GBM}}^*$  if  $f(\pi_{\text{GBM}}^*) > 0$ . Note that the first integral in the expression of  $f(\pi_{\text{GBM}}^*)$  is positive, while the second is negative. Where to put the most of your fortune depends on the parameters of the specific model in question. In Benth, Karlsen, and Reikvam [6] we have compared numerically geometric Brownian motion with the normal inverse Gaussian model proposed by Barndorff-Nielsen [3].

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$$a = \left( \pi M \left( (1 + \pi m - 1))^2 - 1 - \pi m (1 - \pi m - 1) \right) \right) + \int_{m_1(0)} (1 + \pi m - 1)^2 \left( (1 - \pi m - 1) + 1 \right) = 0$$

We thus conclude that

where  $k(\gamma)$  is defined in (2.8). Note that the condition  $\delta > k(\gamma)$  impravel in Section 2 implies that K is positive

We state a condition easuring the existence of a unique solution  $r \in [0, 1]$  to (3.2). To this end, define the function

$$f(\pi) = (a - \tau) - (1 - \tau)\sigma^{2}\pi + \int_{\pi A_{1}(0)} ((1 + \pi)\sigma + 1))^{-1} (a - \tau) - (a - \tau) = (\pi A_{1})^{-1} (a - \tau) = (a - \tau)^{-1} (a - \tau$$

insorties a re 0 and r = L we obtain

$$f(0) = f_{1} - r > 0$$

bas

$$S(3) = (3 - 2) - (1 - 2)\sigma^2 + \int_{a_1(0)} \left(\sigma^{-1}(1 - 2) - (n^2 - 1)\right) d^2 d^2$$
$$= (3 - 2) - (1 - 2)\sigma^2 - \int_{a_1(0)} (1 - n^2)(n^2 - 1) d^2 d^2$$

is order to have a solution in [0,1], we need f(1) < 0, i.e.

This solution is unique since

$$L(\alpha) = -(1 - \lambda) \{ \alpha_2 + \int^{\alpha_1(\alpha)} (1 + \mu(\alpha_1 - 1)) \sum_{i=1}^{n} (\alpha_i - 1) \sum_{i=1}^{n}$$

It is well known that in the case of a geometric Brownian motion, S<sub>2</sub> = S<sub>2</sub> any(*A* + 1, *A*), the optimal allocation of money in the portiolio is independent of three neuroly

On the other hand, we have seen that S given as in (2,1) also gives a constant leaction, denoted by r. , when solves (5.2). A straightfurward calculation shows (3.2)

$$M = Constant = \int_{anton} (a_{1}^{2} - b_{1} - a_{1}^{2}) + (a_{2}^{2}) + (a_{2}^{2})$$

Thus,  $\pi_1^* < \pi_{0,0}^* = 0$ ,  $(\pi_{0,0,0}) < 0$  and  $\pi_1^* > \pi_{0,0,0}^* = 0$ ,  $(\pi_{0,0,0}) > 0$ . Note that the first integral in the expression of  $f(\pi_{0,0,0})$  is positive, while the second in negative. Where to put the most of voir forture depends on the parameters of the specific model in gravitor. In Benth, Katisan and Exilorem [0] we have compared annexically geometric Brownian motion with the momenic Brownian model proposed by Baradouli-Neden [0].

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# 6. Other models and concluding remarks

Instead of modeling the price process  $S_t$  directly as in (2.1) or (1.1), one can let  $S_t$  be the solution of a stochastic differential equation with jumps

(6.1) 
$$dS_t = \mu S_t \, dt + \sigma S_t \, dB_t + S_{t-1} \int_{-1}^{\infty} z \, \tilde{N}(dt, dz).$$

Note that  $S_t$  is positive due to the restriction of the jump size to be greater than -1. As noted by Eberlein and Keller [8], it is the large jumps that are responsible for the empirically observed heavy tails of the logreturn data. Therefore, (6.1) may not be a good model if heavy tails are to be accounted for in the model. Assuming a price dynamics defined by (6.1), condition (2.3) must be substituted by

(6.2) 
$$\int_{1}^{\infty} z \,\nu(dz) < \infty.$$

Under this restriction on the Lévy measure we can show, by arguing as before, that the value function V(x, y) is the unique constrained viscosity solution of the Hamilton-Jacobi-Bellman equation

$$\max\left\{\beta v_{y} - v_{x}; U(y) - \delta v - \beta y v_{y} + \max_{\pi \in [0,1]} \left[ (r + (\mu - r)\pi) x v_{x} + \frac{1}{2} \sigma^{2} \pi^{2} x^{2} v_{xx} \right] \right\}$$
$$\int_{-1}^{\infty} \left( v(x + \pi xz, y) - v(x, y) - \pi xz v_{x}(x, y) \right) \nu(dz) \right] = 0 \text{ in } \mathcal{D}.$$

The condition (6.2), which ensures that (6.3) is well defined for all sublinearly growing  $v \in C^2$ , is satisfied for the normal inverse Gaussian Lévy process discussed in Section 2 and for all  $\alpha$ -stable Lévy processes with  $\alpha > 1$ .

In Framstad, Øksendal and Sulem [11], the price model (6.1) is chosen for the analysis of Merton's problem with consumption. Using a verification theorem, they show that the value function in Merton's problem with consumption (see Section 5) is a unique classical solution of (6.3) under condition (6.2) and  $\nu(\{(-1,\infty)\}) < \infty$ . Honoré [13] has developed estimation techniques for price processes of the type (6.1). This opens for a numerical comparison of the different stock price models for financial data.

Except for a few special cases such as those considered in Sections 4 and 5, the Hamilton-Jacobi-Bellman equation (2.10) cannot be solved explicitly and one has to consider numerical approximations. The construction and analysis of numerical schemes for (first and second order) integro-differential variational inequalities will be reported in future work (see also [9]).

Finally, we mention that the portfolio model studied in this paper is generalized to account for transaction costs in [5].

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#### 6. OTHER MODELS AND CONCERNME REMARKS

instead of modeling the price process S directly as in (2.1) or (1.1), one can let S, he the solution of a stochastic differential equation with junca

$$dS_{1} = \mu S_{1} dI + \sigma S_{1} dI S_{2} = S_{1} dI + \sigma S_{1} dS_{2} + S_{1} dS_{2} + S_{1} dS_{2} + S_{2} + S_{2} dS_{2} + S_{2} +$$

Note that S, is positive due to the restriction of the jump are to be preserved as -1. As acted by Ebericin and Keller [8], it is the large point what are responsible for the construction observed heavy tails of the logreture data. Therefore, (6.1) may not be a construct H heavy tails are to be accounted for in the model. Assuming a proceder dynamics defined by (6.1), condition (2.3) must be substituted for in the model.

Under this restriction on the Lévy measure we can show, by signing as before, that the value function I (a, y) is the unique constrained viscoury solution of the Hamilton-Jacobi Bellania constitution

$$\max \left\{ (a_{1}, \dots, a_{n-1}, (a_{n-1}, a_{n-1}, (a_{n-1}, a_{n-1}, (a_{n-1}, a_{n-1}, a_{n-1},$$

The condition (6.2), which ensures that (0.3) is well defined for all sublinearly graving  $\alpha \in G^*$ is satisfied for the normal inverse Gaussian Lévy process discussed in Section 2 and for all  $\alpha$ -stable Lévy processes with  $\alpha > 1$ .

In Framestad, Observatal and Sulem [11], the price model (0.1) is denoted for the analysis of Merton's problem with consumption. Using a varification theorem, they show that the value function in Merton's problem with consumption (see Section 6) is a mateur chamical solution of (6.3) under condition (6.2) and  $\nu(\{(-1, \infty)\}) < \infty$  due likenore [13] has developed estimation techniques for price processes of the type (6.1). This opens for a magazini comparison of the different stock price models for fitnancial data.

Except ion when regular cases such as these considered in Sections, i and 3, the Richilson-Jacobi-Bolineau equation (2,10) cannot be solved expiritily and one has to consular annerical approximations. The construction and analysis of annerical schemes for (6 as and neord order) integradifierratial variational inequalities will be reported in (6 tara work (see also (9)).

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