Department of

APPLIED MATHEMATICS

SOLUTION OF A STATIONARY FOKKER-PLANCK EQUATION.

by

Tore Leversen and Jacqueline Naze Tjøtta

Report No. 29

June 1971



UNIVERSITY OF BERGEN Bergen, Norway



SOLUTION OF A STATIONARY FOKKER-PLANCK EQUATION.

by

Tore Leversen and Jacqueline Naze Tjøtta

Report No. 29

June 1971

Abstract.

We solve a stationary, linearized and inhomogeneous Fokker-Planck equation describing the electrons of a weakly coupled and weakly inhomogenous plasma in a magnetic field at times large compared to the effective electron - electron collision time.

MIAMOTTATE A TO MOTHERS
MOTESTER TOWARD AND ADDRESS OF

- . .

adia; Pessil enlicepeat but nearevel eser

TYPE and

· Sun Confe

we got to a chartonary, innounted and inhomogeneous rection-Planck equation decembing the electrons of a vector counted and usainty inhomogeneous pressure in a magneticalisation of times inuge compared to one wiferstive precision - electron colorus times.

1. Introduction.

We propose ourselves to solve a stationary Fokker-Planck equation. Motivation for this study is to be found in reference [1], [2], where evolution of a weakly coupled and weakly inhomogeneous plasma in a magnetic field is studied by the multiple-time-scale method. The electron-ion mass ratio and a weak inhomogenity parameter being introduced as small parameters, kinetic equations for electrons and ions are obtained at different orders of approximation in these parameters. These equations appear as non-secularity conditions in the multiple-time-scale expansion, and they are valid at times which are large compared to the effective electron-electron collision time. As is always the case when applying the multiple-time-scale method to kinetic theory, some assumptions are made, which are difficult to give a strict justification. Therefore it is of some importance to show that equations obtained as non-secularity conditions do have solutions which are physically reasonable.

The kinetic equations for electrons obtained in reference[2]contain, in addition to the linearized Fokker-Planck operator, a diffusion term which is due to the fact that electrons have a greater velocity than ions, and a magnetic field term. The kinetic equations for ions [2] do contain only the Fokker-Planck operator. We will here concentrate on the equation for electrons. However, similar results are easily obtained for the equation for ions (by making $\tilde{\gamma}$ =0, B=0). As to the possibility of extending the results to equations where the right hand

Contract the

has before given a larger willows were all and company

The sines of the contract was and the country of th

sides are of more general form than the ones we consider, see comments in section [4].

In section 2, we expand the distribution function in series of surface spherical harmonics. Thus we reduce the problem to get the solution of an infinite set of ordinary integro-differential equations. The number of inhomogeneous equations is determined by the order of anisotropy of the right-hand side. In our case this order is finite. It seems to be very important to make use of this property. Two parameters, α and γ , are introduced; the choice $\alpha = \frac{1}{2}$ and $\gamma = 0$ corresponds to the case treated by Su [3] and McLeod and Ong [4].

In section 3 we show that if the relations $2\alpha = \gamma + 1$ and $\gamma < 1$ hold, we have: The obtained integral operators $e^{-2}L_{2\ell}$, are symmetric and completely continuous, and the second-order differential operators, $e^{\gamma c^2}L_{1\ell}$, are selfadjoint. Thus $e^{\gamma c^2}L_{1\ell}$ and $e^{\gamma c^2}(L_1, + L_{2\ell})$ have the same essential spectrum. The choice $\gamma = 0$ gives an essential spectrum ranging from $-\infty$ to 0, while $0 < \gamma < 1$ gives a negative, discrete spectrum.

In section 4 we localize the spectrum to obtain the necessary information on the inverse operators. We conclude that the solutions of the integro-differential equations under consideration, do exist, under suitable conditions.

We also touch upon a corresponding plasma model where ions are neglected, and we give results in this case.

In section 5 we show that the solutions are twice differentiable. We also give results concerning the asymptotic behaviour of the solutions in different cases.

sided ere of word general form that the case we consider,

In section 2, we expend the distribution function in series of surface appearing a sagmenter. Thus we reduce the problem to get the solveton of an infinite set of ordinary integro-differential equations. The number of innocessarius equations is questioned by the order of emisorappy of the reactions to deep this order is finite. It requests to be very important to make use of this property.

Two parameters, a and y are introduced, the shales of the property.

Two parameters, a and y are introduced, the shales and property.

In section 5 we anow the to to relations for y the sections of the year operators and the section of the continuous and the section of the se

in section twe localite the aperbranco ontain the northune management in the northune operators, the northune that the localities that the localities and the localit

erady lebes amening principles and upon a copyright of the charge of the

cates Justella of the solutions of the s

2. Reformulation of the problem.

The kinetic equation to be solved is [2], Eq. (2.50)

$$FP_{11} \left[f_{1M}^{\circ}(\underline{c}_{1}) f_{1M}^{1}(\underline{c}_{1}) + f_{1M}^{\circ}(\underline{c}_{1}) f_{1M}^{1}(\underline{c}_{1}) \right] - \frac{e_{1}}{m_{1}} \underline{c}_{1} \times \underline{B} \cdot \frac{\partial f_{1M}^{1}}{\partial \underline{c}_{1}} + D_{1} (f_{1M}^{1}(\underline{c}_{1})) = f_{1M}^{\circ}(\underline{c}_{1}) \left[\frac{m_{1}c_{1}^{2}}{2kT_{1}^{\circ}} - \frac{5}{2} \right]_{T_{1}^{\circ}}^{1} \frac{\partial T_{1}^{\circ}}{\partial \underline{r}} + \frac{e_{1}}{kT_{1}^{\circ}} \left(\frac{m_{1}c_{1}}{e_{1}} + \frac{e_{0}c_{1}}{e_{1}} + \frac{e_{0}c_{1}}{e_{1}} \right) \right] \cdot \underline{c}_{1} \cdot (1)$$

As long as nothing else is indicated, index 1 refers to electrons and 2 to ions. \underline{C}_1 , e_1 , m_1 are the peculiar velocity, charge and mass of electrons, n_1^0 , p_1^0 , T_1^0 are the density, pressure and temperature of electrons, and \underline{c}_0^0 is the total mass transport velocity at zeroth order of approximation. f_{1M}^0 is the Maxwell distribution function at density n_1^0 and temperature T_1^0 , $f_{1M}^0 + f_{1M}^1$ is the distribution function of electron velocity at first order of approximation. \underline{B} is an external magnetic field, \underline{F}_1 an external force. The Fokker-Planck operator FP_{11} describing electron-electron interactions and the diffusion operator D_1 of electrons by heavy ions are defined by

$$FP_{11} = \frac{1}{m_1^2} \frac{\partial}{\partial \underline{C}_1} \cdot \int d\underline{C}'_{1} \underline{\Phi}^{11} (\underline{C}_1 - \underline{C}'_{1}) \cdot (\frac{\partial}{\partial \underline{C}_1} - \frac{\partial}{\partial \underline{C}'_{1}})$$

$$D_1 = \frac{n_2^0}{m_1} \frac{\partial}{\partial \underline{C}_1} \cdot (\underline{\Phi}^{12} (\underline{C}_1) \cdot \frac{\partial}{\partial \underline{C}_1})$$
(2)

The kinetic equation to be solved as [2], Eq. (2.50

As long as nothing else is indicated, index i refers to

electrons and C to lone. Q, c, m, are the peculiar velocity, charge and mass of electrons, n, b, b, l, l, are the density, pressure and temperature of electrons, and Q, is the total mass transport velocity at sencih order of approximation. In the Maxwell distribution function of electron velocity at lie the distribution function of electron velocity at liret erder of a spondination. B is an external suggestic field. E, an external suggestic section of electron-electron interactions and the difficulty.

Tensors Φ^{11} and Φ^{12} are given by

$$\underline{\Phi}^{\text{ij}}(\underline{w}) = \int dx_{\text{ij}} \frac{\partial \phi_{\text{ij}}}{\partial \underline{x}_{\text{j}}} \int_{0}^{\infty} d\tau \frac{\partial \phi_{\text{ij}}}{\partial \underline{x}_{\text{ij}}} [\underline{x}'_{\text{ij}} = \underline{x}_{\text{ij}} - \underline{w}\tau], \text{ i,j} = 1, 2.$$

Specifying ϕ_{ij} to the Colomb potential and making the appropriate cutoffs, we obtain, see for instance [3],

$$\Phi^{ij}(\underline{w}) = 2\pi e_i^2 e_j^2 \ln \Lambda \frac{w^2 \underline{I} - \underline{w} \underline{w}}{w^3}$$

where I is the unit tensor and

$$\Lambda = \frac{3\lambda_{D}kT_{1}^{0}}{2e_{1}^{2}} \qquad \qquad \lambda_{D}^{-2} = 4\pi \frac{e_{1}^{2}n_{1}^{0} + e_{2}^{2}n_{2}^{0}}{kT_{1}^{0}}$$

A new unknown function Φ is defined by $f_{1M}^1 = f_{1M}^0 \Phi$ and the non-dimensional velocity $\underline{c} = (\frac{m_1}{2kT_1^0})^{\frac{1}{2}}\underline{c}_1$ is introduced. After some calculations the following forms are obtained for FP_{11} , [3], and D_1 :

$$FP_{11} \left[f_{1M}^{o}(C_{1}) f_{1M}^{o}(C_{1}) (\Phi(\underline{C}_{1}) + \Phi(\underline{C}_{1})) \right] = \frac{8\pi^{2} m_{1} n_{1}^{o2} e_{1}^{4} \ln \Lambda}{(2\pi k T_{1}^{o})^{3}} e^{-c^{2}} FP'_{11}(\Phi)$$
(3)

$$D_{1}\left[f_{1M}^{o}(C_{1})\Phi(\underline{C}_{1})\right] = \frac{8\pi^{2}m_{1}n_{1}^{o2}e_{1}^{4} \ln \Lambda}{(2\pi kT_{1}^{o})^{3}} e^{-c^{2}}D'_{1}(\Phi)$$
(4)

$$S_{1} = \int dx_{1} \frac{\partial x_{1}}{\partial x_{2}} \int dx_{3} \frac{\partial x_{1}}{\partial x_{3}} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial x_{2}} \int dx_{3} \frac{\partial x_{2}}{\partial x_{3}} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial x_{2}} dx_{3} = \int dx_{3} \frac{\partial x_{2}}{\partial x_{3}} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial x_{2}} dx_{3} = \int dx_{3} \frac{\partial x_{2}}{\partial x_{3}} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial x_{2}} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial x_{1}} dx_{3} = \int dx_{3} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial x_{1}} dx_{3} = \int dx_{3} \frac{\partial x_{1}}{\partial$$

Specifying $\phi_{i,j}$ to the Colomb potential and making the appropriate outoffs, we obtain, see for instance $\{5\}$.

$$\frac{C}{\mathbb{R} \cdot \mathbb{R} - \mathbf{1}_{\mathbb{R}^{N}}} \vee \operatorname{ur}_{S}^{f} \circ \mathbf{j}_{S} = (\mathbb{R}) \cap \mathbf{1}^{d}$$

bus dosned thus end at I otenwo

A new unknown function φ is defined by $\Gamma_{[1]} = \Gamma_{[M]} \Phi$ and the non-dimensional volcety $\underline{c} = (\frac{\overline{c}_{[1]}}{2})^{\frac{1}{2}}\underline{c}_{[1]}$ is introduced. After some calculations the following forms are obtained for $FF_{[1]}$ [3], and $F_{[2]}$:

$$= \left[((, 12) \circ + (, 2) \circ) (, 10) \frac{9}{m} 2 (, 0) \frac{9}{m} 2 \right]_{1} 2 q$$

$$(8)$$

$$= \frac{8}{m^{2} n_{1} n^{2} \circ \frac{1}{n}} \frac{1}{n} \frac{\Lambda}{n} = \frac{8}{n^{2} n_{1}} \frac{1}{n^{2}} \frac{1}{n} \frac{\Lambda}{n} = \frac{8}{n^{2} n_{1}} \frac{1}{n^{2}} \frac{1}{n^{2}$$

(a)
$$(0)_{1}(0)_{2}(0)_{3}(0)_{4}(0)_{5}(0)$$

$$\begin{aligned} &\operatorname{FP'}_{11}(\Phi) = \left\{ \left[\frac{\operatorname{erf}(c)}{2c^{3}} (2c^{2} - 1) + \frac{\operatorname{e}^{-c^{2}}}{2c^{2}} \right] \mathbb{I} - \left[\frac{\operatorname{erf}(c)}{2c} (2c^{2} - 3) + \frac{\operatorname{e}^{-c^{2}}}{2c^{2}} \right] \mathbb{I} - \left[\frac{\operatorname{erf}(c)}{2c} (2c^{2} - 3) + \frac{\operatorname{e}^{-c^{2}}}{2c^{2}} \right] \mathbb{I} \right\} \\ &+ \frac{3\operatorname{e}^{-c^{2}}}{2c^{2}} \left[\frac{1}{2} \frac{\partial^{2} \Phi}{\partial \underline{c} \partial c} - \underline{c} \frac{\partial \Phi}{\partial \underline{c}} \right] + \\ &+ \left(\frac{\operatorname{e}^{-c^{2}}}{c^{2}} - \frac{\operatorname{erf}(c)}{c^{3}} \right) \underline{c} \cdot \frac{\partial \Phi}{\partial \underline{c}} + 2\operatorname{e}^{-c^{2}} \Phi(\underline{c}) + \\ &+ \frac{1}{\pi} \int d\underline{c}_{1} e^{-c^{2}} \frac{(c^{2} - 1)g^{2} - \underline{c} \cdot \underline{g}}{g^{3}} \Phi(\underline{c}_{1}) \end{aligned}$$

$$D'_{1}(\Phi) = \tilde{\gamma}e^{c^{2}}\frac{\partial}{\partial \underline{c}} \cdot \left[e^{-c^{2}}\frac{c^{2}\underline{I}-\underline{c}}{c^{3}}\cdot\frac{\partial \Phi}{\partial \underline{c}}\right]$$

$$\tilde{\gamma} = \frac{\sqrt{\pi} \frac{n_{2}^{0}e_{2}^{2}}{4n_{1}^{0}e_{1}^{2}}}{e^{-x^{2}}dx}$$

$$erf(c) = \int_{0}^{c} e^{-x^{2}}dx$$

$$(6)$$

Thus Eq. (1) writes

where
$$\underline{B'} = \frac{(2\pi k T_1^0)^{\frac{3}{2}}}{8\pi^2 \sqrt{m_1 n_1^0 \ln \Lambda}} \underline{B}$$
 (7)

and h·c represents the right-hand side of Eq.(1) divided by $8\pi^2 m_1 n_1^{02} e_1^4 \ln \Lambda (2 \text{ kT}_1^0)^{-3} e^{-c^2}$. Thus it is a known quantity. In order to solve Eq.(7) with respect to $\Phi(\underline{c})$, we introduce spherical polar coordinates c, θ, χ , the fixed

 $+(\xi^{-9}_{-9})^{\circ}(2)\frac{(2)^{3}}{9} - \xi^{-\frac{9}{10}} + (\xi^{-9}_{-9})^{\circ}(2)\frac{(2)^{3}}{9} - (5)^{-\frac{9}{10}}$

 $+ \left[\frac{26}{26} + \frac{26}{2666} \right] \left[\frac{2}{8} \right] \left[\frac{2}{8} \right] \left[\frac{2}{8} \right]$ (a)

(a) $\left[\frac{06}{2} \cdot \frac{2 \cdot 2 \cdot 1^{\circ}_{3}}{2} \cdot \frac{8}{3} - \frac{1}{3}\right] \cdot \frac{6}{3} \cdot \frac{8}{3} = (6), 10$

 $\exp(c) = \int_{0}^{c} e^{-x^2} dx$

Thus Eq. (1) writes

(a) - 2.2 = 25 · 12 × 2 - (a) / 12 + (a) / 14 = 2

where <u>B</u> ending and a sum of the second

and in removerate the right-hald side of Eq. (1) divided by Sr end of eq. (1) divided by Sr end of eq. (1) divided by Sr end of eq. (1) with respect to $\Phi(c)$.

The introduce spherical point descriptingtes of ext. the fixed

polar axis being directed along the magnetic field \underline{B} . We may formally expand Φ in series of surface spherical harmonics [3], [4]

$$\Phi(\underline{c}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{c} e^{\alpha c^2} \Phi_{\ell}^{m}(c) Y_{\ell}^{m}(\theta, \chi)$$
 (8)

The factor $c^{-1}e^{\alpha c^2}$, where α is a constant, unspecified so far, is introduced for mathematical purposes. Let us assume for the moment that the summation and the differential operators in Eq.(7) do commute. Using Eqs.(7), (8), Parseval's theorem and Eqs.(9), (10), we obtain after some calculations the set of equations Eqs.(11), (12), (13), (14), (15)

$$\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} Y_{\ell}^{m} \right) + \frac{1}{\sin^{2}\theta} \frac{\partial^{2}}{\partial \chi^{2}} Y_{\ell}^{m} = -\ell(\ell+1) Y_{\ell}^{m}$$
 (9)

$$\frac{\partial}{\partial x} Y_{\ell}^{m} = im Y_{\ell}^{m} \tag{10}$$

$$e^{\gamma c^2} (L_{10} + L_{20}) \Phi_0^0 = 0 \tag{11}$$

$$e^{\gamma c^2} (L_{11} + L_{21} - i2B') \Phi_1^{-1} = -(h_1 - ih_2) \sqrt{\frac{2\pi}{3}} 2c^2 e^{(\gamma - \alpha)c^2}$$
 (12)

$$e^{\gamma c^2} (L_{11} + L_{21}) \Phi_1^0 = h_{3} \sqrt{\frac{4\pi}{3}} 2c^2 e^{(\gamma - \alpha)c^2}$$
 (13)

$$e^{\gamma c^2} (L_{11} + L_{21} + i2B!) \Phi_1^1 = -(h_1 + ih_2) \sqrt{\frac{2\pi}{3}} 2c^2 e^{(\gamma - \alpha)c^2}$$
 (14)

$$e^{\gamma c^2} (L_{1\ell} + L_{2\ell} + i2mB') \Phi_{\ell}^{m} = 0, -\ell \le m \le \ell, \ell = 2,3,4,...$$
 (15)

odice ands being directed along the magnetic field E. We now formally expend of in series of anniado spierical

(6) $(x,a)^{n}_{x}(a)$

The factor of end, where a is communicative weapenfiled as far, as introduced for mathematical numbers. Her was as as some some for the summer los and the differential communication and the differential communication and factor and

(ci) The second of the second

(any many solumn) so so the particular of the solution of the

Here the functions $h_i(c)$, i=1,2,3, are the projections of $\underline{h}(c)$ along three orthogonal vectors \underline{e}_i , i=1,2,3, \underline{e}_3 being parallel to the polar axis, \underline{e}_1 and \underline{e}_2 corresponding to the $\theta=0$ and $\theta=\frac{\pi}{2}$ directions. The $L_{1\ell}$ and $L_{2\ell}$ operators are defined by

$$\begin{split} & L_{1\ell} \psi = \left(\frac{\text{erf}(c)}{c^{3}} - \frac{e^{-c^{2}}}{c^{2}}\right) \frac{d^{2} \psi}{dc^{2}} + \left[3\frac{e^{-c^{2}}}{c^{3}} - 3\frac{\text{erf}(c)}{c^{4}} + \right. \\ & + \left. (4\alpha - 2)\frac{\text{erf}(c)}{c^{2}} + 4(1 - \alpha)\frac{e^{-c^{2}}}{c}\right] \frac{d\psi}{dc} + \left[3\frac{\text{erf}(c)}{c^{5}} - 3\frac{e^{-c^{2}}}{c^{4}} + \right. \\ & + 2(1 - 2\alpha)\frac{\text{erf}(c)}{c^{3}} - 4\alpha(1 - \alpha)\frac{\text{erf}(c)}{c} - 4(1 - \alpha)\frac{e^{-c^{2}}}{c^{2}} + \\ & + 4(1 + \alpha(2 - \alpha))e^{-c^{2}} - \frac{\ell(\ell + 1)}{2c^{4}}\left(\frac{\text{erf}(c)}{c}(2c^{2} - 1) + e^{-c^{2}}\right) - \frac{2\gamma\ell(\ell + 1)}{c^{3}}\right] \psi \end{split}$$

$$L_{2\ell} \Psi = 4 \int_{0}^{\infty} cc_{1} e^{-(1-\alpha)c_{1}^{2}-\alpha c_{1}^{2}} K_{\ell}(c,c_{1}) \psi(c_{1}) dc_{1}$$

 $K_{\ell}(c,c_1)$ is a symmetric kernel defined by [3], [4]

$$K_{\ell}(c,c_{1}) = \frac{2}{2\ell+1} \left\{ \frac{c_{1}^{\ell}}{c^{\ell+1}} \left[\frac{(\ell+1)(\ell+2)}{2\ell+3} c_{1}^{2} - 1 \right] - \frac{\ell(\ell+1)}{2\ell-1} \frac{c_{1}^{\ell}}{c^{\ell-1}} \right\}$$

for $c_1 < c$.

 $L_{2\ell}\psi$ comes from the integral part of FP'_{11} , Eq.(5). The term with coefficient $\tilde{\gamma}$ in $L_{1\ell}$ comes from the diffusion operator D'_{1} , Eq.(6), while the remaining part of $L_{1\ell}$ is provided by the differential part of FP'_{11} , Eq.(5). As it can be seen, both sides of Eqs.(11)-(15)

Here the two three terms of the stress one of the projections of the

 $T_{2,k} = i \int_{-\infty}^{\infty} c_{0,k} e^{-(1-\phi)_{0,k}^{2} - c_{0,k}^{2}} \times_{c_{0}(0,0)} |\psi(c_{1})| dc_{1}$

[4] (E) We bentled the deliver of without it as (page) X

(10.0) x

The test as spossible in the Range of the remaining part diffusion specific in the Range of the remaining part diffusion specific in the Range of the remaining part diffusion specific in the Range of the remaining part diffusion of the remaining part dif

Eq. (5). As at ean be seen, posts sides of Eqs. (5).

have been multiplied by $e^{\gamma c^2}$, where γ is a constant, unspecified so far. This is done for mathematical convenience and helps only to invert Eq.(13). As it does not complicate arguments, we keep this factor also in the other equations in order to unify the notations. We will now proceed to the solution of Eqs(11)-(15).

3. Properties of the operators.

We find by inspection that the right-hand sides in the equations all belong to $L^2(o,\infty)$ when $\alpha>\gamma$. Thus we investigate $e^{\gamma c^2}L_{1\ell}$ and $e^{\gamma c^2}L_{2\ell}$ in this space. It is easily seen that the kernel $H_\ell(c,c_1)$ of $e^{\gamma c^2}L_{2\ell}$ is symmetric ans satisfies

$$\int_{0}^{\infty} \int_{0}^{\infty} H_{\ell}^{2}(c,c_{1}) dc dc_{1} < \infty$$

if

$$2\alpha = \gamma + 1$$
 and $\gamma < 1$ (17)

We will throughout assume that α and γ satisfy this relation. It follows that $e^{\gamma c^2}L_{2\ell}$ is selfadjoint and completely continous in $L^2(0,\infty)$. Symmetry of $e^{\gamma c^2}L_{1\ell}$ follows from Eq.(17), and selfadjointmess when $\ell \neq 0$ is shown in appendix 1. Thus $e^{\gamma c^2}L_{1\ell}$ and $e^{\gamma c^2}(L_{1\ell}+L_{2\ell})$, $\ell \neq 0$, will have the same essential spectrum. A study of

have been sulvigiled by e. where a is a constant, unspecified so far. This is done for mathematical canventence and helps only to invertigal (13). At it does not compileate arguments, we keep this factor also in the other equations in order to unity the notations. We will now proceed to the solution of Eqs(11)-(15).

educated of the contact of

equations all belong to L²(s.s.) when or y.y. Thus we squations all belong to L²(s.s.) when o y.y. Thus we investigate e²⁰ L, and e²¹ L, it to be space. In the season that the terms L.(s.e) of e²² L, is season the terms L.(s.e) of e²² L, is season that the terms L.(s.e) of e²² L, is season that the terms L.(s.e) of e²² L, is season the terms L.(s.e) of e²² L, is season that the terms L.(s.e) of e²² L, is season the terms L.(s.e) of e²² L, is sea

The transfer of a second

We will throughout sasume that a sud y satisfy this color and y satisfy this color and called the condition of the continue of the complete of the continue of

the differential operator $e^{\gamma c^2}L_{1\ell}$, $\ell \neq 0$, shows that the essential spectrum is void when $0 < \gamma < 1$ and is the negative semi-axis when $\gamma \leq 0$, see appendix 2 for the proof. Finally, when $0 < \gamma < 1$ $e^{\gamma c^2}(L_{1\ell}+L_{2\ell})$, $\ell \neq 0$, shows to be negative, as expected for physical reasons, and such that every eigenvalue λ satisfies

$$\lambda < -2\widetilde{\gamma} \ell(\ell+1)M \tag{18}$$

where M is a strictly positive constant, see appendix 3. This result justifies the introduction of the factor $e^{\gamma c^2}$: it is only when $0<\gamma<1$ that it is possible to invert Eq.(13), as well as those of the following equations corresponding to $\ell>1$ and m=0.

4. Existence of solutions.

It follows directly from Eq. (A.3.2) that Eq. (11) has only solutions of the form

$$\Phi_0^0 = ce^{-\alpha c^2} (k_1 + k_2 c^2)$$
 (19)

where k_1, k_2 are constants. They correspond to conservation of mass and energy in the interactions which are considered. The only solution of Eq.(15) is zero: where $m \neq 0$, this follows from the fact that the operator is negative definite. Accordingly, there are only a finite number of terms in

the differential operator of $L_{\chi\chi}$, χ 0, shows that the essential spectrum is void when $0 < \gamma < 1$ and is the negative semi-axis when $\gamma < 0$, see appendix 2 for the proof. Finally, when $0 < \gamma < 1$ eV $(L_{\chi} + L_{\chi\chi})$, $\chi \neq 0$, shows to be negative, as expected for physical reasons, and such that every eigenvalue χ satisfies

(81),

B(1+3) & (S- > K

where M is a strictly positive constant, see appendix 3 this result justifies the introduction of the factor $e^{\gamma c}$:

It is only when $0 < \gamma < 1$ that is possible to inverting only when $0 < \gamma < 1$ that is possible to inverting only when $0 < \gamma < 1$ that is possible to inverting to $0 < \gamma < 1$ and $0 < \gamma < 1$ are approximately and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma < 1$ and $0 < \gamma < 1$ are all $0 < \gamma <$

enokimica to eoneterz ;

only solutions of the form

(1) $(x_1, x_2)^{3}$

where k,k, are comptants. They correspond to conservation of mass and energy in the interpartions which are considered.

The only solution of Eq.(15) is zero: where m # 0, this follows from the fact that the operator is negative definite.

Accordingly, there are only a lintum number of terms in

expansion Eq.(8). It was easy to predict this result due to the fact that the right-hand side in Eq.(1) has an order of anisotropy equal to one, and since interaction—and magneto-field operators do conserve the order of anisotropy for physical reasons. Thus the problem can be reduced by studying the restriction of the operator in Eq.(1) to subspaces of functions with zeroth and first order of anisotropy. If the right-hand side of Eq.(1) had not a finite order of anisotropy, an infinite number of Eq.(15) would be inhomogeneous. As we have seen, it is always possible to solve such an equation provided the right-ahnd side belongs to $L^2(0,\infty)$. However, one would have to prove the convergence in some meaning, of the corresponding series in Eq.(8) to achieve the solution of Eq.(1). This problem has not been treated so far.

The spectrum of $e^{\gamma c^2}(L_{11}+L_{21})$ being real, as shown in section 3, the left-hand side of Eqs(12) and (14) can be inverted and these equations have a unique solution in $L^2(0,+\infty)$. Zero is a regular value for the operator in Eq.(13) where one chooses $0 < \gamma < 1$ and when $\tilde{\gamma}$ is different from zero (see Eq.(18)) i.e. provided election-ion interactions are taken into account. Thus Eq.(13) has a unique solution in $L^2(0,+\infty)$. When $\tilde{\gamma} = 0$, and $0 < \gamma < 1$, zero is an isolated eigenvalue of $e^{\gamma c^2}(L_{11}+L_{21})$, see appendix 3. Then Eq.(13) has solutions in $L^2(0,+)$ if and only if the right-hand side is ort ogonal to $c^2e^{-\alpha c^2}$; then solutions are determined to a $c^2e^{-\alpha c^2}$ near: To check up that the orthogonality condition indeed is fullfilled, one has to turn back to the original eqiation in [2] since making $\tilde{\gamma} = 0$ modifies the form of the right-hand side (see [2], p.38

expension Eq.(8). It was easy to predict this wenuit due to the fact that the right-hand side in Eq.(1) has in another of entarteepy equal to one, and since interaction— and aspectation of the operations of conserve the order of entarchappy for physical reasons. Thus the problem can be reduced by studing the physical reasons with seroth and first error of entarchapy. If the functions with seroth and first error of entarchapy. If the elementary with seroth and first error of entarchapy. If the elementary number of Eq.(1) had not a timite order of entarchaps and infinite number of Eq.(1) would be inhomogeneous. As we have seen, it is always paratible to colors such an equation of the servent order to prove the convergence is some meaning of the servent ording state in Eq.(2) to schize the solution of the servent of excite in Eq.(2) to schize the solution of the spectrum of ever the convergence is some meaning of the servent of every of the servent of every the source of Eq.(2) to schize the solution of the spectrum of every the source of Eq.(3) to schize the solution of Eq.(3), the spectrum of every the source of Eq.(3) and (14) can be sent on the left-hand side of Eq.(3) and (14) can be

section 3, the left-need aids of Eqs(12) and (14) can be inverted and those equations have a unique solution in L^(0,+w). Zero is a regular value for the operator in Eq.(13) where one phooses 0 < y < 1 and when y is different from sero (see Eq.(18)) i.e. provided alection-ion interactions are taken into account. Thus Eq.(13) as a unique solution in L^(0,+w). Then Fq.(13) as a unique solution in L^(0,+w). Then Fq.(14) as a unique solution in L^(0,+w). Of syo (L_1+L_2), see appendix 3. Then Eq.(13) has solutions in L^2(0,+) if and only is the right-hand side is ort ogenal to described up that the orthogonality condition indeed is rularilled one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn back to the original eqiation in (2) since saking one has to turn form of the right-hand side (see [2], p.38).

for an analogue).

We have thus shown that Eq.(1) has solutions depending on arbitrary constants $k_{\dot{1}}$ (two if $\tilde{\gamma}\neq 0$, three if $\tilde{\gamma}=0)$ given by

$$f_{1M}^{1}(\underline{c}_{1}) = h_{1}^{0} \left(\frac{m_{1}}{2\pi k T_{1}^{0}}\right)^{\frac{3}{2}} \left\{ \frac{1}{c} e^{-\frac{\gamma-1}{2}c^{2}} \left(\Phi_{1}^{0}(c)\cos\theta + \Phi_{1}^{-1}(c)\sin\theta\sin\chi + \Phi_{1}^{1}(c)\sin\theta\cos\chi\right) + e^{-c^{2}\left[k_{1}+k_{2}c^{2}+(1-\delta_{\gamma,0})k_{3}c\cos\theta\right]} \right\}$$

$$(20)$$

 \underline{c} is the nondimensional velocity, $\underline{c} = (\frac{m_1}{2kT_1^0})^{\frac{1}{2}}\underline{c}_1$, Φ_1^{-1} , Φ_1^0 and Φ_1^1 are solutions of Eqs.(12), (13), (14), respectively, in $L^2(0,+\infty)$. (c,θ,χ) are the spherical polar coordinates of \underline{c} , the polar axis being directed along \underline{B} . Thus the solutions, Eq.(20) are such that

$$\int d\underline{c} \, e^{(1-\gamma)c^2} |f_{1M}^1(\underline{c}_1)|^2 < \infty , \quad 0 < \gamma < 1.$$

5. Properties of solutions.

Further properties of solutions of Eq.(1) are obtained. First we establish differentiality of solutions of Eqs(11)-(15). We see by inspection that $e^{\gamma c^2}L_{2\ell}\psi(c)$ is continous in $(0,+\infty)$. Further a straight forward analysis gives a rough estimation of $e^{\gamma c^2}L_{2\ell}\psi(c)$:

$$|e^{\gamma c^2}L_{2\ell}\psi(c)| < (Nc^{\frac{3}{2}}+Pc^{\frac{3}{2}})e^{\frac{\gamma-1c^2}{2}} \left[\int_{0}^{\infty} |\psi|^2 dc\right]^{\frac{1}{2}}$$
 (21)

for an analogue).

We have this shown that Eq. (1) has columnant dependent on subtrary constants Eq. (two 1f V d.O. thise 1f V n Q) given by

In $E^2(0, +\omega)$, when nonlimentational velocity, $g = (\frac{m_1}{2(\pi^2)})^2 Q_1$, ψ_1^2 , ψ_2^2 , ψ_3^2 , and ψ_3^2 are solutions of $\mathbb{E}_{\{0,+\omega\}}$, (13), (14), respectively, in $E^2(0,+\omega)$, $(0,0,\chi)$ are the opherical polar coordinates of g, the polar sale being directed alone \mathbb{R}_2 . Thus the solutions, \mathbb{R}_2 . (20) are such that

- Proceedings of sectional

First we satabile aliferentiality of solutions of Eq.(1).

First we satabile aliferentiality of solutions of Eq.(11).

(15). We see by Largeotion fault see For Magnetions.

(15). We find there a steadiest Venezard analysis gives a rough section of eV. L. y(c):

for all c in $(0,+\infty)$, where N and P are independent of c and ψ . Defining $(e^{\gamma c^2}L_{2\ell}\psi)_{c=0}=0$, we get that $e^{\gamma c^2}L_{2\ell}\psi(c)$ is continuous everywhere when ψ belongs to $L^2(0,+\infty)$; since the right-hand sides of Eqs.(11)-(15) are continuous everywhere, and zero is the only singular point of $e^{\gamma c^2}L_{1\ell}$ at finite distance, it follows that the solution of these equations (which we know belong to $L^2(0,+\infty)$) are twice continuously differentiable on $(0,+\infty)$. Continuity and differentiality of solutions at c=0 follow from a study of asymptotic properties.

Using Eq.(21), we estimate the non-differential terms in Eqs.(11)-(15) to be of order $O(c^2)$ as $c\to 0$. For $\ell=1$, relevant equations in the neighbourhood of zero are

$$y'' - \frac{6}{5}cy' - \frac{6\tilde{\gamma}}{c^{3}}y - \frac{2}{c^{2}}y + 2imB'y = C_{1}c^{\frac{3}{2}}$$

$$m = 0, \pm 1; C_{1} \text{ constant}$$

Asymptotic solutions of this equation are obtained by using the method of variation of coefficients and asymptotic expansions Eq.(A.1.4) of the solutions of the corresponding homogeneous differential equations. We get $\Phi_1^m(c) = O(c^2)$ when $\tilde{\gamma} > 0$ and $\Phi_1^m(c) = O(c^2)$ when $\tilde{\gamma} = 0$ as $c \to 0$.

Using the same method, asymptotic behaviour of solutions may be obtained for large c. Relevant equations in the neighbourhood of $c=\infty$ are

$$y'' + [2(2\alpha-1)c-3c^{-1}]y' + [4\alpha(\alpha-1)c^{2}-imBc^{3}]y = c_{2}c^{7}e^{-\alpha c^{2}}$$

In all o in (0,+0), where H and F are independent of and y and y. Setining (e^{ye} loyy) composed the continuous everywhere when y belongs to the L²(0,+0); aimoe the right-hand sides of Eqs(11)-(15) are continuous everywhere, and sero is the only singular point of e^{ye} los timite distance, it follows that the schutics of these equations (which we know belong to L²(0,+0)) are tides continuously differentiable on (0,+0). Continuity and cifferentiality as solutions as e = 0 follow from a rucky of caracteric properties.

using Eq. (21), we estimate the non-differential lemme in Eq. (11)-(15) to be of order $v(e^2)$ as e = 0. For e = 1, relevant squations in the neighbourhood of zero ene

depleases .5 : 14 .0 = m

Asymptotic solutions of bull equation are obtained by walks the method of variation of coefficients and saymptotic expansions Eq. (A. i. 4) of the solutions of the corresponding howegeneous differential equations. We get $\Phi_{i}^{2}(c) = 0(c^{2})$ when $\hat{\varphi} > 0$ and $\Phi_{i}^{2}(c) = 0(c^{2})$ when $\hat{\varphi} > 0$ and $\Phi_{i}^{2}(c) = 0(c^{2})$ when $\hat{\varphi} > 0$ are seen sethod, regarded being floor of solution

and the formulation to be a fine for the fine formulation to be a fine

San-100 = v1 (DEM1-Soft-what] + tuf - a5-a(1-a5)51 + "v

When $B \neq 0$, we find that $\Phi_1^0(c) = O(c^7 e^{-\alpha c^2})$ and $\Phi_1^{\pm 1}(c) = O(c^4 e^{-\alpha c^2})$. When B = 0, we find that $\Phi_1^m(c) = O(c^7 e^{-\alpha c^2})$, m = 0, ± 1 .

Summarizing the results, we have shown that Eq.(1) has solutions given by Eq.(20). These solutions are twice continuously differentiable everywhere and are such that

$$\int d\underline{c} e^{(1-\gamma)c^2} |f_{1M}^1(\underline{c}_1)|^2 < \infty , \quad 0 < \gamma < 1 .$$

When $c \to \infty$, they tend towards zero at least as fast as $c^6e^{-c^2}(c^3e^{-c^2})$ if the magnetic field and/or gradients parallel to the magnetic field are equal to zero). When $c \to 0$, the part of the solution arising from the inhomogeneous term tend towards zero at least as fast as c^2 (c if election-ion interactions are not taken into account).

when B # 0, we rind that $g^0(c) = 0(c^7 e^{-\alpha c^2})$ and $g^0(c) = 0(c^4 e^{-\alpha c^2})$. When B = 0, we find that $g^0(c) = 0(c^7 e^{-\alpha c^2})$.

Summarining the results, we have shown that Eq.(1) has solutions given by Eq.(20); There colditions are twice continuously differentiable everywhere and are such that

From tend towards zero of least as last as $e^{C_{\rm e}} = e^{C_{\rm e}} =$

Appendix 1.

We determine the number of boundary values needed to make $e^{\gamma c^2}L_{1\ell}$ selfadjoint,[5], p.1306, i.e. we study the equation

$$e^{\gamma c^2}L_{1\ell}\psi = -\lambda\psi$$
, $Im\lambda \neq 0$, $\ell \neq 0$. (A.1.1)

We show that Eq.(A.1.1) has exactly one solution which is square integrable near c=0 and one near $c=\infty$. Hence the operator is selfadjoint with no boundary condition imposed.

Equation (A.1.1) becomes near c = 0

$$y'' - \frac{6}{5} cy' - \left[3\tilde{\gamma} \ell (\ell+1) \frac{1}{c^3} + \ell (\ell+1) \frac{1}{c^2} \right] y = -\frac{3}{2} \lambda e^{-\gamma c^2} \psi \qquad (A.1.2)$$

We consider the following two cases $\tilde{\gamma} = 0$ or $\tilde{\gamma} > 0$.

(i) $\tilde{\gamma}=0$. We get,[3], [4], two linearly independent solutions of (A.1.2) which for small c behave as $c^{\ell+1}$ and $c^{-\ell}$. Hence only one of them is square integrable near c=0. (ii) $\tilde{\gamma}>0$. We substitute

$$z = a c^{-1}$$
, $y = u z^{-1}$ (A.1.3)

where a is a constant, a = $3\tilde{\gamma}l(l+1)$. For large z, Eq.(A.1.1) becomes

$$u'' - z^{-1}u = 0$$
.

This equation has subnormal solutions, [6]. For small c, the two linearly independent solutions of Eq. (A.1.1) are

Appendix 1 ...

we december the number of boundary values needed to make. State of the state of the

(C.1.4) OFFICE OFFICE WASHINGTON

Wer show that Eq. (A.1.1) has example one solution without a square integrable near c.e. C. and. one hear c.e. Harar the operator is selfedjoint with no foundary cendibles imposeds.

Nearton (A.1.1) becomes dear c.e. O

We consider the following the cases: West Traces

(()) 3 = 0. We get [53]. Lelipse lineship independent

solutions of (M.1.2) which is equal to behave at off and

off. Hence only one or what is equal integrable near c = 0.

(11) 3 > 0. We substitute

((6.1.4)) B = (6.1.3)

Merre a le a constante, a es 5/1(1+1)). Nor lange si.

This constitution has subnowned such blanks. [6] . For small of the two lanearity transparents of the confidences of the confidence of the co

asymptotically represented by

$$\psi \simeq \left(\frac{c}{a}\right)^{\frac{3}{4}} \exp\left(\pm 2\sqrt{\frac{a}{c}}\right) \sum_{k=0}^{\infty} c_k \left(\frac{c}{a}\right)^{\frac{k}{2}}, \quad c_0 \neq 0$$
 (A.1.4)

Only one of them is square integrable in the neighbourhood of c=0.

Investigating the solutions of Eq.(A.1.1) for large c, we find that the relevant equation is

$$y'' + [2(2\alpha-1)c - 3c^{-1}]y' + 4\alpha(\alpha-1)c^2y = 0$$

Assuming $\alpha > \frac{1}{2}$ and substituting

$$y = uv \quad u = exp[-(\alpha - \frac{1}{2})c^2]$$

the relevant equation for v near $c = \infty$ is

$$v^{11} - c^2 = 0$$

Asymptotic solutions of this equation may be obtained [7] in the form

$$v(c) = e^{\pm \frac{c^2}{2}} c^{-\frac{1}{2}} \sum_{k=0}^{\infty} c_k c^{-k} c_0 \neq 0$$

Since $\gamma < 1$, we find that only one of the solutions of Eq.(A.1.2) hence of Eq.(A.1.1), is square integrable near $c = \infty$. The case $\alpha = \frac{1}{2}$ is treated similarly with the same result. We have thus shown that $e^{\gamma c^2} L_{1\ell}$, $\ell \neq 0$, is selfadjoint.

d Astronomy vileotto tomyas

only eas of them is squere interesting in the meighbourhood.

Investigating the solutions of Eq. (b. 1.1) for large to

0 = 2 (2(2(-1))0. + 301 (100 + 401(0-1))0 x = 0

unifulfaction from t < m portopeed

I section Agree - u vu - v

the nelevant equation for a sear of a la

Asymptotic selections of this emission may be obtained: [7]

There y < 1, we find that only one of the analthone of the manthone of the Child (A-1) (A-1), to square integrable near or w o. The case or = } to the another almillarly with the same remult. We have thus some that some that should be a first and the self-

Appendix 2.

We write

$$e^{\gamma c^2}L_{1\ell} = \frac{d}{dc}(p(c)\frac{d}{dc}) + q_{\ell}(c)$$

where p and q $_\ell$ are defined by this relation and Eq.(16). In order to study the spectrum of $e^{\gamma c^2}L_{1\ell}$ we examine the eigenvalue problem of the Sturm-Liouville equation

$$e^{\gamma c^2} L_{1\ell} \Psi = -\lambda \Psi$$
.

To reach standard form we apply the Liouville transformation

$$U_{\ell} = [p(c)]^{\frac{1}{4}} \psi; x = \int_{0}^{c} [p(\xi)]^{-\frac{1}{2}} d\xi$$
or
$$x = \int_{0}^{c} e^{-\frac{\gamma}{2} \xi^{2}} \left(\frac{erf(\xi)}{\xi^{2}} - \frac{e^{-\xi^{2}}}{\xi^{2}} \right)^{-\frac{1}{2}} d\xi$$

to get the following equation

$$\frac{\mathrm{d}^2}{\mathrm{dx}^2} \mathbf{U}_{\ell} + [\lambda - Q(\mathbf{x})] \mathbf{U}_{\ell} = 0 \quad \mathbf{x} \in (0, A)$$

$$e^{-\gamma c^2}Q(x) = \frac{erf(c)}{c} \left(1 - \frac{\gamma^2}{4}\right) + e^{-c^2} \left(\frac{\gamma^2}{4} - \frac{\gamma}{2} - 8\right) + \frac{\gamma erf(c)}{4c^3} - \frac{\gamma^2}{4c^3}$$

$$-\frac{\gamma e^{-c^{2}}}{4c^{2}} - \frac{l(l+1)(erf(c)(2c^{2}-1) + e^{-c^{2}}) - \frac{2\gamma l(l+1)}{c^{3}} - \frac{c}{16} - \frac{(3\frac{e^{-c^{2}}}{c^{2}} - 3\frac{erf(c)}{c^{3}} + 2e^{-c^{2}})^{2}}{(erf(c) - ce^{-c^{2}})}$$

Appendix 2

ed write

(a) = + (a) (a) (a) = 1 1 2 ave

where p and q are defined by this relation and Eq. (16).

In order to study the spectrum of e^{VC} L₁ we examine the eigenvalue problem of the Sturm-Licaville equation

every = +x e

To readh, standard form we apply the blantille, transformation

nolumpe galwollpleads, seg of

 $(A_{\xi}Q) = x \quad (0 = x) = (0) = (0) = (0)$

9(34-20 + (0)372 5 - 32 0)

((Comes - (A) (228)

We have two cases

- (i) when $\gamma > 0$, then $\lim_{C \to \infty} x = A < \infty$
- (ii) when $\gamma \leq 0$, then $\lim_{C \to \infty} x = \infty$
- (i) When $x \to A$ the dominant term in Q(x) is $(1 \frac{\gamma^2}{4}) \frac{1}{c} e^{\gamma c^2}$ and $\lim_{x\to A} Q(x) = +\infty$ when $0 < \gamma < 2$. On the other side

$$\lim_{x\to\infty} Q(x) \cong \lim_{c\to\infty} \frac{2\ell(\ell+1)}{3c^2} + 2\gamma \frac{\ell(\ell+1)}{c^3} = +\infty.$$

Noticing that the essential spectrum of $e^{\gamma c^2}L_{1\ell}$ is the union of the essenstial spectrum of $e^{\gamma c^2}L_{1\ell}$ on $(0,A_0]$ and $[A_0,A)$, $A_0 < A$, we get (see [5], p.1594 and p.1599) that the essential spectrum of $e^{\gamma c^2}L_{1\ell}$ is void when $0 < \gamma < 1$. Hence the operator has only a discrete spectrum in this case.

(ii) We have still $\lim_{\substack{x\to 0 \\ x\to 0}} Q(x) = +\infty$ while $\lim_{\substack{x\to 0 \\ x\to \infty}} Q(x) = 0$. For the same reasons as before we have now that the essential spectrum of $e^{\gamma c^2}L_{1\ell}$ is the negative semi-axis $(-\infty,0]$.

seaso ovi evan'av

(c) when y > 0, then LE X = A < =

when y \$ 0, then its x = m

(1) then x = A the dominant term in Q(x) is $(1 - \frac{1}{V})_0^{1/V^2}$ and this Q(x) = x when $0 < y < \theta$. On the other side x = x

in the second se

Noticing that the essential spectrum of $e^{\gamma C}$ L, is the mass of the essential spectrum of $e^{\gamma C}$ L, on $(0,A_0)$ and (A_0,A) . A < A, we get (see [5], p.159% and p.1599) beat the sessivial spectrum of $e^{\gamma C}$ L, is void seen $C < \gamma < 1$. Hence the operator has only a discrete spectrum in this case. (iii) we have still lim $O(x) = \frac{1}{2}$ we calle lim O(x) = 0. For the case case that the left case reasons as pefore we have one that the description of the sessition of the case of the case the sessition of the case of

10 mm) show-large everyoned and as 10 month and

Appendix 3.

We study now the sign of $e^{\gamma c^2}(L_{1\ell}+L_{2\ell})$. To do so we examine the quantity

$$I = \int \underline{dc} \Psi(\underline{c}) FP_{11}[f_{1M}^{o}(\underline{c})f_{1M}^{o}(\underline{c})(\Psi(\underline{c}) + \Psi(\underline{c}_{1}))] \qquad (A.3.1)$$

It is standard work to show that

$$I \leq 0 \tag{A.3.2}$$

for all Ψ which are bounded, $\frac{\partial \Psi}{\partial \underline{c}}$ bounded and $\frac{\partial^2 \Psi}{\partial \underline{c}^2}$ continuous (see [8] for an analogue) and that I=0 if and only if

$$\Psi(\underline{c}) = \rho + c^2 + \underline{k} \cdot \underline{c}$$

where ρ,δ are constants and k a constant vector.

Further, let us specify $\Psi(\underline{c})=c^{-1}\mathrm{e}^{\gamma c^2}\psi(c)Y_\ell^m$. Using Eqs(3), (4), (5), (6), (16) and (A.3.1) and assuming $0<\gamma<1$ we get

$$\int_{0}^{\infty} dc \overline{\psi} e^{\gamma c^{2}} (L_{1\ell} + L_{2\ell}) \psi \leq -2 \widetilde{\gamma} \ell (\ell+1) \int_{0}^{\infty} \frac{e^{\gamma c^{2}}}{c^{3}} |\psi|^{2} dc$$

$$\leq -2\widetilde{\gamma}\ell(\ell+1)M\int_{0}^{\infty}|\dot{\psi}|^{2}dc$$

where $M=\inf\frac{e^{\gamma c^2}}{c^3}$, $c\in[0,+\infty)$. Thus the operator is semibounded on a set of functions ψ which is dense in $L^2(0,\infty)$, and it can be extended [9] to an operator which is selfadjoint and semibounded with the same bound. Thus

E xibmegoA

We study now the sign of eve (Ingles). To do so we sattle the quantity

(1.5.A) E((,2))+ (0) (,2), (2), (3(2)), (31), (3(2)) (0) } = I

tend work of arow bushness at at

(S. E. A)

for all which are bounded, $\frac{25}{50}$ bounded and $\frac{25}{2}$ continuous for all wind that 1 = 0 15 and only if

2.4 + 80 + 0 + (9) 70

where ρ_{i} are constants and E constant vector. Where ρ_{i} and ρ_{i}

OF THE CONTRACT OF THE PROPERTY OF THE PROPERT

opsiel provide - s

where $\mathbb{N} = \inf \frac{e^{\gamma_0}}{\sqrt{2}}$, $c \in [0, +\infty)$. Thus the operator is sense in semilbounded on a set of functions φ which is dense in $\mathbb{N}^2(0,\infty)$, and it can be extensed [2] to an epositor which is self-adjoint and resultangular rich the same bound. Thus

 $e^{\gamma c^2}(L_{10}+L_{20})$ is negative definite when $\tilde{\gamma} \neq 0$ and $\ell \neq 0$. When $\tilde{\gamma} = 0$, i.e. when interactions with ions are not taken into account, total momentum of electrons is conserved during electron-electron interactions, and zero is an eigenvalue for $\ell = 1$. $e^{\gamma c^2} (L_{11} + L_{21})$ is thus negative. Since $0 < \gamma < 1$ is assumed, the essential spectrum is void, see appendix 2, and zero is isolated. It is thus possible to invert $e^{\gamma c^2}(L_{11}+L_{21})$ on the subspace of element which are orthogonal to $c^2e^{-\alpha c^2}$ (corresponding to $\Psi(\underline{c}) = \underline{k} \cdot \underline{c}$, \underline{k} constant vector). When l = 0, $e^{\gamma c^2} (L_{10} + L_{20})$ also is negative, independent of the value of $\tilde{\gamma}$ and B. Indeed zero is eigenvalue as can be seen from Eqs(A.3.1), (3), (4), (5) and (6). This corresponds to conservation of mass and total kinetic energy during electronelectron interactions, to conservation of mass and energy of electrons during electrons-heavy ions interactions (see Eq. (6)) together with the fact that the magnetic field operator $-\frac{c_1}{m_1} c_1 \times \underline{B} \cdot \frac{\partial}{\partial C_1}$ is a differential rotation operator.

References.

- [1] A.H. Øien, J. Naze Tjøtta Phys. Fluids, to appear.
- [2] J. Naze Tjøtta, A. H. Øien "Kinetic Theory of a Weakly Coupled and Weakly Inhomogeneous Plasma in a Magnetic Field". Rep. No. 20, Dept. Appl. Math. Bergen (1969)
- [3] C.H. Su, J. Math. Phys. <u>8</u>, 248 (1967)
- [4] J.B. McLeod, R.S.B. Ong, J. Math. Phys. <u>8</u>, 240 (1967)
- [5] N. Dunford, J.T. Schwarz, "Linear Operators" Part II Interscience Publishers, Inc. New York (1963)
- [6] J. Dettman, "Applied Complex Variables", Chap. 10, The Macmillan Company, New York (1965)
- [7] E. Kamke, "Differential Gleichungen. Lösungsmethoden und Lösungen", I, p.100. Akademische Verlagsgesellschaft. Geest & Partig K. G. Leipzig (1956)
- [8] D.C. Montgomery, D.A. Tidman, "Plasma Kinetic Theory".

 McGraw-Hill, New York (1964)
- [9] F. Riesz, B.Sz. Nagy "Leçons d'analyse fonctionnelle".

 Académie des Sciences de Hongrie, Budapest (1953).

- 09 -

III A.H. Diens, J. Maie Tiptta Phys. Fluids, to appear.

(2) J. Mace Timber, A. H. Mine the Cheory of a Meakly

Coupled and Westly Inhomogeneous Plasma in a Magnauto

Misad", Sop. No. 20, Dept. Appl. Math. Bergen (1969)

(3) c. ft. Smy it Mach. Phys. <u>8</u> 248 (1937)

(15) J. B. Milson, R. S. B. Cag. J. Hatin Inst. E. 240 [1967]

IS IN Sunrand, D.S. Schwarz, "Lineau Opendore" Part II

interesting on A. Stere . Inc. New York 1.1965)

To a Derbitan, "selbio reov xelono centrol", dereved to paj:

The state was exampled on times a spri

(F) E. Mandrey "Daffishential Cleichungen. Dicumpenelloden

Coses & Percela W. - C. Married & Joseph

(8) D.D. Montgonery, D.A. Trunan, "France Minerile Phoory".

[9] F. Hass, E.Sz. Nagy "Esques dismalyde fignotionnelle".

ACCIONAL PERGRAPA DI MANDE SE SECRETO SE SINSTEMA



