# CP Violation in the Two Higgs Doublet Model

Master Thesis in Theoretical Particle Physics
June 2010

*Author:*
Are Flataker Viken

*Supervisor:*
Professor Per Osland

Department of Physics and Technology
University of Bergen
Bergen, Norway

# Acknowledgements

I would like to thank my supervisor Professor Per Osland for all his help during the time this thesis has been written. Thank you for your never-ending patience and all the proof-reading you have done for me. Also, I am truly grateful for the countless discussions and your achievement to always stay calm no matter how confused I have been. Your door is always open, and this reflects your open persona, together with your good sense of humor and dedication to help others. Thank you.

I would also like to thank my family for financial and cheerful support during my studies.

I am also grateful to my fellow students for talks on physics, good laughs and smiles. Special thanks to Mahdi for our funny talks on physics and life.

To the Administration of the Department of Physics and Technology, thank you for all the help I have gotten during these years. Especially I am thankful to Villy for providing me with hard laughs, supplying me with umbrellas on rainy days and for just being a happy person.

# Contents

# Chapter 1

# Introduction

The Standard Model is the current model we have for describing the elementary particles, i.e. quarks, leptons and the gauge bosons. In the Standard Model we have twelve gauge bosons: the photon, three weak bosons and eight gluons. A gauge theory is a field theory where we have a Lagrangian invariant under a group of *gauge transformations*. To ensure the invariance of the Lagrangian under these gauge transformations, a gauge *field* is introduced. The gauge bosons are the particles associated with these fields. In Chapter 2 we give a brief introduction to gauge theory.

Baryons are the particles made up of three quarks, such as the proton and neutron. Of all the visible matter in the Universe, baryons make up most of this, hence called *baryonic matter*. From the Big Bang we assume that there has been produced an equal amount of baryons and anti-baryons, but today we observe an asymmetry between the amount of these. *Baryogenesis* tries to explain how this asymmetry has occured. CP Violation might be able to solve this problem, and in Chapter 3 we introduce symmetries and transformation necessary for our discussion. In the literature there are described two types of CP Violation: Explicit and Spontaneous CP violation and both are briefly described in Chapter 3.

The Electroweak theory is explained in Chapter 4. This theory is a unification of the electromagnetic and weak forces of nature. We also look at the concept of Flavour Changing Neutral Currents.

In the Standard Model there is only one Higgs field and one associated Higgs boson, which is the only particle of the Standard Model yet to be observed. What makes all the magic happen is that the vacuum expectation value of the Higgs field is non-zero. The mass of the elementary particles in the Standard Model is explained through what we call *the Higgs Mechanism* and *Spontaneous Symmetry Breaking*, and this is where the non-zero vacuum expectation value of the Higgs field comes into play. In any gauge theory the gauge bosons become massive through the Higgs Mechanism, and Chapter 5 gives a brief introduction to the Higgs Mechanism and Spontaneous Symmetry Breaking, while Chapter 6 goes on to describe the Higgs model.

The Two Higgs Doublet Model introduces another Higgs field, which gives us the possibility of CP violation beside the known CKM matrix. This model predicts a total of five Higgs bosons, three neutral and two charged bosons. Chapter 7 explains the Two Higgs Doublet Model. We introduce the basic model and the symmetries under which the physics of the model is invariant. Also, we present the mass squared matrix of the neutral Higgs bosons.

The main focus of this thesis is to debate whether there is a possibility that Explicit and Spontaneous CP violation can in fact be two faces of the same coin, that there exists no physical difference. It has been pointed out in the literature that the complexity of the non-zero vacuum expectation value of the Higgs field(s) has no physical meaning and can be transformed away[1], but at the same time in the concept of Spontaneous CP violation this complexity plays a role. This seems, at first sight, ambiguous.

Chapter 8 goes on to describe CP violating properties of the model. In relation with a discussion on Yukawa couplings certain restrictions on the model are introduced, called the Two Higgs Doublet Model Type-II, and we continue our work under these restrictions throughout the thesis. We go on to discuss invariants of the model, which is a nice tool to study CP Violation. A model introduced by Lee in 1973 is discussed, which is a model created to achieve Spontaneous CP Violation.

In Chapter 9, we discuss measurable quantities in the Two Higgs Doublet Model, and basically describe thought experiments on how to obtain experimentally the results needed to calculate the parameters of the Higgs potential. If there indeed is a difference between Explicit and Spontaneous CP violation, we discuss whether it is possible to actually measure this.

[1]We read in [7] that this has been discussed in e.g. [1]

# Chapter 2

# Gauge Theories

From the Maxwell equations in classical electromagnetism we have for the electric and magnetic fields

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{2.1}$$

and

$$\mathbf{E} = -\nabla V - \frac{\partial}{\partial t}\mathbf{A}, \tag{2.2}$$

with the vector potential $\mathbf{A}$ and scalar potential V, which we can write as the four-vector $A_\mu(x) = (V, \mathbf{A})$. But the potentials $\mathbf{A}$ and V are not unique for the fields $\mathbf{E}$ and $\mathbf{B}$, and this is the basis of gauge invariance; transformations on the potentials without changing $\mathbf{E}$ and $\mathbf{B}$ are called gauge transformations, while the invariance, in this case of the Maxwell equations, is called gauge invariance. These transformations, using the four-vector potential $A_\mu \equiv (V, \mathbf{A})$, are specified by

$$A^\mu \to A'^\mu = A^\mu - \partial^\mu \chi. \tag{2.3}$$

Writing the Maxwell equations as

$$\partial_\mu F^{\mu\nu} = j^\nu_{em}, \tag{2.4}$$

where the field strength tensor

$$F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu \tag{2.5}$$

and

$$j^\nu_{em} = (p_{em}, \mathbf{j}_{em}) \tag{2.6}$$

we see that the Maxwell equations are gauge invariant under the transformations above.

Charge conservation is related to an invariance of the electromagnetic potential under a transformation by a constant factor, making the change the same everywhere —hence global gauge transformation. A local gauge transformation is a transformation where the change in the potential is not the same everywhere, but compensation by another local change keeps the physical equations unchanged. Dynamical theories based on local invariance principles are called gauge theories. The demand that a theory is invariant under local gauge transformation, and that this dictates the form of the interaction, is the basis of the "gauge principle". For example, let us take the free-particle Schrödinger equation

$$\frac{1}{2m}\left(-i\nabla\right)^2\psi(\mathbf{x},t) = i\frac{\partial\psi(\mathbf{x},t)}{\partial t}. \tag{2.7}$$

If we take the local gauge transformation

$$\psi(\mathbf{x},t) \to \psi'(\mathbf{x},t) = e^{i\alpha(\mathbf{x},t)}\psi(\mathbf{x},t) \tag{2.8}$$

we see that the free-particle Schrödinger equation is not invariant upon this transformation. Thus, we must modify the equation to be invariant under the local gauge transformation above. This new equation will then not describe the same physics, that is a free particle from before, and we must therefore introduce a new force field in which the charged particle moves. We know from calculations not to be discussed here, that with $\alpha = q\chi$ this local phase transformation is the transformation associated with the electromagnetic gauge invariance, such that the Schrödinger equation now becomes

$$\frac{1}{2m}\left(-i\nabla - q\mathbf{A}\right)^2\psi = \left(i\frac{\partial}{\partial t} - qV\right)\psi, \tag{2.9}$$

when $\psi \to \psi'$. We see that the phase invariance dictates the interaction, and a vector field like $A^\mu$ which is being "dictated", is called a "gauge field".

# Chapter 3

# Symmetries and special transformations

This is a brief discussion on some different symmetries and transformations we will discuss in this thesis, mostly based upon the introductory chapters of the book “CP Violation” by Branco, Lavoura and Silva [1].

## 3.1 Charge, Parity and Time symmetry

Parity symmetry is the invariance of some physics under a discrete transformation usually denoted by P, in which the sign of the space coordinates changes. By a mirror reflection of a coordinate-plane, followed by a rotation by an angle $\pi$ of the axis perpendicular to this plane, one achieves the same thing as a parity transformation. Since physics is invariant under a rotation from the assumption of isotropy of space, we note that parity symmetry is here equivalent to mirror symmetry.

Charge-conjugation symmetry, usually denoted by C, is related to the existence of an antiparticle for every particle, such as the positron for the electron. For every complex field $\phi$ one can relate both positively and negatively charged particles, thus there exists a C-transformation transforming $\phi \to \phi^\dagger$ with opposite U(1)-charges. Positive and negative charges, and left and right, are just mere conventions, but do they differ in some intrinsic physical way?

Time symmetry, or T symmetry is related to the time reversal of some physical event. Is the physical event watched backwards possible? In classical physics it seems obvious, but how does this relate to particle physics?

### 3.1.1 Transformation operators

We represent the Charge, Parity and Time transformations by the operators $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$, respectively, where the two first are unitary and the latter antiunitary. But how do we define and construct these operators, considering for example that the weak interactions do not obey these symmetries? Since we realize that the electromagnetic interactions are C and P invariant, and together with the experimental indication that the strong interactions are too, we can construct satisfying operators. Then, we probe other interactions with the same operators, and see whether they obey the same symmetries. In other words, we define $\mathcal{C}$ and $\mathcal{P}$ to be invariant under the kinetic and electromagnetic part of the Lagrangian, and then compare to the other parts of the Lagrangian to determine whether we have violation of $\mathcal{C}$ and $\mathcal{P}$ or not. Not going into detail, we set the quantum numbers associated with both $\mathcal{C}$ and $\mathcal{P}$ (respectively C-parity and parity) to be +1 and -1. These quantum numbers are not additive such as for momentum, but rather multiplicative. The $\mathcal{T}$ operator does not have meaningful eigenvalues.

## 3.2 CP transformation

The problem of distinguishing matter from antimatter can only be solved by removing our left and right convention, and the composite transformation CP must thus be violated. The charge asymmetry in $K_{l3}$ decays [1] is our clearest evidence of CP violation. The neutral particle (kaon) $K^0$ has a well-defined mass and decay width, is its own antiparticle, and decays in two different ways, one slightly less often than the other. We have both C and CP violation. CP violation is also of great theoretical importance, as it may also explain the observed baryon asymmetry of the Universe, i.e. much more baryonic matter than antimatter. Also CP violation makes it possible for elementary particles to have electric dipole moments.

## 3.3 CPT theorem

The "CPT theorem" states that a quantum field theory, assuming the correctness of the general properties which quantum field theory is based on, such as Lorentz invariance and local (anti-)commutation properties obeying the spin-statistics connection, must be CPT invariant.

[1] leptonic decays of the K meson into three particles, i.e. a lepton, pion and neutrino

## 3.4 Chirality and Helicity

The helicity of a particle is Right(Left)-handed, or positive(negative), if the direction of its spin is in the same(opposite) direction as its momentum. Its operator is $\Pi^{\pm}(\mathbf{p}) = \frac{1}{2}(1 \pm \sigma_p)$, where $\sigma_p = \sigma \cdot \mathbf{p}/|p|$. While helicity is related to handedness, chirality is related to weak charge, as we shall see later. For massless particles, chirality is the same as helicity. Its operator is $\frac{1}{2}(1 \pm \gamma_5)$. Chirality is Lorentz invariant, while helicity is not for $v \neq c$, because one can then always change to a different frame where the momentum has opposite direction. Helicity is a pseudoscalar since the spin $\mathbf{s}$ is an axial vector, or pseudovector, and momentum $\mathbf{p}$ is a polar vector. This means that under a parity transformation the helicity changes sign.

## 3.5 Explicit and Spontaneous CP Violation

As previously discussed, when defining a general CP transformation we require the kinetic part of the Lagrangian to be CP invariant under this transformation. If it so happens that the potential part of the Lagrangian is not CP invariant under this transformation, we say we have explicit CP violation (ECPV). Generating this CP transformation, we must also take into account Spontaneous Symmetry Breaking (SSB) and the values of the vacuum expectation values (VEVs). When CP is a symmetry of the original Lagrangian (both the kinetic and the potential part), but after SSB no CP symmetry remains, we have Spontaneous CP Violation (SCPV). This means that there is no CP transformation conserving the symmetry of both the Lagrangian and the vacuum. This idea comes from Lee (1973) [5].

# Chapter 4

# Electroweak theory

Unification of the electromagnetic and weak interactions was a big breakthrough. This "electro-weak" theory describes the weak interactions transmitted by heavy vector bosons $W$, like photons for electromagnetic forces. Particles involved in the weak interactions are hadrons, and leptons and neutrinos. Hadrons, e.g. n,p,$\pi$,$\Lambda$ participate in the strong interaction too, and neutrinos only in the weak interactions.

For purely leptonic processes the weak interaction Hamiltonian density is constructed from the leptonic currents

$$J_\alpha(x) = \sum_l \overline{\psi}_l(x)\gamma_\alpha(1-\gamma_5)\psi_{\nu_l}(x) \tag{4.1}$$

and

$$J_\alpha^\dagger(x) = \sum_l \overline{\psi}_{\nu_l}(x)\gamma_\alpha(1-\gamma_5)\psi_l(x), \tag{4.2}$$

$l$ labelling the various charged lepton fields and $\nu_l$ the corresponding neutrino fields. Thus, the Intermediate Vector Boson theory, IVB, is described by:

$$\mathcal{H}_I(x) = g_W J^{\alpha\dagger}(x)W_\alpha(x) + g_W J^\alpha(x)W_\alpha^\dagger(x), \tag{4.3}$$

$g_W$ being a dimensionless coupling constant and $W_\alpha(x)$ a field describing the $W$ particles. The Electroweak theory contains another neutral current, which we will come back to. Analagous to QED, the field $W_\alpha(x)$ is coupled to the leptonic vector current. The interaction Hamiltonian above can be rewritten if we write the current as

$$J^\alpha(x) = J_V^\alpha(x) - J_A^\alpha(x), \tag{4.4}$$

with vector current

$$J_V^\alpha(x) = \sum_l \overline{\psi}_l(x)\gamma^\alpha\psi_{\nu_l}(x) \tag{4.5}$$

and axial vector current

$$J_A^\alpha(x) = \sum_l \overline{\psi}_l(x)\gamma^\alpha\gamma^5\psi_{\nu_l}(x). \tag{4.6}$$

Since the axial vector current transforms as a pseudo-vector, it is not invariant under the parity transformation, while the vector current is invariant. Because of the small mass of the neutrinos, we make the approximation $m_{\nu_l} \approx 0$. From our discussion of chirality and helicity, we know that for $m$=0, the chirality operator $\frac{1}{2}(1 \pm \gamma_5)$ is equal to the helicity operator. From the leptonic current $J_\alpha(x)$, we see that the helicity operator is "working its magic" on $\psi_{\nu_l}$, thus only annihilation of negative helicity neutrinos and creation of positive helicity anti-neutrinos is present in our interaction. For high energy charged leptons, $E \gg m_l$, we may make the same approximation such that we also here only have the left-handed fields involved. We can now write

$$J_\alpha(x) = 2\sum_l \overline{\psi}_l^L(x)\gamma_\alpha\psi_{\nu_l}^L(x), \tag{4.7}$$

where

$$\psi_{\nu_l}^L(x) \equiv \frac{1}{2}(1-\gamma_5)\psi_{\nu_l}(x) \tag{4.8}$$

and

$$\psi_l^L(x) \equiv \frac{1}{2}(1-\gamma_5)\psi_l(x) \tag{4.9}$$

For a massive vector boson (spin 1) field the Proca equation yields:

$$\Box W^\alpha(x) - \partial^\alpha(\partial_\beta W^\beta(x)) + m_W^2 W^\alpha(x) = 0 \tag{4.10}$$

with Lorentz condition

$$\partial_\alpha W^\alpha(x) = 0, \tag{4.11}$$

reducing the equation to

$$\Box W^\alpha(x) + m_W^2 W^\alpha(x) = 0 \tag{4.12}$$

The corresponding free field Lagrangian density is

$$\mathcal{L}(x) = -\frac{1}{2}F_{W\alpha\beta}^\dagger(x)F_W^{\alpha\beta}(x) + m_W^2 W_\alpha^\dagger(x)W^\alpha(x) \tag{4.13}$$

with

$$F_W^{\alpha\beta}(x) \equiv \partial^\beta W^\alpha(x) - \partial^\alpha W^\beta(x) \tag{4.14}$$

and

$$W^{\alpha}(x) = \sum_{\mathbf{k}r} \left(\frac{1}{2V\omega_k}\right)^{\frac{1}{2}} \epsilon_r^{\alpha}(\mathbf{k})[a_r(\mathbf{k})e^{-ikx} + b_r^{\dagger}(\mathbf{k})e^{ikx}] \tag{4.15}$$

where $\epsilon_r^{\alpha}(\mathbf{k})$ forms a complete set of orthonormal polarization vectors.

## 4.1 Gauge theory of weak interactions

### 4.1.1 Invariance of QED

The simplest gauge theory is QED, and by making a simple approach to local gauge invariance of QED, we can use this to formulate the theory of weak interactions as a gauge theory as well. We have the free-field Lagrangian density for electrons,

$$\mathcal{L}_0 = \overline{\psi}(x)(i\partial\!\!\!/_{\mu} - m)\psi(x). \tag{4.16}$$

We demand invariance under the local phase transformations

$$\begin{aligned} \psi(x) \to \psi'(x) &= \psi(x)e^{-iqf(x)} \\ \overline{\psi}(x) \to \overline{\psi'}(x) &= \overline{\psi}(x)e^{iqf(x)}, \end{aligned} \tag{4.17}$$

and to cancel out the new terms arising from these transformations we must introduce a gauge field. This gauge field, $A_\mu(x)$ is associated with the matter field $\psi(x)$ and transforms as

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu f(x). \tag{4.18}$$

The interaction between these two fields is given by the minimal substitution, i.e. replacing the ordinary derivative by the covariant derivative:

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu(x), \tag{4.19}$$

such that we get

$$\mathcal{L}_0 \to \mathcal{L} = \overline{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) = \mathcal{L}_0 - q\overline{\psi}(x)\gamma^\mu\psi(x)A_\mu(x) \equiv \mathcal{L}_0 + \mathcal{L}_I \tag{4.20}$$

which is invariant under the local gauge transformations above.

### 4.1.2 Invariance of the weak interactions

We now use this same approach to formulate a theory for the weak interactions, not giving a detailed discussion, but giving the important results.

We start by assuming the particles we are discussing are massless, as we shall see later it is the Higgs mechanism which gives particles mass. Thus, from QED, the free-lepton Lagrangian density is

$$\mathcal{L}_0 = i[\overline{\psi_l}(x)\partial\!\!\!/\psi_l(x) + \overline{\psi}_{\nu_l}(x)\partial\!\!\!/\psi_{\nu_l}(x)], \tag{4.21}$$

and implied summation over the different leptons. With our previous discussion of the weak interactions, where only the left-handed lepton and neutrino fields are contributing, and knowing that for massless particles the chirality operator equals the helicity operator, we write (4.21) in terms of left- and right-handed fields, using

$$\frac{1}{2}(1 \mp \gamma_5)\psi(x) \equiv \begin{cases} \psi^L(x) = P_L\psi(x) \\ \psi^R(x) = P_R\psi(x) \end{cases} \tag{4.22}$$

and combining $\psi_l^L$ and $\psi_{\nu_l}^L$ into the two-component field

$$\mathbf{\Psi}_l^L(x) = \begin{pmatrix} \psi_{\nu_l}^L \\ \psi_l^L \end{pmatrix} \tag{4.23a}$$

$$\overline{\mathbf{\Psi}}_l^L(x) = \begin{pmatrix} \overline{\psi}_{\nu_l}^L(x) & \overline{\psi}_l^L(x) \end{pmatrix} \tag{4.23b}$$

we get

$$\mathcal{L}_0 = i[\overline{\mathbf{\Psi}}_l^L(x)\partial\!\!\!/\mathbf{\Psi}_l^L(x) + \overline{\psi}_l^R(x)\partial\!\!\!/\psi_l^R(x) + \overline{\psi}_{\nu_l}^R(x)\partial\!\!\!/\psi_{\nu_l}^R(x)] \tag{4.24}$$

This Lagrangian density is left invariant when the bilinears (4.23) transform under a two-dimensional global phase transformation, such that the free-lepton Lagrangian density (4.21) also is invariant, leading to conservation of the weak currents. The asymmetry between the left and right fields leads to different transformation properties, respectively SU(2) and U(1) transformations. The unitary operator which transforms the bilinears is

$$U(\boldsymbol{\alpha}) \equiv \exp(i\alpha_j\tau_j/2), \tag{4.25}$$

where $\tau_j$ are the Pauli matrices and $\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \alpha_3)$ are real numbers. We then have the set of transformations which leave (4.24) invariant

$$\begin{aligned} \mathbf{\Psi}_l^L(x) \to \mathbf{\Psi}_l^{L'}(x) &= U(\boldsymbol{\alpha})\mathbf{\Psi}_l^L(x) \equiv \exp(i\alpha_j\tau_j/2)\mathbf{\Psi}_l^L(x) \\ \overline{\mathbf{\Psi}}_l^L(x) \to \overline{\mathbf{\Psi}}_l^{L'}(x) &= \overline{\mathbf{\Psi}}_l^L(x)U^\dagger(\boldsymbol{\alpha}) \equiv \overline{\mathbf{\Psi}}_l^L(x)\exp(-i\alpha_j\tau_j/2). \end{aligned} \tag{4.26}$$

We define the right-handed lepton field to be a weak isoscalar, thus invariant under any SU(2) transformation. SU(2) transformations are 2×2 unitary operators, or matrices, like $U(\boldsymbol{\alpha})$, with the special property that det $U(\boldsymbol{\alpha}) = +1$. The set of all these SU(2) transformations forms the SU(2) group. Depending on the commutation of the elements which constitute the group, we call a group Abelian if they commute, and non-Abelian if they do not commute. Since the Pauli matrices do not commute, the SU(2) group is non-Abelian.

The invariance of $\mathcal{L}_0$ (4.24) leads to three conserved currents. Not just two as stated before in the IVB-theory,

$$J_i^\alpha(x) = \frac{1}{2}\overline{\boldsymbol{\psi}}_l^L(x)\gamma^\alpha\tau_i\boldsymbol{\psi}_l^L(x), \quad i = 1, 2, 3, \tag{4.27}$$

where $\tau_i$ are the Pauli matrices. These three conserved isospin currents have corresponding isospin charges,

$$I_i^W = \int d^3\mathbf{x} J_i^0(x) = \frac{1}{2}\int d^3\mathbf{x}\boldsymbol{\psi}_l^{L\dagger}(x)\tau_i\boldsymbol{\psi}_l^L(x) \tag{4.28}$$

The third current, $i$=3 is neutral, thus coupling electrically neutral or electrically charged leptons, while the first two currents are charged, thus coupling electrically neutral with electrically charged leptons. The two currents from which the IVB-theory was formulated, we can reproduce by linear combinations of $J_1^\alpha(x)$ and $J_2^\alpha(x)$. We get

$$\begin{aligned} J^\alpha(x) &= 2[J_1^\alpha(x) - iJ_2^\alpha(x)] = \overline{\psi}_l(x)\gamma^\alpha(1-\gamma_5)\psi_{\nu_l}(x) \\ J^{\alpha\dagger}(x) &= 2[J_1^\alpha(x) + iJ_2^\alpha(x)] = \overline{\psi}_{\nu_l}(x)\gamma^\alpha(1-\gamma_5)\psi_l(x). \end{aligned} \tag{4.29}$$

We also define a new current, the weak hypercharge current

$$J_Y^\alpha(x) = s^\alpha(x)/e - J_3^\alpha(x) = -\frac{1}{2}\overline{\boldsymbol{\psi}}_l^L(x)\gamma^\alpha\boldsymbol{\psi}_l^L(x) - \overline{\psi}_l^R(x)\gamma^\alpha\psi_l^R(x), \tag{4.30}$$

where we have used the electromagnetic current $s^\alpha(x) = -e\overline{\psi}_l(x)\gamma^\alpha\psi_l(x)$. We have implied conservation of the hypercharge current and the following hypercharge

$$Y = Q/e - I_3^W. \tag{4.31}$$

This is because of the conservation of the electric charge $Q$ and of the weak isocharge $I_3^W$. The conservation of weak hypercharge also follows from the invariance of the free-lepton Lagrangian density (4.24) under global transformations of the fields

$$\psi(x) \to \psi'(x) = e^{i\beta Y}\psi(x),\ \overline{\psi}(x) \to \overline{\psi}'(x) = \overline{\psi}(x)e^{-i\beta Y}, \tag{4.32}$$

and $\psi(x)$ denoting either $\psi^L_{\nu_l}, \psi^L_l, \psi^R_{\nu_l}$ or $\psi^R_l$, with $Y$ being the hypercharge. The left-handed states have hypercharge $Y = -\frac{1}{2}$ and for the right handed lepton states $Y = -1$, and for the right handed neutrino states $Y = 0$. We have until now discussed two global transformations, the SU(2) transformations (4.26) and the U(1) transformations (4.32). Like in the example of QED we will in the next step generalize these transformations from global to local phase transformations.

### 4.1.3 Local phase transformations

The following discussion will be simplified, and will mostly contain the results. By generalizing the global SU(2) transformation (4.26) we get

$$\begin{aligned}
&\mathbf{\Psi}^L_l(x) \rightarrow \mathbf{\Psi}^{L'}_l(x) = \exp[ig\tau_j\omega_j(x)/2]\mathbf{\Psi}^L_l(x) \\
&\overline{\mathbf{\Psi}}^L_l(x) \rightarrow \overline{\mathbf{\Psi}}^{L'}_l(x) = \overline{\mathbf{\Psi}}^L_l(x)\exp[-ig\tau_j\omega_j(x)/2] \\
&\psi^R_l(x) \rightarrow \psi^{R'}_l(x) = \psi^R_l(x), \quad \psi^R_{\nu_l}(x) \rightarrow \psi^{R'}_{\nu_l}(x) = \psi^R_{\nu_l}(x) \\
&\overline{\psi}^R_l(x) \rightarrow \overline{\psi}^{R'}_l(x) = \overline{\psi}^R_l(x), \quad \overline{\psi}^R_{\nu_l}(x) \rightarrow \overline{\psi}^{R'}_{\nu_l}(x) = \overline{\psi}^R_{\nu_l}(x)
\end{aligned} \tag{4.33a}$$

and for small $\omega_j(x)$ the $W^\mu_i(x)$ fields transform as

$$W^\mu_i(x) \rightarrow W^{\mu'}_i(x) = W^\mu_i(x) - \partial^\mu\omega_i(x) - g\varepsilon_{ijk}\omega_j(x)W^\mu_k(x) \tag{4.33b}$$

Generalizing the U(1) transformations (4.32) to the corresponding local transformations we get

$$\begin{aligned}
&\psi(x) \rightarrow \psi'(x) = \exp[ig'Yf(x)]\psi(x) \\
&\overline{\psi}(x) \rightarrow \overline{\psi}'(x) = \overline{\psi}(x)\exp[-ig'Yf(x)]
\end{aligned} \tag{4.34a}$$

where $g'$ is a real number not yet determined, $f(x)$ an arbitrary real differentiable function, and $Y$ is the weak hypercharge associated with the different fields, just as in (4.32). The real gauge field $B^\mu(x)$ transforms like

$$B^\mu(x) \rightarrow B^{\mu'}(x) = B^\mu(x) - \partial^\mu f(x) \tag{4.34b}$$

We now obtain the leptonic Lagrangian density by replacing the ordinary derivatives in (4.24) by the different covariant derivatives which preserve the invariance under the local SU(2) and U(1) transformations. By requiring that the gauge fields $W^\mu_i$ and $B^\mu$ are SU(2)×U(1) invariant, the Lagrangian density is thus SU(2)×U(1) gauge invariant

$$\mathcal{L}^L = i\left[\overline{\mathbf{\Psi}}^L_l(x)\not{D}\mathbf{\Psi}^L_l(x) + \overline{\psi}^R_l(x)\not{D}\psi^R_l(x) + \overline{\psi}^R_{\nu_l}(x)\not{D}\psi^R_{\nu_l}(x)\right], \tag{4.35}$$

where the covariant derivatives of the different fields are

$$D^\mu \Psi_l^L(x) = [\partial^\mu + ig\tau_j W_j^\mu(x)/2 - ig' B^\mu(x)/2]\Psi_l^L(x) \tag{4.36a}$$

$$D^\mu \psi_l^R(x) = [\partial^\mu - ig' B^\mu(x)]\psi_l^R(x) \tag{4.36b}$$

$$D^\mu \psi_{\nu_l}^R(x) = \partial^\mu \psi_{\nu_l}^R(x). \tag{4.36c}$$

For the last part of our discussion, we shall do the following: We divide the new Lagrangian density (4.35) into a free part $\mathcal{L}_0$ and an interacting part $\mathcal{L}_I$. Focusing on the latter, we shall rewrite the fields as linear combinations of other fields, hence introducing the electromagnetic field and reaching the SU(2)×U(1) gauge invariant interaction part first introduced by Glashow (1961), which describes the electromagnetic and weak interactions of leptons.

$$\mathcal{L}_I = -gJ_i^\mu(x)W_{i\mu}(x) - g' J_Y^\mu(x) B_\mu(x). \tag{4.37}$$

Using (4.29) and analogously introducing the non-Hermitian gauge field

$$W_\mu(x) = \frac{1}{\sqrt{2}}[W_{1\mu}(x) - iW_{2\mu}(x)] \tag{4.38}$$

and its adjoint, together with

$$\begin{aligned} W_{3\mu}(x) &= \cos\theta_W Z_\mu(x) + \sin\theta_W A_\mu(x) \\ B_\mu(x) &= -\sin\theta_W Z_\mu(x) + \cos\theta_W A_\mu(x) \end{aligned} \tag{4.39}$$

and by skipping the calculations, together with the requirement

$$g\sin\theta_W = g'\cos\theta_W = e, \tag{4.40}$$

we can rewrite (4.37) to get

$$\begin{aligned} \mathcal{L}_I = &- s^\mu(x) A_\mu - \frac{g}{2\sqrt{2}}[J^{\mu\dagger}(x)W_\mu(x) + J^\mu(x)W_\mu^\dagger(x)] \\ &- \frac{g}{\cos\theta_W}[J_3^\mu(x) - \sin^2\theta_W s^\mu(x)/e]Z_\mu(x). \end{aligned} \tag{4.41}$$

## 4.2 Flavour Changing Neutral Currents

Another concept we have to pay attention to is Flavour Changing Neutral Currents (FCNC), a quark changing flavor by a neutral current. This is highly suppressed in the Standard Model, as it is only the charged-current interactions that connect fermions with different flavors. We can see from (4.41) and (4.27), together with the fact that the third Pauli matrix is diagonal, that this interaction does not mix fermions of different flavors.

# Chapter 5

# Introduction to the Higgs mechanism

The Higgs Mechanism is what gives mass to all elementary particles in the Standard Model. This is a brief introduction.

## 5.1 The Goldstone Model and Spontaneous Symmetry Breaking (SSB)

We have a system with a certain Lagrangian $\mathcal{L}$, which has a particular symmetry, that is, the Lagrangian is invariant under the related symmetry transformations. Now, we take a look at the energy-levels of the system. If a certain energy-level is non-degenerate, the corresponding energy eigenstate is thus unique. It is also invariant under the symmetry transformations of $\mathcal{L}$. This differs when the energy-level is degenerate —the corresponding energy eigenstates are not invariant under the same transformations, but transform linearly among themselves. If this energy-level is the ground state, we have no unique eigenstate, and if we pick out one of these ground eigenstates to represent the ground state, it follows that it does not share the symmetries of the Lagrangian. We have obtained a non-symmetric ground state, and this is called spontaneous symmetry breaking (SSB). Relating this to field theory, where the state of lowest energy is the vacuum, then the vacuum-state cannot be unique for SSB to occur. Thus, for a certain system, we can characterize/pick out a particular vacuum state that is not invariant under symmetry transformations as the ground state. The expectation value of the field in the vacuum state is then non-vanishing.

The Goldstone model has Lagrangian density

$$\mathcal{L} = [\partial^\mu \phi^*(x)][\partial_\mu \phi(x)] - \mu^2 |\phi(x)|^2 - \lambda |\phi(x)|^4, \tag{5.1}$$

with

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + i\phi_2(x)], \tag{5.2}$$

with $\mu^2$ and $\lambda$ real. This theory exhibits SSB. To show this, we consider the global U(1) transformation

$$\phi(x) \to \phi'(x) = \phi(x) e^{i\alpha} \tag{5.3}$$

and its complex conjugated. Considering $\phi(x)$ as a classical field, and using $\pi_r(x) = \partial \mathcal{L}/\partial \dot{\phi}_r$ and

$$\mathcal{H}(x) = \pi_r(x)\dot{\phi}_r(x) - \mathcal{L}\left(\phi_r, \frac{\partial \phi}{\partial x^\alpha}\right) \tag{5.4}$$

we get for the Hamiltonian density:

$$\mathcal{H}(x) = [\partial^0 \phi^*(x)][\partial^0 \phi(x)] + [\nabla \phi^*(x)][\nabla \phi(x)] + \mathcal{V}(\phi), \tag{5.5}$$

and for the potential energy density

$$\mathcal{V}(\phi) = \mu^2 |\phi(x)|^2 + \lambda |\phi(x)|^4. \tag{5.6}$$

For the case $\mu^2 > 0$, $\mathcal{V}(x)$ has an absolute minimum for the unique value $\phi(x) = 0$, and SSB can not occur. But for $\mu^2 < 0$ we get a local maximum for $\mathcal{V}(x)$ at $\phi(x) = 0$, and the absolute minima on the circle

$$\phi(x) = \left(\frac{-\mu^2}{2\lambda}\right)^{\frac{1}{2}} e^{i\theta}, \tag{5.7}$$

$0 \leq \theta < 2\pi$. SSB is possible, because the vacuum state is not unique. Since the Lagrangian density is invariant under the global gauge transformation, we can choose $\theta = 0$ and absolute minimum at

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} v. \tag{5.8}$$

Rewriting the field $\phi(x)$ in terms of the deviations $\sigma(x)$ and $\eta(x)$ from the equilibrium ground state $\phi_0$, we get

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x) + i\eta(x)]. \tag{5.9}$$

We have

$$\langle 0|\phi(x)|0\rangle = \phi_0, \tag{5.10}$$

which is the condition for SSB in quantized field theory.

## 5.2 Remark on SSB

The only way for gauge-quanta to acquire mass is if the symmetry of the (massive) field equations is hidden — or spontaneously broken. For the Higgs Mechanism to give mass to a gauge field quantum, the physical vacuum state must be such that the expectation value of the Higgs field in vacuum is not zero, thus we must have SSB.

# Chapter 6

# The Higgs Model

Replacing the ordinary derivatives in the Goldstone Lagrangian density by the covariant derivatives

$$D_\mu = [\partial_\mu + iqA_\mu(x)], \tag{6.1}$$

where $A_\mu(x)$ is a gauge field, we can show that the Goldstone model is invariant under U(1) gauge transformations. We add the Lagrangian density of the free gauge field,

$$-\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x), \tag{6.2}$$

where

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x), \tag{6.3}$$

and get the "new" Lagrangian density:

$$\mathcal{L}(x) = [D^\mu\phi(x)]^*[D_\mu\phi(x)] - \mu^2|\phi(x)|^2 - \lambda|\phi(x)|^4 - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x). \tag{6.4}$$

This new Lagrangian density defines the Higgs Model, and is invariant under the U(1) gauge transformations (hence "Abelian Higgs Model"):

$$\begin{aligned} \phi(x) &\to \phi'(x) = \phi(x)e^{-iqf(x)}, \\ \phi^*(x) &\to \phi'^*(x) = \phi^*(x)e^{iqf(x)}, \end{aligned} \tag{6.5}$$

and

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu f(x). \tag{6.6}$$

Analogous to the Goldstone Model, we get SSB for $\mu^2 < 0$. Rewriting the field as we did before, (5.9), then using the unitary gauge, we can write

$$\phi(x) = \frac{1}{\sqrt{2}}[v + \sigma(x)]. \tag{6.7}$$

Separating the quadratic terms of the Lagrangian density from the higher order interaction terms, we get for the free Lagrangian density:

$$\begin{aligned}\mathcal{L}_0(x) = &\frac{1}{2}[\partial^\mu \sigma(x)][\partial_\mu \sigma(x)] - \frac{1}{2}(2\lambda v^2)\sigma^2(x)\\ &- \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}(qv)^2 A_\mu(x)A^\mu(x).\end{aligned} \tag{6.8}$$

We interpret the first line of this free Lagrangian density as the free field Lagrangian density of a real Klein Gordon field $\sigma(x)$ giving rise to a neutral scalar boson of mass $\sqrt{2\lambda v^2}$, and the second line as a real massive vector field $A_\mu(x)$ giving rise to neutral vector bosons of mass $|qv|$. One of the degrees of freedom of the Higgs field $\phi(x)$ has been taken up by the vector field $A_\mu(x)$, making it massive. This is the phenomenon known as the Higgs mechanism; a vector boson acquires mass without destroying the gauge invariance of the Lagrangian density, and we call this scalar boson the Higgs boson.

# Chapter 7

# 2HDM - Two Higgs Doublet Model

The Standard Model (SM) fails to explain the baryon asymmetry of the universe, as it is only through a complex phase in the CKM matrix we get CP violation. By extending this model it allows for more CP violation. Spontaneous CP violation (SCPV) occurs when CP is a symmetry of the original Lagrangian, but after spontaneous symmetry breaking, there is no CP symmetry remaining. This is not possible in the SU(2)×U(1) gauge theory with only one Higgs doublet [1], hence the extension.

## 7.1 Higgs Potential

The two Higgs doublets can be written as

$$\boldsymbol{\phi}_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix} \quad i = 1, 2, \tag{7.1}$$

where the second doublet $\boldsymbol{\phi}_2$ generally comes with a complex phase $e^{i\xi}$, but which we have transformed away. This will be discussed later. We have

$$\begin{aligned} v_1 &= v\cos\beta \\ v_2 &= v\sin\beta, \end{aligned} \tag{7.2a}$$

and from the SM

$$v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2 \tag{7.2b}$$

with

$$\tan\beta = v_2/v_1 \quad (0 \leq \beta \leq \pi/2). \tag{7.2c}$$

The latter is the ratio of the vacuum expectation values, VEV's, of the two doublets, and the angle $\beta$ rotates the CP-odd and the charged scalars into their mass eigenstates. It is the extremes of the potential which define the VEV's, since the potential has its absolute minima at the fields VEV's. Thus,

$$\frac{\partial V}{\partial \phi_1}\Big|_{\substack{\phi_1=\langle\phi_1\rangle \\ \phi_2=\langle\phi_2\rangle}} = 0, \qquad \frac{\partial V}{\partial \phi_2}\Big|_{\substack{\phi_1=\langle\phi_1\rangle \\ \phi_2=\langle\phi_2\rangle}} = 0. \tag{7.3}$$

The most general solution to these equations for a physical neutral vacuum, that is, a vacuum where we have conserved U(1) symmetry such that we have a massless photon and positive eigenvalues of the mass squared matrix (which we shall discuss later), is given by

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \tag{7.4}$$

but note that we, as briefly mentioned (and will be discussed later), transform away the phase $\xi$, such that we get

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{7.5}$$

The 2HDM is defined in terms of the potential, and depending on the potential chosen the neutral Higgs sector may or may not lead to CP violation. The potential can take the form [2]

$$\begin{aligned} V =& \frac{\lambda_1}{2}\Big[(\phi_1^\dagger\phi_1) - \frac{v_1^2}{2}\Big]^2 + \frac{\lambda_2}{2}\Big[(\phi_2^\dagger\phi_2) - \frac{v_2^2}{2}\Big]^2 + \lambda_3\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ &+ \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5{}^*(\phi_2^\dagger\phi_1)^2] \\ &- \frac{1}{2}\Big[[(\lambda_3 + \lambda_4 + \Re(\lambda_5)] - 2\nu\Big][v_2^2(\phi_1^\dagger\phi_1) + v_1^2(\phi_2^\dagger\phi_2)] \\ &- v_1 v_2[2\nu\Re(\phi_1^\dagger\phi_2) - \Im(\lambda_5)\Im(\phi_1^\dagger\phi_2)] \end{aligned} \tag{7.6}$$

where $\lambda_5$ can be complex and

$$\nu = \frac{1}{2v_1v_2}\Re(m_{12}^2). \tag{7.7}$$

Note that there are more general potentials [3, 4], such as

$$\begin{aligned}V =&\frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)\\&+ \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[(\lambda_5(\phi_1^\dagger\phi_2)^2 + h.c.]\\&+ \left\{[(\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)](\phi_1^\dagger\phi_2) + h.c.\right\}\\&- \frac{1}{2}[m_{11}^2(\phi_1^\dagger\phi_1) + [m_{12}^2(\phi_1^\dagger\phi_2) + h.c.] + m_{22}^2(\phi_2^\dagger\phi_2)],\end{aligned}\tag{7.8}$$

where $\Re(\phi_1^\dagger\phi_2) = \frac{1}{2}(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)$. Since the vacuum state is assumed to be a stability point,

$$\frac{\partial V}{\partial \phi_i} = 0 \quad i = 1, 2,\tag{7.9}$$

we get certain relations between the parameters of the potential. Using these relations we can rewrite the potential (7.8) as (7.6). We must then use the relations

$$\begin{aligned}m_{11}^2 =&(\lambda_1 v_1^3 + \lambda_3 v_1 v_2^2 + \lambda_4 v_1 v_2^2 + \Re(\lambda_5)v_1 v_2^2 - \Re(m_{12}^2)v_2)/(v_1),\\m_{22}^2 =&(\lambda_2 v_2^3 + \lambda_3 v_1^2 v_2 + \lambda_4 v_1^2 v_2 + \Re(\lambda_5)v_1^2 v_2 - \Re(m_{12}^2)v_1)/(v_2),\\\Im(m_{12}^2) =&v_2 v_1 \Im(\lambda_5),\end{aligned}\tag{7.10}$$

where $\nu$ is defined in (7.7), and we set $\lambda_6 = \lambda_7 = 0$. Even if $\lambda_5$ is non-zero and real, CP violation can arise from nonzero imaginary values of $\lambda_6, \lambda_7,$ and $m_{12}^2$. Actually, CP violation is absent if all coefficients, of a potential with a real vacuum, are real [5]. From a discussion of Wu and Wolfenstein [6] it is a problem with the multi-Higgs boson models that they create the possibility of Flavor Changing Neutral Currents (FCNC). We shall see that this problem is fixed if we impose a certain symmetry making $\lambda_6 = \lambda_7 = m_{12}^2 = 0$. But this removes the explicit CP violation from the potential, and we are only left with CP violation from the complex Yukawa-couplings. We return to this discussion later.

We define the field

$$\eta_3 = -\chi_1 \sin\beta + \chi_2 \cos\beta,\tag{7.11a}$$

orthogonal to the neutral Goldstone boson

$$G^0 = \chi_1 \cos\beta + \chi_2 \sin\beta.\tag{7.11b}$$

### 7.1.1 Reparametrization and rephasing invariance

Following a discussion by Ginzburg and Krawczyk [7], since the two fields have identical quantum numbers, the model can be described by fields obtained through a global unitary transformation of the old fields $\phi_i$.

$$\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \mathcal{F} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{7.12}$$

with

$$\mathcal{F} = e^{-i\rho_o} \begin{pmatrix} \cos\theta e^{i\frac{\rho}{2}} & \sin\theta e^{i\frac{\tau-\rho}{2}} \\ -\sin\theta e^{-i\frac{\tau-\rho}{2}} & \cos\theta e^{-i\frac{\rho}{2}} \end{pmatrix} \tag{7.13}$$

Even though the Higgs model does not change under such a transformation, the coefficients change. We get a particular case of this transformation if we set $\theta = 0$. This can be treated as a rephasing transformation of the fields, leading to a change of phase of some of the coefficients in the Lagrangian. The transformation

$$\begin{aligned} \phi_i \to e^{-i\rho_k}\phi_i, \quad (i = 1, 2), \quad \rho_1 = \rho_0 - \frac{\rho}{2}, \\ \rho_2 = \rho_0 + \frac{\rho}{2}, \quad \rho = \rho_2 - \rho_1. \end{aligned} \tag{7.14a}$$

gives a phase change to the parameters of the potential,

$$\lambda_5 \to \lambda_5 e^{-2i\rho}, \quad \lambda_{6,7} \to \lambda_{6,7} e^{-i\rho}, \text{ and } m_{12}^2 \to m_{12}^2 e^{-i\rho}, \tag{7.14b}$$

while the ones not mentioned are left unaltered. We see from (7.14a) that we have the possibility of rephasing the fields, such that the phase $\xi$ in the VEV of $\langle\phi_2\rangle$ disappears. We do this by choosing $\rho = \xi$ in the phase transformation. This means that the phase difference $\xi$ between the VEV's has no physical meaning at all.

There exists a basis for the scalar doublets, which we call "the Higgs basis" [1]. This basis is in particular useful because it is defined such that only one of the doublets in this basis has a Vacuum Expectation Value (VEV), and this VEV is real and positive. First we define

$$\begin{aligned} x_1 &= \phi_1^\dagger \phi_1 \\ x_2 &= \phi_2^\dagger \phi_2 \\ z &= \phi_2^\dagger \phi_1 \\ z^\dagger &= \phi_1^\dagger \phi_2. \end{aligned} \tag{7.15}$$

We reach the Higgs basis in the 2HDM by the following unitary transformation of the Higgs doublets [1]

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \frac{1}{v} \begin{pmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{7.16}$$

where $H_i$ denotes the doublets in the Higgs basis. Comparing this unitary transformation with the unitary transformation $\mathcal{F}$, we see that $\mathcal{F}$ provides the same result, adjusting for a change in sign, by setting $\theta$ in $\mathcal{F}$ equal to $\beta$, and setting the global phases equal to zero. We get the new coefficients of the potential in the Higgs basis by using the algebraic computer program Reduce [8]. Defining the corresponding expressions of (7.15) in the Higgs basis,

$$\begin{aligned} x_1' &= H_1^\dagger H_1 \\ x_2' &= H_2^\dagger H_2 \\ z' &= H_2^\dagger H_1 \\ z'^\dagger &= H_1^\dagger H_2, \end{aligned} \tag{7.17}$$

and using (7.16) they can be expressed in terms of the Higgs-basis invariants as

$$\begin{aligned} x_1 =& c^2 x_1' + s^2 x_2' + cs(z'^\dagger + z'), \\ x_2 =& s^2 x_1' + c^2 x_2' - cs(z'^\dagger + z'), \\ z =& cs(x_1' - x_2') + s^2 z'^\dagger - c^2 z', \\ z^\dagger =& cs(x_1' - x_2') + s^2 z' - c^2 z'^\dagger, \end{aligned} \tag{7.18}$$

where $c$ and $s$ are abbreviations for $\sin\beta$ and $\cos\beta$. Then, in our code of the Higgs potential, we replace the expressions of (7.15) with (7.18) and extract the coefficients

$$
\begin{aligned}
\lambda_1' &= c^4\lambda_1 + s^4\lambda_2 + 2(cs)^2[\lambda_3 + \lambda_4 + \Re(\lambda_5)] + 4c^3s\Re(\lambda_6) + 4cs^3\Re(\lambda_7) \\
\lambda_2' &= c^4\lambda_2 + s^4\lambda_1 + 2(cs)^2[\lambda_3 + \lambda_4 + \Re(\lambda_5)] - 4cs^3\Re(\lambda_6) - 4c^3s\Re(\lambda_7) \\
\lambda_3' &= c^4\lambda_3 + s^4\lambda_3 + (cs)^2[\lambda_1 + \lambda_2 - 2\lambda_4 - 2\Re(\lambda_5)] \\
&\quad + 2[cs^3 - c^3s]\Re(\lambda_6 - \lambda_7) \\
\lambda_4' &= c^4\lambda_4 + s^4\lambda_4 + (cs)^2[\lambda_1 + \lambda_2 - 2\lambda_3 - 2\Re(\lambda_5)] \\
&\quad + 2[cs^3 - c^3s]\Re(\lambda_6 - \lambda_7) \\
\lambda_5' &= c^4\lambda_5 + s^4\lambda_5^* + (cs)^2[\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4] - 2c^3s(\lambda_6 - \lambda_7) \\
&\quad + 2cs^3(\lambda_6^* - \lambda_7^*) \\
\lambda_6' &= -c^4\lambda_6 + s^4\lambda_7^* + (cs)^2[\lambda_6 + 2\lambda_6^* - 2\lambda_7 - \lambda_7^*] \\
&\quad + c^3s[\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5] + cs^3[-\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5^*] \\
\lambda_7' &= -c^4\lambda_7 + s^4\lambda_6^* - (cs)^2[2\lambda_6 + \lambda_6^* - \lambda_7 - 2\lambda_7^*] \\
&\quad + c^3s[-\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5] + cs^3[\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5^*] \\
(m')^2_{11} &= c^2m^2_{11} + csm^2_{12} + csm^{2*}_{12} + s^2m^2_{22} \\
(m')^2_{22} &= c^2m^2_{22} - csm^2_{12} - csm^{2*}_{12} + s^2m^2_{11} \\
(m')^2_{12} &= -c^2m^2_{12} + csm^2_{11} - csm^2_{22} + s^2m^{2*}_{12}
\end{aligned} \tag{7.19}
$$

For later it will be useful to present the result for $\lambda_6 = \lambda_7 = 0$, we get

$$
\begin{aligned}
\lambda_1' &= c^4\lambda_1 + s^4\lambda_2 + 2(cs)^2[\lambda_3 + \lambda_4 + \Re(\lambda_5)] \\
\lambda_2' &= c^4\lambda_2 + s^4\lambda_1 + 2(cs)^2[\lambda_3 + \lambda_4 + \Re(\lambda_5)] \\
\lambda_3' &= c^4\lambda_3 + s^4\lambda_3 + (cs)^2[\lambda_1 + \lambda_2 - 2\lambda_4 - 2\Re(\lambda_5)] \\
\lambda_4' &= c^4\lambda_4 + s^4\lambda_4 + (cs)^2[\lambda_1 + \lambda_2 - 2\lambda_3 - 2\Re(\lambda_5)] \\
\lambda_5' &= c^4\lambda_5 + s^4\lambda_5^* + (cs)^2[\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4] \\
\lambda_6' &= c^3s[\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5] + cs^3[-\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5^*] \\
\lambda_7' &= c^3s[-\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5] + cs^3[\lambda_1 - \lambda_3 - \lambda_4 - \lambda_5^*],
\end{aligned} \tag{7.20}
$$

noting that even though $\lambda_6 = \lambda_7 = 0$ in one basis, this may not be true in another basis. For the special case $\tan\beta = 1$ we have

$$
\begin{aligned}
\lambda_1' &= \frac{1}{4}\lambda_1 + \frac{1}{4}\lambda_2 + \frac{1}{2}[\lambda_3 + \lambda_4 + \Re(\lambda_5)] \\
\lambda_2' &= \frac{1}{4}\lambda_2 + \frac{1}{4}\lambda_1 + \frac{1}{2}[\lambda_3 + \lambda_4 + \Re(\lambda_5)] \\
\lambda_3' &= \frac{1}{2}\lambda_3 + \frac{1}{4}[\lambda_1 + \lambda_2 - 2\lambda_4 - 2\Re(\lambda_5)] \\
\lambda_4' &= \frac{1}{2}\lambda_4 + \frac{1}{4}[\lambda_1 + \lambda_2 - 2\lambda_3 - 2\Re(\lambda_5)] \\
\lambda_5' &= \frac{1}{2}\Re(\lambda_5) + \frac{1}{4}[\lambda_1 + \lambda_2 - 2\lambda_3 - 2\lambda_4] \\
\lambda_6' &= \frac{1}{4}[\lambda_1 - \lambda_2 - 2\Im(\lambda_5)] \\
\lambda_7' &= \frac{1}{4}[\lambda_1 - \lambda_2 + 2\Im(\lambda_5)].
\end{aligned} \tag{7.21}
$$

We note that $\tan\beta$, previously defined as the relation between the VEVs of the two doublets, has no meaning in the Higgs basis, in which only one doublet has VEV. Haber and O'Neil discuss this in [9] and point out that in a general 2HDM $\tan\beta$ is in fact an unphysical parameter with no significance. However, $\tan\beta$ can be promoted to a physical parameter in specialized versions of the 2HDM, such as the 2HDM type-I and type-II, to be discussed later. These specialized versions constrain the Yukawa couplings of the Higgs bosons and fermions, and in these couplings we can measure $\tan\beta$.

## 7.2 Mass Squared Matrix

We get the mass squared matrix $\mathcal{M}^2$ of the neutral sector by differentiating the potential $V$ twice with respect to the different $\eta_i$ fields. Next we set the different fields equal to zero, leaving us with the elements in the mass squared matrix. Such that the positions are

$$\mathcal{M}_{ij}^2 = \frac{\partial^2 V}{\partial\eta_i \partial\eta_j} \tag{7.22}$$

The (symmetric) mass squared matrix for the neutral sector of the potential in (7.6) is then

$$\begin{aligned}
\mathcal{M}^2_{11} &= v^2(c^2\lambda_1 + s^2\nu) \\
\mathcal{M}^2_{22} &= v^2(s^2\lambda_2 + c^2\nu) \\
\mathcal{M}^2_{33} &= v^2\Re(-\lambda_5 + \nu) \\
\mathcal{M}^2_{12} &= csv^2\Re(\lambda_3 + \lambda_4 + \lambda_5 - \nu) \\
\mathcal{M}^2_{13} &= -\frac{1}{2}v^2 s\Im(\lambda_5) \\
\mathcal{M}^2_{23} &= -\frac{1}{2}v^2 c\Im(\lambda_5).
\end{aligned} \tag{7.23}$$

Also note that here $\lambda_6 = \lambda_7 = 0$. In the basis of $\eta_i$, $i$=1,2,3, we can diagonalize the mass squared matrix $\mathcal{M}^2$ to the physical states $(H_1, H_2, H_3)$ with masses $M_1 \leq M_2 \leq M_3$. We do this with a rotation matrix $R$, such that (and note that $H_i$ here denotes the physical Higgs fields, not the Higgs basis discussed before)

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \tag{7.24}$$

Thus,

$$R\mathcal{M}^2 R^T = \mathcal{M}^2_{\text{diag}} \tag{7.25}$$

because the masses of the physical Higgs fields $H_i$ must be the eigenvalues of the diagonalized mass matrix $\mathcal{M}^2_{ii}$ and to allow for $R$ to mix all $\eta_i$, we need three angles and the parametrization becomes

$$\begin{aligned}
R &= R_1 R_2 R_3 \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_3 & \sin\alpha_3 \\ 0 & -\sin\alpha_3 & \cos\alpha_3 \end{pmatrix} \begin{pmatrix} \cos\alpha_2 & 0 & \sin\alpha_2 \\ 0 & 1 & 0 \\ -\sin\alpha_2 & 0 & \cos\alpha_2 \end{pmatrix} \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 \\ -\sin\alpha_1 & \cos\alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_1 c_2 & s_1 c_2 & s_2 \\ -(c_1 s_2 s_3 + s_1 c_3) & c_1 c_3 - s_1 s_2 s_3 & c_2 s_3 \\ -c_1 s_2 c_3 + s_1 s_3 & -(c_1 s_3 + s_1 s_2 c_3) & c_2 c_3 \end{pmatrix},
\end{aligned} \tag{7.26}$$

with the same abbreviations for the sine and cosine functions as used in $\mathcal{M}^2$.

# Chapter 8

# CP violation in the 2HDM

## 8.1 Yukawa couplings

A Yukawa coupling, or Yukawa interaction, is a coupling of a scalar field $\boldsymbol{\phi}$ with a fermion field $\psi$, of the form $g\overline{\psi}\boldsymbol{\phi}\psi$, where $g$ is some coupling constant. If the scalar field is the Higgs field, then through spontaneous symmetry breaking, the fermions acquire mass. This is how mass is introduced in the Standard Model, and it is proportional to the Higgs field's VEV. We can write the interaction in terms of the quark and lepton mass-eigenstate fields as [6]

$$\begin{aligned} -\mathcal{L}^Y = & \overline{\psi}_q^L g_1^d \boldsymbol{\phi}_1 \psi_d^R + \overline{\psi}_q^L g_1^u \widetilde{\boldsymbol{\phi}}_1 \psi_u^R \\ & + \overline{\psi}_q^L g_2^d \boldsymbol{\phi}_2 \psi_d^R + \overline{\psi}_q^L g_2^u \widetilde{\boldsymbol{\phi}}_2 \psi_u^R \\ & + \overline{\psi}_l^L g_1^e \boldsymbol{\phi}_1 \psi_l^R + \overline{\psi}_l^L g_2^e \widetilde{\boldsymbol{\phi}}_2 \psi_l^R + h.c. \end{aligned} \tag{8.1}$$

The Type-II models of the 2HDM constrain all quarks with the same quantum number to couple to the same (scalar) Higgs field. One Higgs field $\phi_2$ couples to the up-type quarks ($I_3 = 1/2$) and the other, $\phi_1$, couples to down-type quarks and leptons ($I_3 = -1/2$). This is called a discrete symmetry or $Z_2$ symmetry, and was proposed by Glashow and Weinberg [10], making the Lagrangian invariant under a change of sign of one of the Higgs fields:

$$\begin{aligned}
&\phi_1 \leftrightarrow -\phi_1, \\
&\phi_2 \leftrightarrow \phi_2, \\
&\psi_d^R \leftrightarrow -\psi_d^R, \\
&\psi_u^R \leftrightarrow \psi_u^R
\end{aligned} \tag{8.2}$$

and as noted before, this implies that in the more general potential we have to set $\lambda_6 = \lambda_7 = m_{12}^2 = 0$. As briefly mentioned above, this does suppress FCNC, but also suppresses CP violation from the potential. That is, we are left with CP violation from the complex Yukawa couplings — same as in the SM. Our main reason for extending the SM to the 2HDM was to allow for more CP violation, we must therefore find a way around this problem. And there are various ways proposed [6], some of which are:

1. The discrete symmetry is softly violated by the term proportional to $m_{12}^2$. It is called soft violation since the terms that break the symmetry are of second order in the fields. This is referred to as explicit CP violation when $m_{12}^2$ and $\lambda_5$ are complex. At small orders of pertubation theory, this type of violation in fact respects the discrete symmetry, such that FCNC are suppressed in these cases.

2. The discrete symmetry is violated in both Yukawa couplings and the potential, but these violations are small

3. One abandons the discrete symmetry, and assume FCNC is suppressed by other mechanisms.

4. The terms which break the symmetry are of dimension two and four, that is, $\lambda_6$ and $\lambda_7$ are complex. This is called hard symmetry breaking.

## 8.2 2HDM type-II

Due to different ways of coupling the two Higgs doublets to quarks and leptons, we have different versions of the 2HDM. The 2HDM type-II, as mentioned above, couples $\phi_1$ to down-type quarks and $\phi_2$ to the up-type quarks. Thus, the type-II Yukawa Lagrangian for the quarks (the lepton coupling is left out, for now) is

$$\begin{aligned}
-\mathcal{L}_{II}^Y = &+g_1^d[\overline{\psi}_q^L \psi_d^R \boldsymbol{\phi}_1 + \overline{\psi}_d^R \boldsymbol{\phi}_1^\dagger \psi_q^L] \\
&+g_2^u[\overline{\psi}_q^L \psi_u^R \widetilde{\boldsymbol{\phi}}_2 + \overline{\psi}_u^R \widetilde{\boldsymbol{\phi}}_2^\dagger \psi_q^L]
\end{aligned} \tag{8.3}$$

where

$$\widetilde{\boldsymbol{\phi}}_i(x) = -i[\boldsymbol{\phi}_i{}^\dagger(x)\tau_2]^T \tag{8.4a}$$

changes sign of the upper component of the $i$th Higgs doublet, complex conjugates and switches the places of the two components:

$$\boldsymbol{\phi}_i = \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix} \Rightarrow \widetilde{\boldsymbol{\phi}}_i = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_i + \eta_i - i\chi_i) \\ -\varphi_i^- \end{pmatrix} \tag{8.4b}$$

## 8.3 Top and Bottom Yukawa Couplings

We shall now apply the following procedure: Expand the Higgs field in the Yukawa Lagrangian. Find the mass of the up quark and down quark by identifying the coefficients of the quadratic quark field terms. Rewriting the coupling constants in terms of the quark masses, and transforming the Higgs fields into their physical basis using the rotation matrix, we rewrite the Yukawa Lagrangian. Last, we identify the desired coupling of the Higgs fields and two quarks, $H_j\overline{u}u$.

Expanding the Higgs field in the Yukawa Lagrangian (8.3), we get

$$\begin{aligned} -\mathcal{L}_{II}^Y = &+ g_1^d \Big[ \overline{\psi}_u^L \psi_d^R \varphi_1^+ + \overline{\psi}_d^R \psi_u^L \varphi_1^- \\ &+ \frac{\overline{\psi}_d^L \psi_d^R}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) + \frac{\overline{\psi}_d^R \psi_d^L}{\sqrt{2}}(v_1 + \eta_1 - i\chi_1) \Big] \\ &+ g_2^u \Big[ -\overline{\psi}_d^L \psi_u^R \varphi_2^- - \overline{\psi}_u^R \psi_d^L \varphi_2^+ \\ &+ \frac{\overline{\psi}_u^L \psi_u^R}{\sqrt{2}}(v_2 + \eta_2 - i\chi_2) + \frac{\overline{\psi}_u^R \psi_u^L}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \Big] \end{aligned} \tag{8.5}$$

Setting the fields equal to zero, $v_1$ and $v_2$ "survive", we get Spontaneous Symmetry Breaking (SSB) and non-zero VEV's, and can identify the mass terms

$$\begin{aligned} -\mathcal{L}_{II\text{mass}}^Y &= \frac{g_1^d}{\sqrt{2}}(\overline{\psi}_d^L \psi_d^R + \overline{\psi}_d^R \psi_d^L)v_1 + \frac{g_2^u}{\sqrt{2}}(\overline{\psi}_u^L \psi_u^R + \overline{\psi}_u^R \psi_u^L)v_2 \\ &= \frac{g_1^d}{\sqrt{2}}(\overline{\psi}_d \psi_d)v_1 + \frac{g_2^u}{\sqrt{2}}(\overline{\psi}_u \psi_u)v_2, \end{aligned} \tag{8.6}$$

We have here used the following relations between the left and right handed fields

$$\begin{aligned}
\overline{\psi}\psi &= (\overline{\psi}^L + \overline{\psi}^R)(\psi^L + \psi^R) \\
&= \frac{1}{2}\left[\overline{\psi}(1+\gamma^5) + \overline{\psi}(1-\gamma^5)\right]\frac{1}{2}\left[(1-\gamma^5)\psi + (1+\gamma^5)\psi\right] \\
&= \overline{\psi}^L\psi^R + \overline{\psi}^R\psi^L = \overline{\psi}\psi^R + \overline{\psi}\psi^L \qquad (8.7)
\end{aligned}$$

which we will also be using later for simplification. We can now identify the masses as the coefficients of the terms quadratic in the fields, and using (8.6) we get

$$m_d = \frac{g_1^d}{\sqrt{2}} v \cos\beta \qquad (8.8a)$$

$$m_u = \frac{g_2^u}{\sqrt{2}} v \sin\beta \qquad (8.8b)$$

We will now extract the parts of (8.5) which contribute to the mass terms. We use the relations of $\chi_i$, inverting (7.11a),

$$\chi_1 = G^0 \cos\beta - \eta_3 \sin\beta \qquad (8.9)$$

and

$$\chi_2 = G^0 \sin\beta + \eta_3 \cos\beta, \qquad (8.10)$$

and using the fact that the rotation matrix (7.26) is unitary, thus

$$(R^{-1})_{ij} = (R^T)_{ij} = R_{ji}, \qquad (8.11)$$

we get

$$\eta_i = R_{ji} H_j \qquad (8.12)$$

We shall be changing the indicies from the up and down quark to the top and bottom quark, because of their much larger masses, thus more relevant for our discussion of CP violation. We get for the $H_j\overline{t}t$ coupling

$$\begin{aligned}
-\mathcal{L}^Y_{IItop} =& g_2^t \left[\frac{\overline{\psi}_t^L\psi_t^R}{\sqrt{2}}(\eta_2 - i\chi_2) + \frac{\overline{\psi}_t^R\psi_t^L}{\sqrt{2}}(\eta_2 + i\chi_2)\right] \\
=& \frac{m_t}{v\sin\beta}[\overline{\psi}_t^L\psi_t^R(\eta_2 - i\chi_2) + \overline{\psi}_t^R\psi_t^L(\eta_2 + i\chi_2)] \\
=& \frac{m_t}{2v\sin\beta}[\overline{\psi}_t(1+\gamma^5)\psi_t(R_{j2}H_j - i\cos\beta R_{j3}H_j) \\
& + \overline{\psi}_t(1-\gamma^5)\psi_t(R_{j2}H_j + i\cos\beta R_{j3}H_j)] \\
=& \frac{m_t}{v\sin\beta}[\overline{\psi}_t\psi_t R_{j2}H_j - i\cos\beta\overline{\psi}_t\gamma^5\psi_t R_{j3}H_j] \qquad (8.13)
\end{aligned}$$

Using the exact same approach, we get for the $H_j\bar{b}b$ coupling

$$\begin{aligned}
\mathcal{L}^{Y}_{IIbot} =& g_1^b\left[\frac{\overline{\psi}_b^L\psi_b^R}{\sqrt{2}}(\eta_1+i\chi_1)+\frac{\overline{\psi}_b^R\psi_b^L}{\sqrt{2}}(\eta_1-i\chi_1)\right]\\
=&\frac{m_b}{v\cos\beta}[\overline{\psi}_b^L\psi_b^R(\eta_1+i\chi_1)+\overline{\psi}_b^R\psi_b^L(\eta_1-i\chi_1)]\\
=&\frac{m_b}{2v\cos\beta}[\overline{\psi}_b(1+\gamma^5)\psi_b(R_{j1}H_j-i\sin\beta R_{j3}H_j)\\
&+\overline{\psi}_b(1-\gamma^5)\psi_b(R_{j1}H_j+i\sin\beta R_{j3}H_j)]\\
=&\frac{m_b}{v\cos\beta}[\overline{\psi}_b\psi_bR_{j1}H_j-i\sin\beta\overline{\psi}_b\gamma^5\psi_bR_{j3}H_j]
\end{aligned} \tag{8.14}$$

Note that we only used the $\eta_3$ component of $\chi$, and dropped the $G^0$ term. We can now express the couplings (relative to the SM coupling) as

$$H_j\bar{b}b: \quad \frac{1}{\cos\beta}[R_{j1}-i\gamma^5\sin\beta R_{j3}] \tag{8.15a}$$

$$H_j\bar{t}t: \quad \frac{1}{\sin\beta}[R_{j2}-i\gamma^5\cos\beta R_{j3}] \tag{8.15b}$$

## 8.4 Invariants

In a model with more than one Higgs-doublet, one can perform a Higgs-basis transformations (HBT), also called a Reparametrization transformation, without altering the physical content of the model, such as a transformation to “the Higgs basis”, which we previously discussed. We also noted that the couplings do in fact change from one basis to another, thus presenting some ambiguities as to what kind of CP violation one has in a particular presentation of the model. Motivated by this, one searches for HBT invariants which imaginary parts are constrained to vanish from CP invariance. That is, what are the HBT-invariant conditions for CP invariance of the Lagrangian [11]? We have a theorem stating that if and only if there exists a basis in which all the parameters of the potential are real, a so-called real basis, then we have explicit CP conservation of the potential. Using CPT invariance, Gunion and Haber give a proof of this in [12]. The invariants constructed from the parameters of the potential are just another way of presenting this theorem; if the imaginary parts of these invariants (note again, they are constructed from the parameters of the potential) are nonzero, we have explicit CP violation, and there shall exist no real basis. But even though such a real basis might exist, it does not exclude the possibility of SCPV.

In this thesis we constrain ourselves to only two doublets, but in general one can write the Higgs Lagrangian with $n$ doublets as [11]

$$\mathcal{L} = Y_{ab}\phi_a^\dagger\phi_b + Z_{abcd}(\phi_a^\dagger\phi_b)(\phi_c^\dagger\phi_d), \tag{8.16}$$

where the indices denote the $n$ doublets, and repeated indicies are summed over. By demanding hermiticity of the Lagrangian, one also gets certain constraints

$$(Y_{ab}\phi_a^\dagger\phi_b)^\dagger = Y_{ab}^*\phi_b^\dagger\phi_a \;\Rightarrow\; Y_{ab}^* = Y_{ba} \tag{8.17a}$$

and analogously

$$Z_{abcd}^* = Z_{badc} \tag{8.17b}$$

One can define a HBT by [11]

$$\begin{aligned}
&\phi_a \xrightarrow{\text{HBT}} \phi_a' = V_{ai}\phi_i, \\
&\phi_a^\dagger \xrightarrow{\text{HBT}} (\phi')_a^\dagger = (V\phi)_a^\dagger = (\phi^\dagger V^\dagger)_a = (\phi^\dagger)_i(V^\dagger)_{ia} = V_{ai}^*\phi_i^\dagger,
\end{aligned} \tag{8.18}$$

where $V$ is unitary and $n \times n$, acting on the Higgs doublets. The sub-index $a$ outside a parenthesis points to the component $a$ of the quantity inside the parenthesis. If it is a product of two matrices inside the parenthesis, one can move the sub-index $a$ inside, such that the product only includes that which becomes the component $a$ of the product. For example

$$\begin{aligned}
\phi_1' &= (V\phi)_1 = V_{11}\phi_1 + V_{12}\phi_2, \\
\phi_1'^\dagger &= \left[(V\phi)^\dagger\right]_1 = (\phi^\dagger V^\dagger)_1 = V_{11}^*\phi_1^\dagger + V_{12}^*\phi_2^\dagger.
\end{aligned} \tag{8.19}$$

We noted that under such a HBT the physics of the Lagrangian stays the same, but the coefficients change. We shall now calculate how these change under a HBT using (8.18), where the primes denote a transformed field or coefficient. Repeated indices are still summed over,

$$\begin{aligned}
&Y_{ab}'(\phi'^\dagger)_a\phi_b' = Y_{ab}'(\phi^\dagger)_i(V^\dagger)_{ia}V_{bj}\phi_j, \\
&Y_{ab}'(V^\dagger)_{ia}V_{bj} = Y_{ij} \qquad\qquad \Rightarrow Y_{ab} \xrightarrow{HBT} Y_{ab}' = V_{ai}Y_{ij}(V^\dagger)_{jb}
\end{aligned} \tag{8.20a}$$

and

$$\begin{aligned}
&Z_{abcd}'(\phi'^\dagger)_a\phi_b'(\phi'^\dagger)_c\phi_d' = Z_{abcd}'((\phi^\dagger)_i(V^\dagger)_{ia})(V_{bj}\phi_j)((\phi^\dagger)_l(V^\dagger)_{lc})(V_{dk}\phi_k), \\
&Z_{abcd}'(V^\dagger)_{ia}V_{bj}(V^\dagger)_{lc}V_{dk} = Z_{ijlk} \\
&\Rightarrow Z_{abcd} \xrightarrow{HBT} Z_{abcd}' = Z_{ijlk}(V^\dagger)_{kd}V_{cl}(V^\dagger)_{jb}V_{ai},
\end{aligned} \tag{8.20b}$$

where we have used the fact that $V_{iy}(V^\dagger)_{yj} = I_{ij}$, where $I$ is the identity matrix. The $V$ matrices with indicies are merely matrix components, that is numbers, which can be moved around as we please.

Now, we wish to display how the couplings $Z$ and $Y$ change under a general CP transformation of the Higgs fields, with the restriction that the kinetic part of the Lagrangian must be invariant. We have [11]

$$\phi_a \xrightarrow{CP} U_{ai}\ \phi_i^* \quad ; \quad \phi_a^\dagger \xrightarrow{CP} U_{ai}^*\ \phi_i^T, \tag{8.21}$$

$U$ being an $n \times n$ unitary matrix. The conditions for CP conservation of the Lagrangian are [11]

$$(Y^*)_{ab} = {U^\dagger}_{am} Y_{mn} U_{nb} \tag{8.22a}$$

$$(Z^*)_{abcd} = {U^\dagger}_{am} {U^\dagger}_{cp} Z_{mnpq} U_{nb} U_{qd} \tag{8.22b}$$

These conditions are also HBT-invariant, but as they require a search for an unitary matrix $U$ they are not very practical. This is why HBT-invariants are useful.

### 8.4.1 J- and I-invariants

Different invariants in the 2HDM can be constructed, and with different claimed properties, which will be discussed in detail. Three invariants are made of the VEV and the parameters of the potential, and four are just made of the parameters.

From Eq.(18) in [13] we get [1]

$$J_1 = M_{12}M_{13}(M_{22} - M_{33}) + M_{23}[(M_{13})^2 - (M_{12})^2], \tag{8.23}$$

where $M$ is the mass squared matrix of the neutral scalars, before diagonalization, denoted by $\mathcal{M}^2$ in (7.22). We can express $J_1$ as a function of the parameters of the potential, and expressed in the Higgs basis one gets from Eq.(22) in [13]

$$J_1 = -8v^6 \Im(\lambda_5'^* \lambda_6'^2). \tag{8.24a}$$

It is also possible to form two other similar invariants

$$J_2 = -4v^4 \Im(\lambda_5'^* \lambda_7'^2), \tag{8.24b}$$

from Eq.(23) in [13], and

$$J_3 = 2\sqrt{2} v^3 \Im(\lambda_6' \lambda_7'^*), \tag{8.24c}$$

[1] Lavoura and Silva stress that this quantity is introduced by Méndez and Pomarol [14]

from Eq.(24) in [13]. If any of these invariants is non-zero, this implies CP violation. But if all are zero, it implies CP conservation. We call these invariants $J$-invariants. From Eq.(22.54) in [1], with a different notation (there denoted $I_2$ and $I_1$), two of these invariants in the most general form are

$$J_1 \equiv V_{ab} Y_{bc} V_{de} Y_{ef} Z_{cafd} \tag{8.25a}$$

and

$$J_3 \equiv Y_{ab} Z_{bccd} V_{da}, \tag{8.25b}$$

where [see [1], Eq. (22.9)]

$$V_{ab} = V_{ba}^* = v_a v_b e^{i(\theta_a - \theta_b)} \tag{8.25c}$$

We can express these $J$-invariants in the standard basis, transforming the invariants in Higgs basis directly to the standard basis, by using the Higgs basis parameters expressed in terms of standard parameters (7.20), with $\lambda_6 = \lambda_7 = 0$ in the standard basis,

$$\begin{aligned} J_1' = &- 8v^6(c^2 s^2 \Im(\lambda_5)) \Big[ -c^4 \lambda_1^2 + s^4 \lambda_2^2 \\ &+ (1-2c^2)\big[(\lambda_3+\lambda_4)^2 - |\lambda_5|^2\big] \\ &+ 2(c^4\lambda_1 - s^4\lambda_2)(\lambda_3+\lambda_4) - 2c^2 s^2(\lambda_1-\lambda_2)\Re(\lambda_5) \Big] \end{aligned} \tag{8.26a}$$

$$\begin{aligned} J_2' = &- 4v^4(c^2 s^2 \Im(\lambda_5)) \Big[ s^4 \lambda_1^2 - c^4 \lambda_2^2 \\ &+ (1-2c^2)\big[(\lambda_3+\lambda_4)^2 - |\lambda_5|^2\big] \\ &+ 2(-s^4\lambda_1 + c^4\lambda_2)(\lambda_3+\lambda_4) + 2c^2 s^2(\lambda_1-\lambda_2)\Re(\lambda_5) \Big] \end{aligned} \tag{8.26b}$$

$$J_3' = 2\sqrt{2} v^3 (c^2 s^2 \Im(\lambda_5))(\lambda_2 - \lambda_1) \tag{8.26c}$$

There also exist four other invariants called $I$-invariants, from Eqs.(13) and (20) in [11] and Eqs.(23)-(26) in [12],

$$I_{Y3Z} \equiv \Im(Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}), \tag{8.27}$$

$$I_{2Y2Z} \equiv \Im(Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}), \tag{8.28}$$

$$I_{6Z} \equiv \Im(Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}), \tag{8.29}$$

$$I_{3Y3Z} \equiv \Im(Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}), \tag{8.30}$$

where the tensors $Y$ and $Z$ are from (8.16), and $Z^{(1)}_{a\bar{d}} \equiv Z_{a\bar{b}b\bar{d}}$. The barred indicies are for keeping track of the indicies that transform with $V^\dagger$, see (8.18), while the unbarred indicies transform with $V$. The three first $I$-invariants (8.27), (8.28) and (8.29) are automatically zero for the isolated point $\lambda_1 = \lambda_2$ and $\lambda_7 = -\lambda_6$ (see Eqs.(28)-(30) in [12]) and in this case only the last invariant (8.30) needs to be considered, because now this is the only potentially nonzero invariant. Except for this special isolated point, it is only necessary to consider the three first $I$-invariants, because there is no other such point where the three first $I$-invariants are simultaneously vanishing (see page 5 in [12]). The simultaneous vanishing of these imaginary parts of the four invariants must be satisfied for an explicitly CP conserving 2HDM potential. If there exists a basis in which all parameters are real and all imaginary parts of the $I$-invariants disappear, the Higgs potential may still be CP violating if the VEVs are complex [12]. We see from (8.25a) that if this is the case then the $J$-invariants will still have non-vanishing imaginary parts. Or we can put this in another way, with real parameters of the potential, the VEV must violate time-reversal if it is to be CP violating — from the CPT theorem.

## 8.5 Lee model and SCPV

We wish to look into the possibility that SCPV and ECPV are just different mathematical constructs of the same matter and have no intrinsic physical differences. Of course they differ from the fact that the parameters of the potential are different in the two cases, but thinking that something exceptional happens if we have SCPV is not necessarily true. In the following we shall use the model of Lee (1973)[2] [1] (Ch.23: Spontaneous CP Violation). The Lee model is built to achieve SCPV. It has scalar potential given by

$$\begin{aligned} V =& a_1(\phi_1^\dagger\phi_1)^2 + a_2(\phi_2^\dagger\phi_2)^2 + a_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ &+ a_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + [(a_5(\phi_1^\dagger\phi_2)^2 e^{i\delta_5} + h.c.] \\ &+ a_6(\phi_1^\dagger\phi_1)[e^{i\delta_6}(\phi_1^\dagger\phi_2) + h.c.] + a_7(\phi_2^\dagger\phi_2)[e^{i\delta_7}(\phi_1^\dagger\phi_2) + h.c.] \\ &+ m_1(\phi_1^\dagger\phi_1) + m_3[e^{i\delta_3}(\phi_1^\dagger\phi_2) + h.c.] + m_2(\phi_2^\dagger\phi_2)], \end{aligned} \tag{8.31}$$

[2]Original reference is [5] ("A theory of Spontaneous T Violation"), but we have used the notation of Branco et al. in [1]

from [1] (section 22.3) (the complex parts of the potential are explicitly written out). Comparing to the potential in (7.8), we see that

$$\begin{aligned}
\lambda_1/2 =& a_1 \\
\lambda_2/2 =& a_2 \\
\lambda_3 =& a_3 \\
\lambda_4 =& a_4 \\
\lambda_5/2 =& a_5 e^{i\delta_5} \\
\lambda_6 =& a_6 e^{i\delta_6} \\
\lambda_7 =& a_7 e^{i\delta_7} \\
-m_{11}^2/2 =& m_1 \\
-m_{22}^2/2 =& m_2 \\
-m_{12}^2/2 =& m_3 e^{i\delta_3}
\end{aligned} \tag{8.32}$$

The Lee model has

$$e^{2i\delta_3} = e^{2i\delta_5} = e^{2i\delta_6} = e^{2i\delta_7} = 1, \tag{8.33}$$

such that all parameters are real. We have a complex VEV of the second doublet (7.4)

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \tag{8.34}$$

where we assume preservation of the unbroken U(1) gauge symmetry of electromagnetism, like in the SM. We will refer to the potential in (8.31), with constraints in (8.33) and complex VEV of the second doublet, as the "Lee potential". Using Eq. (23.8) in [1], we have for the minimum of the potential

$$\begin{aligned}
V_0 \equiv \langle 0|V|0\rangle = m_1 v_1^2 + m_2 v_2^2 + a_1 v_1^4 + a_2 v_2^4 \\
+ a_3 v_1^2 v_2^2 + (a_4 - 2a_5) v_1^2 v_2^2 + 4a_5 v_1^2 v_2^2 (\cos\xi - 2\Delta)\cos\xi
\end{aligned} \tag{8.35a}$$

with

$$\Delta \equiv -\frac{m_3 + a_6 v_1^2 + a_7 v_2^2}{4a_5 v_1 v_2} \tag{8.35b}$$

There is a theorem, see Theorem 23.2 in [1], stating that there is a range of parameters of the Lee model potential for which the minimum is not invariant under the CP transformation

$$(\mathcal{CP})\phi_a(t,\mathbf{r})(\mathcal{CP})^\dagger = \phi_a^*(t,-\mathbf{r}), \tag{8.36}$$

and since $\partial V_0/\partial\xi = 0$ for the minimum of the potential, we must have $\cos\xi = \Delta$ (with constraints $a_5 > 0$ and $2a_5 > a_4$). We read in [1] that this implies non-conservation of CP symmetry after SSB. We emphasize that it is the VEV of the second doublet that breaks CP symmetry, and not the minimum of the potential, $V_0$, in (8.35a). Lee also stresses this: the total Lagrangian is CP invariant, but the physical solutions are not.

- This means that the minimum of the potential, $V_0$, in a model with SCPV, is doubly degenerate.

We note that (8.36) is not the most general CP transformation, but above we are now operating under the assumption that we only have to consider the CP invariance of the Lagrangian and do not have to consider the explicit values of the VEVs when choosing a CP transformation. After SSB, one then checks if the VEVs are CP invariant or not under the CP transformation used on the original Lagrangian.

But is SCPV really physically different from ECPV? Starting again, with the Lee model, we can remove the complex part of $\langle\phi_2\rangle$ by a U(1) phase rotation of $\phi_2$, making some coefficients complex, like in (7.14a). Thus, an equivalent of the Lee model potential (which has complex VEV) is the potential (with $\lambda_6 = \lambda_7 = 0$)

$$\begin{aligned} V =& a_1(\phi_1^\dagger\phi_1)^2 + a_2(\phi_2^\dagger\phi_2)^2 + a_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ &+ a_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + [(a_5(\phi_1^\dagger\phi_2)^2 e^{2i\xi} + h.c.] \\ &+ m_1(\phi_1^\dagger\phi_1) + m_3[e^{i\xi}(\phi_1^\dagger\phi_2) + h.c.] + m_2(\phi_2^\dagger\phi_2)], \end{aligned} \tag{8.37}$$

and real VEVs. Comparing to the potential in (7.8), we have

$$\begin{aligned} \lambda_1/2 =& a_1 \\ \lambda_2/2 =& a_2 \\ \lambda_3 =& a_3 \\ \lambda_4 =& a_4 \\ \lambda_5/2 =& a_5 e^{2i\xi} \\ -m_{11}^2/2 =& m_1 \\ -m_{22}^2/2 =& m_2 \\ -m_{12}^2/2 =& m_3 e^{i\xi}, \end{aligned} \tag{8.38}$$

which is a subset of the parameters in (8.32). Now, these are just two ways of writing out the same model, both describing exactly the same physics. If one is able to measure the real and imaginary parts of the parameters of the potential (8.37), then in the case of SCPV we see that

$$\frac{\Im(\tilde{a}_5)}{\Re(\tilde{a}_5)} = \tan(2\xi)$$
$$\frac{\Im(\tilde{m}_3)}{\Re(\tilde{m}_3)} = \tan(\xi), \tag{8.39}$$

where we have used e.g. $\tilde{a}_5 = a_5 e^{2i\xi}$, not writing out the transformed phases explicitly. We have only performed a global phase transformation, under which the physics is to be the same. With SCPV in the Lee model, and a global phase transformation, we end up with complex parameters (and a real VEV).

We must note again that this is precisely the same model, because the potential is the same before and after the global phase transformation. It is just a matter of choice if the phase $\xi$ follows the parameters or the VEV. An instructive task now might be to reconstruct the mass squared matrix $\mathcal{M}^2$ in (7.23) for the Lee model potential (8.31). We set $a_6 = a_7 = 0$ and rewrite the symmetric mass squared matrix by explicitly writing out the real and imaginary parts of the pararameters in (7.23). Note that in most cases when writing any mass squared matrix, the phase $\xi$ following the second doublet, in (7.4), is transformed into the parameters of the potential by a global phase transformation (7.14a), and not explicitly shown in the mass squared matrix, to make the matrix more pleasing to the eye. But now we want to show the contribution of the phase $\xi$. We have here used the following convention for the two Higgs doublets,

$$\boldsymbol{\phi'}_1 = \boldsymbol{\phi}_1 = \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix} \tag{8.40}$$

$$\boldsymbol{\phi'}_2 = e^{i\xi}\boldsymbol{\phi}_2 = e^{i\xi} \begin{pmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix}, \tag{8.41}$$

and the mass squared matrix (7.23) becomes

$$\begin{aligned}
\mathcal{M}'^2_{11} =& v^2 \left[2c^2 a_1 + s^2 \cos(\xi)\nu'\right] \\
\mathcal{M}'^2_{22} =& v^2 \left[2s^2 a_2 + c^2 \cos(\xi)\nu'\right] \\
\mathcal{M}'^2_{33} =& v^2 \left[-2\cos(2\xi) a_5 + \cos(\xi)\nu'\right] \\
\mathcal{M}'^2_{12} =& v^2 cs \left[a_3 + a_4 + 2\cos(2\xi) a_5 + \cos(\xi)\nu'\right] \\
\mathcal{M}'^2_{13} =& - s\sin(2\xi) a_5 \\
\mathcal{M}'^2_{23} =& - c\sin(2\xi) a_5,
\end{aligned} \tag{8.42}$$

where we now have

$$\nu' = -m_3/(v_1 v_2), \tag{8.43}$$

since (see (7.7), and remember $e^{i\delta_{3,5}} = \pm 1$ from (8.33))

$$\begin{aligned}\nu =& \frac{1}{2v_1v_2}\Re(m_{12}^2) = -\frac{1}{v_1v_2}\Re(m_3 e^{i\xi}) \\ =& -\frac{1}{v_1v_2}\cos(\xi)m_3 = \cos(\xi)\nu'\end{aligned} \tag{8.44}$$

We see quite clearly that it is the phase $\xi$ of the second doublet that is the reason for non-zero $\mathcal{M}'^2_{13}$ and $\mathcal{M}'^2_{23}$. But it is also possible to construct the mass squared matrix of the Lee potential with another convention for the Higgs doublets,

$$\boldsymbol{\phi}_1 = \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix} \tag{8.45}$$

$$\boldsymbol{\phi}_2 = \begin{pmatrix} \varphi_2'^+ \\ \frac{1}{\sqrt{2}}(e^{i\xi}v_2 + \eta_2' + i\chi_2') \end{pmatrix}, \tag{8.46}$$

and note the ' after the fields, e.g. $\varphi_2'^+$, means that the field is rotated by

$$\varphi_2'^+ = e^{i\xi}\varphi_2^+. \tag{8.47}$$

The mass squared matrix becomes after derivation, using Reduce [8] and (7.22),

$$\begin{aligned}\mathcal{M}''^2_{11} =& 2v^2[c^2a_1 + s^2\cos^2(\xi)a_5] \\ \mathcal{M}''^2_{22} =& 2v^2[s^2\cos^2(\xi)a_2 + c^2a_5] \\ \mathcal{M}''^2_{33} =& 2s^2v^2\sin^2(\xi)[c^2a_2 + s^2a_5] \\ \mathcal{M}''^2_{12} =& \cos(\xi)csv^2(a_3 + a_4) \\ \mathcal{M}''^2_{13} =& \sin(\xi)sv^2[c^2(a_3 + a_4 - 2a_5) - 2s^2\cos(\xi)a_5] \\ \mathcal{M}''^2_{23} =& 2\sin(\xi)cs^2v^2[\cos(\xi)a_2 - a_5]\end{aligned} \tag{8.48}$$

Previously, when first writing out the mass squared matrix (7.23), we specified some relations between the coefficients, see (7.10). Take special note of the

$$\Im(m_{12}^2) = v_2v_1\Im(\lambda_5) \tag{8.49}$$

relation (note that we have in our discussion changed notation: $-m_{12}^2/2 = m_3$ and $\lambda_5/2 = a_5e^{i\delta_5}$). If we assume we have a potential of the form (8.31), with the phases written out explicitly, and a complex VEV of the second doublet (7.4), we can transform the phase of the second doublet into the parameters

$a_5 e^{i\delta_5}$ and $m_3 e^{i\delta_3}$. We now have real VEVs and the relation above becomes, writing out the imaginary parts of (8.49) explicitly,

$$m_3 \sin(\delta_3 + \xi) = -2a_5 v_2 v_1 \sin(\delta_5 + 2\xi). \tag{8.50}$$

It is tempting to suggest that $a_5$ and $m_3$ are independent parameters. If this is the case then from (8.50) we must conclude that $\delta_3 = -\xi$ and $\delta_5 = -2\xi$. But this removes the phases in the potential, and we are left with a real potential and no CP Violation. Thus, to keep the Higgs potential CP Violating, $m_3$ and $a_5$ must be dependent parameters. We therefore have, in general, no constraints on the phases $\delta_3$ and $\delta_5$.

- Therefore, it is only in the case of SCPV we have constraints on these phases, not in the case of ECPV. More precisely, in the case of SCPV we must have a basis with $\delta_3 = 2\delta_5$, as this is equivalent to finding a real basis with complex VEV.

So, if we find a basis with such a relation between $\delta_3$ and $\delta_5$ we know we have a model with SCPV. But this is trivial, since we can transform the phases into the VEV and obtain a real basis with complex VEV, which basis most literature describe leading to SCPV. In the most general Higgs potential there exists at most four potentially complex parameters and under a Higgs-basis transformation, or Reparametrization transformation, these parameters change (like e.g. (7.19)). Such a transformation is given in (7.13), where the change of parameters is given in Eq. (2.3) in [7].

The reparametrization induces a change in the parameters. If it is possible to find a basis where these potentially complex parameters are all real, or satisfy $2\delta_5 = \delta_3$, we have SCPV. If not, we have ECPV, and the difference between ECPV and SCPV is a basis independent, or physical, difference.

Since the off-diagonal elements $\mathcal{M}'^2_{13}$ and $\mathcal{M}'^2_{23}$ are non-zero, we have mixing of all the $\eta_i$ fields in (7.24). We see that this is because of the phase $\xi$. The $\eta_i$ fields have different CP properties, i.e. under a CP transformation they transform as

$$\begin{aligned}
\eta_1 &\to \eta_1 \\
\eta_2 &\to \eta_2 \\
\eta_3 &\to -\eta_3.
\end{aligned} \tag{8.51}$$

Because of this mixing of all the fields $\eta_i$ to make up the physical Higgs fields, the physical Higgs fields have indefinite CP quantum numbers. More explicitly written, if the mass squared matrix has non-zero off-diagonal terms, the rotation matrix (7.26) needs all three angles $\alpha_i$ to diagonalize the mass squared matrix, thus mixing all three unphysical fields $\eta_i$ to make up the physical fields in (7.24). This can be seen by

$$\begin{aligned}
&\begin{pmatrix}\eta_1 & \eta_2 & \eta_3\end{pmatrix} R^T R \mathcal{M}^2 R^T R \begin{pmatrix}\eta_1 \\ \eta_2 \\ \eta_3\end{pmatrix} \\
&= \begin{pmatrix}H_1 & H_2 & H_3\end{pmatrix} R \mathcal{M}^2 R^T \begin{pmatrix}H_1 \\ H_2 \\ H_3\end{pmatrix} \\
&= \begin{pmatrix}H_1 & H_2 & H_3\end{pmatrix} \mathcal{M}^2_{\text{diag}} \begin{pmatrix}H_1 \\ H_2 \\ H_3\end{pmatrix}
\end{aligned} \tag{8.52}$$

If we have explicit CP Violation in a Higgs potential (8.31) with $a_6 = a_7 = 0$, there are two terms possibly violating CP. These are the $m_3$-term and $a_5$-term. Put in another way, we cannot find a CP transformation simultaneously conserving CP symmetry of both of these terms. We find an example of this in section 22.7 (CP violation in the scalar potential: simple examples) of [1], which we will discuss in detail later. The breaking of the symmetry comes from the clashing of the complexity of the parameters of the potential (given by the phases $\delta_3$ and $\delta_5$) and the phase $\xi$ of the VEV, under a CP transformation. But even though in the case of SCPV we do not have explicit CP Violation of the quadratic or quartic terms in the potential, the interactions from these same terms will in fact exhibit CP Violation, because the physical Higgs fields have indefinite CP quantum numbers. Of course the amount of CP Violation in these terms will differ between ECPV and SCPV. It is interesting that in the case of ECPV in (8.31) $V_0$ takes the form (setting $a_6 = a_7 = 0$)

$$\begin{aligned}
V_0 \equiv & m_1 v_1^2 + m_2 v_2^2 + 2 m_3 v_1 v_2 \cos(\delta_3 + \xi) + a_1 v_1^4 \\
& + a_2 v_2^4 + (a_3 + a_4) v_1^2 v_2^2 + 2 a_5 v_1^2 v_2^2 \cos(\delta_5 + 2\xi),
\end{aligned} \tag{8.53}$$

and will change value after a CP transformation. The clashing of the phases under a CP transformation will cause $V_0$ to violate CP.

In a Corollary in [15] we read that if we have a discrete symmetry of the Higgs potential, minima that conserve and break this symmetry cannot coexist in the 2HDM. So for the Lee model potential, which is CP invariant, we cannot have another minimum which breaks CP, beside the one in (8.35a). But for the case of (8.53), whether there exists a deeper minimum of the potential which is not CP violating, might be concluded with from the following: If after a reparametrization (see [7]) there exists a basis with all real parameters, then CP is a symmetry of the potential in this basis, and the corresponding minimum $V_0'$ of the reparametrized potential will be CP invariant. By the Corollary above such a CP invariant minimum $V_0'$ cannot coexist with a CP violating minimum (such as (8.53)), thus if we have explicit CP violation of a potential, there does not exist a real basis with a CP invariant minimum. We can not find this conclusion reached explicitly in [15], [16] or [17]. In [16] ("3.The stationary points of the 2HDM") there is a discussion regarding a potential with explicit CP violation when the VEV of the fields are either real or complex. The two minima for the real and complex VEVs are different in depths, and the difference is given by eq. (13) in [16]. But both these minima must be CP violating if the potential is explicitly CP violating, and from [15] (see Proposition 1, page 7) there are at most two local minima, thus no CP invariant minima exists beside the two discussed CP violating ones. What the consequence of a CP violating global minimum of the potential is, we have no idea.

## 8.6 Example of Explicit CP conservation and violation

We mentioned section 22.7 (CP violation in the scalar potential: simple examples) of [1], and based on this we shall give examples of Explicit CP conservation and violation.

### 8.6.1 Explicit CP conservation

If we have the general potential in (8.31) with a discrete symmetry, or $Z_2$-symmetry (see (8.2)), then

$$m_3 = a_6 = a_7 = 0, \tag{8.54}$$

such that the only potentially complex parameter left is $a_5$. From (8.53) $V_0$ now becomes

$$V_0 \equiv m_1 v_1^2 + m_2 v_2^2 + a_2 v_2^4 + (a_3 + a_4) v_1^2 v_2^2 + 2a_5 v_1^2 v_2^2 \cos(\delta_5 + 2\xi). \tag{8.55}$$

We see we get the minimum when

$$\cos(\delta_5 + 2\xi) = -1, \tag{8.56}$$

because $a_5$ is positive by definition. From Eq. (22.38) in [1] we get the CP transformation

$$(\mathcal{CP})\phi_a(t,\mathbf{r})(\mathcal{CP})^\dagger = U_{ab}^{CP} \phi_b^{\dagger T}(t,-\mathbf{r}), \tag{8.57}$$

where

$$U^{CP} = \mathrm{diag}(1, e^{2i\xi}), \tag{8.58}$$

such that the complex VEV of the second doublet in (7.4) is left invariant. The $a_5$-term of (8.31) therefore changes the following way under the CP transformation (8.57)

$$(\mathcal{CP})\left(\phi_1^\dagger\phi_2\right)^2(\mathcal{CP})^\dagger = e^{4i\xi}\left(\phi_2^\dagger\phi_1\right)^2, \tag{8.59}$$

and for the potential to be CP invariant we then have the requirement

$$\begin{aligned} e^{i(\delta_5+4\xi)} &= e^{-i\delta_5} \\ \Rightarrow e^{i(\delta_5+2\xi)} &= e^{-i(\delta_5+2\xi)} \\ \Rightarrow \cos(\delta_5+2\xi) &= \pm 1, \end{aligned} \tag{8.60}$$

which actually implies (8.56). Therefore CP is conserved. There is only one complex term in the potential, and it is possible to construct a CP transformation which leaves the terms explicitly CP invariant.

### 8.6.2 Explicit CP violation

As briefly mentioned, softly broken discrete symmetry is when the terms which break the symmetry have dimension two. In the 2HDM this allows for the term with parameter $m_3e^{i\delta_3}$ in (8.31). In addition to CP invariance of the term with parameter $a_5e^{i\delta_5}$ (see (8.59)), we now also must require CP invariance of the soft breaking term

$$(\mathcal{CP})\left(\phi_1^\dagger\phi_2\right)(\mathcal{CP})^\dagger = e^{2i\xi}\left(\phi_2^\dagger\phi_1\right), \tag{8.61}$$

and CP invariance of this term now requires

$$e^{i(2\xi+\delta_3)} = e^{-i\delta_3}. \tag{8.62}$$

Thus, from (8.59) and (8.62), requirements of CP invariance are

$$\begin{aligned} e^{2i(\delta_5+2\xi)} =& 1 \\ e^{2i(\delta_3+\xi)} =& 1. \end{aligned} \tag{8.63}$$

We reproduce (8.50), or Eq. (22.17) in [1] (with $a_6 = a_7 = 0$), which is a constraint on the parameters of the potential,

$$m_3\sin(\delta_3+\xi) = -2a_5v_2v_1\sin(\delta_5+2\xi). \tag{8.64}$$

It is clear that the CP invariance requirements in (8.63) together with (8.64) both are satisfied when

$$\delta_3 + \xi = \delta_5 + 2\xi = 2n\pi \quad n = 0, 1, 2... \tag{8.65}$$

But in general this may not be true, and then we have ECPV of the potential. That is, at least one of the complex terms in (8.59) and (8.61) may explicitly break CP invariance.

# Chapter 9

# Thought Experiments to measure parameters of the potential

## 9.1 Physical Higgs Masses

In QED, the insertion of a fermion self-energy correction in a fermion propagator, see Figure 9.1, leads to a propagator representing the interacting physical fermion with a mass $m = m_0 + \delta m$, instead of the non-interacting fermion with mass $m_0$. The pole in the propagator of the interacting physical fermion is thus required to be at $\not{p} = m$. The rest mass of the real fermion differs from the rest mass $m_0$ of the non-interacting fermion due to the interaction of the fermion field and the electromagnetic field. This replacement of of $m_0$ by $m$ is called mass renormalization, and the experimentally determined mass must be expressed as $m$ and these are important considerations to make when comparing theory to experimental results.

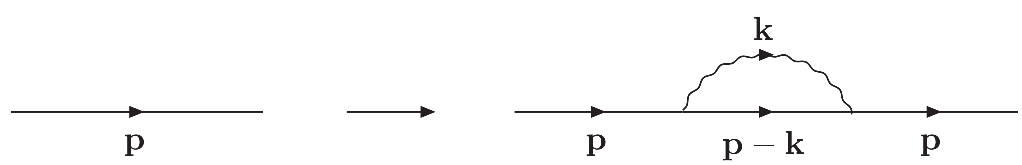


Figure 9.1: Fermion propagator second order correction

In the same spirit, we can imagine taking physical masses as input, as well as mixing angles, and consider the parameters of the potential as *derived* quantities. This imposes no difficulty, since the masses are expressed as linear combinations of the parameters of the potential, so we can choose to take the physical masses as input.

## 9.2 Yukawa couplings and CP Violation in top quark production

The Yukawa couplings of quarks and neutral Higgs bosons include the elements of the rotation matrix $R$, consisting of the three angles $\alpha_i$, together with the angle $\beta$. By determining the mass of the neutral Higgs bosons together with these Yukawa couplings, it can be possible to determine the value of the parameters of the Higgs potential.

We reproduce (8.15),

$$\begin{aligned} H_j\bar{t}t: &\quad \frac{1}{\sin\beta}[R_{j2} - i\gamma^5\cos\beta R_{j3}] \equiv a + i\tilde{a}\gamma_5 \\ H_j\bar{b}b: &\quad \frac{1}{\cos\beta}[R_{j1} - i\gamma^5\sin\beta R_{j3}]. \end{aligned} \tag{9.1}$$

Note that $a$ has nothing to do with the parameters in (8.31). Writing out explicitly for $H_1$, i.e. $j = 1$,

$$H_1\bar{t}t: \quad \frac{1}{\sin\beta}[R_{12} - i\gamma^5\cos\beta R_{13}] \tag{9.2a}$$

$$H_1\bar{b}b: \quad \frac{1}{\cos\beta}[R_{11} - i\gamma^5\sin\beta R_{13}], \tag{9.2b}$$

we see that there are in total 3 unknowns, counting the three matrix elements $R_{11}, R_{13}$ and $R_{12}$ (which consist of the two angles $\alpha_1$ and $\alpha_2$), and the angle $\beta$. In general there will be more variables than equations to solve them for. Note that the measurable quantity is the squared of the Yukawa couplings, i.e.

$$\frac{1}{\sin^2\beta}[R_{i2} - i\gamma_5\cos\beta R_{i3}]^\dagger[R_{i2} + i\gamma_5\cos\beta R_{i3}] = \frac{1}{\sin^2\beta}[(R_{i2})^2 + \cos^2\beta(R_{i3})^2], \tag{9.3}$$

for $H_i\bar{t}t$.

Focusing on the Yukawa coupling of the Higgs and top-quarks, $H_j\bar{t}t$, we follow [18]. The product of the $H_j\bar{t}t$ scalar and pseudoscalar couplings $a$ and

$\tilde{a}$, respectively, in $[a + i\tilde{a}\gamma_5]$, is a quantity defined as

$$\gamma_{CP} = -a\tilde{a}. \tag{9.4}$$

In [18] we learn that in the cross section of a process (depending on the $t\bar{t}$-spins)

$$pp \to t\bar{t} + X, \tag{9.5}$$

the CP violating part will be proportional to the quantity $\gamma_{CP}$. In [18] $gg \to t\bar{t}$ is discussed, and Figure 2 on page 3 describes the lowest-order QCD Feynman diagrams of this reaction, shown in Figure (9.2)

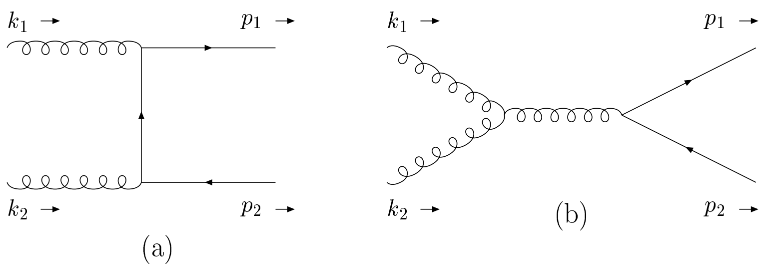


Figure 9.2: Lowest-order QCD Feynman diagrams of $gg \to t\bar{t}$, Figure 2 in [18]

Now, there are higher order corrections to these two diagrams with exchanges of the neutral Higgs bosons $H_i$. These are listed in Figure 3 in [18]. As the cross section of these higher order correction diagrams have contributions from all three neutral Higgs bosons, we can not really separate e.g. $H_1\bar{t}t$ from $H_2\bar{t}t$ in a measurement. If it was possible to consider two gluon beams with a center-of-mass energy equal to the mass of one of the Higgs bosons $H_i$, we would mainly have contribution from Figure 9.3. This is not realistic as we cannot regulate the energy of the gluons, but it is a nice thought experiment. So, continuing this thought experiment, it could be possible to determine the three $\gamma_{CP}$ for the Yukawa couplings $H_i\bar{t}t$:

$$\gamma^i_{CP} = -a\tilde{a} = \frac{\cos\beta}{\sin^2\beta} R_{i2} R_{i3}. \tag{9.6}$$

$$\gamma^1_{CP} = \frac{s_1 s_2 c_2}{\tan\beta\sin\beta} \tag{9.7a}$$

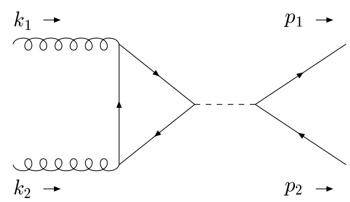


Figure 9.3: Figure 3 (h) in [18]

$$\gamma_{CP}^2 = \frac{c_1c_2c_3s_3 - c_2s_1s_2s_3^2}{\tan\beta\sin\beta} \tag{9.7b}$$

$$\gamma_{CP}^3 = \frac{-c_1c_2c_3s_3 - s_1s_2c_2c_3^2}{\tan\beta\sin\beta} \tag{9.7c}$$

But these three $\gamma_{CP}^i$ contain four variables, namely $\beta$ and $\alpha_i$. So we must then determine the relation between the $\gamma_{CP}^i$ such that we cancel out the $\beta$-dependence. We see that it is idealistically possible to determine the angles of the rotation matrix $R$, and by knowing the mass of the neutral Higgs bosons, we can calculate the parameters of the Higgs potential. The ratios between the $\gamma_{CP}^i$ can be written as

$$\frac{\gamma_{CP}^2}{\gamma_{CP}^1} = \frac{c_1c_2c_3s_3 - c_2s_1s_2s_3^2}{s_1s_2c_2} = \frac{c_1c_3s_3}{s_1s_2} - s_3^2, \tag{9.8}$$

$$\frac{\gamma_{CP}^3}{\gamma_{CP}^1} = \frac{-c_1c_2c_3s_3 - c_2s_1s_2c_3^2}{s_1s_2c_2} = \frac{-c_1c_3s_3}{s_1s_2} - c_3^2, \tag{9.9}$$

and

$$\frac{\gamma_{CP}^2}{\gamma_{CP}^3} = \frac{c_1c_2c_3s_3 - c_2s_1s_2s_3^2}{-c_1c_2c_3s_3 - s_1s_2c_2c_3^2}. \tag{9.10}$$

We see that the ratios are independent of $\beta$. But the last ratio $\frac{\gamma_{CP}^2}{\gamma_{CP}^3}$ is dependent on the two other ratios, (9.8) and (9.9), so we do not have enough information to determine all the angles $\alpha_i$. Note that we require experimental results on the $\gamma_{CP}^i$, and not just the Yukawa couplings (8.15). If we knew the angles of the rotation matrix, we could calculate the parameters of the potential by e.g. (Eq. (4.14) in [18])

$$\begin{aligned}\mathcal{M}_{13}^2 =& R_{11}R_{13}M_1^2 + R_{21}R_{23}M_2^2 + R_{31}R_{33}M_3^2,\\ \mathcal{M}_{23}^2 =& R_{12}R_{13}M_1^2 + R_{22}R_{23}M_2^2 + R_{32}R_{33}M_3^2,\end{aligned} \tag{9.11}$$

where $M$ is the mass eigenvalues of the neutral Higgs bosons, and $R$ is the rotation matrix. We do the same for the rest of the mass squared matrix elements, and are able to construct the whole mass squared matrix.

## 9.3 Trilinear Higgs self-couplings

Since it may not be possible to determine by inspection if it is possible to transform an arbitrary Higgs basis, with 4 potentially complex parameters, into a real basis, we need experimental values for the parameters, or combinations of them. We can get this from trilinear Higgs self-couplings, see Figures 9.4, 9.5 and 9.6. Following and using the results of [19] we write the trilinear self-couplings of the neutral Higgs bosons as

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}, \tag{9.12}$$

and we note that the fields are the physical neutral Higgs fields. By using a useful relation between the $\eta_i$-basis and $H_i$-basis obtained through the rotation matrix (7.26),

$$\frac{\partial}{\partial H_i} = \frac{d\eta_j}{dH_i}\frac{\partial}{\partial \eta_j} = R_{ij}\frac{\partial}{\partial \eta_j}, \tag{9.13}$$

we can express the trilinear couplings through the unphysical fields $\eta_i$ and the rotation matrix

$$\begin{aligned}\lambda_{ijk} =& \sum_{m\le n\le o=1,2,3}^{*} R_{i'm}R_{j'n}R_{k'o}\frac{-i\partial^3 V}{\partial\eta_m\partial\eta_n\partial\eta_o}\\ & \sum_{m\le n\le o=1,2,3}^{*} R_{i'm}R_{j'n}R_{k'o}a_{mno}.\end{aligned} \tag{9.14}$$

The $*$ denotes sum over all permutations of $i', j', k'$. At tree level, quoting the results of [19], the trilinear couplings $a_{mno}$ are

$$\begin{aligned}
a_{111} =&\frac{1}{2}(\cos\beta\lambda_1 + \sin\beta\Re(\lambda_6)),\\
a_{112} =&\frac{1}{2}(\sin\beta\Re(\lambda_{345} + 3\cos\beta\Re(\lambda_6)),\\
a_{113} =&-\frac{1}{2}\left[\cos\beta\sin\beta\Im(\lambda_5) + (1+2\cos^2\beta)\Im(\lambda_6)\right],\\
a_{122} =&\frac{1}{2}(\cos\beta\Re(\lambda_{345}) + 3\sin\beta\Re(\lambda_7)),\\
a_{123} =&-\Im(\lambda_5) - \cos\beta\sin\beta(\Im(\lambda_6) + \Im(\lambda_7)),\\
a_{133} =&\frac{1}{2}\Big\{\cos\beta(\sin^2\beta\lambda_1 + \cos^2\beta\Re(\lambda_{345}) - 2\Re(\lambda_5))\\
&+\sin\beta\left[(\sin^2\beta - 2\cos^2\beta)\Re(\lambda_6) + \cos^2\beta\Re(\lambda_7)\right]\Big\}\\
a_{222} =&\frac{1}{2}(\sin\beta\lambda_2 + \cos\beta\Re(\lambda_7)),\\
a_{223} =&-\frac{1}{2}\left[\cos\beta\sin\beta\Im(\lambda_5) + (\cos^2\beta + 3\sin^2\beta)\Im(\lambda_7)\right],\\
a_{233} =&\frac{1}{2}\Big\{\sin\beta(\cos^2\beta\lambda_2 + \sin^2\beta\Re(\lambda_{345}) - 2\Re(\lambda_5))\\
&+\cos\beta\left[\sin^2\beta\Re(\lambda_6) + (\cos^2\beta - 2\sin^2\beta)\Re(\lambda_7)\right]\Big\},\\
a_{333} =&\frac{1}{2}(\cos\beta\sin\beta\Im(\lambda_5) - \sin^2\beta\Im(\lambda_6) - \cos^2\beta\Im(\lambda_7)).
\end{aligned} \tag{9.15}$$

For example, for $\alpha_2 = \alpha_3 = 0$

$$\begin{aligned}
\lambda_{112} =&iv\Big[3c_1s_1(c_1c_\beta\lambda_1 - s_1s_\beta\lambda_2)\\
&-[c_1^3s_\beta - s_1^3c_\beta + 2c_1s_1(c_1c_\beta - s_1s_\beta)]\Re\lambda_{345})\Big],
\end{aligned} \tag{9.16}$$

with $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. For our work it is interesting to reproduce (9.15) with $\lambda_6 = \lambda_7 = 0$,

$$\begin{aligned}
a_{111} &= \frac{1}{2}\cos\beta\lambda_1, \\
a_{112} &= \frac{1}{2}\sin\beta\Re(\lambda_{345}), \\
a_{113} &= -\frac{1}{2}\cos\beta\sin\beta\Im(\lambda_5), \\
a_{122} &= \frac{1}{2}\cos\beta\Re(\lambda_{345}), \\
a_{123} &= -\Im(\lambda_5), \\
a_{133} &= \frac{1}{2}\cos\beta\left[\sin^2\beta\lambda_1 + \cos^2\beta\Re(\lambda_{345}) - 2\Re(\lambda_5)\right] \\
a_{222} &= \frac{1}{2}\sin\beta\lambda_2, \\
a_{223} &= -\frac{1}{2}\cos\beta\sin\beta\Im(\lambda_5), \\
a_{233} &= \frac{1}{2}\sin\beta\left[\cos^2\beta\lambda_2 + \sin^2\beta\Re(\lambda_{345}) - 2\Re(\lambda_5)\right], \\
a_{333} &= \frac{1}{2}\cos\beta\sin\beta\Im(\lambda_5).
\end{aligned} \tag{9.17}$$

We can see that it is possible to determine the parameters of the potential by experimental results. Although we have come a long way by measuring the Yukawa couplings, we need more experimental results to measure the parameters. These results might come from the trilinear couplings. Without putting any restrictions on the angles of the rotation matrix, we can calculate the trilinear couplings $\lambda_{ijk}$ using Reduce [8], (9.17) and (9.14), but unfortunately these trilinear couplings are neither aesthetic nor compact. As we see, there are eleven potential trilinear couplings to be measured. These trilinear couplings consist of the parameters of the potential: the 5 quartic (we have set $\lambda_6 = \lambda_7 = 0$) and 1 quadratic parameter (the real part of $m_{12}^2$: the imaginary part is given by $\lambda_5$). We also have the three angles $\alpha_i$ in the rotation matrix, and the angle $\beta$. In total that is 10 parameters. So, experimental results on the 11 trilinear couplings might give us enough information to calculate the mentioned parameters. This requires, though, that we mainly have contributions from the diagrams in Figures 9.5 and 9.6, because it is these two that contain the trilinear couplings, and not a quadratic coupling like in Figure 9.4. We can achieve this if we have measurements with center-of-mass energy close to the mass of the Higgs bosons $H_k$, such that the dominant contribution are from diagram 9.5. Denoting the Feynman amplitudes corresponding to the diagrams in Figures 9.4, 9.5 and 9.6 with $\mathcal{M}_0$, $\mathcal{M}_1$ and $\mathcal{M}_2$ respectively, the cross section with such a center-of-mass energy will become

$$\sigma \propto |\mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2|^2 \overset{(p_1+p_2)^2=M_k^2}{\approx} |\mathcal{M}_1|^2 \tag{9.18}$$

The Feynman amplitude $\mathcal{M}_1$ contains the product of two trilinear couplings from the two vertices in the Feynman diagram in Figure 9.5. What we would measure would therefore be products of the trilinear couplings.

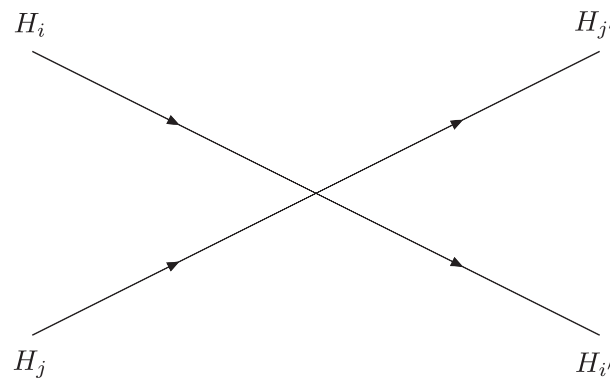


Figure 9.4: Feynman diagram $\mathcal{M}_0$

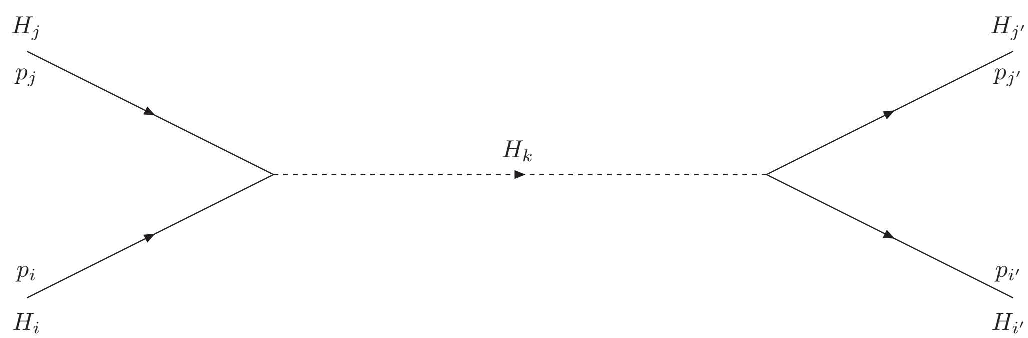


Figure 9.5: Feynman diagram $\mathcal{M}_1$

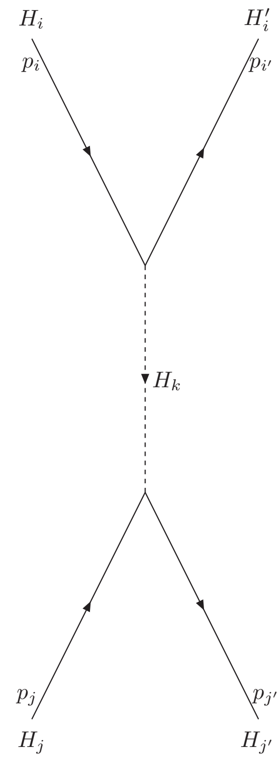


Figure 9.6: Feynman diagram $\mathcal{M}_2$

For the scattering of two $H_1$-bosons with center-of-mass energy equal to for example the mass of $H_2$, we will have the Feynman diagrams in Figure 9.7. The trilinear couplings in both vertices are then $\lambda_{112}$, and we see from (9.18) that the cross section will get contributions proportional to $(\lambda_{112})^2$. From theoretical scenarios like this, however unlikely, we can possibly determine the trilinear couplings with identical procedures for the rest of the couplings.

If we were able to measure $\lambda_{112}$, we could use this result together with results from (9.6), in total 4 equations, to determine the 4 angles $\alpha_i$ and $\beta$. But note that $\lambda_{112}$ is expressed in terms of the parameters of the potential, and we must therefore express the parameters in terms of the Higgs masses and angles $\alpha_i$ and $\beta$. From Eq. (3.1)-(3.6) in [20] we write their results:

$$\begin{aligned}
\lambda_1 &= \frac{1}{c_\beta^2 v^2}[c_1^2 c_2^2 M_1^2 + (c_1 s_2 s_3 + s_1 c_3)^2 M_2^2 \\
&\quad + (c_1 s_2 c_3 - s_1 s_3)^2 M_3^2 - s_\beta^2 \mu^2], \quad (9.19)\\
\lambda_2 &= \frac{1}{s_\beta^2 v^2}[s_1^2 c_2^2 M_1^2 + (c_1 c_3 - s_1 s_2 s_3)^2 M_2^2 \\
&\quad + (c_1 s_3 + s_1 s_2 c_3)^2 M_3^2 - c_\beta^2 \mu^2], \quad (9.20)\\
\lambda_3 &= \frac{1}{c_\beta s_\beta v^2}\{c_1 s_1[c_2^2 M_1^2 + (s_2^2 s_3^2 - c_3^2) M_2^2 \\
&\quad + (s_2^2 c_3^2 - s_3^2) M_3^2] + s_2 c_3 s_3 (c_1^2 - s_1^2)(M_3^2 - M_2^2)\} \\
&\quad + \frac{1}{v^2}[2M_{H^\pm}^2 - \mu^2], \quad (9.21)\\
\lambda_4 &= \frac{1}{v^2}[s_2^2 M_1^2 + c_2^2 s_3^2 M_2^2 + c_2^2 c_3^2 M_3^2 + \mu^2 - 2M_{H^\pm}^2], \quad (9.22)\\
\Re\lambda_5 &= \frac{1}{v^2}[-s_2^2 M_1^2 - c_2^2 s_3^2 M_2^2 - c_2^2 c_3^2 M_3^2 + \mu^2], \quad (9.23)\\
\Im\lambda_5 &= \frac{-1}{c_\beta s_\beta v^2}\{c_\beta[c_1 c_2 s_2 M_1^2 - c_2 s_3 (c_1 s_2 s_3 + s_1 c_3) M_2^2 \\
&\quad + c_2 c_3 (s_1 s_3 - c_1 s_2 c_3) M_3^2] + s_\beta[s_1 c_2 s_2 M_1^2 \quad (9.24)\\
&\quad + c_2 s_3 (c_1 c_3 - s_1 s_2 s_3) M_2^2 - c_2 c_3 (c_1 s_3 + s_1 s_2 c_3) M_3^2]\},
\end{aligned}$$

where $c_\beta = \cos\beta$, $s_\beta = \sin\beta$. Assuming we have measured the masses of both the neutral and charged Higgs bosons, we then have a set of equations from which we can calculate the angles $\alpha_i$ and $\beta$.

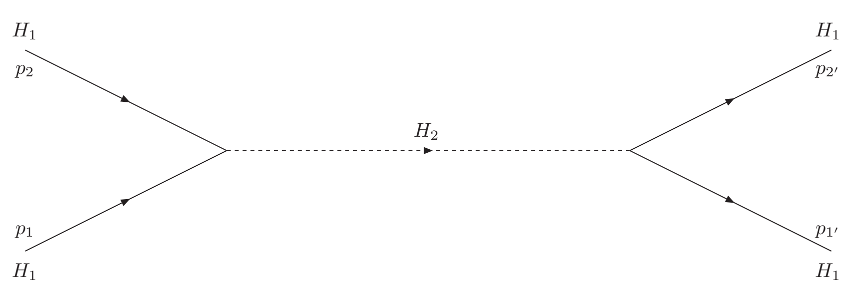


Figure 9.7: Two $H_1$-bosons with center-of-mass energy equal to the mass of $H_2$

# Chapter 10

# Conclusion

Since it is so hard to determine if there exists a real basis in the Two Higgs Doublet Model (2HDM), we have discussed the 2HDM in both bases with ECPV and SCPV, and from there studied their physical properties.

In this thesis we searched for the possibility of unifying Spontaneous CP Violation (SCPV) and Explicit CP Violation (ECPV). We found that there must be a relation $\delta_3 = 2\delta_5$ between the phases of the complex parameters $\lambda_5 = 2a_5 e^{i\delta_5}$ and $m_{12}^2 = -2m_3 e^{i\delta_3}$ in the case of SCPV, and that this is equivalent to a basis with all real parameters. But as pointed out in Chapter 8, it might be possible to go from a basis with ECPV to a basis with SCPV, and if this is the case, there is no basis independent, physical, difference between ECPV and SCPV. This is a complex task to solve. It is because of this complex task basis invariant tensors in the 2HDM have been introduced, and we discussed these invariants in Chapter 8. They can be important in determining what kind of CP Violation we have, but will require experimental results.

We showed that the mass matrix in both SCPV and ECPV lead to mixing of the neutral eigenstates, thus leading to CP Violation. In the case of SCPV it is the complex phase of the vacuum expectation value (VEV) of one of the Higgs doublets which leads to mixing of the eigenstates, so this phase does in fact have a physical role in the mass squared matrix of the neutral Higgs bosons.

Even though in a basis with SCPV we can transform away the phase of the doublets, to make both VEVs of the Higgs doublets real, the phase will be transformed into some of the parameters in the potential making them complex. But we can still find a CP transformation in which the potential is CP invariant. In literature we read that in case of SCPV it is the minimum of the potential which violate CP, but this is poorly formulated. It is actually the solutions, the VEVs of the Higgs doublets, which violate CP. And since the minimum of the potential is CP invariant, it means that the minimum of the potential is doubly degenerate.

An interesting feature of ECPV is the CP violating minimum of the potential, but as to what physical significance this has we have proposed no solution.

# Bibliography

[1] G. C. Branco, L. Lavoura, J. P. Silva, "CP Violation" (Oxford Univ. Press, 1999).

[2] A. W. El Kaffas, W. Khater, O. M. Ogreid and P. Osland, Nucl. Phys. B **775** (2007) 45 [arXiv:hep-ph/0605142].

[3] R. A. Diaz, arXiv:hep-ph/0212237.

[4] J. Gunion, H. Haber, G. Kane, and S. Dawson, "The Higgs Hunters Guide" (Addison-Wesley, New York, 1990).

[5] T. D. Lee, Phys. Rev. D **8** (1973) 1226.

[6] Y. L. Wu and L. Wolfenstein, Phys. Rev. Lett. **73** (1994) 1762 [arXiv:hep-ph/9409421].

[7] I. F. Ginzburg and M. Krawczyk, Phys. Rev. D **72** (2005) 115013 [arXiv:hep-ph/0408011].

[8] Homepage: http://reduce-algebra.com/

[9] H. E. Haber and D. O'Neil, Phys. Rev. D **74** (2006) 015018 [arXiv:hep-ph/0602242].

[10] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958

[11] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, Phys. Lett. B **614** (2005) 187 [arXiv:hep-ph/0502118].

[12] J. F. Gunion and H. E. Haber, Phys. Rev. D **72** (2005) 095002 [arXiv:hep-ph/0506227].

[13] L. Lavoura and J. P. Silva, Phys. Rev. D **50** (1994) 4619 [arXiv:hep-ph/9404276].

[14] A. Méndez and A. Pomerol Phys. Lett. B**272**, 313 (1991)

[15] I. P. Ivanov, Phys. Rev. D **77** (2008) 015017 [arXiv:0710.3490 [hep-ph]].

[16] A. Barroso, P. M. Ferreira and R. Santos, Phys. Lett. B **652** (2007) 181 [arXiv:hep-ph/0702098].

[17] I. F. Ginzburg and K. A. Kanishev, Phys. Rev. D **76** (2007) 095013 [arXiv:0704.3664 [hep-ph]].

[18] W. Khater and P. Osland, Nucl. Phys. B **661** (2003) 209 [arXiv:hep-ph/0302004].

[19] P. Osland, P. N. Pandita and L. Selbuz, Phys. Rev. D **78** (2008) 015003 [arXiv:0802.0060 [hep-ph]].

[20] A. W. El Kaffas, P. Osland and O. M. Ogreid, Nonlin. Phenom. Complex Syst. **10** (2007) 347 [arXiv:hep-ph/0702097].