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Fast 3D Modeling of the CSEM Response of Petroleum Reservoirs

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SUMMARY

The geometry of a petroleum reservoir is usually irregular so that accurate modeling may require a very large number of grid cells. This may severely limit the applicability of 3D CSEM solvers for this application both with respect to computational complexity and memory requirements. With finite difference (FD) (and finite element (FE)) methods the computational domain is much larger than the reservoir itself, increasing the number of grid cells further. An advantage, however, is that the corresponding coefficient matrix is sparse. With integral equation (IE) methods only the reservoir itself needs to be discretized. A disadvantage (particularly with a large number of grid cells) is that the corresponding coefficient matrix is dense. Recently, a novel approximate hybrid method has been shown to produce excellent accuracy in modeling the CSEM response from petroleum reservoirs (Bakr and Mannseth (2009)). This method – termed simplified IE (SIE) modeling – fully utilizes that the low-frequency CSEM response from a thin resistive body is dominated by the galvanic effect. The dense-matrix part of IE can then be replaced by sparse-matrix calculations corresponding to solving a Poisson equation for the reservoir. In this paper, we quantify the computational complexity of SIE and compare its performance to IE and FD in computing the CSEM response from petroleum reservoirs with a large number of grid cells. It is shown that the number of floating point operations (flops) per iteration with SIE is an order of magnitude smaller than the number of flops per iteration with IE, and much smaller than the number of flops per iteration with FD. Numerical examples demonstrate that the number of iterations required with SIE is an order of magnitude smaller than the number of iterations with IE. It is demonstrated that SIE can be applied to solve problems that are too large for IE.

INTRODUCTION

Marine CSEM has become an important complementary tool for offshore petroleum exploration. A typical target has a much (20-100 times) lower electric conductivity than its surroundings, and is characterized by a relatively large extension in two spatial directions (x and y) and a much smaller extension in the third (z) direction. The method then exploits lossy guiding of EM energy in thin resistive bodies within more conductive media in an attempt to detect hydrocarbon reservoirs. For an in-line source-receiver geometry, the response from a thin resistive target is much more due to galvanic effects than to inductive effects (Eidesmo et al. (2002); MacGregor and Sinha (2000)). Typically, very low source frequencies (0.05 – 1 Hz) are applied since the petroleum reservoir can be buried more than 1 km below the sea floor.

To detect a potential petroleum reservoir, 3D electromagnetic (EM) modeling results based on several geoelectric models are compared to EM data acquired in sea floor receivers. The geoelectric model is then changed until a satisfactory match between the modeling results and the data is obtained. Such inversion of electromagnetic data thus requires a number of repeated solves of the 3D EM model. The computational efficiency of the solver will therefore have great impact of the computational efficiency of the inversion process. Various types of solvers, like finite difference (FD), finite element (FE), integral equation (IE), and hybrid methods, have been applied. The different types of methods have different computational advantages and disadvantages (see, e.g., Avdeev (2005)), as will be discussed later.

The geometry of a petroleum reservoir is usually irregular. Reasonably accurate modeling of such a geometry can require use of a very large number of grid cells. This may severely limit the applicability of 3D solvers both with respect to computational complexity and memory requirements. Potential use of marine CSEM for production monitoring (Lien and Mannseth (2008); Orange et al. (2009)) will further emphasize the need for accurate modeling of reservoir geometry.

Recently, a novel hybrid method (Bakr and Mannseth (2009)) was proposed as a solver for marine CSEM as applied to petroleum exploration. This method, termed *simplified IE* (SIE) modeling, is based on rigorous IE modeling, but replaces the computationally intensive part of rigorous IE by an approximate method. The accuracy of the approximation was found to be excellent for low-frequency scattering from thin resistive targets (Bakr and Mannseth (2009)). In the current paper, we assess the computational performance of SIE when applied to model the low-frequency CSEM response of a petroleum reservoir with many grid cells, and compare to those of rigorous IE and FD.

COMPUTATIONAL ASPECTS OF TRADITIONAL MODELING APPROACHES

Figure 1 shows a 2D illustration of the different computational domains used by the different modeling approaches, with the target itself being denoted by D. It is assumed (for convenience) that all methods use orthogonal Cartesian uniform grids. Furthermore, it is assumed that the preconditioned BiCGSTAB (van der Vorst (1992)) iterative method is applied to solve the linear systems arising after discretization. (The relative computational merits of the different methods would not change much if another Krylov-subspace method had replaced BiCGSTAB.) The dominating computational effort is associated with the Krylov iteration step since this is repeated for each iteration, while the work associated with constructing the preconditioner is done only once. Therefore, we consider only the work involved in a single preconditioned BiCGSTAB sweep in our comparisons of number of floating point operations (flops), F. We will, however, also compare the number of iterations required, as well as the CPU times, for a set of numerical exam-

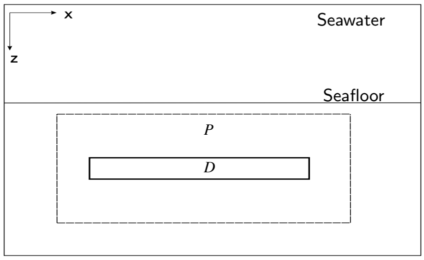

Figure 1: Sketch of the different computational domains.

ples (involving only IE and SIE).

Finite-difference approach

With FD, a 3-vector PDE for the electric field, **E**, is derived from Maxwell's equations and solved. The computational domain is much larger than the target region, D, in order to be able to apply homogeneous boundary conditions. The outermost lines on Figure 1 illustrate the boundaries of the domain for FD modeling.

Let N_x, N_y, N_z, denote the number of grid cells in the three coordinate directions, respectively. 3D FD discretization results in a linear system of equations where the coefficient matrix, A_{FD}, has dimensions $3N \times 3N$, where $N = N_x \times N_y \times N_z$. A_{FD} is, however, sparse, so that the number of nonzero entries is smaller than $21 \times 3N$ when applying the standard 7-point discretization stencil. The number of flops per iteration is $F_{FD} = 84 \times 3N$ (Spitzer and Wurmstich (1999)).

Integral-equation approach

With rigorous IE (see, e.g., Hursán and Zhdanov (2002)), the computational effort is dominated by solving an integral equation for the anomalous electric field

$$\mathbf{E}^a(\mathbf{r}') = \int_D \mathbf{G}_E(\mathbf{r}'|\mathbf{r})\sigma^a(\mathbf{r})(\mathbf{E}^b(\mathbf{r}) + \mathbf{E}^a(\mathbf{r}))dV \qquad (1)$$

for $\mathbf{r}' \in D$, see, for example, Zhdanov (2002). (For the computation of $\mathbf{G}_E$ and $\mathbf{E}^b$ in D, as well as the computation of $\mathbf{E}^b$ and $\mathbf{E}^a$ in the receivers when equation 1 has been solved, we refer to, e.g., Zhdanov (2002).) Here, $\mathbf{G}_E$ denotes the electric Green's tensor, σ denotes the electric conductivity, **E** denotes the electric field, while the superscripts b and a denote background and anomalous quantities, respectively. (Hence, the conductivity , σ, equals $\sigma^b + \sigma^a$ in D, while the conductivity outside D equals σ^b.)

Let n_x, n_y, n_z, denote the number of grid cells within D in the three coordinate directions, respectively. 3D IE discretization results in a linear system of equations where the coefficient matrix, A_{IE}, has dimensions $3n \times 3n$, where $n = n_x \times n_y \times n_z$. Note that with a similar grid resolution in D for FD and IE, the dimension of A_{IE} is significantly smaller than that of A_{FD}. A_{IE} is, however, dense, so that the number of nonzero entries is $3n \times 3n$, leading to $F_{IE} = 36n^2 + 60n$ for straightforward application of IE modeling.

As stated in the introduction, accurate modeling of the geometry of a petroleum reservoir can require use of a very large number of grid cells. As an example, a reservoir with $n = 1.5 \times 10^6$ is considered in (Endo et al. (2008)). In such cases, straightforward application of IE modeling becomes prohibitively computationally intensive. (Also, the storage requirements become huge, but we will not focus on that issue here.)

For a uniform grid with uniform electric conductivity, it is, however, possible to reduce the computational complexity of IE significantly by applying the fast Fourier transform (FFT) in the two horizontal directions (Hursan (2001)), resulting in $F_{IE-FFT} = (36 \times \log_2(n_x) \times \log_2(n_y) \times n_z + 60)n$. For very large models, however, also F_{IE-FFT} represents a huge computational work load. (In Endo et al. (2008), parallel processing on 32 CPU's was applied to run IE-FFT on the reservoir model with $n = 1.5 \times 10^6$, resulting in a run time of 34 minutes.)

COMPUTATIONAL ASPECTS OF SIE MODELING

SIE replaces the computationally intensive calculation of $\mathbf{E}^a$ in D from equation 1 by solving

$$\nabla \cdot (\sigma \nabla U) = \nabla \cdot (\sigma^a \mathbf{E}^b), \qquad (2)$$

$$\mathbf{E}^a = -\nabla U, \qquad (3)$$

for $\mathbf{E}^a$ in D. When $\mathbf{E}^a$ in D has been calculated, **E** is found in the receivers with low computational effort, using standard equations for IE modeling. We refer to (Bakr and Mannseth (2009)) for the derivation of equations 2 and 3, as well as a discussion about their range of validity. We remark, however, that these equations constitutes no more than a quantitative utilization of the well-known fact (see, e.g., Eidesmo et al. (2002); MacGregor and Sinha (2000)) that the galvanic effect dominates the inductive effect in the low-frequency CSEM response from a thin resistive target (i.e., assuming a horizontal electric dipole source and an in-line source-receiver geometry).

The dominating computational effort of SIE lies in the calculation of U from the variable-coefficient Poisson equation, 2. We solve for U in the region P (see, Figure 1). The quasi analytical (QA) approximation (Zhdanov et al. (2000)) of equation 1 is applied to compute boundary conditions on ∂P. The QA approximation is fast enough to contribute only negligibly to the computational effort. It is, however, relatively inaccurate for realistic conductivity contrasts, so that ∂P must be kept at some distance from ∂D in order to obtain a sufficiently accurate U in D. An increasingly coarser grid may be applied when moving away from ∂D to save computer resources without renouncing on the accuracy of U in D.

Let m_x, m_y, m_z, denote the number of grid cells in P in the three coordinate directions, respectively. 3D FD discretization results in a linear system of equations where the coefficient matrix, A_{SIE}, has dimensions $m \times m$, where $m = m_x \times m_y \times m_z$. A_{SIE} is sparse, so that the number of nonzero entries is smaller than $7m$ when applying the standard 7-point discretization stencil. The number of flops per iteration is $F_{SIE} = 76m$. Clearly, m is significantly smaller than $3N$, and ,hence, F_{SIE} is significantly smaller than F_{FD}. In addition, one must expect

Table 1: Summary of the operation count per one iteration for different discretization.

n_x	n_y	n_z	F_{IE-FFT}	F_{SIE}
100	60	8	54.5×10^7	0.7296×10^7
100	60	16	217.6×10^7	1.4592×10^7
200	120	8	293.1×10^7	2.9184×10^7
200	120	16	1170.1×10^7	5.8368×10^7
200	240	16	2678.2×10^7	11.6736×10^7
400	240	16	6056×10^7	23.3472×10^7

that the number of iterations for SIE will be smaller than the number of iterations for FD, since better preconditioners exist for Poisson's equation than for Maxwell's equations. This means that SIE can be expected to significantly outperform FD, computationally.
To compare SIE to IE with respect to theoretical computational performance, let $m_x = n_x + r_x$, $m_y = n_y + r_y$, $m_z = n_z + r_z$, so that $m = (n_x + r_x)(n_y + r_y)(n_z + r_z)$, and assume that $r_x = \mathcal{O}(r_y) = \mathcal{O}(r_z) = \mathcal{O}(n_z) \ll n_x = \mathcal{O}(n_y)$. These assumptions correspond well with a reservoir with large horizontal dimensions and a small vertical dimension, and with our experience with the required number of grid cells between ∂D and ∂P. They lead to $m = n_x n_y (n_z + r_z) + \mathcal{O}(r_z^2) \approx 2n$, and hence, to $F_{SIE} \approx 152n$. A comparison of F_{SIE} and F_{IE-FFT}, using this approximation, is presented in Table 1. It is seen that F_{IE-FFT} is about an order of magnitude larger than F_{SIE}.

The number of required iterations and CPU times for some simulations will be compared in the Examples-section.
We remark that some apparent advantages of SIE with respect to IE has not been considered here. IE-FFT is applicable only for uniform grids with uniform electric conductivity, while SIE can be applied for non-uniform grids and non-uniform electric conductivity. (Obviously, also FD can be applied for non-uniform grids and non-uniform electric conductivity.)

EXAMPLES

We compare computational results obtained with SIE to results obtained with rigorous IE-FFT (Hursán and Zhdanov (2002)). Results (extracted from a more extensive numerical comparison) are shown for a test model consisting of a homogeneous half space with electric conductivity $\sigma^b = 0.5$ S/m, under a 1350 m thick sea water column with electric conductivity 3.33 S/m. The reservoir has a complex geometry and contains three layers: a gas-filled layer with electric conductivity $\sigma_D = 0.001$ S/m, an oil-filled layer with electric conductivity $\sigma_D = 0.01$ S/m, and a water-filled layer with electric conductivity $\sigma_D = 2$ S/m, a vertical section and plan view of the geoelectrical model is shown in Figure 2. The dimensions and location of D are selected as $x \in [19,29]$ km, $y \in [2,8]$ km, and $z \in [2.55, 2.56]$ km. The source is a 270 m long, 1000 A, x-directed horizontal electric dipole and located at the point with horizontal coordinates $x = 24$ km and $y = 5$ km. The elevation of the source is 50 m above the sea bottom and it is operate at a frequency of 0.25 Hz.

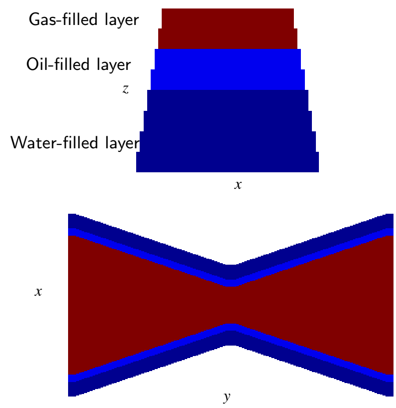

Figure 2: Top: side view of the reservoir in x and z directions, bottom: plan view of the reservoir in x and y directions.

Table 2: Number of iterations, I and CPU times in seconds for IE-FFT and SIE.

				IE-FFT		SIE	
n_x	n_y	n_z	n	I	CPU	I	CPU
100	60	8	48×10^3	150	196	7	12
100	60	16	96×10^3	160	688	22	35
200	120	8	192×10^3	173	1144	9	50
200	120	16	384×10^3	**	**	45	200

** Out of memory in MATLAB.

In our numerical study, the modeling domain D corresponds to the location of the reservoir, and this domain is discretized in n_x, n_y, and n_z, cells, corresponds to rigorous IE-FFT. With SIE we used the same number of cells within the domain D as for IE-FFT and applied coarse grids outside the body. We considered different values of n_x, n_y, and n_z, as shown in Table 2.

To demonstrate the accuracy, Figure 3 shows corresponding results for $|E_x^a|$ (top plot) and $|E_z^a|$ (bottom plot) in an array of electric receivers located at the sea floor, along the line between $(x,y) = (14,5)$ km and $(x,y) = (34,5)$ km. In both plots, the solid line corresponds to rigorous IE, the dash-dot line corresponds to SIE. Figure 3 confirms that SIE is a very good approximation to IE for low frequencies and resistive targets.

Now, we go back to the computational efficient. The results can be seen in Table 2. The first, second and third columns are the number of cells n_x, n_y, and n_z in x, y, and z directions, respectively for the different discretization. The fourth column consists of the total numbers of cells, n. The fifth and sixth

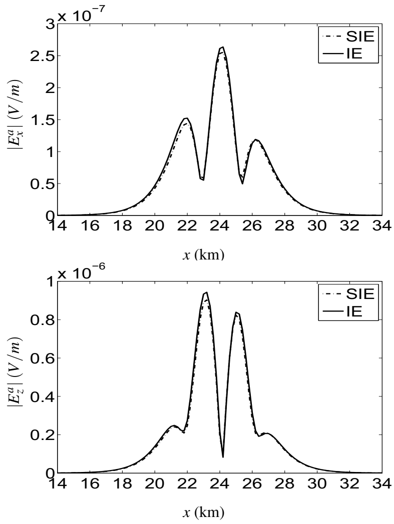

Figure 3: Top: $|E_x^a|$, bottom: $|E_z^a|$ using IE (solid line) and SIE (dash-dot line).

columns contain the required iteration number, I and the CPU times in seconds by IE. The last two columns are the required iteration number, I and the CPU times in seconds by SIE. One can see that both the iteration number and CPU times for SIE are about an order of magnitude smaller than IE. We can see also that SIE can be applied to solve problems that are too large for IE.

DISCUSSION AND CONCLUSIONS

The geometry of a petroleum reservoir is usually irregular so that accurate modeling may require a very large number of grid cells. This may severely limit the applicability of 3D CSEM solvers for this application both with respect to computational complexity and memory requirements. Recently, a novel approximate hybrid method, SIE, has been shown to produce excellent accuracy in modeling the low-frequency CSEM response from petroleum reservoirs.

In this paper, the excellent accuracy of SIE for this application has been confirmed. Our main focus was, however, to quantify the computational complexity of SIE and compare its computational performance to those of the IE and FD methods when there is a large number of grid cells in the model reservoir. It was found that both theoretical and practical computational performance of SIE is orders of magnitude better than that of IE. Furthermore, the theoretical computational performance of SIE was found to be much better than that of FD.

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EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2009 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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