

## Additional file 1: Details of the model

Notations used :

$a_t (t = 1, 2, \dots, n)$	White noise series normally distributed with mean zero and variance $\sigma^2$
$p$	Order of the non - seasonal autoregressive part of the model
$q$	Order of the non - seasonal moving average part of the model
$d$	Order of the non - seasonal differencing
$P$	Order of the seasonal autoregressive part of the model
$Q$	Order of the seasonal moving average part of the model
$D$	Order of the seasonal differencing
$s$	Seasonality or period of the model
$\phi_p(B)$	AR polynomial of B of order $p$ , $\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$
$\theta_q(B)$	MA polynomial of B of order $q$ , $\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$
$\Phi_P(B^s)$	Seasonal AR polynomial of BS of order $P$ , $\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \dots - \Phi_P B^{sP}$
$\Theta_Q(B^s)$	Seasonal MA polynomial of BS of order $Q$ , $\Theta_Q(B^s) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \dots - \Theta_Q B^{sQ}$
$\Delta$	Differencing operator $\Delta = (1 - B)^d (1 - B^s)^D$
$B$	Backward shift operator with $BY_t = Y_{t-1}$ and $Ba_t = a_{t-1}$
$Z\sigma_t^2$	Prediction variance of $Z_t$
$N\sigma_t^2$	Prediction variance of noise forecasts

A Transfer Function model describing the relationship between the dependent and predictor series has the following form:

$$Z_t = f(Y_t)$$

$$\Delta Z_t = \mu + \sum_{i=1}^k \frac{Num_i}{Den_i} \Delta_i f_i(X_{it}) + \frac{MA}{AR} a_t \quad (1)$$

Dropping the predictor series yields univariate ARIMA:

$$\Delta Z_t = \mu + \frac{MA}{AR} a_t \quad (2)$$

Main features of the model :

- Initial data transformations of the dependent and predictor series
- A constant term  $\mu$
- The unobserved i.i.d., zero mean, Gaussian error process  $a_t$  with variance  $\sigma^2$
- The moving average lag polynomial  $MA = \theta_q(B)\Theta_Q(B^s)$  and the auto-regressive lag polynomial  $AR = \phi_p(B)\Phi_P(B^s)$
- The difference/lag operators  $\Delta$  and  $\Delta_t$
- Predictors are assumed given. Their numerator and denominator lag polynomials are of the form :

$$Num_i = (\omega_{i0} - \omega_{i1}B - \dots - \omega_{iu}B^u) (1 - \Omega_{i1}B^s - \dots - \Omega_{iv}B^{vs}) B^b \text{ and}$$

$$Den_i = (1 - \delta_{i1}B - \dots - \delta_{ir}B^r) (1 - \Delta_{i1}B^s - \dots)$$

- The "noise" series

$$N_t = \Delta Z_t - \mu - \sum_{i=1}^k \frac{Num_i}{Den_i} \Delta_i X_{it} \text{ is assumed to be a mean zero, stationary ARMA process.}$$