

Unveiling teachers' reasons for choosing practical activities in mathematics teaching

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Dissertation for the degree philosophiae doctor (PhD)
at the University of Bergen

2010

Scientific environment

This dissertational work has been completed based on the contribution of two milieus.

First, the Faculty of Psychology at the University of Bergen, where I have received my research education and participated in the research programme *Western Norway Graduate Education Research* (WNGER). I have also participated in the research group *Productive Learning Practice – Reading group* (PLP) and, more recently, the UH-nett Vest research network *Pedagogy, didactics and leadership in school*.

Second, Sogn og Fjordane University College, where I have been employed during the dissertational period and have had my workplace. Sogn og Fjordane University College has financed the dissertational work.

He considered this for a while and then asked: "Do you know chess?"

"Sort of, but please don't ask me to play; I can tell you right now I'm going to lose!"

He smiled. "I wasn't suggesting a game; I just want to give you an example that you'll understand. Look, real mathematics has nothing to do with applications, nor with the calculating procedures that you learn at school. It studies abstract intellectual constructs which, at least while the mathematician is occupied with them, do not in any way touch on the physical, sensible world."

"That's all right with me, I said."

"Mathematicians", he continued, "find the same enjoyment in their studies that chess players find in chess. In fact, the psychological make-up of the true mathematician is closer to that of the poet or the musical composer, in other words of someone concerned with the creation of Beauty and the search for Harmony and Perfection. He is the opposite of the practical man, the engineer, the politician or the..." – he paused for a moment seeking something even more abhorred in his scale of values – "...indeed, the businessman."

*(Quoted from *Uncle Petros and Goldbach's Conjecture* (2000), by Apostolos Doxiadis)*

Acknowledgements

This dissertation builds on the many opinions about school mathematics and specifically on the demands for practical relevance and application of mathematics from various voices in school. Some voices are more restrictive and have doubts about the possible consequences of using practical activities in mathematics teaching. This dissertation about practical activities in mathematics teaching focuses on the teacher's voice. The teacher stands in a central position when asking how mathematics should be taught. During my own teaching of mathematics in compulsory school and in teacher education, dilemmas and challenges about practical activities in teaching have been the subject of many valuable discussions with dedicated compulsory school teachers. For these inspirational discussions, I am thankful.

The dissertational work started in January 2007, when I was granted a research scholarship by Sogn og Fjordane University College (Høgskulen i Sogn og Fjordane, HSF). Throughout my research project, my employer has been a valuable supporter both economically and through its practical adjustments of other assignments and working conditions, and has shown a keen interest, anticipation and faith in my work and project. This has motivated me into making an extra effort, and I am thankful for this support.

As a part of my PhD work, I participated in and completed the doctoral programme directed by the Faculty of psychology at the University of Bergen. I also participated in the research programme Western Norway Graduate Education Research (WNGER). Thank you all for providing me with inspiration for further work. A special thank you goes to my colleague Eirik S. Jenssen at HSF for our small talks and long walks. The encouragement you have shown and additional approaches you have introduced me to, have been vital.

I have been favoured with the best of supervision, for which I am grateful. My supervisors, Professor Kari Smith and Associate Professor Christoph Kirfel, both from the University of Bergen, have been critical readers who offered constructive yet demanding suggestions and guidance. Thank you also to Assistant Professor Ingvill Stedøy-Johansen at the Norwegian Council of Mathematics for valuable suggestions in the early stages of the project.

A special gratitude is devoted to my main supervisor Professor Kari Smith at the University of Bergen. Your faith in me and firm, but gentle way of challenging me to face struggles I have experienced, have meant much to me.

A special thank you goes to everyone who contributed to the collection of data and who, with dignity and open minds, shared your thoughts and experiences. Your contributions have helped realise this project.

Finally, my most sincere gratitude goes to my wife Eli and my two young boys, Viktor and Jakob. You have shown a resilient faith in me and have always been supportive when Dad had to work.

To my parents Inger and Ingvald, and my grandparents Else, Olav, Marry and Ingvald — this is for you.

Hafslo, December 2010
Frode Olav Haara

Summary

The use of practical activities in mathematics teaching has been advocated for some time, and reports from mathematics classrooms show that teachers include a multitude of activities in the teaching of mathematics. However, research shows that a substantial portion of these practical activities are not well thought through and do not relate to other parts of the mathematics lessons. More knowledge is required about the reasons why teachers choose to use practical activities in mathematics teaching. The overall aim of this research project was to contribute to research-based knowledge about the inclusion of practical activities in mathematics teaching. This research project focused on the reasons why teachers include practical activities and identified some of the changes the teacher might make based on professional knowledge-based information when choosing practical activities.

The theoretical foundation of the project concentrated on professional knowledge, beliefs and change in beliefs, and teacher identity. Shulman's theory on teachers' professional knowledge and Handal and Lauvås's practice theory are linked with theories on beliefs and change in beliefs, and Sfard and Prusak's theory on teacher identity. This has helped to identify a cluster of internal constraints that influence the teacher's choice. Influence also stems from a cluster of external constraints. The project had a qualitative design that applied a hermeneutical approach to the collection and analysis of the data. A strategically selected group of eight teachers, considered acknowledged teachers of mathematics, was recruited, and each participated in two of three phases of data collection. During the last collection phase, data from a larger group of acknowledged teachers were also collected. The data were produced using multiple qualitative data production instruments, such as interviews, video recordings, written logs and an open-ended questionnaire.

The research identified three categories of reasons why teachers choose practical activities: the importance of the teacher's professional knowledge, compromises that the teacher feel obliged to make, and practical dilemmas that the teacher experience. The research also identified possible steps that can be taken to narrow the gap between actual and designated practice, and to choose practical activities for professional rather than practical reasons. The discussion and conclusions generate implications about how the choices of practical activities in mathematics teaching can be made to a greater extent for professional knowledge-based reasons and about the autonomous space for teaching that the teacher should have.

List of articles

Article I:

Haara, F. O. & Smith, K. (2009). Practical activities in mathematics teaching—mathematics teachers' knowledge based reasons. *Nordic Studies in Mathematics Education*, 14, 3, 33–54.

Article II:

Haara, F. O. & Smith, K. (under review). Increasing the use of practical activities through changed practice. A case-study examination of the influence of a value based intervention on two teachers' use of practical activities in mathematics teaching. *The Montana Mathematics Enthusiast*.

Article III:

Haara, F. O. (under review). Why use a leek in mathematics teaching? Teachers' choice of using practical activities. *Nordic Studies in Mathematics Education*.

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Chapter 1: Introduction

1.1 The context of the project

This dissertation focuses on teachers' reasons for choosing to use practical activities in mathematics teaching. Today, school mathematics in Norway makes extensive use of practical activities (Klette, 2003; Kjærnsli et al., 2004). Pupils are introduced to various activities such as games and use of concretising materials, and to innovative arrangements such as “mathematical circus” or “The Mathematics Day” (Røsseland, 2004). On the one hand, use of practical activities has developed in accordance with a common acknowledgement of the utility dimension of education, and a distinctively Norwegian moral appreciation of public-minded equality (Skarpenes, 2007; Hestholm, 2010). On the other hand, the educational dimension of mathematics and the transfer of theoretical connections to new utility areas are challenged by the emergence of a changed approach to teaching.

The significance of the focus of the dissertation relates to the surrounding context. A major developmental factor is that society is becoming more complex (Bauman, 2001), and within modern society, each individual experiences individual freedom and increasing opportunities to realise dreams and projects (Ziehe, 1989; Schreiner & Sjøberg, 2005). The opportunities for personal realisation are more varied than ever, but the interests and needs of society are rarely considered when it comes to individual choices. What one finds interesting guides the choice in higher education and not tradition or encouragement from society. Social acceptance, a positive self-image and personal realisation become vital terms.

Mathematics has a fairly high status in parts of society (Niss, 1994), a status that has been persistent for quite some time. Some of the status associated with mathematical competence arises because some people do not master or see the value of mathematics and therefore choose to distance themselves through self-irony or belittling of the subject (McLeod, 1992; Volmink, 1994). Such affective reactions seem to justify some kind of acceptance of an open antipathy towards mathematics (ibid.; Lerman, 2000). Opinions expressed by relatives or other people of mathematics as incomprehensible or unnecessary (e.g. Pehkonen, 2003), together with a steady stream of disagreement with the “almighty answer book” (e.g. Eidsvåg, 2000), strengthen such reactions. History is full of stories about negative experiences with mathematics in school and how people have taken a dislike to a subject that has a strong influence on the continuous development of society.

In society, one can see that individuals experience the need for mathematics to varying extents. Society gives ambiguous signals about the level of mathematical competence each citizen should possess. In Norway, for instance, the tax form has been simplified and will probably disappear in a few years, and opportunities to use credit cards instead of cash are increasing rapidly. Niss (1994) stated that mathematics has an objective relevance for society but a subjective irrelevance for the individual, who can manage with fairly limited mathematical competence. Society contributes in many ways to this understanding of the need for mathematics because many signals indicate the anticipated decrease in the use of mathematics in daily life. Hence, it might be argued that society does not foster the opinion that it is perhaps more important than ever to be mathematically competent. To fulfil societal tasks and comply with public demands, society needs pupils to train in professions that require a high theoretical level of mathematics, such as in the fields of engineering, economics and health.

Whether pupils, upon entering upper secondary school and higher education, choose mathematics and related subjects is based on complex connections of global and local foundations from both objective and subjective perspectives (Højgaard Jensen, Niss & Wedege, 1998). It is accepted that too few individuals choose professions that require a high level of mathematics (Gardiner, 2004; KD¹, 2006b; Olsen, 2006; Rocard et al., 2007) to meet the needs of the society. This makes it difficult to recruit qualified personnel to some professions and positions. In Norway, the Government takes structural measures to increase the number of young people studying mathematics. More time has been earmarked for mathematics in compulsory school² and upper secondary school, extra credits are given for finishing mathematical courses in upper secondary school, and resources are earmarked for in-service education of teachers (KD, 2006b). In addition, the content included in mathematics teaching has been modified through curriculum development and more precise criteria for working methods (KUF³, 1996; KD, 2006a). Viewed retrospectively, such changes

¹ KD (Kunnskapsdepartementet) is the Norwegian Ministry of Education and Research.

² In Norway, compulsory school comprises elementary and lower secondary school. The pupils are aged 6–16 years.

³ KUF (Kirke-, utdannings- og forskningsdepartementet) was the name of the Norwegian Ministry of Church Affairs, Education and Research between 1991 and 2002. In 2002, the name was changed to UFD (Utdannings- og forskningsdepartementet) (Ministry of Education and Research), and in 2006 to KD (Kunnskapsdepartementet).

do not seem to have made a difference (Grønmo et al., 2004; Kjærnsli et al., 2004; Grønmo et al., 2008).

Mathematics has for a long time been rated as an objectively unassailable subject (Volmink, 1994). Rigid mathematical argumentation is difficult if one is not familiar with mathematical terms and series of argumentation. Some well-known historical examples of such use of mathematically supported argumentation in public debates are Euler's deceiving argumentation for the existence of God in a discussion with Diderot (Hogben, 1952: in Botten, 2003), Bentham's suggestion to quantify ethics through a pleasure and pain calculation (Harrison, 1999) and Piaget's attempt to interrelate psychology and abstract algebra⁴. These examples are related to the application of mathematics, and application is a feature of mathematics receiving increasing attention in school mathematics. In society, one looks for the application value of mathematics. Almost 10 years ago, Professor Edvard Befring at the University of Oslo suggested that mathematics should no longer be a mandatory subject in the Norwegian compulsory school (Kristensen, 2008)⁵. He rated the mathematics taught in compulsory school as the present day Latin, an educational subject with legitimacy problems. This initiative did not lead anywhere, but it represents an important signal of the impression of mathematics as an educational subject in school through the prevailing attention to theoretically based mathematics and as a utility subject in society.

1.1.1 Teaching of mathematics

It can be argued that throughout history, the teaching of elementary mathematics has been almost canonical around the world (Volmink, 1994). Essentially, it has been based on a philosophy of teacher explanation and task solving as the way to develop mathematical competence. On the other hand, such an approach to mathematics teaching has proved to be negative for many pupils. In a society that requires documented mathematical competence to a greater extent, this is unacceptable for both the individual and society. Teaching of mathematics today attracts great interest from a didactic research perspective (e.g. Lester, 2007), a social perspective (e.g. Niss, 1994; Haara et al., 2009) and a political perspective

⁴Piaget spent some time working on group structures in relation to statements and logical compositions of statements (Piaget, 1953; Sjøberg, 1982), and thus approached the possibility of describing intellectual processes algebraically, for instance regarding the operation of reversibility in operational thinking.

⁵ Befring first made this suggestion in an interview in the Norwegian newspaper *Aftenposten* on 20 September 1998.

(e.g. KUF, 1996; KD, 2006a; Rocard et al., 2007). The outcome of this attention is the official and focused direction of school mathematics found in national white papers and curriculums.

One can point to the school and teaching—or, frankly, to *bad teaching*—as the reason for the almost canonical negative myths and opinions about mathematics (Frank, 1990). Within such a paradigm, it is not controversial to state that many teachers are not sufficiently qualified to teach mathematics. The teacher has a huge influence on the pupil's learning (e.g. McKenzie et al., 2005; Sowder, 2007; Hattie, 2009), but it is too simplistic to blame only the teacher for the situation of school mathematics described. Other factors, such as the influence of other pupils or relatives, also affect learning. Hence, to blame the situation solely on the quality of the teaching does not sufficiently describe the situation. It is also plausible to say that society has responded to the challenges inherent in teaching school mathematics by “listening to those who cry out”. Society's response has given extra weight to political demands for the addition or change in the methods of teaching of mathematics in compulsory school. The opportunities for parents, school management and others to influence mathematics teaching have increased noticeably. This means that the teacher is to a greater extent the subject of expectations about teaching priorities.

Changing mathematics education is a longitudinal process that has been the subject of an increased focus on practical relevance as a domain of development. This accommodates the opinion of Professor Befring mentioned previously. A feature article written by representatives of the Norwegian Centre of Mathematics defended this development: “In general, measures are taken to make school mathematics more publicly relevant and interesting, through activities that involve the pupil” (my translation) (Bones, Stedøy & Wæge, 2006). An increase in the use of practical activities in mathematics teaching is one of the material changes identified through domestic research in Norway (Klette, 2003; Kjærnsli et al., 2004). On the other hand, such development within school mathematics has been criticised on several occasions. The opinion has been expressed, and results put forward in support, that the use of practical activities in mathematics education has not been considered sufficiently. Klette (2003: 73) wrote that “there has been too little systematic and conclusive reflection about activities when they are actually used in mathematics teaching” (my translation). Kjærnsli et al. (2004) noted that the curriculum includes many practically organised activities in Norwegian schools, but that too little weight is put on learning and professional criteria. Olsen and Grønmo (2006: 55–56) concluded: “...it seems relevant to

reconsider how fruitful the strong emphasis on real-life mathematics has been, and it seems to be relevant to ask whether this emphasis has become too dominant.”

Poor results in mathematics (Grønmo et al., 2004; Kjærnsli et al., 2004) have stimulated criticism of the large-scale introduction of new working methods in mathematics teaching. Concerns have been raised about whether the pupils can learn “proper mathematics” or what is labelled *pure mathematics* (see paragraph 1.2) when practical activities are prioritised (e.g. Johnsen Høines & Rangnes, 2003; Gradovski & Sigmundsson, 2006). In opposition to such concerns, Bones, Stedøy and Wæge (2006) claim that it is easier for children to learn mathematics when the teaching is focused on meaning, for instance through the use of practical activities. They explain the discouraging results in the PISA (Kjærnsli et al., 2004) and TIMSS reports (Grønmo et al., 2004) by referring to the fact that mathematics is taught in Norway the way it has always been taught (Alseth, Breiteig & Brekke, 2003). Bones, Stedøy and Wæge (2006) claim that, rather than there being too much focus on practical activities in school mathematics, the focus has not been comprehensive enough. Thus, there is disagreement in Norwegian research on mathematics teaching about the role and influence practical activities should have.

To put this ongoing discussion into perspective, it is relevant to reflect on the different opinions about how children learn mathematics. From an educational perspective, the changes in the approach to mathematics teaching in school stem from increased emphasis on research on teaching in general and in relation to mathematics. Traditional teacher-dominated teaching has been challenged by the influence of theories about learning, ethno-mathematics and realistic mathematics education. According to realistic mathematics education, mathematics originates from daily life and should be a useful tool when solving problems in real-life situations. Mathematics is seen as an integrated subject in which topics such as geometry, algebra, arithmetic, calculus and statistics are very much related. Some of the aims of “realistic mathematics education” are to develop a critical attitude, understand the underlying concepts and use mathematics in problem-solving situations (van Reeuwijk, 1992). Such challenges have contributed to the increasing influence of *practical activities* (see paragraph 1.2) and therefore *practical mathematics* (see paragraph 1.2) as part of mathematics teaching in school. This developmental process is now recognised in the national curriculum in Norway, both through the general part of the national curriculum (prolonged from its

introduction in 1993 (KUF, 1993)) and in the current curriculum for mathematics (KD, 2006a):

Mathematics in school participates in developing the mathematical competence which society and each and everyone needs. To achieve this, the pupils must be allowed to work both practically and theoretically. The training shifts among investigative, playful, creative and problem solving activities and skill development. Through application within technology and design, and in practical use, mathematics shows its usefulness as a tool subject (my translation) (ibid.: 57).

Hence, the curriculum shows that the political attitude about the priorities in mathematics teaching in Norway has moved towards acknowledging a broader spectrum of approaches and working methods, which include practical activities.

1.2 Practical activities in mathematics teaching

In the Norwegian national curriculum (LK06) (KD, 2006a), mathematics is presented in the context of the need for citizens to have mathematical competence. The arguments also touch upon the subject of mathematics as an educational subject:

Mathematics is the foundation for important parts of our cultural history and for the development of logical thinking. Mathematics therefore has an important role in the general sense of decorum by its influence on identity, ways of thinking and self-awareness (my translation) (ibid.: 57).

Working strategies within problem solving (analysis and transformation of problems, reasoning and communication, evaluation of validity and generalisation of the solution) are part of mathematics, both as an educational subject and as a utility subject. The possibility of generalisation and adaptation to new situations and problems is mathematics' biggest asset.

It is an astonishing feature with mathematics that it has a dualistic character; on the one hand it is an abstract, mental activity dominated by aesthetic and logical principles, and on the other hand a powerful problem solver in the real world (my translation) (Aschehoug & Gyldendals Store Norske Leksikon, 2006a: 220).

Niss (1994: 367) defines mathematics as a science in the following way:

Mathematics ... is a science in an epistemological sense, oriented towards developing, describing and understanding objects, phenomena, relationships, mechanisms, and so forth belonging to some domain. When this domain consists of what we usually think of as mathematical entities, mathematics acts as a pure science. In this capacity, mathematics aims at internal self-development and self-understanding, independent of the world outside... If, on the other hand, the domain under consideration lies outside of mathematics, typically with some other scientific field, mathematics serves as an applied science. In this capacity, mathematics is activated to help understand and develop aspects of various extra-mathematical areas. Needless to say, mathematics as a pure science provides crucial contributions to mathematics as an applied science....

The distinction between pure and applied mathematics is a common distinction. *Pure mathematics* (partially based on an established understanding of mathematics referred to in a mathematical encyclopaedia (Matematikkleksikon, 1997: 292)) is here defined as: *The theory about numbers and space, and generalisations of these concepts that are created by the human intellect, and can be said to have existence independent of any practical application.* Hence, a clear distinction is made between pure and applied mathematics, where *applied mathematics* is recognised as: *Application of pure mathematics within areas outside mathematics itself.* Such applications can for instance be within various professions that attend to societal needs or daily life duties and tasks where mathematics is a relevant tool to apply (Mosvold, 2005).

According to Gardiner (2004), concepts such as relevance and usefulness are important to the observed change in course from a strict priority of pure mathematics to the escalating priority of applied mathematics. This change has led to a more formal incorporation of practical activities in mathematics teaching. Evidence of this change can be found more than 20 years ago both internationally (e.g. NCTM, 1989) and in Norway (KUD⁶, 1987; KUF, 1996). In Norway, changes were made formally through the specific emphasis on practical work in the area of *means of instruction in mathematics* (KUD, 1987) and *working methods of mathematics* (KUF, 1996). Although the current national curriculum (KD, 2006a) is not as specific on this matter as its predecessors, it clearly expresses expectations of the opportunities for practical work by the pupils.

1.2.1 Practical activity

There are many types of activity. The word *active* refers to a kind of action or work. Hence, to be active or to do an activity contains all types of work, for instance thinking, reading, individual work on theoretical mathematical exercises, discussion with others or physical display. The word *practical* can be defined in two ways. In reference to *being something*, it refers to *being handy or suitable*, whereas in reference to *doing something*, it is defined as *doing something actively*. The term *practical activity* therefore brings associations to both the execution of something by possible use of some concrete materials and the visible physical activity of those who perform the activity (Bokmålsordboka, 2005).

⁶ KUD (Kirke- og undervisningsdepartementet) was the name of the Norwegian Ministry of Church Affairs, Education and Research until 1989, when the name was changed to UFD (Utdannings- og forskningsdepartementet) for the first time. The Ministry kept this name until 1991, when it was changed to KUF (Kirke-, utdannings- og forskningsdepartementet).

In a book about workshops in mathematics, Rystedt and Trygg (2005: 3) define the term *laborative activity* to include "...actions where pupils work practically with explorations and experiments, which have a specific meaning for the mathematics teaching" (my translation). This definition demands that the activity has a specific purpose related to teaching. From reports describing the reality of the classroom, it may be concluded that the activities do not always have such an effect (e.g. Klette, 2003; Kjærnsli et al., 2004). The applied activities do not always seem to have a specific meaning for teaching, at least not for the pupils (ibid.). *Practical activity* is defined here in a way that is more inclusive and pragmatic than the definition given by Rystedt and Trygg because their definition excludes some of the activity experiences reported through research. Based on the previously given interpretations of *pure* and *applied mathematics*, and the concepts *active* and *practical*, a *practical activity* here is defined as that which:

Include[s] all forms of engagement⁷ where the pupil uses physical concretes while carrying out the activity at hand (Haara & Smith, 2009).

This definition contains some limitations because it excludes elements that can be categorised by the term *practical mathematics*. Practical mathematics refers to a wider range of possible teaching priorities than a practical activity approach guided by the definition given above. It relates mathematics to real-life situations, whether through application of oral or written exercises or examples with real-life associations, use of practical activities, or actual applications of theoretical mathematics to real-life problems. Hence, practical mathematics is too wide a term to describe what is defined here as a practical activity.

1.3 Research question

Regardless of whether the priority of using practical activities in mathematics teaching is rooted socially, politically, disciplinarily or didactically, it is the teacher who is challenged to determine the priority of such approaches. Based on the described use of practical activities, it is plausible to ask in what way teachers explain their use of practical activities in mathematics teaching. The way these activities contribute to the teaching of mathematics through such means is influenced by tradition, work settings and the teachers' beliefs about the teaching and learning of mathematics (e.g. Raymond, 1997; Kerem Karağac & Threlfall, 2004).

⁷ The practical activity must allow the participants to use both language and mathematical symbols to reveal and pave the way towards achieving the theoretical knowledge goal(s).

Thematically, it is appropriate to discuss the research-based knowledge of the teachers' choice of practical activities in mathematics teaching by asking the main research question of this dissertation:

What reasons do teachers give for choosing practical activities in mathematics teaching?

The main research question is enlightened through sub-studies involving three sub-questions:

1. *How do acknowledged teachers⁸ of mathematics explain the reasons for choosing practical activities in their teaching, and to what extent is this related to their professional knowledge?*
2. *How does the introduction to a values and knowledge education (VaKE⁹)-based teaching approach supported by practical activities influence two elementary school mathematics teachers' use of practical activities in mathematics teaching?*
3. *In what way do teachers' experiences call for an expansion of a system theoretically grounded hierarchy of impact factors regarding the choice to use practical activities in mathematics teaching?*

The main research question is discussed in Chapter 5 in the context of the theoretical background, which is presented in Chapter 2. The examination of three sub-studies are presented in Chapter 4.

1.4 The structure of the dissertation

After the preliminary considerations in Chapter 1, the dissertation continues with the theoretical background for the dissertation in Chapter 2. Practical activities, and teachers' professional knowledge and beliefs are featured subjects in all the three sub-studies that form part of this research project. An overall featured topic of the articles is how teacher learning is exemplified through the development of disciplinary and didactic knowledge and change in beliefs. Chapter 2 has a dual focus. First, the chapter discusses the theoretical foundation for understanding the influences on teachers' choices of teaching activities, with an emphasis on professional knowledge, beliefs and teacher identity, and the potential to change these factors. The chapter then discusses the influences of external constraints, because the theoretical foundation is related to teacher practice and research on practical activities in mathematics teaching. Chapter 3 presents the methodological aspects of the research, with an emphasis on

⁸ An *acknowledged teacher* of mathematics is defined in this dissertation as *a teacher who is viewed as a competent mathematics teacher by the principal and who earns respect from colleagues, pupils and other relevant groups within the working environment* (see Section 3.3).

⁹ Values and knowledge education (VaKE) is a teaching approach that emphasises development of the pupils' moral and ethical values through the acquisition of new disciplinary knowledge within a constructive learning environment (Patry, Weyringer & Weinberger, 2007) (see Section 3.2).

the research paradigm, design, participants, analysis and ethics. Chapter 4 provides a review of the three sub-studies. A general discussion related to the project's main research question is given in Chapter 5, and conclusions and implications are discussed in Chapter 6. Finally, the project's limitations and ideas for future studies are discussed in Chapter 7.

Chapter 2: Theoretical background

This chapter introduces the theoretical background underpinning the dissertation. The theoretical background has been developed as part of the preparation for collecting and interpreting empirical data on which this dissertation is based. Data collection work and interpretations of data have in a dialectical manner called for studies of theory and attention to the processes related to collecting and interpreting data.

The chapter is divided into three parts. Part one introduces the three main focuses of the theoretical foundation of the dissertation. First, Shulman's (1987) theory about professional knowledge is introduced. The practice theory of Handal and Lauvås (1987) then provides a bridge to the theory on beliefs and changes in practice and beliefs. This section focuses on how teachers change (here: learning), seen from both a practice theoretical and a system theoretical perspective. The relationship between inconsistency and professional knowledge development then provides a bridge to Sfard and Prusak's theory on teacher identity (2005a). Part two relates the theoretical perspectives emphasised in part one to teachers' professional experiences with the choices involved in using practical activities in mathematics teaching. This leads to consideration of the role of the teacher's autonomous space of action in teaching of mathematics. Finally, part three is devoted to preliminary attention to issues regarding teachers' reasons for choosing practical activities which need to be added to the field.

2.1 Theoretical foundation

A *practice theoretical perspective* is referred to here as an approach to teacher learning (through a change in beliefs) where a teacher's learning processes are interpreted with reference to practice. The term practice here refers to the act of teaching. In relation to teacher learning, the term practice refers to how a teacher's knowledge and beliefs together with increasing teaching experience lead to changes that improve the quality of teaching.

2.1.1 Professional knowledge

Shulman (1987) presented a practice theoretical perspective for the professionalisation of teaching. This perspective is based on identifying the *knowledge base for teaching* (ibid.: 8). Shulman argued that at least four major sources for such a base are relevant to the teacher's understanding of teaching:

- *Scholarship in content disciplines* (here: disciplinary and didactic knowledge in mathematics and the use of practical activities specifically in mathematics teaching, and beliefs about the impact of these factors and how they can and should develop)
- *Educational materials and structures* (here: external constraints such as curriculum, textbooks, number of pupils, time pressure and teaching cultures in the working environment)
- *Formal educational scholarship* (here: formal didactic competence)
- *Wisdom of practice* (here: practical teaching experience — the wisdom to do the right thing at the right time in the act of teaching).

First of all, mathematics teachers need disciplinary knowledge in mathematics (Hill et al., 2007). Consistent with the interpretation of the curriculum, the mathematics teacher uses his/her disciplinary knowledge to meet the demands (and wishes) of teaching. A high level of disciplinary knowledge increases the teacher's disciplinary confidence. Such confidence allows the teacher to focus on connecting distinct areas of mathematics and to take approaches to the mathematical content outside the textbook. Research shows that teachers with a low level of disciplinary knowledge lead the pupils through the mathematical content mechanically without seeing themselves as capable of leaving the textbook's suggested content, examples and progression (e.g. Gudmundsdottir & Shulman, 1987; Grossmann, Wilson & Shulman, 1989). Secondly, Shulman (1987: 9) stated that "The manner in which that [subject matter] understanding is communicated conveys to students what is essential about a subject and what is peripheral." The importance of the teacher's didactic knowledge is recognised by Shulman as part of both the teacher's content discipline (here: mathematics) and the formal didactic competence. A third element in Shulman's identification of sources of a knowledge base for teaching is the teacher's beliefs about mathematics and about how it is taught and learned. "Perhaps the most enduring and powerful scholarly influences on teachers are those that enrich their images of the possible: their visions of what constitutes good education..." (Shulman, 1987: 10).

Shulman notes specifically the term *pedagogical content knowledge* as crucial for understanding the teacher's methods of teaching. This knowledge of teaching comprises a blend of disciplinary knowledge, didactic knowledge and teaching experience (Gudmundsdottir & Shulman, 1987). According to Shulman, a teacher's knowledge should be understood as a teacher's understanding of how disciplinary content should be arranged for

the student. Fennema and Franke (1992) referred to Shulman's focus on presentation of disciplinary content through transformation as critical to the teacher's choices of the content of teaching. This includes both a short-term consideration of how the disciplinary content should be presented and a longitudinal evaluation of the pupils' development of understanding, skills and beliefs. These evaluations form the foundation for the teacher's arrangement of the disciplinary content for teaching in the best way possible within prevailing constraints. Understanding the forms of representation that are useful for teaching particular topics is a crucial dimension in the teacher's professional knowledge (Hill et al., 2007).

Handal and Lauvås (1987) claim that the teacher's subjective practical theory directs the teaching, and they see the practice theory as a dynamic construction of:

- *personal experience* (an accumulated practice experience regarding teaching and learning)
- *transmitted knowledge, experience and structures* (the external influence on one's practice theory)
- *values* (professional and personal values).

According to Handal and Lauvås, the dynamic construction of the factors mentioned above forms the individual teacher's practice theory. These factors and how they develop are interwoven and cannot be separated into isolated parts. Hence, teaching practice is more than what happens in the meeting between pupils, the teacher and the disciplinary content to be taught.

Both Shulman and Handal and Lauvås see the development of teaching practice as a cyclical process. Several factors influence the foundation upon which teaching is planned and delivered. However, some of these factors relate directly to the teacher and others to the teacher's surroundings. Disciplinary and didactic knowledge are based partly on the teacher's beliefs about the subject. Other factors, such as *educational materials and structures* (Shulman, 1987), or *external constraints*, such as time, curriculum, textbooks, pupils, parents and colleagues are impact factors of the latter kind. Both Shulman and Handal and Lauvås emphasise the influence of the teacher's beliefs on the teacher's choices. Visions about teaching, which are based on such beliefs, are a necessary part of the teacher's acquisition of professional knowledge. The realisation of such a vision depends on the teacher's beliefs and potential to change those beliefs (Shaw, Davis & McCarty, 1991; Pehkonen, 2003).

2.1.2 Beliefs and changes in practice and beliefs

In the research literature, the teacher's *beliefs* are considered crucial to the development of the teacher's practice and potential to change teaching practice (e.g. Fennema & Franke, 1992; Beijaard et al., 2000). Considering the influence of one's prevailing beliefs when one interprets impressions and new knowledge, Pajares (1992) ascribes to beliefs an almost subconscious filtering effect regarding new impulses and identifies the features of beliefs. Based on Pajares's work, Beijaard et al. (2000: 262) defined beliefs through three main features:

- Beliefs are "highly individual, deeply personal, and seem to persist"
- Beliefs are "formed by past experiences"
- Beliefs are represented as "an individual's understanding of reality enough to guide thought and behavior and to influence learning".

There seems to be a dynamic interaction between knowledge and beliefs, in which beliefs are understood as *subjective knowledge* (Bishop, 2001; Philipp, 2007) or in Philipp's (2007: 259) words, "psychologically held understandings, premises, or propositions about the world that are thought to be true." Influenced by feelings, beliefs are materialised through actions and are thereby defined as values. Hence, values are a visualisation of beliefs (Bishop, 2001). The emphasis on beliefs as crucial to the development of professional knowledge and persistent changes in practice supports the interpretations given by Shulman and Handal and Lauvås, and thereby underpins the consensus of beliefs and knowledge as interwoven. The influence of beliefs on the mathematics teacher's teaching has been studied by several researchers (e.g. Fennema & Franke, 1992; Thompson, 1992b; Furinghetti & Pehkonen, 2002; Sztajn, 2003; Mosvold, 2005; Philipp, 2007). A common, characteristic feature is that the teacher's beliefs influence the potential for teacher development. The question is then, "How can teachers' beliefs change?" At the core of this process of change is the relationship between the teacher's beliefs and practice, and the teacher's learning process might be studied from both a system theoretical perspective and a practice theoretical perspective.

A *system theoretical perspective* is characterised by the concepts *system* and *model* (Eide & Eide, 1996; Nordahl, 2007). From a system theoretical perspective, the totality of the teaching situation and the teacher's interaction with the surrounding conditions provide understanding about the teacher's choices in teaching practice. This perspective emphasises that the teacher interacts with a number of social systems. The common factor within social system theory is

that the teacher is part of a system where he/she both influences the totality and is influenced by this totality (Eide & Eide, 1996). A system theoretical approach to learning can be found in the *activity theory* introduced by Lev Vygotsky. Through Vygotsky's approach, the individual (here: the teacher) is interpreted and understood in a perspective that acknowledges the individual's cultural means and society is interpreted and understood according to the individual's actions. Alexei Leont'ev brought the activity theory from an individually focused level to a collective activity system and added the explanation of "the crucial difference between an individual action and a collective activity" (Engeström, 2001: 4). In the perspective of this dissertation, this expansion focuses attention on the complex interrelationship between the teacher and the teacher's surroundings.

The third generation of activity theory, summarised by Engeström (2001) into five principles, shows how complexity is a challenge in a system theory approach. First, "a collective, artefact-mediated and object-oriented activity system, seen in its network relations to other activity systems, is taken as the prime unit of analysis" (ibid.: 6–7). Intentional individual and group actions, and automatic operations, are subordinate units of analysis and understandable only when interpreted against the background of the entire activity system. Second, an activity system is a community of multiple points of view, traditions and interests held by the participants in the system. Third, activity systems are shaped and changed over long periods of time. The fourth principle is the central position of "historically accumulating structural tensions within and between activity systems" (ibid.: 7), or contradictions, as a source of change. When the system adopts a new element (here: increased expectations about the use of practical activities), "it often leads to an aggravated secondary contradiction where some old element (ibid.: 7) (here: traditional teaching of mathematics) collides with the new one. Such contradictions generate disturbances and conflicts, but also innovative attempts to change the activity" (ibid.: 7). The fifth principle is that as the contradictions within an activity system become increasingly aggravating, some of the activity system participants begin to deviate from the norms of the system (here: how to teach mathematics). "In some cases, this escalates into collaborative envisioning and a deliberate collective change effort. An expansive transformation is accomplished when the object and motive of the activity are reconceptualised to embrace a radically wider horizon of possibilities than in the previous mode of the activity" (ibid.: 7).

Relational communication theory, developed by Gregory Bateson (Eide & Eide, 1996), is based on the assumption that communication between participants establishes and develops relationships and that these relationships determine how the communication takes place. The communication comprises the interactions, and the interactional patterns make up the structure of the system (Littlejohn, 1992: in Eide & Eide, 1996). In other words, the teacher and other groups of people relevant to the teaching constitute a system only through their communication. This means that the relationships within the system become established around the teaching structure and interaction with the teacher. This interaction and the system influence each other. In the essay “The Logical Categories of Learning and Communication”, Bateson (1972) linked *learning* to the element of *change*. Through a logical division in levels of learning and communication into levels labelled 0, 1 and 2, he suggested that the influence of personal features on learning processes should be organised hierarchically. Bateson characterised learning at Level 0 as first-order learning, learning at Level 1 as second-order learning, and so forth. From the perspective of a practising teacher, Level 0 is about receiving, understanding and responding to signals and responses from the experienced teaching, and the teacher’s learning at this level will be about developing (more or less) automatic actions (or reactions) based on received signals and responses (Glosvik, 2000). Level 1 relates to the way the teacher acts and is about changing actions (here: choices) to adapt to responses to the performed actions from other groups of people in the system constituted around the teacher. Second-order learning is thereby a revision of actions based on experiences provided by actions at Level 0, which again generate changes at Level 1, and consequently at Level 0. Level 2 is influenced by the teacher’s internal responses to experiences at Level 1 and comprises factors that control second-order learning. Hence, third-order learning is about the teacher’s perceptions and interpretations of new experiences stemming from responses and learning at Level 1, and the subsequent development of alternatives that control changes in learning processes at this level. It can, for instance, be a subconscious change in the teacher’s beliefs about how mathematics should be taught and learned, and about the teaching conditions necessary for such a change.

To summarise using a Batesonian reference, within activity theory learning *change* is rated as activity based. Activity cannot be interpreted or understood outside the experiencing context. This means that when one wants to unveil the reasons teachers give for choosing to use practical activities in their teaching, one must look into the choice as part of the teaching; the

beliefs the teachers represent; their intentions, rules and norms that might influence the choice; and the societal totality the choice is made within.

In a *practice theoretical perspective*, the teacher is ascribed to have greater influence on his/her opportunity to change than in a system theoretical perspective. Within a system theoretical perspective, change of practice is considered to implicitly generate the possibility of change in beliefs, which again might preserve the change in practice. In a practice theoretical perspective, the experiences from practice might make the teacher initiate change on the basis of professional knowledge development. The teacher's reflections about knowledge, beliefs and practice, and a will to change, consistently develop the foundation for the teacher's teaching (Handal & Lauvås, 1987; Shulman, 1987). A thorough and direct attempt to change the teacher's beliefs is given by Kolb (1984). He suggests that an experience (here: practice) generates the teacher's observation and reflection, and general conceptions are developed based on the reflection and tested in new situations. This gives a concrete experience, at a higher level. Korthagen and Wubbels (2001) developed this view further by showing that the process in experiential learning alternates between reflection and action (here: choice of using practical activities in mathematics teaching). Korthagen and Vasalos (2005) took this a step further by noting the specific focus on reflection and action attached to the teacher's core reflections. To change practice permanently, the teacher must change his/her beliefs and actions. This means that the teacher has to identify an inconsistency between prevailing beliefs and current practice, and become motivated to change his/her beliefs to erase the experienced inconsistency. The realisation of this perspective, with the subsequent teacher learning process, influences the teacher's professional knowledge development. This framework for teacher change is described by Shaw, Davis and McCarty (1991) and later by Pehkonen (2003). Shaw, Davis and McCarty (1991) suggest that the teacher must accept the challenges inherent in the inconsistency between attitude and practice, and must feel responsible for doing something about this. Hence, the teacher must have a vision of how the teaching should be (as also put forward by Shulman (1987)) and must prepare a plan to realise this vision.

In their framework, Shaw, Davis and McCarty (1991) acknowledge a system theoretical perspective and the advantage regarding teacher change which is confirmed easier to find within a well-bounded community of practice (Lave & Wenger, 1991; Wenger, 1998; Engeström, 2001). Shaw, Davis and McCarty (1991) emphasise the influence of the

surrounding cultural and working environment both in the initiating phase of teacher change and in the longitudinal developmental process, through which the change in beliefs occurs. They also acknowledge a practice theoretical perspective by placing great emphasis on the teacher's responsibility in initiating the learning (here: changing) process. This is done through the teacher's identification of an inconsistency between beliefs and practice, by commitment to the process and by requiring both a vision of how the teaching should be and how to introduce this into practice. Consistent with the writing of Shulman and Handal and Lauvås, they acknowledge the influence of both the factors attached to the teacher and the external constraints.

2.1.3 Impact of the teacher's identity

Sfard and Prusak (2005a: 15) proposed "the notion of identity [as] a perfect candidate for the role of 'the missing link' in ... the story of the complex dialectic between learning and its sociocultural context." A socio-cultural approach to learning is based on the principle that knowledge is constructed through collaboration or social and cultural activities (Dysthe, 2001). Lave and Wenger (1991) interpret the individual's participation at multiple levels as entailed by membership in a community of practice, and they define a community of practice as:

a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. A community of practice is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage (ibid.: 98),

and because it is:

a context for new insights to be transformed into knowledge (Wenger, 1998: 214).

Hence, the "motivation to learn stems from participation in culturally valued collaborative practices in which something useful is produced" (Engeström, 2001: 12). Therefore, conversation and common activities are crucial features for learning, and the participant's development is recognised by changed participation in the practice situation (Carraher, Carraher & Schliemann, 1985; Matusov, 1998; Sfard, 1998). From a school perspective, this provides a communicative foundation that allows both the pupils and the teacher to influence their own learning. For the teacher, this might be related to learning further about teaching mathematics and eventually about the change in beliefs about mathematics and teaching mathematics (e.g. Moyer, 2001; Skott, 2001).

Sfard and Prusak (2005a) say that learning may be recognised as an attempt to narrow the gap between actual and designated identity. They split the definition of identity into two subsets: “*actual identity*, consisting of stories about the actual state of affairs, and *designated identity*, composed of narratives presenting a state of affairs which, for one reason or another, is *expected* to be the case, if not now then in the future” (Sfard & Prusak, 2005b: 45). This definition is also applicable to the teacher’s identity as a teacher, in the sense that *actual teacher identity* should be recognised as the impression the teacher actually expresses as a teacher, and that *designated teacher identity* represents the impression the teacher would like to give as a teacher. According to Sfard and Prusak (2005a: 18) “[D]esignated identities give directions to one’s actions and influence one’s deeds to a great extent, sometimes in ways that escape any rationalization.” In other words, influence from external constraints might influence the teacher and the teaching. For each teacher, the influences from some constraints may have more impact than others. The critical constraints “are those core elements that, if changed, would make one feel as if one’s whole identity [has] changed” (Sfard & Prusak, 2005a: 18). Such a “perceived persistent gap between actual and designated identities, especially if it involves critical elements, is likely to generate a sense of unhappiness” (ibid.). The gap that Sfard and Prusak refer to aligns with the inconsistency between beliefs and practice put forward by Shaw, Davis and McCarty (1991). It also aligns with Shulman’s emphasis on the teacher’s professional knowledge-based reasons for teaching. Suppression of designated teacher identity and beliefs will eventually have a negative impact on the teacher, which could, for instance, be triggered through representation of elements of bad conscience (Mellin-Olsen, 1996). If the teacher chooses to use practical activities solely on impulse or for reasons relating to collectivism, the quality of the teaching is anchored poorly in the teacher’s designated teacher identity.

Teacher learning to narrow the gap between actual and designated teacher identity can be interpreted in two ways. First, it might imply an increase in disciplinary and/or didactic knowledge as part of professional knowledge. Second, it can be interpreted to take place through a change in beliefs, based on a change in practice (a system theoretical perspective) and/or through a change in practice based on a change in beliefs (a practice theoretical perspective). Independent of this discussion, the view in this dissertation agrees with the views of Pehkonen (2003) and Shaw, Davis and McCarty (1991) that the impact must stem from the teacher’s experience of an inconsistency between beliefs and practice. Shulman (1987) and Handal and Lauvås (1987) see the development of teaching practice as a cyclic

process based on the impression that all impact factors are dynamic. This is a feature that is shared by system theoretical approaches. If one is supposed to change practice, both beliefs and actions must change. Such an impression about change of beliefs is also given by Handal and Lauvås, in a way that on this occasion bridges the relationship between the system and practice theoretical approach (1987: 12):

We experience our own practical efforts very much in the light of structures, concepts and theories transmitted to us in such a way that this may even lead us to change our values and beliefs to some extent.

2.1.4 Summary

This section has discussed the interrelationship between the teacher's professional knowledge, beliefs and identity, and how they may influence the choices made by the teacher. This is shown schematically in Figure 1.

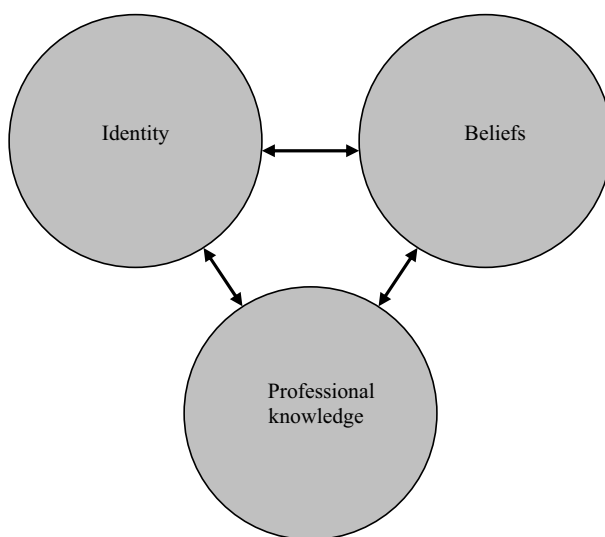


Figure 1: Interrelationships between impact factors attached to the teacher

The teacher's professional knowledge, beliefs and identity influence the choices that the teacher makes about teaching. They represent internal constraints, which exert the initial influence on the teacher's intention to choose to use a practical activity or not, before any external constraints are allowed to influence. They are interrelated in such a way that the influence of one such impact factor is in a dependent relationship with the other two impact factors. The influence of the teacher's professional knowledge in mathematics teaching

implies reliance on beliefs about mathematics and teaching of mathematics and teacher identity. The same relationship applies for all three perspectives. If the teacher chooses to use a practical activity, the teacher's identity, beliefs and professional knowledge all influence the realisation of the choice.

This section has also described the phenomenon of change in the context of the three internal constraints shown in Figure 1. A change in one implies the opportunity for changing the other two. A change in beliefs may lead to the development of professional knowledge, which may increase or decrease the gap between actual and designated teaching. The next step is to relate this theoretical dynamics to the teacher's professional understanding of choices when choosing to use practical activities. The next section discusses the autonomous space in which the teacher experiences the options of making a choice. This discussion includes attention to external constraints that may suppress a preferred choice to the benefit of another teaching approach.

2.2 Autonomous space for choosing practical activities in mathematics teaching

Traditionally, the mathematics teacher has had many opportunities to place a personal structure on teaching (Mellin-Olsen, 1991). However, although teachers retain a considerable degree of autonomy (KD, 2008), external constraints strongly influence teaching. The teacher is officially expected to collaborate with other teachers (KD, 2003; KD, 2008). In addition, the official expectations about the priorities of working methods have become more explicit in the Norwegian national curriculum (KUD, 1987; KUF, 1996). However, in the current national curriculum, the teacher's autonomous space for teaching and classroom activities may be interpreted to have increased (KD, 2006a), when compared to its predecessors. The explicit expectations about the use of practical approaches to the mathematical content in the national curriculum have faded somewhat. On the other hand, theory of learning that emphasises collaboration (Lave & Wenger, 1991; Wenger, 1998) and arguments for viability of school mathematics outside school (Niss, 1994) have sustained expectations of practical application in mathematics teaching. There is pressure from groups that perceive they have legitimacy in decision making about mathematics teaching in school, and therefore should be listened to, when deciding the teaching priorities (Gardiner, 2004). Finally, there is a growing body of research on mathematics teaching (e.g. Grouws, 1992; Lester, 2007) and, specifically, the use of practical activities in mathematics teaching.

2.2.1 Research on the use of practical activities in mathematics teaching

The variety of opinions about the use of practical activities described in Chapter 1 shows a constantly growing body of practical activity material that is accompanied by arguments for the use of various practical activities in mathematics teaching (e.g. McNeil et al., 2009). However, a review of the research gives a more balanced impression of the teaching purposes that practical activities seem to fulfil (Meira, 1995; Moyer, 2001) and the potential pitfalls (Klette, 2003; Kjærnsli et al., 2004; McNeil et al., 2009). Research on the use of practical activities in mathematics teaching focuses on either a pupil-outcome perspective or the teacher's role. Each focus seems to have a period of predominance in the literature. Most research with a mainly pupil-outcome perspective stems from the 1990s, when there was an emphasis on practical activities in the curriculum. Research on the role of the teacher became more prevalent after 2000. Around this time, the perspective changed from what is being taught and what the pupils learn, to how the content is taught and the influence of the teacher.

In an article based partly on a study of the use of concrete materials in elementary mathematics, Thompson (1992a: 123) concluded that although "...the use of concrete materials in elementary mathematics instruction has been widely advocated ... the research literature on effectiveness of instruction involving uses of concrete materials is equivocal at best." In support, he refers to a wide range of references that touch upon using concrete materials in mathematics teaching. Thompson attributes this indistinctness to the fact that the referred studies had different goals, but in a larger context he also conveys the impression of hesitance about the use of practical activities. Thompson refers to Resnick and Omanson (1987) who "observed that students' active participation in a prescribed activity may have little effect if they think that they are following a prescription" (Thompson, 1992a: 124). The doubts expressed by Resnick and Omanson (1987) are also mentioned by Meira (1995), who elaborated on the observations of Resnick and Omanson about the socio-cultural perspective of the importance of the students' participation and understanding of practical activities (see also Moyer, 2001). Meira (1995: 108) noted that:

...while physical devices are generally used in instruction to provide a meaningful context for mathematics, it is the students' activity with mathematical representations that allow them to understand the relationships embodied (by design) in the physical object ... It is certain that actions on the objective (material) device contributed to initiate the whole thing, but it is the students' discursive activity based on mathematical representations that [make] them aware of the object itself.

In other words, pupils must be given the opportunity to make connections between the practical activity and the mathematical relationships that the teacher's choice of practical activity illustrates or reveals (Moyer, 2001). Pupils should not feel that the teacher must maintain their attention and use discursive strategies to reveal the relationship that the practical activity should illustrate. On the other hand, the teacher cannot expect the pupils to learn or be able to grasp theoretical connections solely through working on a practical activity. The effectiveness of an activity seems to depend on the pupils' awareness of the purpose of the activity (Swan et al., 2000; Hiebert & Wearne, 1992: in Moyer, 2001). In addition, time must be devoted to supporting the pupils' ideas and suggestions (e.g. Abrahams & Millar, 2008).

Meira (1995: 102) began the article titled "Mediation by tools in the mathematics classroom" with the following quote:

Classroom use of physical devices (or more generally "concrete materials") is accepted by many mathematics teachers as good practice in mathematics instruction.

He does not underpin this statement with any references, but support can be found in the constantly growing body of literature mentioned, which refers to the prosperity that follows a practical activity approach (e.g. Bell, 1993; Hunter et al., 1993; Houssart, 1997; Bones, Stedøy & Wæge, 2006). Abrahams and Millar (2008) make a similar statement without providing support from any references in the field of school science. Moyer (2001) and McNeil et al. (2009), however, are more explicit when they describe the current position of practical activity approaches in mathematics teaching, as shown earlier in this section. Moyer (2001) emphasised the social-constructivism tradition, stemming from Vygotsky, as important for research on the relationships between the uses of concrete materials and learning in communities of practice (Carraher, Carraher & Schliemann, 1985; Wenger, 1998).

In summary, research on the use of practical activities in mathematics teaching points to two crucial elements. First, the pupils must perceive the practical activity as relevant. Second, there must be a balance between the pupils' striving to understand the connection between practical and pure mathematics and the teacher's help in transferring between practice and theory.

Norwegian research shows that practical activities do not make traditional teaching vanish (Alseth, Breiteig & Brekke, 2003), and that the use of practical activities is not always

sufficiently planned or pursued (Klette, 2003; Kjærnsli et al., 2004). According to Moyer (2001) and McNeil et al. (2009), it is a challenge for the teacher to transform mathematical ideas into representations. Meeting this challenge requires both disciplinary and didactic knowledge of mathematics. With reference to Grant, Peterson and Shojgreen-Downer (1996), Moyer (2001: 178) stated that:

Some teachers use [manipulatives] in an effort to reform their teaching of mathematics without reflecting on how the use of representations may change mathematics instruction.

McNeil et al. (2009: 182) concluded that:

...there may be both costs and benefits to providing students with perceptually rich, realistic objects to help them solve mathematics problems.

Hence, there seems to be a discrepancy in the literature between the commonly accepted opinion about the use of practical activities in mathematics teaching and the impressions from research about the pupils' outcome of this element in teaching and teachers' use of practical activities. The teacher has to attend to both these external constraints described independent of the fact that he/she interprets them as encouraging or restraining.

2.2.2 Influence of external constraints

Although teachers experience the effects of various external constraints on their teaching priorities, research supports the theoretically based emphasis given to internal constraints (see Section 2.1) (e.g. Raymond, 1997; Moyer, 2001). Figure 2 shows that the choice by the teacher is influenced by both internal and external constraints. External constraints can be experienced or interpreted as both encouraging and restraining, depending on the prevailing intention held by the teacher when faced with the impact of an external constraint. If the teacher wants to use a practical activity, he/she will find support in external constraints such as curriculum or school policy expectations of practical approaches to mathematics teaching. On the other hand, he/she will perceive the same constraints to restrain the autonomous teaching space if he/she does not see the appropriateness of the teaching approach. Some external constraints might be interpreted as restraining independent of all possible intentions about the use of practical activities. Practical activities are time consuming for the pupils, demand access to equipment or concrete materials, and can require extra work by the teacher in the planning and follow-up phases (Kerem Karağac & Threlfall, 2004).

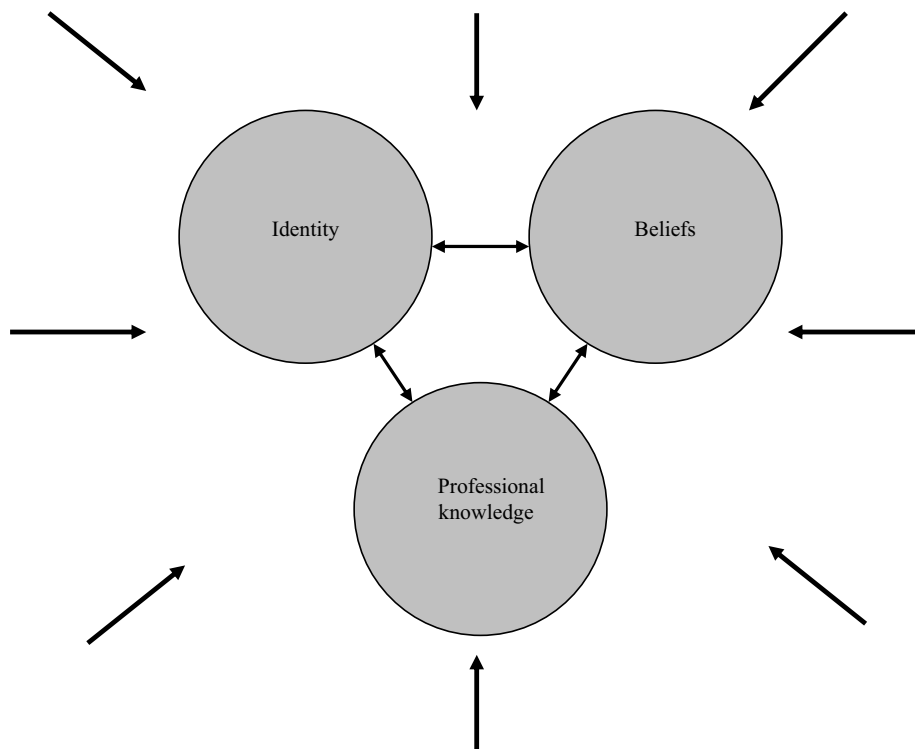


Figure 2: Influence on choice by both internal and external constraints

As noted earlier, the teacher's professional knowledge, beliefs and identity are three main areas of influence on the teacher's choice whether to use practical activities. A fourth main area is the various external constraints that might encourage or restrain the choice, as shown by the arrows pointing towards the internal constraints in Figure 2. The influence from an external constraining impact factor will vary between teachers and situations. The teacher's experience of time pressure, for instance, is not a constant constraint but is rather a dynamic constraint determined by how severe the teacher finds it to fulfil the content demands in the curriculum and the pupils' understanding of the mathematical content.

Because external constraints can differ, three sub-clusters of external constraints can be differentiated in relation to teachers' professional knowledge. The impact factors comprising the main part of professional knowledge (disciplinary knowledge, didactic knowledge and beliefs) (Shulman, 1987) can be referred to as *primary impact factors* because they are impact factors introduced by the teacher. The identification of a dynamic relationship between

various external constraints and teachers' professional knowledge is illustrated in Figure 3. In this figure, the strength of the relationship increases from left to right.

→ Relationship between external and internal constraints →		
Cluster I	Cluster II	Cluster III
Impact factors that the teacher is in a position to influence to a limited extent.	Impact factors that the teacher is in a position to respond to by making changes to the teaching, but still keeping within the range of his/her professional knowledge.	Impact factors that interact directly with or challenge the teacher's internal constraints (primary impact factors).

Figure 3: Various influences of external constraints

In Figure 3, the dynamic differentiation in the three sub-clusters is guided by the teacher's opportunity to respond. Cluster I comprises impact factors such as common thoughts about how mathematics should be taught, the national curriculum and structure of schooling and evaluation, or the physical environment. Cluster II comprises the impact factors of a teaching structural kind, such as the number of pupils, access to equipment that makes it possible to use practical activities, work pressure, or available textbooks. Cluster III comprises factors such as recommendations of in-service education in mathematics, suggestions and inspirations from colleagues, comments on the teaching by pupils and parents, or the teacher's experiences with time pressure. Based on the information contained in Figure 3, external constraints, which are identified through their possible influence on the teacher's choice to use practical activities in mathematics teaching, are appropriate to cluster in one category, *secondary impact factors*. These impact factors are directed towards the teacher either as guidelines or a framework for teaching, or as responses to the delivered teaching.

2.3 Issues relating to the need for empirically based studies

The theoretical background provides an approach to understanding the reasons teachers give for choosing practical activities in mathematics teaching. Primary and secondary impact factors influence the choice the teacher makes. Any change in primary impact factors occurs through a long-term and complex process needed for the teacher to choose to include practical

activities for professionally based reasons. Section 2.2 illustrates the complexity of the choice process (see Figures 2 and 3). This has been done without consideration of the professional attributes or features recognised by teachers who choose to use practical activities in mathematics teaching on a regular basis. Such a consideration is unnecessary because it is treated implicitly through the attention given to didactic knowledge and beliefs in Section 2.1. In the classroom, the teacher acts within an autonomous space when making choices about teaching. This space allows the teacher to influence and guide the teaching in a direction consistent with the teacher's identity, beliefs and disciplinary and didactic knowledge. Therefore, I have avoided categorising mathematics teachers according to preferred teaching style or beliefs about learning of mathematics (see Chapter 3). This dissertation examines a small sample of mathematics teachers in compulsory school and is thus inclusive without attempting to identify groups of teachers who use practical activities more than others. The influence of the teacher's professional knowledge is discussed thoroughly in Chapter 6.

The theoretical background of the dissertation shows that the choices emphasised in this project are the result of a complex process. The process can vary with the levels of influence between teachers and between situations, and can be affected by primary and secondary impact factors. However, through the work with the theoretical background I have identified three issues regarding the reasons teachers give for choosing practical activities in mathematics teaching that have not been studied thoroughly until now, and about which new knowledge is needed:

1. First, little is known about the importance of professional knowledge in the teachers' reasons for choosing practical activities. Empirically teacher-based information on this issue might increase our understanding of why and when practical activities are chosen or not chosen, and the disciplinary and didactic knowledge-based reasons for these choices. Such knowledge might also contribute to confirming or refuting the influence of external constraints.
2. Second, the theoretical background leads to identification of a totality that involves practice, beliefs and reflection, and how these relate to changing teacher practice and the potential for changing a teacher's beliefs. A longitudinal follow-up of some teachers who attempt to change their use of practical activities will provide case-based examples of the role of totality. This will supplement what is currently known about the relationships between the teacher's identity, beliefs, use of practical activities and reasons for using practical activities (here: practice). It will add practical experiences

as teachers attempt to change their practice and thus challenge prevailing beliefs to narrow the gap between actual and designated identity. The experience-grounded contribution provides data relevant to the initiation of a systematic attempt to increase the use of practical activities and to reveal factors that might influence this process. This has received little attention in the current research literature (Shaw, Davis & McCarty, 1991; Moyer, 2001; Pehkonen, 2003).

3. Third, the way in which we interpret the influences on the teacher's choice from a system theoretical and/or practice theoretical approach should be elaborated in more detail. Section 2.1 discusses the understanding from a practice theoretical point of view about the influence of primary and secondary impact factors. To discuss the system theoretical point of view, Section 2.1 suggests Bateson's (1972) theory on the logical categories of learning and communication as a possible interpretation of how impact factors influence each other and the teacher's choice. The possibility of a hierarchical categorising of clusters of impact factors (here: through Bateson's hierarchical interpretation) leads to discussion of the different strengths of some impact factors and their relationship with the process of teacher change. This aspect of teacher change (here: teacher learning) is not addressed explicitly in the current research literature. Only limited attention is given to the hierarchical classification of factors that may influence a teacher's choices (Veal & MaKinster, 1999). Such an empirically data-based categorisation will provide a better overview of the influence of impact factors.

Three such empirically based studies are part of this dissertation (see Chapters 3 and 4); each study focuses on one of the project's three sub-questions (see Section 1.3).

Chapter 3: Methodology

This chapter first discusses the choice of research approach for this project. The second section of this chapter presents the design of the project and the data collection instruments and process. The third section discusses the research participants and the sampling methods. This is followed by the data analysis section, and finally some ethical considerations about the research project are discussed.

3.1 Research paradigm

3.1.1 *Hermeneutics*

Based on historical tradition about the search for understanding through the interpretation of texts and the prevailing consensus, one tries to systematise the use of non-systematic conditions within science. In such an attempt, the objective is put aside as an impossible condition, and the subjective interpretation is treated as an obvious part of science. What is studied, and the meaning constituted through the studies, has a subjective validity that is first understood through the personal horizon of understanding and then becomes part of the horizon. The horizon consists of conscious and subconscious beliefs that one holds (Føllesdal, Walløe & Elster, 1990). Gadamer (1998) speaks of these beliefs as *prejudices*. They include general experiences, cultural and national heritage (for instance, language and tradition-influenced affiliations) and more personal qualities (for instance, interests and personal state of mind). These conditions are part of all attempts to understand. The prejudices do not always coincide with what one tries to understand, and an adjustment of prejudices, and thereby a changed horizon of understanding, is regarded as understanding (Føllesdal, Walløe & Elster, 1990). According to a hermeneutical interpretation of certainty, understanding is acquired through the hermeneutical circle. With a foundation in a preconception¹⁰, a new phenomenon is understood based on one's horizon of understanding (totality), and thus, through prevailing prejudices (parts). Hence, the phenomenon belongs to a totality and must be understood in relation to this totality. The parts are understood based on the totality, and the totality is understood based on the parts (Føllesdal, Walløe & Elster, 1990). New insight is achieved through studies that changes the horizon of understanding, and thereby change the preconception. The new preconception (understanding of totality) offers possibilities for new understanding of the parts, and this understanding continues to reshape the horizon of

¹⁰ From a hermeneutical point of view, every understanding starts with a preconception of the phenomenon at hand. This is a belief or a (vague) totality-related understanding one has of the current phenomenon.

understanding. This process continues until no new insights emerge, and one assumes that one has developed the correct interpretation of the phenomenon. Hence, the researcher's experience, history and understanding of the area of research influence the understanding of the focused research area and priorities during the development of a project. This means that facts, established consensus and personal prejudices influence choices in such a process (Grønmo, 2004). Thus, from a hermeneutical perspective, the researcher's horizon of understanding and thereby the researcher's preconception and prejudices influence how the project develops and the interpretations made during the developmental process¹¹.

The understanding and influence of the horizon of understanding in hermeneutical research have been discussed widely. Traditionally, one believed within hermeneutics and phenomenology that it was possible to study both one's own and others' horizons of understanding. If so, that would mean that it would be possible to understand another's horizon with one's own horizon as the starting point and together agree to adjust the horizons to a common understanding. Neo-hermeneutists such as Heidegger and Gadamer disagree with this (Føllesdal, Walløe & Elster, 1990). One cannot escape one's horizon of understanding as a totality. Even exploration and understanding of one's horizon of understanding has to be done with one's horizon of understanding as the starting point. At best, a part of one's horizon may be used to understand another part of one's horizon, and thus one's horizon cannot be understood as a totality, according to Gadamer (*ibid.*). Critical hermeneutists represent a further discussion of the understanding of one's horizon of understanding. One may be conscious about several aspects that influence one's horizon, but the official ideology to which one belongs and is shaped through is difficult to become conscious of (Føllesdal, Walløe & Elster, 1990). To understand one's own horizon of understanding is really about understanding one's own prejudices liberated from distortions by unconscious collective prejudices. This perspective is the case for both the researcher and the participants. The researcher must emphasise the need to be aware of prejudices that may influence data collection and data analysis. With this awareness of influential prejudices, he/she must then constantly evaluate the information given by the participants. The participants are not necessarily conscious of whether they describe the actual phenomenon at

¹¹ This is also to a large extent the case within mathematic didactics. For example, Alseth and Kobberstad (1997: 20) encourage us "to realise that much of mathematics didactic research will contain strong elements of hermeneutics" [my translation].

hand or a designated interpretation of the phenomenon, or a mix of these (Kvale, 2006; Silverman, 2006).

3.1.2 Choice of a hermeneutics-based qualitative approach

The study described in this dissertation takes a hermeneutics-based qualitative approach because the focus is on the teacher's own reasons for choosing practical activities in mathematics teaching. This was decided to allow for the participating teachers to volunteer their experiences of how various impact factors influence the teacher's choice of practical activities. Qualitative research is applicable if the objective of the study is to seek more understanding of a situation. As a researcher, one seeks totality and to see the studied situation as completely as possible. Qualitative research is favoured because of its flexibility and access to in-depth information from a few participants, but on the altar of totality we must sacrifice the comprehensive possibilities of generalisation, which are ascribed from quantitative studies. The element of transfer value remains though, depending on the validity and reliability of the project's data and analysis. Denzin and Lincoln (2000: 3) made the following attempt to define qualitative research.

Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or to interpret, phenomena in terms of the meanings people bring to them.

This definition shows that it is not easy to define qualitative research. On the other hand, Denzin and Lincoln (2000) specify that qualitative research implies an interpretative approach that replaces a strict demand for objectivity with an element of subjective reflection. It also leads to a variety of attempts to understand the totality of a situation or phenomenon. This is the case for both the collection of data without formal, structured instruments and the interpretative analysis of the participants' interpretations of situations and phenomena. Hence, in qualitative research, one acknowledges the perspectives of the participants and the variety of perspectives represented. Importantly, the preconceptions, prejudices and reflections of the researcher and the participants' reflections are part of the data. Finally, a wide variety of theoretical and methodological approaches is available (Flick, 2006).

The opportunity for generalisation is limited within qualitative research. Because of the interpretative approach, it is important to recognise that the results of qualitative research cannot be subjected to quality criteria in the same way as results of quantitative research can. When arguing that the results of a qualitative study may be generalised to similar situations, one must consider the uniqueness of the participants and the difficulties of conducting a repeated study (Grønmo, 2004). The validity of a generalisation depends on the relevance of the situations being compared, for instance through a strategically selected group of participants (Silverman, 2006) (see Section 3.3). *Analytic generalisation* includes assessing to what extent research findings can be applied as guidance for what might occur in other situations. The generalisation is based on the analysis of similarities and differences (Kvale, 2006). Such a generalisation will probably also include influence by a more *naturalistic generalisation* based on prejudices and personal experiences, which open the possibility of providing expectations rather than formal predictions (Kvale, 2006). However, this does not necessarily lead to a superficial treatment of validity and reliability issues in qualitative research projects. The reliability of a qualitative research project is related to several parts of the research process and requires the development of an unambiguous design and clarity about the data collection method(s) (ibid.). This implies that the project must document thoroughly how and why participants in the project were recruited (Silverman, 2006), and how the data were collected. The validity of the project must be considered constantly. As a researcher, one should always be suspicious of the data one has collected and maintain a continuous verification process throughout the project (Kvale, 2006).

3.2 Research design

The research project was divided into four phases: three phases of qualitative data collection and analysis, and a concluding analysis phase. To search for understanding of the examined phenomenon, a variety of methods were applied.

3.2.1 Data collection process – phase one

The aim of the first phase was to show how mathematics teachers explain their reasons for choosing practical activities in their teaching, and to what extent their reasons relate to their professional knowledge. Eight teachers were invited to participate in semi-structured interviews to allow them to individually report and explain their experiences, feelings and beliefs. The dynamic dimension of the interview, in which it was emphasised that the teacher and his/her experiences was the starting point for the interview, provided a confident

framework (Fossåskaret, 1997; Kvale, 2006). The interviews started with open questions about the teacher's experience and education, and how the teacher saw himself/herself as a teacher of mathematics. These themes were followed by questions about mathematics as a subject. The third section comprised questions and invitations to describe situations relating to the teacher's mathematics teaching. Some of the questions in this section focused specifically on practical activities in mathematics teaching. At the end, the teacher was challenged to envision how the teaching of mathematics should be in the future. The level of detail was shifted to produce data based on both the teacher's overall impressions and one possible element to include in the teaching. The fixed questions (see Appendix I) ensured that I, as the researcher, was able to maintain a thematic relation (*ibid.*) to the teacher's mathematics teaching and use of practical activities. This also allowed each interviewee "to propose his or her own insights into certain occurrences" (Yin, 2003: 90), propositions that helped create the basis for further inquiries.

3.2.2 Data collection process – phase two

In the second phase, two of the teachers interviewed in the first phase were recruited to participate in an 18-month comparative case study. This case study examined the influence of a value-based intervention on the two teachers' use of practical activities. Because the case study focused on a particular example of what influences the teacher's choice to use a practical activity, a "two-case" comparative case study (Yin, 2003; Flick, 2006) based on qualitative data was applied. This approach was chosen because of its appropriateness when investigating "a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2003: 13). In addition, "case studies of teachers can be used intentionally to prompt teachers to reflect upon and examine their own beliefs and practices" (Thompson, 1992b: 143). The data collection instruments were multiple; semi-structured and structured interviews, video-recorded observations of teaching together with the teachers' own reactions and impressions about the content of the recorded lessons, written logs and a questionnaire based on open-ended questions. These instruments are consistent with Yin's statement (2003: 14) that "the case study relies on multiple sources of evidence, with data needing to converge in a triangulating fashion." The importance of multiple sources of evidence offered by a case-study approach has also been emphasised by research reviews on changing mathematics teachers' beliefs about mathematics and mathematics teaching (Thompson, 1992b; Wilson & Cooney, 2003). The use of multiple sources of evidence was chosen to validate the quest for converging lines

of inquiry (Yin, 2003). In a triangulating fashion, multiple methods were combined to produce an accurate, comprehensive and objective representation of the influence of the intervention (Silverman, 2006). This made it possible to collect and compare data while searching for a convergence of data within a longitudinal perspective (Yin, 2003).

In the intervention, the two teachers participated in a 20-hour in-service values and knowledge education (VaKE) course, which I directed. VaKE is a teaching approach that emphasises development of the pupils' moral and ethical values through the acquisition of new disciplinary knowledge within a constructive learning environment (Patry, Weyringer & Weinberger, 2007). VaKE is based on constructive theory of learning with a foothold in both socio-cultural learning theory and radical constructivism, and is influenced by Kohlberg's theory on moral development through social interaction (Kohlberg, 1976). A teacher who wants to follow the VaKE paradigm teaches by introducing a moral dilemma. This implies that the pupils have to choose between two possible decisions. Two groups of pupils are then formed based on the pupils' initial decisions. This is followed by a moral viability check through discussion, first within each group and then between the two groups. The need for new disciplinary knowledge to illuminate better the different aspects of the topic and to provide more coherent arguments through collecting new knowledge is revealed. Rounds of discussion and content viability checks on the arguments are then possible, until both groups are ready to present their conclusions as the final moral and content viability checks¹². The teacher and the class close the sequence by capitalising on the whole process. Accordingly, the teaching is aimed at developing pupils' critical thinking, basic values and ethical principles. The in-service course focused on applying VaKE when teaching mathematics. The course comprised two sessions each of five hours in length. The sessions focused on VaKE, the basis of VaKE (constructivism, value education, moral dilemmas in teaching), and professional learning for teachers. Between sessions, the participants prepared suggestions for themes and dilemmas that could be included in mathematics lessons based on the VaKE method and how practical activities could be included in such mathematics lessons. The first session comprised lectures that presented the course literature and emphasised practical examples to illustrate how teaching of mathematics through a moral dilemma can be supported by a practical activity. The second session focused on changing practice using themes and practical activities suggested by the participating teachers.

¹² See Patry, Weyringer and Weinberger (2007) for a detailed review of each step of the VaKE methodology.

The data collection period started when the two participating teachers were interviewed about six months before the intervention, during the first phase of the data collection process. The interpretation of the interviews (see Section 3.4) contributed to the planning of the intervention. This interpretation offered impressions of how beliefs about mathematics and teaching in general, and more specifically about practical activities in mathematics teaching, formed part of the participants' visions of teaching. Each participant was observed and filmed during three mathematics lessons. The observations took place within a two-week period starting about one month after the intervention. The first observed lesson was typical of the kind of mathematics teaching that each of the participants, in their own opinion, usually practised. The following two were based on the introduction of new mathematical content in a VaKE-based environment, which was supported by a practical activity¹³. The lessons were video-recorded. Immediately after each lesson, the teacher and I watched the video recording together. During these sessions the participating teacher was free to comment on what he/she saw (Jacobs & Morita, 2002). This gave me access to each participant's reflections on and observations of the recent teaching experience, and tensions between observations and teacher comments. Comments and evolving discussions were recorded and transcribed.

The participants also wrote personal logs, starting on the day they received the in-service course information and reading list. The logs cover 12 months of personal impressions about mathematics teaching, the in-service course and experiences of both the observed and independently conducted VaKE lessons. Exactly 12 months after the intervention started, the participants completed an open-ended questionnaire on their beliefs about the factors that influence their use of practical activities in mathematics teaching (Appendix II). The questionnaire was validated independently by three researchers and three mathematics teachers in compulsory school who did not participate in this research project in any other way. The questions did not focus on VaKE but were developed based on interpretations stemming from the analysis of the pre-intervention interviews, observations and video sessions. The initial questions examined the responding teacher's teaching of mathematics and personal definition of what characterises a practical activity. This was followed by

¹³ Because of the emphasis on moral and ethical values, the in-service course in VaKE to some extent includes an educational aspect, both for the teachers participating in the course and the pupils taught later using the VaKE methodology. This is considered among other impact factors mentioned in the discussion of the findings in the article written from this study (see Section 4.2). Further attention to the educational impact of VaKE has been omitted from this dissertation because VaKE was not involved in the other parts of the project.

questions about the influence of primary and secondary impact factors on the teacher's planning and actual teaching, and questions about the influence of secondary impact factors on primary impact factors. The last part of the questionnaire contained questions about demographic facts such as the teacher's age, formal education and teaching experience.

The participants were interviewed once more at the end of the project, about one month after completing the questionnaire. The interview focused on the analysis of the logs, questionnaires and the interpretations of the pre-questionnaire analysis (see Section 3.4). The interviews were structured, and the interview guide (Appendix III) was divided into three main parts:

- The teacher's beliefs about mathematics and practical activities in mathematics
- The teacher's response to the value-based intervention
- The influence of the intervention on the teacher's teaching of mathematics.

The structured interview form was chosen to allow the interviews to corroborate or invalidate interpretations established through the preceding data analysis (Yin, 2003; Silverman, 2006). During these interviews, the interviewee answered the questions without any encouraging or restraining influence from me, and the answers were not followed up by additional questions or comments (ibid.). I felt it was important not to influence the answers by asking leading questions or expressing any reactions to the answers given.

3.2.3 Data collection process – phase three

In the third phase, 25 teachers anonymously answered the same open-ended questionnaire as the two teachers had answered in the second phase of the project. An open-ended questionnaire allowed for a broader, but not as in-depth, collection of qualitative data (Grønmo, 2004) about what mathematics teachers perceive as impact factors influencing the choice to use practical activities. Open-ended questions prevented the potential for leading answer alternatives in the questionnaire and preserved the interpretational possibilities for registration and consideration of nuances in the answers from the responding teachers (ibid.). The recruitment of teachers started with a random selection of three Norwegian counties. Next, schools within the three counties were selected randomly. The selection process was monitored by an independent observer. For each county, a letter in the alphabet was selected randomly, and the principals of the first 10 compulsory schools starting with that letter, sorted alphabetically, were contacted by me. Each principal was informed about the study and asked to participate by recruiting from the school one teacher recognised by the principal as an

acknowledged teacher of mathematics (see Section 3.3) and who would agree to respond anonymously to an open questionnaire. All the principals agreed to participate in the study, and the acknowledged teachers of mathematics were recruited. In addition, the six teachers remaining from the eight teachers interviewed in the first phase of the data collection process were asked to answer the same questionnaire as the two participants had in the second phase of the data collection process. Each of the 30 principals received an envelope containing the questionnaire, a letter of information to the principal, a blinded letter of information about the questionnaire, and a stamped envelope. The principal was asked to give the questionnaire and the blinded letter of information to the teacher recruited by the principal. The teacher returned the completed questionnaire to the principal who then mailed the questionnaire to me. All 30 principals assured me that they would do their best to support the study, but 11 schools did not participate in the end. Hence, I received 25 completed questionnaires (70% response rate) including the questionnaires from the additional six teachers, who had confirmed in advance their willingness to participate in the study. They were instructed to follow the same mailing procedure as the principals, and their anonymity remained the same as for the other 19 responding teachers.

On the basis of the results of the questionnaires, two of the teachers who participated in the first phase were interviewed using structured interviews (Kvale, 2006). The interviews were organised in the same way as the structured interviews performed in the second phase. These interviews focused on the two teachers' reactions to interpretations of the answered questionnaires; in other words, this comprised a process of confirmation or invalidation of the interpretations made (Yin, 2003; Silverman, 2006). Each interview comprised four clusters of questions (Appendix IV). The interviews started with a cluster of questions about the influence of impact factors related to the teacher's use of practical activities. The main part of the interviews comprised three clusters of questions, each about one of three interpretations of the analysis of the questionnaires:

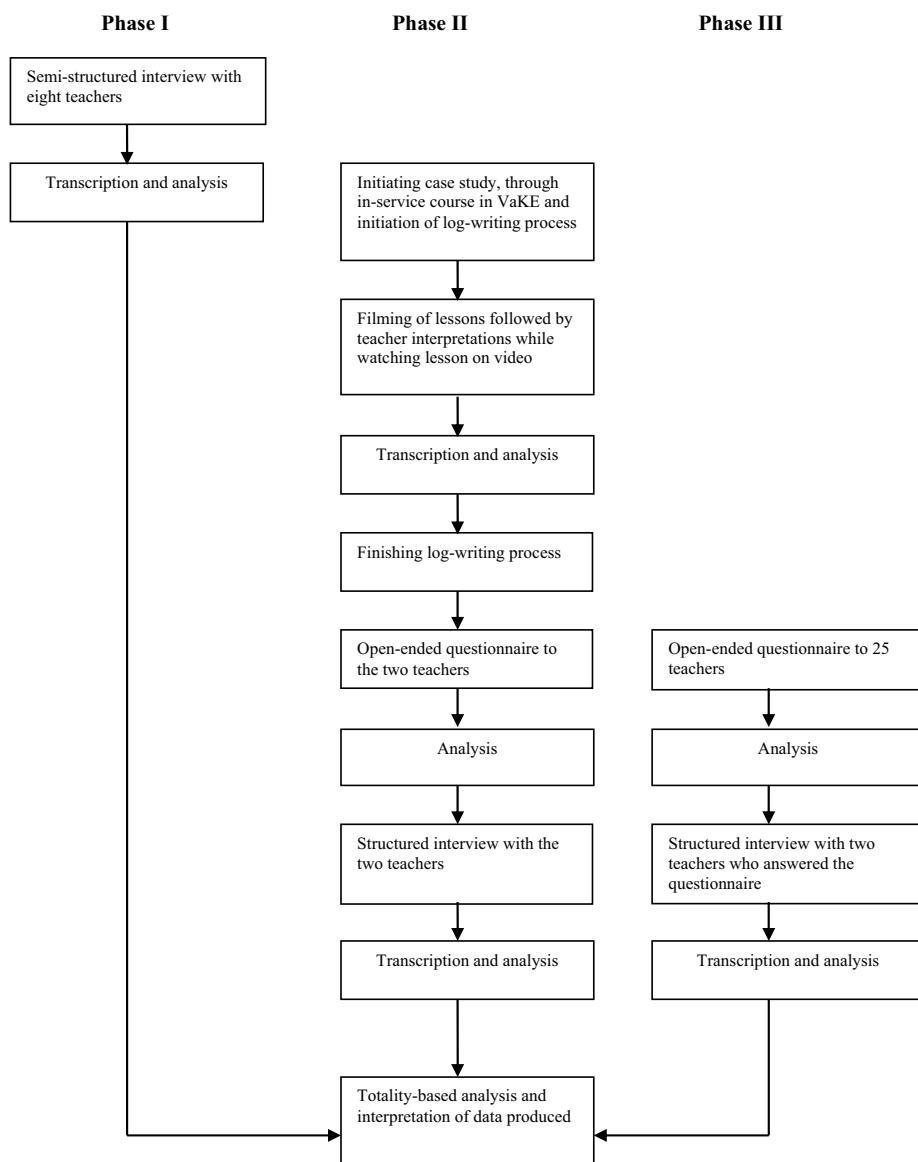
- The teacher's everyday life experiences influence the frequency of possibilities for using practical activities in mathematics teaching
- The teacher's knowledge about pupils' everyday life experiences influences the frequency of possibilities for using practical activities in mathematics teaching
- The teacher's use of practical activities is related to the teacher's conscience.

The structured interview was piloted with a teacher from the remaining group of four teachers not selected for interview.

3.2.4 Data collection process – summary

Even though many methods are used in the data collection process, this does not necessarily ensure understanding from a hermeneutical perspective. The basis for the research design therefore requires the researcher to attempt to control the influences of preconception and prejudices by both the researcher (see Chapter 8) and participating teachers (see Section 3.3). Eight teachers participated in the long-term data collection process. They participated in two phases. All eight teachers participated in the first phase, two of them in the second phase, and the remaining six in the third phase. Hence, the design includes awareness of the importance of both my and the participating teachers' preconceptions and prejudices, and the effects of these interpretations as they developed during the project. The design of the project is shown in Figure 4.

Figure 4: Design of the research project



3.3 Research participants

3.3.1 Acknowledged teacher

The research literature included many labels to describe teaching expertise: *the expert teacher* (Gudmundsdottir & Shulman; 1987; Berliner, 1992; Berliner, 2004), *the experienced teacher*

(Hoekstra et al., 2007), *the good teacher* (Korthagen, 2004) and *the accomplished teacher* (Hill, Rowan & Loewenberg Ball, 2005; Lieberman & Pointer Mace, 2009). Some have referred to both the *expert* and *experienced teacher* (Berliner, 1986; Shulman, 1987)¹⁴. Regardless of the label used, expert and/or experienced teachers of mathematics can be found. However, when the aim is to study the reasons teachers give for choosing practical activities, relevant informants do not necessarily need a high level of professional expertise or extensive experience. In this research project, it was more important to collect data that represent reality than to produce data that coincide more naturally, for instance by recruiting only highly skilled mathematics teachers with long teaching experience. I believed it was important to include the thoughts and impressions of a variety of mathematics teachers teaching in compulsory schools.

A teacher may be brilliant at work without being recognised as influential among subjective stakeholders (pupils, parents, colleagues) who informally evaluate teacher performance. On the other hand, a teacher might in reality start as a new and inexperienced teacher every year (Handal & Lauvås, 1987; Shulman, 1987). Thus, expert or experienced teachers do not represent the variety of teachers who teach mathematics in compulsory school. Many teachers are either inexperienced or lack formal disciplinary and/or didactic education in mathematics (e.g. Grønmo, et al., 2004). Nevertheless, they teach mathematics, some of them with great success and acknowledgement from their principal and other groups (stakeholders) interested in their teaching. Acknowledgement of the teacher within the working environment is relevant to the development of the professional environment (e.g. Lohman & Woolf, 2001). Some teachers have a more distinct influence on their surroundings than others, in the positive sense of the phrase. They are acknowledged within their environment both for their teaching and how they influence their working environment. In designing this project, I defined a new term – *acknowledged teacher* – and all teachers recruited to the project were recognised as acknowledged teachers. An *acknowledged teacher* of mathematics is defined in this dissertation as: *A teacher who is viewed as a competent mathematics teacher by the principal and who earns respect from colleagues, pupils and other relevant groups within the working environment.* The acknowledged teacher is on many occasions an expert and/or experienced teacher in the way these terms are defined (e.g. Berliner, 1986; Shulman, 1987), but adds qualities to his/her position through the positive impact he/she has on the school environment.

¹⁴ Bereiter and Scardamalia (1993; in Tsui, 2003) do on the other hand emphasise that experience is often mistaken for expertise.

Regardless of the level of formal disciplinary or didactic competence or years of experience, his/her actions and experiences lend validity to the analysis of changes in direction or the preservation of the current direction with respect to the content and working methods in the professional environment.

3.3.2 Population

To examine the reasons teachers give for choosing practical activities in mathematics teaching, the participants were recruited from a group of potential informants who I expected to offer relevant data (Silverman, 2006). I have attached this relevance to the recruited teachers through their acknowledgement as mathematics teachers in their own working environment. To use only defined expert and/or experienced teachers would narrow the variety of opinions and experiences among the informants for the benefit of clarity. The influence on the recruitment of teachers as informants has been explored in other projects that recruited mathematics teachers in compulsory education for didactic research (Mellin-Olsen, 1991; Mosvold, 2005). Mellin-Olsen (1991: 100) refers directly to his reflections regarding this recruitment challenge:

Therefore I must secure that it is likely that the teachers who I speak with have something interesting to tell... How am I supposed to find such teachers? ... Who do I want to be informed by?

Teachers were recruited on three occasions in the project, each identified through the three phases of data collection. A plan for the use of a strategic group of eight participants was designed to strengthen the interpretative influence of acknowledged teachers in the project. They were presented with the entire data collection plan as part of the recruiting process. The longitudinal participation strengthened their confidence in the data collection and generally led to a higher level of trust and commitment towards sharing their thoughts and beliefs. The eight members of the strategic group were recruited according to the following criteria.

- As a group, they represented teaching experience from lower and upper grades in the Norwegian compulsory school system.
- The group included teachers of both genders with varying levels of formal education in mathematics¹⁵ and varying levels of practical teaching experience.

¹⁵ Four of the teachers graduated from the teacher education programme after 1994 and therefore have at least 30 ETCS in mathematics. Two of the teachers graduated between 1978 and 1990, and did not choose to study mathematics in their teacher education. One of them chose not to because of the high disciplinary level of mathematics in upper secondary school. The other two teachers in the study graduated from the teacher education programme before 1975 and had completed a university programme.

- All teachers were recognised as acknowledged teachers in the school where they worked.

The recruiting phase was initiated through direct contact with six acknowledged teachers in compulsory school. They were chosen based on my knowing several compulsory school teachers through my work as a lecturer in mathematics and Head of Studies in teacher education for compulsory school at *Sogn og Fjordane University College*. An official request for their participation was sent to each principal involved after each teacher had agreed to participate in the project. Two other teachers were recruited to the group of participants in addition to the first six teachers. They were recruited by their principals upon my request for one acknowledged teacher in mathematics from their respective schools¹⁶.

Twenty-five acknowledged teachers were recruited for the third phase. Nineteen teachers were recruited especially for this purpose through the principal at the schools where they worked. The six teachers from the group of eight recruited to longitudinal participation in the project who remained after two of them finished their participation after the second phase made up the rest of the recruited teachers. The intention was to interview two acknowledged teachers of mathematics about the interpretations made from the analysis of data from the open-ended questionnaire. I thought it important that these two interviewees were familiar with the questionnaire when being interviewed. However, strict anonymity of the 19 participants recruited through the principals prohibited contact with any of them. The identity of the participants had to be known to recruit the two interviewees. Therefore, the two interviewed teachers were selected randomly from the group of six. This secured the recruitment of interviewees without further harming the anonymity of participants. At the same time, this provided a strengthened respondent validation of data (Silverman, 2006).

¹⁶ These two principals were contacted because their schools were supposed to participate in an international project whose aim was to test *VaKE* in science teaching. The project did not make it through the final stage when competing for an EU-FP7 (EU's Seventh Framework Programme for Research and Technological Development) grant but would have involved two teachers who were interested in developing their teaching of mathematics. Such interest also suited this research project because, in the second phase of the data collection process, I wanted to offer the participating teachers an approach to mathematics teaching that was new to them (see Section 4.2).

Table 1: The teacher participation in the three data collection phases

Instrument/Phase	Phase I	Phase II	Phase III
Semi-structured interview	All eight teachers from the strategically selected group of eight teachers		
Case study		Two teachers from the strategically selected group of eight teachers*	
Open-ended questionnaire			- Nineteen acknowledged teachers recruited by their principals - Six teachers remaining from the strategically selected group of eight teachers after phase II
Structured interview			Two teachers from the remaining six teachers of the strategically selected group

* The case study involved several data collection instruments, including the open-ended questionnaire answered in phase III.

3.4 Analysis

All the analyses were influenced by the preconceptions, horizon of understanding and prejudices I brought into the project (see Chapter 8). This required me to switch between a focus on totality and on parts within each research phase and also in the continuous interpretation process involved in the entire project, as shown in Figure 4. The research project totality was the focus of the main research question: *What reasons do teachers give for choosing practical activities in mathematics teaching?* I focused on the parts when trying to illuminate an aspect of the main question through the sub-questions in phases I to III of the project.

3.4.1 Intention and intentional explanation

Intention is intentionality recognised as referring to how consciousness is aimed towards an object favoured with attention (in an act of consciousness), for instance through an act of will, judgement or remembrance (*Aschehoug & Gyldendals Store Norske Leksikon*, 2006b). Therefore, the teacher's intention when choosing a practical activity here refers to the teacher's consciousness about what aim he/she has for making such a choice.

A teacher who chooses to use a practical activity in his/her teaching has a purpose for using the activity in the teaching and wants to express something through the use of the activity (Føllesdal, Walløe & Elster, 1990). According to Wimsatt and Beardsley (1946), it is necessary to make clear what one wants to try to understand when focusing on intention.

When looking into the reasons teachers give for choosing a practical activity, the focus should not be on the activity itself because that would imply that all interpretations depend on the pupils' participation in the activity. The research must address the teacher's purpose for choosing to use a practical activity and what he/she wants to express through the choice.

An *intentional explanation* tries to explain human actions by stating the reasons that made the acting person carry out the action at hand (Føllesdal, Walløe & Elster, 1990). In an intentional explanation, both the acting person's beliefs and consideration of available alternatives and consequences are included. When seeking to unveil the reasons teachers have for choosing to use practical activities, the researcher may try to state the cause(s) for such a choice. This attempt will be difficult and probably inexhaustible because of the complexity and variety of reasons and conditions available to each teacher. A causal explanation requires scientific law representation and logical deduction from cause and scientific law. However, a causal hypothesis could be reached. One or only a few impact factors might be identified as the prevailing impact factors influencing the choice, for instance, if the teacher is ordered by the school management to use a practical activity in each mathematics lesson. The causal hypothesis might then be that the teacher wants to remain loyal to the school management. However, such an example could also be put forward with regard to an intentional explanation. Through an intentional explanation, the teacher's choice will be explained by stating the reasons why the teacher chose to use practical activities. This will require attention to what considerations the teacher gave to available teaching alternatives. Such considerations are based on the teacher's professional knowledge, beliefs, identity and external constraints (see Chapter 2). In other words, the decision is not based solely on the teacher's loyalty towards the school management. This means that a causal explanation (or at least a causal hypothesis) can be used to underpin an intentional explanation (Føllesdal, Walløe & Elster, 1990).

Insight into the intentions of the participants through intentional explanations is important to understanding their choices to use practical activities. An intentional explanation is developed through a hermeneutical approach and starts with the performed choice. All aspects of such a choice cannot be understood completely, but to reach new understanding, two elements of the teacher's horizon of understanding must be interpreted. First, the teacher's intention is to achieve something. Second, the teacher has reasons for the choice he/she has made. This implies that the teacher is the starting point for the explanation of the phenomenon of

choosing practical activities in mathematics teaching. In other words, the explanation must be traced back to the internal constraints brought to the decision process by the teacher (see Chapter 2). This is methodological individualism, an explanation principle that here works as the basis for bringing new understanding to aspects related to teachers as a group. Methodological individualism means that collective intentions and actions can be traced back to and understood from concurrent individual intentions and actions (Guneriussen, 1996; Hovi & Rasch, 1996; Gilje & Grimen, 1998; Grimen, 2004).

By applying a strictly methodological individualism-based explanation principle, the unveiling of the teachers' reasons for choosing practical activities is approached only through explanations stemming from the research participants. Interpretation of structurally influenced conditions for such choices is ignored. This would call for a phenomenological approach (Grønmo, 2004). On the other hand, everyone is influenced by the surrounding society. Taking the phenomenon *choosing practical activities in mathematics teaching* as the relevant example, this means that the teacher will be influenced by the social systems (school, society) he/she is part of. The choice to use practical activities has a perspective only in relation to the conditions made possible by school and society. Any consideration of a choice requires that the teacher belongs to a community that is open to different choices. Hence, it is not useful to make a sharp distinction between methodological individualism and methodological collectivism (Hovi & Rasch, 1996; Gilje & Grimen, 1998). *Methodological collectivism* implies that individual actions are seen in relation to qualities identified with social phenomena (Hovi & Rasch, 1996). The teacher influences the system he/she is part of as well as being influenced by the system. This implies that the teacher might unconsciously produce data that are descriptions of the actual phenomenon or a designated interpretation of the phenomenon. Therefore, the teacher's intentions and choices are interpreted together with the institutional conditions allowing teacher choice. Within the institutional and cultural conditions established by school and society, this means that the teacher is trusted and given autonomy, responsibility and opportunities to make choices about the teaching. This underpins the need for including the teacher's intention when making choices and implies heavy reliance on the teacher's professionally based reasons and external constraints as factors that can influence choice. Thus, elements of methodological collectivism that originate in external constraints should be acknowledged in analysing the process of the teachers' reasons for choosing practical activities in mathematics teaching.

3.4.2 Interpretation of data

The interpretations of data comprise mainly my interpretations of the collected data, but also include the participants' interpretations of the data collection processes and my interpretations of the data obtained in these self-interpretation processes. This requires attention to both the collected data produced by the project participants and the contributions that I bring to the project through my preconceptions and prejudices (see Chapter 8). In addition, a second researcher validated the analysis in all three data collection phases (Kvale, 2006) through discussions of the data, analytical approaches and findings based on the analysis. A hermeneutic approach has been applied in the analysis (Grønmo, 2004), which includes awareness of interpretational challenges related to my "involved position and ... preferences regarding [my] object of study and modes of inquiry" (Tillema, Orland-Barak & Mena Marcos, 2008: 50). In addition, elements of a phenomenological approach have strengthened the totality of the hermeneutical approach by introducing an element of intimacy to the participants' own explanations and interpretations.

The data from the semi-structured interviews collected during the first phase of the project were transcribed fully. I then concentrated on extracting essences of meaning from each interview (Kvale, 2006). First, I read through the entire interviews aware of my preconceptions of the teachers' use of practical activities in mathematics teaching as the starting point. I then categorised the interviewed teachers according to their level of disciplinary knowledge and teaching experience, and I then identified and marked similarities and discrepancies within and between the categories in separate interviews. The interpretations within each category focused on each teacher's basis for and beliefs about using practical activities. The following two sequence examples show that the categorisation helped identify nuances within different categories. Both sequences are from interviews with teachers I characterised as inexperienced, but the teacher in the second sequence (labelled T3) has a higher level of formal disciplinary mathematics knowledge than the teacher in the first sequence (labelled T1):

T1: I think that on my behalf it is extremely important...with the concrete approach...the practical approach. To go from working at a very concrete, a very visual level,...use the senses one actually can activate, so to speak. I am not that bookish, I feel that working...the book is a working tool to me and the pupils, and I think that working...they ought to work in their books as well, but to me it is equally important to put the book aside, because I feel that when the pupils get that book in front of them, mathematics instantly becomes a much more boring subject. Because then they are in a way supposed to sit and work in the book...But if one can make mathematics teaching more...more

fun! What I mean is a bit meaningful, and that one works in a hands on manner and visually, and with things that you can grasp and feel, and things like that...that is important!

T3: First of all it [mathematics] is something that can be made concrete and practical. When I went through my teacher education, I really enjoyed the element of practical activities [in mathematics], like when the teacher pulled up a leek, and cut it up with diameter and circumference in mind, and...that I really liked. So...such small, simple, fine practical activities are fine, I think... But then, when you are supposed to teach on your own, then it is not so easy to come up with...how are we supposed to go into this? Then it would have been nice...to maybe get a tip from somewhere, for instance from a booklet, for inspiration...for practical activities related to the textbook that is used for the class you are teaching.

T1 reveals beliefs about mathematics as an applied subject, which requires inspiring teaching and meaningful content. Moreover, the focus should be on the application and meaningfulness of mathematics through practical activities in the teaching. This teacher emphasised the importance of activating pupils and ensuring that mathematics is experienced as interesting and fun. Mathematics as a subject is not ascribed any self-motivating qualities. The teacher is supposed to motivate the pupils to want to learn mathematics. Practical activities are chosen because they are perceived as illustrative of the usefulness of mathematics, and they play an important role in maintaining and developing pupils' motivation towards the subject. T3 also notes that mathematics is a subject that will be enriched by use of practical activities, although he does not use such activities that much. He sees disciplinary-based obstacles in the didactic challenges to introduce the mathematical content through an activity and pave the way for the transfer to theoretical mathematics.

The nuances within and between the categories were interpreted with awareness of the presence of my prejudices and those of the participants, and the possibility of identifying both actual descriptions of the phenomenon and designated interpretations of the phenomenon by the participants. This provided a matrix of similarities and discrepancies to discuss in relation to the research question at hand.

In the second phase of the project, the participants' comments made while watching the video recording of their own teaching were used to register possible tensions between video-recorded observations and their comments. This added a dimension to the interpretation process. The comments about the video sessions were transcribed and coded when I watched the videos once more while listening to the audiotaped comments from the teacher. From the comments made by the teacher, I created units (Grønmo, 2004) that were categorised as

“positive”, “negative” or “neutral” (Jacobs & Morita, 2002). Units that included discussion of practical activities, isolated or within the progress of the VaKE methodological structure, were divided into five subcategories and interpreted according to the teacher’s comments as: “positive – unconscious”, “positive – conscious”, “neutral”, “negative – conscious”, and “negative – unconscious”. This is consistent with how people are conscious about some reactions and prevented from being conscious about other exhibited reactions. Unconscious reactions are difficult to explain. In other words, the observing teachers’ reactions could be separated similar to the distinction between conscious and unconscious beliefs (Bishop, 2001). The same categorising system used for the video sessions was used in the analysis of the two logs, but was based on systematic extraction of the meaning of sequential content organised in a matrix (Grønmo 2004), which was structured by a timeline and the participants. The data collected from the open-ended questionnaire were treated in the same way as the logs, except that the matrix was structured by the questions and participants. The structured interviews, which ended the second phase of data collection, were transcribed fully and then read through with my revised preconceptions as the starting point. The interviewees’ answers were interpreted in relation to the interpretations I had made based on the video observations. Hence, the logs, questionnaires and interviews served as data-producing devices in the triangulating check for points of refutation and confirmation of the pre-questionnaire interpretations.

In the analysis, I applied a hermeneutical approach that also included an element of a phenomenological approach (Grønmo, 2004). The phenomenological approach may be used as the basis for analysing the two teachers’ experience with the intervention programme because their interpretations of what they watched on video became part of the data to analyse. Their involvement allowed for immediate interpretations of the specific intervention at hand and its influence on their practice of the use of practical activities to increase the possibility of exposing possible tensions between their visions and beliefs (see Section 4.2). The hermeneutical approach was reflected in the influence of the intervention compared with the pre-intervention situation and in the similarities and discrepancies between the two teachers’ beliefs about teaching mathematics. My preconception about using practical activities was influenced by the discussion of findings in phase I of the research design. Phase II required that this preconception and my unconscious prejudices were tested in the context of the data from the interventional study. (see Section 3.1 and Chapter 8).

The analysis of the data from the third phase of the project included a hermeneutic phenomenological analysis (Grønmo, 2004; Kvale, 2006). The teachers answered the open-ended questionnaire in handwriting. The handwritten answers were typed and then structured and compared through a matrix (Grønmo, 2004). A systematic extraction of meaning of the answers was made. In the sense that the interpretations were based on the participants' experiences with impact factors that influence the use of practical activities, this should be considered a phenomenological interpretation process (Grønmo, 2004; Kvale, 2006). This approach was chosen to focus on the participants' own impressions and interpretations of their experiences. The two structured interviews were planned and completed to confirm or invalidate hypotheses-based interpretations from the first round of analysis. The interviews were transcribed fully and read through. Extracts of meaning (Kvale, 2006) were identified from the transcriptions and interpreted to find points of support or invalidation of the proposed hypotheses. This allowed for a hermeneutical approach (Grønmo, 2004; Flick, 2006) to be applied in the second round of interpretation of the data from the questionnaires and interviews and a new round of preconception development and testing of preconception and unconscious prejudices in the context of the data from the questionnaires.

The analysis was closed by agreeing with an impression that understanding is inexhaustible, but that an analysis process needs to end at some stage. The starting point for the closure was that my preconception of the phenomenon at hand was influenced by the findings in phases I–III of the project, and the closure implied that the preconception was tested in discussion with the findings from phases I–III. Therefore, the hermeneutical approach continued in order to develop a holistic understanding about the reasons teachers give for choosing to use practical activities in mathematics teaching. Chapter 5 shows the results of this process.

3.5 Ethical considerations

The Norwegian committee of research ethics (NESH) has published ethical guidelines for research in social sciences, the humanities, law and theology (NESH, 2010). These guidelines are valid for all parts of this project. The principals of all the schools involved through teacher representations were informed about and approved the forthcoming data collection process. The eight participants recruited for the entire project were given both oral and written information about the project before they consented to participate. The 19 teachers who answered the open-ended questionnaire anonymously consented to participate by answering the questionnaire and letting their principal mail the answered questionnaire. The request for

the participants' informed consent noted that they were free to end their participation in the project at any time and that they could restrict the application of produced data in the project. Before filming the two teachers who tried out a new approach to mathematics teaching (Phase II of the data collection process) the parents of each involved pupil were informed about the project in writing and asked to return a signed consent form allowing their child to participate.

As the person responsible for the project, I was aware that the data collection entailed contact between people and registration of peoples' actions and utterances. Some of the information that had to become accessible through the production of data was emotionally or personally revealing, and I viewed the protection of the participants' confidentiality as crucial. I took seriously my responsibility to maintain scientific reliability without challenging the prevailing ethical guidelines. All participants in the project have been treated anonymously, and the dissertation and papers produced refer only to demographic facts considered necessary to ensure the validity and reliability of the participant selection and information about the frames for the work as teacher. The collected data has been stored safely. Data in handwriting, printouts, film and audiotapes have been secured. Computer files containing the transcriptions of data have been saved but access to them is restricted by username and password in the computer system of *Sogn og Fjordane University College*. In addition, I have stored backup files on a private laptop to which no one else has been given access.

I applied for approval of the data collection process to the Norwegian Social Science Data Services (NSD) on 1 October 2007. In a letter dated 3 December 2007 from NSD, the project was considered not to be notifiable in terms of issues of protecting the privacy of the participants in the project (Appendix V).

Chapter 4: Presentation of articles

This chapter presents the three articles published from the data collected in this dissertation, with emphasis on reviewing the findings in each article. As established in Chapter 1, this research project focused thematically on the factors that influence a teacher's choice to use practical activities in mathematics teaching. Based on the identification of teachers' use of practical activities as an area in need of further research, the main question for the dissertation was stated in Chapter 1 (Section 1.3) as:

What reasons do teachers give for choosing practical activities in mathematics teaching?

Each of the articles focuses on an issue identified in Chapter 2 as needing further research (see Section 2.3). Each article focused on one of the three phases of empirical data collection discussed in the review of the project's research design in Section 3.2. The research questions for the articles are the sub-questions shown in Chapter 1 (Section 1.3). As articles, they can be treated as independent research projects, but they also form parts of the hermeneutical interpretation process for the entire dissertation.

4.1 Article I: Practical activities in mathematics teaching – mathematics teachers' knowledge-based reasons

This article focuses on the teachers' knowledge-based explanations and reasons for choosing practical activities in the teaching of mathematics. This article addresses the following research question:

How do acknowledged teachers of mathematics explain the reasons for choosing practical activities in their teaching, and to what extent is this related to their professional knowledge?

Eight acknowledged mathematics teachers, with varying levels of disciplinary knowledge of mathematics and accumulated teaching experiences, who work in the Norwegian compulsory school were interviewed. The teachers with a low level of formal education in mathematics explained the use of practical activities by referring to didactic and psychological dimensions such as interest, motivation, variation and fun. The mathematical content played a minor role in decisions about activities. They saw many possibilities for using practical activities, but their analysis of the context of the choices was somewhat superficial. Arguments related to mathematical content were used rarely. Teachers with a low level of formal education in

mathematics but with a developed pedagogical content knowledge (developed through practice) had a more critical and restrictive perspective towards including practical activities in teaching. Development of professional knowledge led to change in beliefs about teaching mathematical content. Teachers with a higher level of disciplinary knowledge of mathematics but with little teaching experience were the teachers least able to find space and priority for practical activities. However, these teachers claimed that they would like to use practical activities more often but that they experienced a lack of vigour in exploring the possibilities. The teachers with a higher level of disciplinary knowledge in mathematics and pedagogical content knowledge (developed through practice) were positive towards using practical activities. However, they made content demands about the possible use of an activity. The activity had to offer new perspectives or at least concretise the mathematical content in a way that enhances learning beyond the possibilities of traditional teaching. They did not look mainly to the usefulness of mathematics or didactic dimensions but rather considered the mathematical content as the primary basis for using a practical activity.

The article concludes that there is a difference between experienced and inexperienced teachers in the use of practical activities. The experienced acknowledged teachers do not embrace a practical activity as something fulfilling in its own. They consider a practical activity to be relevant only when it seems appropriate to concretise the mathematical content and proves to be equally or more useful than other approaches. A noticeable distinction is that the inexperienced teachers did not have clear knowledge-based explanations or reasons for using practical activities in the same way as the experienced teachers did.

4.2 Article II: Increasing the use of practical activities through changed practice

The article examined what influence a practical activity supported by value-based intervention had on two teachers' use of practical activities in teaching of mathematics. This was addressed through the following research question:

How does the introduction to a values and knowledge education (VaKE)-based teaching approach supported by practical activities influence two elementary school mathematics teachers' use of practical activities in mathematics teaching?

The two teachers participated in an 18-month case study. During this period, they participated in an in-service course that emphasised the use of VaKE and was supported by practical activity application in mathematics teaching, tried VaKE in their teaching and shared their experiences with and interpretations of the new approach. The introduction to a new approach

in mathematics teaching offered them an alternative, which increased the use of practical activities. However, the article also showed how good intentions of changing practice might be restrained or constrained by beliefs and previous experience.

The two teachers experienced the teaching approach in different ways, which led to different outcomes. One teacher (Vivian¹⁷) maintained her initial enthusiasm about practical activities that were supported by a value-based approach. The other teacher (Walter) did not. The main reason for this is the different starting points of the two teachers. The article concludes that Vivian's beliefs were not challenged to the same extent as Walter's beliefs. Her vision of how teaching should be proved possible for her to reach. The discrepancy between Walter's beliefs and experiences of constraints given by his regular teaching practice and the value-based approach was too wide. The article concludes that several reasons can explain the two teachers' different responses to their experiences with a new approach to mathematics teaching intended to increase the use of practical activities. The research suggests that the two teachers' beliefs about teaching mathematics and their didactic knowledge are crucial impact factors. In addition, external constraints such as the in-service course and the pupils' response to the new teaching approach also influenced Vivian's and Walter's acceptance of the use of practical activities supported by VaKE. The article concludes that a change in beliefs is necessary to induce sustained change in practice; if not, practice drifts back to its initial pattern.

4.3 Article III: Why use a leek in mathematics teaching?

This article focuses on the possibility of identifying a hierarchy of impact factors that influence a teacher's choice to use practical activities. This was addressed through the following research question.

In what way do teachers' experiences call for an expansion of a system theoretically grounded hierarchy of impact factors regarding the choice to use practical activities in mathematics teaching?

The article first suggests, in accordance with Bateson's (1972) relational communication approach, a theoretically grounded hierarchy of primary and secondary impact factors that influence a teacher's choice to use practical activities. Data were obtained from an open-ended questionnaire completed by 25 acknowledged mathematics teachers and interviews

¹⁷ The two teachers are referred to through the pseudonyms Vivian and Walter.

with two of the teachers who had answered the questionnaire. The interviews were based on interpretations of the data provided by the questionnaire. The findings suggested that a hierarchy of impact factors must include more than just a binary clustering of primary and secondary impact factors. The findings show that teachers' everyday life experiences, their knowledge about the pupils' everyday life experiences and their conscience are impact factors with the potential to influence strongly the choice to use practical activities. These factors are based on knowledge and beliefs, and are at the same time dependent on secondary impact factors. Hence, it is necessary to expand the Batesonian-inspired theoretical hierarchy based on teacher experience.

The article discusses the difficulty in aligning a Batesonian-inspired hierarchy of clusters of impact factors and teacher learning when introducing an intermediate level in the suggested hierarchy. Nevertheless, the main part of the discussion addresses the influence on the teacher's choice to use practical activities of the three identified impact factors. This influence requires expansion of the theoretically grounded hierarchy. The article concludes that teachers' experiences of practice did not invalidate a Batesonian hierarchical clustering of impact factors but led to a necessary expansion of the theoretically grounded hierarchy. On the basis of this conclusion, this article suggests that clusters of impact factors that influence the teacher's choice to use practical activities can be organised in a hierarchical structure. The article acknowledges that changes should occur on a higher level in a hierarchy of impact factors if the teacher is going to develop further the reasons for choosing to use practical activities. This agrees with the Batesonian hierarchical thinking about learning. However, the article also concludes that such an acknowledging emphasis on the influence of the three impact factors cannot be identified clearly through only a binary clustering of impact factors. Hence, a model based on a Batesonian hierarchy of learning and communication does not satisfy completely the experience-based impressions of impact factors that influence the teacher's choice of practical activities. The incorporation of an intermediate level of impact factors in the hierarchy is required, and this requires a more complex visualised structure of impact factors.

4.4 Summary of the main findings

The articles focus on three important aspects of the main question of this dissertation and can be summarised as follows. First, the teacher's professional knowledge is an important influential constraint on the reasons that teachers give for choosing to use practical activities.

Second, the examination of two teachers' experiences with a new approach to mathematics teaching showed how an attempt to change practice, and thus challenge prevailing beliefs, confirms empirically the influence of internal and external constraints on the teacher's choice. The articles discuss the challenging process to change beliefs to sustain changed practice. An attempt to increase the use of professional knowledge-based reasons for choosing to use practical activities calls for the development of internal constraints. Third, the possibility of hierarchical categorising of clusters of impact factors illustrates how teachers can change and therefore increase professional awareness about the choice of practical activities. The hierarchical categorising provides an overview of the influence of impact factors, although the experience of the influence of encouraging and restraining constraints is a personal experience for the teacher. Such hierarchical categorising is complex and difficult to interpret unambiguously because of the variety of ways experience can influence the teacher and because some impact factors are interrelated. Taken together, the articles relate the main research question to the findings in three areas:

4.4.1 Identification of three classifications of reasons for choosing practical activities

First, a teacher who is confident about the mathematical content and about how to teach it chooses to use practical activities based on an intention that is robust and less affected by constraints. Teachers with a high level of professional knowledge can give mathematical content-related reasons for using practical activities. An activity is used only when it is appropriate from a professional knowledge perspective or, in other words, when it is assessed to have a greater effect than other available teaching approaches. Second, after considering any inconsistency between the practised teaching and the vision for the teaching, the teacher might compromise his/her own understanding of how the teaching should be. The third identified classification of reasons is the teacher's experience of practical dilemmas.

4.4.2 Change through the gap between designated and actual teacher identity

The gap between actual and designated teacher identity (Sfard & Prusak, 2005a) can influence how much internal and external constraints affect the choices the teacher makes about the teaching. Fulfilling what can be interpreted both as personal expectations that contrast with personal beliefs and as expectations of others about teaching of mathematics can be challenging for the teacher. Attempting to increase the use of professional knowledge-based reasons for choosing to use practical activities requires the development of internal constraints. The teacher may attempt to develop his/her professional knowledge by changing

beliefs through changed practice in mathematics teaching to narrow the gap between actual and designated teacher identity. However, such an attempt may prove too great an endeavour, and the teacher may return to a teaching style that accords with the prevailing beliefs about mathematics and mathematics teaching.

4.4.3 Numerous influential reasons categorised hierarchically

The influence of impact factors varies between teachers and between situations. This can mean that there are numerous influential reasons for choosing to use practical activities, reasons that depend on both internal and external constraints, which can be interpreted as both encouraging and restraining. A hierarchical categorising of clusters of impact factors means that some constraints are favoured. This illustrates how teacher change can increase professional awareness about the choice of practical activities. However, such a hierarchy must be more complex than that provided by an attempt to construct a hierarchy through Bateson's (1972) relational communication theory. The teacher's experience of the influence of impact factors as either encouraging or restraining constraints is personal, some impact factors are interrelated, and the various levels of the influence of constraints make it difficult to include influence in such a hierarchy.

Chapter 5: General discussion

The discussion comprises four parts. First, the decision process that the teacher uses from intention to action is recapitulated. This section also discusses the findings on the identification of a hierarchical structure between impact factors that influence the choice to use practical activities and possible ways to reduce the complexity of this process. The second section discusses the three identified classifications of reasons that teachers give for choosing practical activities: professional knowledge reliance, compromise with one's own understanding and the teacher's experience of practical dilemmas. The third section discusses the findings about the reasons teachers give for choosing activities in the context of the potential gap between actual and designated teacher identity. The fourth section summarises the chapter through a general discussion of the main research question.

5.1 A complex decision process from intention to action

How to teach mathematics is often questioned with regard to both content and form. Teachers now meet more expectations about their teaching priorities than ever before (e.g. Sowder, 2007). Chapter 1 and Section 2.2 show that the increased emphasis on practical relevance and practical activities in mathematics teaching is part of this process. These sections also show that the increase in the use of practical activities has proven questionable and somewhat equivocal (Thompson, 1992a; Klette, 2003; Gardiner, 2004; Kjærnsli et al., 2004; Olsen & Grønmo, 2006). Before the actual application of a practical activity, a teacher must decide to use the activity. Chapter 2 discussed the complexity of the decision-making process, which is influenced by internal and external constraints that can encourage or restrain the teacher's intention. This process is illustrated in two phases in Figure 5.

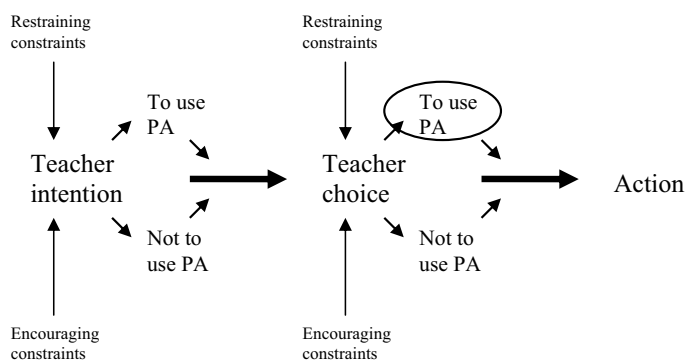


Figure 5: Illustration of the decision process from intention to action (here: use of a practical activity (PA))

The teacher either intends to choose to use a practical activity or not. This intention is influenced by both internal and external constraints. In the next phase this intention, which now works as a constraint, is either supported or challenged in a new round of influence by constraints. Then the choice is made.

Chapter 2 established that a separation between internal and external constraints could be treated as a separation between primary and secondary impact factors. From the practice theoretical perspective, this separation and clustering of impact factors was found to be suitable because of the teacher's role in the developmental processes. The teacher's knowledge and beliefs together with a growing body of teaching experience may lead to a quality-improving change in practice. On the basis of the practice theoretical perspective, the teacher's professional knowledge constitutes the primary impact factors (or internal constraints). Constraints with an external origin, such as time pressure, curriculum, textbooks and pupils, are secondary impact factors (external constraints). In an article on the complexity of teaching and options available to the teacher, Nordahl (2007) states that the teacher's understanding of possible choices and the realisation of choices depend on the way in which the teacher observes his/her surroundings. The teacher has to make choices that are related to the surroundings. According to Nordahl (2007), when the surroundings are experienced as complex, the choices made by the teacher will be about reducing this complexity and thereby establishing more control. Teaching is a complex activity, and the teacher will try, through experiences, reflections and choices, to reduce the complexity of the teaching situation. As the teacher increases his/her knowledge and experience, the surroundings will be experienced as less complex (Rasmussen, 2004). Through the levels of increasing complexity, Bateson's (1972) hierarchical categorisation of learning and communication offers such a perspective to explain the teacher's aim of reducing the surrounding complexity. Change on one level will generate change on the lower levels in the hierarchy, and because of the dynamic nature of the system, at least some impact upwards in the hierarchy.

My attempt to adapt the numerous influential reasons to Bateson's hierarchical structure, and thereby to offer a commonality between a practice theoretical and a system theoretical perspective, was based on identification of primary and secondary impact factors. The study concluded that, from a system theoretical perspective, it is possible to make a hierarchical categorisation of clusters of impact factors. On the other hand, such a hierarchy must be more complex than what a hierarchy construction attempt based on Bateson's (1972) theory can

provide. The study showed that some impact factors were not solely teacher based or did not stem from sources external to the teacher. The teacher's conscience about the choice he/she made whether to use a practical activity or not is one such impact factor. Hence, the data in the study required the introduction of a new level of impact factors in the hierarchical structure. This was not possible to implement while retaining the Batesonian hierarchical structure. In other words, the complexity of the teacher's choice process within the activity system (Engeström, 2001) requires a more complex model of the influential reasons for choosing practical activities in mathematics teaching.

The attempt to establish a hierarchy of impact factors through clusters of impact factors did not produce a satisfactory overview of the various impact factors that might influence the teacher's choice to use practical activities. On the other hand, Section 4.4 presents important findings about the issues identified as needing further empirically based studies (see Section 2.3); these findings should be discussed in relation to the main research question.

5.2 Teachers' reasons for choosing practical activities in mathematics teaching

5.2.1 Knowledge-based reasons

The findings confirm that teachers put great emphasis on disciplinary knowledge, didactic knowledge and previous experience using practical activities when deciding whether to use an activity or not. These internal constraints are related to the teacher's beliefs about the teaching, as shown in Chapter 2. One of the sub-studies of this project (see Section 4.1) suggested a continuous development of the relationship between the teacher's beliefs about using practical activities and disciplinary and didactic knowledge of mathematics teaching. This suggestion shows how a pattern of beliefs about practical activities for knowledge-based reasons seems to evolve for acknowledged teachers in terms of the continuous development of disciplinary and didactic knowledge (Figure 6).

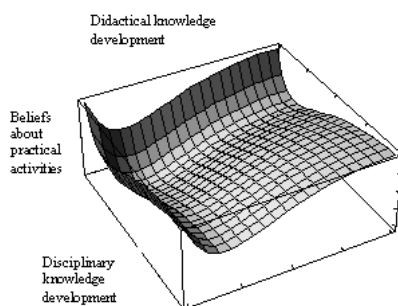


Figure 6: Beliefs about practical activities in mathematics teaching for knowledge-based reasons

The hypothesis suggested in Figure 6 is illustrated through metaphorical use of mathematics. It is based on the findings from the interviews with the eight teachers in phase I of the project and is supplemented by my general experiences and impressions (my preconception towards the end of phase I of the project) about teachers' beliefs about practical activities. The hypothesis indicates how the teacher's experiences with his/her own level of disciplinary knowledge and/or didactic knowledge seem to influence the teacher's beliefs about using practical activities. Teachers who do not have disciplinary or didactic knowledge in mathematics believe that practical activities are very important in the teaching of mathematics, but they do not argue for such an element in the teaching for knowledge-based reasons. The arguments are more affective and are related to motivational qualities such as interest and fun. An increase in disciplinary and/or didactic knowledge seems to decrease beliefs about the use of practical activities for knowledge-based reasons; this decrease is illustrated with darker shades in Figure 6. The areas with the darkest colour indicate where the steepest decrease in encouraging beliefs about the use of practical activities probably can be found. Experiences of falling short on introducing and administering a practical activity (e.g. Haara & Jenssen, 2007; Abrahams & Millar, 2008) and experiences of influence from external constraints influence the teacher's reflections about the choice to use a practical activity. Figure 6 also indicates that with development of didactic and/or disciplinary knowledge, practical activities are considered more positive and relevant to mathematics teaching. However, the model also shows that this is not the case for all teachers. A teacher who does not develop pedagogical content knowledge (Shulman, 1987) on the basis of increased disciplinary and/or didactic knowledge (independent of time) will not seek practical activities as a highly relevant element of teaching.

Figure 6 illustrates how increased disciplinary and/or didactic knowledge in mathematics strengthens the impact of the teacher's knowledge-based reasons for choosing to use a practical activity. This will make the decision less susceptible to influence by external constraints, and an activity will be used when assessed to have a greater effect on the learning process than other available teaching approaches. Professional knowledge in mathematics teaching is therefore crucial to the teacher's decision to choose a practical activity approach without feeling obliged to make compromises against his/her own understanding of how the teaching should be.

5.2.2 Compromise of one's own understanding of how the teaching should be

The internal constraints formed by the relationship between the teacher's professional knowledge, beliefs and identity influence the teacher's intention towards the choice to use a practical activity. In the case study where two teachers tried a new teaching approach in their teaching, this was based on identification of a discrepancy between their actual and designated teacher identity. They had a vision for their teaching (Shaw, Davis & McCarty, 1991; Pehkonen, 2003), but they felt that they failed to fulfil their own understanding of how the teaching should be. This inconsistency increased their exposure to the influence of restraining or encouraging constraints compared to if they had had a more coherent relationship between actual and designated teacher identity. The attempt to change practice led to different outcomes for the two teachers. One teacher sustained the change, whereas the other teacher ended up advocating the practice that he attempted to distance himself from. In other words, the teacher can change or fulfil beliefs about how the teaching should be but may also end up with suppressing beliefs (Wilson & Cooney, 2003). To fulfil personal designated expectations that contrast with personal beliefs and expectations from others, may influence the reasons for choosing practical activities. If so, this means that the reasons are founded on the teacher's experience of expectations of an actual opinion about mathematics teaching in society and not on professional knowledge. This will cause the teacher to experience practical and conscience dilemmas about the teaching offered (Mellin-Olsen, 1996), and the reasons for choosing to use practical activities will be based on the influence of constraints.

5.2.3 The experience of practical dilemmas

A third source of reasons for choosing practical activities is the constant confrontation between the teacher's experiences of practical dilemmas stemming from the application of practical activities. For a teacher with positive beliefs about practical approaches, it can be frustrating to experience that one cannot use such approaches as often as desired because of actual or perceived external constraints. For a teacher with restraining beliefs, it can be equally frustrating to experience that others can make decisions while the teacher cannot. Either way, this leads to influence from "bad" conscience, a companion that seems to follow the mathematics teacher throughout teaching (e.g. Mellin-Olsen, 1996), or in the words of one of the project's participating teachers:

...in a way I feel that there are nuances related to being a very professionally confident mathematics teacher ... that is, I think that conscience, it can become intolerable, almost detrimental, if you are a mathematics teacher who is not discipline or didactically confident, because ... I have experienced these dilemmas myself, that you can have a bad conscience no matter what you do ... but this only gets worse

if you are discipline or didactically unconfident. Then you will almost develop a bad conscience no matter what. You will, probably, if you use a lot of practical activities ... you have been to courses and you have heard that it is the right thing to do ... and then you do it, and realise that you are losing time, and feel badly about that ... because you believe that you have done much wrong Or you can choose to keep up a steady pace getting through the whole textbook, and you cannot do [practical activities] ... and then you get a bad conscience because of that. So, I feel that disciplinary and didactic overview is extremely important in mathematics.

In this sequence, the teacher describes how the influence of external constraints challenges the teacher's professional knowledge and nurtures the state of bad conscience as the teacher continually faces the choice to use a practical activity. The level of professional knowledge in mathematics teaching influences strongly to what extent external constraints are allowed to affect the choices that are made. An increase in the teacher's professional knowledge strengthens the teacher's confidence about his/her teaching (Hill et al., 2007). This intended finding is confirmed through the findings of this project. In addition, when teachers give reasons related to overconsumption or shortage of practical activity use, they tend to relate to specific external constraints, for instance, textbook suggestions, time pressure or the pupils' mathematical skills. However, the origin of such reasons and what consequences a teacher's bad conscience about the use of practical activities might bring to the teaching are more important than particular external constraints (Moyer, 2001; Klette, 2003; Kjærnsli et al., 2004). When the teacher chooses to use a practical activity, the perception relating to the bad conscience is based originally on the teacher's professional knowledge and thereby his/her experiences of previous teaching, specifically his/her previous use of practical activities. External constraints are not important, but they are made important, and they become constraints that the teacher can distance himself/herself from.

5.3 Narrowing the gap between actual and designated teacher identity

A beliefs-changing process is a longitudinal and complex process in which the prevailing beliefs watch over, interpret and filter the signals stemming from experiences the teacher receives from practice. In addition to experiences, several external constraints might influence the teacher's choice of practice and interpretations of what might be changed. However, such constraints are not related to the teacher's beliefs, although from a system theoretical perspective they are likely to influence what develops into a "silent murmur"¹⁸ that eventually

¹⁸ This phrase is derived from a paper about developing a new discourse related to control of knowledge, by Mellin-Olsen (1992), where it appears in a quote taken from Foucault's (1985) "The Archaeology of Knowledge".

materialises through a paradigmatic change of beliefs (Engeström, 2001). From a more practice theoretical perspective, the teacher's change in beliefs seems to depend on an initiative from the teacher himself/herself, although the degree of consciousness of such an initiative is difficult to determine. If one is conscious about making comprehensive changes in teaching practice, it is plausible that the beliefs about teaching and the teaching practice are already inconsistent. In other words, one cannot be forced to change beliefs (Pehkonen, 2003; Wilson & Cooney, 2003). One can be forced to change practice, for instance by one or more influencing constraints, but this does not necessarily lead to a change in beliefs.

To minimise the influence of external constraints, there seems to be two main strategies to narrow the gap between actual and designated teacher identity in relation to choosing to use practical activities. Both strategies originate at a higher hierarchical influential level than the external constraints. First, the teachers' impressions of their own teaching priorities and ability to change their teaching, as shown in this project, call for reliance on the development of further disciplinary and/or didactic knowledge. In addition, another possibility for change is through the teacher's change in beliefs about mathematics teaching. This may lead to the development of reasons for choosing practical activities that are more coherent with the teacher's designated identity about mathematics teaching than before the initiation of the change process. Suppression of a designated teacher identity for the benefit of fulfilling an actual teacher identity, which one finds difficult to acknowledge, materialises as blaming restraining constraints for the lack of fulfilment of expectations provided by oneself or others. The reasons for choosing to use practical activities are then given on a basis of a wish to fulfil expectations that follow a teacher identity that is mirrored in society's expectations (as a compromise towards one's own understanding) or because of bad conscience. Practical dilemmas occur as consequences of both reasons. The reasons are given not on professional, but on practical grounds. The teacher can suppress his/her beliefs for some time, but has to figure out how to adapt either his/her own beliefs about the teaching or the teaching itself. This is a complicated process whose outcome cannot always be predicted, as shown in one of the sub-studies of this research project (see Section 4.2).

5.4 Choice of practical activities for professional reasons

Mathematics is not a subject that will always prove itself to be fun, and it is rarely possible to learn without hard work and concentration (Hadamard, 1954). But mathematics can provide moments of fun through the understanding of mathematical connections or solving of

exercises or problems. Increased emphasis on practical applications in the Norwegian national curriculum (KUF, 1996; KD, 2006a) is one initiative incorporated to enhance pupils' interest in mathematics through momentary usefulness and understanding. Several studies in the research literature on mathematics teaching and learning, as mentioned in Section 2.2, assume that practical activities boost pupils' learning of mathematics (e.g. Thompson, 1992a; Meira, 1995); however, such enthusiasm has faded in recent years (Moyer, 2001; McNeil et al., 2009). The development of use and appropriate use of practical activities has proven a challenge for many teachers (Klette, 2003; Kjærnsli et al., 2004). Such a situation may lead to the use of practical activities in mathematics for the wrong reasons.

The use of practical activities is an approach to mathematics that frames a large part of what mathematics as a utility subject is really about, and this approach is encouraged in the national curriculum (KD, 2006a). However, some teachers find it difficult, or do not make it a priority, to give disciplinary or didactically grounded reasons for including practical activities in their teaching. Expressions such as "it is fun" and "they do not think about it as mathematics" show a lack of professional teacher knowledge. External constraints or practical reasons are allowed to decide the choice. This project shows that some teachers struggle to find professional arguments or reasons for choosing to use practical activities, and therefore choose not to use them. This might not be what the teacher considers to be the ideal mathematics teaching situation, but the teaching is nevertheless in accordance with a coherent plan for both the short and long term.

In a feature article written in 2003 (Haara, 2003), I claimed that "Many pupils do not see beyond the activity they are involved in, and do not manage to attach the exemplification represented by the practical activity to a general theoretical connection." Based on the findings of this project, I am tempted to rewrite my own phrase and claim that "*Many teachers do not see beyond the activity they are involved in, and do not manage to attach the exemplification represented by the practical activity to a general theoretical connection.*" This is a rather serious claim to make, and it is based on the findings that there are differences between the professional reasons for using practical activities given on the basis of professional knowledge and the practical reasons given on the basis of external constraints. This means that one might criticise the conditions that many mathematics teachers work under in a society that acknowledges mathematics teaching based on the momentary utility effect.

Chapter 6: Conclusions and implications

The aim of this research project was to provide further research-based knowledge about the reasons teachers give for choosing practical activities in mathematics teaching. This chapter comprises two parts that together present the project's contributions of new knowledge about teachers' choice of practical activities in mathematics teaching. The first part presents the conclusions of the project. The second part discusses the implications that should follow the conclusions on the didactic, political and social levels, with priority on the implications for teacher education and teacher practice.

6.1 Conclusions

The conclusions of this project relates to two domains: the teachers' reasons for choosing practical activities in mathematics teaching and the more normative question of choosing practical activities for the "right" reasons.

6.1.1 Teachers' reasons for choosing practical activities in mathematics teaching

The analysis of the reasons given by teachers for choosing practical activities led to three main areas of reasons.

1. The importance of the teacher's professional knowledge.

Teachers put great emphasis on their professional knowledge when deciding whether to use a practical activity or not. The internal constraints brought to the decision process by the teacher are all attached to his/her beliefs. The findings indicate that beliefs about practical activities in mathematics teaching depend on the teacher's disciplinary and didactic knowledge. The teacher's level of disciplinary and didactic knowledge and the teacher's beliefs encourage or restrain the choice to use practical activities. This leads to both professionally and practically based reasons for using practical activities. Increasing professional knowledge will strengthen the impact of knowledge-based reasons. This will make the decision more resistant to the influence of external constraints and more content based. Practical activities will be used only when assessed to have a greater effect than alternative teaching approaches.

2. Compromises that the teacher might feel obliged to make against his/her own understanding of how the teaching should be.

On occasions the teacher experiences discrepancies between his/her beliefs about how mathematics should be taught and influences of internal and external constraints. This inconsistency means that the teacher's exposure to the influence of restraining or encouraging constraints is greater than it would be if the relationship between actual and designated teacher identity was coherent. When this influence leads to suppression of beliefs in order to fulfil personal designated expectations and/or expectations of others about choosing to use practical activities, the choice is based on practical reasons. The teacher feels obliged to make a compromise against his/her understanding of how the teaching should be.

3. Practical dilemmas that the teacher might experience because of the influence of constraints and experiences from previous teaching of mathematics.

The teacher's experiences with dilemmas associated with the use of practical activities in the teaching because of influence of internal and/or external constraints often results in an experience of bad conscience about the choice to use a practical activity. The reasons related to personal opinions such as a shortage or overconsumption of practical activity approaches are related by teachers to specific external constraints, such as time pressure or textbook suggestions.

6.1.2 Practical activities in mathematics teaching for the right reasons

KD (2006b) has already taken some measures to increase the professional knowledge of mathematics teachers, including increasing the emphasis on disciplinary and didactic knowledge of mathematics. If practical activities are to be chosen mainly for professional reasons and not practical reasons, additional measures must be taken. The teacher's autonomous space for the teaching needs to be expanded by reducing the expectation that teachers will use a homogeneous approach to mathematics teaching; this should be implemented in the national school policy, by school management and throughout society in general. Doing so would allow teachers to be able to prioritise more in accordance with both personal beliefs about mathematics teaching and the prevailing curriculum guidelines for mathematics teaching (KD, 2006a). Such autonomy should be given the necessary space, as articulated by one of the project's participating teachers.

Totally independent of my own skills, I mean that it is [the abstracting from a practical activity to theoretical content] really important, because it...is important that the pupils do not experience mathematics only as a lot of enjoyable happenings, instead of a lot of boring exercises. That would leave us in separate trenches, in my opinion, because these enjoyable happenings are supposed to help

the pupils understand, and motivate them to work with mathematics...but the mathematical competence is the ability to apply the terms and the calculation techniques, or in other words the theoretical competence. It is supposed to show itself in the ability to use mathematics, both related to written traditional exercises and related to day-to-day situations. And the situations they meet in their lives do not seem to be more similar to plastic pieces or other activities than they are to the written exercises which pupils used to work with. So, I am really concerned that if the mathematics is supposed to be limited to the classroom, then it is indifferent to me if they are related to plastic pieces or other pupil-centred activities. But if the mathematics does not move outside, so that the pupils can solve challenges on their own, then I believe that the result will be equally poor. Whether one is bored to death or amused to death, so to speak, becomes irrelevant.

On the other hand, if a teacher does not act in accordance with his/her beliefs when teaching mathematics, and particularly when choosing practical activities, there is a gap between actual and designated teacher identity. Such a gap sustains and escalates the influence of external constraint-based reasons for choosing practical activities. Professional confidence gained through an acknowledged level of professional knowledge makes it possible to narrow this gap. An alternative is to attempt to change beliefs about mathematics teaching to make the reasons for choosing to use practical activities more consistent with the actual teacher identity shown before the initiation of the beliefs-changing process, in other words making teaching more professional knowledge based. Then the designated teacher identity is changed.

6.2 Didactic, political and social implications

The journal editors' introduction to the article about mathematics teachers' knowledge-based reasons for using practical activities (see Section 4.1) focused on "an imbalance between the intended curriculum and the actual mathematics teaching taking place" (NOMAD, 2009: 4). They found that this "calls for further investigations of the reform process that has led to the current curriculum and of possible ways of supporting teachers' professional development in the process of implementing the curriculum" (ibid.: 4). Such a reflection shows that acting in accordance with the conclusions of this research project's implications should be considered from didactic, political and social viewpoints.

Basing a larger part of teachers' reasons for choosing practical activities in mathematics teaching on professional knowledge requires more emphasis on the process of implementing practical activities in school mathematics. This will strengthen the planning, use and follow-up of practical activities. If teachers are to choose practical activities for their teaching, they should be allowed to do so in a way that does not require them to ask whether these are an

appropriate use of time and resources or to make a questionable attempt to connect an activity to pure mathematics. Such emphasis reflects a didactic approach that should be given greater emphasis than it is given today in teacher education and in further education of mathematics teachers.

From the political viewpoint, measures are needed in at least two areas. First, further education for mathematics teachers within both the disciplinary and didactic knowledge of mathematics should be emphasised. Based on the hypothesis illustrated in Figure 6, extensive disciplinary and didactic knowledge are required for teachers to develop positive beliefs about practical activities for knowledge-based reasons.

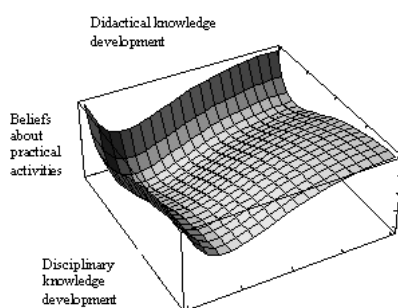


Figure 7: Replica of Figure 6: Beliefs about practical activities in mathematics teaching for knowledge-based reasons

This calls for a long-term strategy whose aim is to educate in-service mathematics teachers and teacher education students to higher disciplinarily and didactic levels to increase their professional knowledge in mathematics teaching (KD, 2006b). The other strategy is to provide the background to allow the mathematics teacher the autonomous space in which he/she with good conscience can consider the use of practical activities when appropriate. This calls for less emphasis on the control of how teaching of mathematics should be done and more autonomy for the teacher's decisions about teaching. The current Norwegian national curriculum (KD, 2006a) has already started such a process by liberating teaching priorities and didactic approaches compared with its predecessor (KUF, 1996). However, the current situation in school mathematics shows that there is still an imbalance between the mathematics teaching found in the practice field and the intended curriculum.

In a feature article written in an early phase of this research project (Haara, 2007), I argued for changing the opinion of mathematics teaching in school at the societal level. In the feature article, I made the following conclusion (my translation).

Most people need to master a limited level of mathematics in order to function in the life one is living. Society has taken the responsibility to organise welfare, and this brings anticipations about what tasks society should take care of on behalf of the inhabitants in society. Therefore, everybody has a personal interest in a well-functioning society. At the same time, we want to protect our opportunities for self-realisation and free choices. The challenge in this is obviously to unite these interests, and the school and the teacher become important parts of such a challenge. The school must to a greater extent put themes on the agenda and to a lesser extent settle for being a contributor in the organisation of family and working life. The teaching of mathematics must consider mathematics as a competitive subject when striving for the pupils' attention and teachers must be able to bring forward the possibilities for self-realisation, choices and personal development that are available through, for instance, mathematical competence. On some level, this will have consequences for how the teacher teaches mathematics and reflects about the interest that actually changes the teaching. But the pupils are not at school all the time. It is necessary to develop trustworthy and elevating associations between mathematics and society and between mathematics and individuals within society. This is why we all have to think about how we express ourselves when we refer to our experiences with mathematics and not undermine the attitudinal work done in school. Sometimes it is not the concrete problem that is the problem.

This impression of society's contributions to the challenges that mathematics as a subject and mathematics teachers as a group are facing has strengthened during the research project. The increasing reliance on practical activities in mathematics teaching will continue to meet societal requests about school mathematics. The positive, yet inadequately, documented consensus about this change (see Section 2.2) seems to give impetus so that the trend will not be reversed. Society and individuals must allow and support teachers' experiments with the implementation of practical activities in mathematics teaching that are based on professional knowledge of mathematics teaching. This requires changes in attitudes about school mathematics both inside and outside of school.

Chapter 7: Closure

This final chapter comprises two parts. The first part presents the limitations of the research project. The second part suggests some future studies to further understanding of teachers' reasons for using practical activities.

7.1 Limitations

From a research point of view, one must be neutral towards the research one is about to undertake. The researcher has a responsibility to act objectively and base the research on the interpretation of data in a way that secures validity and reliability. This means that “the researcher has an individual responsibility for the research activity, research theme and method, and for the quality of results” [my translation] (NENT, 2007: 14). A research project based on a hermeneutical approach shows that the researcher's position can be influential because of the possible active participation in the data collecting processes (see Section 3.2) and in the data interpretation (see Section 3.4). Although the researcher continuously strives for neutrality when producing and interpreting the data, these methodological processes are influenced by the researcher's preconceptions. These preconceptions are based on a horizon of understanding comprising an undefined number of prejudices (see Section 3.1). As the project moves forward, the preconceptions will change. In relation to this project, several prejudices could have been put forward as vaguely influential parts of my preconceptions, but the following two prejudices have probably been particularly influential.

1. *A personal scepticism about some teachers' use of practical activities that are not part of a general plan for the teaching and follow-up of the pupils in and after work with practical activities* (Haara, 2003; Haara, 2004)¹⁹.
2. *An impression of society as welcoming an increased orientation towards the usefulness of mathematics, with a lack of appreciation of mathematics as an educational subject and an emerging reliance on practical experience attached to mathematics as consequences* (Haara, 2003). The influence of the relevance paradox of mathematics is considerable: mathematics has an objective relevance for society but a subjective irrelevance for the individual, who can manage fine without much mathematical competence (Niss, 1994).

¹⁹ I have taught mathematics in the Norwegian compulsory school and upper secondary school and since 2001 have worked in teacher education training for compulsory school in Norway, and my preconceptions at the start of the project were to a considerable extent built on experiences in these situations.

I acknowledge that the influence of my subconscious preconceptions and prejudices is a limitation of this project. The researcher's role in qualitative data collection and hermeneutical interpretative processes (see Chapter 3) can influence the data production and interpretations of the data. This includes my role in this project, although I have strived to maintain an open-minded and neutral position in all parts of the project.

The choice of a strategic selection of participating teachers (Grønmo, 2004) is a methodological limitation of the project, which needs to be mentioned specifically. The data collection process chosen for the project was based on choices made with the aim of providing the best possible access to valid and reliable data within the framework set for the project and the sub-questions to be addressed by each sub-study. The most crucial choice in the methodological part of this project has been the process of recruiting participants, and from that perspective the possibility of a more random recruitment of interviewees. I decided to secure the recruitment of a broadly-based, but relatively small group of informants with different beliefs about mathematics and teaching of mathematics. Because I aimed to gain a perspective about the teachers' own reasons for their choices of practical activities in mathematics teaching, I believed that including a variety of informants was important for identifying different perspectives. As a researcher, I made the methodological choice to use a strategically selected group based on my professional knowledge about qualitative research. My prejudices probably influenced the choice through my conviction that the random recruitment of teachers might generate a homogeneous sample of teachers or might not produce interesting or relevant data. Thus, I influenced the selection of the participants in a convenience sample of teachers. I wanted access to teachers who I assumed would contribute to a deeper understanding of teachers' reasons for choosing practical activities in mathematics teaching.

In retrospect, the possibility of strengthening the reliability by randomly recruiting participants measured against the possibility of securing a relevant group of informants has some ethical implications. In the end, the recruitment included both my influence and that associated with random recruiting (see Section 3.3). From a self-realisation point of view, it is understandable retrospectively that this process is questionable. The choice was made based on an overall ethical evaluation of the recruitment process as the right one for this project. The influence of the two prejudices identified above guided the choice towards the direction of wanting to recruit a sufficiently representative group (here: the definition of an

acknowledged teacher (see Section 3.3)) and to maintain validity and reliability of data. I can only believe that my horizon of understanding on this occasion has included my own moral judgement as one of the other prejudices that influenced my decision.

Throughout the work on this project, I have tried to be critical of my interpretations and to be aware of possible biases rooted in my preconception and prejudices. Elements of communicative validity have been emphasised in the project. Data, analysis approaches and findings have been discussed with a second researcher, and in all the three data collection phases respondent validity (Kvale, 2006; Silverman, 2006) has been applied. In the first phase, the interviewed teachers read the transcribed interviews and were encouraged to approve the content or suggest changes. In the second and third phases, teachers were challenged to participate in the interpretations of the data. In the second phase, the two participating teachers interpreted the video-recorded observations of their teaching. In the third phase, the two teachers who were interviewed based on my interpretations of the answers to the open questionnaire, contributed by making adjustments to validate the interpretations. Hence, in the work on this dissertation, I have tried to present the voice of the participants including the interpretational influence by the participants. It is my belief that this has strengthened the validity of the findings of the project. Nevertheless, the data should be interpreted with caution. The validity will always be open to challenge because the data may be designated interpretations of the phenomenon. Because of the unconscious impact of personal prejudices and horizons of understanding, the participants might communicate their beliefs, for instance about the teaching of mathematics, that do not accord with their non-cognitive knowledge (van Manen, 1999).

7.2 Future studies

This dissertation identifies at least three directions for further research. First, it provides research-based confirmation of the “common agreement” that the emphasis on practical activities in mathematics teaching has a positive influence on the pupils’ learning of mathematics. Further studies are needed about how we can provide better frameworks for using such activities in school. We know that not all the practical activity work in mathematics teaching in school has been thought through well or assimilated into the rest of the mathematics lessons (Klette, 2003; Kjærnsli et al., 2004). We also know that teachers sometimes struggle to give professional knowledge-based reasons for choosing to use practical activities in mathematics teaching and what causes this uncertainty.

A second unresolved issue needing further investigation is the suggested relationship between acknowledged mathematics teachers' disciplinary and didactic knowledge and beliefs about practical activities (see Figure 6). The suggestion of a possible evolution of beliefs about practical activities related to the teacher's disciplinary and didactic knowledge is currently only a qualified hypothesis. It needs to be tested on a larger scale to determine whether it is a reliable basis for continuous development.

The third and final direction needing further research attention is the interpreted imbalance between the intended curriculum and the actual practice in mathematics lessons. Alseth, Breiteig and Brekke (2003) reported that many of the activities in the Norwegian mathematics classroom are still guided by traditional teaching approaches to mathematical content. Klette (2003), Kjærnsli et al. (2004) and Olsen and Grønmo (2006) reported that the use of practical activities has become a considerable part of the teaching in mathematics lessons and that the quality is sometimes questionable. This research project's findings contribute to the understanding of teachers' choice to use practical activities in mathematics teaching. We need to know more about the mathematics the pupils actually learn through the inclusion of practical activities in mathematics teaching.

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