

Increasing the use of practical activities through changed practice

A case-study examination of the influence of a value based intervention on two teachers' use of practical activities in mathematics teaching

Frode Olav Haara & Kari Smith

This study sets out to examine the influence of a value based intervention on two elementary school teachers' use of practical activities in mathematics teaching. The intervention was a "Values and Knowledge Education" (VaKE) based in-service course which introduced the two teachers to a practical activity supported value based approach to mathematics teaching. Interviews prior to the intervention made the teachers aware of an inconsistency between the desired and actual practice of their own teaching. The intervention provided them with a possibility to narrow the gap between vision and practice by changing practice. Qualitative data show how the VaKE-approach offered an alternative which opened up for increased use of practical activities in the teaching of mathematics, but also how good intentions of changing practice might be restrained or hindered by beliefs and previous experience.

Introduction

Today the educational policy in Norway (KD 2006) encourages the use of practical approaches to mathematics teaching¹. Using practical activities² is one way of doing this. However, Norwegian research shows that teachers find it difficult to change existing practice (Klette 2003; Kjærnsli et al. 2004) and that teachers of mathematics do not necessarily acknowledge the theoretical consensus supporting practical activities (Alseth, Breiteig & Brekke 2003; Haara & Smith 2009). If a teacher is going to use more practical activities, the teacher has to believe that such an approach supports students learning.

¹ From a mathematical didactical perspective the traditional teacher dominated teaching has been challenged by influence from theories about teaching and learning, ethno-mathematics and realistic mathematics education. In addition, the development of mathematics teaching on the Norwegian venue is influenced by societal factors. The Norwegian society needs to increase the part of the population which enter higher education in mathematics and science, a realisation which has given extra weight to the political and societal demands for development of additional or even change of working methods in teaching of mathematics in elementary school. This is a longitudinal and manifold process which has shown itself by an increased focus on practical relevance and use of practical activities in school mathematics as one domain of development. For a more thorough introduction to the background for changes in mathematics teaching for educational policy reasons, see Haara et al. (2009).

² In Haara and Smith (2009) we define a practical activity to include all forms of engagement where the pupil uses physical concretes while carrying out the activity at hand. That means the including of the opportunity for physical activity, and not just the use of artefacts or material found in the nature.

Values³ and Knowledge Education (*VaKE*)

VaKE is a teaching approach which emphasises developing students' moral and ethical values through acquisition of new disciplinary knowledge within a constructive learning environment (Patry, Weyringer & Weinberger 2007). Based on constructive theory of learning with foothold in both socio-cultural learning theory and radical constructivism, and influenced by Kohlberg's theory on moral development through social interaction (Kohlberg 1976), the teacher who wants to follow the *VaKE*-paradigm teaches through the introduction of a moral dilemma. This implies that the students have to choose between two possible decisions. Two fractions of students are then formed based on the students' decisions. This is followed by a moral viability check through discussion, first within each fraction and then between the two fractions. The need for new disciplinary knowledge to better illuminate different aspects of the topic and provide more coherent arguments through collecting new knowledge is revealed. Rounds of discussion and content viability checks on arguments are then possible, until both fractions are ready to present their conclusions as the final moral and content viability checks⁴. The teacher and the class close the sequence by capitalising on the whole process. Accordingly, the teaching is aimed at developing students' critical thinking, basic values and ethical principles.

Research question

In this article we examine the influence of the introduction to a value based intervention on two teachers' use of practical activities in mathematics teaching, based on the following two assumptions: First, elements of value and viability in regard to application of mathematics are not commonly used in order to increase the use of practical activities in school mathematics. It might therefore offer a new approach to the use of practical activities in mathematics teaching and initiate reflective processes regarding beliefs (Lerman 2002) about using practical activities in mathematics teaching. Experience with a different setting for practical activities might stimulate reflection regarding own beliefs, which is essential for a lasting change of practice (Wilson & Cooney 2002). Secondly, the introduction of new mathematical content in a *VaKE*- based learning environment entails a socio-cultural approach. In socio-cultural learning theory the construction of knowledge takes place through interaction or activities of social and cultural kind (Dysthe 2001). Conversation and joint activities are

³ The term *Values* in *VaKE* refers to the emphasis given to moral and ethical aspects through the use of dilemmas which challenge the students' opinion of right and wrong. Hence, in *VaKE* there is not an explicit element of value regarding the application of mathematics (Skovsmose 2002).

⁴ See Patry, Weyringer and Weinberger (2007) for a detailed review of each step of the *VaKE*-methodology.

crucial to learning, and each individual's development is recognized by changed participation in the practical situation. Communities of practice are important for the development of knowledge, and social factors become more than a frame surrounding the learning situation (Wenger 1998). Such features characterize an encouraging environment for practical activity based teaching (Bell 1993; Meira 1995; Wæge 2007). Therefore, an unmodified application of the *VaKE*-method can be applied when introducing new mathematical content supported by practical activities in an attempt to influence the teacher's use of practical activities. Based on the described prevailing situation, and the assumptions presented, our research question is:

How does the introduction to a VaKE-based teaching approach supported by practical activities influence two elementary school mathematics teachers use of practical activities in mathematics teaching?

Theoretical background

Beliefs

The *Teachers Matter* report (OECD 2005) confirms the important role teachers play in students' learning. According to the work of Shulman (1987) and Handal and Lauvås (1987) teachers' professional knowledge, which combines disciplinary knowledge, didactical knowledge and beliefs, is regarded as the most fundamental impact factor on teachers' professional choices. Furthermore, beliefs, values and attitudes can be seen as part of an individual belief system where the conviction about an issue or task often develops into "values, which house the evaluative, comparative, and judgemental functions of beliefs and replaces predispositions with an imperative to action" (Rokeach 1968: in Pajares 1992: 314). Such views imply that teachers' beliefs are fundamental factors influencing the teacher's practice, and that they influence disciplinary and didactical choices made by the teacher. Factors which make an impact on teachers' professional knowledge are dynamic features (e.g. Korthagen & Vasalos 2005), but the teacher's beliefs are seen as an impact factor which have been found to be difficult to challenge and to change (Thompson 1992; Borasi et al. 1999; Chin, Leu & Lin 2001; Wilson & Cooney 2002; Pehkonen 2003; Philipp 2007). Furthermore, the change of all other impact factors is more or less regarded as superficial and temporary if they are not in accordance with the teacher's prevailing beliefs (Lloyd 1999; Pehkonen 2003; Day 2004). It seems that if teachers are to make sustainable change in the teaching practice, their beliefs need to be challenged (Wilson & Cooney 2002).

Rokeach (1968) and Pehkonen (2003) look at different degrees of knowledge as subsumed in personal beliefs. Beliefs which are in accordance with an objective coherence in the surroundings are established as knowledge. Beliefs which remain as subjective knowledge are disputable, and therefore susceptible to be influenced by feelings (Grelland 2005) and personal evaluation of good or bad consequences (values) when transformed into action. In a review of research on teacher's beliefs Pajares (1992) identifies several commonalities concerning beliefs, summed up by Beijaard et al. (2000: 262) who suggest three common features of beliefs:

1. "They are highly individual, deeply personal, and seem to persist.
2. They are formed by past experiences.
3. They represent an individual's understanding of reality enough to guide thought and behaviour and to influence learning."

The understanding of beliefs as subjective knowledge influenced by feelings materialised through actions, and thereby defined as values, seems to be recognised as the way beliefs are visualised (Bishop 2001). Moreover, through the fundamental influence which beliefs have on interpretation of impressions and new knowledge, Pajares (1992) ascribes beliefs a filtrating effect on new impulses. This is in accordance with the fundamental position of beliefs emphasised in the research literature on beliefs in mathematics teaching (Pehkonen 2003). Beliefs are influenced by new impulses and make an impact on how impulses are interpreted (Feiman-Nemser & Remillard 1996).

Changing practice and changing beliefs

According to Kerem Karağac and Threlfall (2004: 137), with reference to Lerman (2002), the assumption within research on teachers' beliefs about mathematics teaching and learning has been "that awareness of a difference between beliefs and practice would result in some attempt to change". Within this research there is a growing body of research though, which report cases where the teacher either do not try to change even though he/she is aware of a difference between beliefs and practice (Kerem Karağac and Threlfall 2004) or simply do not become aware of such a discrepancy (Raymond, 1997). Hence, a discrepancy between beliefs and practice does not always call for an attempt to change.

However, change of beliefs increases the possibility to develop practical knowledge (Beijaard et al. 2000), but due to the presence of feelings, beliefs are found to be resistant to change (Pehkonen 2003). Independently of the content in presented arguments or experienced

practice, efforts are made to interpret the impressions to support prevailing beliefs. If it proves impossible to make such interpretations, the arguments or practice experiences are neglected or refused due to the influence of feelings, for instance by ignorance, irritation or even anger (ibid). Pehkonen further states that if a person's beliefs are supposed to change, it is a long process which demands personal engagement. Based on Shaw, Davis and McCarty (1991) Pehkonen (2003) suggests that the teacher must accept to be challenged with a problem, doubt or an inconsistency in attitude and practice and feel responsible to do something about it. The teacher must also have a vision of how teaching ought to be and prepare a plan for how the vision may be realised.

Shulman (1987) and Handal and Lauvås (1987) see the development of teaching practice as a cyclic process based on the impression that all impact factors are dynamic. According Kolb (1984) teachers' practical experiences generate observation and reflection and are based on general notions which are tested and developed in new situations. This provides the teacher with experiences at a higher level. The developmental process (*experiential learning*) is cyclic (Kolb 1984), as a helix. This process alternates between reflection and action (Korthagen & Wubbels 2001). Korthagen and Vasalos (2005) develop this further by focusing specifically on teachers' reflection and action attached to fundamental beliefs and views (*core reflections*). If one is supposed to change practice, both beliefs and actions must be changed. Such an impression on change of beliefs is also presented by Handal and Lauvås (1987: 12): "we experience our own practical efforts very much in the light of structures, concepts and theories transmitted to us in such a way that this may even lead us to change our values and beliefs to some extent". Teachers' *pedagogical content knowledge* (Shulman 1987) and teachers' professional development are influenced during and by practice.

In the essay "The Logical Categories of Learning and Communication" Bateson (1972) links *learning* to the element of *change*. According to Bateson a logical hierarchy of learning and communication can be identified and applied to suggest what priorities are relevant for change of teacher practice. The hierarchy consists of different levels of influence, with the levels of the hierarchy labelled 0, 1, 2 and so forth. With regard to change of practice and beliefs level 0 in the hierarchy is about receiving and developing actions (here: practice) based on internal or external signals received by the teacher. Level 1 relates to how the teacher acts and to changed actions in accordance with responses to experienced practice. Level 2 focuses on the teacher's internal responses to the experiences at Level 1. Level 2 then relates to change of

beliefs based on experiences initiated by practice (Level 0) and change of practice (Level 1). Hence, existing beliefs need to be challenged to create a permanent change of practice.

Independently of the chicken and egg discussion about what comes first, practice or beliefs, we agree with Pehkonen (2003) and Shaw, Davis and McCarty (1991) that the impact must stem from an experienced inconsistency between vision and practice. Transferred to the mathematics classroom this means that teachers must be given the opportunity to initiate change in teaching practice if change of beliefs is to be facilitated.

Methods

In this article we examine the influence of the introduction to a value based intervention on two teachers' use of practical activities in mathematics teaching. Since we wanted to focus on this particular excerpt of what might influence teachers' use of practical activities, we decided to apply a "two-case" comparative case study (Yin 2003; Flick 2006) to collect qualitative data. The approach was chosen due to its appropriateness when investigating "a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin 2003: 13). Secondly, we find that "case studies of teachers can be used intentionally to prompt teachers to reflect upon and examine their own beliefs and practices" (Thompson 1992: 143).

The data were collected from two teachers over a period of about eighteen months. Data collection instruments were multiple; interviews, video-recorded observations of teaching together with the teachers' own reactions and impressions about the content of the recorded lessons, log-writing and a questionnaire based on open ended questions. This is in accordance with Yin (2003: 14) who states that "the case study relies on multiple sources of evidence, with data needing to converge in a triangulating fashion". The importance of multiple sources of evidence offered by a case-study approach is also emphasised by research reviews on change of mathematics teachers' beliefs about mathematics and mathematics teaching (Thompson 1992; Wilson & Cooney 2002; Philipp 2007).

The two teachers, *Vivian* and *Walter* (pseudonyms), were recruited to the study by their respective principals upon our request of recruiting a teacher from their respective schools. We contacted these two schools because they were supposed to participate in an EU-FP7

project which aimed to try out *VaKE* in science teaching, but which did not make it to the final stage in competing for an EU-FP7 grant⁵. We asked the school principal to find a teacher recognised as an acknowledged teacher by the work environment (Haara & Smith 2009)⁶, and who was interested in developing his/her teaching of mathematics. Vivian has been teaching mathematics and other subjects in the Norwegian upper primary school (9 to 13) for ten years, and Walter has been teaching mathematics and other subjects in the Norwegian lower secondary school (13 to 16) for five years. They are about the same age, and both have 30 ETCS (*European Credit Transfer and Accumulation System*) in mathematics from their Norwegian teacher education.

An intervention was designed for the case studies (Lane et al. 2006). The intervention was a twenty hour long in-service course in *VaKE* held by one of the two researchers responsible for the research project, focusing on applying *VaKE* when teaching mathematics. The course consisted of two gatherings of two five hours long course days each, and focused on *VaKE*, areas on which *VaKE* is based (constructivism, value education, moral dilemmas in teaching), and on professional development of teachers. In-between the two gatherings the course participants prepared suggestions for themes and dilemmas for mathematics lessons based on the *VaKE*-method and how practical activities could be included in the mathematics lessons. The first gathering consisted of lectures presenting the course literature, and there was an emphasis on practical examples allowing for teaching of mathematics through moral dilemma supported by a practical activity. An example of which is focusing on airlines overbooking policy or salary payments for completed work. The second gathering focused on change of practice using themes and practical activities suggested by the two participating teachers, for instance about choosing between refreshing the playground at the school and expanding the computer facilities for the students of one class, or about delivering a tender for a house building contract.

The data collection period started when Vivian and Walter were interviewed about six months prior to the intervention. The interviews focused on their opinions on mathematics and school mathematics in general and their present and future teaching practice. Each semi-structured

⁵ EU-FP7 is EU's 7th framework programme for research and technological development, and the *VaKE*-project was one of the 8 finalists for the grant (Patry et al. (2007).

⁶ In Haara and Smith (2009), acknowledged teachers of mathematics are defined to be *teachers who are viewed as competent mathematics teachers by the principal and earn respect from colleagues, pupils and other groups of relevance within the working environment.*

interview lasted for approximately 75 minutes and was recorded and transcribed. Essences of meaning were extracted from the transcriptions (Kvale 2006) and interpreted through a hermeneutical approach. The interpretation process contributed to the planning of the forthcoming intervention since it offered impressions of how beliefs about mathematics and teaching in general, and more specifically about practical activities in mathematics teaching, were part of Vivian's and Walter's visions of teaching. These impressions also served as references for comparison in the analysis of data produced after the intervention.

Vivian and Walter were observed and filmed in three mathematics lessons each. The observations took place within a two week period starting about a month after the intervention. Observation data were collected when Vivian taught mathematics in 4th grade (9-10), and Walter taught mathematics in 8th grade (13-14). Respectively, the first lesson was typical for the kind of mathematics teaching which Vivian and Walter traditionally practiced, and the other two were based on the introduction of new mathematical content in a *VaKE*-based environment supported by a practical activity opportunity. Immediately after each lesson the teacher and the researcher who video recorded the lesson, watched it together. During these sessions Vivian and Walter were free to comment on what they saw (Jacobs & Morita 2002). This gave access to Vivian's and Walter's reflections and observations on the recent teaching experience. Comments and evolving discussions were recorded and transcribed.

The transcribed comments from the video-sessions were coded. From the comments made by the teachers we created units (Grønmo 2004) which were then categorised as "positive", "negative" or "neutral" (Jacobs and Morita 2002). Units including discussion of practical activities, isolated or within the progress of the *VaKE*-methodological structure, were divided into five subcategories and given an interpretation according to the teacher's comments: "positive - unconscious", "positive - conscious", "neutral", "negative - conscious", "negative - unconscious". This is in accordance with how people are conscious about some reactions and prevented from being conscious about other exhibited reactions. Unconscious reactions are difficult to explain. In other words, the observing teachers' reactions could be separated similar to the distinction between conscious and unconscious values (Bishop 2001; Grelland 2005).

Vivian and Walter wrote personal logs. They started on the day they received the in-service course information and reading list. The logs cover the last about 12 months of personal impressions about mathematics teaching, the in-service course, and experiences in accordance with both observed and independently conducted *VaKE*-lessons. The same categorising system as with the video-sessions was used in the analysis of the two logs, but based on systematic extraction of meaning of sequential content organized in a matrix (Grønmo 2004), structured by a timeline and the participants.

Exactly 12 months after the intervention started, Vivian and Walter responded to an open-ended questionnaire focusing on beliefs regarding factors with influence on their use of practical activities in mathematics teaching. The questionnaire was validated by three researchers and three mathematics teachers in elementary school, who commented on the relevance and clarity of the questions. The questions did not focus on *VaKE*, but were developed based on interpretations stemming from the analysis of the pre-intervention interviews, observations and video-sessions. The collected data were analysed in the same way as the logs, but the matrix was structured by the questions and participants.

Based on the analysis of the logs and questionnaires and in accordance with the interpretations of the pre-questionnaire analysis, Vivian and Walter were interviewed once more at the end of the project, about one month after responding to the questionnaire. The logs and questionnaires served as data producing devices in a triangulation quest for points of refutation and confirmation of pre-questionnaire interpretations. The interviews were structured, and the interview guide was divided into three main parts:

- The teacher's beliefs about mathematics and practical activities in mathematics.
- The teacher's response to the value based intervention.
- The influence of the intervention on the teacher's teaching of mathematics.

In a hermeneutical perspective our interpretations in the analysis have probably been effected by our unconscious prejudices, although the triangulation process and validation by Vivian's and Walter's interpretations strengthened the viability of our conjecture suggestions and the subsequent discussion of how the intervention influenced the teachers' use of practical activities. Hence, in the analysis we used both a phenomenological approach and a hermeneutical approach (Grønmo 2004). The phenomenological approach is recognised in the use of Vivian's and Walter's experience with the intervention programme as basis for the

analysis. The hermeneutical approach is reflected in the comparison of the influence of the intervention with the pre-intervention situation as well as similarities and discrepancies between the two teachers' beliefs about the teaching of mathematics.

Findings

The findings are reported through a description of beliefs Vivian and Walter had about mathematics and practical activities in mathematics, their response to the value based intervention and use of *VaKE* supported by practical activities in teaching. This follows the pattern of four phases for teacher change, reported by Shaw, Davis and McCarty (1991) and Pehkonen (2003):

- experiencing personal inconsistency
- feeling responsible to do something about the inconsistency
- developing a vision of how teaching ought to be
- making a plan for how the vision can be realised

Vivian

Experiencing personal inconsistency

Vivian was in the pre-intervention interview fairly open about her own lack of understanding of generalized mathematics and that she did not always see the application of theoretical dimensions in real-life situations. She was more focused on mathematics in a strictly real-life context, with an emphasis on practical application of mathematics. Furthermore, she was an active teacher, who enjoyed being the focus of attention and to explain the mathematical content at hand, as she explained during the pre-intervention interview:

Vivian: I think I am very present...and very active. In a mathematics lesson which could actually be boring, I still feel that I am creative, and I feel...I think that my problem maybe is that I am too...ehh...active. So what happens...especially in mathematics...what happens when I am about to explain something...then it is like *Oh yes!* (changes her voice), and then I like to use things which they know. *Imagine!* (changes her voice again)...and then I tell a little story about something...)

Vivian used narratives and relied on the students' imagination when using examples in teaching. In her opinion the teacher had to explain the mathematical content to the students, and then the students had to do quite a lot of exercises to internalise the content. Kuhs and Ball (1986) refer to this "as content-focused with emphasis on conceptual understanding". The students' understanding of ideas and processes is emphasised through the instruction of the mathematical content, and the lessons might vary considerably from lesson to lesson. It

was important to Vivian that the students both have fun and learn, and that they are offered some exiting experiences when learning mathematics. In accordance with Ernest's (1989) recognised pattern for an *Explainer's* use of curricular materials this meant to Vivian that the textbook approach was enriched through her introducing of additional examples, problems and activities of real-life relevance.

The pre-intervention interview revealed that Vivian was confident that her students learned mathematics, but she was not satisfied with her own organizing priorities. She meant the lessons ought to be more varied, and she wanted to be more in accordance with what Kuhs and Ball (1986) refer to as "learner-focused", in the sense of focusing the teaching more on the students' active involvement. She therefore experienced an inconsistency between her teaching and her beliefs about how mathematics ought to be taught.

Feeling responsible to do something about the inconsistency

Vivian was clear about her bad conscience for what she experienced as a lack of variation in her teaching. In her opinion, the content based teaching of mathematics for which she had been an exponent, with emphasis on the progress and approaches suggested by the textbook, ought to be supported by an expanded organisational repertoire, as she stated during the final interview in the project:

Vivian: My mathematics teaching ought to consist of exercises which the students master, exercises which challenge the students, use of the textbook, use of different tools and props, collaboration among the students, individual work, work through theoretical approaches, work through practical approaches, and so fort. I would like my teaching to be varied.

Developing a vision of how teaching ought to be

Vivian wanted her teaching to be more varied and student focused. She also wanted to make her instructive *Explainer*-role less dominant. The introduction of practical activities supported by a *VaKE*-based approach provided her with an opportunity to change her practice, as she concluded during the observation of one of the video-recorded *VaKE*-based lessons:

Vivian: I have missed such an approach in mathematics... I have needed something to change my teaching of mathematics with, and this is what I have been missing!

Making a plan for how the vision can be realised

On two occasions three weeks after the in-service course Vivian used dilemmas which she found relevant to the students' real-life interests. The first dilemma depended on, in terms of mathematics, economical calculations related to choosing between computer accessories for the involved students and a new climbing frame area for all students in school. The second dilemma involved economical and volume calculations related to choosing between a party for the entire school to celebrate the new climbing frame area, and refurbishing the school entrance. The students had access to props. In the first lesson it was fake money, and in the second lesson it was drinking glasses, decilitre and litre measures and free access to water. The dilemmas required the students to work with the four arithmetical operations, money values, estimation, measuring and geometrical figures. The props made it possible to practically systematise information and carry out operations which initiated, simplified and confirmed or refuted the students' calculations.

Vivian was conscious about her neutrality while applying the *VaKE*-approach, but she was really into setting "a conflict zone". The competitive organisation appealed to her. She reorganized the classroom before the lessons, initially grouping the students on the floor. Vivian clarified the moral dilemma and each student made a written, initial decision on the dilemma. Based on the students' decisions she then divided them into two groups, separated by a front-line. "It is you against them!", she said several times referring to the students on the other side, respectively.

When observing the video recording of her own teaching, she reported that she could see that the *VaKE*-approach introduced a new organizational possibility to her mathematics lessons:

Vivian: And that is just what this math builds on. That you actually do not only sit and work on some numbers, you actually go into yourself a bit...because when you start to tear at something inside yourself, you automatically become more motivated, and then you approach the problem in another way than you would do if you just sat there.

At the same time she claimed that the new method occasionally resembled her regular approach:

Vivian: ...and I have got something of a revelation by entering this project, and I now feel that one of my strengths is that I have motivated students...and that the reason for that maybe is because I challenge them in relation to themselves to some extent...

She was familiar with challenging the students and putting them up against each other, but not in such a planned and structured way. This was supported by the video-recordings which

showed that she was comfortable with the organisational demands of the *VaKE*-method and that she was able to let the students and the method set the pace of the lesson.

In the interview at the end of the project Vivian revealed that she believed that her teaching of mathematics and the use of practical activities in the teaching had changed:

Researcher: Did your use of practical activities change after you were introduced to *VaKE*?

Vivian: Yes, it is much more...it is no longer so structured. Now I start trying to make the students curious, investigative and uncertain for a while. I give them a challenge which involves them, and then...they can get a feeling of solving, and I can focus on challenges which occur. So it is a bit different now.

Walter

Experiencing personal inconsistency

Whereas Vivian was content with focusing on practical applications, Walter found it in the pre-intervention interview important to emphasise both the theoretical dimension and the practical applications of theoretically based results:

Walter: Well, it is a theoretical subject, but at the same time one can approach it in a practical way, and I feel that is very important.

Furthermore, Walter and Vivian held different views about how mathematics ought to be taught. In the pre-intervention interview he emphasised, as Vivian did, that the teacher should explain the mathematical content and that this should be followed by the students' work on exercises. But the observation of lesson 1 showed that Walter taught in a more traditional way than Vivian did. He explained the new content and examples to the students before they worked on exercises. Finally Walter gave a summary of the lesson. Whereas Vivian focused on motivating the students, Walter to a larger extent wanted mathematics as a subject to be self-motivating, as he reveals through his description of his mathematics lessons in the pre-intervention interview:

Walter: ...and traditionally school mathematics is kind of a mix between a theoretical review, usually using the blackboard, and a conversation with the students, and then this is combined with solving exercises in the textbook. That is in a way how I have experienced mathematics myself through my own schooling, and how I to a large extent teach myself...although I sometimes perhaps would have wished that I could vary my teaching more.

Walter's teaching seems to be in accordance with a "content-focused view with emphasis on conceptual understanding" (Kuhs & Ball 1986), but it is, to a larger extent than Vivian's

teaching, “content-focused with emphasis on performance” (ibid). In this approach it is assumed that acquiring the content motivates for further studies and practical applications.

In the pre-intervention interview Walter expressed beliefs about mathematics as a general education subject:

Walter: Everybody needs mathematics. That is, a certain basic mathematical knowledge...in order to make reasonable, good choices. And one will be confronted with it no matter what...regardless of profession...if not with pure, formal mathematics, then certainly with a mathematical way of thinking.

Researcher: Are you thinking about the terms which you used earlier [in the interview], like problem solving, logical reasoning, and structuring...?

Walter: Yes! Because I mean that mathematics is an educational subject which structures one's thoughts...which I often miss among the students. If they are given some kind of problem or exercise or something, they are not able to see logical flaws, and in my opinion that has to do with mathematical thinking...

In Walter's opinion the educational subject dimension of mathematics seems to vanish as an argument for maintaining interest in learning mathematics if compared to the legitimacy of the general education dimension in mathematics which he remembered from his own time as a student. He sees mathematics as an educational subject based on concepts like curiousness, logic and persistence, but mathematics proves not to be as self-motivating to the students as he would expect it to be. In fact, he reveals that he always has bad conscience for his lack of practical activity based teaching. A more varied lesson structure would hopefully increase the students' interest in mathematics, as he reveals in this sequence from the pre-intervention interview:

Walter: I do have to say...I have always had an ambition to use practical activities in mathematics because I think it is a very useful approach if you can combine it...with another kind of mathematics teaching, so that the students are given a balance towards...well, solving of exercises and such. And I must admit that I have always had a bad conscience for my lack of practical activity based teaching.

Walter seemed to be influenced by both a “content-focused view with emphasis on conceptual understanding” and a “learner-focused view” (Kuh & Ball 1986), but suppressed the influence due to bewilderment about how to change his teaching, which becomes apparent during the pre-intervention interview:

Walter: It is a bit about...that I am not used to using it, and I spend much more time in preparing such activities. And, obviously, you did not get trained in such teaching during teacher education. And that...and that puts you...and the textbooks do not emphasise such teaching either, and that leaves you to...to your own...oh, what is the word I am looking for?...That is, my own...you have to rethink, maybe be a bit creative, and that...is maybe a bit time consuming in a...well, in the hectic school day.

Walter's traditional teaching is a compromise between his beliefs about how mathematics ought to be taught and his awareness of the advantage of emphasising structure, performance and textbook applications when teaching mathematics, a phenomenon previously shown by, for instance, Cooney (1985), Raymond (1997) and Lloyd (1999). Hence, Walter experienced a personal inconsistency between his beliefs about mathematics teaching and his actual teaching, since his teaching lacked variation and did not prioritise practical activities in the way he wanted.

Feeling responsible to do something about the inconsistency

As Vivian, Walter expressed a kind of guilt feeling for lacking variation in his teaching. Moreover, in the pre-intervention interview he was not entirely willing to accept the students' prevailing opinion which saw mathematics from a utility perspective only:

Walter: For instance, I remember compared to my own schooling, I thought it was really funny to get some practical...the daily puzzle or things like that to work on. But when I try such problems with students...they do not seem to see any point in it...*Well, what is this then? Are we supposed to do...*(changes his voice). Often they do not understand the problem at all. They are not used to think in a...in a mathematical way.

He therefore felt that instead of the rather traditional teaching, he should teach more in accordance with a "learner-focused view" (Kuhs and Ball 1986), and include more practical activities in his teaching.

Developing a vision of how teaching ought to be

Walter did not have the same starting point regarding the *VaKE*-approach as Vivian and based on the organisation of his regular teaching, Walter's vision implied a more radical change of practice. The intervention introduced Walter to an approach which he believed could challenge the present suppressing of his mathematics teaching beliefs, as seen on separate occasions in his log during the in-service course:

"Making teaching more realistic is a massive challenge, especially when compared to one's own view about what teaching is, and ought to be. I believe the *VaKE*-project to be useful in this respect"

"I especially approve of using such a methodology as an approach to teaching mathematical content, and then later on concentrate on the theoretical approach to the mathematical topic at hand. I believe that the students are more easily able to see that what we are supposed to learn is relevant to learn, that this is something which they actually may find useful."

Making a plan for how the vision can be realised

As Vivian, Walter on two occasions about three weeks after completing the in-service course, taught through introducing two dilemmas which he found relevant to the students' real-life interests. The first dilemma depended on economical calculations and the calculation of an area of a planned house where a compounded area consisting of different geometrical shapes represented the new mathematical content. In terms of mathematics the students worked on calculating construction costs. The second dilemma was about a non-regular pyramid shaped box of chocolate pudding and a lack of coherence between the quantity of pudding stated on the package and the measured quantity of pudding in the package. The new mathematical content was represented by a pyramid shaped polyhedron, the theorem of Pythagoras, and the connection between cubic centimetre and decilitre. In the first lesson the practical activity equipments were the traditional compass, protractor and ruler, but in the second lesson these were accompanied by an actual package of the chocolate pudding polyhedron.

The observations of the lessons show that Walter experienced some challenges. He struggled to find his position in the context, and the students were not sure about what was expected of them. They seemed curious and interested at first, but the lessons did not work out the way Walter had planned. The dilemma discussions did not develop as planned for two reasons. One of the discussion groups was outnumbered in both *VaKE*-based lessons, and Walter did not succeed in pushing the two groups to find arguments and discuss in favour of the group's point of view. In the end Walter found the lessons to be rather boring and worthless, an impression which he states explicitly in his log after both *VaKE*-based lessons:

“I had the first *VaKE*-session today, and it was done pretty much the way I had planned. I would perhaps have hoped for more engagement from the students, but it turned out to be rather boring.”

“I had the chocolate pudding session today, and I have to say that the lesson was not a success. I felt that the dilemma at hand engaged only a few of the students.”

As the observations of the video-recordings proceeded, Walter expressed doubts about his loyalty to the *VaKE*-approach in mathematics teaching. In his opinion, he did not seem to be able to make the students aware of the moral aspects of the dilemmas. In fact, he changed his view on the *VaKE*-approach as he gained more experience with it. When observing on video himself and the class in the second lesson he stated that the *VaKE*-approach would be appropriate to use after the mathematical content had been introduced in another way, instead of combining the introduction of mathematical content and the value emphasis:

Walter: In general I mean that such approaches...*VaKE*-approaches in relation to mathematics, would be best to

have when you have finished a mathematical topic. Because then you can use the knowledge, put it into a setting which in a way creates engagement and shows that you need mathematics in daily life.

(...) Because...if you do it when you are introducing a mathematical topic, I believe...that the students will find it difficult to do the necessary calculations, and then the foundations disappear for some of the arguments which they may put forward...)

In his opinion, the calculations which the students would need to do in order for the dilemma discussion to become active, were too complicated, a situation which is also described by Lloyd (1999). It would therefore be better to revisit mathematics which they had learned in a traditional manner by applying it in a *VaKE*-based context. In the end Walter argued for his usual teaching to be a way to make the students better prepared or disciplinary skilled enough in a mathematical theme before relying on mathematical arguments in discussions focusing on moral dilemmas, as shown in this concluding comment from the same video observation session:

Walter: As a way of teaching it obviously brings along more noise, and it becomes a bit more difficult to see what each student actually does. If they are seated at separate desks it gives me a much better overview... what each student does, if he is disturbing or not... (...) Students who work in groups often make teaching more complicated than when students work individually.

Discussion

The pre-intervention interviews revealed that both Vivian and Walter claimed that they believed in applying practical activities in mathematics teaching, and that they were interested in changing their practice in order to increase the use of practical activities. They did not find their current teaching to be in accordance with personal visions, and they struggled to find personal acceptance for increased reliance on practical activities in mathematics teaching. A change of practice towards an increased use of practical activities would therefore only be temporary or superficial unless the change made an impact on their beliefs and didactical knowledge (Bateson 1972; Wilson & Cooney 2002).

Vivian was enthusiastic about the theoretically supported approach to mathematics teaching provided by the value based intervention. It acknowledged elements of her previous teaching, and she referred both to how she was influenced and how she experienced excitement among the students. "I have probably never seen the students this engaged!", she said during the observation of the first *VaKE*-based lesson. Vivian became more aware of her own role and about making the students engaged without her direct involvement and guidance. Hence, her role as a facilitator became more important (Ernest, 1989), and she personally felt that she had

experienced a kind of revelation by participating in the study. Finally, her impression of the students' work with mathematical content was also influenced. She experienced the group work as coherent with her opinion about students being active learners, an opinion which she reported she had not been able to include in mathematics teaching in the same way as she had done when teaching other subjects.

Walter was also enthusiastic at first, but developed a resistance towards the thought of introducing new mathematical content through the *VaKE*-approach supported by practical activities as the experience with the approach increased. Walter experienced that the positive expectations which followed the in-service course disintegrated when he applied the *VaKE*-approach in his own teaching. This feeling was reinforced by watching video-recordings of his lessons, all of which provided a setback regarding his vision of how to change practice. Making his suppressed beliefs about how mathematics ought to be taught explicit once more seemed to capitulate to the prevailing and familiar way of teaching mathematics. Similar situations are described by Raymond (1997) and Kerem Karağac and Threlfall (2004), but the case of Walter refers to a situation where the teacher actually attempted to change practice. He experienced constraints which prevented him from further consideration of the new approach as a possibility to learn mathematics through a new perspective and increase the use of practical activities. His rather modest level of didactical knowledge of mathematics, revealed in the pre-intervention interview through his bewilderment about how to arrange for appropriate use of practical activities, and the response from the students to his new approach to teaching strengthened this impression. Hence, he withdrew to the established form of teaching familiar to himself and the students.

Vivian and Walter experienced the *VaKE*-approach in different ways, which led to different outcomes. Vivian maintained her enthusiasm about a value based approach supported by practical activities. Walter did not. The main reason for this, in our opinion, is found in the different starting points of the two teachers. Vivian's beliefs were not challenged to the same extent as Walter's beliefs were. Her vision of teaching proved to be within an approachable reach. The discrepancy between Walter's beliefs and experiences of constraints given by his teaching practice of mathematics and the actions used in the value based approach was too wide, and in a way he "broke" the cycle of reflection and action necessary to change practice and beliefs (Kolb 1984; Korthagen & Vasalos 2005). In the in-service course it was created a community of learning for Vivian and Walter (Wenger 1998). Vivian entered a productive

moderation process since her beliefs were not severely challenged and her students did not meet a teaching approach which was totally different from what they had experienced before. Vivian and her students were able to explore the new approach together. Walter's beliefs were deeply challenged and his students met a teaching approach which was quite alien to them. Walter therefore lacked the moderation process which Vivian so successfully benefitted from. Having said this, though, professional growth can take the form of maintaining present beliefs after having had the courage to challenge them. Walter tried to change his practice and had the courage to challenge his beliefs about using practical activities for teaching mathematics, but this did not lead to change due to the influence from what he experienced as restraining constraints.

Conclusions

Changing beliefs about teaching of mathematics is an extensive and longitudinal process (e.g. Wilson & Cooney 2002; Pehkonen 2003). Change of beliefs and change of practice can be independent of each other, they are not synonymous. However, change of beliefs and change of practice are often tangled in such a way that when one is changed it will cause change to the other. In this study we aimed at examining the influence of the introduction to a value based intervention on two teachers' use of practical activities in mathematics teaching. The two teachers, Vivian and Walter, were introduced to a value based approach to teaching mathematics that opened for practical activity support opportunities which implied a change of practice for them both. From this study we can note that Vivian approved of the alternative practice both as a teaching approach and as a possibility to increase the use of practical activities, while Walter did not. A more thorough examination of the study reveals, however, that the change of practice challenged both Vivian's and Walter's beliefs about how to teach mathematics and the possibilities for using practical activities. It is a common impression that beliefs have a filtering effect on new impulses (Pajares, 1992; Beijaard et al., 2000; Philipp 2007), and since the applied change of practice was not too controversial in relation to Vivian's prevailing beliefs, her positive attitude towards an increased use of practical activities and student involvement was strengthened. Walter found the change of practice to be too controversial in relation to his prevailing beliefs, and instead of maintaining the positive attitude towards increasing the use of practical activities through applying the value based approach nurtured by the offered intervention, he returned to the previously established teaching practice as the preferred way of teaching mathematics.

Regardless of the tangled question whether a change of practice implies change of beliefs, or if change of beliefs implies change of practice (e.g. Bateson 1972; Kolb 1984), we are left with the impression that Vivian managed to offer the new teaching approach to the students in a way which appealed to them, while Walter did not. There might be several encouraging or restraining constraints which paved the way for such a course of event, and the impact from different constraints are not necessarily similar for Vivian and Walter. Nevertheless, we find that three constraints on this occasion need to be mentioned on behalf of both teachers. First of all, we would like to mention the two teachers' beliefs about teaching mathematics and their didactical knowledge as crucial impact factors. Second, the impact of the intervention in which Vivian and Walter participated must be acknowledged. Third, the students' response to the new teaching approach probably also played a role in forming Vivian's and Walter's acceptance of the use of a value based teaching approach supported by practical activities. In order to maintain a changed practice, it seems that the changed practice must also lead to a change of beliefs. If not, practice will eventually drift back to its initial pattern or to something less radical than the alternative practice. The isolated findings in this study show that Vivian entered a process which might lead to increased use of practical activities in her future teaching, whereas Walter in the end found his traditional way of teaching to suit him better. For Walter this return implied staying faithful to the explicit practice he upheld when entering this study, as his professional conscience did not allow for increased use of practical activities.

In this article we base our cautious suggestions on interpretations of data stemming from the cases of two teachers' experiences with the introduction to a value-based approach to change practice in mathematics teaching. The interpretations have been validated by the two teachers through a triangulating process. Our temporary interpretations were tested and reformulated in the light of their logs and responses to an open questionnaire. Finally the interpretations were validated by conducting individual interviews with the two respondents which allowed for their personal interpretations. Hence, the contextual interpretations are based on multiple data sources and we believe the interpretations to be well justified, despite the limitations of basing a study on a relatively small and narrow empirical source (Yin 2003).

We want to conclude that the in-service course which emphasised the use of practical activities in mathematics teaching through a value based approach to new mathematical

content, influenced one of the participating teacher's beliefs about teaching mathematics and increased the space given to practical activities in her teaching. Furthermore, we add another study to the body of research which confirms that awareness of difference between beliefs and practice will result in some attempt to change (Lerman 2002; Kerem Karaağac & Threlfall 2004). But the case of Walter shows that the influence from restraining constraints might result in an aborted attempt to change. We hope that Vivian's and Walter's reported struggles and challenges with the correspondence between beliefs and practice will bring about further research on persistent change of teachers' practice.

References

- Alseth, B., Breiteig, T. & Brekke, G. (2003). *Evaluering av Reform 97. Endringer og utvikling ved R97 som bakgrunn for videre planlegging og justering – matematikkfaget som kasus* [Evaluation of Reform 97. Changes and development with R97 as the background for further planning and adjustment – the case of school mathematics; in Norwegian]. Notodden: Telemarkforskning.
- Bateson, G. (1972). The Logical Categories of Learning and Communication. In G. Bateson, *Steps to an ecology of mind*. Chicago: The University of Chicago Press, 279 - 308.
- Beijaard, D., Verloop, N., Wubbels, T. & Feiman-Nemser, S. (2000). The Professional Development of Teachers. In R. J. Simmons et.al. (Eds.). *New Learning*, Dordrecht: Kluwer Academic Publishers, 261 – 274.
- Bell, A. (1993). Principles for the Design of Teaching. *Educational Studies in Mathematics*, 24, 5 – 34.
- Bishop, A. (2001). Educating Student Teachers about Values in Mathematics Education. In F.-L. Lin & T. J. Cooney (Eds.). *Making Sense of Mathematics Teacher Education*, Dordrecht: Kluwer Academic Publishers, 233 – 246.
- Borasi, R., Fonzi, J. Smith, C. & Rose, B. J. (1999). Beginning the process of rethinking mathematics instruction: A professional development program. *Journal of Mathematics Teacher Education*, 2, 49 – 78.
- Chin, C., Leu, Y.-C. & Lin, F.-L. (2001). Pedagogical Values, Mathematics Teaching, and Teacher Education: Case Studies of Two Experienced Teachers. In F.-L. Lin & T. J. Cooney (Eds.). *Making Sense of Mathematics Teacher Education*. Dordrecht: Kluwer Academic Publishers, 247 – 269.
- Cooney, T. J. (1985). A Beginning Teacher's View of Problem Solving, *Journal for Research in Mathematics Education*, 16(5), 324 – 336.
- Day, C. (2004). *A Passion for Teaching*. London and New York: RoutledgeFalmer.
- Dysthe, O. (Ed.) (2001). *Dialog, samspel og læring* [Dialogue, interaction and learning: in Norwegian]. Oslo: Abstrakt forlag.
- Ernest, P. (1989). The Impact of Beliefs on the Teaching of Mathematics, In P. Ernest (Ed.), *Mathematics Teaching: The State of the Art*. London: Falmer Press, 249 – 254.
- Feiman-Nemser, S. & Remillard, J. (1996). Perspectives on Learning to Teach, *The Teacher Educator's Handbook: Building a Knowledge Base for Teaching*, San Francisco: Jossey-Bass Publishers, 63 – 91.

- Flick, U. (2006). *An Introduction to Qualitative Research, 3rd Edition*. London: Sage.
- Grelland, H. H. (2005). *Følelsenes filosofi* [The philosophy of feelings: in Norwegian]. Oslo: Abstrakt forlag.
- Grønmo, S. (2004). *Samfunnsvitenskapelige metoder* [Methods in social science; in Norwegian]. Bergen: Fagbokforlaget.
- Haara, F. O. & Smith, K. (2009). Practical activities in mathematics teaching—mathematics teachers' knowledge based reasons. *Nordic Studies in Mathematics Education, 14, 3, 33 -54*.
- Haara, F. O., Stedøy-Johansen, I., Smith, K. & Kirfel, C. (2009). Mathematics between society and the individual. In C. Winsløw (Ed.), *Nordic Research in Mathematics Education. Proceedings from NORMA08 in Copenhagen, April 21st – April 25th 2008*, 353 - 357. Rotterdam: Sense Publishers.
- Handal, G. & Lauvås, P. (1987). *Promoting Reflective Teaching: supervision in action*. Milton Keynes: Open University Educational Enterprises Ltd.
- Jacobs, J. K. & Morita, E. (2002). Japanese and American Teachers' Evaluations of Videotaped Mathematics Lessons. *Journal for Research in Mathematics Education, 33(3)*, 154 – 175.
- KD [Ministry of Education and Research] (2006). *Læreplanverket for kunnskapsløftet* [The curriculum for the primary and secondary school; in Norwegian]. Oslo: KD.
- Kerem Karağac, M. & Threlfall, J. (2004). The tension between teacher beliefs and teacher practice: The impact of the work setting. In *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 3*, 137 – 144.
- Kjærnsli, M., Lie, S., Olsen, R. V., Roe, A. & Turmo, A. (2004). *Rett spor eller ville veier? Norske elevers prestasjoner i matematikk, naturfag og lesing i PISA 2003* [Right track or wrong direction? The achievements of Norwegian students in mathematics, science and reading in PISA 2003; in Norwegian]. Oslo: Universitetsforlaget.
- Klette, K. (Ed.) (2003). *Klasserommets praksisformer etter reform 97* [The practice styles of the classroom after reform 97; in Norwegian]. Oslo: Pedagogisk forskingsinstitutt.
- Kohlberg, L. (1976). Moral stages and moralization: The cognitive-developmental approach. In T. Lickona (Ed.), *Moral Development and Behavior: Theory, Research and Social Issues*. NY: Holt, Rinehart and Winston, 31 – 53.
- Kolb, D. A. (1984). *Experiential Learning: Experience as the Source of Learning and Development*. Englewood Cliffs, NJ: Prentice Hall.
- Korthagen, F. & Vasalos, A. (2005). Levels in reflection: core reflection as a means to enhance professional growth. *Teachers and Teaching: theory and practice, 11(1)*, 47 – 71.
- Korthagen, F. & Wubbels, T. (2001). Learning from Practice. In F. Korthagen (Ed.) *Linking Practice and Theory*, Mahawah, NJ: Lawrence Erlbaum, 32 – 50.
- Kuhs, T. M. & Ball, D. L. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills, and dispositions*. East Lansing: Michigan State University, Center on Teacher Education.
- Kvale, S. (2006). *Det kvalitative forskningsintervjuet*, 8. utgave [Interviews. An Introduction to Qualitative Research Interviewing, 8th Edition; in Norwegian]. Oslo: Gyldendal akademisk.
- Lane, K. L., et al. (2006). Illustrations of Function-Based Interventions Implemented by General Education Teachers: Building Capacity at the School Site. *Education and Treatment of Children, 29(4)*, 549 – 571.
- Lerman, S. (2002). Situating research on mathematics teachers' beliefs and on change. In G. C. Lehrer, E.

- Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education*, 233 – 243. Dordrecht: Kluwer.
- Lloyd, G. M. (1999). Two Teachers' Conceptions of a Reform-oriented Curriculum: Implications for Mathematics Teacher Development, *Journal of Mathematics Teacher Education*, 2, 227 – 252.
- Meira, L. (1995). *Mediation by tools in the mathematics classroom*. In Proceedings of the 19th annual conference of the International Group for the Psychology of Mathematics Education, Volume 1, 102 – 111.
- OECD (2005). *Teachers Matter. Attracting, Developing and Retaining Effective Teachers*. OECD Publishing.
- Pajares, M. F. (1992). Teachers' Beliefs and Educational Research: Cleaning Up a Messy Construct. *Review of Educational Research*, 62(3), 307 – 332.
- Patry, J.-L., et al. (2007). Values and Knowledge Education in Science Teaching. *Proposal for EU FP7 Capacities Work Programme: Science in Society*. Salzburg: University of Salzburg.
- Patry, J.-L., Weyringer, S. & Weinberger, A. (2007). Combining values and knowledge education. In Aspin, D. & Chapman, J. (Eds.) *Values Education and Lifelong Learning: Philosophy, Policy, Practices*. Dordrecht: Springer Press, 160 - 179.
- Pehkonen, E. (2003). Lærere og elevers oppfatninger som en skjult faktor i matematikkundervisningen [Teachers' and students' beliefs as a hidden factor in the teaching of mathematics; in Norwegian]. In B.Grevholm (Ed.). *Matematikk for skolen* [Mathematics for the school; in Norwegian] (pp. 154 – 181). Bergen: Fagbokforlaget.
- Philipp, R. A. (2007). Mathematics Teachers' Beliefs and Affect. In F. Lester Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*, 257 – 315. Charlotte, NC: Information Age Publishing.
- Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education*, 28, 550 – 576.
- Rokeach, M. (1968). *Beliefs, attitudes, and values: A theory of organization and change*. San Francisco: Jossey-Bass.
- Shaw, K. L., Davis, N. T. & McCarty, B. J. (1991). *A cognitive framework for teacher change*. In Proceedings of the 13th annual meeting in PME-NA, 2, 161 – 167.
- Shulman, L. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review*, 57(1), 1 – 22.
- Skovsmose, O. (2002). Matematikken er verken god eller dårlig [Mathematics is neither good nor bad: in Norwegian], *Tangenten*, 13(3), 22 – 26.
- Thompson, A. G. (1992). Teacher's Beliefs and Conceptions: A Synthesis of the Research. In D. G. Grouws (Ed.): *Handbook of Research on Mathematics Teaching and Learning*, NY: Macmillan, 127 – 146.
- Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge University Press.
- Wilson, M. & Cooney, T. (2002). Mathematics Teacher Change and Development: The Role of Beliefs. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A Hidden Variable in Mathematics Education?*, Seacaucus, NJ: Kluwer Academic Publishers, 127 – 147.
- Wæge, K. (2007). *Elevenes motivasjon for å lære matematikk og undersøkende matematikkundervisning* [The

students' motivation for learning mathematics and investigative mathematics teaching; in Norwegian]
(Ph. D. dissertation). Trondheim: Norwegian University of Science and Technology.
Yin, R. K. (2003). *Case Study Research: Design and Methods, 3rd Edition*. London: Sage.

Acknowledgement

The authors would like to thank Dr. Christoph Kirfel for his comments during the writing process.