Pupils collaborating in pairs at a computer in mathematics learning: investigating verbal communication patterns and qualities

Rune Herheim

Dissertation for the degree philosophiae doctor (PhD)

at the University of Bergen

2012

Dissertation date: 19.06.2012

A willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them. (Wells 1999, p. 121)

Scientific environment

The dissertation is written at the Department of Education, Faculty of Psychology at the University of Bergen (UoB). I have been a member of the research group Digital Learning Communities (DLC), and I have participated in the research program Western Norway Graduate School of Educational Research (WNGER).

Acknowledgements

To be a PhD-Candidate is to be in a privileged position. I have had four good years at the University of Bergen with the best of working conditions. It has been important to me to be self-supported, but one cannot underestimate the importance of favourable framework conditions. I had a good start, with one week at Gran Canaria with the research group Productive Learning Practice. I did my research education at WNGER, I have been a member of the research group DLC, and I have taught at UoB's teacher education together with good colleagues at the Department of Education. I am pleased that I have been able to take advantage of this opportunity, and that I have been ahead of schedule throughout the process.

I would like to thank my supervisors Rune Krumsvik, professor at the University of Bergen, and Marit Johnsen-Høines, professor at Bergen University College. Rune's area of expertise is ICT and learning while Marit's area of expertise is communication and mathematics learning. The discussions with Rune on research design, his knowledge of academia, and his good overview of a PhD-candidate's assignments, have been important. The discussions with Marit on data interpretation and analysis, and her critical reading of my writings, have increased the quality of the dissertation.

I am grateful to the pupils and teachers who participated in the empirical studies. Your participation as co-researchers has been very important.

The most important contributions to every aspect of my life, including my dissertational work, are made by my wonderful wife Ragnhild and our two fantastic children Johanne and Ola. And without my parents I would not be here. Family is everything.

Abstract

The dissertation concerns 9th grade pupils' verbal communication in mathematics learning when they work in pairs at a computer. The dissertation includes three sub-studies, involving three sub-questions. The first sub-study is a research literature overview, which addresses the research questions: What are the important aspects in promoting pupils' talk and reflections in small group settings using a computer? The second sub-study is an empirical study, which addresses the research question: What characterises pupils' verbal communication at a stand-alone computer in a mathematics lesson? The third sub-study is an empirical study, which addresses the research question: What characterises communication qualities, if any, that can develop a pair of pupils' communication and mathematics learning at a computer?

The method of the overview study is based on developing an effective search. This involved finding and refining search terms, effective inclusion-exclusion criteria, and a trustworthy quality assessment. The methodological approach of the two empirical studies is design-based research. The most important methodical characteristics are the collaborative research perspective, the iterative cycles of design-interventions-analysis-redesign, and the intertwining of designing learning environments and developing theory. Descriptions, analyses, and interventions are conducted in iterative cycles based on collaborative analysis of communication potential. To bridge educational theory, practice, and research through joint research reflections between pupils, teachers, and researcher has been a guideline for the design of the empirical studies. The data material is collected through an extended use of video recording, screen recording, observation, and "watch-and-talk" sessions where the pupils, the teacher, and the researcher interpret and analyse video recordings together.

The first sub-study, the overview of the research literature, identifies four focus areas in the research of communication and learning at a computer: to establish and develop a common ground, communication characteristics, roles, and software design/task structure. The communication analysis in the second sub-study identifies six

communication patterns that can be summarised in terms of two interrelated main aspects: thinking aloud and building a mutual language. To address each other, speak in chorus, and use the same linguistic turns are distinct patterns for successfully setting and taking each other's perspectives. "Huey, Dewey, and Louie" talk represents the communication where pupils compose sentences together by alternatively making short contributions. These communication patterns help the pupils build a mutual language and a communicative common ground. The major outcome of the dissertation is made in the third sub-study on the connections between communication qualities and managing differences. The pupils have two quite different approaches on how to solve a mathematics task, yet they manage to have a productive collaboration. They develop communication qualities that make the collaboration possible despite their individual differences.

List of figures

List of publications

Article I:

Herheim, R. (2010). Communication and learning at computers: an overview. *Nordic Studies in Mathematics Education*, *15*(2), 69-94.

Article II:

Herheim, R. & Krumsvik, R. (2011). Verbal communication at a stand-alone computer. *Journal for Educational Research Online*, *3*(1), 29-55.

Article III:

Herheim, R. (submitted). Managing differences by developing communication qualities: Pupils learning mathematics in pairs at a computer.

Contents

1. Introduction

The dissertation concerns 9th grade pupils' verbal communication in mathematics learning when they work in pairs at a computer, and consists of this synopsis, three articles, and four appendices. The synopsis is a meta-text that clarifies and gives reasons for the background of the dissertation and the choices made during the research processes.

This first chapter starts by describing the research context and the rationale for the dissertation, including the ontological and epistemological basis. Next, there is a discussion of relevant research, before the goal and the research questions are presented. The chapter is completed by a discussion of the coherence of the dissertation. Chapter two presents a *dialogic* approach to communication as the theoretical foundation of the dissertation. Chapter three describes the methodological approach, the methods, and the principles of analysis. The chapter is completed by reflections on trustworthiness and ethical considerations. The findings are presented in chapter four, and chapter five finishes the synopsis by discussing the main findings, how they complement each other, and how they relate to and contribute to previous research. Limitations, challenges, and implications are pointed out at the end.

The three articles are closely related but have different purposes. Article number I, Herheim (2010), refers to an overview study of relevant research literature. This study is an important part of the discussion of relevant research in chapter 1 in this synopsis, and articulates a background for the research questions addressed in article number II and III. Article number II, Herheim and Krumsvik (2011), presents the first empirical study, and article number III, Herheim (submitted), presents the second empirical study. The synopsis aims to bind the elements together – to summarise and synthesise – and to show how the overview study and the two empirical studies are interrelated and cumulative.

1.1 Research context

The two empirical studies investigate the research context where pairs of 9th grade pupils work together on a mathematical task and share a computer; see figure 1. The main focus is on the pupils' communication. The expression "communication *at* a computer", adapted from Crook's (1994, p. 148) expression "interaction *at* computers", distinguishes this context of communication from e.g. written, asynchronous, and long distance communication. It implies that the pupils are physically sitting next to each other in order to work together on a joint task at the same time. This research context concerns face-to-face, non-networked communication, and therefore belongs to a small segment of the field of research on Computer-Supported Collaborative learning.

Figure 1: Snapshot from a recording of a work segment.[1]

The pairs of pupils have been video recorded one at a time in an authentic classroom setting. There are four active units in this context: the two pupils, the computer, and the teacher. The software is the venue for the pupils' mathematical inquiries. The

[1] The use of pictures is approved by the parents/guardians and the Norwegian Social Science Data Services.

teacher creates the tasks and is involved in the recorded pupils' work every now and then as she walks around the classroom and helps the other pupils. The main focus of this dissertation is on the verbal communication between the pupils. The non-verbal communication is valuable in order to improve the understanding of the pupils' verbal communication and to strengthen and validate the communication interpretations and analyses. The extended use of video recording as a central part of the research design provides access to non-verbal action. The role of the teacher and the computer is taken into consideration in order to understand and develop the quality of pupils' communication and learning.

A key part of the research process is the joint discussions between two pupils, their teachers, and me. We[2] analyse a pair's communication together based on the video recording of their work. These discussions are also video recorded in their entirety, see figure 2.

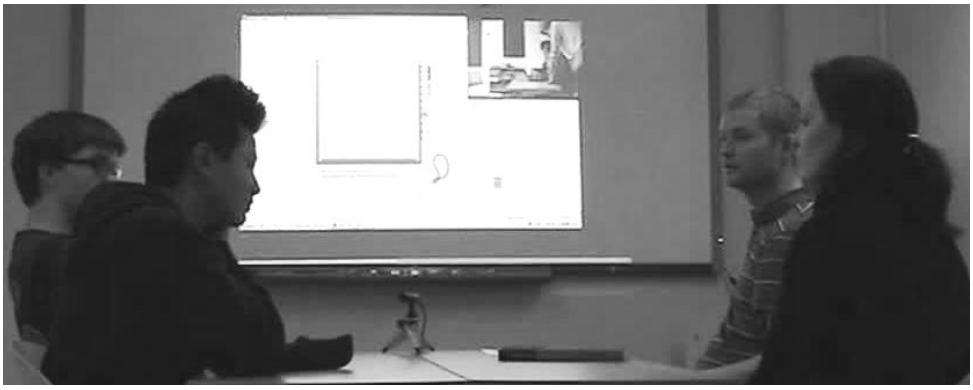

Figure 2: Pupils and teacher as co-researchers

The pupils and teachers are co-researchers in these joint discussions. They take part in the interpretation and analysis of the data material and make substantial contributions

[2] The use of "we" in the rest of the dissertation refers, if not stated otherwise, to the pupils, the teacher, and me.

to the findings. The pupils and the teachers adopted, carved, and operationalised the tentative research questions and goals that I introduced. It was a goal from the very beginning of the research to include the pupils and the teachers as genuine co-researchers.

The personal and practical goals described in the next section, together with the intellectual goal presented in section 1.5, underpin a choice of design where pupils and teachers are co-researchers. Understanding and developing pupils' subject matter collaboration and communication call for the involvement of those who are at the centre of the process of development, namely the pupils themselves and the teachers. Involving those who shall effectuate changes is reasonable in itself, but it is also important in order to study the complexity of such a research context.

1.2 Personal and practical goals

Maxwell (2005) distinguishes between three types of goals in research design: personal (one's personal reasons for doing a study), practical (what one wants to accomplish), and intellectual (what one wants to understand). In this section I will first describe a personal and then the practical goals in the dissertation. The intellectual goal is presented in section 1.5.

I have a bachelor's degree in mathematics (and physics), and mathematics has always been a field of special interest to me. On top of my bachelor degree I have a one-year teacher's education. My interest in mathematics education was confirmed, and I decided to do a master's degree in mathematics education. Because of a growing interest in communication and mathematics learning I focused on full-class communication in an upper secondary mathematics class in my master thesis. After working for one year as a 6^{th} grade teacher, I worked for five years as a teacher's educator in mathematics. My research still focused on communication, but now in group contexts at the primary level. The use of computers was becoming more prevalent in my teaching as well as in my research, particularly when computers made

it possible to achieve mathematical activities and insights that would have been difficult to achieve by using traditional teaching methods. My personal interests in mathematics, mathematics learning, communication, and computers were influential for my choice of research field.

The second goal is a practical one. The ability to communicate orally is highlighted as one of five basic competencies in the national curriculum in Norway by the Ministry of Knowledge (2006). Raising questions and communicating ideas are features that are highlighted. However, international research (e.g. Galton, Hargreaves, Comber, Wall, & Pell, 1999; Newton, Driver, & Osborne, 1999; Wegerif, 1996) as well as national research (e.g. Alseth, Breiteig, & Brekke, 2003; Grønmo, Onstad, & Pedersen, 2010; Vavik et al., 2010) show that there is not much time devoted to pupils' subject matter communication in schools. Furthermore, Mellin-Olsen (1991) identified what he called the *exercise discourse* when mathematics teachers described their teaching of mathematics. The teachers used phrases such as "to get through the curriculum", "to drive on/keep going", "to be one step ahead", or "to fall behind". It was a discourse in which learning mathematics was regarded as a journey, and "to drive on" and maintain speed were emphasised. During the last decade I have supervised student teachers in their practicum, and I have observed that common questions between pupils are "how many tasks have you done?" and "what answer did you get on task number …" My experiences confirm Mellin-Olsen's claims concerning the exercise discourse and a lack of subject matter discussions in the mathematics classroom. This is the background for the goal of increasing the amount and quality of pupils' mathematical communication.

Pupils' ability to use digital tools is also highlighted as a basic competence in the national curriculum. Pupils have, according to the research by Hægeland, Kirkebøen, and Raaum (2009), as well as and Vaage (2009), a growing accessibility to computers, both in and outside of school. A publicly accessible information system for primary and lower secondary schools in Norway (https://www.wis.no/gsi) shows that there were 7.7 pupils per computer in 2003–04. This ratio is changing rapidly,

and in 2011–12 there were 2.92 pupils per computer. These figures vary from municipality to municipality and from school to school. The computer density is higher at the lower secondary level than at the primary level (Hatlevik, Ottestad, Skaug, Kløvstad, & Berge, 2009). At the upper secondary level every pupil has a portable personal computer, and the expenses are mainly covered by the schools. The research of Hatlevik et al. (2009) showed, however, that the degree to which computers were used, and how they were used, varied between schools, classes, and teachers. The national survey by Vavik et al. (2010) concerning the lower secondary level showed that there was only one subject in which computers are used less than in mathematics, namely physical education. Vavik et al. (2010) also found that dynamic geometry software and graphing software were used the least – more than 60 % of the teachers rarely if ever used software of this kind. This latter result confirms OECD (2005), who reported that the use of educational software such as mathematics programs was, out of 12 ICT uses, the one that fewest (13 %) reported to be using frequently. Hence, it has been a practical goal to develop a fruitful learning context by using dynamic geometry software. We did not want to "electrify traditional methods" (Krumsvik, 2007) but to use computers in ways that paper and pencil cannot.

Krumsvik (2008) described two practical goals for using computers. First, computers can be used to increase pupils' abilities to use computers or particular software, and second, computers can be used to increase pupils' subject matter knowledge, e.g. mathematics. The ability to use spreadsheet software is a competence aim in the national curriculum, and the teachers in the first empirical study referred to the curriculum when they explained why the pupils used such software. To use computers to increase mathematical understanding was a goal for the teacher and me when we planned the pupils' work segments in the second empirical study. The first empirical study showed that the pupils found the spreadsheet software insufficiently transparent (transparent software is easy to access and use). In the second empirical study the teacher and I therefore decided to find software that we believed would be transparent, that offered educational possibilities that paper and pencil could not offer, and that could facilitate pupils' mathematical discussions.

1.3 Ontology and epistemology

Ontology is the study of conceptions of reality and the nature of being, what reality is fundamentally like, and what sorts of things there are (Craig, 2002). Packer and Goicoechea (2000) argued for the importance of considering the ontological framework of a study; for being more articulate about the starting premises of a research study. The purpose of focusing on ontological and epistemological stances is to raise consciousness about the underlying assumptions behind the research questions, the theoretical foundation, the design, and the interpretations and analyses, and thereby to give readers a foundation on which they can base their understanding of this work and in relation to which they can consider transferability issues.

This dissertation is in accordance with the semantic view of theories (Kvernbekk, 2005), and is written based on the fundamental assumption that social sciences differ from natural sciences; human interaction differs from natural phenomena. Skjervheim (1959) was a pioneer in his critique of positivism (scientism), a tradition which applied the natural science research model to all other sciences, including social science. Skjervheim (1996c) accentuated the fact that man has language, and he (1996a) argued that since man cannot be explained as an object, the model developed for the natural sciences cannot be employed where it does not belong.[3] This dissertation is written in light of a research approach associated with social sciences.

The distinction between subjectivism and objectivism is a core difference between social sciences and natural sciences. A subjective stance acknowledges that there are several interpretations of a phenomenon and that observations are influenced by one's

[3] To use the work of Skjervheim in research on collaborative learning of mathematics might seem problematic for two reasons in particular. First, if one takes into consideration the (partly) behaviouristic-inspired learning and competence aims of mathematics in the national curriculum one could say that mathematics is associated with post-positivism. Second, some misinterpret Skjervheim's critique of positivism as discrediting mathematics and science as such. However, Skjervheim's critique is a critique of positivism only. Furthermore, this dissertation concerns research on mathematics education, which belongs to the social sciences, and not the nature of mathematics itself.

theoretical framework. Eisner and Peshkin argued that "thoughts have consequences: how one thinks about subjectivity and objectivity affects research procedure because these issues typically are embedded in the broader framework, albeit most often implicitly, that directs the conduct of our inquiry" (1990, p. 15). Subjectivity differs from objectivity in the view on ontology, epistemology, and methodology. The three latter aspects are causally related, where "ontological assumptions will give rise to epistemological assumptions which have methodological implications for the choice of particular data collection techniques" (Hitchcock & Hughes, 1995, p. 21). This fundamental chain of action from ontology and epistemology to methodology and methods will be discussed in what follows and in section 3.2.

This dissertation is based on acknowledging the importance of human relations and the learning potential of the existence of a dialogic space *between* interlocutors. Wegerif (2011c) said that dialogic space is a real phenomenon. He goes on to suggest that a dialogic approach is not only a way to gain knowledge but also a way of being. It is an ontological matter as well as an epistemological matter. One exists in relation to others, and what one is, what one says, and what one does, are defined and developed through interaction. When persons collaborate there will always be an aspect of diversity; they have different preferences concerning how to do things and they use different words to describe the same phenomenon, or the same words in different ways. Reality is fluid; it is not a set of fixed, uniform, and easily explainable units. One cannot define reality since there will always be an element of contextual and historical change and an element of subjectivity. The inclusion of pupils and teachers as co-researchers is based on the view that their participation is indispensable in order to understand the complexity of this educational context.

The use of computers can also be reflected on in light of ontology. A dual ontological status of computers has been identified (Monaghan, 2005; Wegerif, 2004). On the one hand the computer is a non-judgemental object without emotions or expectations. The computer can, on the other hand, be interpreted subjectively (pupils refer to it as "he" or "him") when responding to inputs in such a way that pupils can feel obliged to

justify their responses. The computer as a quasi-human agent can, according to Monaghan and Wegerif, facilitate and direct pupils' communication towards subject matter learning. This adds to the rationale for using computers described in the practical goals in the previous section.

Questions such as "what is knowledge?" and "how does one acquire knowledge?" are epistemological questions. When looking at mathematical knowledge, e.g. intuition, proofs, and reasoning, there is a distinction between an absolutist and a fallibilist paradigm. The absolutist paradigm has been dominant for more than two thousand years but is now being challenged by fallibilism (Ernest, 1991). It is essential, from a fallibility perspective, to acknowledge that new arguments can cause a person to change or modify his or her understanding. Moreover, knowledge is not considered conclusive and indisputable, but rather as constantly changing and subjective. This is an important basis for this dissertation's focus on how pupils can discuss their understanding. If knowledge had been absolute only, then the pupils could just "tell each other" about the various elements in their mathematical curriculum.

Investigations of the teaching and learning of mathematics have shown how some issues are emphasised more than others: speed over meaning (Mellin-Olsen, 1991), rote over meaningful, procedural over conceptual – rules without reasons (Skemp, 1976), and instrumental understanding over relational understanding (Mellin-Olsen, 2002; Skemp, 1976). This type of teaching and learning, if it is the only approach, will, according to Piaget and Inhelder (1973), generate static, inflexible, and fragmental knowledge. The TIMSS 2007 international mathematics report (Mullis, Martin, & Foy, 2008) does indicate that procedural learning and drill are important, particularly in elementary arithmetic. However, more complex areas, such as the one that this dissertation addresses, namely geometric properties and their relations, require a deeper understanding. Knowledge is regarded as something that changes and develops continuously through interplay between individual construction and interaction with others. As Stahl (2005) argued, there is knowledge that is built

through interaction that "may not be attributable as originating from any particular individual" (p. 81).

Thus, this dissertation investigates pupils' ability to express mathematics; the ability to participate in mathematical discussions where mathematical tasks and tensions between perspectives are managed. This is the perspective on mathematical knowledge addressed in this dissertation. One learns by means of language – the use of language breeds language: "every conversation presupposes a common language, or better, creates a common language" (Gadamer, 2004, p. 371). One acquires knowledge through talking with others. This means that to communicate is a means and a goal of knowledge construction (Sidorkin, 1999). Differences between perspectives are regarded as a fruitful underlying assumption for generating meaning through human relationships. Pupils' ability to reach some sort of flexible agreement is another aspect of collaboration. A mutual answer might be based on differing opinions and explanations. These aspects of difference and agreement are elaborated upon in section 2.5.

Ontological and epistemological matters constitute the background when research is carried out. They influence which research questions are being raised and how these questions are expressed, what kind of theories and methods are being used and how they are used, what kind of knowledge is sought, and how the methods, interpretations, and results are legitimated.

1.4 Relevant research

Boote and Beile (2005) argued that a thorough and sophisticated literature review is a necessary foundation for any useful research. Based on Boote and Beile one could argue that every academic work can be assessed according to how well it relates to previous research. Acknowledging the importance of others' research can be compared to the importance of acknowledging the importance of one's ontological and epistemological stance.

Maxwell (2006), in a commentary on Boote and Beile's article, acknowledged the importance they attach to literature reviews in research preparation. However, he criticised their strong emphasis on thoroughness and comprehensiveness. Maxwell distinguishes between reviews *of* and *for* research, and underlines the centrality of *relevance* (how relevant earlier studies are for one's own study) as a key issue for conducting the latter type of review. A key consequence of Maxwell's arguments is that every single study that is included in a selective review should inform or support one's study, that is, the justification for the study, the goals, the theory, the conceptual framework, the design, or/and the principles of analysis.

Based on Boote and Beile's (2005) concepts *coverage*, *synthesis*, *methodology*, *significance*, and *rhetoric*, and Maxwell's (2006) commentary, I conducted a literature overview on research concerning small groups' (principally pairs) communication at a computer. The overview, disseminated in article number I, helped me, as Maxwell pointed out, to crystallise my focus, design, and principles of analysis. This article describes in detail the literature search strategy, the development of criteria, and key terms for inclusion and exclusion of articles, and the quality assessment of the included research. The overview generated several valuable findings, and the main findings are also incorporated in the discussion of relevant research in this section.

The overview identified four focus areas, or – to use Lagrange, Artigue, Laborde, and Trouche's (2001) term – "problématiques", in the research literature. The first of the four focus areas is the *common ground* aspect introduced by Clark and Brennan (e.g. 1991). A common ground is a shared frame of reference, a body of shared knowledge (Teasley & Roschelle, 1993). A common ground involves a common focus of attention (Crook, 1994), common goals (Healy, Pozzi, & Hoyles, 1995), a base of common language and communication, and a base of common subject matter knowledge. Teasley and Roschelle (1993) found that collaboration involves two concurrent activities, namely to solve a task and to build a common ground. However, Wegerif, referring to Bakhtin (1986), argued that "it is the difference between us in a

dialogue that makes the meaning flow; if you fill this difference in with 'common ground' then the flow of meaning will stop" (Wegerif, 2011b, p. 86). This statement highlights two important points. First, it underlines the importance of differences in perspectives for achieving fruitful communication. Second, to establish and develop a common ground is not a matter of building as much common ground as possible or reaching a completely uniform understanding; rather, it is a matter of having *sufficient* common ground to be able to collaborate and communicate (Herheim, submitted; Stahl, 2005). Wegerif (2011c) goes on to say that there will always be differences, *a dialogic gap*, between two persons' perspectives since they have different bodies and different histories. To generate meaning requires an ability to see something from another point of view. A common ground can help pairs of pupils to develop their knowledge and collaboration. Such a common ground can be characterised by growing knowledge and acknowledgement of the perspectives of the other.

The second focus area, the most common focus of attention, is *communication characteristics*. It concerns the identification and investigations of different communication characteristics and how these characteristics influence pupils' communication and learning. The following characteristics' positive influence on learning is well supported by empirical examples and analyses. The first is to engage critically in each other's perspectives. This is a core of Mercer and Wegerif's (e.g. 1998) discussions of *explorative talk*. Another overarching characteristic is to think aloud, to verbalise ideas and suggestions (e.g. Kieran, 2001; Monaghan, 2005). Collaboration is dependent on pupils' verbal accompaniment of what they do on the computer screen. An important way of thinking aloud is asking questions (e.g. Monaghan, 2005; Staarman, Krol, & van der Meijden, 2005) and framing statements as questions (Teasley & Roschelle, 1993). Few questions and many overconfident statements do not promote collaboration. Collaboration is more than composing well formulated arguments. Equally important, or probably more important, is exhibiting unfinished ideas. In this respect Alrø and Skovsmose (2002) emphasise the willingness to take risks. It takes courage to present incomplete understanding. The last communication characteristic highlighted in this paragraph is listening.

Successful communication requires not only different aspects of thinking aloud but also an ability to listen actively. This latter characteristic is an important part of Alrø and Skovmose's (2002) inquiry cooperation model.

The third focus area is pupils', teachers', and computers' *roles* in this context. Research (e.g. Nussbaum et al., 2009; Wegerif, Littleton, & Jones, 2003) identifying positive learning outcomes emphasises pupils as collaborators and talkers who, in line with the communication characteristics described in the previous paragraph, think aloud and listen to each other. The pupils make arguments, raise questions, and explain ideas. The pupils who manage to take advantage of collaboration take the role of participants; see Skjervheim's (1996c) triangular relationship in section 2.5.

The teacher can, from a communication perspective, act as a guide, a role model, a motivator, a conductor, and a facilitator. The teacher can guide and model pupils' communication during class or arrange separate lessons on communication (e.g. Monaghan, 2005; Wegerif, Mercer, & Dawes, 1998), and motivate and encourage the pupils to think aloud and verbalise their ideas (Nussbaum, et al., 2009; Panselinas & Komis, 2011). The importance of the teacher is generally acknowledged in the research. As Nussbaum et al. (2009) highlight, the teacher can orchestrate pupils' collaboration and facilitate and stimulate pupils to raise questions, express thoughts, and reason aloud.

The computer can be regarded as a servant that releases pupils from time-consuming calculations and thereby facilitates their subject matter discussions (Ainley, Nardi, & Pratt, 2000; Åberg-Bengtsson, 2006). The computer's role as "scaffolder" for pupils' communication is pointed out by Mavrou, Lewis, and Douglas (2010), and by Nussbaum et al. (2009). The dual ontological status for the computer as described in section 1.3 is also a significant element. The pupils in the studies by Lavy and Leron (2004), and by Teasley and Roschelle (1993), regard the computer as a third participant, a partner. The computer screen serves as a shared frame of reference, a mediator; the pupils use screen images to make explanations and to understand each

other's explanations. The computer scaffolds, it provides language when pupils' vocabularies are insufficient.

The forth focus area presented in the overview is *software design and task structure*. Much of the research focuses on how the characteristics of the tasks and the design of the software influence the quality of pupils' talk. Software that is particularly designed for supporting subject matter talk has generated positive results (e.g. Wegerif, 1996; Wegerif, et al., 1998). Two design features are particularly emphasised, namely to include elements on the screen that are easy to point at and that help the pupil's reasoning, and that the tasks are sufficiently complex so that joint reflection and discussion is necessary. There is also a common distinction between open and closed software – between the number of options, the degree of prompting, and the end-user tailorability. The research by Anderson, McAteer, Tolmie, and Demissie (1999) supports the seminal work of Fisher (1993), who found that open software generates more questions, more explaining, and more turning to the other. In mathematics education the software discussion centres on the positive effects of dynamic manipulations in geometry (Ruthven, Hennessy, & Deaney, 2008) and graphing (Åberg-Bengtsson, 2006). The opportunity to quickly change angles and side lengths, to move graphs around, or to change intersection points in no time, has substantially increased the quality of pupils' subject matter talk.

The publications included in the overview focused on one or more of these four focus areas. The focus areas are interrelated theoretical categorisations of the literature. For instance, the role of computers is closely connected to software design and task structure. The exclusion criteria are just as important as the inclusion criteria. Research that is related but still regarded as being outside the focus concerns interaction in whiteboard and calculator contexts. Research on written long-distance communication and web-based e-learning is also outside the focus of this dissertation. Qualitative and mixed methods research conducting in-depth communication analysis on pairs' communication at a computer is included, while quantitative studies are only used to set the scene in the introduction.

The two seminal publications that conducted qualitative analyses of pupils' communication in pairs at a computer were Fisher (1993) and Teasley and Roschelle (1993). Fisher focused on characteristics of children's talk at the computer, and Teasley and Roschelle investigated pupils' construction of a joint problem space at a computer. The data collection and analyses of pupils' communication in these two seminal works, and in the research by Kieran, Lavy, Sinclair, and Wegerif, have considerably influenced my goals, research questions, research design, and data analysis.

The practical goals in section 1.2 describe a gap between the emphasis on oral and technological abilities in policy documents and what researchers actually find in the classrooms. This gap, together with research on common ground, communication characteristics, and software design/task structure from the overview presented above, constitutes the background for this dissertation's focus on communication patterns and qualities when pupils work with mathematics in pairs at a computer.

1.5 Intellectual goal and research questions

The personal and practical goals described in section 1.2 gave grounds for the interest in learning more about pupils' communication when they work in pairs at a computer. The following intellectual goal has been a guideline throughout the research process: *to gain knowledge of pupils' communication quality and mathematics learning when they work in pairs at a computer*. The dissertation emphasises in particular the understanding of and development of pupils' communication. The use of computers and the learning of mathematics constitute the context in which the communication takes place. The intellectual goal is enlightened through sub-studies involving three sub-questions:

The overview of relevant research presented in article number I address the research question: *What are the important*[4] *aspects in promoting pupils' talk and reflections in small group settings using a computer?* The overview is incorporated in the discussion on relevant research in section 1.4. The aim of this study is to provide an overview of a complex research area and to identify relevant research aspects and challenges in order to crystallise my own research questions and methods.

The research questions in the two empirical sub-studies are questions of description and classification. Flick (2006) differentiated between two main types of qualitative research questions: *describing state* and *describing process*. The first study, presented in article number II, address the research question: *What characterises pupils' verbal communication at a stand-alone computer in a mathematics lesson?* To identify and illuminate communication patterns is of special interest. In this study the aim is to describe and analyse the state of pupils' communication, and to be able to characterise pupils' communication by identifying communication patterns.

The second empirical sub-study, presented in article number III, address the research question: *What characterises communication qualities, if any, that can develop a pair of pupils' communication and mathematics learning at a computer?* The aim was to improve the quality of pupils' communication and learning of mathematics at a computer by making interventions in several cycles based on joint data interpretations and analyses between the pupils, the teacher and the researcher, see section 3.3 and 3.4. According to the findings in the overview study and the first empirical study we decided to start off by particularly emphasising the two general communication aspects of thinking aloud and listening actively. The two pupils developed several

[4] To use a positively loaded word such as 'important' can be problematic in research questions, and it might make it a somewhat leading question. However, this is an overview research question and the focus was to identify areas regarded as important in the research literature. One would assume that it is not overly problematic to think that researchers are arguing that there are important aspects to their research.

communication qualities during the research process, and some unforeseen but interesting differences between them appeared as well.

1.6 Coherence

There are four main elements in this dissertation: the overview article of relevant research literature; the first empirical article referring to the descriptive-analytical study; the second empirical article presenting the intervention study; and finally the synopsis, which encompasses and binds the three studies together. The most important coherence aspects are how the overview study serves as a basis for the two empirical studies, and how the first empirical study forms the basis for the second empirical study.

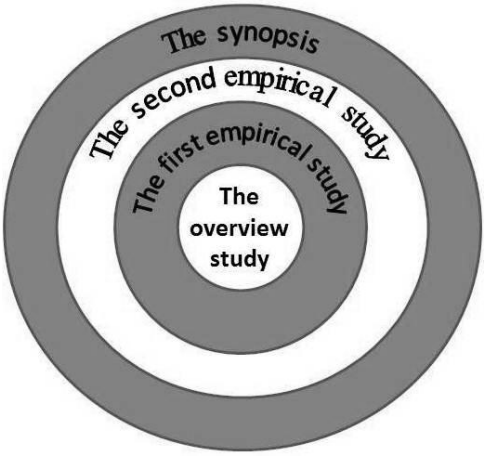

Figure 3: Overview of the four elements in the dissertation.

The work on the dissertation started with the overview study. The overview is the core of the dissertation, the hub around which the rest of the work rotates. In the process of searching, reading, analysing, and disseminating the literature overview I developed insights into several aspects of this particular field of research, including the articulation of research questions, the methodology and methods, the theoretical

foundation, relevant projects, key journals, writing styles, and terminology. It was, as previously mentioned, important in order to position my own work.

The overview identified a need for qualitative micro studies on pairs of pupils' face-to-face communication at a computer. There is only a limited amount of research on this context (exceptions are e.g. Fisher, 1993; Mavrou, et al., 2010; Teasley & Roschelle, 1993; Wegerif, 2004), and much of this literature has a more general learning approach (e.g. games or logic). The research on learning mathematics through collaborative use of digital technology is also limited, and the emphasis on the use of computers and conducting in-depth communication analyses is rare (exceptions are e.g. Kieran, 2001; Lavy, 2006; Sinclair, 2005). This, together with the four focus areas identified in the overview study, constituted the background for the research focus in the two empirical studies in this dissertation. In the second empirical study the teacher and I chose software and constructed tasks that supported pupils' mathematical communication. This was based on the findings in the overview study and on the finding in the first empirical study to the effect that insufficiently transparent software generated more technological than mathematical communication. Equivalently, the communicative focus on thinking aloud and listening in the second empirical study was based on the importance of these two aspects identified in both the overview study and the first empirical study.

The overview study and the two empirical studies are all focusing on communication at a computer – the identification and discussions of communication aspects – and the overview and the second empirical study investigate how communication aspects influence pupils' learning. The research context is the same in all three studies: pairs' (some triads in the overview study) communication at a computer. The two empirical studies are conducted within the same methodological framework, namely designed-based research. All three studies are conducted and written in line with the formulations of the research questions, the research design, and the theoretical stance in the project proposal. Adjustments have been a natural and necessary part of the collaborative research process, see section 5.2 on limitations and challenges, although

the initial plan was quite thorough and feasible. The three sub- studies are designed and analysed within the theoretical perspective of a dialogic approach, and the following chapter elaborates on key aspects of this approach in the form of language, how to meet others, and communication.

2. Theoretical foundation

The dissertation is written from a social perspective on learning and knowledge. The role of language and communication in learning mathematics at a computer is the focus of the overview study as well as of the two empirical studies. I acknowledge the individual part of learning, the cognitive structuring of knowledge where language is used as a thinking tool, as important. However, the emphasis in this dissertation is on collaborative learning, on the expression of knowledge where language is used to communicate – the joint thinking aloud of mathematical understanding. A dialogic approach to communication serves as a theoretical framework for the dissertation.

The theoretical foundation for the dissertation is discussed in five sections. The first concerns the theory of a dialogic approach, followed by a discussion of ontological and epistemological aspects of this approach. The fundamental role of language is elaborated on in the third section before a discussion on "ways of meeting others" in the fourth section. The last section highlights six different aspects of communication.

2.1 A dialogic approach

The notion of *dialogic* is here ascribed to Bakhtin, and is also referred to as "dialogism" by many (e.g. Dysthe, 1996). Dysthe, influenced by Bakthin and Rommetveit in particular, accentuated the idea that interlocutors have different backgrounds, knowledge, language, opinions, and values. The "mutuality of differences" is to be seen as the key to understanding Bakhtin's concept of dialogue (Holquist, 2002, p. 41). A dialogic point of view acknowledges the coexistence of diverging understandings. Therein lays an important difference between dialogic and dialectic, since a goal in the latter is to merge viewpoints into a compromise or some state of agreement.

When I started working on my dissertation I was influenced by Vygotsky[5] and his notion of *the zone of proximal development* and his emphasis on language as the most important mediating tool for an individual's productive interaction with others. I still retain much sympathy with Vygotsky (and activity theory), but during the dissertational period the work of Vygotsky and his successors have been overshadowed by ideas developed by Bakhtin, Gadamer, Rommetveit, and Skjervheim.[6] Their theories are products of a historical development, and arise from different discursive levels than pupils' communication at a computer. Thus, when applying such theoretical concepts in the discussion of pupils' communication at a computer I do not pretend that a small communication excerpt can represent the diversity of these concepts. The concepts are used to show a possible interpretation of them and to show how a theoretical idea can be used to enrich the reflections on classroom communication. The contributions made by these four scholars constitute this dissertation's most significant theoretical roots.

The Vygotsky-based activity theory and the idea of concepts as tools, artefacts, and mediation may appear to be an adequate theory since I investigate pupils' communication when they use computers. I have, however, experienced that the reflections concerning human relations and continuous communication at a computer has been a bit out of place in this kind of framework. This dissertation is, to use an argument from Roth and Lee (2007), one of those projects that relate to activity theory, but adhere more to a semiotic perspective drawing on Bakhtin. Wegerif and De Laat (2011, p. 317) explained that "where in the neo-Vygotskian sociocultural tradition technology is conceptualised as a mediating means for cognition, from this more dialogic perspective technology is seen as a facilitator opening and shaping dialogic spaces that would not otherwise be there". The quote illustrates my view on

[5] Vygotskian theories are applied in article number II. This is because this article is written early in the dissertational process when the influence of Vygotsky was stronger.

[6] Skjervheim's contributions apply to theory of science as well as to theory of dialogic. He is therefore referred to in the introductory chapter and in this theoretical chapter.

the role of the computer as having the potential to facilitate and support pupils' subject matter discussions. Consequently, my work belongs to a dialogic tradition and is particularly influenced by the work of Bakhtin, Gadamer, Rommetveit and Skjervheim, who, in turn, are in debt to the founder of the philosophy of dialogue, namely Buber.

Buber made pioneering reflections on how to meet others through dialogue, and especially influential is his famous little book *I and Thou* (2004). Buber set up a clear distinction between two opposite ways of being or engaging with others: the I–It relation and the I–Thou relation. The I–It relation often represents the relation one has to things or to one's mental representations – to things one can experience. The problematic part starts when this relation is transferred to the relation between persons. In an I–It relation one regards the other as an object that can serve one's interests.

Buber contrasts the I–It relation with the I–Thou relation. The Thou is not an object. The Thou is in a human relation to the I; the I–Thou is a subject–subject relation. The I and the Thou, if it is to constitute a genuine I–Thou relation, approach each other in an absorbing and open manner. Key aspects are *meeting* and *addressing*, and the relation is mutual, direct, and present. "All real living is meeting," Buber (2004, p. 17) argued. Knowledge arises here-and-now, in the present. It is neither within the I nor the Thou; it is mutual and it is between. The Thou-orientation also involves a risk. One has to engage in the meeting with one's whole being. If one withholds something or reduces one's engagement, one will distance oneself from, or leave, the I–Thou relationship. This distance is what Buber would call *remoteness* from one's Thou. Buber's work serves as an in-depth effort to clarify the ontology and epistemology of a dialogic framework.

2.2 Ontology and epistemology of a dialogic approach

Theunissen (1984) argued that Buber is the primary proponent of a social ontology. Buber's reflections on relations are not only an account of a dialogic framework; they also illustrate the ontological and epistemological grounds of this framework. For Buber the ontology of *being* equates with the ontology of *the between* – the human reality consists of relations, our relation to the Thou, the meeting between the I and the Thou.

The ontology and epistemology of a dialogic approach is in this dissertation tied to Rommetveit's (1992) emphasis on the social nature of Man. Bakhtin (1984), for instance through his conception of *unfinalisability*, is in line with a view of man and reality as something that can change and never be fully revealed or understood. Bakhtin (1981) argued that a word is half someone else's. It becomes one's own when one populates it with one's own intentions and accent – when one appropriates it:

> Prior to this moment of appropriation, the word does not exist in a neutral and impersonal language … but rather it exists in other people's mouths, in other people's contexts, serving other people's intentions: it is from there that one must take the word, and make it one's own. (pp. 293-294)

This contrasts the view that reality, words, and knowledge exist independently of man. The moment words are spoken they are printed with the speaker's signature: intonations, intentions, dialects, and so forth. Every word one says is inescapably intertwined with what is said before us and with what is going to be said in the future. Bakhtin (1984) argued also for the *open-ended dialogue*:

> The single adequate form for *verbally expressing* authentic human life is the *open-ended dialogue*. Life by its very nature is dialogic. To live means to participate in dialogue: to ask questions, to heed, to respond, to agree, and so forth. (p. 293)

This quote illustrates that Bakhtin, in line with Buber, does not just explain a dialogic perspective by using ontology and epistemology, they both view a dialogic aspect *as*

an ontology and an epistemology. Holquist (2002, p. 15) discusses Bakhtin's concept of dialogism as a pragmatic epistemological theory whereby human behaviour is understood "through the use humans make of language". The research questions, the research design, and the interpretations and analyses in this dissertation reflect this fundamental view that pupils' learning can be understood through investigating their use of language.

2.3 Language

Language is a cornerstone of a dialogic approach. The evolution of language is by many considered as "the hardest problem of science" (Christiansen & Kirby, 2003, p. 1). I will not discuss the evolution of language, but follow Skjervheim's (1996c) emphasis on language and how language makes us human. We are language users and our language can take many forms: written, verbal, gestures, sign language, drawings, music, and so forth. We say, do, and make things to express ourselves. We use language to think and to communicate. It is the primary means by which we can gain insights into other's reflections and knowledge. There are degrees of linguistic development and sophistication where knowledge of a field's language can evolve from that of a novice to a gradually developed, rich, and functional vocabulary.

The importance of language is highlighted by many well-known thinkers. Gadamer said that "language is the universal medium in which understanding occurs" (2004, p. 390), and Bakhtin (1981) focused on how language can enhance dialogue. Rommetveit (1992) talked about the importance of bringing matters into language, and Skjervheim's (1996c, p. 127) point of departure was "the ways one meets others in language". Skjervheim also underlined that human interaction and development takes place mainly by *means* of language and *in* the language.[7] Researchers as Kieran

[7] Other thinkers emphasise the role of language as well. Wittgenstein proposed that "language itself is the vehicle of thought" (2009, p. 113), and "the limits of my language means the limits of my

(2001), Monaghan (2005), Nussbaum et al. (2009), Staarman et al. (2005), and Wegerif (1996) have during the last decades underlined the fundamental role of language for small group communication in computer settings. Language makes elaboration, explanation, evaluation, exploration, and clarification possible: to verbalise ideas through sharing and explaining. Language is regarded as "the foundation of the acquiring of understanding" and "the process of understanding is actuated by linguistic utterances" (Herheim, 2010, p. 71).

The studies in this dissertation have focused on pupils' verbal language at a computer. An aspect of language that has proven to be influential is the concept of mutual language. Mutual language includes building a mutual language and having a sufficient amount of shared language in order to be able to collaborate fruitfully. This is closely associated with the common ground concept discussed in section 1.4. Teasley and Roschelle (1993) addressed the same issue in their discussion of the duality of building and maintaining a joint problem space at a computer. The two empirical studies in this dissertation concern different aspects of mutual mathematical language, inspired by Mellin-Olsen's (2002) focus on mathematics as a language, regardless of whether it is elementary arithmetic or advanced geometry.

In the research literature (e.g. Lavy & Leron, 2004; Teasley & Roschelle, 1993) it is discussed how the computer can provide a language when the pupils lack a technical vocabulary. Typically, the pupils' utterances are in such situations strongly based on images on the computer screen (e.g. Lavy, 2006; Wyndhamn & Säljö, 2009). These utterances are often a combined use of verbal and non-verbal language. Computers have the potential to release and scaffold pupils' language (e.g. Mavrou, et al., 2010; Nussbaum, et al., 2009). Several aspects of language are displayed through communication. The focus on linguistic details in this dissertation's empirical studies is to be understood within this framework. Language and communication are the

---

world" (1990, p. 149). Habermas (1991) argued that shared understanding is constructed through language.

connecting links between individual and social learning. One uses language to meet others; language makes communication possible.

2.4 How one meets others

A core aspect of a dialogic approach is captured in the phrase *how one meets others*. Theoretical reflections on how two pupils can meet relate to the discussion of language in the previous sections, and to the discussion of aspects of communication in the next section.

Theoretically, the dissertation is, as mentioned earlier, largely based on the work of Bakhtin, Gadamer, Rommetveit, and Skjervheim. All four scholars relate to Buber's (2004) primary notions of I–It and I–Thou.[8] Buber's reflections on how to meet others are the root of my theoretical roots. His reflections are not easy accessible, but the lights of Bakthin, Gadamer, Rommetveit, and Skjervheim have concretised and operationalised his theories. They have all made substantial theoretical contributions to understanding the relationship between the I and the other – one's orientation to the other, in a similar contrasting manner as Buber. Each and one of their contrasts offer valuable details on dialogic communication, on different ways of meeting others, and between different modes of communication. Figure 4 shows the four scholars' "adapted contrasts" of Buber's two primary notions.[9]

---

[8] Bakhtin has explained how strongly Buber has influenced him (Friedman, 2001). He rarely refers to Buber in his writings, but one exception is in *Forms of Time and of the Chronotope in the Novel* (1981). Gadamer does not refer to Buber in *Truth and Method* (2004), but his reflections share a distinct resemblance with Buber's two relationships. Gadamer distances his work from Hegel's dialectic, and he said in an interview that he was clearly inspired by Buber (Stewart, 1985). Skjervheim refers to Buber in the essay *The instrumental fallacy* (1996b). "Rommetveit developed similar ideas to Bakhtin's dialogism starting in the late sixties before Bakhtin was known in the West" (Dysthe, 1966, p. 390), but later Rommetveit refers to both Buber and Bakhtin several times (e.g. Rommetveit 1992).

[9] One could also add related contrasts, for instance Freire's (2000) contrast between oppression and empowerment, and Habermas' (1991) contrast between strategic and communicative rationality.

Buber	I–It	I–Thou
Bakhtin	Monologic, the authoritative word, isolated utterances	Dialogic, mutuality of differences, multivoicedness, the internally persuasive word, continuous interaction
Gadamer	Inauthentic dialogue, apparent questions, argue the other down	Authentic dialogue, real questions, openness, want to know, consider the other's opinion
Rommetveit	Monologic, representational-computational cognition and communication	Dialogic, reciprocal perspective setting/taking, attunement, fusion of minds, bring matter into language
Skjervheim	Be spectator, to persuade, no mutual attention or engagement	Be participant, to convince, mutual attention and engagement in a mutual subject matter, genuine dialogue

Figure 4: Contrasts between how one meets others.

To develop such a table of contrasts, particularly when taking into considerations the diversity and amount of the productions of these scholars, one needs to make certain simplifications. There are also some possible objections. Buber, Bakhtin, and Rommetveit are all considered dialogical thinkers, while Gadamer and Skjervheim are strongly influenced by dialectics. Buber and Bakhtin have both rejected dialectics; in fact, they compared dialectic with monologic practise. However, some of Gadamer's work can be regarded as dialogic based on his emphasis on conversation and how he holds many of the common views shared by most dialogical thinkers (Vessey, 2005). The same argument goes for Skjervheim. Both of them developed valuable contributions on communication theory and on how to meet others that are compatible with a dialogic approach. Another objection concerns how Gadamer distanced himself from the notion of the I–Thou relationship. However, Gadamer's work bears several similarities with Buber's, and Gadamer has confirmed that he knew Buber well and that he regarded Buber's discussions on *the other* as "the most poetic, if not the deepest analysis on that topic" (Stewart, 1985, p. 333). Their theories are also developed in different contexts. Bakhtin dealt with literature, Gadamer with

philosophical hermeneutics, Rommetveit with social psychology, and Skjervheim with, among other things, the relation between social and natural sciences. However, all of them have transfer value to the research context addressed in this dissertation.

Rommetveit (1974, 1992) and several researchers (e.g. Kieran, 2001; Monaghan, 2005) emphasise verbalising and thinking aloud – to bring ideas into language. But what does this look like? What kind of communication aspects can be regarded as important in pupils' articulation of mathematical language at a computer? The following section describes six aspects of communication extracted from the theories of Bakhtin, Gadamer, Rommetveit, and Skjervheim.

2.5 Communication

Communication between individuals is regarded as the fundamental element for development in this dissertation. I consider the following communication aspects, mainly based on the work of Bakhtin, Gadamer, Rommetveit, and Skjervheim, as vital for a dialogic approach: listening, asking questions, willingness to take risks, reciprocal perspective setting and taking, agreement and difference, and mutual engagement. The four contrasts between how to meet others in figure 4 are incorporated in the discussion of these communication aspects.

Fiske and Jenkins (2011) distinguish between two communication models: *the transmission model* and *the semiotic model*. The former reduces the process of communication to merely transmitting information or meaning. It is an instrumental approach by which communication is regarded as a tool for the sender to transmit a message with as little communicative noise as possible. The emphasis is on the receiver's ability to understand the intentions and meanings put into the message by the sender. In the latter, the semiotic communication model, the focus is not on the sender's intentions but on the *exchange of meaning*. The roles of context and mutuality of communication are taken into consideration. The semiotic model describes communication as a dynamic process of joint meaning making. The

discussions of pupils' communication at a computer in this dissertation are in line with the semiotic communication model.

Listening

Listening must be regarded as a key condition for successful communication. Without listening there is no dialogic space; it is rather more like Piaget's (1997) description of the *collective monologue* or *parallel conversation* identified in the first empirical study of this dissertation. Bakhtin (1984, p. 293) described the monologic approach, in its extreme form, as "finalised and deaf to the other's response, does not expect it and does not acknowledge it in any *decisive* force".

To listen involves the ability to *hear* what the other says and, not the least, the ability to enter the other's thinking and relate it to one's own thinking. Gadamer (2004) emphasised the aspect of listening to purpose or reason when he discussed the *fusion of horizons*. Alrø and Skovsmose (2002), in their inquiry cooperation model, added the concept of active listening to the discussion of listening. Alrø and Skovmose emphasise listening as an active process of getting in contact. To grasp the other's view, to tune in to each other by active listening, is regarded as the first condition of mutual inquiry. Active listening means asking questions and displaying a willingness to investigate a subject matter. Such openness and abilities to listen to others' utterances require a particular focus towards questions.

Questions

The amount of questions, and how questions are active, can reveal a great deal about people's collaboration. People's use of questions can illustrate Bakhtin's (e.g. 1984) concepts *monologic* (homophonic), *dialogic* (polyphonic), and *continuous and open-ended dialogue*, and the difference between Gadamer's (2004) concepts *inauthentic* and *authentic dialogue*, that is, between different modes of communication, and between different ways of meeting others.

Bakhtin (1986) emphasised that dialogic communication includes questions and a questioning attitude: “If an answer does not give rise to a new question from itself, it falls out of the dialogue and enters systemic cognition, which is essentially impersonal” (p. 168). He goes on to say that dialogic communication “always includes a question, an address, and the anticipation of a response, it always includes two (as a dialogic minimum)” (p. 170).

Gadamer (2004) argued that a question is not always a question. There are *apparent questions* and *real questions*. Apparent questions are used to prove that one is right and generate inauthentic dialogue. Real questions are used to gain insight and “to bring into the open. The openness of what is in question consists in the fact that the answer is not settled” (Gadamer, 2004, p. 357). Bakhtin correspondingly emphasised that “questions must be serious and sincere” (Bakhtin, 1986, p. 7). Researchers such as Monaghan (2005) and Staarman et al. (2005) are in line with Gadamer when they emphasise pupils’ abilities to ask questions in order to create a communicative space.

Gadamer (2004) accentuated that not only are questions an essence of authentic dialogue; so is framing utterances in a questioning manner. This is in line with Bakhtin’s focus on the questioning attitude, “an open, dialogic attitude” (Bakhtin, 1984, p. 251), an attitude that makes continuous interaction possible. The research by Teasley and Roschelle (1993) supports this by finding that many utterances can actually be framed as questions. Thus, questions and utterances can be placed on a continuum with inauthentic and monologic questions/utterances at one end, and dialogic and real questions/utterances at the other (Alrø & Johnsen-Høines, 2012).

Gadamer (2004, p. 357) said that “in order to be able to ask, one must want to know, and that means knowing that one does not know”. To be inquisitive, open-minded, and to have the ability to listen therefore requires a particular focus on questions. “The differentia between methodological sterility and genuine understanding is imagination, that is, the capacity to see what is questionable in the subject matter and to formulate questions that question the subject matter further” (Gadamer & Linge, 1977, p. xxii). This is closely related to Bakhtin’s (1986) concept of continuous

interaction. However, this inquisitiveness and open-mindedness is dependent upon a willingness to take risks.

Willingness to take risks

To go outside the box, to discuss subject matters one has not discussed before, involves an element of risk. If one is afraid of showing incompetence, or if one finds it difficult to make a mathematical utterance unless one is confident that there is nothing wrong with it, then it will be difficult to create a dialogic space. In this respect, learning involves risks (Alrø & Skovsmose, 2002).[10]

Willingness to take risks is, as Gadamer (2004) underlined, related to openness, a desire to learn, and real questions are those to which the answers are not settled. It is demanding to raise questions directed towards something one does not know. "The questions by which one transgresses one's zone of familiarity and safety are the ones that most crucially require openness and willingness to take risks" (Herheim, submitted). Such communication is unpredictable and one does not have any guarantee that it will turn out to be a success. Another reason is that it involves exhibiting unfinished ideas. Openness, and knowing that one does not know, is important in order to achieve what Gadamer terms a fusion of horizons and what Rommetveit terms reciprocal perspective setting and taking.

Reciprocal perspective setting and taking

Reciprocal perspective setting and taking is Rommetveit's (1992) terminology, and by this he highlights several important aspects of a dialogic approach. The aspect of reciprocity shows how a dialogic context involves two (or more) persons, and that it concerns both parties. Two persons have to set and take perspectives in order to achieve dialogic collaboration. They have to take the risk of presenting their own

[10] If learning involves risks, then classroom activities need to address trust-building. Trust issues are not dealt with here because it is beyond the scope of this dissertation.

views, and they need the ability to take the perspectives set by the other. To take the perspective of the other requires an "attunement to the attunement of the other" (Rommetveit, 1992, p. 19). When a view is introduced by one participant it has to be jointly attended to by both of them. This is what Rommetveit termed "a dyadic state of intersubjectivity" (1992, p. 23), in which participants' perspectives have to fuse. This focus on response and perspective taking is also addressed by Bakhtin. "Any understanding of live speech, a live utterance, is inherently responsive … Any understanding is imbued with response and necessarily elicits it in one form or another: the listener becomes the speaker" (Bakhtin, 1986, p. 68). In this quote Bakhtin also points to the interplay between listening and speaking, and between taking and setting perspectives. Two interlocutors play the roles of speaker and listener in a continuous interplay.

Wegerif (2007, p. 4) wrote that "dialogic space opens up when two or more perspectives are held together in tension". A deconstruction of this quote into the following words describes key aspects of a dialogic approach: "space", "open up", "perspectives", "hold together", and "tension". These words are interesting because they illustrate much of the essence of how one can meet others in language in a dialogic mode. Bakhtin's (1981) concept *time-space* concerns the temporal and spatial dimension of a language and a conversation, and can among other things illustrate the dynamics between past experiences, the here-and-now, and what is yet-to-come of classroom interaction. A dialogic space that "opens up" indicates that it does not exist before or after, but that it can open up here-and-now. Wegerif talks about perspectives and to hold them together, and this is much in line with Rommetveit's reciprocal perspective setting and taking. The last word, "tension", illustrates that dialogic communication is not a type of communication characterised by extensive agreement. Equally fruitful, or possibly more fruitful, is the tension between different perspectives.

Agreement and difference

The discussion on agreement versus difference is in some sense a discussion about dialectic versus dialogic. The dialectical reasoning in which a thesis and an antithesis shall end up in an agreement (possibly dependent on compromises) is widespread in educational research as well as by scholars like Gadamer and Skjervheim. Skjervheim (1996a), when discussing the difference between *to convince* and *to persuade*, argued that to convince others does not only concern "the other's insight, but rather that insight that, if it is real insight, has to be the same for both of them" (p. 224, my translation), and Gadamer (2004, p. 292) stated that "the task of hermeneutics has always been to establish agreement". In research one frequently finds phrases such "to reach a shared understanding", "joint knowledge", and "mutual understanding". The focus on people's ability to reach consensus can be too dominant, although agreement will always have to be an ingredient of collaboration. If not, the communication will turn out as what Barnes and Todd's (1978) described as *disputational talk*, characterised by disagreement, individualism, and an unwillingness to take the perspective of the other.

A dialogic stance focuses on how differences exist. Such differences can be valuable parts of people's communication and learning. Bakhtin's (1981) notions of *speech diversity* and *multi-languagedness* illustrate some of the differences and diversity that can come into play when people communicate. The voices of two persons who collaborate will, to various degrees, have different historical and cultural backgrounds. Their voices have different intentions, intonations, and accents. Two persons can describe the same mathematical connection using different words, or even using the same words with different contents. The task of adding two two-digit numbers will most likely yield as many strategies as there are people doing it. Rommetveit's (1992) focus on the importance of taking the perspective of the other is to be understood in this context. If there are no differences, or tension, as Wegerif (2007) emphasised, then it is not necessary to spend energy on discussing how to take the perspective of the other.

Differences can be too big to cope with, or people might not be able or willing to manage them. This relates to the common ground discussion in section 1.4. People need to have a sufficiently common language and enough common knowledge in order to be able to take advantage of any tension between their perspectives. A flexible view on agreement, whereby people understand and acknowledge their similarities and differences, and where they can agree to disagree, will always be an important part of collaboration. Diverging opinions can be allowed to co-exist. However, a rigid view on agreement, to praise agreement uncritically, can imply that people agree too much and too fast. This is what Barnes and Todd (1978) would term *cumulative talk*. Cumulative talk might be a limiting factor for people's learning as their understanding can differ in many aspects. Understanding is also developed through resistance and conflicting views, and that might be undermined by a too strong focus on agreement. People apply different methods to the same tasks, they use different strategies within the same method, and they use different words to articulate their knowledge. This variety might be subverted if there is a one-sided focus on likeness and consensus rather than acknowledging the possibilities of diversity. Barnes and Todd's (1978) concept of *explorative talk* describes a third type of talk in which different perspectives of interlocutors are investigated with a critical and mutual engagement.

Mutual engagement

Skjervheim (1996c) distinguishes two opposite ways of meeting the other. In a *dual relationship* a person simply registers that the other is saying something and does not engage in this something. In a triangular relationship the persons turn their attention to a joint subject matter. They are both participants and they have a mutual engagement in a shared topic. Bakhtin (1986, p. 7) also focused on mutuality, engagement, and the tension between perspectives: "A meaning only reveals its depth once it has encountered and come into contact with another, foreign meaning: they engage in a kind of dialogue".

Rommetveit (1974, p. 37) argued, equivalently, for "how *some commonality is being established by the very fact that two persons engage in a dialogue* … a temporarily established *we* engaged in that particular act of communication as opposed to *all* others (he, she, they) who are not so engaged". Rommetveit, in line with Bakhtin and Skjervheim, accentuated the importance of engagement between interlocutors and also how the matter of engagement separates those who participate from those who do not.

Comments on communication

In this section I have discussed six aspects of communication based on the theories of Bakhtin, Gadamer, Rommetveit, and Skjervheim. The first aspect of communication was *listening*. The next five were: questions, risks, perspectives, agreement and difference, and engagement. These latter five aspects are all different aspects of *thinking aloud*. The relevance of thinking aloud is highlighted as a practical goal in section 1.2, in connection to the emphasis on pupil's ability to communicate orally in the national curriculum. To express oneself, to exhibit ideas, to articulate suggestions, to verbalise one's reasoning – to make thinking public – illustrate vital facets of thinking aloud. The ability and willingness to think aloud is a prerequisite for the five communication aspects discussed in this section. This discussion of communication aspects builds on and overarches the focus on communication in the sub-studies. The presentation might appear too harmonious if one takes into consideration the substantial differences between Bakhtin, Gadamer, Rommetveit, and Skjervheim. However, I do not aim to show their diversity but to extract concepts and notions from their theories that are valuable for the dialogic approach in this dissertation.

3. Methodology and methods

This chapter accounts first for the approach of the overview study. The three following sections concern the methodological and the methodical approach in the two empirical research studies. The fourth section elaborates on the principles of analysis for both the overview and the empirical studies. Reflections on trustworthiness and ethics finish the chapter.

Article number I presents the overview study. A clear thematic focus, functional inclusion and exclusion criteria, systematics, and thoroughness were emphasised in order to make the overview valuable as an overview in itself, but also for the empirical studies. The thematic focus was on research that concerned small group communication at a computer. Well-defined criteria and key terms were developed to conduct the mapping and inclusion–exclusion process. Articles could be included if they focused on primary and secondary school, applied qualitative or mixed-methods, and belonged to a dialogic view on communication or a sociocultural learning perspective. Lack of in-depth communication analysis and a focus on long-distance e-communication were the most effectual exclusion criteria. Initially, the time span was "the last ten years". However, it became evident during the process that the thematic focus, combined with continuously more fine-grained search criteria, generated quite few relevant articles. There was also a distinct starting point in the 90s for this particular field of research. This gave grounds for adjusting the time span in order to include the important seminal works of Fisher (1993) and Teasley and Roschelle (1993).

The articles had to pass a three-step quality assessment based on a collaborative process with colleagues in order to be included in the overview. They had to be: 1) well-written and reflective; 2) transparent with regard to methodology, methods, and

theory; and 3) published in well-recognised peer-refereed journals.[11] The articles were also assessed according to relevance, and categorised according to their focus areas. Four such focus areas emerged as categories through a code-and-retrieve process. This process of analysis is further explained in section 3.4 and in article number I.

Article number II refers to the study that applied the first part, the descriptive-analytical part, of a design-based research approach. Article number III refers to a design-based research study that carried out descriptions, analyses, and interventions in iterative cycles based on collaborative analysis of communication potential. To bridge educational theory, practice, and research through joint research reflections between pupils, teachers, and researcher has been a guideline for the design of the empirical studies. Both studies are imbued with a dialogic mode of communication as described in chapter two. This focus on communication is a vital part of the sessions where pupils, teacher, and researcher interpret and analyse video recordings together. The collaborative research perspective is a key asset of the design-based research approach and will be discussed in further detail in the following sections.

Figure 5 displays the titles of the articles and the research questions, and gives a brief overview of the methodology, methods, analysis, and samples for the three articles/studies.

[11] The search process also located digitally accessible book sections. Two of the 27 included articles are such book sections.

Article title	Research question	Methodology, methods, analysis, sample
Article I: Communication and learning at computers: an overview	*What are the important aspects in promoting pupils' talk and reflections in small group settings using a computer?*	An overview survey, search strategy: criteria and key terms for inclusion–exclusion, quality assessment, identification of focus areas through a code-and-retrieve process. Sample: the search strategy
Article II: Verbal communication at a stand-alone computer	*What characterises pupils' verbal communication at a stand-alone computer in a mathematics lesson?*	A first step of design-based research: mapping and understanding the current situation, collaborative research, video recording, loops of joint descriptions and analyses, identification of communication patterns through categorisation. Sample: representative, extremities, co-research
Article III: Managing differences by developing communication qualities: Pupils learning mathematics in pairs at a computer	*What characterises communication qualities, if any, that can develop a pair of pupils' communication and mathematics learning at a computer?*	Design-based research, collaborative research, video recording, iterative loops of design-interventions-analysis-redesign, analysis of communication qualities and differences between pupils. Sample: representative, potential, co-research

Figure 5: Overview of the three articles.

3.1 Design-based research (DBR)

The empirical studies belong methodologically to what Hoadley (2002) termed "design-based research". DBR stems from the seminal works of Brown (1992) and Collins (1992), and "blends empirical educational research with the theory-driven design of learning environments" (The Design-Based Research Collective, 2003, p. 5). Kelly (2003, p. 3) proposed that DBR "attempts to be both scientific and educational". DBR is, according to Sandoval and Bell (2004), a research design particularly developed for innovative educational research, often including educational technology, and for studies of learning in context through systematic, iterative designs. The Design-Based Research Collective (2003) proposed five characteristics for good DBR: 1) the intertwining of designing learning environments and developing theory; 2) continuous cycles of design, enactment, analysis, and redesign; 3) the generation of applicable knowledge for teachers and other researchers; 4) the accounting for how designs function, and documenting interactions that refine the understanding of the learning issues involved – in authentic settings; and 5) using methods that can document and link interventions to outcomes. Section 3.3 on methods and 3.4 on analytical principles elaborate on how the empirical research in this dissertation is conducted according to these characteristics.

The Design-Based Research Collective (2003, p. 6) goes on to underline that "in design-based research, practitioners and researchers work together to produce meaningful change in contexts of practice … Such collaboration means that goals and design constraints are drawn from the local context as well as the researcher's agenda". Wang and Hannafin (2005) confirm the special focus on context sensitiveness and collaborative research in DBR. The design of the empirical studies in this dissertation was initially, in line with another of Wang and Hannafin's characteristics of design-based research, insufficiently detailed so that progressive design refinements could be made according to emerging design experiences and findings.

I consider the DBR approach, based on its characteristics and participative research perspective, to be an appropriate methodological framework whereby functional research methods could be effectuated to find answers to the research questions presented in section 1.5. The research questions in the empirical studies were tentatively outlined by me and then operationalised and refined during the research partnership. The pupils and the teachers did not reformulate the research questions, but they participated based on a joint understanding of the tentative research questions developed in our discussions prior to the data collection period. The participative perspective of DBR reflects well the goal to include pupils and teachers as co-researchers.

3.2 Ontology and epistemology of DBR in these studies

The choice of design-based research as the methodological approach reflects the view on ontology and epistemology described in section 1.3. A view of reality and knowledge as something changing, to acknowledge subjectivism and a dialogic approach as ways of being, has several methodological and methodical implications, as Hitchcock and Hughes (1995) argued. The intellectual goal in this dissertation concerns the ability to characterise and understand social interaction and thus reflect Buber's social ontology and Bakhtin's notion of open-ended dialogue. Bakhtin's concept of dialogic also accounts, according to Holquist (2002), for an epistemological view according to which human interaction can be understood based on their use of language.

The pupils' and teachers' roles as co-researchers in this dissertation are in accordance with Johnsen-Høines' (2010) emphasis on participants' empowerment in collaborative research settings. The co-researching perspective also relates to the ontological and epistemological view described in section 1.3, according to which reality and knowledge is changing and diverse. Hoadley (2004) argued that universality is rare in educational research. The collaborative research approach

makes it possible to understand some of the diversity and complexity that characterises pupils' communication at a computer. It is not a goal to produce universal knowledge, and the fact that the samples are small and non-random reflects this. Rather, there is a particular emphasis on the perspectives of the participants. Distinctive features such as the authenticity of the context and thick descriptions distinguish this qualitative research approach from quantitative research studies.

The pupils' communication, what they say and do, is studied in order to understand and develop their learning processes. Communication is a natural focus for a social science such as this. The research questions reflect my view that the ability to express mathematics is a vital part of what should be reckoned to be mathematical knowledge. They are also based on the ontological assumption that non-physical objects such as thought and reasoning exist just like palpable aspects such as verbal communication and externalisation of thoughts on computer screens do. This assumption implies that it is possible to gain knowledge of pupils' thinking and learning based on what they say and do. The extensive uses of video recording, which captures both verbal and non-verbal activity, is grounded in the view that it is possible to gain knowledge on thinking based on what the participants say and do.

3.3 Methods

This section describes the methodical designs in the two empirical studies. The research question addressed in the first empirical study concerns characteristics of pupils' communication at a computer, and the research questions addressed in the second study concerns characteristics of communication qualities that can develop pupils' communication and mathematics learning at a computer. A design, inspired by Almås and Krumsvik (2008), based on extensive video recording, analytical loops, and collaborative analyses of the video recordings of pupils' work segments is used to find answers to these research questions. The schools, the teachers, and the pupils who participated in the studies can be described as a representative sample. However,

the pupils and the teachers in both studies are not only someone who is "researched on"; they are also co-researchers. This means that the pupils' and the teachers' interest in the teaching–learning of mathematics and in taking part in collaborative analyses of pupils' work was an important sampling criterion.

The notion of a *work segment* (ws) has been invoked to represent settings where two pupils' work at a computer as displayed in figure 1 (page 2). A ws typically lasted between 20 to 30 minutes. To capture the pupils' actions I used a video camera to record the pupils perpendicularly from the side. An external microphone was situated on the pupils' desk to generate quality recordings of their oral activities and screen-recording software was used to capture what the pupils did on the computer.

The designs in the two empirical studies have much in common, but there are also some differences between them.[12] Nine pairs of pupils and three teachers participated during the four months of duration of the first study. One ws of each of the pairs was observed and video recorded, and in all of these the pairs worked with the same task and the same software. The recordings were conducted when the classes worked with mathematics in pairs at the computers as per the teachers' plans. This is in line with the emphasis on authentic settings in DBR. The teachers had decided to use spreadsheet software, and they had created tasks that challenged the pupils' understanding of the technical functions of the software and their geometrical understanding related to perimeter and area.

The second study involved two[13] pairs of pupils, a teacher, and me, and it was carried out during five intensive weeks in January–February 2011. I was present several times during the two months prior to the research period in order to build and sustain the collaborative partnership that the Design-Based Research Collective (2003)

[12] Some of the design details and argumentation behind methodical choices in the sub-studies, and also the descriptions of the software and the mathematics tasks, are described in the articles and are therefore not repeated in this synopsis.

[13] Article number III concerns only one of the pairs.

emphasised. In these meetings we planned the research and we discussed the roles we were going to assume in the collaborative process of analysing the ws's. Three ws's were recorded for each pair. The same software was used throughout the period, but they worked with different tasks in each ws.

In this second study the teacher and I chose the software and developed the tasks together based on the knowledge from the research literature presented in section 1.4, from the first study, and from the teacher's knowledge about her pupils and the subject. The criteria were as follows: 1) transparency – avoid "noise" in the task formulations and use easy accessible software; 2) complexity – collaboration should be required; 3) support – software and tasks should have potential to support pupil's mathematical communication; and finally, 4) suitability – software and tasks should fit in with the pupils' curricular goals and the teacher's plan. We decided on a virtual geoboard where the pupils can explore geometric concepts such as area and perimeter by making different polygons. The pupils can measure and change angles, side lengths, and areas with just a few mouse clicks. The software is transparent and easy to use, and it provides elements on the screen that the pupils can use in their language and refer to in their discussions. A teacher can design tasks for the geoboard with a complexity that encourages collaborative investigations. The main mathematical challenges were to discuss the relation between number of sides and the sum of interior angles in polygons, and to discuss why one can change the perimeter of a triangle without changing the area. In the last ws there was a mathematical language exercise.

The setting in which the pupils, the teachers, and I watched and discussed the ws-videos has been termed *watch-and-talk sessions* (wt-session). Our joint discussions of pupils working in authentic settings are in line with Wang and Hannafin's (2005) emphasis on context-sensitiveness and collaborative research in DBR. These sessions were also video recorded in their entirety with a video camera and an external microphone. The camera captured all four of us as well as the ws-recording we were watching; see figure 2 (page 3). A wt-session typically lasted 40–50 minutes.

In the wt-sessions in the first study we watched the recording of the pupils and a printed version of their spreadsheet document. These wt-sessions were conducted in two loops. The first loop of analysis consisted of the wt-sessions in which six pairs analysed their ws together with me. Four of these six ws's were also analysed in wt-sessions by the teacher involved and me. Two out of these four ws's were regarded as particularly interesting and were hence scrutinised in a second loop of analysis (see the next section for more details on how the analyses were conducted).

In the second study we watched a picture-in-picture (PIP) movie, see figure 6. The screen recording was the main picture, while a recording of the pupils, perpendicular from the side, was synchronised as a PIP effect in the top right corner. The pupils, the teacher, and I analysed the ws's together in these wt-sessions. There was a cyclic design typical of DBR: three ws's were conducted and during the following day there was a wt-session in which we analysed a ws and agreed on a few communication issues to emphasise in the next ws.

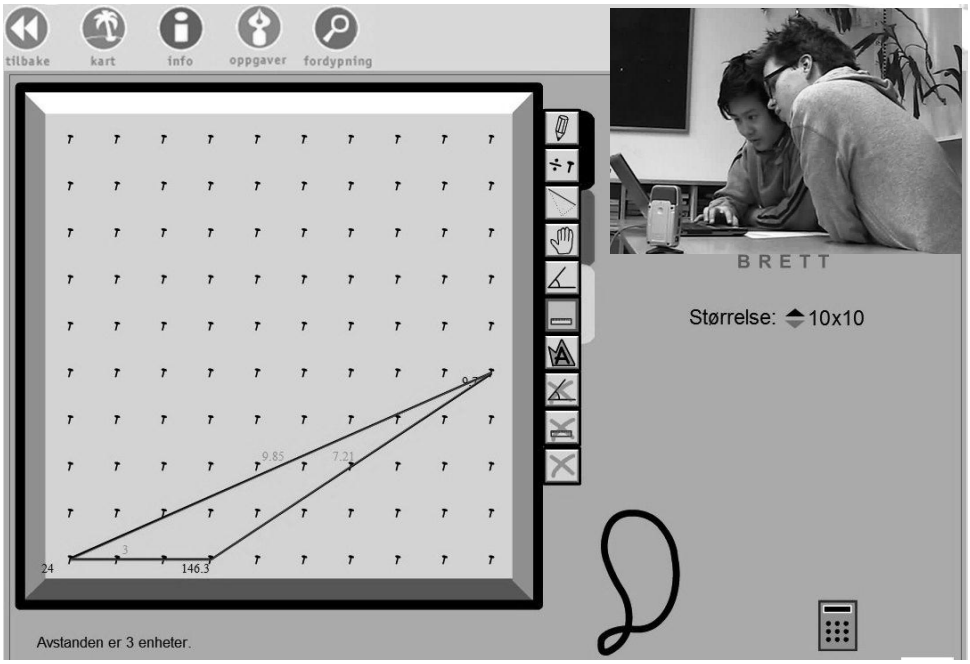

Figure 6: Snapshot of a video we watched in a watch-and-talk session.

3.4 Principles of analysis

This section concerns the analytical principles of the overview and the two empirical studies. First, the overview was designed to identify which aspects the research literature on small group communication at a computer highlighted as important. The search process is described at the beginning of this chapter and in the overview article. The analysis was as a code-and-retrieve process with the goal of identifying focus areas. A systematic code-and-retrieve process created an overview and made it possible to generate categories according to the research question based on the coding (Merriam, 1998). The articles were sorted and labelled according to the focus area they addressed (most of the articles addressed more than one focus area). Four focus areas stood out as stable, meaningful units as the process of constant comparison advanced.

In the first empirical study there were two analytical loops. In the first loop we watched the recordings almost in their entirety. We stopped the recording every time we found something interesting that could seem like an important characteristic of the pupils' communication, and otherwise when we needed to clarify or discuss something. The ws's of two pairs were chosen for a follow-up, second loop of analysis. The collaboration in these two pairs appeared, based on the discussions in the first loop, to be contrasting examples of how pairs of pupils communicate at a computer. The second loop was a collaborative process of analysis, in the same manner as the process in the first loop. However, in the second loop we selected key episodes from the ws's in order to generate more in-depth discussions on the emerging patterns. Working with contrasting examples evolved as a fruitful way of effectuating the analytical process of locating episodes in which relevant and characteristic patterns occurred. It gave us grounds for describing pupils' verbal communication at a computer.

The second empirical study applied a complete DBR approach with a cyclic process of identification and implementation of interventions. As described in section 3.3 we conducted three ws's for each of the two involved pairs. The day after every ws we

arranged a wt-session. This session was conducted in the same collaborative manner as the wt-sessions in the first empirical study, except that the pupils and the teacher were together and not in separate sessions. In the analysis in these wt-sessions we identified collaboration and communication aspects and potentials that we believed could prove important in order to develop the pupils' communication and learning in the next ws. The unit of analysis was the communication between the pupils, the teacher (when present), and the computer. The aim of our analyses was to single out a few key communication issues that the pupils would try to effectuate in the next ws. These issues concerned either something that was good and thus important to promote, or something that seemed to have an unrealised potential and that could be improved. The analyses in these wt-sessions can be regarded as the first level of analysis. The wt-sessions in both empirical studies were a particularly important asset of the research design because this was the arena in which the pupils and the teacher, as Johnsen-Høines (2010) underlined, could assume ownership of the study.

There was also a second level of analysis in the second empirical study, carried out when I, as a researcher, and retrospectively after the data collection had finished, analysed the ws's, the wt-sessions, and the reciprocity between them. The analyses of communication qualities in the ws's were carried out similarly to what we did in the wt-sessions, and the research question was the same. The main contribution of the second level of analysis lies in the analysis of the wt-sessions and in the analysis of the interrelations between the ws's and the wt-sessions. The inclusion of the second level of analysis provided an opportunity to elaborate on and to extend the discussions from the first level. Insufficient, inadequate, or erroneous analysis from the first level could be reanalysed. It was the second level of analysis that led to the investigation of how communication qualities can be important in order to manage differences fruitfully.

3.5 Trustworthiness and ethics

A core underlying assumption for the qualitative research in this dissertation is that "reality is holistic, multidimensional, and ever-changing; it is not a single, fixed, objective phenomenon waiting to be discovered, observed, and measured as in quantitative research" (Merriam, 1998, p. 202). This dissertation is conducted according to Lincoln and Guba's (1985, p. 290) notion of trustworthiness whereby the aim of trustworthiness is to assure that the findings are "worth paying attention to". The progress of qualitative research has resulted in several reconceptualisations concerning the trustworthiness of research: 1) internal validity to *credibility*; 2) reliability to *dependability*, methodological transparency, or audit trail; and 3) external validity/generalisation to *transferability*, authenticity, or contextualisation. Furthermore, the focus on rigor has changed from identifying indicators of precision and the appropriateness of the measures to accenting the importance of an unending and systematic reflective cycle. Generating and understanding data is emphasised, as is presenting this understanding compellingly, together with a method description (Fischer, 1994). All of these reconceptualisations relate to how qualitative research builds on subjectivity rather than objectivity – how reality and knowledge is regarded as something changing. Lincoln and Guba (1985) introduced in this respect a fourth aspect of trustworthiness: *confirmability*. How the aspects of credibility, dependability, transferability, and confirmability are addressed throughout the process, from the tentative plans to the dissemination process, is discussed in the following.

The trustworthy and ethical use of references has relevance for the dissertation in general and for the overview study in particular. The importance of accurate and valid use of research literature is stressed by Davidson, Kock, Loch, and Clarke (2001), and by Levy and Ellis (2006). I have paid particular attention to avoiding namedropping and taking material out of context, to quote correctly, and to aim for the most relevant reference. These ethical guidelines were an integrated part of the process of creating the overview article.

Credibility

The credibility of a study concerns the degree to which the findings are credible interpretations of the data material (Lincoln & Guba, 1985). The question of credibility has been subject to a systematic and continuous process throughout the dissertation. From a subjectivist standpoint there is no reality that exists independent of man; the reality is changing. This has consequences for how to address credibility. The uses of multiple investigators (the pupils, the teachers, and I), multiple sources (screen recording, video recording, observation), and collaborative analytical loops have facilitated extensive triangulation. Excerpts, interpretations, and analyses are also presented to colleagues and to other researchers at conferences. This triangulation has decreased credibility challenges such as researcher "bias" and reactivity (Maxwell, 2005). Building relationships with the pupils and the teachers and establishing a positive atmosphere in the wt-sessions are additional measures taken to reduce reactivity. The Design-Based Research Collective (2003, p. 7) argued that "design-based research relies on techniques … like thick descriptive datasets, systematic analysis of data with carefully defined measures and consensus building within the field around interpretations of data". The articles provide thick descriptions of the ws's and the wt-sessions through the extensive use of excerpts and the corresponding interpretations and analyses of each excerpt. The collaborative and cyclic approach of DBR strengthens the credibility because the participatory mode of research includes member checks as an integrated part of the research design. The credibility of tentative interpretations and analysis could be clarified instantly in the wt-sessions. In the wt-sessions we tried to develop a mutual understanding of what we saw on the video recordings, something that reflects the Design-Based Research Collective's focus on consensus building in the quote above.

Transferability

Lincoln and Guba (1985) described transferability as the degree to which findings apply or transfer beyond the bounds of the project, to other contexts, and/or with other participants. Qualitative research applies an every-situation-unique perspective. The

aim is to understand a specific context in depth and in its uniqueness, and not to reveal "the general laws" of human interaction. Merriam (1998, p. 210) argued that "what we learn in a particular situation we can transfer or generalise to similar situations subsequently encountered". Although the studies in this dissertation aim to generate concrete and contextualised knowledge, there has been an aim to conduct research that has transfer value for both teachers and researchers.

In all three studies it is a goal to provide rich, thick descriptions of the methods, the data material, and the analyses. Both empirical articles offer ample amounts of excerpts from the ws's and the wt-sessions. The dissemination of the data material through excerpts enhances the authenticity and, in turn, the transferability of the studies. It is a question of providing sufficient information to make readers able to estimate how relevant these studies are compared to theirs. This includes information about how representative the research context is and information about the pupils' computer, mathematics, and communication abilities.

Dependability

Dependability is a quality assessment of the data collection process, the analysis, and the theory generation. Dependability is not a measure for explaining how likely it is that a new, identical study will generate equivalent results. The dynamics of reality requires a criterion by which one can discuss to which degree our findings are consistent with our data material. An important aspect for promoting the dependability in this respect is to have a methodical transparency, to have what Lincoln and Guba (1985) termed an *audit trail*. The complex interventions conducted in the empirical studies involved hundreds of decisions by the participants. The research processes are therefore described thoroughly, together with my own theoretical stance. Thick descriptions about my background, the research design, and how the analyses are conducted should make the reader able to assess the dependability of the studies. The design involved methods that documented the interventions in order to warrant the reflections on why the outcomes occurred. The extensive use of video recording, the transcriptions of ws's and wt-sessions, and the

many excerpts in the articles, are measures taken not only to increase the credibility but also the dependability of the dissertation. An aspect such as causality can be a challenge since it is only possible to pursue some of the many factors involved, and identical replications are not feasible in such intervention studies. However, the triangulation through multiple investigators, sources, and methods, the iterative analysis and interventions, and the video recording, has made discussions on causality possible and enhanced the dependability of the studies.

Confirmability

Confirmability concerns how well findings are supported by the data material and that findings are not generated as a result of a researcher's biases. Confirmability is related to a researcher's ability "to understand oneself as part of a community and one's work as part of a project" (MacIntyre, 1978, p. 37). A central issue of confirmability is therefore to be conscious of one's role as researcher, how one, and one's possibly held assumptions, can influence the situation. The research design in the empirical studies includes multiple investigators, multiple sources of data, and multiple methods. This makes an extensive triangulation possible that enhances not only validity and credibility but is also vital to address the challenge of confirmability. The methods, the collaborative processes in the wt-sessions where we analysed and arrived at our findings, are accounted for. This provides an audit trail by which readers can follow the trails and authenticate the results.

Brinkmann and Kvale (2005, p. 161) emphasised that "scientific objectivity is about letting objects object" to a researcher's methods and interpretations. It was an aim in these studies to recognise "the perspectives of those at the heart of the transition process – that is, the pupils" (Jones & Stanley, 2008, p. 32), to give the pupils a voice and listen to what they have to say (Cullingford, 2002). The collaborative research approach provided good opportunities for the pupils and the teachers to influence methodically and to take part in the interpretations and analysis. This increases the confirmability of the studies.

Wegerif (2011a, p. 212) proposed, in line with the emphasis on giving pupils a voice in the previous paragraph, that "there is an ethical imperative in a dialogic approach which suggests that a more genuine engagement with the perspectives of practitioners and children is required in research on educational practice". A genuine engagement with the perspectives of the pupils and the teacher has been a key essence in the research design of this dissertation, both ethically and methodically. Additional ethical considerations are addressed in the following.

Ethics

There are three key aspects that have served as important ethical guidelines during the work on the dissertation: honesty, common sense, and systematics. These aspects may look somewhat basic. They have, however, been valuable and important guidelines, both ethically and otherwise, throughout the whole research process, from the selection of excerpts from the transcriptions, through choosing which findings to pursue, and, not least, to the dissemination process. To disseminate results honestly, in accordance with the data material and with respect for the participants, is both a matter of trustworthiness and ethics.

Another important ethical issue is the researcher–participant relationship (Merriam, 1998). In the empirical studies the participants are more than merely oriented by the goals of the studies. They also take part in the process of setting the goals and deciding how to reach these goals. The success of the research design, the wt-sessions in particular, is dependent upon a good relationship between the participants and me. The importance of this relationship is particularly visible in the unsuccessful study described in section 5.2. The researcher–participant relationship concerns essentially what kind of communication we develop in the wt-sessions. The fact that the pupils and the teacher have insight into and take part in what we are trying to achieve is an important foundation for our communication. Their informed consents are very informed. An essential part of my role as a researcher is to include the participants in the project and to be genuinely interested in taking their perspectives. The joint

analysis of ws's in the wt-sessions, the mutual engagement in a joint subject matter, is based and dependent on a positive collaborative relationship.

The question of anonymity is taken into consideration in all parts of the dissertation, including the appendices. All information that might harm the participants' anonymity, such as the names of the schools, is removed. The school principals, the teachers, the pupils, and the parents/guardians have given their informed consents, and the research is approved by the Norwegian Social Science Data Services. This includes an approval for the use of video recording during the studies, for the use of video sequences in presentations, and for the use of pictures in publications.

4. Findings

This chapter chronologically presents the findings from the three studies. I do, however, regard the excerpts and the interpretations and analysis of the excerpts in the articles as equally valuable as the findings. They are an important part of the findings.

The overview of the research literature identifies four focus areas in the research of communication and learning at a computer: to establish and develop a common ground, communication characteristics, roles, and software design/task structure. These focus areas are discussed in section 1.4, where they are included in the discussion on relevant research in this synopsis. An additional finding from the overview study is how the studies by Fisher (1993) and Teasley and Roschelle (1993) are identified as the seminal works in this field of research. Hence, the overview shows that this field of research is relatively young, and the overview also shows that the number of relevant studies is low. The latter is a consequence particularly of the focus on face-to-face communication rather than e-communication, and because of the request for transcription excerpts and in-depth analysis of communication.

The overview finds that micro studies and pupils' and teachers' voices in research are not much focused on in the current research. The dual approach of the overview towards education in general and mathematics education in particular reveals a lack of interdisciplinary communication. If one adds that this field of research is complex and lacks a mutual terminology, it is evident that a complete coverage is not possible to achieve. There is also, from a mathematics education perspective, a paradoxical finding: the research on how digital dynamic possibilities in graphing and geometry outrank the traditional paper and pencil provides the best documented results of how computers can contribute to pupils' learning of mathematics (e.g. Ainley, et al., 2000; Ruthven, et al., 2008; Åberg-Bengtsson, 2006). Still, the use of computers in mathematics is, according to Artigue (2000) and Fuglestad (2009), scarce, and in fact particularly so when it comes to dynamic geometry software and graphing software (OECD, 2005; Vavik, et al., 2010).

The communication analysis in the first empirical study identifies six communication patterns:

	Pair one	**Pair two**
1	Verbal style, think aloud, "talk and write" Many (short) utterances, in rapid succession	Write directly on computer Long periods of silence
2	Drive (mutual), progression: "Let's go on!"	Individual progression Teacher as driving force
3	Teacher often addresses both of the pupils Pupils use "we" and "us"	Teacher often addresses *one* pupil Pupils rarely use "we" or "us"
4	Speak "in chorus" "Huey, Dewey and Louie" talk	Two individual/parallel conversations
5	Repeats what the other says Uses the same linguistic turns	Little amount of mutual language
6	Laughter/humour, speak positively about their work. Supportive	No/little amount of laughter and support

Figure 7: The six communication patterns from the first empirical study.

These patterns can be summarised in terms of two interrelated main aspects: thinking aloud and building a mutual language. To address each other, speak in chorus, and use the same linguistic turns are distinct patterns for successfully setting and taking each other's perspectives. "Huey, Dewey, and Louie" talk represents the communication where pupils compose sentences together by alternatively making short contributions. These communication patterns help the pupils build a mutual language and a communicative common ground. When these patterns are absent, when the pupils make individual contributions without thinking aloud or building a mutual language, we observe what we term "parallel conversation". An additional finding is that although several pairs communicate well, there is clearly a lack of mathematical focus in their communication. The communication is on task but it is more technological than mathematical. The spreadsheet software is not sufficiently transparent, and the software design and the tasks do not initiate and support pupils' mathematical discussions well enough.

The study also generates descriptions of the contrasts between two pairs of pupils based only on how they sit at the computer. The pupils in one of the pairs are leaning towards each other and the computer. They have two, three and even four hands on the keyboard simultaneously. The teacher, when describing the pupils and the computer, said that they were "as in a bubble". The pupils establish and maintain this mode of collaboration through mutual engagement in the task, and when they enter this mode they seem to be able to ignore their surroundings and focus entirely on their own subject matter discussions. The pupils in the other pair sit as far apart as possible within the frames of still calling it something other than "individual work". The computer is clearly associated with one pupil at a time. It is angled towards the pupils controlling it and moved a little away from the other pupil. One of the pupils describes their work as follows: "It was not very much talking. It was more like we divided the work between us". The pupils are not able to develop a collaborative mode in which they can bring matters into language, ask questions or present and take each other's perspectives.

In the second empirical study the two communication qualities of listening and thinking aloud are investigated. In the beginning the two pupils' communication is halting. They do not follow up the other's suggestions; they pursue their own ideas instead. However, the pupils develop several communication qualities as the collaborative research process progresses. The qualities of listening and thinking aloud gain content during the process. We found that listening implies the ability to hear, understand, and relate the other's reasoning to one's own. The pupils introduce their ideas, they continue on the other's reasoning, and they adopt each other's linguistic turns multiple times. This is in line with several of the patterns developed in the first empirical study. All of these "thinking aloud" qualities are dependent on active listening. Important underlying factors are the pupils' willingness to set and take perspectives and their ability to ask questions. *What* they express and *how* they express it are communication factors that influence the learning outcome of collaboration. The pupils establish and develop a communication space in which they share ideas in an inquiring manner. Two reasons for the pupils' achievement of such a

collaborative space are that approximately 25 % of their utterances are real questions and that they repeatedly use including pronouns such as “we” and “us”.

The analysis also identifies fundamental differences between the two pupils. One of the pupils emphasises details, accuracy, and counting, while the other applies a holistic approach. The analyses from the second level of analysis, see section 3.4, show how the pupils’ development of communication qualities makes it possible for them to manage differences fruitfully. The pupils themselves underline two crucial communication qualities when we summarised the research process in the last wt-session: “we used the same words … we had mutual concepts” and “we didn’t stick to what we wanted ourselves. We were kind of open-minded to what the other said”. The two pupils develop a mutual language with humour and high spirits, which is identified as one on the patterns in the first empirical study. Their different ways of being and their different approaches to learning mathematics turn out to be positive for their learning because their ways of understanding are challenged and carved in a space of mutual engagement and reciprocal perspective setting and taking.

In this study the teacher and I set up the software and the tasks to facilitate the pupils’ mathematical inquiries and discussions. We emphasised using transparent software that offers dynamic possibilities such as dragging, and to make the tasks open and complex enough for collaboration to be valuable. In the task formulations we specifically ask the pupils to investigate and discuss the different mathematical tasks collaboratively. The importance of such preparations is illustrated by the ways in which the pupils collaborate and how their communication concerns mathematics and does not exhibit unclarities related to technology or task formulations.

5. Discussion

The chapter comprises four sections. The first discusses findings and design in relation to previous research and theory. The second concerns the possible limitations of the research and the challenges we faced during the research process. The third points out implications of this dissertation for mathematics teaching and for future research in mathematics education, and the last section presents some concluding remarks.

5.1 Findings and design in relation to previous research and theory

The four focus areas in the overview study constitute an important foundation for the goals and design of the two empirical studies. The empirical studies are in this respect closely related to previous research. Six communication patterns are identified in the first empirical study, and the labelling, descriptions, and examples of these patterns give grounds for describing differences between pairs of pupils' communication at a computer. The importance of thinking aloud, to express oneself, discussed in this study is also identified in some of the research included in the overview study (e.g. Kieran, 2001; Monaghan, 2005). Two pupils that participated in both loops in the first empirical study said the following in a wt-session: "Can't read each other's thoughts … It's not possible … just think aloud instead". The study supports and adds to the focus on verbalisation in previous research. The novelty in these results lies particularly in how the pupils' and the teachers' views are influential in the identification of the patterns. The interpretations and analyses do not only generate findings; they also illustrate the collaborative aspect of the research design. The findings emerge during the research process; they are interwoven with the design. In fact, more than half of the patterns are closely linked to analytical utterances made by the pupils and the teachers. An example is how the comments made by the pupils in the quote above play an important role for the identification and labelling of pattern

number 1 in figure 7. Another example is how the pattern concerning mutual engagement and progression is based on analysis initiated and developed with the teacher in charge, see for instance excerpt 3 in article number II.

Research on the link between building a mutual language and completing each other's sentences in this educational context is rare, although there are exceptions such as Kieran (2001), Sinclair (2005), and Teasley and Roschelle (1993). This study strengthens this link and provides examples and analysis of three related aspects: speaking in chorus, alternately composing sentences together ("Huey, Dewey, and Louie" talk) and using the same linguistic turns. The link between these three latter aspects and mutuality is elucidated. Pupils are not able to develop such communication patterns unless they build a mutual language. The excerpts and analyses show that building a mutual language is not only a result of, but also a part of, the process of thinking aloud. This confirms Teasley and Roschelle's (1993) research on constructing and maintaining a joint problem space.

The study also provides descriptions of two different ways of working together only based on analysis of visual signs from the video recording. Contrasting descriptions concerning issues such as how do the pupils sit, where do they look, and who uses the keyboard and the mouse, are valuable for the understanding of this educational context because they provide distinct differences between collaboration and individual work in pairs.

The aspect of questions is addressed in both empirical studies in terms of numbers, form, and content. The first study contrasts two pairs' communication. A main difference between the pair in which the members are able to communicate dialogically and the other pair is the amount or the proportion of real questions. Dialogic communication is not just about speaking in chorus or completing each other's sentences; it is also about asking questions and exhibiting a questioning attitude. In the other pair the members rarely ask questions. On the contrary, they make several authoritative utterances. The connection between having trouble with engaging in fruitful communication and the amount of utterances without further

explanation or any invitation to question the underlying reasoning is also identified in the research by Kieran (2001). The investigation of questions and a questioning attitude by discussing contrasting examples complements previous research on the role of questions (e.g. Monaghan, 2005; Staarman, et al., 2005; Teasley & Roschelle, 1993).

The findings in the second empirical study concern the communication qualities of thinking aloud and listening. These two qualities constitute the initial, broad focus of attention, which we operationalise and concretise during the research process. The pupils develop communication qualities such as continuing each other's reasoning, echoing, addressing, and asking questions. These qualities add to the results of Alrø and Skovsmose's (2002) research on thinking aloud, and to Mercer and Wegerif's (1998) research on explorative talk in which pupils build on each other's reasoning with critical engagement. To follow up each other's utterances by using the same linguistic turn is a clear sign of mutual engagement and of building on the other's reasoning. It is a communication nuance that has not been the focus in much of the research in this field, but one can say that it expands upon Teasley and Roshcelle's (1993) research on the link between restatements and how shared the nature of two pupils' work is. This way of investigating and sharing ideas relies on the listener's responsibility of listening actively in order to understand and question the content of the speaker's utterances (Alrø & Skovsmose, 2002). Much of the reason for the pupils' ability to develop these different communication qualities can be ascribed to their questioning attitude and to the fact that 25 % of their utterances are real questions.

The pupils' development of communication qualities turned out to be more influential than expected. The second level analysis reveals fundamental differences between the two pupils. One of them adheres to a holistic approach without much need for confirmation while the other adheres to a detailed, counting approach and to frequently give and request confirmations. The study makes a major contribution to this field of research by showing how pupils can learn mathematics collaboratively

through communicating with someone different from themselves. The process ends with the two pupils acknowledging and to a certain degree even adopting each other's approaches. It is not a matter of overcoming the differences between them, but to manage them fruitfully, as Wegerif (2011a) argued.

The degree to which the pupils are able to communicate mathematics is emphasised in both empirical studies. The first study shows that the pupils' communication was more technological than mathematical. The focus on technology is linked to how the software lacks transparency, which Ainley et al. (2000) also identified, and how the software is not specifically designed for teaching and learning mathematics. The importance of not only designing software to support talk but also to direct it to particular curriculum areas is identified by Wegerif et al. (1998). In the second empirical study the pupils achieve a distinct mathematical focus in their communication. This is related to the communication qualities discussed above, but also to the fact that the teacher and I could choose educational software that is constructed and adapted exactly for this particular age group and for this particular curriculum. The characteristics of the software and the pupils' communication qualities confirm Lavy and Leron's (2004) finding that the software's ability to support and provide language and to facilitate subject matter discussion also applies to the context of learning mathematics. The tasks are created in collaboration with the teacher in order to adapt the degree of difficulty, complexity, and choice of words to the pupils' mathematical, communicative, and linguistic abilities. The focus on how task complexity and software characteristics can provide a context in which pupils are stimulated to communicate mathematics builds on the research by Wegerif et al. (1998).

The findings in these empirical studies confirm, complement, and add to existing research that is conducted in settings comparable to the context addressed in this dissertation. The excerpts, the interpretations, and the analyses add to a small data base of pairs' communication at a computer in mathematics learning. The design involving analytical loops, video recording, and a collaborative research approach has

a touch of methodical innovation and can be a valuable addition to previously applied research designs in this field. The decision to focus on contrasting differences has worked out well. One example from the first empirical study concerns how the isolated utterance “I got it” is contrasted with the verbal style of the other pair. In the second empirical study there are multiple exemplifications of the differences between the holistic and the counting approach. Such contrasting is effective in order to generate distinct communication qualities.

The dissertation utilises and combines several theoretical concepts and notions developed by Bakhtin, Gadamer, Rommetveit, and Skjervheim in the discussions of pupils’ communication. Using such theoretical concepts lets us make two achievements: First, some of the content of valuable theoretical contributions concerning how to communicate and meet others is illustrated; and second, the understanding and analyses of communication in a classroom context are enriched by potent theories with long historical roots. The following examples show some of the most influential ways in which these theories are used to improve the understanding of pupils’ communication at a computer. Pupils build a mutual language, a common ground for collaboration, by addressing each other and by consistently referring to themselves by “we” and “us” rather than “I” and “me”. This use of pronouns, as well as the contrast between a questioning and an authoritative attitude, is reflected upon by using Bakhtin’s (e.g. 1984) contrast between a dialogic and a monologic approach. To use the same linguistic turns, identified as a pattern in the first empirical study and as a communication quality in the second empirical study, is illuminated by Bakhtin’s (1986) notion of echoing. The tension between different perspectives is discussed by Rommetveit’s (1992) focus on reciprocal perspective setting and taking as well as Bakhtin’s (1981) focus on the gap between the authoritative word and the internally persuasive word. To establish a joint focus is elaborated by Skjervheim’s (1996c) emphasis on a mutual engagement in a shared subject matter. Thinking aloud is related to Rommetveit’s (1992) emphasis on bringing perspectives into language, and the aspects of questions and questioning attitudes are analysed by using ideas from Bakhtin (1986) and Gadamer (2004). There is without doubt more to these concepts

and notions than what these studies show. The implementation of the theoretical concepts mentioned in this paragraph is to be regarded as possible concretisations and exemplifications of the theories of these thinkers.

5.2 Limitations and challenges

Conducting research requires, among other things, an ability to establish delimitations, deal with challenges, and to be aware of and reduce the impact of limitations. Important delimitations, the boundaries of the dissertation, are the goals, the research context, the research questions, and the sampling. In the following I discuss the limiting aspects of two delimitations, some other limitations that may place restrictions on the findings, and the most significant challenges to the research.

There are especially two delimitations that can be viewed as limitations. First, the focus in this dissertation has been on face-to-face communication. The potential of new ways of communicating and learning through e-communication, the development of Castell's (2010) idea of *the network society*, has not been investigated. In this respect the dissertation lacks an innovative aspect, but researchers like Dillenbourg and Evans (2011) and Overdijk and Diggelen (2009) argued for a growing importance of face-to-face communication since it is the context in which pupils most often participate. Second, during the research processes we have focused more on communication than on the role of the computer and the learning of mathematics. This can be regarded as a limitation because a more distinct focus on these two latter aspects can be valuable in itself and could also have increased our understanding of pupils' communication.

The focus has been on collaborative learning. Some might argue that investigating collaborative learning at a computer can feel a bit forced when taking into consideration the "individual nature" of computers: there is one keyboard, one mouse, and one screen. One of the most distinct collaborative problems for one of the pairs in the first empirical study was how the computer was linked to one pupil at a time. The

computer screen was not a shared frame of reference for them. The pupils in the collaborative pair, on the other hand, were, according to their teacher, as in a bubble at the computer. The computer screen served as a particularly vital component for promoting their collaboration. There were frequently three and even four hands at the keyboard simultaneously; they shared the keyboard and the touchpad. This is in line with how Lavy and Leron's (2004) and Teasley and Roschelle's (1993) studies highlighted the role of the computer screen as a shared frame of reference.

Pupils have different learning preferences. Some prefer collaborative learning and others prefer working on their own. Pupils' communication has in this dissertation been analysed from a dialogic perspective and within a particular ontological and epistemological framework. When two pupils have not been able to collaborate, the interpretations and analyses have focused on what they might lack in terms of collaborative communication qualities. The focus has not been on how their individual reasoning contributed to their individual learning. Similarly, successful communication has not been criticised by a perspective that defends an individual approach to learning.

A limitation of the overview study concerns how difficult it is to develop search strategies that capture the most relevant literature. Search terms such as "ICT" and "communication" generate enormous amounts of hits. All of the following terms are used in research that concerns aspects that are relevant for this dissertation's focus on communication and learning: "collaboration", "cooperation", "talk", "discussion", "conversation", "dialogue", and "interaction". There is a lack of mutual terminology.

There are also contradictory results regarding the degree to which use of ICT enhances pupils' learning. Machin, McNally, and Silva's (2006) research showed positive learning outcomes by use of ICT in English, less so in science, and not at all in mathematics, while OECD (2005) found a positive association between the length of time of ICT use and students' performance in PISA mathematics tests. In a research literature review of ICT and attainment Cox and Abbot (2004) found that research focusing on specific uses of ICT, such as the second empirical study in this

dissertation, generates the most convincing results, while research without a clearly defined use of ICT generates unclear findings. The pupils in the first empirical study learned some of the functionalities of the spreadsheet software but did not learn much mathematics. The teachers explained this by pointing to the fact that the use of spreadsheet software is a qualification requirement in the national curriculum. The use of computers was not integrated with the other aspects of teaching and learning; it was more like what Richards (2005) described as "an add-on". The second empirical study therefore focused on specific use of software directed towards curricular goals, and on integration between computer use and other activities in order to achieve a more holistic teaching–learning approach.

This dissertation can be criticised according to the view that all research, no matter if it is qualitative or quantitative, has to argue for how generalisable its findings are. The empirical studies in this dissertation are small-scale studies conducted over a relatively short period of time and with a single researcher. They do not encompass a wide range of settings and participants. Such studies have, according to Baron and Bruillard (2007), often strong contextual dependencies and therefore also produce results with limitations. Large, long-term studies have the opportunity to identify patterns over time and to increase trustworthiness, for instance by conducting intercoder reliability. However, this dissertation emphasises transferability and thick descriptions rather than generalisability. The pupils' and the teachers' roles as co-researchers have increased the transferability of these studies by providing ample possibilities to conduct member checks and triangulation.

The nine pairs and the three teachers in the first loop of the first empirical study can be regarded as representative since they come from a typical Norwegian lower secondary school and the pupils' collaborative, mathematical and technological abilities vary between being above, on, and below average. The context and the software are also representative, since working in pairs with spreadsheet software a few lessons a year is common at the lower secondary level. The two pairs who took part in the second loop did so because their communication was *information rich*

(Patton, 1990) and represented two extremities. Their communication did not diverge much from the seven other pairs who took part in the first loop, but it consisted of distinct communication patterns. In the second empirical study the pupils and the teachers participated because they showed interest in taking part in such a study. The focus was to illustrate the communication and learning potential of the context, and the teacher and I created tasks and chose software with this in particular in mind. This context is therefore not representative; rather it is an example of what is possible to achieve. It is then possible to argue for contextual dependencies and low generalisability. However, the aim of this dissertation has been to enhance contextualisation and to give readers the opportunity to consider the transfer value by providing thick descriptions.

One can also talk about limitations concerning a gap between theory and reality. The first concerns the development of patterns and categories. A different research team will probably not generate identical patterns, or they will at least not develop the same labels as we did. Also, in such a process of theorising one can create an unwanted distance to reality and a rigid view of reality. Pupils' communication does not belong to a pattern or a category; the patterns are not their communication (Merriam, 1998). The patterns are representations of pupils' communication, a communication that can quickly change from one pattern to the other. The categorisations of the four focus areas in the overview study can be similarly problematised. Many of the articles included in the overview study have more than one focus area, and their focuses might also be somewhere in between categories. Second, it is possible to argue that there is a gap between research on expressing mathematical knowledge at a computer and a classroom reality that can be heavily influenced by an educational system that acknowledges textbook reproduction and national and international test rankings.

Research on communication at a computer is a young field of research, and DBR is a young methodological approach. This means that the dissertation's computer context and its methodology have not gone through centuries of development and quality enhancements. Dede (2004) pointed to this by saying that:

> At this early stage of its evolution less ambitious claims about DBR, more careful delineation of its limited role in the spectrum of experimental methods, the evolution of collective standards for what constitutes quality in DBR, and the focusing of the DBR community's efforts on issues of concern to practitioners and policymakers all seem appropriate next steps. (p. 105)

The most significant challenge for this dissertation in this respect has been the wt-sessions. The Design-Based Research Collective (2003) called attention to this issue when they brought forth a logistical challenge for design-based research, namely to maintain and sustain a collaborative partnership. It is an ambitious enterprise for pupils, teachers, and researchers to analyse data together. The wt-session is a communicatively demanding component of the cyclic research design. Article number III is said to be referring to the second empirical study. However, it actually refers to the third study because the second study unfortunately did not work out. The problems started when the most heavily involved teacher in the first school was moved to another class. This was the main reason why the second study was moved to a different school.

In the first study the pupils' work was recorded, and I watched the recording together with them before I watched it together with the teacher. These collaborative analyses, the pupils' and the teacher's roles as co-researchers, worked out well for several reasons. By splitting up the pupils and the teacher in separate wt-sessions we avoided challenges related to roles and relationships between the teacher and the pupils. Another influential factor for the pupils' participative approach was that they "outnumbered" me. Since there were two pupils, and I was the only adult, we avoided a context were a single pupil is "interrogated" by several adult researchers. Then there was the practical issue that we had a room at our free disposal. This gave me the opportunity to take my time organising the computer, the video projector, the video camera, the microphone, and all the wiring. After I had checked that everything worked fine, I could spend 15 minutes composing my thoughts. I would then go to the classroom, and the two pupils and I would walk-and-talk during the five minutes it took us to get to the "research room". These three factors: the research room being

ready, I being mentally ready, and the small talk between the pupils and me, was more important for our collaborative research approach than I was aware of at that time.

One year later, in the unsuccessful study, there was no room at our free disposal. I got a key to a random room and had ten minutes to set up the equipment. I did not have any extra time to organise my thoughts or the room. This time the pupils and the teacher took part together in the same wt-session, something that, among other things, added the dimension of the teacher–pupil relationship to the wt-session context. The pupils (and the teacher) came by themselves, which meant that we had to break the ice surrounded by wires and a buzzing camera and a video projector. An additional challenge was that in this study the pupils used dynamic software in the ws. This meant that in order to understand what had happened in the pupils' ws we needed to watch both the video recording of the pupils and the screen recording video. I tried to solve this by playing the two videos in two separate windows and to achieve as good a synchronisation as possible with respect to stopping and starting the two videos. This combination of a general technological overkill, mental overload for me, and relationships that were not sufficiently well established was not favourable at all for our collaborative analyses and discussions. Consequently, there had to be a third study.

Based on the knowledge gained from the two previous studies, the following was emphasised in the third study: to build relationships between the pupils, the teacher, and me, to discuss roles, and to reduce the technological impact. The first two aspects were addressed in particular in the discussions when I visited the class several times during the two months prior to the study. The third aspect was solved by having a room at our free disposal with a video projector permanently attached to the ceiling. This gave me the same opportunity to get organised as I had the first study, and the informal walk-and-talks again proved to be a positive start to the wt-sessions. I also made picture-in-picture movies, see figure 6, where the screen recording and the recording of the pupil are synchronised into a single movie. These may appear to be

small details, but taken together they made the difference between success and failure for the wt-sessions.

To transcribe the ws's was a challenge, at least until I became used to the pupils' voices and their ways of speaking. The wt-sessions were very challenging to transcribe. It was quite time-consuming to transcribe four enthusiastic voices, the sometimes overlapping topics, and with the voices from the ws video in the background. Particularly when one takes into consideration that communication analysis requires functional transcriptions of high quality.

A last challenge worth mentioning is a flaw in the Geoboard software: one could only do measures on "activated" figures. The teacher and I thought we had mapped the software thoroughly, but the pupils faced a problem we did not foresee. When they tried to measure something on a non-activated figure they got a faulty error message: "This is not a vertex". This shows how demanding it is to ensure quality by testing software and pretending that one can imitate what pupils might do. It did not take long before the pupils clicked in ways that we had not, and unfortunately a minor bug in the software occurred. It puzzled all of us, but after a few minutes the pupils had figured it out.

5.3 Implications

This section discusses implications of this dissertation for future research, and for the teaching and learning of mathematics. Article number I refers to the overview study and points to some of these implications and they are therefore not repeated here. The overview study surveyed education in general and mathematics education in particular. This dual focus revealed a lack of interdisciplinary communication and a diverging terminology. It is possible for researchers to take more into consideration research that is "outside" their own particular field of research. Furthermore, the diverging use of key terms, which can partly explain the former point, can be converted into a more consistent conceptual framework.

Researchers (e.g. Cuban, 2001; Hennessy, Ruthven, & Brindley, 2005; Warschauer, 2007) fight against a naive optimism concerning the computers' role in education. Alrø and Skovsmose (2002) and Sfard and Kieran (2001) argue correspondingly that there is no straightforward link between collaboration and learning. This dissertation confirms these viewpoints. A computer does not generate learning by itself, and collaboration does not guarantee success. The overview study identified a need for research that focuses on the use of specifically arranged educational software to solve specifically arranged tasks. The link between lack of mathematical communication and insufficiently transparent software in the first empirical study, and between the pure mathematical communication and specifically developed software in the second empirical study, has implications. One should investigate further how pupils collaboratively can learn mathematics by taking advantage of the educational possibilities that dynamic software, particularly in graphing and geometry, can offer. This means using educationally transparent software with well proven effects, and which is directed specifically towards curricular goals, as Cox and Abbot (2004) also underlined.

The communication patterns identified in the first empirical study and the communication qualities developed in the second empirical study provide new insights into pupils' communication at a computer, and they also show some of the complexity of the issue. This dissertation's findings concerning how pupils' communication, software characteristics, and task formulation relate to a particular mathematical topic generate knowledge for the teaching of mathematics. Mishra and Koehler's (2006) notion of teachers' technological pedagogical content knowledge displays well the multifaceted demands that rest upon teachers. Computers and collaboration require additional expertise from teachers. It is not enough for teachers to possess knowledge about mathematics and pedagogy; they also need a digital competence. Teachers have to be able to locate relevant software and know how to use such software. They have to develop tasks and motivate and challenge pupils in ways that can help pupils take advantage of the possibilities computers offer. Teachers are conductors who can model and facilitate pupils' communication and

mathematics learning at a computer. Nussbaum et al. (2009, p. 150) focused on "when and how the teacher should intervene" when discussing some of the challenges teachers are facing when pupils use digital tools. The possibility of understanding and assessing what pupils are doing and thereby encourage their persistence is clearly different in computer contexts than in traditional teaching. The teacher in the second empirical study pointed to such an issue: the action on the screen that the pupils had discussed was no longer available when she arrived. The communication patterns and qualities investigated in this dissertation can help teachers assess and develop pupils' communication and learning of mathematics. To be familiar with such patterns and qualities is essential for teachers' abilities to understand and promote pupils' abilities to think aloud and build a mutual language.

The cyclic design, in which pupils and teachers are involved as co-researchers and which incorporates extensive use of video recording, is a methodical design that deserves attention in future research as well. The design made a joint analytical process possible. It also helped the pupils gain meta-knowledge about collaboration, communication, and mathematics learning. Wegerif argued that "the ideal of 'teaching' learning to learn through promoting dialogue as an end in itself is the most distinctive and important contribution that a dialogic perspective brings to the debate about education" (2011a, p. 211). To include pupils in meta-talk about their learning and communication of mathematics at a computer is a core underpinning of the collaborative design and the wt-sessions. It is not realistic for teachers to do this in their regular teaching to the same extent that we did in these studies, but the importance of pupils' learning to learn and communicating about communication should not be underestimated.

5.4 Concluding remarks

The intellectual goal of this dissertation has been to gain knowledge of pupils' communication quality and mathematics learning when they work in pairs at a

computer. The first study maps relevant literature in this field of research and identifies four aspects that are highlighted as important on this topic. Reviews and overviews are important studies because they provide reflexive meta-analyses that can structure the knowledge within a field and establish a mutual point of departure for future research. Subsequently the communication of two pairs is contrasted in the first empirical study, and the connections between communication qualities and the contrasting perspectives of two pupils are discussed in the second empirical study. An important contribution made by this dissertation in this respect is the descriptions and analyses of what, when, and how pupils communicate in this context.

The most significant strengths of this dissertation are the following. The major outcome is the discussion presented in article III on the connections between communication qualities and managing differences. The discussion on how the pupils' communication qualities made the collaboration possible despite the individual differences between the pupils is a valuable contribution to the field. The design with collaborative analyses in several loops and cycles in the wt-sessions and extensive use of video recording is a research approach that is useful and informative. The multiple links between the overview study and the empirical studies, and between the first and the second empirical study, underline the coherence of the dissertation. These principles of design and coherence have transfer value for other researchers.

A dialogic way of being runs as a thread through the dissertation, through my reading of theory and research, the empirical studies, the wt-sessions, and the dissemination process. But as Gadamer (2004, p. 581) said: "I will stop here. The ongoing dialogue permits no final conclusion. It would be a poor hermeneuticist who thought he could have, or had to have, the last word". Maybe Bakthin would agree on this, or just comment that there is neither a first nor a last word.

(Freire, 2000; Friedman, 2001; Habermas, 1991; Hjørdevar, 1990b; Wittgenstein, 2009; Wittgenstein & Ogden, 1990)

6. References

Ainley, J., Nardi, E., & Pratt, D. (2000). The construction of meanings for trend in active graphing. *International Journal of Computers for Mathematical Learning, 5*(2), 85-114.

Almås, A. G., & Krumsvik, R. (2008). Teaching in Technology-Rich Classrooms: is there a gap between teachers' intentions and ICT practices? *Research in Comparative and International Education, 3*(2), 103-121.

Alrø, H., & Johnsen-Høines, M. (2012). Inquiry - without posing questions? *The Mathematics Enthusiast, 9*(3).

Alrø, H., & Skovsmose, O. (2002). *Dialogue and Learning in Mathematics Education. Intention, Reflection, Critique* (Vol. 29). Dordrecht: Kluwer Academic Publishers.

Alseth, B., Breiteig, T., & Brekke, G. (2003). *Endringer og utvikling ved R97 som bakgrunn for videre planlegging og justering: Matematikkfaget som kasus*. [Changes and development in R97 as the background for further planning and adjustment: The case of school mathematics]. Notodden: Telemarksforsking.

Anderson, A., McAteer, E., Tolmie, A., & Demissie, A. (1999). The effect of software type on the quality of talk. *Journal of Computer Assisted Learning, 15*(1), 28-40.

Artigue, M. (2000). *Instrumentation issues and the integration of computer technologies into secondary mathematics teaching.* Paper presented at the Annual Meeting of the GDM, Potsdam.

Bakhtin, M. (1984). *Problems of Dostoevsky's Poetics*. Minneapolis: University of Minnesota Press.

Bakhtin, M. (1986). *Speech Genres and Other Late Essays*. Austin, Tx: University of Texas Press.

Bakhtin, M., & Holquist, M. (1981). *The dialogic imagination: four essays*. Austin: University of Texas Press.

Barnes, D., & Todd, F. (1978). *Communication and Learning in Small Groups*. London: Routledge and Kegan Paul.

Baron, G.-L., & Bruillard, E. (2007). ICT, educational technology and educational instruments. Will what has worked work again elsewhere in the future? *Education and Information Technologies, 12*(2), 71-81.

Boote, D. N., & Beile, P. (2005). Scholars Before Researchers: On the Centrality of the Dissertation Literature Review in Research Preparation. *Educational Researcher, 34*(6), 3-15.

Brinkmann, S., & Kvale, S. (2005). Confronting the ethics of qualitative research. *Journal of Constructivist Psychology, 18*(2), 157-181.

Brown, A. L. (1992). Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings. *Journal of the Learning Sciences, 2*(2), 141-178.

Buber, M. (2004). *I and Thou*. London: Continuum.

Castells, M. (2010). *The rise of the network society* (Vol. 1). Oxford: Blackwell.

Christiansen, M. H., & Kirby, S. (2003). Language evolution: The hardest problem in Science? In M. H. Christiansen & S. Kirby (Eds.), *Language evolution* (pp. 1-15). Oxford New York: Oxford University Press.

Clark, H., & Brennan, S. (1991). Grounding in communication. In L. Resnick, J. Levine & S. Teasley (Eds.), *Perspectives on Socially-Shared Cognition* (pp. 127-149). Washington, DC: American Psychological Association.

Collins, A. (1992). Towards a design science of education. In E. Scanlon & T. O'Shea (Eds.), *New directions in educational technology* (pp. 15-22). Berlin: Springer.

Cox, M., & Abbott, C. (2004). *ICT and attainment: A review of the research literature*. Coventry and London: Becta, DfES.

Craig, E. (2002). *Philosophy: a very short introduction*. New York: Oxford University Press.

Crook, C. (1994). *Computers and the collaborative experience of learning*. London and New York: Routledge.

Cuban, L. (2001). *Oversold and Underused: Computers in the Classroom*. Cambridge, Mass.: Harvard University Press.

Cullingford, C. (2002). *The best years of their lives? Pupil's experiences of school*. London: Kogan Page Limited.

Davidson, R. M., Kock, N., Loch, K. D., & Clarke, R. (2001). Research ethics in information systems: would a code of practice help? *Communications of the Association for Information Systems, 7*(4), 1-39.

Dede, C. (2004). If Design-Based Research is the Answer, What is the Question? A Commentary on Collins, Joseph, and Bielaczyc; diSessa and Cobb; and Fishman,

Marx, Blumenthal, Krajcik, and Soloway in the JLS Special Issue on Design-Based Research. *Journal of the Learning Sciences, 13*(1), 105-114.

Dillenbourg, P., & Evans, M. (2011). Interactive tabletops in education. *International Journal of Computer-Supported Collaborative Learning, 6*(4), 491-514.

Dysthe, O. (1996). The Multivoiced Classroom. *Written Communication, 13*(3), 385-425.

Eisner, E. W., & Peshkin, A. (1990). *Qualitative inquiry in education: the continuing debate*. New York: Teachers College, Columbia University.

Ernest, P. (1991). *The philosophy of mathematics education*. London: Falmer Press.

Fischer, C. T. (1994). Rigor in qualitative research: Reflexive and presentational. *Methods: A journal for human science* 21-27.

Fisher, E. (1993). Characteristics of children's talk at the computer and its relationship to the computer software. *Language and Education, 7*(2), 97-114.

Fiske, J., & Jenkins, H. (2011). *Introduction to communication studies*. London: Routledge.

Flick, U. (2006). *An introduction to qualitative research, 3rd ed*. London: Sage Publication Ltd.

Freire, P. (2000). *Pedagogy of the oppressed*. New York: Continuum.

Friedman, M. (2001). Martin Buber and Mikhail Bakhtin: The Dialogue of Voices and the Word That Is Spoken. *Religion & Literature, 33*(3), 25-36.

Fuglestad, A. B. (2009). ICT for Inquiry in Mathematics: A Developmental Research Approach. *The Journal of Computers in Mathematics and Science Teaching, 28*(2), 191-202.

Gadamer, H.-G. (2004). *Truth and Method*. London: Continuum.

Gadamer, H.-G., & Linge, D. E. (1977). *Philosophical hermeneutics*. Berkeley, Calif.: University of California Press.

Galton, M., Hargreaves, L., Comber, C., Wall, D., & Pell, T. (1999). Changes in Patterns of Teacher Interaction in Primary Classrooms: 1976-96. *British Educational Research Journal, 25*(1), 23-37.

Grønmo, L. S., Onstad, T., & Pedersen, I. F. (2010). *Matematikk i motvind: TIMSS advanced 2008 i videregående skole*. [Mathematics in headwind: TIMSS advanced 2008 in upper secondary school]. Oslo: Unipub.

Habermas, J. (1991). *The Theory of Communicative Action* (Vol. 2). Cambridge: Polity Press.

Hatlevik, O. E., Ottestad, G., Skaug, J. H., Kløvstad, V., & Berge, O. (2009). *ITU Monitor 2009 – Skolens digitale tilstand*. [ITU Monitor 2009 – The digital conditions in school]. Oslo: Universitetsforlaget.

Healy, L., Pozzi, S., & Hoyles, C. (1995). Making Sense of Groups, Computers, and Mathematics. *Cognition and Instruction, 13*(4), 505 - 523.

Hennessy, S., Ruthven, K., & Brindley, S. (2005). Teacher perspectives on integrating ICT into subject teaching: commitment, constraints, caution, and change. *Journal of Curriculum Studies, 37*(2), 155-192.

Herheim, R. (2010). Communication and learning at computers: an overview. *Nordic Studies in Mathematics Education, 15*(2), 69-94.

Herheim, R. (submitted). Managing differences by developing communication qualities: Pupils learning mathematics in pairs at a computer.

Herheim, R., & Krumsvik, R. (2011). Verbal Communication at a Stand-alone Computer. *Journal for Educational Research Online, 3*(1), 29-55.

Hitchcock, G., & Hughes, D. (1995). *Research and the Teacher. A Qualitative Introduction to School-based Research, 2nd edition*. London: Routledge.

Hoadley, C. M. (2002). *Creating context: Design-based research in creating and understanding CSCL*. Paper presented at the Computer Support for Cooperative Learning (CSCL 2002), Boulder, CO.

Hoadley, C. M. (2004). Methodological Alignment in Design-Based Research. *Educational Psychologist, 39*(4), 203-212.

Holquist, M. (2002). *Dialogism: Bakhtin and his world*. London: Routledge.

Hægeland, T., Kirkebøen, L. J., & Raaum, O. (2009). *Øre for læring – Ressurser i grunnskole og videregående opplæring i Norge 2003-2008*. [Resources invested in elementary school and upper secondary school in Norway 2003-2008]. Oslo: Ragnar Frisch Centre for Economic Research.

Johnsen-Høines, M. (2010). Interpretative research as collaborative inquiry. In B. Sriraman, C. Bergsten, S. Goodchild, G. Pálsdóttir, B. Dahl & L. Haapasalo (Eds.), *The first sourcebook on nordic research in mathematics education* (pp. 109-123). Charlotte, N.C.: Information Age publishing.

Jones, M., & Stanley, G. (2008). Children's lost voices: ethical issues in relation to undertaking collaborative, practice-based projects involving schools and the wider community. *Educational Action Research, 16*(1), 31-41.

Kelly, A. E. (2003). Theme Issue: The Role of Design in Educational Research. *Educational Researcher, 32*(1), 3-4.

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics, 46*(1), 187-228.

Krumsvik, R. (2007). *Skulen og den digitale læringsrevolusjonen*. [The school and the digital learning revolution]. Oslo: Universitetsforlaget.

Krumsvik, R. (2008). Situated learning and teachers' digital competence. *Education and Information Technologies, 13*(4), 279-290.

Kvernbekk, T. (2005). *Pedagogisk teoridannelse: insidere, teoriformer og praksis*. [The growth of pedagogical theory: insiders, theory types, and practice]. Bergen: Fagbokforlaget.

Lagrange, J. B., Artigue, M., Laborde, C., & Trouche, L. (2001). A meta study on IC technologies in education. Towards a multidimensional framework to tackle their integration into the teaching of mathematics. *Proceedings of the 25th Conference of the International Group for the Psychology of Mathematics Education., I*, 111-125.

Lavy, I. (2006). A case study of different types of arguments emerging from explorations in an interactive computerized environment. *The Journal of Mathematical Behavior, 25*(2), 153-169.

Lavy, I., & Leron, U. (2004). The Emergence of Mathematical Collaboration in an Interactive Computer Environment. *International Journal of Computers for Mathematical Learning, 9*(1), 1-23.

Levy, Y., & Ellis, T. J. (2006). A systems approach to conduct an effective literature review in support of information systems research. *Informing Science Journal, 9*, 181-212.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage Publications, Inc.

Machin, S., McNally, S., & Silva, O. (2006). *New technology in schools: Is there a payoff?* Bonn: Institute for the study of labor.

MacIntyre, A. (1978). Objectivity in morality and objectivity in science. In H. Tristram Engelhardt Jr. & D. Callahan (Eds.), *Morals, science and sociality* (pp. 21-39). Hastings-on-Hudson, NY: Institute of Society, Ethics and the Life Sciences.

Mavrou, K., Lewis, A., & Douglas, G. (2010). Researching computer-based collaborative learning in inclusive classrooms in Cyprus: The role of the computer in pupils' interaction. *British Journal of Educational Technology, 41*(3), 486-501.

Maxwell, J. A. (2005). *Qualitative Research Design: An Interactive Approach*. Thousand Oaks, Calif.: Sage Publications.

Maxwell, J. A. (2006). Literature Reviews of, and for, Educational Research: A Commentary on Boote and Beile's "Scholars Before Researchers". *Educational Researcher, 35*(9), 28-31.

Mellin-Olsen, S. (1991). *Hvordan tenker lærere om matematikkundervisning?* [How do teachers think about mathematics teaching?]. Bergen: Høgskolen i Bergen.

Mellin-Olsen, S. (2002). *The politics of mathematics education*. New York: Kluwer Academic.

Mercer, N., & Wegerif, R. (1998). Is exploratory talk' productive talk? In K. Littleton & P. light (Eds.), *Learning with computers: Analysing Productive Interactions* (pp. 79-101). London: Routledge.

Merriam, S. B. (1998). *Qualitative Research and Case Study Application in Education*. San francisco: Jossey-Bass Publishers.

Ministry of Knowledge. (2006). *Curriculum work for the 13-year primary and secondary schools*. Oslo: Statens forvaltningstjeneste.

Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record, 108*(6), 1017-1054.

Monaghan, F. (2005). 'Don't think in your head, think aloud': ICT and exploratory talk in the primary school mathematics classroom. *Research in Mathematics Education, 7*(1), 83-100.

Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). *TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eight Grades*. Boston: TIMSS and PIRLS Intarnational Study Center.

Newton, P., Driver, R., & Osborne, J. (1999). The place of argumentation in the pedagogy of school science. *International Journal of Science Education, 21*(5), 553-576.

Nussbaum, M., Alvarez, C., McFarlane, A., Gomez, F., Claro, S., & Radovic, D. (2009). Technology as small group face-to-face Collaborative Scaffolding. *Computers & Education, 52*(1), 147-153.

OECD. (2005). *Are students ready for a technology-rich world? What PISA studies tell us*. Paris: OECD.

Overdijk, M., & Diggelen, W. (2009). Computer support for problem solving discussions in the classroom. In M. B. Ligorio, J. Andriessen, M. Baker, N. Knoller & L. Tateo (Eds.), *Talking over the Computer. Pedagogical scenarios to blend computer and face to face interaction* (pp. 65-82).

Packer, M. J., & Goicoechea, J. (2000). Sociocultural and Constructivist Theories of Learning: Ontology, Not Just Epistemology. *Educational Psychologist, 35*(4), 227-241.

Panselinas, G. E., & Komis, V. (2011). Using educational software to support collective thinking and test hypotheses in the computer science curriculum. *Education and Information Technologies, 16*(2), 159-182.

Patton, M. Q. (1990). *Qualitative evaluation and research methods*. Newbury Park: Sage.

Piaget, J. (1997). *The language and thought of the child* (Vol. 5). London: Routledge.

Piaget, J., & Inhelder, B. (1973). *Memory and intelligence*. London: Routledge and Kegan Paul.

Richards, C. (2005). The design of effective ICT-supported learning activities: exemplary models, changing requirements, and new possibilities. *Language Learning & Technology, 9*(1), 60-79.

Rommetveit, R. (1974). *On message structure: A framework for the study of language and communication*. London: John Wiley & Sons. Ltd.

Rommetveit, R. (1992). Outlines of a Dialogically based Social-Cognitive Approach to Human Cognition and Communication. In A. Wold (Ed.), *The Dialogical Alternative: Towards a Theory of language and Mind* (pp. 19-45). Oslo: Scandinavian Press.

Roth, W.-M., & Lee, Y.-J. (2007). "Vygotsky's Neglected Legacy": Cultural-Historical Activity Theory. *Review of Educational Research, 77*(2), 186-232.

Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: A study of the interpretative flexibility of educational software in classroom practice. *Computers & Education, 51*(1), 297-317.

Sandoval, W. A., & Bell, P. (2004). Design-Based Research Methods for Studying Learning in Context: Introduction. *Educational Psychologist, 39*(4), 199-201.

Sfard, A., & Kieran, C. (2001). Cognition as Communication: Rethinking Learning-by-Talking Through Multi-Faceted Analysis of Students' Mathematical Interactions. *Mind, Culture, and Activity, 8*(1), 42-76.

Sidorkin, A. M. (1999). *Beyond discourse: education, the self, and dialogue*. Albany, N.Y.: State University of New York Press.

Sinclair, M. P. (2005). Peer interactions in a computer lab: reflections on results of a case study involving web-based dynamic geometry sketches. *The Journal of Mathematical Behavior, 24*(1), 89-107.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching, 77*, 20-26.

Skjervheim, H. (1959). *Objectivism and the study of man*. Oslo: Universitetsforlaget.

Skjervheim, H. (1996a). Eit grunnproblem i pedagogisk filosofi. [A fundamental problem in the philosophy of pedagogy]. In H. Skjervheim (Ed.), *Deltakar og tilskodar og andre essays*. [Participant and spectator and other essays] (pp. 214-229). Oslo: Idè og tanke, Aschehoug.

Skjervheim, H. (1996b). The Instrumentalist Fallacy. In H. Skjervheim (Ed.), *Selected essays. In Honour of Hans Skjervheim's 70th Birthday*. Bergen: The Department of Philosophy.

Skjervheim, H. (1996c). Participant and Spectator. In H. Skjervheim (Ed.), *Selected essays. In Honour of Hans Skjervheim's 70th Birthday* (pp. 127-141). Bergen: The Department of Philosophy.

Staarman, J. K., Krol, K., & van der Meijden, H. (2005). Peer Interaction in Three Collaborative Learning Environments. *The Journal of Classroom Interaction, 40*(1), 29-39.

Stahl, G. (2005). Group cognition in computer-assisted collaborative learning. *Journal of Computer Assisted Learning, 21*(2), 79-90.

Stewart, J. (1985). Martin Buber's central insight: Implications for his philosophy of dialogue. In M. Dascal (Ed.), *Dialogue: an interdisciplinary approach* (pp. 321-335). São Paulo: John Benjamins Publishing Company.

Teasley, S. D., & Roschelle, J. (1993). Constructing a Joint Problem Space: The Computer as a Tool for Sharing Knowledge. In S. P. Lajoie & S. J. Derry (Eds.), *Computers as Cognitive Tools* (pp. 229-258). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

The Design-Based Research Collective. (2003). Design-Based Research: An Emerging Paradigm for Educational Inquiry. *Educational Researcher, 32*(1), 5-8.

Theunissen, M. (1984). *The other: studies in the social ontology of Husserl, Heidegger, Sartre, and Buber*. Cambridge, Mass.: MIT Press.

Vaage, O. F. (2009). *Norsk Mediebarometer 2009*. [The Norwegian Mediabarometer 2009]. Oslo: Statistisk Sentralbyrå.

Vavik, L., Andersland, S., Arnesen, T. E., Arnesen, T., Espeland, M., Flatøy, I., et al. (2010). *Skolefagsundersøkelsen 2009: utdanning, skolefag og teknologi*. [The school subject survey 2009: Education, school subject, and technology]. Stord: Høgskolen Stord/Haugesund.

Vessey, D. (2005). Gadamer's account of friendship as an alternative to an account of intersubjectivity. *Philosophy Today, 49*(5), 61-67.

Wang, F., & Hannafin, M. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology Research and Development, 53*(4), 5-23.

Warschauer, M. (2007). The paradoxical future of digital learning. *Learning Inquiry, 1*(1), 41-49.

Wegerif, R. (1996). Using computers to help coach exploratory talk across the curriculum. *Computers & Education, 26*(1-3), 51-60.

Wegerif, R. (2004). The role of educational software as a support for teaching and learning conversations. *Computers & Education, 43*(1-2), 179-191.

Wegerif, R. (2007). *Dialogic education and technology: expanding the space of learning*. New York: Springer-Verlag.

Wegerif, R. (2011a). From dialectic to dialogic. In T. Koschmann (Ed.), *Theories of learning and studies of instructional practice* (pp. 201-221). New York: Springer.

Wegerif, R. (2011b). My personal responses to the school of the dialogue of cultures. *Journal of Russian and East European Psychology, 49*(2), 83-89.

Wegerif, R. (2011c). Towards a dialogic theory of how children learn to think. *Thinking Skills and Creativity, 6*(3), 179-190.

Wegerif, R., & De Laat, M. F. (2011). Using Bakhtin to re-think the teaching of higher-order thinking for the network society. In S. Ludvigsen, A. Lund, I. Rasmussen & R. Säljö (Eds.), *Learning across sites: New tools, infrastructures and practices* (pp. 313-329). Abingdon: Routledge.

Wegerif, R., Littleton, K., & Jones, A. (2003). Stand-alone computers supporting learning dialogues in primary classrooms. *International Journal of Educational Research, 39*(8), 851-860.

Wegerif, R., Mercer, N., & Dawes, L. (1998). Software design to support discussion in the primary curriculum. *Journal of Computer Assisted Learning, 14*(3), 199-211.

Wittgenstein, L. (2009). *Philosophical Investigations*. Oxford: Wiley-Blackwell.

Wittgenstein, L., & Ogden, B. (1990). *Tractatus logico-philosophicus*. New York: Routledge.

Wyndhamn, J., & Säljö, R. (2009). Meaning-Making and the Appropriation of Geometric Reasoning: Computer Mediated Support for Understanding the Relationship between Area and Perimeter of Parallelograms. In R. J. Krumsvik (Ed.), *Learning in the Network Society and the Digitized School* (pp. 21-40). New York: Nova Science Publishers, Inc.

Åberg-Bengtsson, L. (2006). "Then you can take half ... almost" – Elementary students learning bar graphs and pie charts in a computer-based context. *The Journal of Mathematical Behavior, 25*(2), 116-135.