

## NUMERICAL SIMULATION OF THE NONLINEAR COUPLED JAULENT-MIODEK EQUATION BY ELZAKI TRANSFORM-ADOMIAN POLYNOMIAL METHOD

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ABSTRACT. The Elzaki transform which is an integral transform used to obtain solutions of linear differential equations is coupled with Adomian polynomial to solve nonlinear coupled Jaulent-Miodek (JM) equation. The Adomian polynomial is used to linearise the nonlinear functions in the partial differential equation before the scheme of the Elzaki transform was used to iteratively generate each term of the series solution. The solutions obtained were compared with the exact solutions and were found to give a very small error, the graphical representation of the solutions which give the shape of the solitons also agree with that of the Adomian decomposition method when a comparison is made. The method is powerful and effective as it does not involve large computer memory and does not involve discretizing the independent variables to achieve the required solution.

### 1. INTRODUCTION

In recent years, multi-scale problems, scientific problems as well as engineering problems occur nonlinearly. Most time these problems are represented by nonlinear partial differential equations [5], and most of them do not have specific analytical solutions except for a few number of them. However, various field of science such as fluid mechanics [37], condensed matter physics [34], plasma physics [4] and optics [25] have committed to studying them over the years.

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The theory of fractional calculus has been around for a while, it can be traced back to 1695 [32], and that is why it is regarded as an old mathematical concept. However, in recent decades, remarkable progress has been made. In the past forty years, the theory of fractional calculus has procured remarkable concentration from the researchers owing to its substantial capability to successfully explain various inconsistent phenomena and complicated operations in natural science and engineering [26]. Diverse anomalous occurrence in real life situations have been mathematically modeled by fractional differential or integral equations to verify the theory of fractional calculus [27, 30, 35]. Obtaining the analytical solutions of these type of equations is generally very hard or occasionally impossible. Several researchers have endeavored to ascertain and develop effective semi-analytical and numerical procedures to handle fractional models [1, 15, 24, 28, 31].

This research work is devoted to exploring the following challenging physical model, the time-fractional coupled Jaulent-Miodek equation associated with the energy-dependent Schrodinger potential [13, 26, 33]

$$(1.1) \quad \frac{\partial^\alpha v}{\partial t^\alpha} + \frac{\partial^3 v}{\partial x^3} + \frac{3}{2}w \frac{\partial^3 w}{\partial x^3} + \frac{9}{2} \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} - 6v \frac{\partial v}{\partial x} - 6vw \frac{\partial w}{\partial x} - \frac{3}{2}w^2 \frac{\partial v}{\partial x} = 0,$$

$$(1.2) \quad \frac{\partial^\alpha w}{\partial t^\alpha} + \frac{\partial^3 w}{\partial x^3} - 6w \frac{\partial v}{\partial x} - 6v \frac{\partial w}{\partial x} - \frac{15}{2}w^2 \frac{\partial w}{\partial x} = 0,$$

where  $\frac{\partial^\alpha}{\partial t^\alpha}(\cdot)$  represents the Riemann-Liouville partial derivative operator of order  $\alpha$  (where,  $0 < \alpha \leq 1$ ) with respect to the variable  $t$ , which is given as

$$\frac{\partial^\alpha}{\partial t^\alpha} v(x, t) = \begin{cases} \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_0^t \frac{v(x, \tau)}{(t-\tau)^\alpha} d\tau, & 0 < \alpha < 1, \\ \frac{\partial}{\partial t} v(x, t), & \alpha = 1, \end{cases}$$

and  $\Gamma(\cdot)$  is the gamma function. Our attention will be focused on when  $\alpha = 1$ . In this case, the model given in equations (1.1) and (1.2) transforms to the coupled classical Jaulent-Miodek (JM) given by [11, 21]

$$(1.3) \quad v_t + v_{xxx} + \frac{3}{2}w w_{xxx} + \frac{9}{2}w_x w_{xx} - 6v v_x - 6v w w_x - \frac{3}{2}v_x w^2 = 0,$$

$$(1.4) \quad w_t + w_{xxx} - 6v_x w - 6v w_x - \frac{15}{2}w_x w^2 = 0,$$

with the initial conditions  $v(x, 0) = g_1(x)$  and  $w(x, 0) = g_2(x)$ . Several semi-analytical and computational methods have been applied to analyze this classical Jaulent-Miodek system, for instance, the time-stepping scheme coupled with the

spectral method [29], the Adomian decomposition method [3, 23], the tanh-coth method [36], the variational iteration method [12], the exp-function technique [14], the homotopy perturbation method [2], and the sine-cosine method [22].

Recently, the use of the Elzaki transform to solve the nonlinear partial differential equations has comes to the limelight [6, 8, 9, 38], though this method cannot handle the nonlinearity, and this lead to using other techniques like Adomian polynomial, Differential transform, Homotopy perturbation method to decompose the nonlinear terms. This work is a continuation of our research, where we studied several nonlinear equations such as third-order KdV equations [17], fifth-order KdV equations [20], Klein-Gordon equations [18] and Sine-Gordon equations [19] using aforementioned technique.

The Elzaki transform and Adomian polynomial method is implemented for obtaining the traveling wave and approximate analytic solutions of the Jaulent-Miodek equations. The JM solution is efficiently obtained by executing the aforementioned method instead of the usual methods of obtaining the exact solutions, without suffering conventional difficulty.

This paper is organized as follows: Section 2 comprises the definitions, properties, as well as the analysis of the Elzaki transform and Adomian polynomial method in solving coupled Jaulent-Miodek equation. In Section 3, we consider the application of this method in solving two problems to show its efficiency. Discussion of results is presented in Section 4, and Section 5 contains the conclusion.

## 2. THE ANALYSIS OF THE ELZAKI TRANSFORM AND ADOMIAN POLYNOMIAL METHOD IN SOLVING COUPLED JAULENT-MIODEK EQUATION

The Elzaki transform [6, 8, 9, 16] is defined for the functions of exponential order [7] as given below

$$A = \left\{ f(t) : \exists M, c_1, c_2 > 0, |f(t)| < M e^{\frac{|t|}{c_j}}, \text{ if } t \in (-1)^j \times [0, \infty) \right\}.$$

For any given function in the set A defined above, the constant  $c_1, c_2$  could either be finite or infinite, while  $M$  is always infinite.

Based on the findings of Tarig [7], he defined the Elzaki transform as:

$$E[f(t)] = u^2 \int_0^\infty f(ut)e^{-t}dt = T(u), \quad t \geq 0, \quad u \in (c_1, c_2),$$

or

$$(2.1) \quad E[f(t)] = u \int_0^{\infty} f(t)e^{-\frac{t}{u}} dt = T(u), \quad t \geq 0, \quad u \in (c_1, c_2).$$

**Theorem 2.1.** *Let the Elzaki transform of the function  $f(t)$  be denoted by  $T(u)$  in such a way that  $E[f(t)] = T(u)$ , then:*

- (i)  $E[f'(t)] = \frac{T(u)}{u} - uf(0)$ ,
- (ii)  $E[f''(t)] = \frac{T(u)}{u^2} - f(0) - uf'(0)$ ,
- (iii)  $E[f^{(n)}(t)] = \frac{T(u)}{u^n} - \sum_{k=0}^{n-1} u^{2-n+k} f^{(k)}(0)$ .

*Proof.* (i) Considering (2.1),  $E[f'(t)]$  is given as

$$(2.2) \quad E[f'(t)] = u \int_0^{\infty} f'(t)e^{-\frac{t}{u}} dt = T(u).$$

Using the integration by part on the right hand side, we have

$$\begin{aligned} \int_0^{\infty} f'(t)e^{-\frac{t}{u}} dt &= e^{-\frac{t}{u}} f(t) \Big|_0^{\infty} + \frac{1}{u} \int_0^{\infty} f(t)e^{-\frac{t}{u}} dt, \\ &= -f(0) + \frac{1}{u} \int_0^{\infty} f(t)e^{-\frac{t}{u}} dt. \end{aligned}$$

Substituting this result back into (2.2) yields

$$\begin{aligned} E[f'(t)] &= u \left[ -f(0) + \frac{1}{u} \int_0^{\infty} f(t)e^{-\frac{t}{u}} dt \right], \\ &= -uf(0) + \int_0^{\infty} f(t)e^{-\frac{t}{u}} dt. \end{aligned}$$

Using definition (2.2) in this expression yields

$$E[f'(t)] = \frac{T(u)}{u} - uf(0).$$

(ii) Again, from (2.1), replacing the function  $f(t)$  by  $f''(t)$ , we have

$$E[f''(t)] = u \int_0^{\infty} f''(t)e^{-\frac{t}{u}} dt.$$

By integrating by part, this yields

$$E[f''(t)] = u \left[ -f'(0) + \frac{1}{u} \int_0^{\infty} f'(t)e^{-\frac{t}{u}} dt \right].$$

Now using definition given in (2.2), we have

$$\begin{aligned} E[f''(t)] &= -uf'(0) + \frac{E[f'(t)]}{u}, \\ &= \frac{E[f'(t)]}{u} - uf'(0). \end{aligned}$$

Using the result obtained in (i) here, this relation yield

$$E[f''(t)] = \frac{T(u)}{u^2} - f(0) - uf'(0).$$

(iii) The general form of this theorem is proved by induction. Given that

$$(2.3) \quad E[f^{(n)}(t)] = \frac{T(u)}{u^n} - \sum_{k=0}^{n-1} u^{2-n+k} f^{(k)}(0), \text{ for all } n \geq 1.$$

**Step 1:** When  $n = 1$  in (2.3), we have

$$E[f'(t)] = \frac{T(u)}{u} - uf(0).$$

Here, it is evident that (2.3) holds for  $n = 1$ .

**Step 2:** Assume that equation (2.3) holds for  $n = N$  i.e.

$$(2.4) \quad E[f^{(N)}(t)] = \frac{T(u)}{u^N} - \sum_{k=0}^{N-1} u^{2-N+k} f^{(k)}(0).$$

This is true for all the values of  $N$ .

**Step 3:** It is proved that it is equally held for  $N + 1$ , i.e

$$E[f^{(N+1)}(t)] = \frac{T(u)}{u^{N+1}} - \sum_{k=0}^N u^{2-(N+1)+k} f^{(k)}(0).$$

By making use of STEP 1 above,

$$\begin{aligned} E[f^{(N+1)}(t)] &= E[(f^{(N)}(t))'], \\ &= \frac{E[f^{(N)}(t)]}{u} - uf^{(N)}(0). \end{aligned}$$

Using equation (2.4), so

$$\begin{aligned} E[f^{(N+1)}(t)] &= \frac{T(u)}{u^{N+1}} - \sum_{k=0}^{N-1} u^{2-N+k-1} f^{(k)}(0) - u f^{(k)}(0), \\ &= \frac{T(u)}{u^{N+1}} - \sum_{k=0}^N u^{2-(N+1)+k} f^{(k)}(0). \end{aligned}$$

Therefore, the last expression corresponds to equation (2.3) when  $n = N + 1$ .  $\square$

$E[f(t)] = T(u)$  implies that  $T(u)$  is the Elzaki transform of the function  $f(t)$ , so,  $f(t)$  will now be the inverse of Elzaki transform  $T(u)$  as given below

$$f(t) = E^{-1}[T(u)].$$

The algorithm of Elzaki transform, which is directly applicable to JM equation is described in this section. Application of Elzaki transform into the equations (1.3) and (1.4) is given in the form

$$(2.5) \quad E[v_t] = -E \left[ v_{xxx} + \frac{3}{2} w w_{xxx} + \frac{9}{2} w_x w_{xx} - 6v v_x - 6v w w_x - \frac{3}{2} v_x w^2 \right],$$

$$(2.6) \quad E[w_t] = -E \left[ w_{xxx} - 6v_x w - 6v w_x - \frac{15}{2} w_x w^2 \right],$$

since [10]

$$\begin{aligned} E[v_t] &= \frac{V(x, u)}{u} - uv(x, 0), \quad E[v_{xxx}] = \frac{d^3}{dx^3} E[v], \\ E[w_t] &= \frac{W(x, u)}{u} - uw(x, 0), \quad E[w_{xxx}] = \frac{d^3}{dx^3} E[w]. \end{aligned}$$

Using these definitions in conjunction with the given initial conditions, equations (2.5) and (2.6) become

$$(2.7) \quad V(x, u) = u^2 v(x, 0) + E^{-1} \left\{ -u \frac{d^3}{dx^3} E[v] - u E \left[ \frac{3}{2} w w_{xxx} + \frac{9}{2} w_x w_{xx} - 6v v_x - 6v w w_x - \frac{3}{2} v_x w^2 \right] \right\},$$

$$(2.8) \quad W(x, u) = u^2 w(x, 0) + E^{-1} \left\{ -u \frac{d^3}{dx^3} E[w] + u E \left[ 6v_x w + 6v w_x + \frac{15}{2} w_x w^2 \right] \right\}.$$

Applying the inverse Elzaki transform to both sides of the equations (2.7) and (2.8) and simplifying yield

$$(2.9) \quad v(x, t) = g_1(x) + E^{-1} \left\{ -u \frac{d^3}{dx^3} E[v] - uE \left[ \frac{3}{2} N_1(v, w) + \frac{9}{2} N_2(v, w) - 6N_3(v, w) - 6N_4(v, w) - \frac{3}{2} N_5(v, w) \right] \right\},$$

$$(2.10) \quad w(x, t) = g_2(x) + E^{-1} \left\{ -u \frac{d^3}{dx^3} E[w] + wE [6N_6(v, w) + 6N_7(v, w) + \frac{15}{2} N_8(v, w)] \right\}.$$

The linear terms  $v(x, t)$  and  $w(x, t)$  can be decomposed by an infinite series of components

$$v(x, t) = \sum_{n=0}^{\infty} v_n(x, t), \quad w(x, t) = \sum_{n=0}^{\infty} w_n(x, t),$$

in which the nonlinear operators  $N_1(v, w)$ ,  $N_2(v, w)$ ,  $N_3(v, w)$ ,  $N_4(v, w)$ ,  $N_5(v, w)$ ,  $N_6(v, w)$ ,  $N_7(v, w)$ , and  $N_8(v, w)$  can be decomposed by the infinite series:

$$N_i(v, w) = \sum_{n=0}^{\infty} A_{i,n}, \quad i = 1, 2, \dots, 8.$$

This implies that all the nonlinear terms, that is,  $ww_{xxx}$ ,  $w_x w_{xx}$ ,  $vv_x$ ,  $vw w_x$ ,  $v_x w^2$ ,  $v_x w$ ,  $vw_x$ , and  $w_x w^2$  are denoted by Adomian polynomials which is the series  $A_{i,n}$ ,  $i = 1, 2, \dots, 8$ .

Next, we want to determined  $v_n(x, t)$  and  $w_n(x, t)$ ,  $n \geq 0$  which are the components of  $v(x, t)$  and  $w(x, t)$ . Therefore, using equations (2.9) and (2.10), the components series can be determined by the recursive relation

$$(2.11) \quad v_0 = g_1(x), \quad w_0 = g_2(x),$$

$$(2.12) \quad v_{n+1} = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[v_n] - uE \left[ \frac{3}{2} A_{1,n} + \frac{9}{2} A_{2,n} - 6A_{3,n} - 6A_{4,n} - \frac{3}{2} A_{5,n} \right] \right\},$$

$$(2.13) \quad w_{n+1} = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[w_n] + wE \left[ 6A_{6,n} + 6A_{7,n} + \frac{15}{2} A_{8,n} \right] \right\},$$

where  $n \geq 0$ . The Adomian's polynomials  $A_{i,n}$  needed for nonlinear decomposition of functions in equations (2.12) and (2.13) is generated by using the formula

$$(2.14) \quad A_{i,n} = \frac{1}{n!} \left[ \frac{d^n}{d\lambda^n} N_i \left[ \sum_{j=0}^n \lambda^j v_j, \sum_{j=0}^n \lambda^j w_j \right] \right]_{\lambda=0}, \quad n \geq 0.$$

Using equation (2.14), the first few Adomian polynomials are given as

$$A_{1,0} = w_{0xxx}w_0,$$

$$A_{1,1} = w_{0xxx}w_1 + w_{1xxx}w_0,$$

$$A_{1,2} = w_{0xxx}w_2 + w_{1xxx}w_1 + w_{2xxx}w_0,$$

$$A_{1,3} = w_{0xxx}w_3 + w_{1xxx}w_2 + w_{2xxx}w_1 + w_{3xxx}w_0,$$

$$A_{1,4} = w_{0xxx}w_4 + w_{1xxx}w_3 + w_{2xxx}w_2 + w_{3xxx}w_1 + w_{4xxx}w_0,$$

$$A_{2,0} = w_{0xx}w_{0x},$$

$$A_{2,1} = w_{0xx}w_{1x} + w_{1xx}w_{0x},$$

$$A_{2,2} = w_{0xx}w_{2x} + w_{1xx}w_{1x} + w_{2xx}w_{0x},$$

$$A_{2,3} = w_{0xx}w_{3x} + w_{1xx}w_{2x} + w_{2xx}w_{1x} + w_{3xx}w_{0x},$$

$$A_{2,4} = w_{0xx}w_{4x} + w_{1xx}w_{3x} + w_{2xx}w_{2x} + w_{3xx}w_{1x} + w_{4xx}w_{0x},$$

$$A_{3,0} = v_0v_0,$$

$$A_{3,1} = v_0v_1 + v_{1x}v_0,$$

$$A_{3,2} = v_0v_2 + v_{1x}v_1 + v_{2x}v_0,$$

$$A_{3,3} = v_0v_3 + v_{1x}v_2 + v_{2x}v_1 + v_{3x}v_0,$$

$$A_{3,4} = v_0v_4 + v_{1x}v_3 + v_{2x}v_2 + v_{3x}v_1 + v_{4x}v_0,$$

$$A_{4,0} = v_0w_0w_{0x},$$

$$A_{4,1} = v_1w_0w_{0x} + v_0w_1w_{0x} + v_0w_0w_{1x},$$

$$A_{4,2} = v_2w_0w_{0x} + v_1w_1w_{0x} + v_0w_2w_{0x} + v_1w_0w_{1x} + v_0w_1w_{1x} + v_0w_0w_{2x},$$

$$A_{4,3} = v_3w_0w_{0x} + v_2w_1w_{0x} + v_1w_2w_{0x} + v_0w_3w_{0x} + v_2w_0w_{1x} + v_1w_1w_{1x}$$

$$+ v_0w_2w_{1x} + v_1w_0w_{2x} + v_0w_1w_{2x} + v_0w_0w_{3x},$$



$$\begin{aligned}
 A_{4,4} &= v_4 w_0 w_{0x} + v_3 w_1 w_{0x} + v_2 w_2 w_{0x} + v_1 w_3 w_{0x} + v_0 w_4 w_{0x} + v_3 w_0 w_{1x} \\
 &\quad + v_2 w_1 w_{1x} + v_1 w_2 w_{1x} + v_0 w_3 w_{1x} + v_2 w_0 w_{2x} + v_1 w_1 w_{2x} + v_0 w_2 w_{2x} \\
 &\quad + v_1 w_0 w_{3x} + v_0 w_1 w_{3x} + v_0 w_0 w_{4x}, \\
 A_{5,0} &= v_{0x} w_0^2, \\
 A_{5,1} &= v_{1x} w_0^2 + 2v_{1x} w_0 w_1, \\
 A_{5,2} &= v_{2x} w_0^2 + 2v_{1x} w_0 w_1 + 2v_{0x} w_0 w_2 + v_{0x} w_1^2, \\
 A_{5,3} &= v_{3x} w_0^2 + 2v_{2x} w_0 w_1 + 2v_{1x} w_0 w_2 + 2v_{0x} w_1 w_2 + 2v_{0x} w_0 w_3 + v_{1x} w_1^2, \\
 A_{5,4} &= v_{4x} w_0^2 + 2v_{3x} w_0 w_1 + v_{2x} w_1^2 + 2v_{2x} w_0 w_2 + 2v_{1x} w_1 w_2 + 2v_{1x} w_0 w_3 \\
 &\quad + 2v_{0x} w_1 w_3 + 2v_{0x} w_0 w_4 + v_{0x} w_2^2, \\
 A_{6,0} &= v_{0x} w_0, \\
 A_{6,1} &= v_{0x} w_1 + v_{1x} w_0, \\
 A_{6,2} &= v_{0x} w_2 + v_{1x} w_1 + v_{2x} w_0, \\
 A_{6,3} &= v_{0x} w_3 + v_{1x} w_2 + v_{2x} w_1 + v_{3x} w_0, \\
 A_{6,4} &= v_{0x} w_4 + v_{1x} w_3 + v_{2x} w_2 + v_{3x} w_1 + v_{4x} w_0, \\
 A_{7,0} &= w_{0x} v_0, \\
 A_{7,1} &= w_{0x} v_1 + w_{1x} v_0, \\
 A_{7,2} &= w_{0x} v_2 + w_{1x} v_1 + w_{2x} v_0, \\
 A_{7,3} &= w_{0x} v_3 + w_{1x} v_2 + w_{2x} v_1 + w_{3x} v_0, \\
 A_{7,4} &= w_{0x} v_4 + w_{1x} v_3 + w_{2x} v_2 + w_{3x} v_1 + w_{4x} v_0, \\
 A_{8,0} &= w_{0x} w_0^2, \\
 A_{8,1} &= w_{1x} w_0^2 + 2w_{1x} w_0 w_1, \\
 A_{8,2} &= w_{2x} w_0^2 + 2w_{1x} w_0 w_1 + 2w_{0x} w_0 w_2 + w_{0x} w_1^2, \\
 A_{8,3} &= w_{3x} w_0^2 + 2w_{2x} w_0 w_1 + 2w_{1x} w_0 w_2 + 2w_{0x} w_1 w_2 + 2w_{0x} w_0 w_3 + w_{1x} w_1^2, \\
 A_{8,4} &= w_{4x} w_0^2 + 2w_{3x} w_0 w_1 + w_{2x} w_1^2 + 2w_{2x} w_0 w_2 + 2w_{1x} w_1 w_2 + 2w_{1x} w_0 w_3 \\
 &\quad + 2w_{0x} w_1 w_3 + 2w_{0x} w_0 w_4 + w_{0x} w_2^2.
 \end{aligned}$$

These polynomials is enough for the purpose of this work, they can be constructed further using the same approach.

## 3. APPLICATIONS

**3.1. Illustration 1:** Here, the Elzaki transform and Adomian polynomial explained above is applied to obtain the approximate analytical solution of coupled JM equation given in equations (1.3) and (1.4) with the initial conditions [23]

$$(3.1) \quad v(x, 0) = \frac{c}{2} + 2c \operatorname{sech}^2(kx), \quad w(x, 0) = 2k \operatorname{sech}(kx).$$

Incorporating equation (2.14) into equations (2.11), (2.12) and (2.13) for the coupled JM equations (1.3) and (1.4) with the given initial conditions (3.1) yield

$$(3.2) \quad v_0 = \frac{c}{2} + 2c \operatorname{sech}^2(kx), \quad w_0 = 2k \operatorname{sech}(kx).$$

Considering when  $n = 0$ , then the recursive relations (2.12) and (2.13) become

$$v_1 = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[v_0] - uE \left[ \frac{3}{2} A_{1,0} + \frac{9}{2} A_{2,0} - 6A_{3,0} - 6A_{4,0} - \frac{3}{2} A_{5,0} \right] \right\},$$

$$w_1 = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[w_0] + wE \left[ 6A_{6,0} + 6A_{7,0} + \frac{15}{2} A_{8,0} \right] \right\},$$

and these yield

$$(3.3) \quad v_1 = \frac{1}{\cosh^5(kx)} \left( 4tk \sinh(kx) \left( -6k^4 \cosh^2(kx) - ck^2 \cosh^2(kx) + 3c^2 \cosh^2(kx) + 18k^4 + 30ck^2 + 12c^2 \right) \right),$$

$$(3.4) \quad w_1 = \frac{2tk^2 \sinh(kx)}{\cosh^4(kx)} \left( -k^2 \cosh^2(kx) + 3c \cosh^2(kx) + 36k^2 + 36c \right).$$

Similarly, when  $n = 1$ , the recursive relations (2.12) and (2.13) become

$$v_2 = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[v_1] - uE \left[ \frac{3}{2} A_{1,1} + \frac{9}{2} A_{2,1} - 6A_{3,1} - 6A_{4,1} - \frac{3}{2} A_{5,1} \right] \right\},$$

$$w_2 = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[w_1] + wE \left[ 6A_{6,1} + 6A_{7,1} + \frac{15}{2} A_{8,1} \right] \right\},$$

which yield

$$\begin{aligned}
 v_2 = & \frac{576t^3k^3 \sinh(kx)c^4}{\cosh^5(kx)} + \frac{6048t^3k^3 \sinh(kx)c^4}{\cosh^7(kx)} + \frac{9216t^3k^3 \sinh(kx)c^4}{\cosh^9(kx)} \\
 & - \frac{23040t^3k^3 \sinh(kx)c^4}{\cosh^{11}(kx)} - \frac{72t^3k^7 \sinh(kx)c^2}{\cosh^3(kx)} - \frac{8976t^3k^9 \sinh(kx)c}{\cosh^5(kx)} \\
 & - \frac{15272t^3k^7 \sinh(kx)c^2}{\cosh^5(kx)} - \frac{384t^3k^5 \sinh(kx)c^3}{\cosh^5(kx)} - \frac{549936t^3k^9 \sinh(kx)c}{\cosh^7(kx)} \\
 & - \frac{214296t^3k^7 \sinh(kx)c^2}{\cosh^7(kx)} + \frac{15552t^3k^5 \sinh(kx)c^3}{\cosh^7(kx)} + \frac{2338560t^3k^9 \sinh(kx)c}{\cosh^9(kx)} \\
 & + \frac{1209216t^3k^7 \sinh(kx)c^2}{\cosh^9(kx)} + \frac{72192t^3k^5 \sinh(kx)c^3}{\cosh^9(kx)} - \frac{2039040t^3k^9 \sinh(kx)c}{\cosh^{11}(kx)} \\
 & - \frac{1146240t^3k^7 \sinh(kx)c^2}{\cosh^{11}(kx)} - \frac{115200t^3k^5 \sinh(kx)c^3}{\cosh^{11}(kx)} + \frac{48t^3k^9 \sinh(kx)c}{\cosh^3(kx)} \\
 & + \frac{16t^2k^6c}{\cosh^2(kx)} - \frac{48t^2k^4c^2}{\cosh^2(kx)} - \frac{3960t^2k^6c}{\cosh^4(kx)} - \frac{1176t^2k^4c^2}{\cosh^4(kx)} + \frac{15720t^2k^6c}{\cosh^6(kx)} \\
 & + \frac{5880t^2k^4c^2}{\cosh^6(kx)} - \frac{12600t^2k^6c}{\cosh^8(kx)} - \frac{5040t^2k^4c^2}{\cosh^8(kx)} - \frac{8t^3k^{11} \sinh(kx)}{\cosh^3(kx)} + \frac{7000t^3k^{11} \sinh(kx)}{\cosh^5(kx)} \\
 (3.5) \quad & - \frac{328440t^3k^{11} \sinh(kx)}{\cosh^7(kx)} + \frac{1192320t^3k^{11} \sinh(kx)}{\cosh^9(kx)} - \frac{984960t^3k^{11} \sinh(kx)}{\cosh^{11}(kx)} \\
 & + \frac{1959552t^4k^6c^4}{\cosh^{14}(kx)} + \frac{1260t^4k^{12}c}{\cosh^4(kx)} - \frac{1620t^4k^{10}c^2}{\cosh^4(kx)} - \frac{972t^4k^8c^3}{\cosh^4(kx)} - \frac{54834t^4k^{12}c}{\cosh^6(kx)} \\
 & + \frac{972t^4k^6c^4}{\cosh^4(kx)} - \frac{95058t^4k^{10}c^2}{\cosh^6(kx)} + \frac{25218t^4k^8c^3}{\cosh^6(kx)} - \frac{1253190t^4k^{12}c}{\cosh^8(kx)} + \frac{41310t^4k^6c^4}{\cosh^6(kx)} \\
 & + \frac{230634t^4k^{10}c^2}{\cosh^8(kx)} + \frac{1076166t^4k^8c^3}{\cosh^8(kx)} + \frac{11394828t^4k^{12}c}{\cosh^{10}(kx)} + \frac{404190t^4k^6c^4}{\cosh^8(kx)} \\
 & + \frac{11053116t^4k^{10}c^2}{\cosh^{10}(kx)} + \frac{4042116t^4k^8c^3}{\cosh^{10}(kx)} - \frac{20865600t^4k^{12}c}{\cosh^{12}(kx)} + \frac{338904t^4k^6c^4}{\cosh^{10}(kx)} \\
 & - \frac{25883712t^4k^{10}c^2}{\cosh^{12}(kx)} - \frac{13960512t^4k^8c^3}{\cosh^{12}(kx)} - \frac{2744928t^4k^6c^4}{\cosh^{12}(kx)} + \frac{10777536t^4k^{12}c}{\cosh^{14}(kx)} \\
 & + \frac{8817984t^4k^8c^3}{\cosh^{14}(kx)} + \frac{14696640t^4k^{10}c^2}{\cosh^{14}(kx)} + \frac{4044924t^4k^{14}}{\cosh^{10}(kx)} + \frac{96t^2k^8}{\cosh^2(kx)} - \frac{7560t^2k^8}{\cosh^8(kx)} \\
 & - \frac{813960t^4k^{14}}{\cosh^8(kx)} - \frac{6197472t^4k^{14}}{\cosh^{12}(kx)} + \frac{27396t^4k^{14}}{\cosh^6(kx)} + \frac{10080t^2k^8}{\cosh^6(kx)} \\
 & + \frac{2939328t^4k^{14}}{\cosh^{14}(kx)} - \frac{216t^4k^{14}}{\cosh^4(kx)} - \frac{3024t^2k^8}{\cosh^4(kx)},
 \end{aligned}$$

$$\begin{aligned}
w_2 = & \frac{8100t^4k^{11} \sinh^2(kx)c}{\cosh^9(kx)} - \frac{58320t^4k^{11} \sinh^2(kx)c}{\cosh^9(kx)} - \frac{4860t^4k^9 \sinh^2(kx)c^2}{\cosh^7(kx)} \\
& - \frac{291600t^4k^9 \sinh^2(kx)c^2}{\cosh^9(kx)} - \frac{14580t^4k^7 \sinh^2(kx)c^3}{\cosh^7(kx)} - \frac{174960t^4k^7 \sinh^2(kx)c^3}{\cosh^9(kx)} \\
& - \frac{2099520t^4k^{11} \sinh^2(kx)c}{\cosh^{11}(kx)} - \frac{2099520t^4k^9 \sinh^2(kx)c^2}{\cosh^{11}(kx)} - \frac{699840t^4k^7 \sinh^2(kx)c^3}{\cosh^{11}(kx)} \\
& - \frac{135t^4k^{11} \sinh^2(kx)c}{\cosh^5(kx)} + \frac{405t^4k^9 \sinh^2(kx)c^2}{\cosh^5(kx)} - \frac{405t^4k^7 \sinh^2(kx)c^3}{\cosh^5(kx)} \\
& + \frac{270t^4k^{11} \sinh^4(kx)c}{\cosh^7(kx)} - \frac{810t^4k^9 \sinh^4(kx)c^2}{\cosh^7(kx)} \\
& + \frac{810t^4k^7 \sinh^4(kx)c^3}{\cosh^7(kx)} - \frac{21600t^4k^{11} \sinh^4(kx)c}{\cosh^9(kx)} \\
& + \frac{194400t^4k^{11} \sinh^4(kx)c}{\cosh^{11}(kx)} + \frac{12960t^4k^9 \sinh^4(kx)c^2}{\cosh^9(kx)} + \frac{972000t^4k^9 \sinh^4(kx)c^2}{\cosh^{11}(kx)} \\
& + \frac{38880t^4k^7 \sinh^4(kx)c^3}{\cosh^9(kx)} + \frac{583200t^4k^7 \sinh^4(kx)c^3}{\cosh^{11}(kx)} + \frac{8398080t^4k^{11} \sinh^4(kx)c}{\cosh^{13}(kx)} \\
(3.6) \quad & + \frac{8398080t^4k^9 \sinh^4(kx)c^2}{\cosh^{13}(kx)} + \frac{2799360t^4k^7 \sinh^4(kx)c^3}{\cosh^{13}(kx)} - \frac{288t^3k^4 \sinh(kx)c^3}{\cosh^4(kx)} \\
& - \frac{4608t^3k^4 \sinh(kx)c^3}{\cosh^6(kx)} - \frac{13824t^3k^4 \sinh(kx)c^3}{\cosh^8(kx)} + \frac{7296t^3k^8 \sinh(kx)c}{\cosh^6(kx)} \\
& - \frac{4800t^3k^6 \sinh(kx)c^2}{\cosh^6(kx)} - \frac{55296t^3k^8 \sinh(kx)c}{\cosh^8(kx)} - \frac{48384t^3k^6 \sinh(kx)c^2}{\cosh^8(kx)} \\
& - \frac{1360t^3k^8 \sinh^3(kx)c}{\cosh^6(kx)} - \frac{25536t^3k^8 \sinh^3(kx)c}{\cosh^8(kx)} - \frac{480t^3k^6 \sinh^3(kx)c^2}{\cosh^6(kx)} \\
& + \frac{16800t^3k^6 \sinh^3(kx)c^2}{\cosh^8(kx)} + \frac{720t^3k^4 \sinh^3(kx)c^3}{\cosh^6(kx)} - \frac{16128t^3k^4 \sinh^3(kx)c^3}{\cosh^8(kx)} + \\
& + \frac{248832t^3k^8 \sinh^3(kx)c}{\cosh^{10}(kx)} + \frac{217728t^3k^6 \sinh^3(kx)c^2}{\cosh^{10}(kx)} + \frac{62208t^3k^4 \sinh^3(kx)c^3}{\cosh^{10}(kx)} \\
& + \frac{544t^3k^8 \sinh(kx)c}{\cosh^4(kx)} + \frac{192t^3k^6 \sinh(kx)c^2}{\cosh^4(kx)} + \frac{t^2k^7}{\cosh(kx)} + \frac{480t^3k^{10} \sinh^3(kx)}{\cosh^6(kx)} \\
& - \frac{26208t^3k^{10} \sinh^3(kx)}{\cosh^8(kx)} + \frac{93312t^3k^{10} \sinh^3(kx)}{\cosh^{10}(kx)} + \frac{7488t^3k^{10} \sinh(kx)}{\cosh^6(kx)} \\
& - \frac{20736t^3k^{10} \sinh(kx)}{\cosh^8(kx)} - \frac{192t^3k^{10} \sinh(kx)}{\cosh^4(kx)} + \frac{4320t^4k^{13} \sinh^4(kx)}{\cosh^9(kx)}
\end{aligned}$$

$$\begin{aligned}
 & - \frac{194400t^4k^{13} \sinh^4(kx)}{\cosh^{11}(kx)} + \frac{2799360t^4k^{13} \sinh^4(kx)}{\cosh^{13}(kx)} - \frac{30t^4k^{13} \sinh^4(kx)}{\cosh^7(kx)} \\
 & - \frac{1620t^4k^{13} \sinh^2(kx)}{\cosh^7(kx)} - \frac{699840t^4k^{13} \sinh^2(kx)}{\cosh^{11}(kx)} + \frac{58320t^4k^{13} \sinh^2(kx)}{\cosh^9(kx)} \\
 & + \frac{15t^4k^{13} \sinh^2(kx)}{\cosh^5(kx)} - \frac{3}{2} \frac{t^2k^5c}{\cosh(kx)} - \frac{456t^2k^5c}{\cosh^3(kx)} + \frac{2412t^2k^5c}{\cosh^5(kx)} - \frac{2160t^2k^5c}{\cosh^7(kx)} \\
 & + \frac{4920t^2k^7}{\cosh^5(kx)} - \frac{4320t^2k^7}{\cosh^7(kx)} - \frac{992t^2k^7}{\cosh^3(kx)}.
 \end{aligned}$$

Therefore, the series solution is given as

$$(3.7) \quad v(x, t) = v_0 + v_1 + v_2 + v_3 + \dots$$

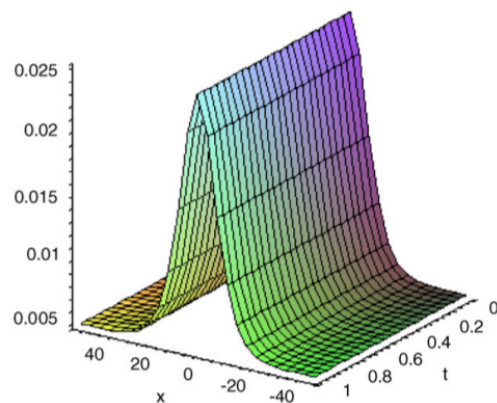
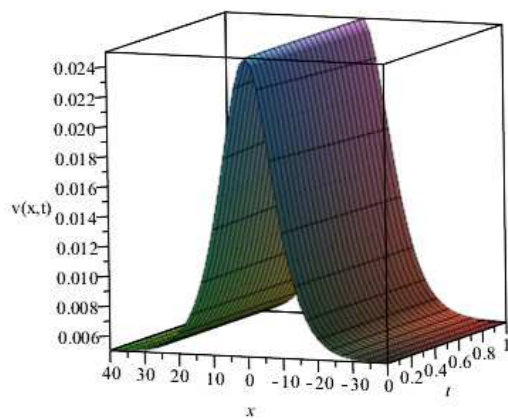
$$(3.8) \quad w(x, t) = w_0 + w_1 + w_2 + w_3 + \dots$$

Substituting equations (3.2), (3.3), (3.4), (3.5) and (3.6) into equations (3.7) and (3.8) give the required approximate solutions associated with the initial conditions of the given nonlinear coupled Jaulent-Miodek equation.

The closed form solutions to equations (1.3) and (1.4) in connection to initial conditions (3.1) are obtained by [11] as

$$(3.9) \quad v(x, t) = \frac{c}{2} + 2c \operatorname{sech}^2(k(Rt + x)), \quad w(x, t) = 2k \operatorname{sech}(k(Rt + x)),$$

where  $R = \frac{1}{2}(b^2 + c)$  and  $b, c, k$  are taken as the arbitrary constants.



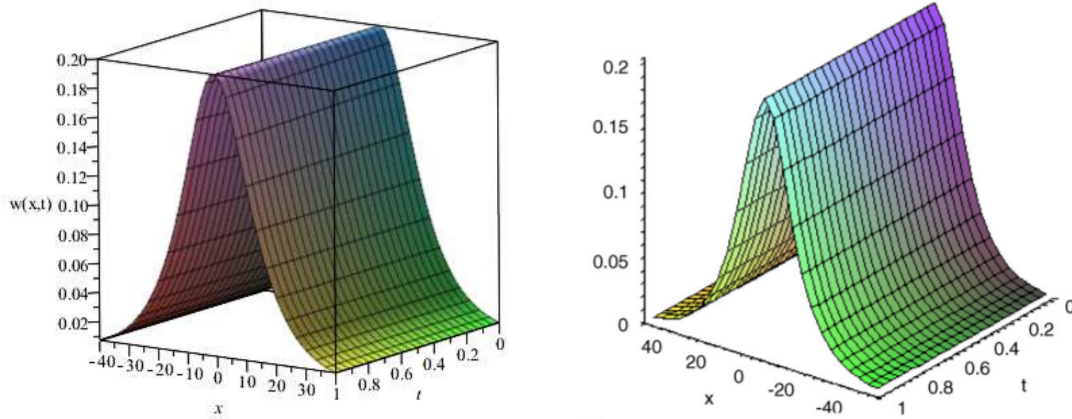


FIGURE 1. The upper and lower left panel show the solitary wave solutions  $v(x, t)$  and  $w(x, t)$  of coupled nonlinear Jaulent-Miodek equation (1.3) and (1.4) with the given initial conditions (3.1) using Elzaki transform and Adomian polynomial as seen in equations (3.7) and (3.8), while the solitary wave solutions  $v(x, t)$  and  $w(x, t)$  of coupled nonlinear Jaulent-Miodek equation (1.3) and (1.4) with the given initial conditions (3.1) using Adomian decomposition method [12] is shown in the upper and lower right panel when  $b = c = 0.01$  and  $k = \sqrt{c}$ .

TABLE 1. Error obtained when the approximate analytical solution to the illustration 1 is compared to the exact solution for  $v(x, t)$ , taken  $b = c = 0.01$  and  $k = \sqrt{c}$ .

| x   | t=0.2                 | t=0.4                 | t = 0.6               | t = 0.8               |
|-----|-----------------------|-----------------------|-----------------------|-----------------------|
| 0.2 | $8.38 \times 10^{-6}$ | $1.55 \times 10^{-5}$ | $2.13 \times 10^{-5}$ | $2.57 \times 10^{-5}$ |
| 0.4 | $1.74 \times 10^{-5}$ | $3.34 \times 10^{-5}$ | $4.83 \times 10^{-5}$ | $6.18 \times 10^{-5}$ |
| 0.6 | $2.63 \times 10^{-5}$ | $5.13 \times 10^{-5}$ | $7.51 \times 10^{-5}$ | $9.75 \times 10^{-5}$ |
| 0.8 | $3.50 \times 10^{-5}$ | $6.88 \times 10^{-5}$ | $1.01 \times 10^{-4}$ | $1.33 \times 10^{-4}$ |
| 1.0 | $4.35 \times 10^{-5}$ | $8.59 \times 10^{-5}$ | $1.27 \times 10^{-4}$ | $1.67 \times 10^{-4}$ |

**3.2. Illustration 2:** As in illustration 1, we applied the same method in obtaining the solution of the coupled JM equation given in equations (1.3) and (1.4) with

TABLE 2. Error obtained when the approximate analytical solution to the illustration 1 is compared to the exact solution for  $v(x, t)$ , taken  $b = c = 0.01$  and  $k = \sqrt{c}$ .

| x   | t=0.2                 | t=0.4                 | t = 0.6               | t = 0.8               |
|-----|-----------------------|-----------------------|-----------------------|-----------------------|
| 0.2 | $6.11 \times 10^{-5}$ | $1.27 \times 10^{-4}$ | $1.97 \times 10^{-4}$ | $2.73 \times 10^{-4}$ |
| 0.4 | $1.19 \times 10^{-4}$ | $2.44 \times 10^{-4}$ | $3.73 \times 10^{-4}$ | $5.07 \times 10^{-4}$ |
| 0.6 | $1.77 \times 10^{-4}$ | $2.59 \times 10^{-4}$ | $5.46 \times 10^{-4}$ | $7.38 \times 10^{-4}$ |
| 0.8 | $2.34 \times 10^{-4}$ | $4.74 \times 10^{-4}$ | $7.17 \times 10^{-4}$ | $9.66 \times 10^{-4}$ |
| 1.0 | $2.91 \times 10^{-4}$ | $5.86 \times 10^{-4}$ | $8.86 \times 10^{-4}$ | $1.11 \times 10^{-4}$ |

the initial conditions [23]

(3.10)

$$v(x, 0) = -\frac{1}{4}b_0^2 + \frac{1}{4}c - \frac{1}{2}b_0k\text{sech}(kx) - \frac{3}{4}c\text{sech}^2(kx), \quad w(x, 0) = b_0 + k\text{sech}(kx).$$

Incorporating equation (2.14) into equations (2.11), (2.12) and (2.13) for the coupled JM equations (1.3) and (1.4) with the given initial conditions (3.10) yield

(3.11) 
$$v_0 = -\frac{1}{4}b_0^2 + \frac{1}{4}c - \frac{1}{2}b_0k\text{sech}(kx) - \frac{3}{4}c\text{sech}^2(kx),$$

(3.12) 
$$w_0 = b_0 + k\text{sech}(kx).$$

When  $n = 0$ , then the recursive relations (2.12) and (2.13) become

$$v_1 = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[v_0] - uE \left[ \frac{3}{2}A_{1,0} + \frac{9}{2}A_{2,0} - 6A_{3,0} - 6A_{4,0} - \frac{3}{2}A_{5,0} \right] \right\},$$

$$w_1 = E^{-1} \left\{ -u \frac{d^3}{dx^3} E[w_0] + wE \left[ 6A_{6,0} + 6A_{7,0} + \frac{15}{2}A_{8,0} \right] \right\},$$

and these give

(3.13) 
$$v_1 = -\frac{\sinh(kx)k^4b_0t}{\cosh^2(kx)} + \frac{15 \sinh(kx)k^3ct}{2 \cosh^3(kx)} + \frac{9 \sinh(kx)k^4b_0t}{4 \cosh^4(kx)}$$

$$- \frac{99 \sinh(kx)k^3ct}{4 \cosh^5(kx)} - \frac{3 \sinh(kx)k^2b_0^3t}{2 \cosh^2(kx)} + \frac{3 \sinh(kx)k^2b_0ct}{4 \cosh^2(kx)}$$

$$- \frac{6 \sinh(kx)k^5t}{\cosh^3(kx)} - \frac{9 \sinh(kx)k^3b_0^2t}{2 \cosh^3(kx)} - \frac{9 \sinh(kx)kc^2t}{4 \cosh^3(kx)} - \frac{9 \sinh(kx)k^2b_0ct}{4 \cosh^4(kx)}$$

$$+ \frac{18 \sinh(kx)k^5t}{\cosh^5(kx)} + \frac{27 \sinh(kx)kc^2t}{4 \cosh^5(kx)},$$

$$\begin{aligned}
 w_1 = & 6\operatorname{sech}(kx) \tanh^3(kx)k^4t - 5\operatorname{sech}(kx) \tanh(kx)k^4t \\
 & - 3\operatorname{sech}(kx) \tanh(kx)k^2b_0^2t - 9\operatorname{sech}^2(kx) \tanh(kx)k^3b_0t \\
 & + 9\operatorname{sech}^2(kx) \tanh(kx)kb_0ct + \frac{27}{2}\operatorname{sech}^3(kx) \tanh(kx)k^2ct \\
 (3.14) \quad & - \frac{3}{2}\operatorname{sech}(kx) \tanh(kx)k^2ct - \frac{15}{2}\operatorname{sech}^3(kx) \tanh(kx)k^4t.
 \end{aligned}$$

Therefore, the series solution is given as

$$(3.15) \quad v(x, t) = v_0 + v_1 + v_2 + v_3 + \dots$$

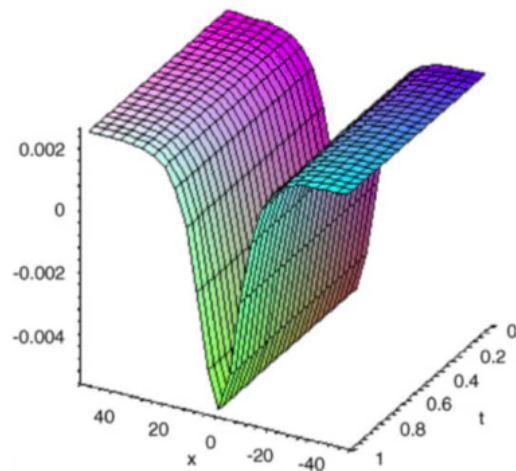
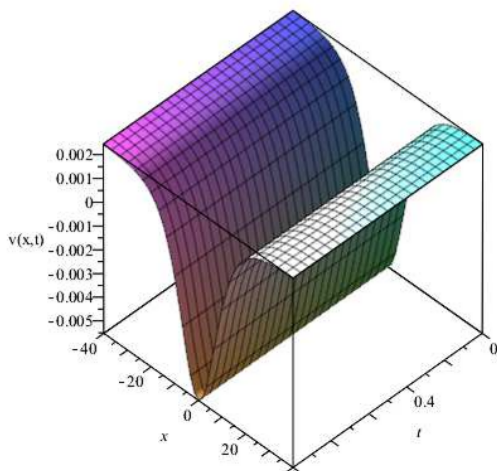
$$(3.16) \quad w(x, t) = w_0 + w_1 + w_2 + w_3 + \dots$$

Upon substitution of equations (3.11), (3.12), (3.13), and (3.14) into equations (3.15) and (3.16), the approximate solutions of the given nonlinear coupled Jaulent-Miodek equation is obtained.

The closed form solutions to equations (1.3) and (1.4) in connection to initial conditions (3.10) are obtained by [11] as

$$\begin{aligned}
 v(x, t) = & s - \frac{bk \operatorname{sech}(k(Rt + x))}{2} - \frac{3c \operatorname{sech}^2(k(Rt + x))}{4}, \\
 (3.17) \quad w(x, t) = & b + k \operatorname{sech}(k(Rt + x)),
 \end{aligned}$$

where  $R = \frac{1}{2}(b^2 + c)$ ,  $s = \frac{1}{4}(c - b^2)$  and  $b, c, k$  are taken as the arbitrary constants.





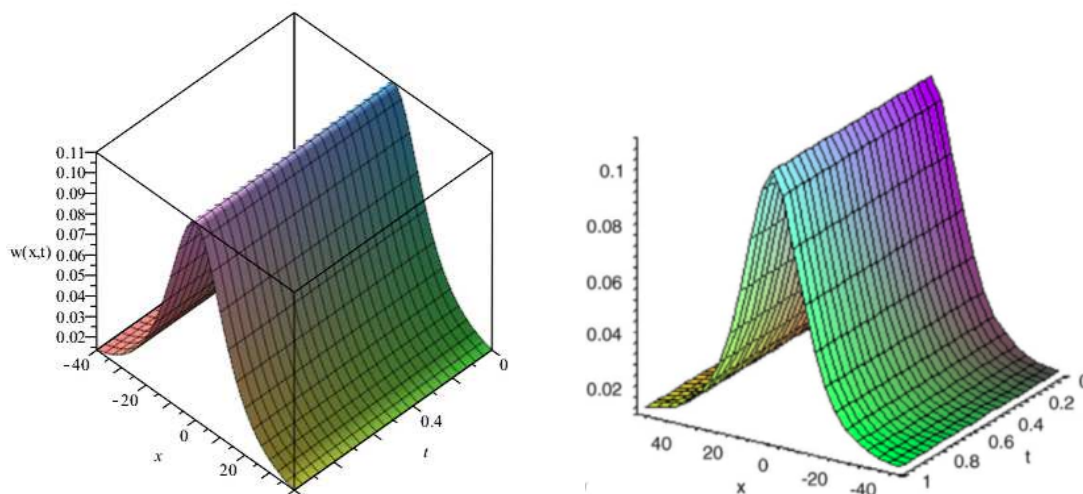


FIGURE 2. The upper and lower left panel show the solitary wave solutions  $v(x,t)$  and  $w(x,t)$  of coupled nonlinear Jaulent-Miodek equation (1.3) and (1.4) with the given initial conditions (3.10) using Elzaki transform and Adomian polynomial as seen in equations (3.15) and (3.16), while the solitary wave solutions  $v(x,t)$  and  $w(x,t)$  of coupled nonlinear Jaulent-Miodek equation (1.3) and (1.4) with the given initial conditions (3.10) using Adomian decomposition method [12] is shown in the upper and lower right panel when  $b = c = 0.01$  and  $k = \sqrt{c}$ .

TABLE 3. Error obtained when the approximate analytical solution to the illustration 2 is compared to the exact solution for  $v(x,t)$ , taken  $b = c = 0.01$  and  $k = \sqrt{c}$ .

| x   | t=0.2                   | t=0.4                   | t = 0.6                 | t = 0.8                 |
|-----|-------------------------|-------------------------|-------------------------|-------------------------|
| 0.2 | $6.2922 \times 10^{-8}$ | $1.2341 \times 10^{-7}$ | $1.8147 \times 10^{-7}$ | $2.3710 \times 10^{-7}$ |
| 0.4 | $1.2687 \times 10^{-7}$ | $2.5133 \times 10^{-7}$ | $3.7339 \times 10^{-7}$ | $4.9304 \times 10^{-7}$ |
| 0.6 | $1.9042 \times 10^{-7}$ | $3.7847 \times 10^{-7}$ | $5.6417 \times 10^{-7}$ | $7.4750 \times 10^{-7}$ |
| 0.8 | $2.5337 \times 10^{-7}$ | $5.0444 \times 10^{-7}$ | $7.5321 \times 10^{-7}$ | $9.9960 \times 10^{-7}$ |
| 1.0 | $3.1554 \times 10^{-7}$ | $6.2886 \times 10^{-7}$ | $9.3994 \times 10^{-7}$ | $1.2480 \times 10^{-6}$ |

TABLE 4. Error obtained when the approximate analytical solution to the illustration 2 is compared to the exact solution for  $w(x, t)$ , taken  $b = c = 0.01$  and  $k = \sqrt{c}$ .

| x   | t=0.2                  | t=0.4                  | t = 0.6                | t = 0.8                |
|-----|------------------------|------------------------|------------------------|------------------------|
| 0.2 | $3.590 \times 10^{-8}$ | $7.080 \times 10^{-8}$ | $1.048 \times 10^{-7}$ | $1.375 \times 10^{-7}$ |
| 0.4 | $4.570 \times 10^{-8}$ | $9.040 \times 10^{-8}$ | $1.340 \times 10^{-7}$ | $1.767 \times 10^{-7}$ |
| 0.6 | $5.530 \times 10^{-8}$ | $1.096 \times 10^{-7}$ | $1.629 \times 10^{-7}$ | $2.153 \times 10^{-7}$ |
| 0.8 | $6.470 \times 10^{-8}$ | $1.285 \times 10^{-7}$ | $1.913 \times 10^{-7}$ | $2.531 \times 10^{-7}$ |
| 1.0 | $7.400 \times 10^{-8}$ | $1.470 \times 10^{-7}$ | $2.190 \times 10^{-7}$ | $2.900 \times 10^{-7}$ |

#### 4. DISCUSSION OF RESULTS

In this paper, the coupling of both Adomian polynomial and Elzaki transform is used to solve the nonlinear coupled Jaulent-Miodek equations (1.3) and (1.4). In this study, two examples were demonstrated and the solutions were presented in series form as seen in equations (3.7), (3.8), (3.15) and (3.16). The solutions to the two examples considered show the accuracy of this method when compared with the results obtained when the Adomian decomposition method is used [12]. This is shown graphically in Figures 1 and 2, the two solutions appear to agree. In both cases, the upper and lower left panel show the solitary wave solutions  $v(x, t)$  and  $w(x, t)$  of coupled nonlinear Jaulent-Miodek equation with the given initial conditions using Elzaki transform and Adomian polynomial, while the upper and lower right panel show the solitary wave solutions  $v(x, t)$  and  $w(x, t)$  of coupled nonlinear Jaulent-Miodek equation with the same initial conditions using Adomian decomposition method when  $b = c = 0.01$  and  $k = \sqrt{c}$ .

Moreover, to affirm the accuracy of Elzaki transform method, we considered the exact solutions to the aforementioned problem as established by [11] in equations (3.9) and (3.17), and the absolute errors is computed, these errors turn out to be very small with the values of  $x$  and  $t$  chosen as shown in Tables 1, 2, 3 and 4.

#### 5. CONCLUSION

The Adomian polynomial is incorporated into the Elzaki transform to obtained the approximate traveling wave solutions of the coupled nonlinear Jaulent-Miodek

equation. Coupling of aforementioned methods has been successfully applied to two examples in which different initial conditions were applied to the coupled nonlinear Jaulent-Miodek, it is noted that this method is an effective method for solving these problems because this is demonstrated by the agreement of the results obtained using the Elzaki transform and Adomian decomposition method. Furthermore, the smaller errors obtained when the difference between the exact solutions and approximate analytic solutions of this problem were computed is another evidence that shows how powerful this method is.

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