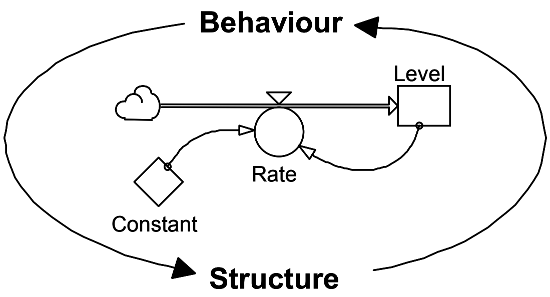
WORKING PAPERS IN SYSTEM DYNAMICS

WPSD 3/03 **ISSN 1503-4860**

Policy Sensitivity Analysis: simple versus complex fishery models

by

Erling Moxnes



The System Dynamics Group

Department of Information Science
University of Bergen, Norway
Postbox 7800, N-5020 Bergen, Norway

POLICY SENSITIVITY ANALYSIS: SIMPLE VERSUS COMPLEX FISHERY MODELS

by Erling Moxnes*

System Dynamics Group, Department of Information Science,

University of Bergen, Post-box 7800, Bergen N-5020, Norway,

Erling.Moxnes@ifi.uib.no

and SNF, Breiviksveien 40, N-4045 Bergen, Norway

Abstract

Sensitivity analysis is often used to judge the sensitivity of model behaviour to uncertain assumptions about model formulations and parameter values. Since the ultimate goal of modelling is typically policy recommendation, one may suspect that it is even more useful to test the sensitivity of policy recommendations. A major reason for this is that behaviour sensitivity is not necessarily a reliable predictor of policy sensitivity. Policy sensitivity analysis is greatly simplified if one can find optimal policies. Then one can simply see how the optimal policy changes when the model assumptions are altered. Our case is a fishery model. We investigate how (near-to) optimal policies change when we correct for a typical estimation bias in an aggregate model, when we substitute the aggregate model with a cohort representation of the same fishery, and when we switch from assuming variable to assuming constant fish prices and per unit variable costs. Normally these assumptions follow from the analyst's school of thought without testing. The most surprising result is that while assumptions about the fish price and the per unit variable costs matter a lot, the choice between an aggregate and a cohort model is of little importance.

Key words: Policy sensitivity, uncertainty, dynamics, fishery

---

* I am grateful for helpful discussions with Sigurd Tjelmeland at the Institute of Marine Research, Bergen and with Pål I. Davidsen and Mohamed M. Saleh at the System Dynamics Group, Department of Information Science, at the University of Bergen. The project has been financed by the Research Council of Norway under the program Marine Resource Management.

1. Introduction

When building simulation models, there is always uncertainty regarding model formulations and parameter assumptions. Two questions follow naturally: can policy recommendations based on such models be trusted, and if necessary, what assumptions should one try to get better information about? To answer these questions, behaviour sensitivity analysis is typically used. However, while such tests quantify the effect of assumptions on model behaviour over time, they do not answer the question about policy sensitivity. Does the fact that model behaviour is sensitive to an assumption mean that the policy is also sensitive? Does the fact that the model is insensitive imply that the policy is insensitive? When policy recommendations represent the ultimate goal of a model analysis, the question about policy sensitivity seems more important than the question about behaviour sensitivity.

In this paper we present a method that can be used to study policy sensitivity in complex, dynamic models. The case is the Northeast Arctic cod fishery in the Barents Sea, and the policy in focus is the quota strategy. We identify three important assumptions for which we explore the policy sensitivity. First, using an aggregate biological model we observe a potential bias in the model's parameter estimates and explore the sensitivity of the quota policy to this bias. Second, current practise differ with respect to the level of aggregation in biological models, we investigate how the quota policy changes when we replace the aggregate biological model with a more complex age-class (or cohort) model. Third, the economic mechanisms in our model (the fish price and the per unit variable costs depend on the harvesting rate) are often ignored in analyses of quota policies. We investigate the policy sensitivity to these assumptions.

In the next section we define what we mean by policy sensitivity and we consider under what conditions policy sensitivity should be preferred to traditional behaviour sensitivity analysis. Then the fishery models with their assumptions are presented. In the fourth section we discuss the method used to find optimal or near-to-optimal policies. Then we present the results. Quota policies are surprisingly insensitive to aggregate model parameters and to the choice between an aggregate and a cohort model. Quota strategies are much more sensitive to the assumptions about the economics. Finally, we conclude.

2. Policy sensitivity analysis

The traditional and frequently used form of sensitivity analysis has been to vary model parameters and to observe how behaviour changes. This is a very useful procedure for model testing, learning, and validation. Using optimisation, one can in addition observe how the optimal policy changes due to variations in model parameters. This is what we do here, and what we refer to as policy sensitivity analysis. Below we give a motivation for the use of policy sensitivity testing and we discuss limitations of the approach.

The main purpose of modelling is problem solving. In the light of double-loop learning models, Argyris and Schön (1978), problem solving can take quite different forms. At one level, the main challenge is to convince managers, politicians or their electorate, that a problem exists and that improvements are possible. Once the problem is acknowledged and proper institutions are in place, the problem is often dealt with at another level where more detailed and advanced policy analysis may be appropriate.

For these two modes of problem solving, different types of sensitivity analysis are needed. To convince that a problem exists, sensitivity analysis could be used to show that basic (problem) behaviour modes are insensitive to large variations in model parameters. This is for instance the type of sensitivity analysis Jay W. Forrester refers to when discussing his world dynamics model, Senge (1973) p.5-18. The underlying assumption is that as long as the basic problem behaviours persist, proper policies stay approximately the same. For example, regarding the management of most renewable resources, it is crucial that decision makers have a basic grasp of the underlying resource dynamics, Moxnes (2000), and of the commons problem, Gordon (1954) and Hardin (1968).

At the level of more detailed and fine-tuned policy analysis, the assumption that policies stay the same becomes more questionable. The fact that model behaviour is sensitive to parameter change, may or may not imply that appropriate (optimal) policies are sensitive to the same parameters. This can be a difficult question to answer because it may be even harder to intuitively identify proper policies than to intuitively predict behaviour in complex dynamic models.

A complex case illustrates. When non-linear dynamics are combined with stochasticity and measurement error, it is both intuitively and analytically very difficult to identify optimal policies. In such an environment, Moxnes (2003) uses traditional sensitivity analysis, keeping the harvesting policy constant, to find that the total payoff from the fishery is very sensitive to the amount of measurement error. The apparent policy conclusion is that first priority should be to increase measurement accuracy, even if this may be costly. However, this conclusion relies heavily on the assumption that the quota policy remains fixed. When the policy is allowed to vary with the error level, performing a policy sensitivity analysis, around ¾ of the earlier estimate of the value of accuracy disappears. Still accuracy is valuable, however, for a start, it is a cheaper option to produce an estimate of the error level and to adapt the harvesting policy to this error level.

As already indicated, it is preferable to use optimisation when testing the sensitivity of policies to model assumptions. First consider policies found without optimisation, using some manual calibration scheme. Then one would make comparisons of the type

$$f_1(x,a_1,\varepsilon_1) \text{ versus } f_0(x,a_0,\varepsilon_0)$$

where x represents the state variables of the system, f_1 is the policy found for assumption a_1 and f_0 is the policy found for the reference assumption a_0. The manual and presumably imprecise calibration procedure, however, leads to policy errors of respectively ε_1 and ε_0 in the two cases. Since there is no guarantee that ε_1 is equal to ε_0, one cannot conclude firmly that a difference between policy f_1 and f_0 is due to the change in model assumptions, a_0 to a_1. There is room for conscious or unconscious manipulation of the results by modellers, and consequently reason for uneasy feelings and outright accusations among readers that results have been manipulated.

Using optimisation, there will be no random element when comparing policies, ε_1=ε_0=0. Thus, the comparison will be "fair", showing truly optimal policies for all model assumptions.

Since this type of policy sensitivity test requires that optimal policies can be found, the direct application of the method breaks down in complex models. One possibility is to resort to near-to-optimal policies. Such policies will have an element of error since they represent simplifications. While subjective, random errors due to an imprecise manual procedure are removed, one cannot rule out that the remaining systematic error is a function of the tested model assumption, $\varepsilon_i=f(a_i)$. If this is the case, one will get a biased estimate of the policy sensitivity, a bias that cannot be removed without a better knowledge about the truly optimal policy.

From a more practical viewpoint, if the simplified policy is the best one can do, and it is the policy one will use in practical management, the bias is of less concern. Then it is interesting to see how the practical policy varies with changes in the model assumptions. Still the comparison is "fair" with no room for manipulation. In Section 4 we present an optimisation method that can be used to find both truly optimal solutions and near-to-optimal solutions.

One can also think of types of policy sensitivity analyses where optimisation is not needed by definition. An examples is presented in Andersen (1980). In two cases, policies resulting from two different models were compared. Andersen found that the policy conclusions were sensitive to the choice of modelling paradigm. When comparing the models, Andersen did not compare optimal policies. Rather, he took for granted the policies suggested by those who performed the original studies. Comparing modelling paradigms it seems fair to include potential shortcomings of the analysts. However, if one compares a small number of studies from each modelling paradigm, comparisons could be biased if the analysts are not representative of their own schools of thought.

Finally, policy sensitivity analysis can be a useful tool for model simplification in order to produce transparent, understandable models which are still highly useful. In order to improve on traditional behaviour sensitivity analysis, Eberlein (1989) conducts an eigenvalue-based linear analysis. This method helps identify and select the subset of feedback loops in a complex model that explain most of the model's behaviour. Eberline points out that: "To allow understanding, the variables in the simplified model must be easy to interpret relative to those in the original model." This is, however, not easy to do when we will compare an aggregate and a cohort model of the same fishery. Not only does the aggregate model collapse all the age classes of the cohort model, it also combines recruitment, growth, and mortality into one single variable for surplus growth. Using policy sensitivity analysis we can see how the optimal policy varies with the choice of modelling concept.

3. The fishery models

Figure 1 gives an overview of the fishery model in terms of a stock and flow diagram. The goal is to maximise the expected future value of bank deposits, which in this case is equivalent to maximising the expected net present value of future profits[1]. Profits are made up by revenues minus costs. Costs depend on the effort spent, which in turn depends on both the harvest and the catch per unit effort. The latter depends on the size of the fish stock. The revenue depends on the harvest and the fish price. The latter declines when the supply of fish in the market (the harvest) increases. The Barents Sea cod fishery is managed by yearly quotas. Therefore the decision variable is the yearly harvest, which depends on the fish stock and is guided by a quota policy[2]. Figure 1 shows the fish stock and the growth rate as they are described in the aggregate model. Note the randomness influencing the net fish growth, implying that stochastic rather than deterministic optimisation is needed. Alternatively the biological part of the model is described by a cohort model. The cohort model and the aggregate model are described in the following subsections. However, first we describe the common economic structure.

The common economic part is characterised by the following equations, first, the expected net present value[3]

$$V = E\sum_{t=0}^{\infty}\rho^t \left\{(p_0 - p_1 H_t)H_t - (c_0 + (c_1 - c_0)(\frac{e_t}{e_0})^{\alpha})e_t - c_2 e_0\right\} \tag{1}$$

---

1 This is correct if the discount factor ρ is determined by the interest rate r, ρ=1/(1+r).

2 A function and not just a constant as Figure 1 may suggest.

3 Since we operate with an infinite horizon, it is more intuitive to focus on the net present value than on the bank deposits which tend towards infinity. The latter is probably the more intuitive representation in the stock and flow diagram.

The discount factor is denoted by ρ, e_t is the applied fishing effort, and e_0 reflects the fishing capacity. The price of fish is a linear function of harvest H_t with parameters p_0 and p_1. Unit variable costs equal c_0 at zero effort, and they equal c_1 when effort equals capacity e_0. Increasing per unit variable costs are ensured by assuming $\alpha>0$. The per unit leasing cost of capacity is c_2. We explicitly avoid maximising a social welfare function for the fishing nation. Most of the harvest is exported and domestic prices reflect export prices.

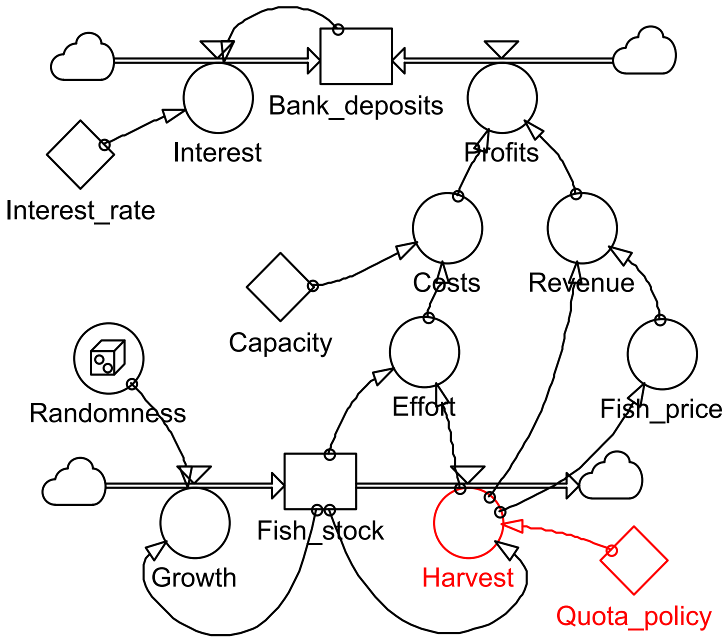


Figure 1: Overview of the fishery model, with the aggregate representation of the biological part.

The assumption that the price of fish depends on the harvest rate and the assumption that per unit variable costs increase with capacity utilisation are only infrequently made in economic models of fisheries, and they are of course never part of purely biological models. Increasing per unit variable costs are probably left out of many early fishery models because they complicate the search for an optimal harvesting policy. Overcapacity in most fisheries in most years also imply that it is difficult to get data that verify increasing per unit variable costs empirically. The assumption about a flexible price does not reflect the domestic demand curve for cod. Since most of the cod is exported, there is a modest effect of domestic harvests on international prices and a probably much larger effect on prices paid by the domestic fish processing industry. Also this industry has increasing per unit variable costs, such that the fishing industry's profit margin decreases with increasing harvests if the fish price is held constant. Both these assumptions will be subjected to a policy sensitivity analysis.

In the following we use capital letters to denote fish in biomass terms (million tons), while lower case letters are used to denote numbers (billion fish). The instantaneous harvest is given as $h = e(X / X_0)^{\beta}$, where $(X / X_0)^{\beta}$ denotes the catch per unit effort relationship. $X = X_0$ is the biomass for which instantaneous harvest is equal to instantaneous effort. To get an expression for total yearly effort

$$e_t = X_0 \{(\frac{X_t}{X_0})^{1-\beta} - (\frac{X_t - H_t}{X_0})^{1-\beta}\} / (1-\beta) \qquad (2)$$

we rearrange the harvest equation to find an expression for *e* and by integrating over *X* from the post catch stock size $X_t - H_t$ to the pre catch stock size X_t, see Clark (1985).[4]

3.1. The cohort model

Cohort models used to find optimal fishing strategies, e.g. Mendelssohn (1978), Naqib and Stollery (1982), Spulber (1983), and Spulber (1985), typically limit themselves to rather simple representations since according to Mendelssohn: "The large increase in analytic complexity caused by the addition of even the simplest interaction term is cause for both consternation and challenge." Since our optimisation method allows for greater model complexity, we introduce "interaction" terms to capture vital feedback mechanisms, e.g. recruitment and weight dynamics. On the other hand, we will, different from the above papers, make the simplifying assumption that harvesters do not target specific age-classes. This is largely consistent with current fishery policies, which do not change restrictions on gear and which do not make major changes in allocations between fishing grounds and vessel segments, from year to year, depending on the stock size.

---

4 Ideally, there should have been a stochastic variable in this equation since the effort needed to catch a given quota is likely to vary from year to year. However, a test shows that such an extra random variable is of little importance for the optimal harvesting policy. The catch per unit effort relationship should also be expected to be related to the selectivity of the gear. This is no problem as long as we keep the selectivity constant.

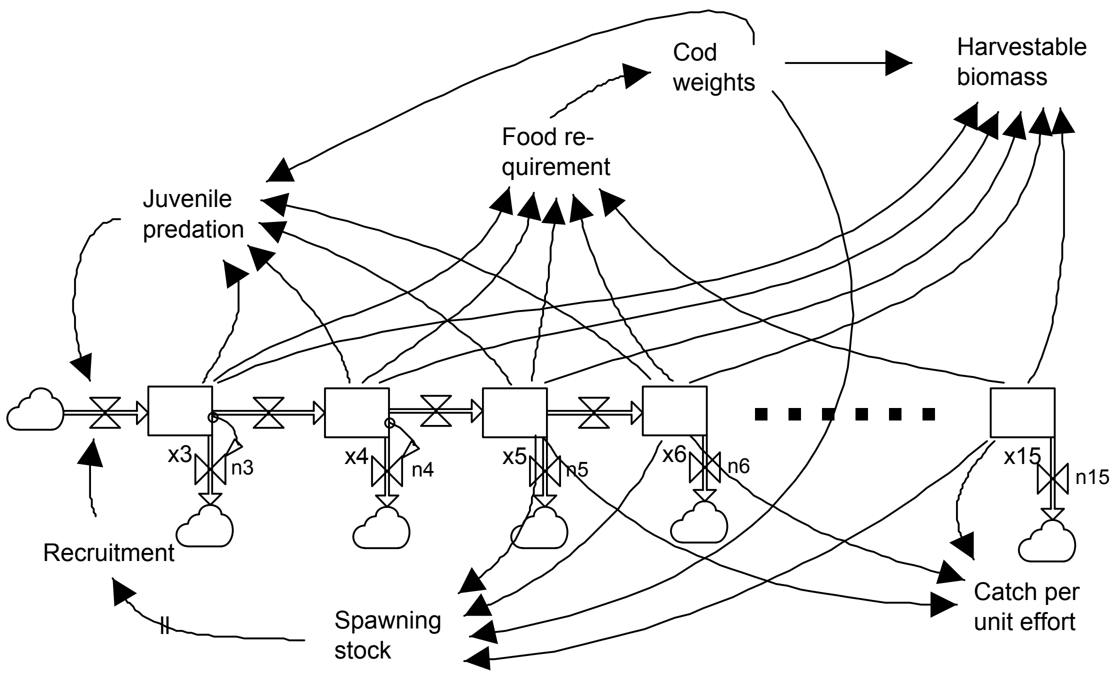


Figure 2: Illustration of cohort representation of the biological part of model, not showing harvest rates going out of each age class.

The cohort model is illustrated in Figure 2. To increase the speed of computation the model is formulated in discrete time with a time step of one year. The fish moves through cohorts as it ages. Different from the aggregate model, net growth consists of recruitment (depends on spawning stock and juvenile predation), weight growth (a vector of cod weights depend on food available and food required) and natural mortality (denoted by the outflows n_i in the figure). Harvestable biomass and catch per unit effort link up to the economic model. Harvest rates from each of the age classes are not shown in the figure. The number of fish in the different age classes are given by the following equations:

$$x_{3,t} = S_{t-3} \exp(r_0 + r_1 S_{t-3} + r_2 J_t + v_{r,t}) < r_s \tag{3}$$

$$x_{i+1,t+1} = x_{i,t} \exp(-m_i v_{i,t}) - h_{i,t} \exp(-m_i v_{i,t} / 2) \qquad i=3,4,...,13 \tag{4}$$

$$\begin{aligned} x_{15,t+1} = {} & x_{14,t} \exp(-m_{14} v_{14,t}) - h_{14,t} \exp(-m_{14} v_{14,t} / 2) + \\ & x_{15,t} \exp(-m_{15} v_{15,t}) - h_{15,t} \exp(-m_{15} v_{15,t} / 2) \end{aligned} \tag{5}$$

where $x_{3,t}$ represents recruitment of three year old cod. S_{t-3} is the biomass of the spawning stock at the appropriate point in time, J_t is a measure of cannibalistic cod juveniles, $v_{r,t} \sim N(0,\sigma_r)$ represents random recruitment variability and r_s is the maximum recruitment. Yearclass harvest is denoted by $h_{i,t}$, m_i is the natural mortality for yearclass *i*, and $v_{i,t} \sim N(1,\sigma_m)$ represents random variations in natural mortality. Even though one might expect natural mortalities for age classes to be influenced by some of the same environmental forces, we disregard this possibility here and assume independence. Suitability matrices (based on stomach content analyses) indicate that there is a certain cannibalism on three year old cod. We ignore this direct relationship since the bulk of cannibalism is captured by the recruitment function.

Due to the choice of total harvest as the decision variable, it is most practical to use age-class harvests and not fishing mortalities in these equations. To facilitate this, we have made use of Pope's approximation, i.e. harvest is assumed to take place in the middle of the year. This approximation is thought to yield good results for cod[5].

Equation 5 shows that the survivors of age class 15 are re-entered into this age class. This is not a perfect way to represent fish older than 15 years of age since fish weight does not increase with further ageing. With normal fishing activity, however, there are very few fish in the upper age classes such that this approximation should be of little concern.

The spawning stock biomass is given by ogives (maturity coefficients) o_i, age class body weights $w_{i,t}$, and age class numbers $x_{i,t}$.

$$S_t = \sum_{i=3}^{15} o_i w_{i,t} x_{i,t} \tag{6}$$

The total biomass of harvestable fish (3 years and older) is

[5] Personal communication with Bjarte Bogstad at the Institute of Marine Research, IMR, Bergen.

$$X_t = \sum_{i=3}^{15} w_{i,t} x_{i,t} \tag{7}$$

Juveniles J_t represent a weighted average of biomass in lower age classes

$$J_t = \sum_{i=4}^{15} s_i w_{i,t} x_{i,t} \tag{8}$$

The weights s_i reflect suitability of pre-recruitment cod for these age groups. The harvest from each age class

$$h_{i,t} = \frac{H_t q_i x_{i,t}}{\sum_{i=3}^{12} q_i x_{i,t} w_{i,t}} \tag{9}$$

is derived from the total harvest H_t, i.e. the policy variable. Here q_i represents the selectivity of the fishing gear. One can easily see that the sum of harvests from individual age classes equals in H_t (multiply by $w_{i,t}$ on each side of the equation and sum over all i) .[6] Based on observed patterns seen in VPA (virtual population analysis) data, harvesting selectivities are given by a logistic function:

$$q_i = e^{u_{i,t}} \,/\, \{1 + \left(q_h / i\right)^{q_e}\} \qquad i=3,4,...,15 \tag{10}$$

For older age classes, q_i tends towards 1.0, q_h denotes the age at which q_i equals 0.5, and the exponent q_e influences the steepness of the function. Selectivities are also influenced by natural variation, $u_{q,t} \sim N(0,\sigma_q)$.

6 When the selectivity varies over age classes, Equation 9 does not ensure that harvests will be less than population numbers in all of the age classes. This problem is most easily seen in the case that $H_t = X_t$. In this case all age classes should be harvested completely. However, harvests in age classes with higher than normal values of q_i will be greater than the corresponding population numbers x_i. The problem is caused by discretization in time. In a continuous world the q_i's could stay constant while the population numbers gradually decrease. In turn the declining populations numbers would serve to limit harvests to what is available. Fortunately this is only a problem when H_t is close to X_t. Since any reasonable harvesting policy will keep a good distance, the weakness of the formulation usually presents no problem.

Nearly all model studies we have come across ignore the effect of intraspecies competition in terms of the effect of own stock biomass on own weight. One exception is Ault and Olson (1996). We assume that the weight of each age class is given by a reference weight for this age class $w_{i,0}$ times a weight index w_t:

$$w_{i,t} = w_{i,0} w_t \tag{11}$$

where the weight index

$$w_t = B_t^{\varphi/(1-\varphi)} e^{v_{w,t}} < w_s \tag{12}$$

depends on the cod biomass and a random variable $v_{w,t} \sim N(0, \sigma_w)$. To avoid simultaneous equations, we have replaced the actual cod biomass X_t by an approximation based on the reference weights for each weight class, $B_t = \sum_{i=3}^{15} w_{i,0} x_{i,t}$. This is not a problem because we need a measure of the food requirement, and not the actual biomass. Weight is assumed to stay below an upper limit w_s in case cod biomass becomes very low. Using one common weight index implies that we ignore possible

differences between age classes. We also ignore time delays in the effect of intraspecies competition. It seems however that the delays are short and of rather little importance.

Tables 1 and 2 give a summary of the parameter values used in the cohort model. The parameter values have been found by a variety of methods, e.g. catch-at-age analysis, OLS and direct observation, see Moxnes (1999).

Table 1: Parameter values in cohort model.

Parameter	Symbol	Value	Unit
Discount factor	ρ	0.95	
Price at zero harvest	p_0	10.0	NOK/kg
Price reduction with harvest	p_1	2.0	
Lower unit variable cost	c_0	3.7	NOK/kg
Unit variable cost when $e_t = e_0$	c_1	4.5	NOK/kg
Leasing cost of capital	c_2	1.8	NOK/kg
Exponent for variable costs	α	2.0	
Biomass where effort equals harvest	X_0	1.0	Mill.tons
Exponent for catch per unit effort	β	0.6	
Recruitment, constant	r_0	0.85	
Recruitment, effect of spawning stock	r_1	0.7	per mill.tons
Recruitment, effect of juveniles	r_2	-0.25	per mill.tons
Recruitment, maximum	r_s	2.0	Billion
Recruitment, standard deviation	σ_r	0.64	
Mortality	m	0.2	
Mortality, standard deviation	σ_m	0.35	
Selectivity, half value	q_h	4.75	
Selectivity, exponent	q_e	6.25	
Selectivity, standard deviation	σ_q	0.25	
Weight, elasticity w.r.t. biomass	φ	-0.2	
Weight, standard deviation	σ_w	0.34	
Maximum weight index	w_s	1.3	

Table 2: Parameters that are distributed over age classes.

Age class	Reference fish weights [kg] $w_{i,0}$	Spawning stock ogives o_i	Juvenile distribution s_i	Expected initial populations [Billions] $x_{i,00}$
3	0.8	0.00	0.30	0.500
4	1.3	0.02	0.80	0.403
5	1.9	0.08	1.00	0.305
6	2.7	0.28	1.00	0.207
7	3.8	0.57	1.00	0.129
8	5.2	0.79	0.63	0.077
9	6.8	0.90	0.26	0.045
10	8.3	0.96	0.00	0.026
11	9.8	1.00	0.00	0.015
12	11.5	1.00	0.00	0.009
13	12.7	1.00	0.00	0.005
14	13.5	1.00	0.00	0.003
15	16.8	1.00	0.00	0.002

3.2. The aggregate model

A stock and flow diagram of the aggregate model is shown in Figure 1. We use a discrete version of the aggregate surplus growth model, Schaefer (1954):

$$X_{t+1} - (X_t - H_t) = a(X_t - H_t) + b(X_t - H_t)^2 + \xi_t \tag{13}$$

where X_t denotes total biomass and H_t is total harvest measured in biomass. Parameter estimates for Equation 13, based on 51 year long historical time-series, are shown in Table 3, see Moxnes (1999) for details. As can be seen, the estimates are highly significant with large t-ratios.

As a preliminary test of the similarity of the models, we use the cohort model to produce synthetic time-series data from which we can also estimate aggregate model parameters. Actually, we produce two sets of synthetic data because it turns out that the estimates obtained are sensitive to the choice of fishing policy in the cohort model. Parameter estimates are shown in Table 3 for a historical policy ($H_t = 0.28 * X_t$) and for the best possible proportional policy ($H_t = 0.21 * X_t$). To get precise results we used 1000 years of synthetic data, which explains the very high t-ratios. Figure 3 shows all the three surplus growth models.

Table 3: OLS estimates for aggregate models (t-ratios in parentheses).

Data used	a	b	σ_A
Historical data (51 years of VPA data from IMR)	0.89	-0.25	0.30
(later referred to as aggregate-historical)	(11.1)	(-7.5)	
Cohort model output (1000 years, $H_t = 0.28 * X_t$)	0.94	-0.27	0.71
	(26.1)	(-18.9)	
Cohort model output (1000 years, $H_t = 0.21 * X_t$)	1.03	-0.23	1.08
(later referred to as aggregate-simulated)	(28.4)	(-24.6)	

The model obtained from historical data (solid line) is nearly identical to the one obtained from synthetic data with the historical policy (dotted line). The close fit is somewhat arbitrary since there is some uncertainty in the estimated parameters for the historical curve. Clearly, the two curves are not statistically different. We do note however that the estimates of the residuals σ_A are significantly different (Chi square test). One possibility is that there is too much natural variation in the cohort model. Another possibility is that the cohort model produces data that are less consistent with the surplus growth model than what the real system does. Both explanations indicate a certain improvement potential for the cohort model.[7]

7 These possible explanations are strengthened by the fact that we have underestimated another source of variation by using a fixed policy when simulating the cohort model. If we had used a policy with a certain element of randomness (observations of historical policies always deviate

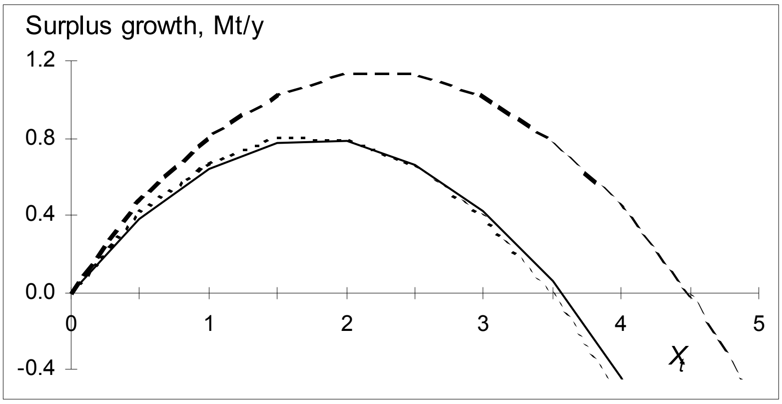


Figure 3: Surplus growth: Historical data: solid line, simulated data with historical policy: dotted line, and simulated data with the best proportional policy: dashed line.

It is interesting to observe that the estimate of the aggregate model from synthetic data is sensitive to assumptions made about the harvesting policy in the cohort model. The best possible proportional harvesting policy, which implies more careful harvesting and higher average fish stocks, leads to a higher estimate of the surplus growth curve (dashed line). This is an unfortunate feature of the aggregate model, at least when its parameters are based on data from historical periods with over- or underfishing compared to a desirable future policy. Thus, in the remaining part of this paper we will consider both the aggregate model based on historical data, and the one based on synthetic data from the cohort model using the best possible proportional policy, referred to as respectively "aggregate-historical" and "aggregate-simulated". Comparing optimal policies for these two models, we will get a sense of the policy sensitivity to this estimation bias.

4. Stochastic optimisation in policy space, SOPS

Cohort models are characterised by a large number of states. Hence, a direct application of stochastic dynamic programming, SDP, is ruled out by the 'curse of dimensionality'. SDP could still be used after some sort of model reduction. However, while such simplifications are conceivable, they would lead to less transparent models and to models that are not familiar to the decision makers. To maintain familiarity and to ensure that important effects are captured by the model, we rely on a method termed

somewhat from what is predicted by an estimated policy), the aggregate model estimates of σ_A would have been even higher.

"stochastic optimisation in policy space", SOPS. In this case it is primarily the policy that is simplified in order to obtain solutions and not the model.

Optimisation in policy space has been proposed, used, and implemented in various settings, e.g. Walters (1986), Bertsekas and Tsitsiklis (1996), Ermoliev and Wets (1988), and Polyak (1987). For deterministic problems with simple policy functions, optimisation in policy space can be performed by simulation programs like Powersim and Vensim with an optimisation option. Here we rely on a practical adaptation to stochastic problems presented in Moxnes (2003).

The basic idea is that one proposes a harvesting policy, see illustration in Figure 4, and simulates the fishery model into a future with unknown, random events. Because the random events cannot be predicted, the model must be simulated over many possible futures, where the goodness of the policy is expressed by the average net present value over many futures. Then one makes systematic changes in the harvesting policy (for example by changing θ_3' to θ_3 in Figure 4) to search for the policy that yields the

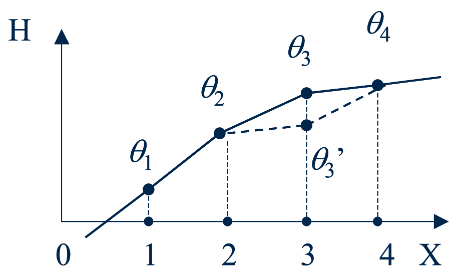


Figure 4: Illustration of policy function with policy parameters θ_i.

highest average net present value. Thus, the stochastic, dynamic optimisation problem is transformed into a problem of non-linear, static optimisation, where there is a deterministic relationship between the policy parameters and the average net present value. More precisely, we want to maximise

$$W(\boldsymbol{\theta}) = \frac{1}{M}\sum_{m=1}^{M} \sum_{t=0}^{T} \rho^t \left\{ \left(p_0 + p_1 H_t\right) H_t - \left(c_0 + (c_1 - c_0)\left(\frac{e_t}{e_0}\right)^{\alpha}\right) e_t - c_2 e_0 \right\} \qquad (14)$$

were the average net present value *W* is an estimate of the expected net present value *V* (Equation 1), and where $\boldsymbol{\theta}$ is the vector of policy parameters in the harvesting strategy[8]

$$H_t = f(X_t, \boldsymbol{\theta}) \geq 0 \tag{15}$$

where X_t represents the biomass of the fish. *W* is produced by *M* Monte Carlo simulations of the fishery model with the proposed fishing strategy implemented. The rest of the fishery model is as described earlier except that the random variables with subscripts *t*, now appear with subscripts *tmn*, e.g. $v_{r,t}$ becomes $v_{r,tmn}$. Thus besides varying with time *t*, the random variables also vary over Monte Carlo runs *m*=1, 2, ..*M* and over *n*=1, 2, ..*N* separate searches for the policy parameters $\boldsymbol{\theta}$. Each new parameter search *n* starts with different initial policy parameters, $\boldsymbol{\theta} = \boldsymbol{\theta}_{\mathbf{0,n}}$, which are drawn from uniform distributions. We use *M*=100 Monte Carlo runs, a time horizon of *T*=50 years, and *N*=20 independent searches.

When using the cohort model each Monte Carlo run starts out with randomly chosen initial age class populations

$$x_{i,0} = x_{i,00} e^{v_{x,0}} \tag{16}$$

where $v_{x,0} \sim N(0, \sigma_x)$ and $\sigma_x = 0.4$. Initial conditions vary similarly for the aggregate model.

$$X_0 = X_{00} e^{v_{X,0}} \tag{17}$$

where $v_{X,0} \sim N(0, \sigma_X)$ and $\sigma_X = 0.4$.

To find the parameter vector $\boldsymbol{\theta}$ that maximises *W*, a hill-climbing search procedure is used (Fletcher-Powell variable metric). The search routine provides accurate parameter values judged by variations between repeated searches with different starting points for the parameter set $\boldsymbol{\theta}_{\mathbf{0}}$ (if necessary ignoring occasional solutions that are not close to the

8 Note that the policy parameter vector $\boldsymbol{\theta}$ also includes the fixed harvesting capacity e_0.

global optimum). Naturally, accurate parameters are only found in subsets of the state space that are visited and where the policy is of importance for the criterion.

Since we do not know what function characterises the optimal solution, we rely on a flexible policy function, which does not restrict the solution very much, see illustration in Figure 4. For the one dimensional policies to be used here, a good numerical approximation can be obtained by interpolating between five grid points and extra-polating beyond the end grid points. Mathematically, the policy H_t is given by

$$0 \le H_t = \theta_k (k - (X_t - \varphi)/\delta) + \theta_{k+1}((X_t - \varphi)/\delta - (k-1)) \le X_t - X_l \qquad (18)$$

where φ is the location of the first grid point, δ is the distance between grid points, and the policy parameter θ_k denotes harvest at grid point k determined by

$$1 \le k = \mathrm{int}((X_t - \varphi)/\delta) + 1 \le 4 \qquad (19)$$

Compared to the discrete representation in dynamic programming, we note that the fish stock X_t and the policy H_t are continuous variables. The grid points denote the kinks in the piecewise linearized policy. Moxnes (2003) shows how linear interpolation can be extended into higher order policy surfaces.

The more complex the model, the greater the need to seek simplifications of the policy. By restricting ourselves to infinite horizon problems, time is left out of the policy function. When all states are measured perfectly, the ideal optimal policy is a function of all states. Since the aggregate model has only one state, its policy will be one-dimensional. The cohort model has many states and the ideal policy is very compli-cated. However, as shown in Moxnes (1999), a one-dimensional policy gives a nearly perfect result compared to higher order policies. The main reason for this is that the fishing selectivities are fixed. In models with targeted harvesting of all age classes, one dimensional policies would not suffice, see Mendelssohn (1978) and Spulber (1983).

By repeated searches and by varying the initial policy parameters, we increase the probability that a global rather than some local optimum is found. The confidence in the method is also increased whenever the method identifies global solutions known *apriori*.

Finally we note that SOPS is an interesting method also from a more practical point of view. The method allows for the use of simulation models and assumptions familiar to decision makers. This is an advantage to the extent that decision makers distrust overly simplified models, Gulland (1991). On the other hand, large models may require that policy functions are simplified. However, this may also be perceived as desirable. According to Walters (1986): "..we will have to find ways to visualise [policy] functions when there are many [state] variables, since it would be silly to expect any real decision maker or manager to blindly plug numbers into such a function and then follow its prescription", p.243. SOPS could be used to find the best possible simplified and "visualizable" policies. This approach may also provide an attractive alternative to the intuitive blending of two or more exact results from simplified models to come up with a best possible policy for a more complex reality. That this can be a complicated task is exemplified by the at times surprising effects of adding new nonlinearities, stochastic variables, and feedbacks to existing models. Empirical evidence of this difficulty is presented in Brekke and Moxnes (2003). If decision makers are not able to untangle complexity, they are left with uncertainty about received results. Such uncertainty is believed to be the major obstacle to diffusion of technologies and policies, Rogers (1995). In this regard, it may also be an advantage of the method that it does not require knowledge of more sophisticated techniques than simulation and search.

By pointing to potential advantages of the method, we do not claim that SOPS is a panacea. For instance, other methods are needed to guide efficient problem formulation, to judge the likelihood that proper solutions are obtained, and to help explain why policies turn out the way they do. In highly complex cases SOPS will only provide improvement, which is also the rationale behind various related methods to tackle highly complex problems, e.g. neuro-dynamic programming and reinforcement learning, Bertsekas and Tsitsiklis (1996).

5. Results

We start by considering the sensitivity of the harvesting or quota policy to a possible estimation bias in the estimate of the aggregate surplus growth curve. Figure 5 and the upper half of Table 4 show the resulting policies. The thin solid line in the figure shows the optimal policy for the aggregate model with parameters based on historical time-series data (aggregate-historical). This policy is very close to the one for aggregate-simulated (thin dashed line), i.e. the policy for the aggregate model based on synthetic data from a cohort model with a near-to-optimal harvesting policy. Hence the likely bias in the estimate of the aggregate model's growth curve has a limited effect on harvesting policies. Looking at the optimal harvesting capacities, e_0 in Table 4, we see that aggregate-simulated has a 15 percent higher capacity than aggregate-historical.

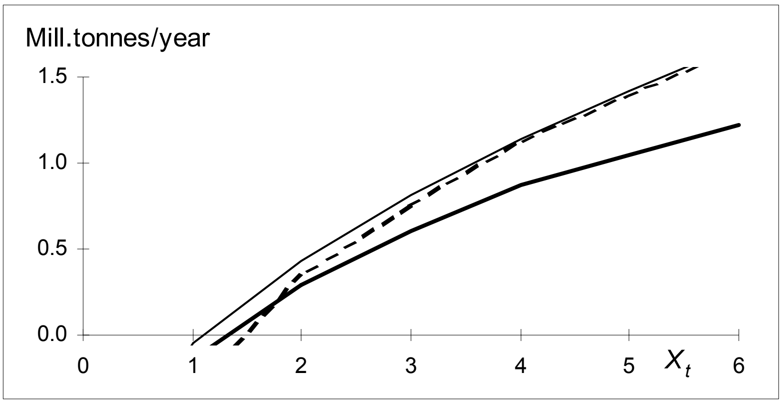


Figure 5: Policies for models with variable price and unit costs. Thick line: Cohort model. Thin solid line: aggregate-historical. Thin dashed line: aggregate-simulated.

Looking at the estimates of expected net present values *W*, differences are large. However, this difference is primarily explained by the different growth curves and not by the policies. To get an estimate of the economic value of the two policies[9], we plug the policies for the two aggregate models into one and the same cohort model. This is not done because we implicitly consider the cohort model superior, but because we see it as a testing ground. It turns out that the policy for aggregate-simulated yields an expected net present value that is 2.6 percent higher than the one for the policy from aggregate-historical. The main impression is that the policy and value sensitivity is limited.

---

9 Policies for both harvesting strategy and capacity.

Table 4: One-dimensional policies for cohort and aggregate models (averages of 20 searches).

Model/policy	θ_1	θ_2	θ_3	θ_4	θ_5	e_0	W
Variable price and unit costs							
Cohort	-0.14	0.30	0.61	0.88	1.05	0.37	73.8
Aggregate-historical	-0.04	0.43	0.81	1.14	1.41	0.40	55.2
Aggregate-simulated	-0.34	0.34	0.76	1.13	1.40	0.46	70.5
Constant price and unit costs							
Cohort*	-2.00	-2.00	-1.50	0.52	1.89	0.5	86.0
Aggregate-historical	-0.84	0.20	1.20	2.19	3.18	0.5	48.3
Aggregate-simulated	-1.75	-0.39	0.59	1.58	2.59	0.5	90.8

* Additional grid point, θ_6=2.88.

Next we turn to the sensitivity of the policy to the choice between the cohort model and the aggregate models. Table 4 and the thick line in Figure 5 show the policy for the cohort model. The main impression is that the policy sensitivity to the choice of model concept is limited. The cohort model policy is somewhat less aggressive than the policies for the aggregate models as the stock increases. The optimal capacity is 7.5 percent lower than in the case with aggregate-historical, and 20 percent lower than in the aggregate-simulated case.

To get a measure of the value sensitivity, we take another look at the above results when the cohort model was used as a testing ground for the aggregate model policies. Using the aggregate-historical policy, rather than the optimal cohort policy, in the cohort model, the expected net present value drops by 7.7 percent. Using the aggregate-simulated policy the value drops by 5.3 percent. To put these numbers in perspective, the expected net present value drops by 13 percent if an estimate of the historical policy ($H_t = 0.28 * X_t$) is used in the cohort model. Hence, if the cohort model had been an exact replication of reality, aggregate model policies would have done better than the historical policy. Losses implied by the historical policy would be reduced by between 40 and 60 percent depending on the degree of estimation bias in the aggregate model. In this example, it is intriguing that the simple, low cost aggregate model policies perform better than the historical policy.

If we leave the unwarranted assumption that the cohort model represents reality, the apparent difference between the aggregate and the cohort model is likely to diminish. In fact, our analysis can not be used to rule out the possibility that the aggregate model is the one closest to reality. True, the estimation bias is a problem with the aggregate model. On the other hand, while the cohort model builds on more correct knowledge about the structure, there are also a larger number of places where errors could sneak in, and the entire cohort model has not been subjected to a thorough and complete test against time-series data. In this connection recall the observed inconsistency in residuals when estimating the aggregate model from synthetic data and from historical data.

Then we proceed by testing the policy sensitivity to the assumptions in Equation 1 that the fish price declines and the per unit variable costs increase with the harvest rate. The two assumptions have similar effects on policies. They both make it less advantageous to catch large harvests and more advantageous to catch small ones. In the sensitivity test we set the fish price constant[10] (p_1=0 and p_0=8.0) and we disallow unit variable costs to vary with capacity utilisation (α=0). When α=0 it is no longer meaningful to search for the optimal fishing capacity, hence we simply set e_0 equal to 0.5 million tons per year. The resulting policies are shown in Table 4 and Figure 6.

Both aggregate models (thin lines) now show the well known constant target escapement policy, Reed (1979). When the stock biomass is above the target, harvest is set such that the biomass is reduced exactly to the target. Compared to the policies shown in Figure 5, we see that at low stock levels less is harvested, and at high stock levels more is harvested, as expected. While the policies in Figure 5 show clear tendencies towards saturation, there is no such tendency in Figure 6.

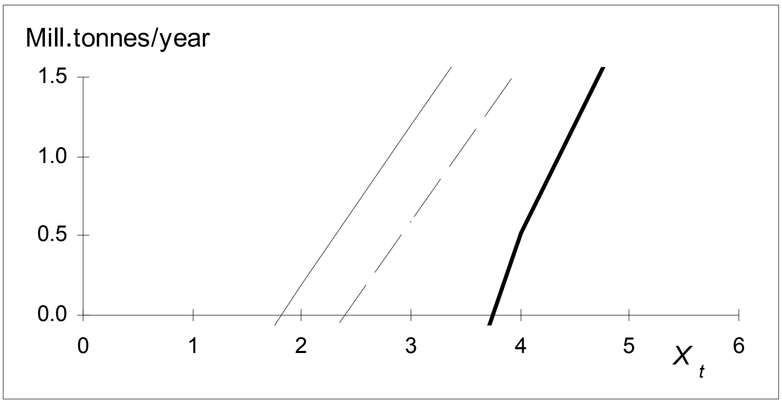


Figure 6: Policies for models with constant price and unit costs: Thick line: policy for cohort model. Thin solid line: policy for aggregate-historical. Thin dashed line: policy for aggregate-simulated.

10 The constant price level is close to the average price observed in earlier tests.

The difference between the policies for aggregate-historical (thin solid) and aggregate-simulated (thin dashed) is more pronounced than when fish prices and unit costs are allowed to vary. The distance between the lines is 0.58 million tons at high stock levels. The horizontal distance between the two curves reflects the horizontal distance between the peaks for the respective surplus growth curves in Figure 2. To indicate value differences, we use the cohort model (now with constant price and constant unit costs) as our testing ground. We find that the policy for aggregate-simulated yields an expected net present value that is 16.1 percent higher than the one for aggregate-historical. This is a much larger value difference than the one found for these policies in the case of variable price/costs (2.6 percent). Thus both the aggregate model policies and the implied values become more sensitive as prices and per unit variable costs are not allowed to vary.

The policy for the cohort model (thick line) portrays the “pulse-fishing” property found in studies of cohort models, Spulber (1983). No fishing takes place for biomasses below 3.7 million tons. For higher biomasses, the harvest increases somewhat faster than the biomass, such that harvests reduce the biomass to levels somewhat below 3.7 million tons. Then it is likely that a period with no fishing is needed before stocks again exceed 3.7 million tons and harvesting again takes place.

Figure 6 and Table 4 show that the harvest can be more than 1.5 million tons higher with aggregate-historical than with the cohort model policy. To approach an explanation, note that the pulse-fishing strategy, with a target stock size around 3.7 million tons, would give a very low average growth rate in the aggregate model, see Figure 3. This can not be the case in the cohort model, the pulse-like harvesting strategy can not be very detrimental to the average growth rate at high and varying stock levels. Thus, the cohort model is allowed to benefit from a high catch per unit effort at high stock levels. This advantage is lost again if the fish price decreases and the per unit variable costs increase at high stock levels due to high harvest rates.

Again using the cohort model (now with constant price and unit costs) as our testing ground, we find that the aggregate-historical policy gives an expected net present value which is 22.3 percent below the value obtained with the optimal policy for this cohort model. The aggregate-simulated policy yields a value reduction of 7.4 percent. These differences are larger than what was found earlier for the models with variable price/costs, respectively 7.7 and 5.3 percent.

Finally, great and important policy sensitivities are found when comparing policies for the cases with constant price/unit costs and cases where price/unit costs are allowed to vary. To get a measure of the value sensitivity, we use the cohort model (with variable price/unit costs) as a testing ground, now for all the policies found for the case with fixed price/unit costs. Since capacity was not optimised in the case with fixed price/costs, it does not seem fair to use an arbitrarily chosen capacity when comparing. Therefore we find capacities that maximise the expected net present value (in the cohort model with variable price/costs) for each of the harvesting strategies found in the case of fixed price/costs. It turns out that in all three cases the maximising capacity is around 0.8 million tons per year, somewhat higher than the assumed 0.5 million tons per year.[11] We find that the pulse-fishing strategy leads to a value loss of 93 percent. The target-escapement policy for aggregate-historical leads to a loss of 66 percent, while the target-escapement policy for aggregate-simulated leads to a similar loss of 66 percent. Clearly, both policies and net present values are highly sensitive to the assumptions about price formation and unit variable costs.

6. Conclusions

By the use of stochastic optimisation in policy space we have been able to find optimal or near-to-optimal policies for dynamic, non-linear fishery models. Thereby, we have been able to perform policy sensitivity analyses, where the sensitivity of policies to changes in model parameters or model structures have been found. Without optimisation the policy sensitivity analysis would have included an element of judgmental error. The case has been a cod fishery, for which we have compared three important assumptions.

With the most complete economic model, we find that the harvesting (or quota) policy is not very sensitive to the choice between an aggregate and a cohort model. This is somewhat surprising because the cheap, aggregate model is often viewed as deficient

---

[11] This makes sense since higher harvesting rates require higher fishing capacities.

compared to the cohort model when it comes to precise policy recommendation. We also find that aggregate model policies are not very sensitive to variations in the estimated surplus growth curve, variations that could be caused by a systematic bias in the estimation of the aggregate model.

Assuming a simplified economic model, with a fixed fish price and fixed per unit variable costs, policies become more sensitive to the choice of biological model (aggregate or cohort) and to the potential estimation bias in the surplus growth curve. If one has this economic model in mind (or no economic model in mind), these choices will seem more important than they actually are.

Harvesting policies are found to be highly sensitive to the assumptions about fish price and per unit variable costs. In the cod fishery at hand, and in general, it seems more realistic to assume that the fish price and the unit costs depend on the harvest rate than to assume that they stay constant. Interestingly, the more realistic assumption is only rarely found in economic fishery models and of course never in purely biological models.

Thus, for analysts and modellers, there seems to be a larger improvement potential in including equations for fish price and per unit variable costs and in obtaining precise estimates of the involved parameters, than to extend the biological model from an aggregate one to a more complex design with cohorts. In fact, the "insignificant" difference between the policies for the two biological models suggests that it would take much further research (validation effort) to find out which biological model is in fact the more appropriate one to use. There is also a certain potential for cheap improvement in the aggregate model. The effect of the estimation bias could be reduced by using data from historical time periods with near-optimal policies in place or by adjusting for suspected over- or underfishing.

Model development costs favour the aggregate model. Fisheries with large potential incomes favour the cohort model, if that is deemed the better model. The choice will move in the direction of the cohort model if one wants to make explicit investigations of policies that change the fishing selectivity over age classes, e.g. by changing the minimum allowable fish size. Regarding knowledge dissemination, the aggregate model seems more potent in terms of its simplicity.

Finally, a few words about the use of policy sensitivity analysis as compared to the more traditional behaviour sensitivity analysis. We have argued that behaviour sensitivity analysis is preferable if one wants to build confidence in the existence of a problem mode, i.e. that over-harvesting leads to stock depletion and lost economic opportunity. In the fishery investigated here, the need for quota control is generally accepted. The challenge is to find a quota or harvesting strategy. The search for such a strategy is not much helped by behaviour sensitivity analysis. Although we have not explicitly used behaviour sensitivity here, we think that it is of limited use because it leaves out the last and complex step of identifying harvesting strategies. By using stochastic optimisation in policy space we are able to carry out this last step.

Even for analysts who do not want to perform optimisation or policy sensitivity analysis, the mere concept of policy sensitivity analysis may be useful. It may help modellers focus on what matters for the choice of policy, leaving out structures of little importance and that complicate the communication of the policy recommendations. Behaviour sensitivity analysis on the other hand may shift the focus towards replicating observed behaviour with high accuracy. To reach this goal it is often tempting to include much detail. In turn this complicates the analysis and the communication of the results. Only if the policy recommendations are also sensitive to these details, the extra detail seems warranted. For these reasons it seems worthwhile to reflect on policy sensitivity already in the conceptualisation phase of a modelling project.

References

Andersen, D.F. (1980). "How Differences in Analytic Paradigms Can Lead to Differences in Policy Conclusions." Pp. 61-75 in *Elements of the System Dynamics Method*, edited by Randers, J. Cambridge: MIT Press.

Argyris, C., and Schön, D. (1978). *Organizational learning: A theory of action perspective*: Addison-Wesley.

Ault, J.S., and Olson, D.B. (1996). "A multicohort stock production model." *Transactions of the American Fisheries Society* **125**(3):343-363.

Bertsekas, D.P., and Tsitsiklis, J.N. (1996). *Neuro-Dynamic Programming*. Belmont Massachusetts: Athena Scientific.

Brekke, K.A., and Moxnes, E. (2003). "Do numerical simulation and optimization results improve management? Experimental evidence." *Journal of Economic Behavior and Organization* **50**(1):117-131.

Clark, C.W. (1985). *Bioeconomic Modelling and Fisheries Management*. New York: John Wiley & Sons.

Eberlein, R.L. (1989). "Simplification and understanding of models." *System Dynamics Review* **5**(1):51-68.

Ermoliev, Y., and Wets, R.J.-B. (1988). "Stochastic programming, an introduction." Pp. 1-32 in *Numerical Techniques for Stochastic Optimization*, edited by Ermoliev, Y. and Wets, R.J.-B. Berlin: Springer-Verlag.

Gordon, H.S. (1954). "The economic theory of a common property resource: The Fishery." *Journal of Political Economy* **62**:124-142.

Gulland, J.A. (1991). "Under what conditions will multispecies models lead to better fisheries management?" *ICES Mar. Sci. Symp.* **193**:348-352.

Hardin, G. (1968). "The tragedy of the commons." *Science* **162**(13 December):1243-1248.

Mendelssohn, R. (1978). "Optimal harvesting Strategies for Stochastic Single-Species, Multiage Class Models." *Mathematical Biosciences* **41**:159-174.

Moxnes, E. (1999). "Near-to-optimal harvesting strategies for a stochastic multicohort fishery." Report No. 57/99. Bergen: SNF.

Moxnes, E. (2000). "Not only the tragedy of the commons: misperceptions of feedback and policies for sustainable development." *System Dynamics Review* **16**(4):325-348.

Moxnes, E. (2003). "Uncertain measurements of renewable resources: Approximations, harvest policies, and value of accuracy." *Journal of Environmental Economics and Management* (forthcoming).

Naqib, F., and Stollery, K. (1982). "Optimal control of a multi-cohort fishery." *Economic Letters* **10**:385-390.

Polyak, B.T. (1987). *Introduction to optimization*. New York: Optimization Software, Publications Division.

Reed, W.J. (1979). "Optimal escapement levels in stochastic and deterministic models." *Journal of Environmental Economics and Management* **6**:350-363.

Rogers, E.M. (1995). *Diffusion of innovations*. New York: The Free Press.

Schaefer, M.B. (1954). "Some aspects of the dynamics of populations important to the management of commercial marine fisheries." *Bulletin of the Inter-American Tropical Tuna Commission* **1**:25-56.

Senge, P. (1973). “Issues in social modeling.” Transcripts of faculty seminars 1972-1973. Boston: Sloan School of Management, MIT.

Spulber, D.F. (1983). “Pulse-fishing and Stochastic Equilibrium in the Multicohort Fishery.” *Journal of Economic Dynamics and Control* **6**:309-322.

Spulber, D.F. (1985). “The Multicohort Fishery Under Uncertainty.” *Marine Resource Economics* **1**(3):265-282.

Walters, C.J. (1986). *Adaptive Management of Renewable Resources*. New York: Macmillian.