

Paper E1

Cyclical behaviour, a function of market complexity? Expanding the cobweb experiment

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ABSTRACT

This paper examines market behaviour in a series of Cournot market experiments with five sellers. Step by step, we add complexity (and realism) to the basic Cobweb market. First, we introduce long lifetimes of production capacity. Second, we introduce a two period investment lag in addition to long lifetimes. Consistent with previous experiments and the rational expectations hypothesis, we find no evidence of cycles in the basic design and in the capacity lifetime treatment. Average prices are close to Cournot Nash equilibrium with a bias towards competitive prices. In the investment lag treatment, however, variance and autocorrelation analysis indicate cyclical tendencies. The observed behaviour in all three treatments may be consistent with one simple adaptive investment strategy.

KEY WORDS: *Commodity cycles, Cournot markets, cobweb markets, bounded rationality, complexity, experimental economics.*

JEL classification: *C9 - Design of Experiments; D01 - Microeconomic Behaviour: Underlying Principles; D43 - Oligopoly and Other Forms of Market Imperfection; D84 - Expectations; Speculations; L10 - Market Structure, Firm Strategy, and Market Performance.*

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1 INTRODUCTION

There seems to be a widespread belief that commodity markets show cyclical behaviour (Spraos, 1990; Cuddington & Urzua, 1989; Cashin *et al* 2002, Cashin & Patillo, 2000; Deaton & Laroque, 1992, 1996, and 2003) and that fluctuations have significant negative effects for consumers, producers and developing countries¹ (Deaton, 1999; Akiyama *et al* 2001; Akiyama *et al* 2003). In spite of this, most modern introductory textbooks in economics either ignore commodity cycles (ex. Mankiw, 2004; Sloman, 2002; Samuelson & Nordhaus, 2001; and Case & Fair, 1996) or they deal with the phenomenon using the highly simplified cobweb model (ex. Lipsey & Chrystal, 2003). With few exceptions, the same is the case in journal articles dealing with commodity cycles. The apparent lack of more elaborate models may be one important reason for several authors to claim that commodity cycles are poorly understood (Cashin *et al* 2002; Deaton & Laroque, 1996 and 2003). Similarly, laboratory experiments of commodity markets have used very simple designs. In this paper, step by step, we add complexity (and realism) to the simple Cobweb or Cournot model and test the effects on behaviour in an accompanying laboratory experiment. Both cyclicality and efficiency are considered.

Most experimental markets do not include dynamic structures and are reset each period (e.g. Plott, 1982; and Smith, 1982). In these experiments, there are no elements carrying over to future periods, such as inventories, capacities, and unfulfilled orders. Dynamics have, however, been introduced by lagged supply models² (Carlson, 1967; Sonnemans *et al* 2004; Holt & Villamil, 1986; Sutan &

¹ About 25% of the world merchandise trade is accounted for primary commodities. Many developing countries are dependent on only one or a few commodities for their export earnings (Cashin and Patillo, 2000).

² Dynamics has been considered to study the formation of speculative bubbles. First, Miller *et al* (1977) took the simplest intertemporal possible market based on the Williams' two season model, where carryovers were allowed only from one season to the next within a period. They found with surprise that the markets worked very

Willinger, 2004) and by repeated play Cournot models (Rassenti *et al* 2000; and Huck *et al* 2002). The predicted cycles of the Cobweb theory do not materialize in these experiments, while some random fluctuations are sustained³ (Miller, 2002).

Our basic experimental treatment (T1) involves the same simple lagged supply model with symmetric constant marginal costs and a linear demand function, i.e. a homogeneous Cournot market under standard conditions (Huck, 2004). Hence, T1 provides a link between the literature and the ensuing treatments. In a second treatment (T2), we relax the assumption that the supply capacity lasts for only one period. Typically physical capital lasts many years, over which repeated investment decisions are made. New capacity is added to existing capacity; vintages are introduced. The third treatment (T3) keeps the vintages and we add an extra delivery delay for investments. Typically capacity additions require a sequence of operations: planning, choice of suppliers, production of parts, transportation, construction, and testing; or time for gestation and growth in biological production systems. In total, capacity additions take several years in most commodity markets. In our case, the lag is such that one new investment decision will be made before the last investment ordered is in place.

We formulate the null hypothesis based on the rational expectations hypothesis (Muth, 1961) and the standard assumption about optimal decision making. The null hypothesis is convergence to a stable Nash equilibrium. Minor and seemingly random variations around the equilibrium value are consistent with this hypothesis. Systematic cyclical tendencies are not consistent with the hypothesis since rational agents could predict these and benefit from countercyclical investment decisions, which over time would stabilize the market.

The alternative hypothesis is formulated based on bounded rationality theory (Simon, 1979), assuming that complex dynamic problems are approached with heuristics (Serman, 1987b; Tversky & Kahneman, 1987). Heuristics are used instead of truly optimal strategies because cognitive capabilities are limited and there are considerable deliberation costs. Such heuristics could lead to optimal results for simple problems (Gigerenzer, 1999), while the results are likely to deteriorate with increasing complexity. A series of experiments show this tendency (Paich & Serman, 1993; Diehl & Serman, 1995; Kampmann, 1992; Serman, 1989, Herrnstein & Prelect, 1991, Moxnes, 2004).

The choice of heuristics is case dependent and may vary over individuals (Tversky & Kahneman, 1987; Conlisk, 1996a). Hence, when there is only one strategy and only one outcome according to rational expectations theory, there are many possible heuristics and outcomes for boundedly rational agents. When formulating heuristics, we will draw on findings from experiments with similar dynamic complexity.

Regarding market efficiency, it is difficult to predict the outcome of bounded rationality. While simplified heuristics are likely to bias outcomes, these biases may be in the directions of both monopoly and perfect competition. Accordingly, the current literature does not present a clear prediction of efficiency for our case (Conlisk, 1996b, and Foss, 2003). Hence, in this regard we consider the experiment exploratory.

Regarding cyclical behavior, we formulate a precise hypothesis. We propose a heuristic that expresses the intended rationality of the subjects. The heuristic is based on adaptive expectations formulated by Nerlove (1958) in his analysis of the Cobweb theorem. Previous experiments lend support to this heuristic for people that are confronted with simple lagged supply decisions (e. g. Carlson, 1967; Serman, 1987b; Heiner, 1989). Our analysis shows that this heuristic leads to fast convergence of markets of type T1 and T2. In T3, the heuristic leads to cyclical behavior. In this case, the adaptive expectation heuristic is inappropriate because of the extra investment lag. Previous studies suggest that people do not adjust their heuristics sufficiently or at all with increasing dynamic complexity. For

efficiently with no signs of instability (Miller, 2002). After, Smith *et al* (1988) considered asset markets for 15 periods that replicated a bubble followed by a crash under a variety of parameters.

³ Asymmetries in costs lead to stronger random fluctuations than symmetric costs (Rassenti *et al* 2000).

instance they seem to ignore or under-estimate supply line delays (Sterman, 1989; Diehl & Sterman, 1995; and Barlas & Günhan, 2004).

Evidence of cyclical tendencies is presented in a number of one player experiments (Sterman, 1987a; Sterman, 1989; Bakken, 1993; Diehl and Sterman, 1995, and Barlas & Günhan, 2004). A first indication that cycles will show up in a market setting is given in Kampmann (1992). Kampmann observed cycles for pricing institutions with fixed and posted prices. Using a market clearing institution, prices tended towards equilibrium over time. Different from our experiment, Kampmann's experiment did not include vintages, had some extra complexity and used a different market clearing mechanism than that implied by the Cournot model.

The next section presents the design and the hypotheses of the experiment. In sections three and four we present the results and the analysis respectively. In T1 and T2, we observe some random fluctuations as in the Cobweb and Cournot experiments. T3 shows cyclical tendencies. Finally, in section five we present the conclusions.

2 EXPERIMENTAL DESIGN AND HYPOTHESES

There are three treatments. Each new treatment builds on the preceding one. Treatment T1 is the Cobweb market with five players and linear demand. T2 introduces vintages and T3 an extra investment lag. Following, we describe the treatments and the rest of the experimental design.

2.1 Treatment T1: Standard five players Cournot market

The first treatment corresponds to a computerized experiment of a Cournot market with linear demand and constant marginal costs, under Huck's standard conditions⁴. There are five symmetric firms in each market, each represented by one player. Each subject chooses production between 0 and 20 units each period. The market price is determined by a linear inverse demand function with a nonnegative restriction. Information about the realized price and profits is given the next period. Thus, there is a one period production lag, which makes the experiment dynamically identical to the traditional Cobweb design. The market price in period t is,

$$P_t = \text{Max} (6 - 0.1 \sum_{i=1}^5 q_{i,t}, 0) \quad (1)$$

where $q_{i,t}$ is the nonnegative production of subject i in period t . Note that $q_{i,t}$ is equal to the investment made by subject i in period $t-1$ ($q_{i,t} = x_{i,t-1}$). The profit function in experimental dollars (E\$) for subject i in period t is,

$$\pi_{i,t} = (P_t - c) q_{i,t} \quad (2)$$

where the constant marginal cost $c=1$ \$/Unit.⁵ The number of periods is large enough to allow learning and eventually convergence, 40 periods with two exceptions.

⁴ Standard conditions (Huck, 2004, p.106): a. Interaction takes place in fixed groups; b. Interaction is repeated over a fixed number of periods; c. Products are perfect substitutes; d. Costs are symmetric; e. There is no communication between subjects; f. Subjects have complete information about their own payoff functions; g. Subjects receive feedback about aggregated supply, the resulting price, and their own individual profits; h. The experimental instructions use an economic frame.

⁵ In previous experiments, similar results have been obtained for increasing (Carlson, 1967, Sutan & Willinger, 2004) and constant marginal costs (Rassenti *et al* 2000; and Huck *et al* 2002)

2.2 Treatment T2: T1 with production capacity lasting four periods

Treatment T2 is equal to treatment T1 except that production capacity is introduced. Production capacity lasts for more than one period. This is the typical case in production sectors of the economy. Treatment T1 represents the special case of agricultural products that are planted in one season and harvested the next. The investments are measured in production units to simplify the task for the subjects. As in T1, it takes one period before new production capacity is in place. Capacity lasts four periods. Full capacity utilization is assumed at all times as is implicitly assumed in T1. Thus, production is equal to the sum of capacities of all four vintages,

$$q_{i,t} = \sum_{j=t-4}^{j=t-1} x_{i,j} \quad (3)$$

where $x_{i,j}$ is the investment decision made in years $j=t-4$ to $j=t-1$. Note that in equilibrium, yearly investments will be one fourth of the desired yearly production.

2.3 Treatment T3: T2 with a one period extra investment lag

This treatment is the same as T2 except for an extra one period investment lag. In many industries the investment lag stretches over several years. This means that there will be a period after an investment decision has been made in which the firm is producing with the existing capacity and in which the firm may make yet another investment decision. This is captured in treatment T3 by lagging capacity by one period such that production is given by,

$$q_{i,t} = \sum_{j=t-5}^{j=t-2} x_{i,j} \quad (4)$$

where $x_{i,j}$ is the investment decision made in years $j=t-5$ to $j=t-2$. Capacity lasts four periods as in T2.

2.4 Experimental Procedure

The experiment follows the standard framework used in experimental economics, with the same procedures in all treatments. Subjects were recruited from the same student population and during the same time period. The subjects were fourth and fifth year Management and Industrial Engineering students at the National University of Colombia, Medellín. Each treatment was run with 5 markets. No subject had previous experience in any related experiment and none of them participated in more than one session. Subjects were told that they could earn between Col \$ 15,000 and Col \$ 40,000 (\$6 –\$16 at that time) in about one hour and a half (circa 1.5 to 2.5 times a typical hourly wage for students). They knew that rewards were contingent on performance, which was measured in cumulative profits.

Upon arrival subjects were seated behind computers. Groups were formed in a random way. There were two or three markets per session, and subjects could not identify rivals in the market. Instructions (in Spanish) were distributed and read aloud by the experimenter. Instructions for treatment T3 can be found in Appendix 2 (In Spanish). Subjects were allowed to ask questions and test out the computer interface.

In all treatments, all parameters of the experiment were common knowledge to all subjects, including the symmetry across firms. The initial condition was a total industry production of 55 Units and individual productions of 11 Units. Thus, the price started out at 0.5 E\$/Unit. These initial values were identical across treatments. Each period the subjects received information about their own production, total production of the rest of the players, total production in the market, market price, marginal profits, profits, and a graph showing the vintages of their capacity. In addition, the software provided a profit calculator, where they could calculate their own profits for different assumptions about own and others' production. Interfaces with initial conditions are presented in Appendix 2 for each treatment.

The subjects were also asked to forecast the price for the next period, except in T3 where they were asked to forecast the price for the period after the next one. Extra reward was given for good forecasting, measured by the accumulated forecasting error. The rewards could vary from 0 for forecast errors above an upper limit to Col \$ 8,000 (around US\$3) for perfect forecasts.

The experiments were run in a computer network using the simulation software Powersim Constructor 2.51. The experimental market recorded all variables including the subjects' decisions. Subjects were also asked to write down their decisions on a sheet of paper to keep a memory of past decisions and to provide a backup of the experiment. The experiment's software is available upon request; the equations are shown in Appendix 2.

2.5 Testable Hypotheses

First, we formulate null hypotheses based on standard economics models with rational expectations. Thereafter, we present alternative hypotheses based on bounded rationality. In each case, we consider equilibrium and cyclicity.

2.5.1 Rational Expectations Hypothesis: Cournot Nash equilibrium

In all treatments there is a unique Cournot Nash equilibrium (CN). Table 1 shows the characteristics of the CN equilibrium, which are derived in Appendix 1. Note that in T2 and T3, investments are one fourth of the investment in T1, due to the introduction of vintages.

Hypothesis 1: Average prices are equal across treatments and equal to the Cournot Nash equilibrium

Table 1. Equilibria of the experimental markets

	Individual Investment [Units] T1 / T2 - T3	Total production [Units]	Price [\$/Unit]	Individual Payoff [\$]	Welfare lost [\$]
Joint maximization	5.00 / 1.25	25.00	3.50	12.50	31.25
Cournot Nash	8.33 / 2.08	41.67	1.83	6.94	3.47
Competition	10.00 / 2.50	50.00	1.00	0.00	0.00

Previous experiments have shown biases from the CN equilibrium towards competition (Huck, 2004; Huck, *et al* 2004). Table 1 also presents the equilibrium values for perfect competition and joint maximization to judge any particular bias of our results.

For this experiment, neoclassical economic theory suggests no cyclical behaviour but stability because market actors with perfect foresight will detect any cyclical tendency and prevent it by countercyclical investments. Accordingly, economic theory normally attributes cyclical behaviour to external shocks, notably in commodity markets (e.g. Cuddington & Urzua, 1989; Cuddington, 1992; Cashin *et al* 2002; Reinhart & Wickham, 1994; and Cashin & Patillo, 2000). Such random variations may occur for a number of reasons, such as discontinuous investments, learning, individual strategic moves, etc. Previously, experiments with Cournot markets have shown that outputs and prices are not exactly equal to the CN equilibrium but very close, typically closer than one standard deviation of the variation over time (Huck, 2004). We consider random shocks generated within a market to be consistent with standard economic theory.

Hypothesis 2. Market prices do not show cyclical tendencies in any of the three treatments while random variations may occur.

2.5.2 Bounded Rationality: Heuristics

The alternative hypotheses are based on bounded rationality theory. Individual investment decision can be seen as consisting of two steps. First, the subjects form expectations about future prices, and next they deliberate on the size of their investment. For instance, Nerlove (1958) models this by

assuming adaptive expectations and by using the inverted marginal cost curve to find the appropriate future supply (and implicit investment). Here we rely on the same assumption about adaptive expectations; however, we have to formulate an explicit investment function because we have assumed constant marginal costs (implying zero or infinite investments).

Proposed heuristic

The heuristic assumes that people are not able to follow the optimal strategy (rational behaviour), instead they adjust capacity towards a desired capacity. The capital is adjusted only through investments; and the investment function is based on the anchoring and adjustment heuristic (Tversky & Kahneman, 1994) and inspired by the application to a related problem (Sterman, 1989). That is, we assume they use a feedback strategy, where the desired capital is indicated by the expected price. The investment function is,

$$x_t = \text{MAX} (0, C_t/\tau + \alpha_C (C_t^* - C_t) + \alpha_{SC} (kC_t^* - SC_t)) \quad (5)$$

where the max-function precludes negative investments, total capacity C_t divided by the average life time of capacity τ denotes normal replacement investments (equal to depreciation), α_C determines how fast capacity is adjusted towards the desired capacity C_t^* , and α_{SC} determines how quickly the supply line is adjusted toward the desired supply line kC_t^* , where k is 1/4 since the investment delay is one fourth of the lifetime. The latter term is only applicable in T3. The desired capital

$$C_t^* = \text{MAX} (0, a + \left(\frac{q^e - a}{P^e} \right) P_t^*) \quad (6)$$

is a linear function of expected price P_t^* . When P_t^* equals the equilibrium price P^e , desired capacity C_t^* equals equilibrium production q^e . The parameter a determines the intercept with the y-axis and the slope. We choose a simple linear model for desired capacity, even though many functions could lead to the same equilibrium, as Stoft (2002) argues for electricity markets. The parameter a is restricted to $a < q^e$ to avoid negative slopes. Also note that C_t^* depends on P_t^* relative to P^e and not relative to the marginal cost c . Hence, the formulation could be used to test different equilibrium assumptions. Here we assume CN equilibrium. Finally, the expected price is given by

$$P_t^* = \beta P_{t-1} + (1-\beta)P_{t-1}^* \quad (7)$$

which represents adaptive expectations, formulated initially by Nerlove (1958) and previously used in other related economic experiments (e. g. Carlson, 1967; Sterman, 1987b and 1989; Frankel and Froot, 1987). The parameter β is referred to as the coefficient of expectations. Following, we provide a simulation analysis of the proposed heuristic.

Differences between treatments?

We simulate the proposed heuristic in each of the three treatments. The initial conditions are similar to those used in the experimental design. The coefficient of expectations is set equal to the average of the values estimated by Sterman (1989) and Carlson (1967), i.e., $\beta=0.53$. The factors for the adjustment of the supply line and the total capacity are taken from Sterman (1989), who estimated parameters in an analogous heuristic with data from an inventory management problem. Average values are $\alpha_{SC}=0.10$ and $\alpha_C=0.26$. We set the parameter a equal to 15. This value produces a standard deviation of the simulated price (with no noise term) similar to the one observed in the experiment (T3). Figure 1 presents the resulting price behaviour. The simulations show a fast convergence to the CN equilibrium for treatment T1, a quite fast convergence for T2, and only mildly damped oscillations for T3.

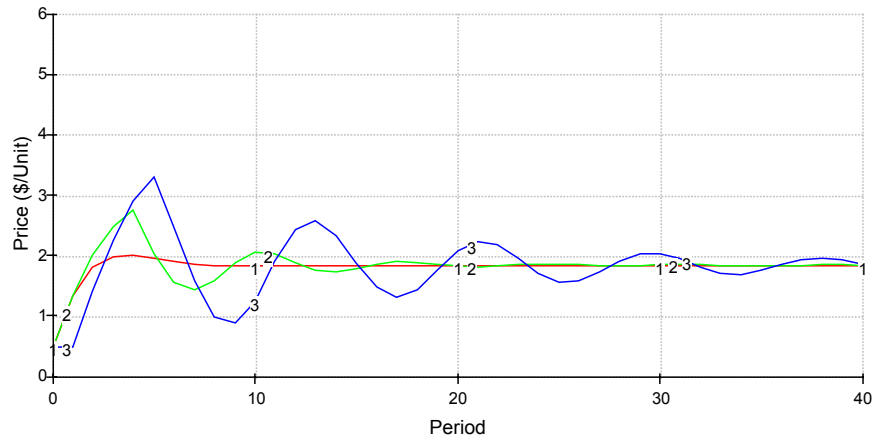


Figure 1. Simulated prices in experimental dollars. The numbers denote treatments.

The effects of random disturbances are presented in Figure 2. We add normally distributed noise u_t (iid) to investments, $u_t \sim N(0, S^2)$. S is set such ($S=2.5$) that the standard deviation of the simulations becomes similar to the average standard deviation estimated from the experimental results of T3 and the standard error of the regressions of the heuristic⁶ becomes the same as in regressions on the experimental results of T3. All treatments are simulated with the same seed and the disturbances serve to keep the dampened cycles alive, Frisch (1933). Still the tendency towards cyclical behaviour seems stronger in T3 than T1.

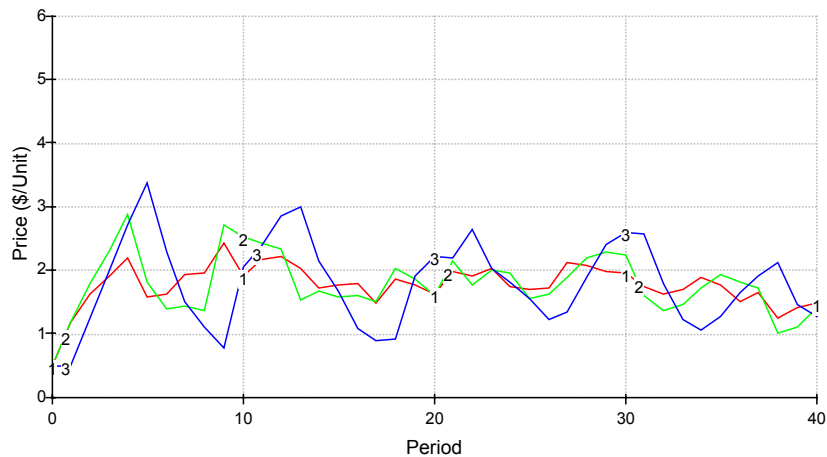


Figure 2. Simulated prices with noise. Numbers denote treatment.

Sensitivity Analysis

Sensitivity analysis of the parameters α_C , α_{SC} , (only for T3) and β is presented in Figure 3 and Figure 4 for treatments T2 and T3 respectively. T1 does not show considerable sensitivity to parameter variations and it is not considered here. We measure the stability of the price series with the standard deviation⁷. Figure 3 (T2) shows that for low values of α_C (lower than 0.4) the standard deviation is low and largely independent of the value of β . The standard deviation increases with α_C except when β is close to its lower limit of 0. Note that the estimative of the standard deviation is discontinuities in the most unstable areas; however, parameters in those ranges are not expected in the experiment. The

⁶ Regressions of the proposed heuristic were performed under different noise conditions and multiple simulations, as presented below in this section.

⁷ Eigenvalue analysis is an alternative methodology which can provide insights about the stability properties of the treatments. It was not considered in this paper, since the simulations provided enough information in this regard.

effects of a on the standard deviation are equal, and values move in the opposite direction than the effects of the parameter α_C .

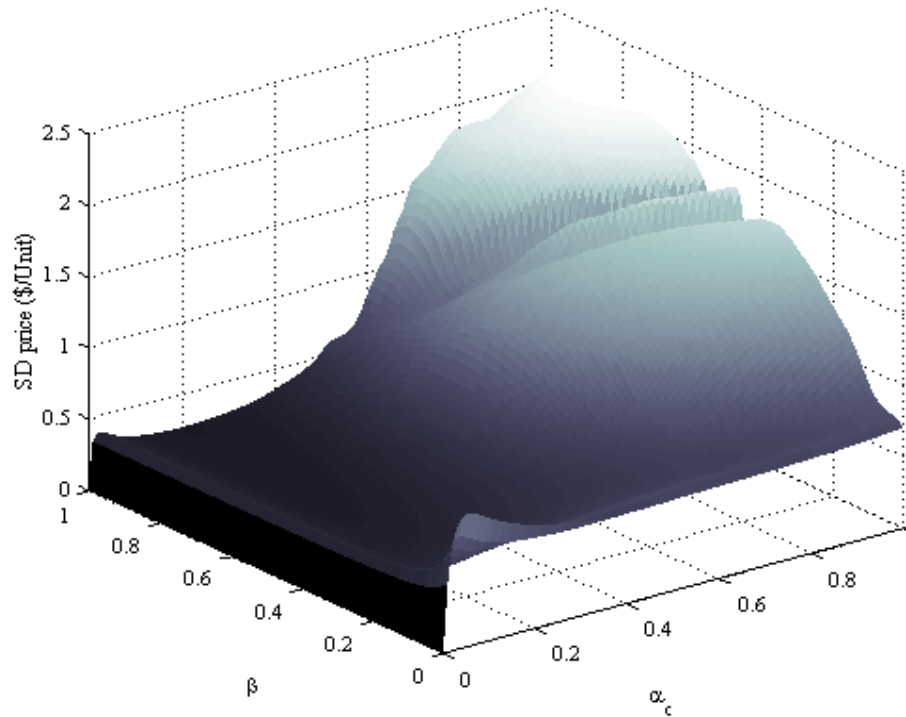


Figure 3. Sensitivity of the standard deviation of price to parameter assumptions, T2.

Figure 4 shows that α_C is an important parameter also in T3. For high values of β , α_{SC} is of little importance. When $\beta=0.0$, α_{SC} matters; low values of α_{SC} combined with high values of α_C give a high standard deviation. The effect of the parameter a is approximately the same as in T2. In conclusion, T1 is not sensitivity to parameter variations, T3 presents tendencies to instabilities when α_C is higher than 0.6; while T4 becomes unstable with different parameter combinations.

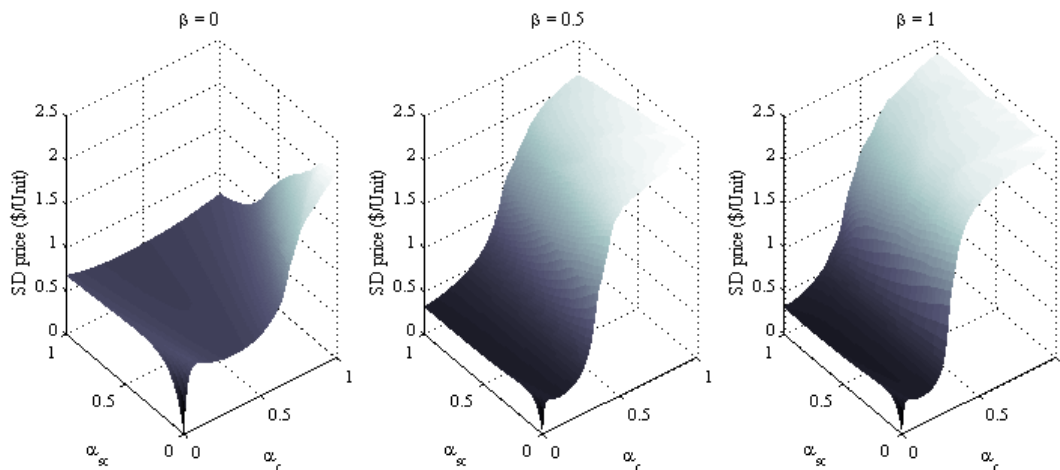


Figure 4. Sensitivity of the standard deviation of price to parameter assumptions, T3.

Introducing a rational agent in T3

The use of the proposed heuristic leads to cyclical behaviour under certain conditions, such as the base case simulation. According to neoclassical economic theory, a rational agent should be able to prevent the occurrence of cycles by countercyclical investments. We test this claim by simulation, assuming that one of the five players behaves rationally within the rules of the experiment while the other four

behave according to the investment function discussed above. According to the rules, the rational agent does not receive information about the current investments of the other players. However, we assume that the rational agent forms perfect expectations about the investment rule used by the other four. The rational agent estimates the expected capacity of the rest of the players at time t , EC_t^R ; and optimizes his/her profits. According to the demand function of eq. (1) and the constant marginal costs, the profits of the rational agent at time t are:

$$\pi_t^{RA} = (6 - 0.1(EC_t^R + q_t^{RA}) - c)q_t^{RA} \quad (8)$$

where q_t^{RA} is the production of the rational agent at time t . For the profit maximization, the first order condition is

$$\frac{d\pi_t^{RA}}{dq_t^{RA}} = 0 \quad \therefore \quad q_t^{RA} = 25 - 0.5 EC_t^R \quad (9)$$

Thus, the rational agent has to adjust his/her capacity to q_t^{RA} through investments. Note that this is consistent with the profit calculator, where subjects assume EC_t^R and find the production that maximizes their profits. The rational actor estimates EC_t^R as,

$$EC_t^R = C_t^R - (L_t^R + L_{t+1}^R) + (ESL_t^R + ET_t^R) \quad (10)$$

where C_t^R is the capacity of the rest at time t , L_t^R is depreciation, ESL_t^R is the expected supply line, and ET_t^R is the expected investment (note that $ESL_{t+1}^R = ET_t^R$). ET_t^R is based on the available information of the market as in the experiments and eq. (5) to (7). Finally, the investment for the rational agent is

$$x_t^{RA} = \text{Max}(q_t^{RA} - (C_t^{RA} + SL_t^{RA} - L_t^{RA} - L_{t+1}^{RA}), 0) \quad (11)$$

where C_t^{RA} denotes the capacity of the rational agent at time t , SL_t^{RA} his/her supply line, and L_t^{RA} his/her depreciation.

A simulation of the market behaviour with the rational agent is presented in Figure 5 together with the base case for T3 from Figure 1. Since there is no noise in this simulation, EC_t^R equals the capacity of the rest of the players C_t^R , hence, the rational agent has perfect knowledge about all conditions in the market. We observe that the actor manages to rapidly stabilize the market.

Table 2 compares the cumulative profits of the rational agent and the average of the rest of the players. The table shows that profits of the rational agent increase considerably with respect to the base scenario, and the average of the rest players is almost at the same level after the inclusion of the rational agent. Thus, there is potential for stabilization if one agent is rational in the sense discussed above.

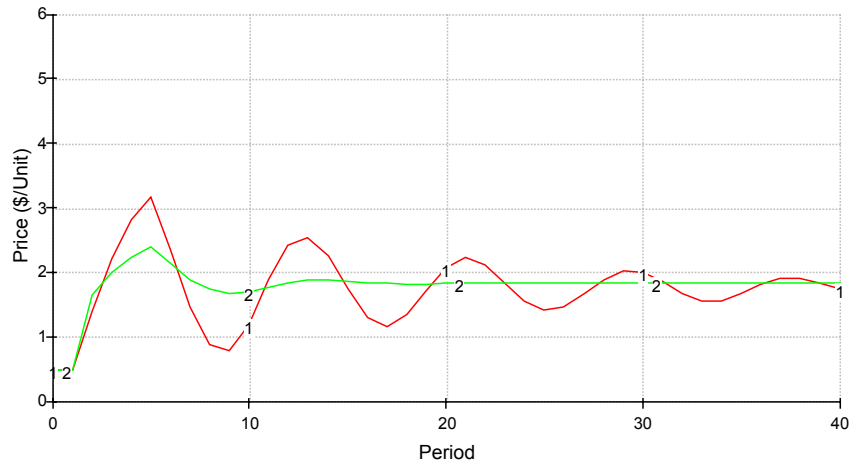


Figure 5. Simulated prices in T3 without (line 1) and with a rational agent (line 2).

Will the rational agent be able to stabilize the market when noise is present? We add the same normally distributed noise u_t (*iid*) to the investments of the rest of the players as before. Figure 6 compares the base scenario to the case with a rational agent, both with the same noise sequence. We observe again that noise does not radically change the dominant behaviour modes from the case without noise. Table 2 shows a reduction of the standard deviation of price from 0.83 \$/Unit in the base scenario to 0.46 \$/Unit for the case with the rational agent (noise unknown).

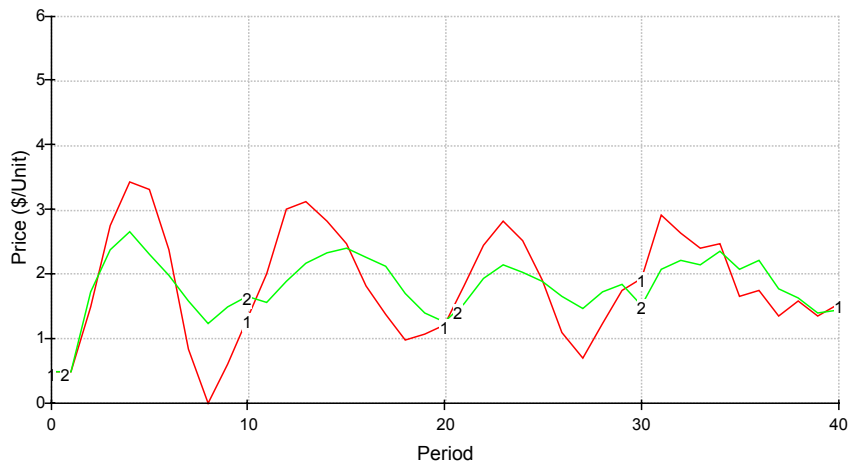


Figure 6. Simulated prices without (line 1) and with rational agent (line 2), with noise known by the rational agent.

Above we assume that the rational agent has no knowledge about the noise introduced in the decision rules of others' heuristic, consistent with the experimental design. Given that actors in real markets may have partial information about the investment plans of competitors, we consider the extreme case where the rational agent has perfect knowledge of the noise in others' decisions. We compare simulated prices assuming unknown and known noise in Figure 7. Perhaps surprisingly, knowledge about the noise term has a negligible effect on price behaviour. In this case, the rational actor has perfect knowledge of the capacity at different vintages and the investment function; hence, there is not uncertainty. Adding up all the certainties, the unknown noise is low compared with the total capacity and the investments without noise.

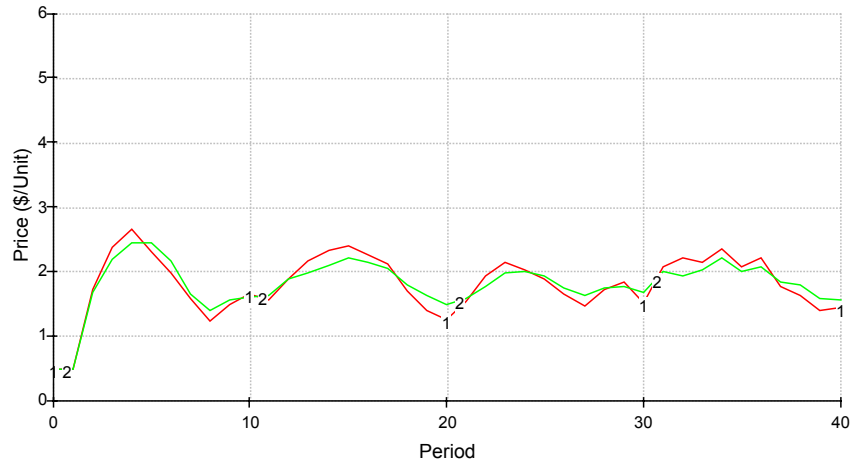


Figure 7. Simulated prices for the inclusion of the rational agent with noise unknown (line 1) and known (line 2) by the rational agent.

Table 2. Performance in the base case and in the case with a rational agent.

Simulation	S_{price} (\$/Unit)	Cumulative profits (average over the rest of agents) (\$)	Cumulative profits (rational agent) (\$)	Cumulative profits (all agents) (\$)
Base scenario, no noise	0.58	255	255	1277
Base scenario, noise*	0.83	227	227	1137
Rational agent, no noise	0.38	256	289	1315
Rational agent, noise unknown*	0.46	251	296	1299
Rational agent, noise known*	0.42	249	303	1300

* Average values of 10 simulations.

To summarize, we present the formal hypothesis for the case of bounded rationality:

Hypothesis 3: *Cycles will not occur in treatments T1 and hardly in T2, only random variations, while T3 will show cycles.*

To test the cyclicity of hypothesis 3, we will consider both aggregated market behaviour and estimated investment functions. The aggregated behaviour is analyzed with spectral analysis and autocorrelation. These methods are discussed below.

Methodologies to test hypotheses about cyclical behaviour

Regular cycles are characterized by their frequency or periodicity, amplitudes and attenuation. However, these measures are not easy to obtain for irregular cycles and consequently analysis by visual inspections might be misleading. Variance measures the dispersion of any data around their average. Nevertheless, this measure does not say anything about frequencies or autocorrelation. To capture these aspects, we turn to spectral analysis and autocorrelation to test for (random and) cyclical behaviour.

Spectral analysis: The frequency decomposition of variance is called the autospectrum or the autospectral density function. Peaks in the autospectrum indicate that the variance is concentrated at certain frequencies⁸. This allows detecting periodicities and estimating length of cycles. For instance, white noise has a uniform autospectrum; a sine wave has an autospectrum concentrated at a single frequency (the period). When both processes are combined, the resulting autospectrum is the sum of the individual spectra.

⁸ Spectral analysis decomposes the time series in orthogonal components. Each component is associated with a particular frequency. The auto spectrum shows the contribution of each band to the total variance (details in Bendat & Piersol, 1980; Box *et al* 1994).

Correlation analysis - autocorrelogram: Based on covariance, the autocorrelogram indicates cyclical behaviour and indicates both amplitude and periodicity. The autocorrelogram measures the correlation of the variable with itself, at different time lags. The autocorrelogram is most directly interpreted as a measure of how well future values can be predicted based on past observations. While random processes have autocorrelation functions rapidly diminishing to zero, cyclicity is observed when there are values significantly different from zero at different time lags.

The insights that these two methodologies provide are illustrated with estimates of the autospectra and autocorrelograms for simulated price series with and without noise across treatments (see Figure 8). The autospectra were estimated with the last 32 data points of the 40 period simulations, since it is recommended to use a power of two for the sample size (Bendat & Piersol, 1980; Box *et al* 1994). T1 and T2 show flat autospectra while T3 shows a peak at a frequency of 0.12, i.e. for a period of 8.3 time steps. The autocorrelograms do not distinguish as clearly between the treatments. However the same tendency is seen, with more significant correlations in T3 than in T2 than in T1. As expected the revealed tendency towards cycles diminishes with the noise: correlations become less significant and the spread around the peak in the autospectrum in T3 increases. Importantly, the test shows that even with a limited sample size the two methods give clear indications of cyclicity.

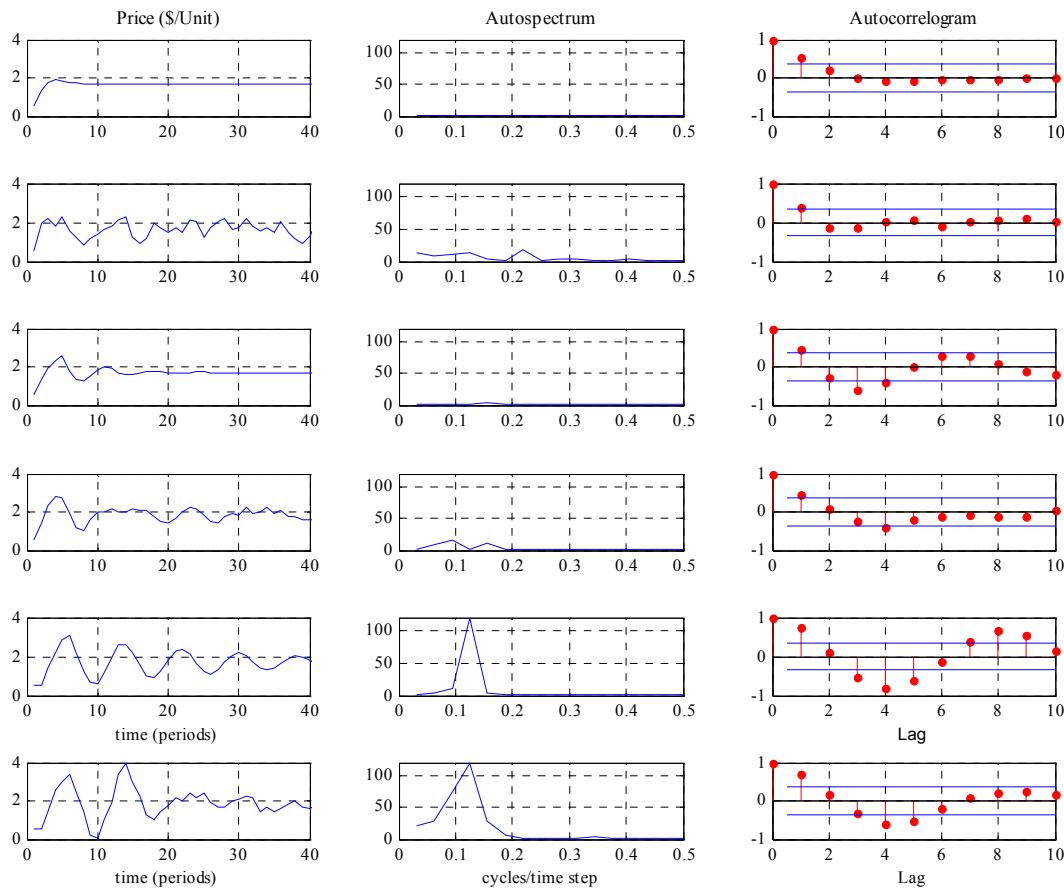


Figure 8. Simulated price behaviours with the corresponding autospectra and autocorrelograms presented in couples with no noise on top and with noise at the bottom in each couple. T1 are the top graphs; T2, the middle ones; and T3 the button graphs.

Tests of the structure that lead to cyclical behaviour

When the outcome of the experiment is available, the proposed heuristic can be tested directly by regressions. Here we regress on time-series data from the simulations to explore the potential of this test. We choose the following linear investment function for the aggregate market.

$$x_t = m_3 P_t^* + m_2 P_t + m_1 SC_t + b + \varepsilon_t \tag{12}$$

where m_i ($i=1,2,3$) and b are parameters to be estimated, and ε_t is *iid* random variable with zero mean and finite variance. This investment function is exactly the same as the proposed heuristic as long as the price falls inside the linear range, i.e., $0 < P_t < 6$ for all $t=1,2,\dots,40$. We perform regressions for simulations with and without noise (see Table 3). Since the price is always in the range $0 < P_t < 6$, the regression of *eq. (12)* for simulations without noise are perfect.

When different amounts of noise are introduced, the average parameters of regressions of *eq (12)* for multiple stochastic simulations are still quite similar to those of the regression without noise. Most of the variation that is seen is due to the randomness (only 10 replications to calculate average coefficients). We have not made an effort to see if there are biases with increasing noise levels. As expected the quality of the regression is reduced due to the introduction of noise with average values of r^2 declining from 1.0 to around 0.3. The coefficients in Table 3 will serve as references for comparisons with the experimental results.

Table 3. Parameter estimation for simulations of T3 with *eq. (12)*. The p-value of each coefficient is presented in parenthesis.

	Eq (12)					S_{price}
	m_3	m_2	m_1	b	r^2	
T1						
No noise	3,67	-7,40		48,30	1,00	0,09
N(0,1.5²)*	5,73	-8,13		45,84	0,33	0,19
N(0,2.5²)*	5,40	-7,51		45,35	0,27	0,30
N(0,3.5²)*	6,08	-8,57		46,33	0,31	0,40
T2						
No noise	3,67	0,10		3,30	1,00	0,23
N(0,1.5²)*	2,21	0,96		4,32	0,24	0,31
N(0,2.5²)*	3,85	0,31		2,77	0,29	0,51
N(0,3.5²)*	3,77	0,33		2,49	0,26	0,62
T3						
No noise	4,02	0,10	-0,10	3,67	1,00	0,51
N(0,1.5²)*	4,57	-0,02	-0,14	3,27	0,60	0,62
N(0,2.5²)*	4,66	-0,50	-0,15	4,02	0,43	0,74
N(0,3.5²)*	3,13	0,74	-0,12	4,04	0,32	0,80

* Parameters are average of estimation of 10 stochastic simulations.

3 RESULTS

Summary statistics for observed prices⁹ are presented in Table 4. In all treatments, the average prices over five markets fall between the competitive (1 \$/Unit) and the CN (1.8 \$/Unit) equilibria. The standard deviations of average values $S_{\bar{x}}$ in treatments T2 and T3 is considerable larger than T1. The averages of standard deviations over groups S have the same magnitude for treatments T1 and T2, but it is higher for T3. This is a first indication of more unstable behaviour in T3. The average autocorrelation over groups α is small in T1 and T2 and considerably higher in T3. This is a first indication of cyclical price behaviour.

Table 4 also shows the same data split on the first 20 periods and on the remaining ones. Average prices do not change much for treatments T1 and T2, while the average price increases over time for T3. Standard deviations tend to fall over time for all treatments. The differences between T3 and the other two treatments with respect to standard deviation and autocorrelation are maintained over time, although somewhat weakened.

⁹ The analysis for prices and quantities are similar. We have focused on prices since it is the usual dealing with commodity market fluctuations.

Table 4. Average statistics over groups*

	Total data					First 20 periods				Remaining periods				
	Inf	\bar{X}	Sup	$S_{\bar{X}}$	\bar{S}	α	\bar{X}	$S_{\bar{X}}$	\bar{S}	α	\bar{X}	$S_{\bar{X}}$	\bar{S}	α
Treatment 1														
G1	1.53	1.78	2.03		0,66	0,27	1,88		0,74	0,24	1,70		0,28	0,41
G2	1.39	1.52	1.66		0,36	0,13	1,63		0,36	-0,08	1,41		0,07	0,39
G3	1.25	1.44	1.64		0,58	0,05	1,54		0,60	0,14	1,37		0,53	-0,19
G4	1.41	1.58	1.76		0,55	0,05	1,51		0,57	-0,11	1,71		0,49	0,20
G5	1.33	1.51	1.69		0,56	-0,21	1,65		0,66	-0,38	1,42		0,37	0,00
Avg T1**	1.41	1.57	1.73	0.13	0,54	0,06	1,64	0.15	0,59	-0,04	1,52	0.17	0,35	0,16
Treatment 2														
G1	0.64	0.84	1.03		0,60	0,17	0,75		0,54	0,53	0,94		0,66	-0,12
G2	1.09	1.21	1.33		0,37	0,24	1,29		0,42	0,29	1,17		0,27	0,09
G3	1.20	1.32	1.45		0,39	0,05	1,29		0,48	0,01	1,39		0,22	0,15
G4	1.84	2.02	2.20		0,56	0,11	2,01		0,65	0,10	2,10		0,31	0,12
G5	0.64	0.84	1.03		0,44	0,34	1,40		0,38	0,39	1,44		0,46	0,32
Avg T2**	0.82	1.36	1.89	0.48	0,47	0,18	1,35	0.52	0,50	0,26	1,41	0.48	0,39	0,11
Treatment 3														
G1	1.10	1.38	1.65		0,86	0,88	0,84		0,78	0,78	1,95		0,50	0,80
G2	0.73	0.96	1.19		0,71	0,71	0,88		0,77	0,76	1,06		0,66	0,62
G3	1.33	1.57	1.80		0,72	0,71	1,31		0,90	0,70	1,87		0,27	0,26
G4	0.75	0.93	1.12		0,58	0,70	0,55		0,42	0,31	1,34		0,45	0,59
G5	2.04	2.26	2.48		0,69	0,63	2,35		0,83	0,78	2,26		0,38	0,17
Avg T3**	0.74	1.42	2.10	0.54	0,71	0,73	1,19	0.71	0,74	0,67	1,70	0.49	0,45	0,49

* \bar{X} : average price over time and groups; $S_{\bar{X}}$: standard deviation of average prices across groups; \bar{S} : average of standard deviations for individual groups; $\bar{\alpha}$: average of autocorrelations for individual groups.

** Confidence interval for the averages of \bar{X} (these is not averages of the limits).

Then we look in more detail at the price development over time. Prices for all markets are presented in Figure 9. The prices vary from zero to price levels close to the joint maximization value (3.5 \$/unit). Thus, except for a few incidents with a price of zero, prices are in the linear range of the demand function. All the experiments begin with over-capacity and a price of 0.5 \$/unit. Thereafter, a simple visual inspection suggests tendencies towards irregular cycles. As a first impression, the fluctuations may seem larger, deeper and perhaps smoother in T3 than in the two first treatments. Particularly in the first 20 periods there are some quite dramatic fluctuations. To explore these tendencies further we consider autospectra and autocorrelograms.

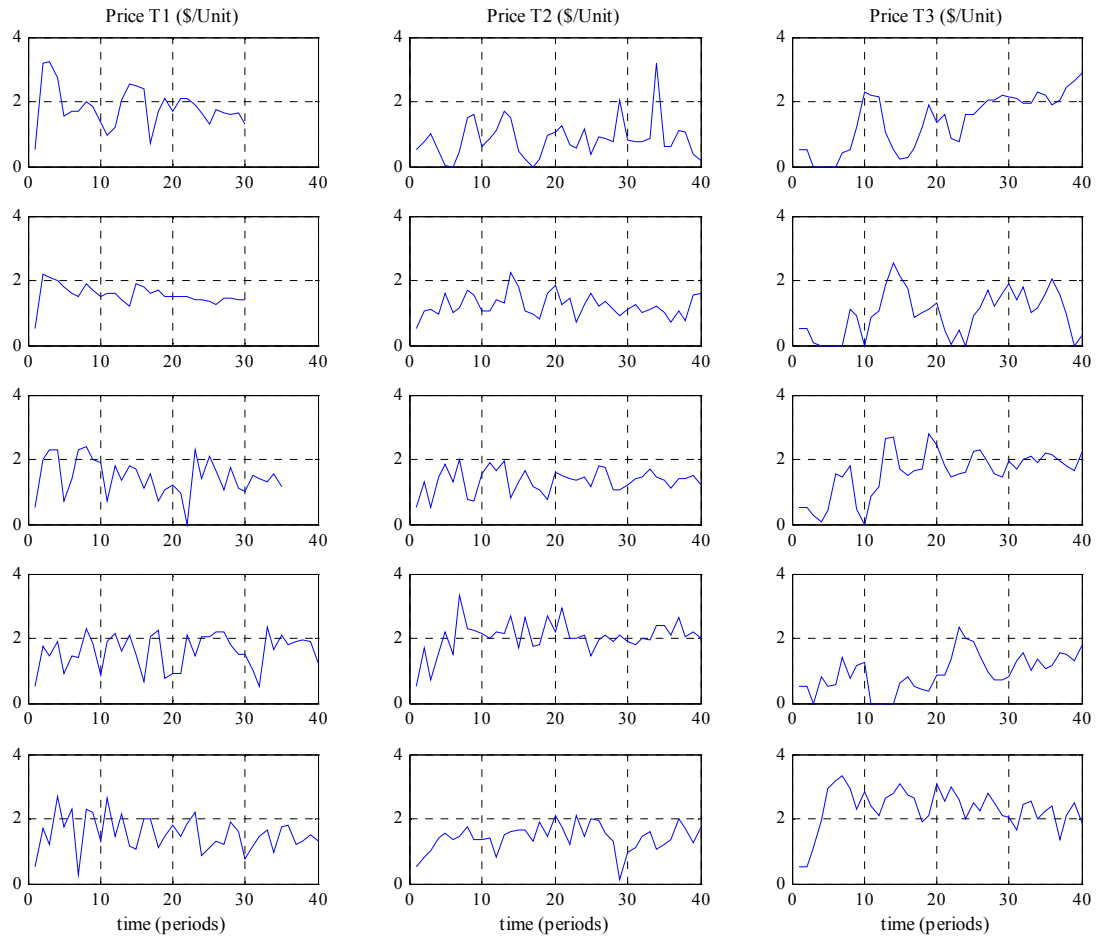


Figure 9. Realized prices in all experimental markets.

Figure 10 shows average autospectra and autocorrelograms¹⁰ over the five independent markets in each treatment (autospectra and autocorrelograms for individual markets are presented in Appendix 3). The shape of the autospectra¹¹ for T1 and T2 are typical of random series; T1 similar to pink noise and T2 similar to white noise. T3 is could also be consistent with pink noise; however, T3 has two minor significant peaks, one at around 0.1 cycles/-5 yr period- (≈ 20 period cycle) and another at around 0.18 cycles/-5 yr period- (≈ 5.5 period cycle). Since T3 is not much different from T1, the autospectra do not suggest a significant difference between the two treatments. Then we turn to the autocorrelograms. They show no significant autocorrelation for any lag for T1 and T2; and significant autocorrelation in T3 for the first and the second lag. Thus, the prices observed in T1 and T2 have features of random series; while prices in T3 have features of cyclical series. Confer Figure 8.

¹⁰ The mathematical properties of autospectra and autocorrelograms allow estimations of confidence intervals for independent series, which increase reliability to the outcomes of the methodologies (Bendat & Piersol, 1980).

¹¹ We have estimated the autospectrum from the last 32 data points only, since the Fourier transform works better with series where the number of observations can be expressed as a power of two (Bendat & Piersol, 1980). By removing the first data points we reduce the effect of the initial disequilibrium.

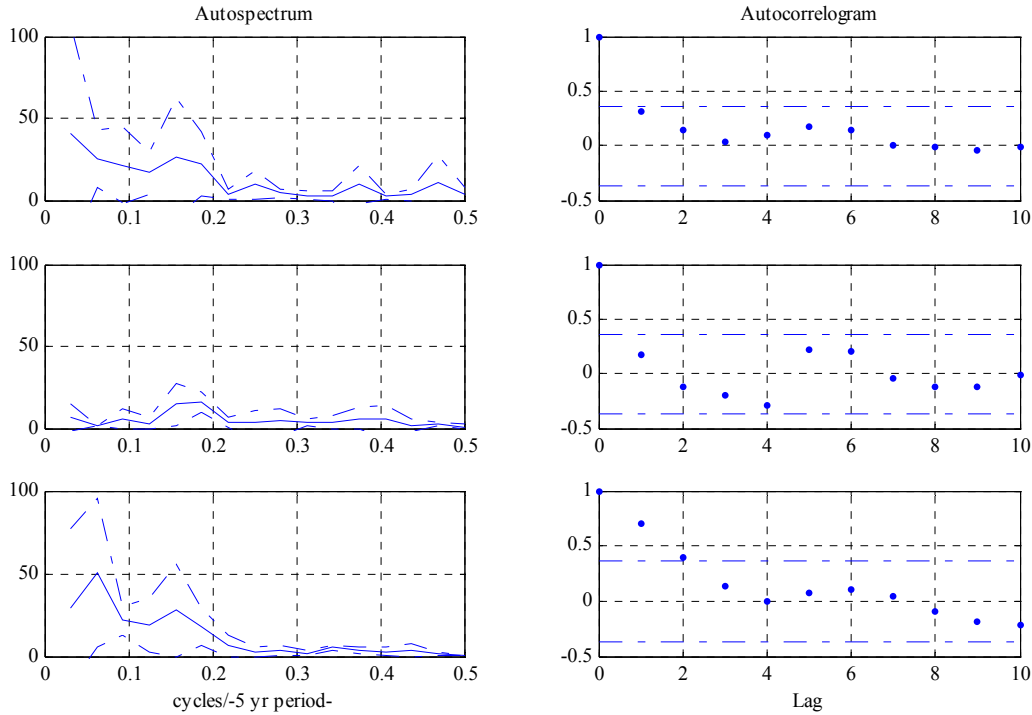


Figure 10. Average autospectra with 95% confidence intervals and average autocorrelograms with the critical values at the 5-percent level (horizontal lines), for T1 (top), T2 (medium), and T3 (bottom).

4 TESTING THE HYPOTHESIS AND DISCUSSION

Following, we perform the formal tests of the hypotheses presented in section 2.

Hypothesis 1: *Average prices are equal across treatments and equal to Cournot Nash equilibrium predictions.*

Table 4 also shows confidence intervals for average prices.¹² When considering averages over all groups, hypothesis 1 is rejected for T1; the average price is below 1.83 \$/unit. It is not rejected for T2 and T3. Since all averages are below the CN equilibrium, we also test against the competitive equilibrium, CE (1.00 \$/unit). The average price for T1 is significantly above, while average prices in T2 and T3 are not significantly different from the CE.

For individual groups, we observe that the confidence intervals for the average prices do not include the CN equilibrium in four out of five groups; all of those four either include or present biases toward the competitive equilibrium for all treatments. Note that there is one group in T2 and one in T3 with an upward bias towards the joint maximization equilibrium. In T2 and T3 there are two groups in each treatment where the CE falls into the confidence interval for average prices.

The bias towards competition observed in treatment T1 is consistent with previous results of Cournot markets under standard conditions and with more than 4 players (see summary in Huck, 2004; and Huck *et al* 2004). Realized prices tend to be between CE and CN equilibrium predictions.

Now, we turn to test cyclicity. We test hypothesis 2 and 3 simultaneously.

¹² For practical reasons we perform the hypotheses tests with confidence bands around the observed value. This gives the same test results as when constructing critical values around the hypothesised values. Our approach makes it easier to test against several hypothesised values (CE, CN, and JM).

Hypothesis 2. Market prices do not show cyclical tendencies in any of the three treatments while random variations may occur.

Hypothesis 3: Cycles will not occur in treatments T1 and hardly in T2, only random variations, while T3 will show cycles.

The main difference between hypothesis 2 and 3 pertains to treatment 3. If we observe cyclicity in treatment 3, that favours hypothesis 3. No cyclicity favours hypothesis 2. Since hypothesis 3 is vague about the outcome of treatment 2, lack of cyclicity in treatment 2 will not lead to a rejection of hypothesis 3 by itself. We consider differences in variance, autospectra and autocorrelograms, and we investigate estimated investment heuristics.

Before we consider the data, note that the following tests cannot distinguish clearly between two different explanations of cyclicity, namely inappropriate heuristics and rational reactions to internally generated randomness. We do observe seemingly random variation in investments in all treatments, as has been observed in previous experiments of standard conditions Cournot markets (Huck, 2004). With random variations, there is a greater challenge to equilibrate the market in T3 than in T1 and T2. This is because of the investment delay. Figure 5 shows that our version of a rational agent needs some time to stabilise the market. This means that the fluctuations in T3 could result both from (rational) reactions to (not so rational) random variations and from heuristics that are not efficient in stabilising the market. Figure 5 suggests that the former effect has a certain potential. This potential is further reduced if we assume that all five players and not just one of the players use the suggested and more rational rule. Hence, if there is a difference between fluctuations in T3 on the one hand and in T1 and T2 on the other hand, this points in the direction of inappropriate heuristics.

Difference in variance across treatments

Table 6 shows the tests of differences between average standard deviations for prices across treatments. While the average standard deviation for T1 is not significantly different from the one in T2, there are significant differences between the average standard deviations in T1 and T2 and the average standard deviations in T3. The only reason for the difference between T2 and T3 is the supply lag we introduce in T3.

Table 5. Tests of the difference between average standard deviations for price.

	$A=X_{S(\text{price})}$	S_A	H_0	$T_{0,05}$	t_{critic}
T1	0,543	0,110	$A_{T1} = A_{T2}$	1,075	2,306
T2	0,471	0,103	$A_{T1} = A_{T3}$	2,570	2,306
T3	0,713	0,098	$A_{T2} = A_{T3}$	3,804	2,306

$X_{S(\text{price})}$: Mean (X) of the Standard deviation (S) of prices.

H_0 : Null hypothesis

$T_{0,05}$: Statistic t with 5% level of confidence

Autospectrum and autocorrelogram

The average autospectra in Figure 10 do not show significant differences between the treatments. T3 is nearly identical to T1. We note that we do not observe the same large peak in the spectrum as was seen in the spectrum produced from simulated data in Figure 8. The test, which is not very conclusive, does not rule out cyclicity; however, it indicates that our experiment does not produce as regular cycles as the simulations do. The autocorrelogram is significantly different from zero in the first two lags for T3. The correlation in T3 is positive and in line with the results in Figure 8. This suggests that although the fluctuations are not very regular, T3 leads to less random behaviour than T1 and T2. T1 and T2 do not produce significant correlations, and are therefore also consistent with Figure 8.

Test of the adaptive expectation hypothesis

The adaptive expectations hypothesis presented in eq. (7), is a linear equation restricted to pass through the origin of the 2D space $(P_{t-1} - P_{t-1}^*, P_t^* - P_{t-1}^*)$. We have relaxed this constraint by postulating a linear function of the form

$$(P^*_t - P^*_{t-1}) = \alpha + \beta(P_{t-1} - P^*_{t-1}) + \varepsilon_t \quad (13)$$

where ε_t is *iid* random variable with zero mean and finite variance. The term α can be interpreted as a bias parameter; and the subject that uses adaptive expectations could retain either an optimistic or a pessimistic bias. The results of estimating α and β are presented in Table 6, for individuals and aggregated markets for all three treatments. We have defined the expected price for aggregated markets to be the average of the expected prices reported by the individuals. The table also includes the arithmetic mean of the coefficients α and β , and the r^2 for all the regressions.

Table 6. Parameter estimation for the adaptive expectations hypothesis for individuals and aggregated markets, corresponding to eq.(13).

Mkt/Player	α (p-value)	β (p-value)	r^2
Individuals - Treatment T1			
1/1	-0.07 (0.39)	0.31 (0.03)	0.18
1/2	0.04 (0.88)	0.28 (0.01)	0.25
1/3	-0.04 (0.66)	0.36 (0.26)	0.05
1/4	0.02 (0.89)	0.54 (0.08)	0.11
1/5	-0.09 (0.43)	0.44 (0.01)	0.24
2/1	0.06 (0.34)	0.84 (0.00)	0.31
2/2	-0.02 (0.48)	-0.04 (0.86)	0.00
2/3	-0.11 (0.04)	1.14 (0.00)	0.50
2/4	0.01 (0.86)	0.19 (0.31)	0.04
2/5	-0.03 (0.63)	0.53 (0.00)	0.29
3/1	0.01 (0.87)	0.17 (0.02)	0.17
3/2	0.00 (0.98)	0.17 (0.26)	0.04
3/3	0.15 (0.19)	0.28 (0.02)	0.16
3/4	0.03 (0.64)	0.15 (0.09)	0.09
3/5	0.48 (0.23)	0.96 (0.00)	0.45
4/1	-0.01 (0.91)	0.03 (0.87)	0.00
4/2	0.08 (0.48)	0.64 (0.00)	0.34
4/3	0.00 (0.98)	0.02 (0.38)	0.02
4/4	0.16 (0.24)	0.41 (0.00)	0.21
4/5	-0.21 (0.01)	0.55 (0.00)	0.48
5/1	0.01 (0.86)	0.36 (0.00)	0.41
5/2	0.08 (0.37)	0.41 (0.00)	0.21
5/3	0.02 (0.81)	-0.03 (0.83)	0.00
5/4	-0.02 (0.77)	0.23 (0.06)	0.10
5/5	0.08 (0.31)	0.42 (0.00)	0.28
Avg	0.03	0.38	
Aggregated Markets T1			
1	0.01 (0.83)	0.40 (0.00)	0.46
2	0.02 (0.53)	0.76 (0.00)	0.59
3	0.13 (0.17)	0.48 (0.00)	0.35
4	-0.01 (0.84)	0.17 (0.12)	0.07
5	0.00 (0.95)	0.02 (0.80)	0.00
Avg	0.03	0.37	
α (p-value)	β (p-value)	r^2	
Individuals - Treatment T2			
0.89 (0.01)	0.56 (0.00)	0.24	
0.04 (0.68)	0.41 (0.00)	0.31	
0.06 (0.66)	0.29 (0.16)	0.05	
-0.06 (0.35)	0.23 (0.05)	0.10	
0.00 (0.97)	0.05 (0.68)	0.00	
0.07 (0.30)	0.11 (0.23)	0.04	
0.23 (0.16)	0.14 (0.09)	0.08	
0.03 (0.38)	0.06 (0.17)	0.05	
0.10 (0.23)	0.35 (0.00)	0.30	
0.05 (0.26)	0.10 (0.12)	0.06	
0.02 (0.75)	0.22 (0.23)	0.04	
-0.01 (0.85)	0.12 (0.30)	0.03	
0.00 (0.92)	0.05 (0.53)	0.01	
0.08 (0.21)	0.42 (0.01)	0.18	
0.10 (0.33)	0.39 (0.14)	0.06	
0.14 (0.19)	0.57 (0.03)	0.13	
0.05 (0.26)	-0.15 (0.18)	0.05	
-0.07 (0.49)	0.39 (0.08)	0.08	
0.01 (0.87)	0.04 (0.77)	0.00	
-0.01 (0.80)	0.21 (0.10)	0.07	
-0.01 (0.85)	-0.03 (0.76)	0.00	
-0.03 (0.41)	-0.13 (0.17)	0.05	
0.00 (0.89)	0.03 (0.74)	0.00	
0.40 (0.03)	0.43 (0.02)	0.15	
0.01 (0.87)	0.00 (0.98)	0.00	
0.08	0.20		
α (p-value)	β (p-value)	r^2	
Individuals - Treatment T3			
-0.01 (0.87)	0.53 (0.00)	0.31	
0.15 (0.08)	0.52 (0.00)	0.33	
0.34 (0.00)	1.32 (0.00)	0.57	
0.11 (0.30)	0.68 (0.00)	0.55	
0.00 (0.90)	0.03 (0.48)	0.01	
0.02 (0.79)	0.47 (0.00)	0.22	
0.10 (0.44)	0.34 (0.01)	0.19	
0.27 (0.12)	0.46 (0.04)	0.12	
0.12 (0.22)	0.21 (0.01)	0.16	
0.21 (0.11)	0.50 (0.00)	0.22	
0.12 (0.23)	0.79 (0.01)	0.17	
0.02 (0.79)	0.16 (0.30)	0.03	
1.01 (0.00)	1.01 (0.00)	0.50	
-0.10 (0.16)	0.42 (0.00)	0.47	
0.14 (0.24)	0.53 (0.04)	0.11	
0.03 (0.67)	0.63 (0.00)	0.53	
0.19 (0.16)	0.57 (0.00)	0.26	
0.08 (0.15)	0.18 (0.05)	0.10	
0.45 (0.17)	0.19 (0.08)	0.08	
1.22 (0.00)	0.39 (0.00)	0.27	
0.10 (0.33)	0.54 (0.20)	0.04	
0.02 (0.80)	0.10 (0.32)	0.03	
0.17 (0.05)	0.53 (0.03)	0.12	
0.03 (0.74)	0.44 (0.04)	0.11	
0.06 (0.20)	0.41 (0.00)	0.40	
0.19	0.48		
α (p-value)	β (p-value)	r^2	
Aggregated Markets T2			
0.15 (0.00)	0.38 (0.00)	0.59	
0.07 (0.14)	0.10 (0.02)	0.13	
0.03 (0.08)	0.42 (0.00)	0.75	
-0.02 (0.24)	0.49 (0.00)	0.79	
0.04 (0.20)	0.16 (0.04)	0.12	
0.05	0.31		
α (p-value)	β (p-value)	r^2	
Aggregated Markets T3			
0.02 (0.63)	0.36 (0.00)	0.54	
0.16 (0.00)	0.40 (0.00)	0.64	
0.11 (0.08)	0.53 (0.00)	0.50	
0.18 (0.08)	0.16 (0.03)	0.12	
0.10 (0.00)	0.59 (0.00)	0.76	
0.11	0.41		

The coefficient of expectations β is postulated to be in a range from zero to one. All the β estimates from aggregate markets fall in this range, and only 8 of the 75 individuals fall outside. In 13 of the 15 estimates from aggregated markets the β values are significant at the 5-percent level. For the individuals, 49 or the 75 estimates are significant at the 10-percent level. Average coefficients of the aggregated markets are not markedly different from the average coefficients for the individuals, and there are not significant differences between the treatments.

The parameter α is significantly different from zero at the 10-percent level for only 9 out of 75 individuals, i.e. approximately what one should expect if the true value is zero. In T3 the estimates for aggregate markets are significantly different from zero in 4 out of 5 markets, $p < 0.08$. A small positive value of α indicates that the players on average have a small optimistic bias in this treatment.

For 8 of the 15 aggregate markets, r^2 -values are greater than 0.5 implying that average forecasts tend to explain quite a large portion of the price variation. In the sample, we see that both α and β are somewhat greater in T3 than in T1 and T2. It would not be surprising if T3 differed from the other two since in T3 the forecast is made over two rather than one time period.

Test of the heuristic

We explore the aggregated investment behaviour by performing regressions of the heuristic described in eq (5) to eq. (7), which takes the form of eq. (12), where the expected price P^* was taken as the average of individual expectations. There is no supply line of capacity, SC_t , in treatments T1 and T2 and therefore we cannot estimate the coefficient m_1 . Regressions are presented in Table 7 together with benchmark values from the simulation model for the deterministic case.

Table 7. Parameter estimation for the proposed heuristic for aggregated markets corresponding to eq.(12) (p-value in parenthesis).

	$m_3 (P^*)$	$m_2 (P)$	$m_1 (SC)$	b	r^2
Treatment T1					
Mkt 1	-4.52 (0.02)	0.15 (0.94)		50.28 (0.00)	0.25
Mkt 2	-8.90 (0.00)	2.16 (0.10)		55.18 (0.00)	0.48
Mkt 3	0.95 (0.72)	-0.68 (0.71)		44.41 (0.00)	0.01
Mkt 4	-7.48 (0.02)	0.03 (0.98)		56.67 (0.00)	0.16
Mkt 5	-5.43 (0.09)	1.52 (0.31)		50.82 (0.00)	0.12
Average	-5.08	0.64		51.47	
Theoretical*	3.67	-7.40		48.30	
Simulations**	3.67	-7.40		48.30	
Treatment T2					
Mkt 1	1.83 (0.64)	8.33 (0.00)		3.37 (0.49)	0.35
Mkt 2	1.70 (0.37)	4.37 (0.00)		3.70 (0.22)	0.32
Mkt 3	5.61 (0.31)	4.21 (0.06)		-2.01 (0.75)	0.28
Mkt 4	-6.76 (0.14)	7.59 (0.01)		7.73 (0.14)	0.26
Mkt 5	-3.86 (0.45)	4.25 (0.03)		11.39 (0.12)	0.13
Average	-0.30	5.75		4.84	
Theoretical*	3.67	0.10		3.30	
Simulations**	3.67	0.10		3.30	
Treatment T3					
Mkt 1	3.40 (0.20)	-2.19 (0.12)	-0.06 (0.72)	9.44 (0.00)	0.07
Mkt 2	4.59 (0.10)	-0.64 (0.67)	-0.09 (0.62)	6.91 (0.03)	0.10
Mkt 3	-0.94 (0.68)	1.32 (0.38)	-0.05 (0.78)	10.54 (0.00)	0.03
Mkt 4	1.82 (0.50)	-0.19 (0.88)	-0.15 (0.35)	10.06 (0.11)	0.03
Mkt 5	2.53 (0.56)	-0.01 (1.00)	-0.15 (0.40)	4.21 (0.40)	0.07
Average	2.28	-0.34	-0.10	8.23	
Theoretical*	4.02	0.10	-0.10	3.67	
Simulations**	4.02	0.10	-0.10	3.67	

* Theoretical: parameters estimated analytically from the equations of the heuristics and parameters values assumed for simulations, considering only the linear part of the functions (see Appendix 5).

** Simulations: parameters estimated from regression of simulations previously discussed.

Despite the potential meaning of the estimations from experiments, we should consider the poor results of the regressions, not only in terms of the r^2 but also in the significance of the estimated parameters. The analysis of results does not allow us to draw conclusions to neither accept nor reject the hypothesis. For T1, we observe that the coefficient m_3 is significant in four out of five cases with a negative average, parameter m_2 is not significant in any case and b is always significant. Thus, it might indicate that subjects use different mental models for forecasting and for investment decisions, as has been observed in other experiments (Broadbent, 1986; Moxnes, 2004). In T2, the coefficient m_2 is always significant with a positive average, which implies that price drive investments; parameter m_3 is not significant in any case and b is always significant. Finally, for T3, we observe low quality in the regressions with not a single significant coefficient, except for b , and r^2 always below 0.1.

We have also explored the individual investment behaviour by performing regressions of the proposed heuristic for all treatments. Appendix 4 shows the results. The significant coefficients are largely consistent with the coefficients for the aggregate markets. T1 produces the largest number of significant parameters ($p < 0.10$) with 10 negative and 2 positive values for m_4 , 12 positive and 3 negative for m_3 , and 11 positive for m_1 . The same puzzling negative effect of forecasted price on investment dominates. The number of significant coefficients declines with an increasing treatment number. T3 has less than half the number of significant coefficients of T1.

Simulations of the estimated aggregated investment functions

Now, we observe the behaviour of the estimated investment function. We plug the estimated aggregate investment function, eq. (12) in the simulation model. Figure 11 shows the resulting behaviour.

Despite the rather poor quality of the estimated coefficients, the simulations provide some support for the hypothesized behaviour produced by the simulation models for the three treatments with one and the same investment heuristic. T1 presents stability, T2 shows fluctuations with modest amplitude, and T3 presents a fairly large deviation from equilibrium followed by a more rapid convergence to equilibrium than produced by the hypothesised heuristic. This may seem surprising for T1 for which treatment we have argued that forecasts do not seem to be used as expected. The resulting behaviour may indicate that the estimated investment equation for T1 is a useful correlation rather than a correct causal representation.

An interesting observation is about the equilibrium level. The equilibrium assumed in the formulation of the heuristic was the CN, but the average prices of the experimental markets are biased towards competition. This bias is reflected in simulations. The investment function of T1 and T2 estimated from experiments leads the price towards equilibrium lower than CN while simulations with assumed parameters lead to CN equilibrium. Thus, it is consistent with the observation about biases towards competition when there are more than 4 players in a CN game (Huck, 2004).

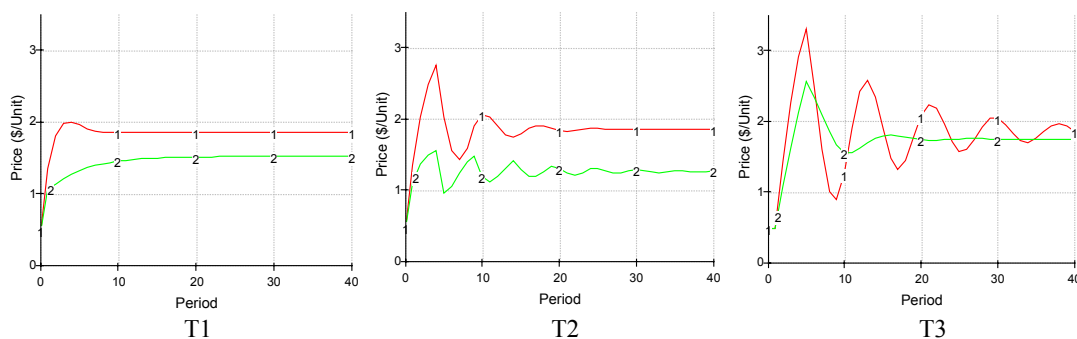


Figure 11. Simulation of investment function with the parameters used in the Testable Hypotheses section (line 1) and the estimations of the experimental results (line 2). Noise is not considered.

In Figure 12 we add normally distributed noise u_t (iid) to investments, $u_t \sim N(0, 3.5^2)$, and perform the similar simulations to those shown in Figure 11. We observe that the tendency towards cyclical behaviour is stronger in T3 than T1, and that the disturbances keep the dampened cycles alive, as is known from some problems in dynamic economics (Frisch, 1933).

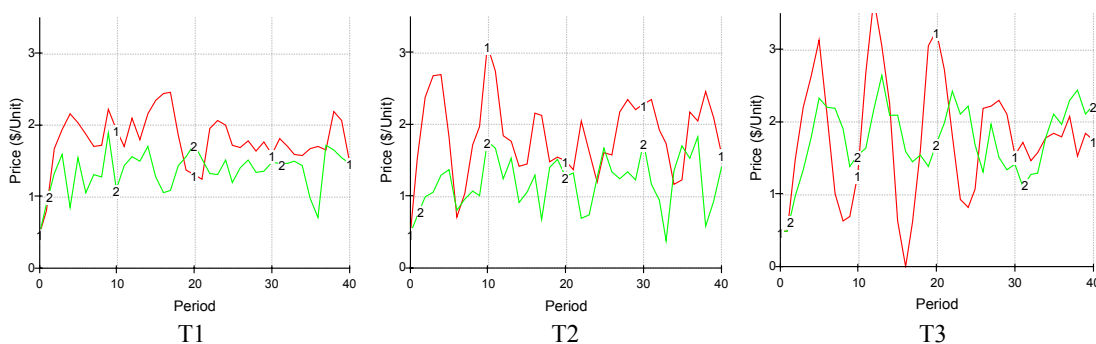


Figure 12. Simulations of investment function with the parameters used in the Testable Hypotheses section (line 1) and the estimations of the experimental results (line 2). Noise is added to investments (noise $\sim N(0, 3.5^2)$).

Random investments?

We also open the possibility of purely random investments. We estimate the average investments and variations for each market. From these values, we take the average and assume it distributes normally. Figure 13 presents some typical behaviour of random stochastic simulations. By visual inspections we observe fast convergence to the equilibrium for T1 and instabilities for T3, which are consistent with previous simulations of the system with the heuristic and the observations of the experiment. However, the salient behaviour of T2 is unstable, with cyclical tendencies, contrary to previous

simulations and the observations of the experimental markets. Thus, random investment can actually create similar behaviour for T1 and T3 but not for T2.

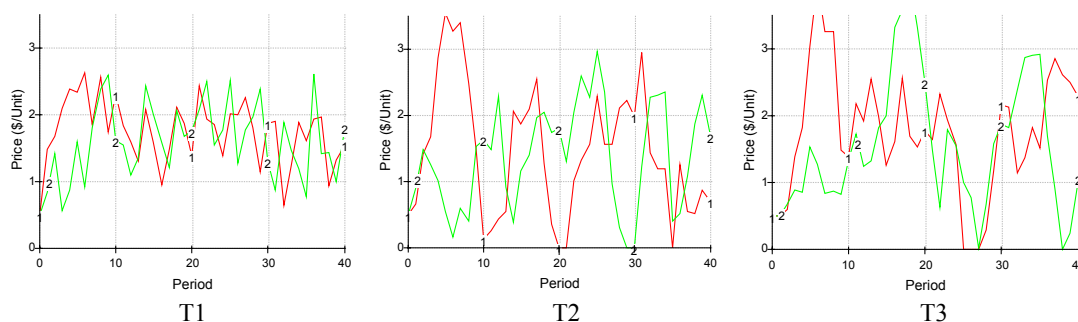


Figure 13. Stochastic simulations with normally distributed investments. Average and standard deviation from the experimental results¹³.

In the attempt to provide explanation for investment behaviour, the tests and analyses of the heuristics failed to reject or support the hypothesis. In addition, simulations have shown that both the estimated heuristic and simple random investment could lead to cyclicity. Thus, it is not necessarily required to have a bias heuristic to create cycles.

Comparison with some commodity prices

The initial motivation of this experiment was to observe the effect of complexity in a commodity market. Complexity was introduced by adding realism to the production system starting from the simplest and well established Cournot market under standard conditions. We turn now to compare what we observe in the experiment with observations of real commodity markets. In order to do so, we take price series of typical commodities and estimate the autospectra and the autocorrelogram as we did with the experimental results (see Figure 14).

Different author have recognized the presence of cycles (Deaton & Laroque 2003, Grilli & Yang, 1988). Cyclicity implies positive autocorrelation, which can be observed with the autocorrelograms of Figure 14. Similarly, positive correlation was observed in T3 but not in T2 or T1. The autospectrums of Figure 14 present larger values for low frequencies, which could be explained by world economic development rather than by oscillations of individual markets. In our experiments, we observe two minor significant peaks for T3 with larger frequencies. We observe, however, weak indications in the spectrums in spite of much data. Some reasons for this could be the need of a filter for the data, e.g. weed out economic long waves, or perhaps the fact that cycles in many commodities are asymmetric as discussed in Arango (2006a).

¹³ Distributions for investments are: for T1 $\sim N(44.06, 5.18^2)$, for T2 $\sim N(11.48, 4.96^2)$, and for T3 $\sim N(11.16, 4.45^2)$.

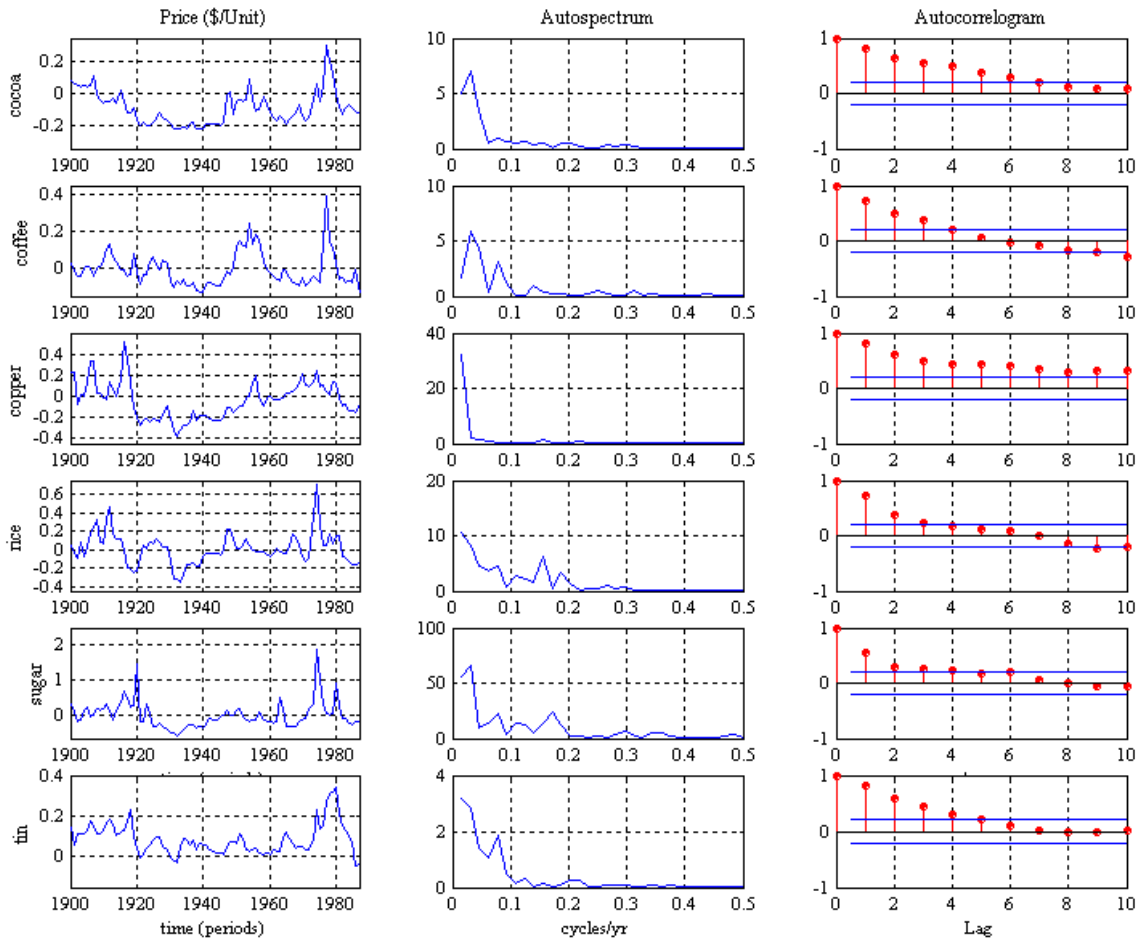


Figure 14. Some commodity prices¹⁴, Autospectrum and Autocorrelogram.

5 CONCLUSIONS

This paper reports a series of Cournot markets with groups of five seller subjects. The subjects decide on production over time and the market price is determined according to a static linear demand curve. Step by step, we add complexity (and realism) to the simple cobweb market (T1): first we introduce lifetimes of capacity extending beyond one period (four vintages) (T2) and then an extra investment lag in addition to the vintages (T3). We explore whether prices converge over time to a unique, static noncooperative Nash equilibrium, and we investigate tendencies towards cyclical behaviour.

The simplest treatment, T1, is considered as Cournot Nash market under standard conditions (Huck, 2004), and also as the Cobweb model. Experimental results show that smoothed prices converge to Cournot Nash equilibrium with biases towards competition, consistent with previous experiments of the Cobweb model (Carlson, 1967; Sonnemans *et al* 2004; Holt & Villamil, 1986; Sutan & Willinger, 2004) and repeated Cournot markets (Rassenti *et al* 2000; and Huck *et al* 2002). Thus, T1 provides a link between the literature and the ensuing treatments. T2 and T3 show similar results regarding the convergence of smoothed prices.

¹⁴ Price data come from the World Bank. Data have been used by Grilli and Yang (1988) and by Deaton and Laroque (2003). We divided the prices by US Consumer Price Index and removed a linear trend.

When it comes to fluctuations and cyclicity, we observe differences between the treatments. In all treatments we find sustained price variations, like in the earlier Cobweb and Cournot experiments. However, T3 shows stronger fluctuations measured by the variance of the price as well as cyclical tendencies measured by autocorrelation. The only factor that makes T3 different from both T1 and T2 is the extra investment lag.

In T3, the autospectrums show that one peak was in the frequency around 0.1 cycles/period and other at 0.18 cycles/periods, i.e. approx. 20 and 5.5 period cycle respectively. By design, the period length was 5 years; therefore, there is one cycle every 100 years and other every 27 years. These results are not realistic compared with empirical evidence of commodity cycles. Some examples of durations (in months) of some commodity cycles are 63 for bananas, 58 for Aluminium, 50 for beef, 56 for cocoa, and 70 for coffee (Cashin *et al* 2002). Five year periods may not be realistic assumption because investment decisions to adjust capital for production have higher frequency, perhaps yearly. Thus, further research is needed to observe the effects of increasing frequency in investment decisions.

To explain the market behaviour, we propose that people use one and the same simple heuristic based on bounded rationality theory. Simulations of the heuristic lead to fast convergence for T1, rapidly converging cycles in T2, and greater and slowly converging cycles in T3. Observed behaviour is consistent with this hypothesised behaviour. However, regressions of the hypothesised investment heuristic indicate that in T1 and T2 subjects use somewhat different heuristics, or at least heuristics with different parameter values from those hypothesised.

T3 estimates are not significantly different from the hypothesised values. On the other hand, the investment function does not explain much of the variation in T3 investments. In all cases we observe a large fraction of seemingly random occurrences, which may difficult the search for patterns of behaviour. Previous Cournot games, similar to T1 (Rassenti *et al*, 2000; Huck, 2002), also fail to explain investments. They have not found completely satisfactory models to explain individual behaviour, and indicate too the need for further research. Nevertheless, we still suspect that there must have been feedback guiding decisions; otherwise smoothed prices would not have been that stable and close to CN. While we have tested one hypothetical investment function, further investigations may reveal more powerful investment functions.

Our results are interesting in that the observed cyclical tendencies in T3 are not generated by exogenous shocks as is often assumed. In our experiment, observed behaviour must be caused by internally generated shocks in combination with heuristics that strengthen, propagate, and bring the frequency of the fluctuations towards the natural frequency of the system. Our simulations with a rational agent suggest that employed heuristics play an important role in addition to the internally generated shocks.

An important research topic of Industrial Organization is that firm-level heterogeneity strongly influences the market behaviour (Ranssenti *et al* 2000). Differences in productivity and firm structure result in differences in profitability, number and size of survivor firms, and market behaviour. According to empirical studies, the factors that cause the firms heterogeneity in markets are inferred under the hypothesis of equilibrium instead of direct observations (Rassenti *et al* 2000). Our experiment suggests the heterogeneity of behavioural strategies could contribute to asymmetric market share distributions, despite the initial homogeneity of the firms. Such heterogeneity in behaviour could also, to some extent, contribute to asymmetries observed in real markets.

Our experiment represents a first attempt at augmenting earlier market experiments with complicating dynamics. Further research should continue this trend. One could for instance introduce: uncertainty in the profit calculator (or remove it altogether), restrictions on investments and financing, capacity utilisation, inventories and backlogs, forward markets, non-linear demand curves, and yearly decision rather than at long time intervals. The latter two are investigated in Arango (2006a, 2006b). Furthermore, more complex and realistic market experiments could be used to test different stabilizing policies, regulations and deregulations.

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Appendix 1. Estimative of the equilibrium points in the market

Following is the notation for the market equilibrium points. Some variables are time dependent, which will be notified if needed. P: market price, c: marginal cost, tc: total cost, C: total supply, q_i : production of the player I, A, B: parameters of the demand curve, π : profits.

Competitive equilibrium: The competitive equilibrium price is the price that equates the quantity demanded and the quantity supplied, with neither surplus nor shortage. The competitive equilibrium is reached when the marginal cost equals the price. The competitive price equilibrium is $P = c = 1$ \$/Unid. The total supply is the sum of individual production ($C_t = \sum q_i, i = 1, 2, \dots, 5$), and there is symmetry across players in the market. Therefore, the total production of competitive equilibrium is distributed symmetrically among players ($S = 5q_i$), given by:

$$S = \frac{A - P}{B} \quad \therefore \quad q_i = \frac{C}{5} \quad \therefore \quad q_i = \frac{A - P}{5 \cdot B} \quad \therefore \quad q_i = 10 \text{ Unid.}$$

Cournot Nash Equilibrium: According to the Cournot Nash model, an oligopolistic market is in equilibrium if each firm produces the same expected production of the other, under conditions of profits maximization. The profit function for each firm is:

$$\pi_i = (P - c) \cdot q_i \quad \text{and} \quad P = A - B \cdot C$$

$$\pi_i = (A - B \cdot C - c) \cdot q_i$$

Every player assumes that the rest of players will produce the same as her/him. The quantity is the result of profit maximization assuming that the other's production q_i for $j \neq i$, is constant, and in the equilibrium the quantity is time independent. The following expression provides the first-order condition for the production q_i (Martin, 2002):

$$\frac{\partial \pi_i}{\partial q_i} = P + q_i \cdot \frac{dP}{dC} - \frac{d \text{tc}(q_i)}{dq_i} \equiv 0$$

Given that $\text{tc}(q_i) = c \cdot q_i$, $S = 5q_i$, the first order conditions becomes:

$$\frac{\partial \pi_i}{\partial q_i} = A - 5 \cdot B \cdot q_i + q_i \cdot (-B) - c \equiv 0$$

$$q_i = \frac{A - c}{6 \cdot B} \quad \therefore \quad q_i = 8,33 \text{ Unid.}; \quad \text{and} \quad P = 1,83 \text{ \$/Unid.}$$

Joint maximization: The "joint maximization" equilibrium is estimated by assuming that each firm (subject) seeks to maximize the total industry profits and divided the joint profits equally. Since all firms are symmetric, it is equivalent to the monopoly equilibrium. Thus, the industry maximizes its total profits with respect to the overall production and divides the profits among firms. The profit function for the total industry is given by:

$$\pi = (P - c) \cdot C$$

$$\pi = (A - B \cdot C - c) \cdot C$$

The first-order condition for the production S is:

$$\frac{d\pi}{dS} = (A - B \cdot C - c) + C \cdot (-B) \equiv 0$$

$$C = \frac{A - c}{2B} = 25 \text{ Unid.} \quad \therefore \quad q_i = 5 \text{ Unid.} \quad \therefore \quad P = 3,5 \text{ \$/Unid.}$$

Appendix 2. Instruction, User interface, and Code for the experiment (The software and the rest of the material is available upon request to the author).

INSTRUCCIONES

PRECAUCIÓN: NO TOQUE EL COMPUTADOR HASTA LA INDICACIÓN PARA HACERLO

Este es un experimento en la economía de toma de decisiones. Varias instituciones han soportado financieramente para realizar el experimento. Las instrucciones son simples, si usted las sigue cuidadosamente y toma buenas decisiones podrá ganar una considerable cantidad de dinero en efectivo después del experimento. En el experimento usted va a jugar el rol de un productor en un mercado. Cada período usted decidirá la producción futura. Su objetivo es maximizar las ganancias en todos los períodos del experimento. A mayores ganancias, mayor será el pago que usted recibirá.

Usted es uno entre 5 productores en un mercado. Usted no sabe quienes son los otros jugadores en su mercado ni su desempeño. Sus ganancias dependen de la producción y del precio de la electricidad menos el costo de producción. La producción debe ser positiva y menor que 20 unidades, el cuál es un límite superior para asegurar un mínimo de competencia en el mercado. El costo unitario es 1 \$/Unid. para todos los productores. El costo incluye los operacionales y los costos de capital, y también el retorno normal al capital. Estoy, si usted vende su producto a 1 \$/Unid. significa que usted está haciendo las ganancias normales en la economía.

El precio de la electricidad está dado para equilibrar la oferta y la demanda. La oferta es la suma de la producción de los 5 jugadores. La demanda es sensitiva al precio y esta dada por la siguiente relación:

$$P = 6 - B*Q \text{ (ver Figura 1), con } A = 6, B = 0,1 \text{ y } Q = \text{Suma de la producción de los 5 productores.}$$

En resumen, a mayor producción total, menor será el precio. Respectivamente, a menor producción total, mayor será el precio. No hay crecimiento económico, lo que significa que la demanda sólo cambia por cambios en el precio. En el inicio del experimento, la producción total 55 unid., el precio es 0,5 \$/Unid. y su producción es 11 Unid.

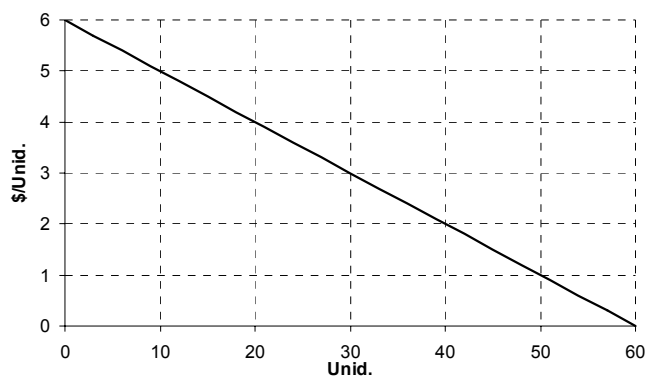


Figura 1. Curva de demanda

For T1: Usted decide cada período la producción para el próximo período. Antes de tomar decisiones, usted obtiene información acerca del precio del producto y de las ganancias del período actual. Cuando el próximo período comienza, este tendrá la producción que usted decidió en el período actual.

For T2 (or T3): Usted decide cada período la producción para el próximo período. Usted decide cada período su producción adicional para el futuro. Antes de tomar decisiones, usted obtiene información acerca del precio del producto y de las ganancias del período actual. Cuando el próximo período comienza, este tendrá la producción que usted ha decidido los últimos 4 períodos (**antes del último**). Cada período usted decide la adición de producción futura. Estas adiciones permanecerán por 4 períodos. La producción adicional está disponible a partir del siguiente período (**después del próximo**). La Figura 2 muestra la figura que se presenta en el software del experimento. Esta muestra las decisiones que usted ha tomado los 3 (**4**)anteriores períodos y la decisión que usted posiblemente tomará en el período actual. Cuando usted halla decidido la producción para el siguiente período (**después del siguiente período**), su decisión no puede ser cambiada cuando usted está en dicho período. En el primer período usted verá las decisiones iniciales hechas antes de usted tomar la compañía. Usted puede hacer clic en la grafica para observar los valores exactos.

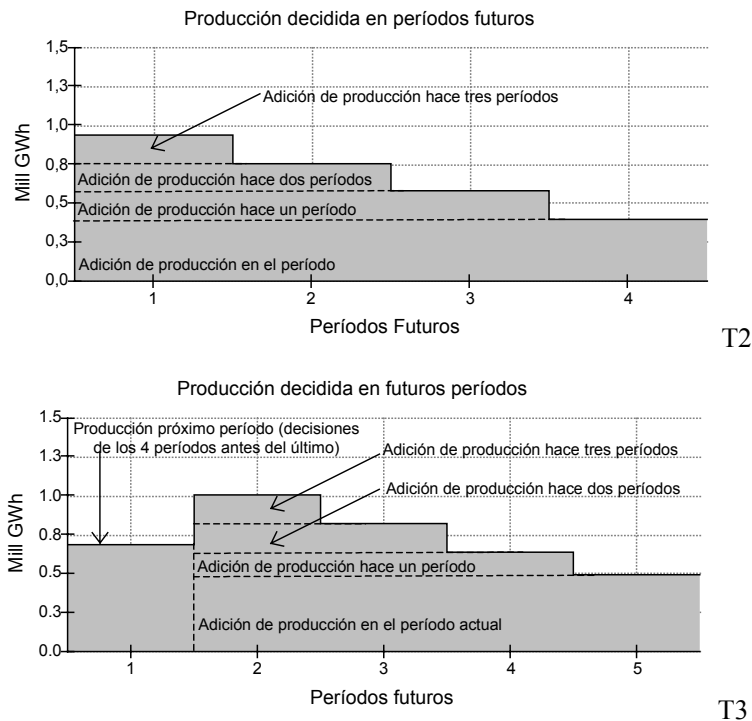


Figura 2. Producción futura decidida.

PAGOS

Usted recibirá un pago según sea su desempeño. Su desempeño es medido por la acumulación de excedentes de ganancias. Si usted obtiene cero excedente de ganancias, su pago será de \$17 000. Si usted acumula mas ganancias recibirá un pago mayor, y si usted acumula menos obtendrá menos pago. Esto, siempre habrá remuneración para hacer lo mejor.

En cada período, también se le solicita hacer el pronóstico del precio para el próximo período (**T3: para el periodo después del próximo**). Usted ganará pago extra dependiendo de la precisión del pronóstico que haga. Si usted hace un pronóstico perfecto en todos los períodos del experimento, usted obtendrá \$8000.

CALCULADORA DE GANANCIAS

La calculadora de ganancias es una ayuda, donde usted asume una decisión de producción propia y otra para el resto de las firmas. Se calcula el precio y las ganancias para estas decisiones.

CORRIENDO EL EXPERIMENTO

Todos los jugadores entran la decisión de producción y el precio pronosticado en el computador, escriben en la hoja de papel correspondiente, y presionan “Accept Decisions”. Cuando todos han tomado sus decisiones, la ventana “Accept Decisions” aparece de nuevo, el juego ha avanzado un período. El tiempo avanza, y los jugadores obtienen los resultados del próximo período. Este es el momento de tomar decisiones de nuevo y así sucesivamente.

Después de 40 períodos, el juego termina. Usted escribe su pago en la hoja de papel y se aproxima al líder del experimento para obtener su pago.

TENGA CUIDADO DE NO PRESIONAR “Accept Decisions” A NO SER DE ESTAR SEGURO DE HACER ESTO. Una vez presione “Accept Decisions” su decisión no puede ser cambiada.

NOTA

De acuerdo con el propósito de los experimentos, se requiere que no compartir ninguna clase de información entre los jugadores (verbal, escrita, gestual, etc.). Por favor, respete estas reglas porque son importantes para el valor científico de los experimento.

Gracias por participar del experimento y mucha suerte!!!

USER INTERFACE

Información

Producción de la firma (Unid.)	11.00
Producción del resto (Unid.)	44.00
Producción total (Unid.)	55.00
Precio (\$/Unid.)	0.50
Costo Unitario (\$/Unid.)	1.00
Margen de ganancia (\$/Unid.)	-0.50
Ganancia del período (\$)	-5.50

Periodo

Decisiones

Producción (Unid.) (próximo período)	0.00
Pronóstico de precio (\$/Unid.)	0.00

Pago
Disponible al final del juego

Desempeño (\$)	0
Pronóstico del Precio (\$)	0
Pago total (\$)	0

Calculadora de Ganancias

Q resto

q propia

Precio

Ganancias

T1

Información

Producción de la firma (Unid.)	11.00
Producción del resto (Unid.)	44.00
Producción total (Unid.)	55.00
Precio (\$/Unid.)	0.50
Costo Unitario (\$/Unid.)	1.00
Margen de ganancia (\$/Unid.)	-0.50
Ganancia del período (\$)	-5.50

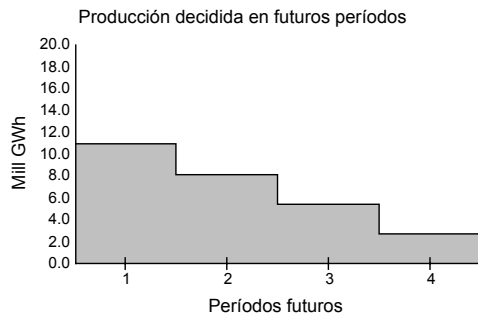
Periodo

Decisiones

Producción adicional (Mill GWh) (en los próximos 4 períodos)	0.00
Pronóstico de precio (\$/kWh)	0.00

Pago
Disponible al final del juego

Desempeño (\$)	0
Pronóstico del Precio (\$)	0
Pago total (\$)	0



Calculadora de Ganancias

Q Resto

q Propia

Producción Acumulada

Precio

Ganancia

T2

Información	
Producción de la firma (Unid.)	11.00
Producción del resto (Unid.)	44.00
Producción total (Unid.)	55.00
Precio (\$/Unid.)	0.50
Costo Unitario (\$/Unid.)	1.00
Margen de ganancia (\$/Unid.)	-0.50
Ganancia del período (\$)	-5.50

Decisiones	
Producción adicional (Unidades) (para los próximos 4 periodos después del próximo)	5.00
Pronóstico de precio (\$/Unidad) (dentro de 2 periodos)	0.00

Pago	
Disponible al final del juego	
Pago por Desempeño (\$)	0
Pago por Pronóstico (\$)	0
Pago total (\$)	0

Período	0
---------	---

Producción decidida en futuros periodos

Calculadora de Ganancias	
Q Resto	24.8
q Propia	16.5
Producción Acumulada	8.25
Precio	1.87
Ganancia	14.37

T3

CODE OF THE BASE PROGRAM FOR T3 (EQUATION WRITTEN IN POWERSIM CONSTRUCTOR 2.51).

```

dim      Acum_Difference = (Players)
init     Acum_Difference = 0
flow     Acum_Difference = +dt*Difference
dim      Bank_account = (Players)
init     Bank_account = 0
flow     Bank_account = +dt*Net_profit
dim      Capacity_lasting_0_periods = (Players)
init     Capacity_lasting_0_periods = Initial_Capacity/4
flow     Capacity_lasting_0_periods = -dt*Retirements
        +dt*Rate_15
dim      Capacity_lasting_1_period = (Players)
init     Capacity_lasting_1_period = Initial_Capacity/4
flow     Capacity_lasting_1_period = -dt*Rate_15
        +dt*Rate_14
dim      Capacity_lasting_2_periods = (Players)
init     Capacity_lasting_2_periods = Initial_Capacity/4
flow     Capacity_lasting_2_periods = -dt*Rate_14
        +dt*Rate_12
dim      Capacity_lasting_3_periods = (Players)
init     Capacity_lasting_3_periods = Initial_Capacity/4
flow     Capacity_lasting_3_periods = +dt*Rate_13
        -dt*Rate_12
dim      ordered_capacity = (Players)
init     ordered_capacity = Initial_Capacity/4
flow     ordered_capacity = -dt*Rate_13
        +dt*Investment
dim      Difference = (Players)
aux      Difference = ABS(Expected_Price-Precio_retardado)/Precio_retardado
dim      Investment = (p=Players)
aux      Investment =
SELECTDECISION(INDEX(p),Investment_Decisions,Simulated,Simulated,Simulated)+IF(TIME=0,Initial_Capacity/4,0)
doc      Investment = AND INDEX(p)=p
dim      Net_profit = (Players)
aux      Net_profit = Revenues-Operational_Cost
dim      Rate_12 = (i=Players)
aux      Rate_12 = DELAYPPL(Rate_13(i),1,Rate_13(i))
dim      Rate_13 = (i=Players)

```

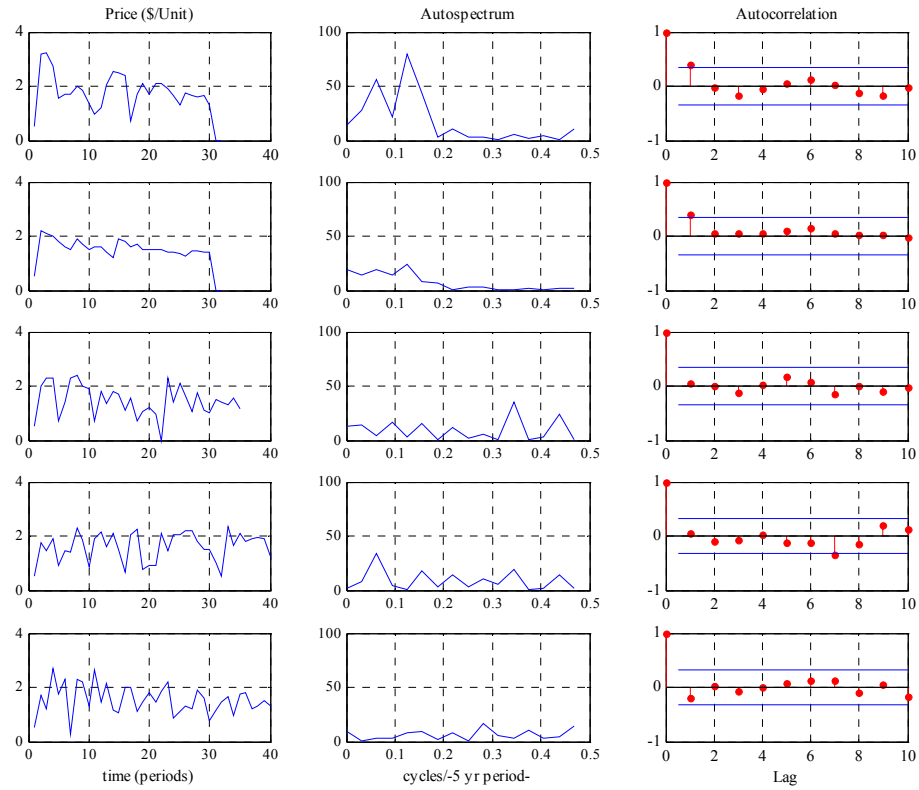


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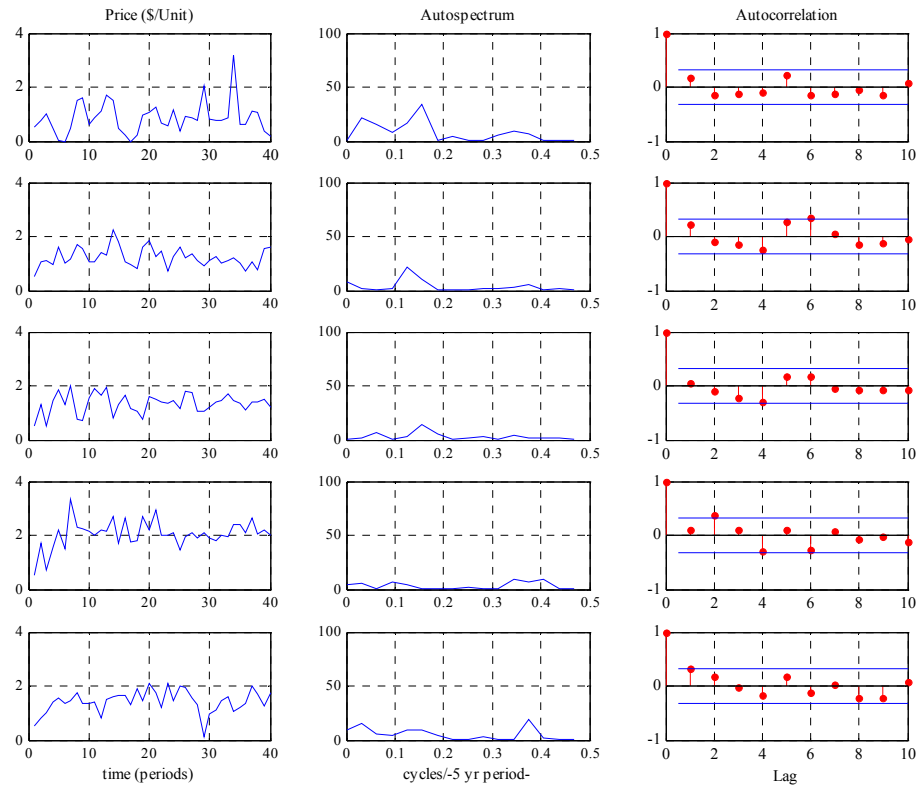
aux      Rate_13 = DELAYPPL(Investment(i),1,Investment(i))
dim      Rate_14 = (i=Players)
aux      Rate_14 = DELAYPPL(Rate_12(i),1,Rate_12(i))
dim      Rate_15 = (i=Players)
aux      Rate_15 = DELAYPPL(Rate_14(i),1,Rate_14(i))
dim      Retirements = (i=Players)
aux      Retirements = DELAYPPL(Rate_15(i),1,Rate_15(i))
dim      Auxiliary_100 = (i=Players, j=1..5)
aux      Auxiliary_100 =
IF(INDEX(j)=1,Capacity_lasting_1_period(i)+Capacity_lasting_2_periods(i)+Capacity_lasting_3_periods(i)+ordered_capacity(i),
IF(INDEX(j)=2,Capacity_lasting_2_periods(i)+Capacity_lasting_3_periods(i)+ordered_capacity(i)+Investment(i),
IF(INDEX(j)=3,Capacity_lasting_3_periods(i)+ordered_capacity(i)+Investment(i),
IF(INDEX(j)=4,ordered_capacity(i)+Investment(i),
IF(INDEX(j)=5,Investment(i),0))))))
dim      Capacity = (i=Players)
aux      Capacity =
Capacity_lasting_3_periods(i)+Capacity_lasting_2_periods(i)+Capacity_lasting_1_period(i)+Capacity_lasting_0_periods(i)
dim      Capacity_Rest = (Players)
aux      Capacity_Rest = ARRSUM(Capacity)-Capacity(1)
aux      Consumption = ARRSUM(Capacity)
dim      Expected_Price = (p=Players)
aux      Expected_Price = SELECTDECISION(INDEX(p),
Decided_Expected_price,Simulated_Expected_price,Simulated_Expected_price,Simulated_Expected_price)
dim      graph_payoff = (Players)
aux      graph_payoff = GRAPH(Bank_account,-
50,26,[20100,20700,20900,21300,21700,22100,22600,23200,23900,24500,25100,25500,26100,26400,27100,28200"Min:15000;Max:30000;
Zoom"])
aux      Margin = Price-Variable_O_and_M_costs
dim      Operational_Cost = (Players)
aux      Operational_Cost = Capacity*Variable_O_and_M_costs
dim      Payoff = (Players)
aux      Payoff = IF(TIME<39,0,1)*graph_payoff
dim      Payoff_Price_Forecasting = (Players)
aux      Payoff_Price_Forecasting = GRAPH(Acum_Difference,0,10,[6000,2790,1130,0"Min:0;Max:6000;Zoom"])*IF(TIME<39,0,1)
aux      Precio_retardado = DELAYPPL(Price, 2,Price)
aux      Price = MAX(A-B*Consumption,0)
dim      Revenues = (Players)
aux      Revenues = Capacity*Price
dim      Simulated = (Players)
aux      Simulated = IF(TIME=0,0,Investment_Decisions(1))
dim      Total_payoff = (Players)
aux      Total_payoff = Payoff+Payoff_Price_Forecasting
dim      Warning_Botton = (Players)
aux      Warning_Botton = IF(Investment_Decisions<0,1,0)
dim      Warning_Top = (i=Players)
aux      Warning_Top = IF(Auxiliary_100(i,2)>Upper_limit_additional_production,1,0)
const   A = 6
const   B = 1/10
dim      Decided_Expected_price = (Players)
const   Decided_Expected_price = 0
const   Initial_Capacity = 55/5
doc     Initial_Capacity = (0.9*90000/5)/1000
dim      Investment_Decisions = (Players)
const   Investment_Decisions = 0
dim      Simulated_Expected_price = (Players)
const   Simulated_Expected_price = 0
const   Upper_limit_additional_production = 20
const   Variable_O_and_M_costs = 1

```

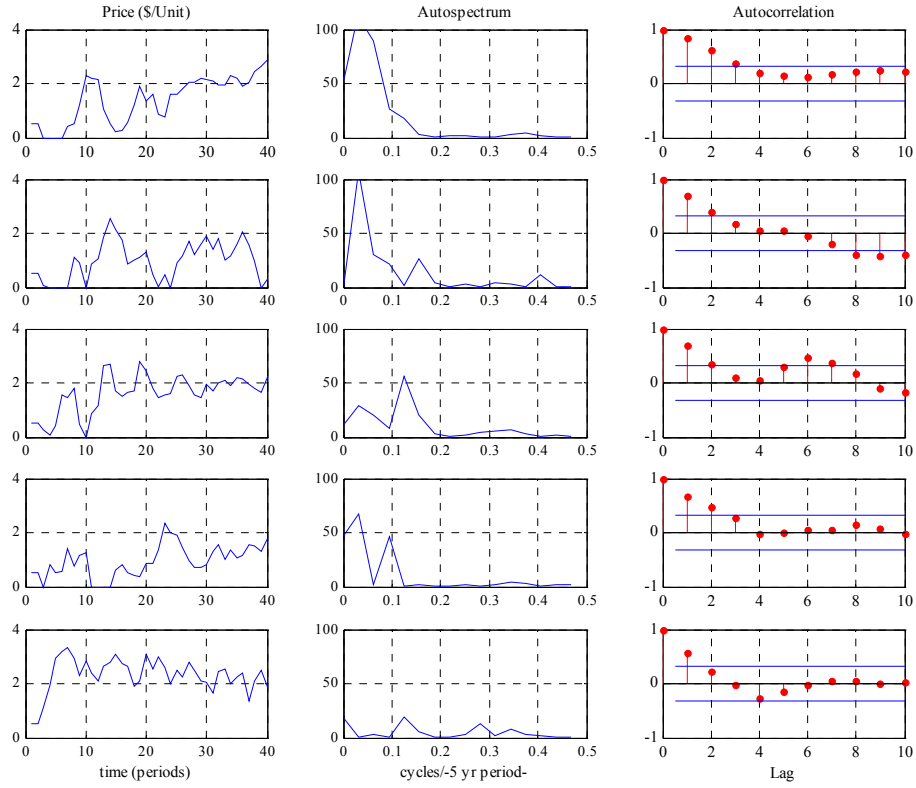
Appendix 3. Price series, Autospectrum, and Autocorrelation



T1



T2



T3

Appendix 4. Parameter estimation for the proposed heuristic for individuals and aggregated market.

Parameter estimation for the proposed heuristic for individuals corresponding to the expression obtained from the linear part of eq (5) to eq. (7), which takes the form,

$$x_t^i = m_4 P^* i_b + m_3 P_t + m_2 SC_b^i + m_1 C_t^i + b + \varepsilon_t$$

Table 8. Parameter estimation for the proposed heuristic for individuals for treatment T1. The p-value of each coefficient is presented in parenthesis.

Mkt/Player	$m_4 (P^*)$	$m_3 (P)$	$m_1 (C)$	b	r^2
1/1	-0.25 (0.77)	0.18 (0.79)	0.26 (0.31)	6.66 (0.07)	0.06
1/2	0.10 (0.35)	0.15 (0.73)	0.22 (0.21)	2.70 (0.04)	0.21
1/3	-1.08 (0.03)	0.78 (0.04)	0.17 (0.28)	7.16 (0.00)	0.18
1/4	-6.73 (0.00)	3.72 (0.02)	0.23 (0.31)	14.37 (0.01)	0.68
1/5	-0.89 (0.12)	1.26 (0.00)	0.89 (0.00)	0.39 (0.85)	0.58
2/1	-0.92 (0.30)	1.41 (0.17)	0.55 (0.00)	1.84 (0.24)	0.42
2/2	-3.67 (0.00)	2.91 (0.00)	0.93 (0.00)	2.29 (0.31)	0.96
2/3	-0.95 (0.01)	-0.01 (0.97)	-0.12 (0.47)	12.61 (0.00)	0.35
2/4	-1.36 (0.02)	2.08 (0.00)	0.38 (0.05)	4.83 (0.03)	0.42
2/5	0.21 (0.65)	0.56 (0.49)	0.20 (0.39)	3.28 (0.12)	0.11
3/1	2.55 (0.00)	-1.87 (0.02)	0.10 (0.60)	10.10 (0.00)	0.45
3/2	-1.16 (0.39)	2.86 (0.00)	0.51 (0.00)	3.59 (0.14)	0.49
3/3	-1.63 (0.00)	0.69 (0.04)	0.26 (0.03)	7.97 (0.00)	0.69
3/4	-4.26 (0.01)	-1.28 (0.09)	0.00 (0.99)	19.42 (0.00)	0.25
3/5	0.56 (0.03)	1.03 (0.33)	0.25 (0.20)	0.04 (0.99)	0.17
4/1	-1.87 (0.00)	1.18 (0.00)	0.74 (0.00)	2.99 (0.01)	0.77
4/2	-0.72 (0.38)	-0.08 (0.95)	0.15 (0.44)	5.14 (0.10)	0.06
4/3	-3.55 (0.00)	1.11 (0.00)	0.70 (0.00)	7.47 (0.00)	0.84
4/4	-1.73 (0.05)	0.06 (0.96)	0.21 (0.24)	11.84 (0.00)	0.13
4/5	-0.44 (0.23)	1.08 (0.08)	0.78 (0.00)	1.29 (0.61)	0.50
5/1	0.38 (0.16)	1.42 (0.00)	0.60 (0.00)	0.38 (0.78)	0.50
5/2	0.11 (0.86)	-1.79 (0.00)	-0.21 (0.16)	11.75 (0.00)	0.27
5/3	-0.51 (0.64)	2.99 (0.00)	0.84 (0.00)	-1.23 (0.66)	0.55
5/4	-0.24 (0.71)	1.17 (0.18)	0.13 (0.57)	5.75 (0.07)	0.07
5/5	-0.94 (0.24)	0.97 (0.24)	-0.10 (0.59)	5.65 (0.02)	0.14
Average	-1.16	0.90	0.35	5.93	0.39

Table 9. Parameter estimation for the proposed heuristic for individuals for treatment T2. The p-value of each coefficient is presented in parenthesis.

Mkt/Player	$m_4 (P^*)$	$m_3 (P)$	$m_1 (C)$	b	r^2
1/1	-0.80 (0.03)	0.70 (0.21)	-0.15 (0.27)	7.19 (0.00)	0.16
1/2	0.49 (0.65)	5.18 (0.00)	-0.01 (0.93)	-1.42 (0.62)	0.33
1/3	2.82 (0.00)	-0.20 (0.75)	0.08 (0.39)	-2.02 (0.06)	0.34
1/4	1.98 (0.04)	1.26 (0.12)	0.06 (0.60)	-1.00 (0.55)	0.22
1/5	1.28 (0.25)	0.29 (0.57)	-0.26 (0.25)	2.68 (0.27)	0.13
2/1	-1.06 (0.30)	3.13 (0.00)	0.06 (0.66)	-0.89 (0.62)	0.33
2/2	-1.87 (0.00)	0.52 (0.07)	-0.56 (0.00)	16.95 (0.00)	0.71
2/3	0.08 (0.93)	0.48 (0.36)	0.02 (0.82)	1.95 (0.30)	0.03
2/4	0.91 (0.00)	-0.06 (0.89)	-0.24 (0.01)	1.28 (0.11)	0.32
2/5	-1.23 (0.01)	1.36 (0.00)	0.06 (0.34)	1.38 (0.15)	0.30
3/1	0.89 (0.02)	0.67 (0.24)	-0.14 (0.24)	0.01 (1.00)	0.25
3/2	0.38 (0.83)	1.39 (0.38)	-0.36 (0.12)	4.73 (0.30)	0.15
3/3	3.14 (0.02)	-0.09 (0.89)	-0.20 (0.09)	2.85 (0.18)	0.19
3/4	0.04 (0.97)	1.35 (0.08)	0.10 (0.40)	-1.01 (0.41)	0.14
3/5	-2.60 (0.04)	1.80 (0.09)	-0.05 (0.71)	4.35 (0.03)	0.12
4/1	-0.99 (0.03)	1.88 (0.00)	0.18 (0.14)	-1.29 (0.38)	0.22
4/2	0.09 (0.97)	1.18 (0.49)	-0.34 (0.07)	3.72 (0.31)	0.23
4/3	0.74 (0.28)	0.40 (0.50)	0.06 (0.62)	-0.88 (0.59)	0.08
4/4	0.32 (0.44)	0.06 (0.82)	0.02 (0.81)	0.53 (0.66)	0.05
4/5	-1.33 (0.18)	0.62 (0.34)	-0.49 (0.10)	8.72 (0.03)	0.11
5/1	0.17 (0.83)	0.27 (0.37)	0.18 (0.00)	-0.36 (0.76)	0.22
5/2	1.33 (0.01)	0.13 (0.74)	-0.24 (0.06)	0.04 (0.97)	0.24
5/3	-1.43 (0.36)	1.93 (0.01)	0.12 (0.10)	1.20 (0.44)	0.28
5/4	-3.91 (0.00)	3.96 (0.00)	0.39 (0.01)	1.28 (0.57)	0.33
5/5	2.66 (0.11)	-0.09 (0.92)	-0.22 (0.40)	-1.32 (0.69)	0.14
Average	0.08	1.12	-0.08	1.95	0.22

Table 10. Parameter estimation for the proposed heuristic for individuals for treatment T3. The p-value of each coefficient is presented in parenthesis.

Mkt/Player	m₄ (P*)	m₃ (P)	m₂ (SL)	m₁ (C)	b	r²
1/1	0.27 (0.14)	-0.14 (0.24)	0.41 (0.00)	0.00 (0.98)	0.08 (0.69)	0.69
1/2	1.00 (0.26)	-0.80 (0.19)	-0.39 (0.03)	-0.33 (0.08)	6.88 (0.02)	0.25
1/3	0.25 (0.68)	0.33 (0.62)	0.39 (0.03)	0.12 (0.15)	-0.82 (0.26)	0.27
1/4	-0.73 (0.03)	0.45 (0.28)	0.35 (0.05)	0.18 (0.03)	0.20 (0.90)	0.32
1/5	8.31 (0.00)	0.77 (0.05)	0.02 (0.87)	0.05 (0.62)	-8.52 (0.02)	0.44
2/1	2.59 (0.01)	-0.13 (0.87)	-0.34 (0.02)	0.08 (0.49)	-0.30 (0.83)	0.34
2/2	1.45 (0.02)	-1.65 (0.05)	-0.04 (0.78)	-0.20 (0.05)	4.21 (0.03)	0.21
2/3	0.48 (0.34)	-0.28 (0.58)	-0.06 (0.73)	-0.14 (0.23)	1.51 (0.16)	0.13
2/4	0.45 (0.25)	0.37 (0.30)	-0.23 (0.24)	-0.21 (0.15)	6.15 (0.01)	0.12
2/5	-0.51 (0.43)	-0.28 (0.64)	-0.34 (0.04)	-0.25 (0.05)	6.69 (0.00)	0.18
3/1	-0.07 (0.93)	-0.36 (0.66)	0.05 (0.76)	-0.02 (0.83)	1.58 (0.21)	0.04
3/2	-2.18 (0.31)	1.27 (0.36)	-0.22 (0.23)	-0.09 (0.68)	5.26 (0.04)	0.07
3/3	-0.29 (0.23)	0.68 (0.19)	0.19 (0.25)	-0.01 (0.96)	1.86 (0.40)	0.15
3/4	-0.71 (0.09)	0.03 (0.95)	0.14 (0.40)	-0.20 (0.04)	5.82 (0.00)	0.24
3/5	1.24 (0.15)	-0.77 (0.27)	-0.34 (0.05)	-0.10 (0.67)	1.69 (0.42)	0.14
4/1	-0.10 (0.86)	0.28 (0.72)	-0.17 (0.34)	-0.02 (0.85)	1.13 (0.32)	0.04
4/2	1.03 (0.19)	-0.59 (0.57)	-0.07 (0.68)	0.04 (0.88)	1.32 (0.74)	0.07
4/3	-0.38 (0.78)	-0.49 (0.61)	-0.14 (0.44)	-0.20 (0.35)	6.14 (0.05)	0.04
4/4	0.01 (0.98)	0.13 (0.83)	-0.10 (0.52)	-0.05 (0.69)	2.74 (0.14)	0.03
4/5	-0.13 (0.75)	0.02 (0.93)	0.01 (0.94)	0.10 (0.21)	2.91 (0.20)	0.06
5/1	0.28 (0.75)	-0.31 (0.74)	-0.07 (0.73)	-0.09 (0.62)	2.22 (0.31)	0.02
5/2	-0.16 (0.82)	0.26 (0.67)	0.07 (0.71)	0.02 (0.91)	1.02 (0.45)	0.01
5/3	0.64 (0.36)	-0.24 (0.74)	-0.04 (0.82)	0.01 (0.97)	0.83 (0.65)	0.04
5/4	0.31 (0.74)	0.31 (0.74)	-0.07 (0.69)	0.07 (0.79)	0.74 (0.81)	0.03
5/5	0.51 (0.41)	-0.67 (0.49)	-0.58 (0.00)	-0.40 (0.04)	5.80 (0.05)	0.33
Average	0.54	-0.07	-0.06	-0.07	2.29	0.17

Appendix 5. Derivation of parameters for the linear part of the decision rule.

We re-state the decision rule with the following equations:

$$x_t = \text{MAX} (0, C_t/\tau + \alpha_C (C_t^* - C_t) + \alpha_{SC} (k C_t^* - SC_t))$$

$$C_t = 60 - 10 P_t$$

$$C_t^* = \text{MAX} (0, a + \left(\frac{q^c - a}{P^e} \right) P_t^*) = a + Y P_t^*$$

$$Y = \left(\frac{q^c - a}{P^e} \right)$$

$$P_t^* = \theta P_{t-1} + (1-\theta) P_{t-1}^*$$

Note that the term $\alpha_{SC} (k C_t^* - SC_t)$ does not exist for T1 and T2. For easier presentation, we eliminate the variable t , which is present in the bold variables. We also take the linear part of the function and the new decision rule is

$$x = C/\tau + \alpha_C (C^* - C) + \alpha_{SC} (k C^* - SC)$$

$$C^* = a + Y P^*$$

$$C = 60 - 10 P$$

Grouping and simplifying:

$$x = C/\tau + \alpha_C C^* - \alpha_C C + \alpha_{SC} k C^* - \alpha_{SC} SC$$

$$x = 60/\tau - 10/\tau P + \alpha_C (a + Y P^*) - \alpha_C (60 - 10 P) + \alpha_{SC} k/\tau (a + Y P^*) - \alpha_{SC} SC$$

$$x = 60/\tau - 10/\tau P + \alpha_C a + \alpha_C Y P^* - \alpha_C 60 + \alpha_C 10 P + a \alpha_{SC} k/\tau + Y \alpha_{SC} k/\tau P^* - \alpha_{SC} SC$$

$$x = P^* [Y \alpha_C + Y \alpha_{SC} k/\tau] + P [\alpha_C 10 - 10/\tau] + [-\alpha_{SC}] SC + [60/\tau + a \alpha_C - 60 \alpha_C + a \alpha_{SC} k/\tau]$$

$$x = P^* [Y \alpha_C + Y \alpha_{SC} k/\tau] + P [\alpha_C 10 - 10/\tau] + SC [-\alpha_{SC}] + [60/\tau + a \alpha_C - 60 \alpha_C + a \alpha_{SC} k/\tau]$$

which is analogous to the expression needed:

$$x_t = m_3 P_t^* + m_2 P_t + m_1 SC_t + b$$

Finally, the coefficient values are

Coefficient	Expression for T1 and T2	Expression for T3
m_3	$Y \alpha_C + Y \alpha_{SC} k/\tau$	$Y \alpha_C + Y \alpha_{SC} k/\tau$
m_2	$\alpha_C 10 - 10/\tau$	$\alpha_C 10 - 10/\tau$
m_1	$-\alpha_{SC}$	$-\alpha_{SC}$
b	$60/\tau + a \alpha_C - 60 \alpha_C + a \alpha_{SC} k/\tau$	$60/\tau + a \alpha_C - 60 \alpha_C + a \alpha_{SC} k/\tau$