The use of heuristics in intuitive mathematical judgment

Rolf Reber[a], Morten Brun[b], and Karoline Mitterndorfer[a]

[a] Cognitive Neuroscience Group, University of Bergen, Norway

[b] Department of Mathematics, University of Bergen, Norway

Psychonomic Bulletin and Review, in press

Word count: 3992

Address of Correspondence:

Rolf Reber, University of Bergen, Department of Biological and Medical Psychology, Cognitive Neuroscience Group, Jonas Lies vei 91, 8th floor, N-5009 Bergen, Norway; E-mail: rolf.reber@psysp.uib.no; Tel +47 55 58 31 81

Abstract

Anecdotal evidence points to the use of beauty as an indication for truth in mathematical problem solving. Two experiments examined the use of heuristics and tested the assumption that participants use symmetry as a cue for correctness in an arithmetic verification task. We presented additions of patterns and manipulated symmetry of the patterns. Speeded decisions about their correctness led to higher endorsements of additions with symmetric patterns, both for correct and incorrect additions. Therefore, this effect is not due to the fact that symmetry facilitates calculation or estimation. We found systematic evidence for the use of heuristics in solving mathematical tasks and we discuss how these findings relate to a processing fluency account of intuition in mathematical judgment.

Mathematicians and scientists reportedly used beauty as a cue for truth in mathematical judgment (Chandrasekhar, 1987; Hadamard, 1954; Stewart, 2007). Hadamard (1954) thought that a sense of beauty seems to be almost the only useful “drive” for discovery in the mathematical field. For example, in “1913, Elie Cartan […] thought of a remarkable class of analytic and geometric transformations in relation to the theory of groups. No reason was seen, at that time, for special consideration of those transformations except just their esthetic character. Then, some fifteen years later, experiments revealed to physicists some extraordinary phenomena concerning electrons, which they could only understand by the help of Cartan’s ideas of 1913.” (Hadamard, 1954, p. 128). However, evidence has been anecdotal, and the nature of the beauty-truth relationship has remained a mystery. We therefore aim at giving a plausible explanation for the heuristic basis of mathematical judgment and at providing first empirical evidence for this hypothesis.

Recent empirical evidence suggests that the common experience underlying both perceived beauty[1] and judged truth is processing fluency, which is the experienced ease with which mental content is processed (R. Reber, Schwarz, & Winkielman, 2004). Indeed, stimuli processed with greater ease elicit more positive affect (R. Reber, Winkielman, & Schwarz, 1998; Whittlesea, 1993; Winkielman, Halberstadt, Fazendeiro, & Catty, 2006), and statements that participants can read more easily are more likely to be judged as being true (Parks, & Toth, 2006; R. Reber & Schwarz, 1999; Unkelbach, 2007). Recently, Topolinski and Strack (2008) demonstrated that intuitive judgments of semantic coherence are based on the affective reaction due to processing fluency. Authors invoked processing fluency to help explain a wide range of phenomena, including judged accuracy of aphorisms (McGlone & Tofighbakhsh, 2000), variations in stock prices (Alter &

Oppenheimer, 2006), brand preferences (Lee & Labroo, 2004), or the lack of reception of mathematical theories that are difficult to understand (McColm, 2007).

Processing fluency increases either through former exposure which render stimuli familiar, such as stimulus repetition (e.g., Jacoby & Dallas, 1981) and associative learning (e.g, Posner & Keele, 1968; A. S. Reber, 1967), or through stimulus features, such as simplicity (Garner, 1974) or symmetry (Palmer, 1991; Royer, 1981), which facilitate perceptual processing. Applied to mathematical reasoning, processing fluency may come from experience with certain types of mathematical stimuli or stimulus features within the task, such as simplicity or symmetry, which in turn increase intuitively judged truth. As a first step towards testing this assumption, we demonstrate in two experiments that symmetry, a feature known to facilitate processing, is used as heuristic cue to correctness in arithmetic problems.

Symmetry has been linked to both beauty and truth in mathematics (Cole, 1998; Stewart, 2007). Preference for symmetry has been observed in humans (Rhodes, Proffitt, Grady, & Sumich, 1998) and a wide variety of other species, including bumblebees, fishes, birds, and primates (see Reber, 2002). One explanation for preference for symmetry is that symmetry in a potential mate signals health (e.g., Gangestad, Thornhill, & Yeo, 1994). This view has been challenged (Kalick, Zebrowitz, Langlois, & Johnson, 1998) and, importantly, preference for symmetry has been found outside mating contexts, which requires a more general explanation, for example in terms of processing fluency (Reber, 2002). Our experimental setup allowed testing the heuristic use of symmetry, but it will leave open how processing fluency, beauty and truth are interrelated.

We examined whether participants without background in professional mathematics intuitively used heuristic bases for truth in speeded arithmetic judgments. In order to speed responses, we trained our participants on a "response window" technique that has been used in research on unconscious semantic priming (Draine & Greenwald, 1998; Greenwald, Draine, & Abrams, 1996). Response time deadlines have been used to examine intuitive processes in complex problem solving (Bolte & Goschke, 2005; Bowers, Regehr, Balthazard, & Parker, 1990). The basic assumption behind using response time deadlines is that it allows studying processes at a stage when participants generate hypotheses, before they know the solution[2]. In a similar vein, we assume that response time deadlines in our experiments allow studying the heuristic basis of hypothesis generation in simple arithmetic tasks at an early processing stage before participants could calculate or estimate the addition.

In both experiments, we manipulated symmetry and examined its role in speeded arithmetic judgments. Participants were presented with additions consisting of symmetric and asymmetric dot patterns (Figure 1). Half of the additions had a correct result, half of them had an incorrect result. As symmetry was an irrelevant attribute in this task, bugs (VanLehn, 1986) or rational errors (BenZeev, 1996) could not explain any observed bias. Such errors are due to the erroneous use of simple, often overlearned computations, as in the so-called "freshman error" (Silver, 1986), where students add numerators and denominators in the addition of fractions (e.g., 1/3+2/7=3/10 instead of 1/3+2/7=13/21). However, if participants use symmetry as cue to correctness, they do not use a correct computation to an erroneous end; symmetry serves as a purely heuristic cue.

Experiment 1

We presented additions with symmetric and asymmetric patterns, together with the results which were either correct or incorrect. We shall report the proportion of endorsements, that is, the proportion with which participants judged a given addition as being correct. If additions are correct and endorsements higher for additions with symmetric patterns than for those with asymmetric patterns, one can not distinguish whether this effect is due to the use of symmetry as heuristic cue or to facilitation of standard mathematical calculation, or estimation. However, if the presented additions are incorrect, one can isolate effects of the use of a heuristic cue from effects of ease of calculation: If participants use symmetry as heuristic cue, they are predicted to endorse incorrect additions with symmetric patterns *more* than incorrect additions with asymmetric patterns. In contrast, if additions with symmetric patterns are just easier to calculate, participants are expected to endorse incorrect additions with symmetric patterns *less* than incorrect additions with asymmetric patterns, yielding higher correct rejection rates for incorrect additions with symmetric patterns, compared to incorrect additions with asymmetric patterns.

Method

Participants: Thirty-eight students at the University of Bergen participated in the experiment. It lasted around 30 minutes, and participants were paid 50 Norwegian Krones (about $8 at that time). Ten students were excluded from analysis: Eight participants uniformly responded “correct” to all patterns or to all symmetric patterns; one reported on the strategy questionnaire that he more probably pressed “correct” for symmetric patterns; one gave less than 50% of the responses within the response time window. We applied the most conservative inclusion criteria; findings essentially were the same with less restrictive criteria.

Materials: Participants were presented with dot pattern additions, one by one. Half of the additions were correct (e.g., 15+18=33), half of them were wrong (e.g., 15+18=27). Incorrect sums were either smaller or greater than the corresponding correct result, but the differences were balanced across symmetry conditions. Each addition was shown twice, once as a symmetric pattern, once as an asymmetric pattern, yielding 96 dot pattern-shaped additions (Figure 1). Symmetric patterns always were rectangles, with three to five rows. Operands with asymmetric patterns always had as many dots and as many rows as the same operand with symmetric patterns, but dots were rearranged so that they possessed neither vertical symmetry nor horizontal symmetry.

Procedure: Participants were given earphones. They sat in front of a computer screen and had a serial response box (Psychology Software Tools) in front of them. They were instructed to verify the correctness of additions and then got instructions for the response time window technique. During pilot testing on verifying additions under time pressure, but with different materials, some participants said that they found it more natural to react to correct solutions with the right index finger and to incorrect solutions with the left index finger. We therefore marked the outmost right key on the response box with green tape and the outmost left key with red tape and instructed participants to press the green key if the solution was correct and the red key if the solution was incorrect. As half of the additions with symmetric patterns and half of those with asymmetric patterns were correct, symmetry was not confounded with side of response. The addition was shown 600 ms. After the addition disappeared, a brief tone was presented via earphones; 600 ms after onset of the first tone, a second brief tone was presented. Participants were instructed to respond after onset of the first tone, but before onset of the second tone; this resulted in a 600 ms response time window. Before the experimental trials started, participants were trained with stimuli not shown in the experimental block. The response time window was progressively shortened: First, participants were trained on a window of 1800 ms until they had responded to at least eight additions within the required time. They were subsequently trained on 1200 ms (at least eight responses within the required time) and on the final response time window of 600 ms (at least 16 responses). Then, the experimental trials started. After the last addition, participants had to complete a strategy questionnaire on paper. First, they had to check whether or not they used a strategy. If yes, they were instructed to describe their strategy in detail. We were interested in whether participants intentionally used symmetry as cue for correctness. After having completed the strategy questionnaire, participants were thanked, debriefed, and dismissed.

Results and Discussion

In all experiments, participants had to give at least 50% of their responses within the response time window in order to be included into the analysis, and only responses provided within this window were analyzed. For the participants included in the analysis, percentage of responses within the response time window was $M = 92.8\%$, $SD = 5.2$, across all conditions.

The findings are shown in the left panel of Figure 2. A 2 x 2-factorial analysis of variance, with the factors symmetry of patterns and task correctness showed that participants were more likely to endorse additions with symmetric patterns ($M = .64$, $SD = .16$) than additions with asymmetric patterns ($M = .49$, $SD = .14$; F (1, 28) = 17.68, $p <$

.001; $r_{effect\ size}$ = .62). Other effects were not significant (*F*s < 1). Please note that we report endorsements; therefore, higher proportion of endorsement means a higher probability of hits for correct additions, a higher probability of false alarms for incorrect additions, or both. This means in terms of accuracy that our participants increased the proportion of hits, but decreased the proportion of correct rejections when additions had symmetric patterns than when additions had asymmetric patterns. Indeed, d'-measures for additions with symmetric and asymmetric patterns did not differ ($d'_{symmetric}$ = .04, $d'_{asymmetric}$ = .04). This finding does not support the notion that symmetry facilitates calculations or estimation, which would have led to higher proportions of both hits and correct rejections.

In sum, participants performed at chance level and relied on symmetry as heuristic cue for correctness. As participants were not able to solve the task, symmetry may have been the only stimulus feature that participants could rely upon.

Experiment 2

Although suggestive, it would be more persuasive to observe the same effect of symmetry on endorsement when accuracy is above chance. This experiment was identical to Experiment 1, with the exception that display times of the additions were increased to 1800 ms, but participants still had to react within the response-time window of 600 ms which followed the presentation of the addition. We used the method outlined in Experiment 1 to isolate the use of a heuristic cue from effects of easier calculation of symmetric patterns.

The extension of presentation time was predicted to render calculation possible, especially for symmetric tasks because participants can use simple strategies, such as estimating surfaces or counting the first row. We expected endorsements to be determined by two processes: First, by symmetry as heuristic cue at an early stage of processing an addition, as observed in Experiment 1, and second by calculation or estimation of the sums when participants were given more time. This would yield both the symmetry main effect observed in Experiment 1 and a symmetry-by-correctness interaction. Participants with extended, but still limited time still use symmetry as a heuristic cue, yielding the main effect, but their ability to perform some calculation or estimation yields the interaction. We expect participants to endorse more correct additions with symmetric patterns than with asymmetric patterns, but to endorse less incorrect additions with symmetric patterns than with asymmetric patterns. The reason for predicting this interaction is that we expect estimation to be easier for additions with symmetric rather than asymmetric patterns.

Method

Participants: Twenty-six students at the University of Bergen participated in the experiment for payment. Two students had to be excluded from analysis because they uniformly pressed “correct”, one for all stimuli and one for correct stimuli with symmetric patterns. Findings essentially were the same with less restrictive inclusion criteria.

Materials and Procedure: Materials and procedure were identical to Experiment 1, with the exception of exposure time: Additions were presented 1800 ms instead of 600 ms; the response time window was 600 ms, as in Experiment 1.

Results and Discussion

The percentage of responses within the response time window was M = 93.9%, SD = 6.4, across all conditions.

The 2 x 2-factorial analysis of variance showed that participants were more likely to endorse additions with symmetric patterns than with asymmetric patterns (F (1, 23) =

33.13, $p < .001$; $r_{effect\ size} = .77$), and more likely to endorse correct than incorrect additions (F (1, 23) = 73.25, $P < .001$; $r_{effect\ size} = .87$). Although performance was at above-chance level, participants were still more likely to endorse additions with symmetric patterns than additions with asymmetric patterns (Figure 2, right panel). The predicted significant interaction term (F (1, 23) = 5.08, $p = .034$) indicated that participants could accurately calculate or estimate some of the correct tasks with symmetric patterns so that the effect of symmetry was more pronounced for correct additions (symmetric patterns: $M = .78$, $SD = .13$; asymmetric patterns: $M = .60$, $SD = .14$; t (23) = 5.66; $r_{effect\ size} = .76$) than for incorrect additions (symmetric patterns: $M = .47$, $SD = .14$; asymmetric patterns: $M = .37$, $SD = .16$; t (23) = 3.16; $r_{effect\ size} = .55$). Importantly, although the effect of symmetry was more pronounced for correct additions than for incorrect additions, suggesting an effect of calculation, both differences were significant. Symmetry still indicated truth for both correct and incorrect additions. Note that the main effect of symmetry was not due to the fact that additions of symmetric patterns were easier to calculate or estimate than additions with asymmetric patterns. Had this been the case, incorrect additions with symmetric patterns would have been endorsed *less* than those with asymmetric patterns, yielding greater accuracy for symmetric than for asymmetric patterns. However, we found that incorrect additions with symmetric patterns were endorsed *more* than those with asymmetric patterns. Moreover, as in Experiment 1, d'-measures for additions with symmetric and asymmetric patterns did not differ significantly ($d'_{symmetric} = .64$, $d'_{asymmetric} = .53$, t (23) = 1.26). Therefore, the results for incorrect additions clearly supported a notion that participants used symmetry as heuristic cue. In sum, participants continued to use symmetry as a cue for correctness even when they calculated or estimated in order to verify the result of the additions, and when their accuracy was above chance.

General Discussion

This study combined existing research in mathematical cognition (see Campbell, 2005; Dehaene, 1997) with research into intuitive judgments (e.g., Bolte & Goschke, 2005; Bowers et al., 1990; Topolinski & Strack, 2008) and, more generally, the heuristics and biases tradition (Gilovich, Griffin, & Kahneman, 2002). Our experiments demonstrated the use of symmetry as heuristic cue in a speeded arithmetic verification task. Symmetry increased the proportion of endorsements in speeded judgments, even if participants performed at above-chance level. Importantly, compared to incorrect additions with asymmetric patterns, symmetry even increased endorsement of incorrect additions with symmetric patterns, supporting the notion that participants used symmetry as a cue for correctness.

We do not claim that people who solve simple arithmetic verification tasks without response deadline always first generate a hypothesis of whether the task is correct or not, although we neither exclude this possibility. What we have shown is that people who do not have enough time to analyze the problem use heuristic cues in order to assess the correctness of a proposed solution. This situation is comparable to a mathematician who has discovered a plausible solution to a problem and now wants a quick assessment of whether this solution "feels" right. In contrast, a mathematician who analyzes the problem thoroughly may take a different route and does not necessarily "feel" whether the solution is correct or not.

Higher mathematics is more complicated than the arithmetic tasks used in our study, and professional mathematicians are more experienced in evaluating hypotheses than our participants. We presented evidence for a plausible mechanism that may underlie intuitive judgments in simple mathematical tasks. Nevertheless, the global feeling of fluency that accompanies the solution of simple arithmetic tasks and complex mathematical problems may be the same.

Our findings suggest a possible solution to the mystery why beauty serves as a cue for truth in the context of mathematical discovery. However, we did not test causal relationships between beauty, fluency, and truth. Theoretically, there are at least three alternatives: First, beauty – which is correlated with processing fluency – may be used as cue for truth. This comes close to what mathematicians and scientists like Hadamard (1954) and Chandrasekhar (1987) claimed. Second, processing fluency may influence perceived beauty, which in turn may be used for judging truth. This would not contradict Hadamard or Chandrasekhar because they did not ponder about where beauty comes from. Third, in accordance to the processing fluency view advocated by R. Reber et al. (2004), processing fluency may influence both perceived beauty and judged truth; the latter are correlated because they have a common underlying mechanism. Beauty in this case would not be causally involved in assessing truth.

Whatever the causal mechanisms might be: Our study has provided strong evidence for the heuristic basis of solving simple additions, and we put forward a plausible explanation for why beauty is truth in mathematical discovery.

References

Alter, A. L., & Oppenheimer, D. M. (2006). Predicting short-term stock fluctuations by using processing fluency. *Proceedings of the National Academy of Sciences of the USA*, **103**, 9369-9372.

Ben-Zeev, T. (1996). When erroneous mathematical thinking is just as “correct”: The oxymoron of rational errors. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking*. Mahwah, NJ: Lawrence Erlbaum.

Bolte, A., & Goschke, T. (2005). On the speed of intuition: Intuitive judgments of semantic coherence under different response deadlines. *Memory & Cognition*, **33**, 1248-1255.

Bowers, K. S., Regehr, G., Balthazard, C., & Parker, K. (1990). Intuition in the context of discovery. *Cognitive Psychology*, **22**, 72-110.

Campbell, J. I. D. (Ed.) (2005). *Handbook of mathematical cognition.* New York: Psychology Press.

Campbell, J. I. D., & Austin, S. (2003). Effects of response time deadlines on adults’ strategy choices for simple addition. *Memory & Cognition*, **30**, 988-994.

Chandrasekhar, S. (1987). *Truth and beauty. Aesthetics and motivations in science*. Chicago: University of Chicago Press.

Cole, K. C. (1998). *The universe in a teacup. The Mathematics of Truth and Beauty*. New York: Harcourt Brace & Company

Dehaene, S. (1997). *The number sense: how the mind creates mathematics*. New York: Oxford University Press.

Draine, S. C., & Greenwald, A. G. (1998). Replicable unconscious semantic priming. *Journal of Experimental Psychology: General*, **127**, 286-303.

Gangestad, S. W., Thornhill, R., & Yeo, R. A. (1994). Facial attractiveness, developmental stability, and fluctuating symmetry. *Ethology & Sociobiology*, **15**, 73-85.

Garner, W. R. (1974). *The processing of information structure*. Potomac, MD: Lawrence Erlbaum Associates, Inc.

Gilovich, T., Griffin, D., & Kahneman, D. (Eds.) (2002). *Heuristics and biases. The psychology of intuitive judgment.* Cambridge: Cambridge University Press.

Greenwald, A. G., Draine, S. C., & Abrams, R. H. (1996). Three Cognitive Markers of Unconscious Semantic Activation. *Science*, **273**, 1699-1702.

Hadamard, J. (1954). *The psychology of invention in the mathematical field*. Mineola, NY: Dover.

Jacoby, L. L., & Dallas, M. (1981). On the relationship between autobiographical memory and perceptual learning. *Journal of Experimental Psychology: General*, **110**, 306-340.

Kalick, S. M., Zebrowitz, L. A., Langlois, J. H., & Johnson, R. M. (1998). Does human facial attractiveness honestly advertise health? Longitudinal data on an evolutionary question. *Psychological Science*, **9**, 8–13.

Lee, A. Y., & Labroo, A. A. (2004). The effect of conceptual and perceptual fluency on brand evaluation. *Journal of Marketing Research*, **41**, 151-165.

McColm, G. (2007). A metaphor for mathematics education. *Notices of the American Mathematical Association*, **54**, 499-502.

McGlone, M. S., & Tofighbakhsh, J. (2000). Birds of a feather flock conjointly (?): Rhyme as reason in aphorisms. *Psychological Science*, **11**, 424-428.

Palmer, S. E. (1991). Goodness, Gestalt, groups, and Garner: Local symmetry subgroups as a theory of figural goodness. In G. R. Lockhead & J. R. Pomerantz (Eds.), *The* *perception of structure* (pp. 23-39). Washington, DC: American Psychological Association.

Parks, C. M., & Toth, J. P. (2006). Fluency, familiarity, aging, and the illusion of truth. *Aging, Neuropsychology, & Cognition,* **13**, 225-253.

Posner, M. I., & Keele, S. W. (1968). On the genesis of abstract ideas. *Journal of Experimental Psychology*, **77**, 353-363.

Reber, A. S. (1967). Implicit learning of artificial grammars. *Journal of Verbal Learning & Verbal Behavior*, **6**, 855-863.

Reber, R. (2002). Reasons for the Preference for Symmetry. *Behavioral & Brain Sciences,* **25**, 415-416.

Reber, R., & Schwarz, N. (1999). Effects of perceptual fluency on judgments of truth. *Consciousness & Cognition*, **8**, 338-342.

Reber, R., Schwarz, N., & Winkielman, P. (2004). Processing Fluency and Aesthetic Pleasure: Is Beauty in the Perceiver's Processing Experience? *Personality & Social Psychology Review*, **8**, 364-382.

Reber, R., Winkielman, P., & Schwarz, N. (1998). Effects of perceptual fluency on affective judgments. *Psychological Science*, **9**, 45-48.

Rhodes, G., Proffitt, F., Grady, J. M., & Sumich, A. (1998). Facial symmetry and the perception of beauty. *Psychonomic Bulletin & Review*, **5**, 659–669.

Royer, F. (1981). Detection of symmetry. *Journal of Experimental Psychology: Human Perception & Performance*, **7**, 1186-1210.

Silver, E. A. (1986). Using conceptual and procedural knowledge: A focus on relationships. In J. Hiebert (Ed.) *Conceptual and procedural knowledge* (pp. 181–198). Hillsdale, NJ: Erlbaum.

Stewart, I. (2007). *Why beauty is truth. A history of symmetry.* New York: Basic Books.

Topolinski, S., & Strack, F. (2008). The analysis of intuition: Processing fluency and affect in judgments of semantic coherence. Manuscript, submitted for publication.

Unkelbach, C. (2007). Reversing the truth effect: Learning the interpretation of processing fluency in judgments of truth. *Journal of Experimental Psychology: Learning, Memory & Cognition,* **33***,* 219-230.

VanLehn, K. (1986). Arithmetic procedures are induced from examples. In J. Hiebert (Ed.) *Conceptual and procedural knowledge* (pp. 133–179). Hillsdale, NJ: Erlbaum.

Whittlesea, B. W. A. (1993). Illusions of familiarity. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, **19**, 1235–1253.

Winkielman, P., Halberstadt, J., Fazendeiro, T., & Catty, S. (2006). Prototypes are attractive because they are easy on the mind. *Psychological Science*, **17**, 799-806.

Acknowledgment

This research was funded by the Norwegian Research Council, Grant #166252, to Rolf Reber. We thank René Westerhausen and Norbert Schwarz for comments on an earlier draft of this manuscript and Marit Knapstad and Linn Vathne Lervik for assistance in running the experiments.

Footnotes

1) Hadamard (1954) and Chandrasekhar (1987) used the term "beauty" broadly and included qualities like "elegance" or "simplicity of a task" which yield mild positive affect. We use beauty in this article in the same broad sense (see also Reber et al., 2004).

2) Response time deadlines can be used for purposes other than the assessment of cognitive processes before the solution is known. In mathematical cognition, such deadlines were used in order to examine retrieval versus procedural strategies during calculating (Campbell & Austin, 2003).

Figure Caption:

Figure 1. Examples of additions used in Experiments 1 and 2; top: symmetric patterns; bottom: asymmetric patterns.

Figure 2. Left: Proportion of speeded endorsements for correct and incorrect additions in Experiment 1. All patterns were shown for 600 ms. Sym600 = symmetric patterns; Asym600 = asymmetric patterns. Right: Proportion of speeded endorsements for correct and incorrect additions in Experiment 2. All patterns were shown for 1800 ms. Sym1800 = symmetric patterns; Asym1800 = asymmetric patterns.

Figure 1:

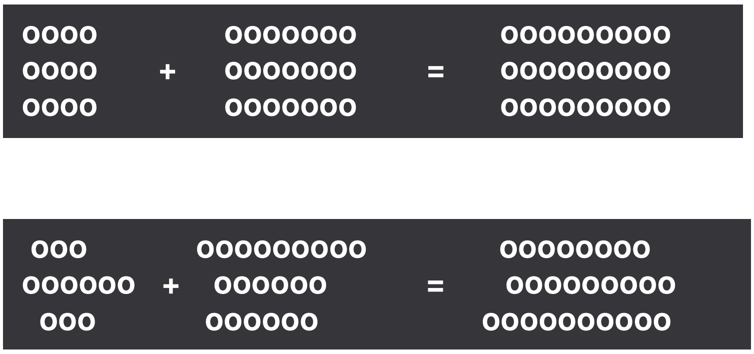
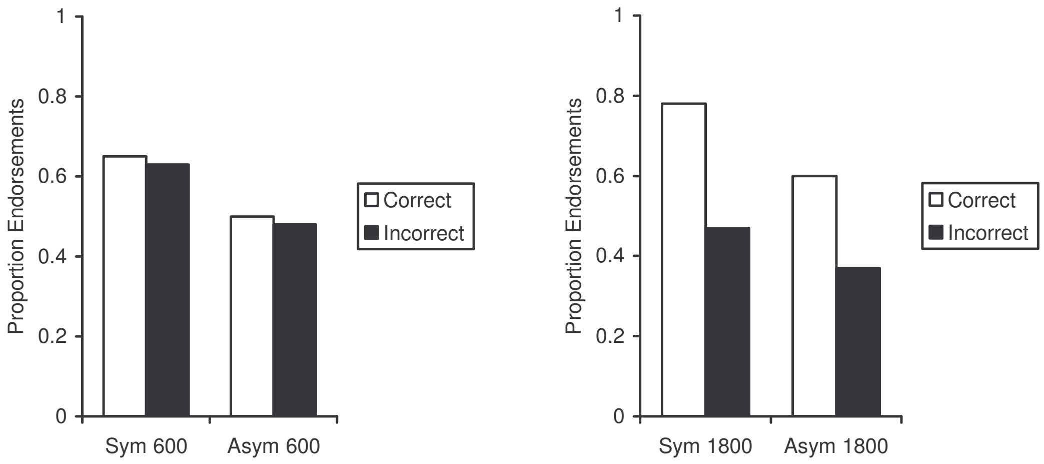


Figure 2:


1
0.8
0.6
0.4
0.2
0
Proportion Endorsements
Sym 600
Asym 600
Correct
Incorrect
1
0.8
0.6
0.4
0.2
0
Proportion Endorsements
Sym 1800
Asym 1800
Correct
Incorrect