Formulation of the planetary boundary layer feedback in the Earth's climate system

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Недавние публикации обратили внимание на факт недостаточной чувствительности современных климатических моделей к наблюдаемым изменениям приземной температуры воздуха. Для объяснения этого факта требуется найти неучтенный в моделях физический механизм обратной связи, который бы оказывал существенное влияние на повышение чувствительности климата к небольшим изменениям внешнего форсинга. В данной работе формулируется гипотеза о том, что повышенная чувствительность климата может быть связана с изменениями в интенсивности турбулентного перемешивания и равновесной толщине планетарного пограничного слоя (ППС). С определенными упрощениями данная гипотеза позволяет провести количественные оценки ограничений на чувствительность климата, налагаемые обратными связями с ППС. Для получения таких оценок использованы теоретические разработки С. Зилитинкевича и соавторов. Парадоксально, но развитие гипотезы об обратных связях в ППС приводит к выводу, что приземные температуры могут не только расти (быстрее или медленнее) при глобальном потеплении климата, но и в отдельных случаях падать. Причем возможность падения температуры обнаружена в часто наблюдаемом интервале параметров. Обратные связи в ППС позволяют также объяснить наблюдаемую асимметрию в климатическом изменении суточных и сезонных максимальной и минимальной температур воздуха.

Introduction

The fourth IPCC (International Panel for Climate Change) assessment report (2007) has quantified uncertainties in climate change simulations by the state-of-the-art models. Those models are sophisticated but yet imperfect. One important imperfection is the model insufficient sensitivity with respect to observed changed in the strength of external forcing. This has been recently revealed by Rahmstorf et al. (2007) from comparisons between the actual global temperature and sea level trends in 1990–2006 and the IPCC simulations. The insufficient sensitivity of the model climate is more obvious in high latitudes as it follows from the sea ice retreat and climate forcing analysis (ACIA, 2004; Stroeve et al. 2007; Chapman and Walsh, 2007). Moreover, comparison with the most computationally expensive cloud-resolving climate simulations (Miura et al., 2005; Wyant et al., 2006), where the representation of the turbulent exchange and the related hydrological cycle are considerably improved, also suggests stronger sensitivity of better resolved earth's climate system.

The observed model imperfection raises a problem of quantification of structural uncertainties in the climate change that are uncertainties introduced by improper account for physical processes and feedbacks in the models (Oppenheimer et al., 2007). The climate system is inherently complex hence there are arguably many structural uncertainties in the climate simulations. However, it has been shown well 40 years ago (Manabe and Strickler, 1964; Manabe and Wetherald, 1967) that processes and feedbacks, related to the turbulent convection or more generally to the vertical turbulent diffusion in the atmosphere-ocean constitute one of the most powerful mechanisms to control the important climate characteristics such as near-surface temperature and low-level clouds. The PBL-feedback can be defined as a response of the surface atmospheric temperature on a change in the magnitude of the vertical turbulent diffusion. The latter is hard to characterize in a simple way since the turbulent diffusion is not measured. In this study, a PBL depth will be utilized as a proper proxy to characterize the diffusion.

The PBL-feedback and its relation to the climate sensitivity in statically unstable, convective planetary boundary layer (CBL), which is mostly observed in daytime, is now reasonably well understood (Hall et al., 1982; Cunningtonand Mitchell, 1990). The understanding was facilitated by two CBL features: absence of vertical gradients in the convective layer; and significant thickness of the convective layer relative to the total depth of the atmosphere. The latter made possible to resolve the CBL not only in idealized radiative-convective models (Moraies et al., 2005) but also in three-dimensional global scale climate models (Medeiros et al., 2005).

The understanding of the PBL-feedbacks in the stably stratified PBL (SBL) is much worse (Mahrt, 1998) and the extrapolation of the CBL-feedback to SBL cases is clearly not justified. At the same time SBL conditions are frequently observed on the Earth especially in high-latitudes and over continents as Figure 1 reveals. It is worth noticing that SBL is observed almost continuously in high latitudes, i. e. in the area of the amplified climate sensitivity (Wang and Key, 2003; Comiso, 2003; Holland and Bitz, 2003).

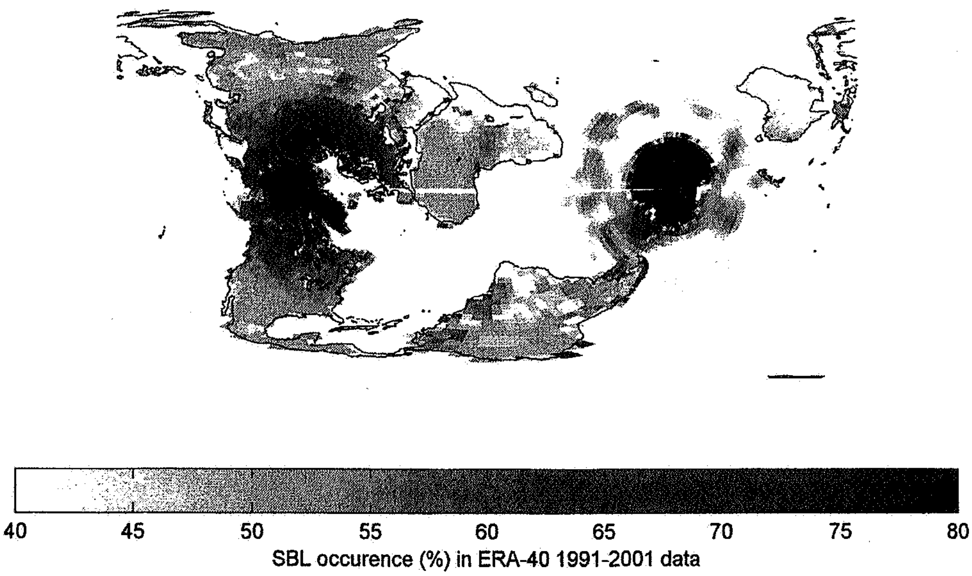


Fig. 1. Occurrence (%) of SBL in ERA-40 re-analysis data during 1991–2001 computed using the sign of the model surface sensible heat flux. The map use "cassini" projection to visualize dominance of the SBL in high-latitudes and on the continents

Recent progress in understanding of the bulk properties of the turbulent exchange in the SBL (Zilitinkevich and Esau, 2002; 2003; 2005; Esau, 2004; Esau and Zilitinkevich, 2006; Beare et al., 2006; Cuxart et al., 2006; Zilitinkevich et al., 2007) justifies this attempt to re-consider PBL-feedback and its role in climate formation and in climate response on the global warming. This paper is focused on an analytical formulation of the PBL-feedback. Although many threads supporting PBL impact on climate could be tested using available observational data, e. g. ERA-40 re-analysis data, this study is limited to the analytical work leaving the data processing for the future.

The paper is structured as following. Section 1 describes physical mechanisms of the PBL-feedback. Section 2 presents analytical formulation of the PBL-feedback. Here in the Section 2a, the CBL-feedback is considered and in the Section 2b, the SBL-feedback is considered. Section 3 falsifies the SBL-feedback against available DATABASE64 large-eddy simulations. Section 5 highlights conclusions.

1. Physical mechanisms of the PBL-feedback

A. Rate of heating/cooling in the PBL. A reasonable assumption is that the adiabatic turbulent diffusion mixes meteorological properties, such as potential temperature or moisture, in the PBL faster than other, non-turbulent processes modify them. Thus, the PBL, unless it is very statically stable, is in equilibrium with its boundary conditions at the surface, $z = 0$ [m], and at the upper boundary, $z = h_{\mathrm{PBL}}$, where h_{PBL} [m] is the thickness of a significantly turbulent layer, known as the PBL depth. Under this assumption, the layer potential temperature, θ [K], which is the measure of the layer thermodynamic entropy (Hauf and Hoeller, 1987), could be characterized by a single value independent of the height (in CBL) or by a non-linear function of the height (Zilitinkevich and Esau, 2005) given the surface air temperature (SAT) is known. Evolution of θ can be related to the PBL depth h_{PBL} and to the of the total heat flux divergence, Q [$\mathrm{W \cdot m^{-2} = J \cdot s^{-1} \cdot m^{-2}}$], as

$$\dot{\theta} = \frac{1}{\rho c_p} \frac{Q}{h_{\mathrm{PBL}}}, \tag{1}$$

where $\dot{\theta} = d\theta/dt$ [$\mathrm{K \cdot s^{-1}}$] is the rate of change of the potential temperature; t [s] is time; ρ [$\mathrm{kg \cdot m^{-3}}$] and c_p [$\mathrm{J \cdot kg^{-1} \cdot K^{-1}}$] are the air density and the air specific heat at constant pressure.

Eq. (1) suggests that θ should change faster in a shallow PBL where h_{PBL} is small than in a deep PBL where h_{PBL} is large. Thus erroneous diagnosis of h_{PBL} in meteorological models would severely affect the surface air cooling/heating rate in a model (Tjernstroem, 2005) and therefore the model climate sensitivity. Typical deficiency of the state-of-the art turbulence parameterizations (Cuxart et al., 2006) is overstatement of h_{PBL}, which results in too warm SAT in the models (e. g. Beesly et al., 2000) given the other parameters in Eq. (1) fixed. IPCC models reveal 1 to 4 degrees cold bias in intercomparisons with ERA-40 re-analysis data (Chapman and Walsh, 2007). ERA-40 data are obtained with ECMWF (European Centre for Medium range Weather Forecast) model and often used as a proxy for observations. Cuxart et al. (2006) showed that the PBL in the ECMWF model is also a way too deep. So the Beesly et al. conclusion is hold. It is reasonable to assume that small h_{PBL} is predominantly a feature of the SBL as it could be expected from the analysis of the turbulence diffusion (Zilitinkevich et al., 2007). ERA-40 re-analysis data support this assumption (not shown). It is worth to mention that Qin the SBL is about order of magnitude smaller and has the opposite sign to Q in the CBL.

B. Relations between PBL stability and the variability of daily temperature extremes. Now let us consider two asymptotic cases of Eq. (1) leading to diurnal maximum $T_{\max}$ and minimum $T_{\min}$ temperatures. Notice that the mean surface air temperature (SAT) — a popular climate characteristic — lays between those two extreme temperatures. On the daytime asymptote, excluding early morning hours, the PBL depth gradually increases (Zilitinkevich, 1991) as

$$h_{\mathrm{PBL}} = \left(\frac{C}{\rho c_p} Qt\right)^{1/2} N^{-1} = \left(C' Qt\right)^{1/2} N^{-1}, \tag{2}$$

where C=1.67 (Fedorovich et al., 2004); N [s^{-1}] is the Brunt–Vaisala frequency of the free atmosphere above the PBL. In result, the diurnal (approximately sinusoidal) heating applied to the developed daytime CBL cannot rise the SAT significantly. It imposes a natural limit on $T_{\max}$ as

$$\dot{\theta} \propto Q/\left(t\,Q\right)^{1/2} \text{and therefore}\, \dot{\theta} \rightarrow \lim_{t \rightarrow \Delta_{\mathrm{CBL}}} \left(t^{-1} \sin(t\pi/\Delta_{\mathrm{CBL}})\right)^{1/2} = 0. \tag{3}$$

Here, Δ_{CBL} is the total duration of the applied heating. So in the CBL, the extreme temperature, $T_{\max}$, is limited by the PBL depth that is in turn a weakly growing function of the duration of heating as well as the non-adiabatic heat flux divergence and stability of the free atmosphere above the PBL. The absolute value of $T_{\max}$ also depends on the temperature at the beginning of heating (at sunrise), which naturally corresponds to $T_{\min}$.

In the SBL, the extreme temperature, $T_{\min}$, is not limited but amplified by h_{PBL} changes. Indeed, the longer non-adiabatic cooling, $Q < 0$, is applied the stronger stability develops. It further reduces h_{PBL} and accelerates lowering of the SAT, which even more strengthens the stability and reduces h_{PBL}. There is however a finite limit for h_{PBL} exists (and reachable) under given equilibrium conditions (Zilitinkevich et al., 2007). The limit exists due to strong dependence between Q and the absolute temperature according to the Stefan—Boltzmann law. During the SBL evolution, the SAT drops, so that the cooling rate decreases to the values that could be sustained by the turbulent heat exchange with the free atmosphere, i. e. by the downward adiabatic heat entrainment. This evolution may be not monotonic. Walters et al. (2007) pointed out that a bifurcation mechanism is possible in the SBL. That is under some combination of the external parameters, the SBL can rapidly transit from cold to warm regimes with a few degrees centigrade temperature difference.

It is important to observe here that with decrease of the absolute values of cooling, $|Q|$, as it is expected under climate change, $T_{\min}$ should response the strongest as it corresponds to the minimum values of h_{PBL} in Eq. (1). Indeed, analysis of global meteorological observations (Easterling et al., 1997; Vose et al., 2005) disclosed that $T_{\min}$ has risen globally at much steeper rate (0.20 K $\cdot$ dec^{-1}) than $T_{\max}$ (0.14 K $\cdot$ dec^{-1}) between 1950 and 2004. Models as found by Stone and Weaver (2002) do not reproduce this remarkable feature increasing both temperature extremes approximately at the same rate so that the daily temperature range (DTR $= T_{\max} - T_{\min}$) changes just by -0.02 K $\cdot$ dec^{-1} as compared with observed changes of -0.07 K $\cdot$ dec^{-1}. Moreover, the obtained $T_{\max}$ trends in the summer and autumn Northern Hemisphere (the seasons with the most intensive convection and therefore with the deepest PBL) observations are so weak that the observed SAT trends should be almost entirely attributed to the trends in $T_{\min}$. Thus, $T_{\min}$ and therefore SBL climatology should contribute boldly to the observed climate sensitivity.

The contribution of the SBL is however downplayed by the climate models in part because of inadequate vertical resolution (Roeckner et al., 2006; Byrkjedal et al., 2007) and in part because of inadequate parameterizations of the SBL turbulence (Beare et al., 2006; Mauritzen et al., 2007; Esau and Byrkjedal, 2007; Zilitinkevich and Esau, 2007). A general drawback in the models is too strong turbulent diffusion in too deep SBL (e. g. Cuxart et al., 2006). Despite of this fact, the model studies have disclosed considerable sensitivity of the results to the details of the SBL parameterization (King et al., 2007), especially in high latitudes (Dethloff et al., 2001).

2. Formulation of the PBL-feedback and its impact on climate sensitivity

Let us define the PBL-feedback as a change of the SAT solely due to the change in the PBL depth, and therefore in the turbulent mixing, in response to perturbations of the lower atmosphere heat flux balance. The effect of other, non-turbulent feedbacks is not considered in this study. The PBL-feedback can be expressed through the following perturbation equation utilizing Eq. (1):

$$\frac{\delta\dot{\theta}}{\delta Q} = \frac{1}{\rho c_p}\frac{\delta}{\delta Q}\left(\frac{Q}{h_{\mathrm{PBL}}}\right) = \frac{1}{\rho c_p}\left(h_{\mathrm{PBL}}^{-1} - Q h_{\mathrm{PBL}}^{-2}\frac{\delta h_{\mathrm{PBL}}}{\delta Q}\right). \tag{4}$$

Integration of Eq. (4) with respect of the duration,Δ, of the heating/cooling will give us the sensitivity of the PBL temperature, and therefore the SAT, to the changes in the heat flux divergence within the PBL:

$$\frac{\delta\theta}{\delta Q} = \frac{1}{\rho\, c_p}\int_0^{\Delta}\left(h_{PBL}^{-1} - Q h_{\mathrm{PBL}}^{-2}\frac{\delta h_{\mathrm{PBL}}}{\delta Q}\right)dt. \tag{5a}$$

In the climate science, the inverse quantity, so called the feedback parameter, is usually used to characterize the climate sensitivity as

$$\lambda = \delta Q/\delta T \sim (\delta\theta/\delta Q)^{-1}. \tag{5b}$$

Following Hansen et al. (1984) and Bony et al. (2006), one can define a feedback gain as amplification or damping of the black body (Plank) response on the temperature change. The feedback gain reads as $g_{\mathrm{CBL}} = \lambda/\lambda_{\mathrm{Plank}}$ where λ_{Plank} is the Plank temperature feedback. Different estimations gives λ_{Plank} from 2.1 $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$ for the mid-latitude winter to 3.4 $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$ for tropics (Huang and Ramaswamy, 2007) and up to 3.8 $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$ in the Stefan—Boltzmann law by equating the outgoing long wave radiation to the fourth power of temperature and assuming the earth's emission temperature of 255 K. In this paper $\lambda_{\mathrm{Plank}} = 3.2\ \mathrm{W\cdot m^{-2}\cdot K^{-1}}$ (Bony et al., 2006) will be used to allow for intercomparisons. Commonly accepted terminology is something odd here. As the matter of fact the largest feedback gain corresponds to the smallest temperature change with respect to the black body response $\delta\theta_{\mathrm{Plank}}$. It could be written as

$$\delta\theta = \left(1 + \sum_{i\neq\mathrm{Plank}} g_i\right)^{-1} \delta\theta_{\mathrm{Plank}}.$$

The feedback gain is a non-dimensional additive quantity. Hence it is convenient for inter-comparisons of feedback strengths. The traditionally quoted feedback gains in the IPCC models (Bony et al., 2006) are 0.563 for the water vapour feedback; −0.263 for the lapse rate feedback; 0.216 for the cloud feedback; and 0.081 for the surface ice-albedo feedback. Thus, the ice-albedo feedback appears to be the strongest feedback in the Earth's climate system. It is necessarily to note that the IPCC values have something different meaning from the values of the PBL-feedback in this paper. The IPCC values have been obtains from models where different feedback mechanisms interact and therefore have a possibility to moderate or to amplify each other. The PBL-feedback in this paper will be considered as a stand-alone feedback as if it does not interact with the other feedback mechanisms.

A. CBL-feedback. The heat flux divergence peaks within the surface layer (Savijarvi, 2007). Therefore the positive heat divergence in the PBL can be associated with the surface heating. It creates a source of the turbulent kinetic energy and enhances the turbulent mixing. Hence, the depth of the CBL is monotonically growing function of time as it is represented in Eq. (2). It allows the following explicit formulation

$$\frac{\delta h_{\mathrm{PBL}}}{\delta Q} = \left(C't\right)^{1/2} N^{-1} \frac{1}{2Q^{1/2}} = \frac{h_{\mathrm{PBL}}}{2Q}. \tag{6}$$

Substitution of Eq. (6) into Eq. (4) gives

$$\frac{\delta \dot{\theta}}{\delta Q} = \frac{1}{\rho\, c_p}\left(h_{\mathrm{PBL}}^{-1} - Q h_{\mathrm{PBL}}^{-2}\frac{h_{\mathrm{PBL}}}{2Q}\right) = \frac{1}{2\rho\, c_p} h_{\mathrm{PBL}}^{-1}. \tag{7}$$

Hence the sensitivity of the warming rate in the CBL is simply reciprocal to the PBL depth. Observe that any other external parameters disappear from the sensitivity equation while the PBL depth itself is indeed a parametric function of the state of the atmosphere. Integration of Eq. (7) assuming sinusoidal variation in the radiation divergence gives the daytime climate sensitivity defined in Eq. (4a) as

$$\frac{\delta \theta}{\delta Q} = \frac{1}{2\rho\, c_p}\int_0^{\Delta_{\mathrm{CBL}}} h_{\mathrm{PBL}}^{-1}(t)dt = \frac{1}{2\rho\, c_p}\frac{t_{\mathrm{noon}}^{1/2}}{\left(C' Q_{\mathrm{noon}} t_{\mathrm{noon}}\right)^{1/2} N^{-1}}\int_0^{\Delta_{\mathrm{CBL}}}\left(t \sin\left(\frac{t}{\Delta_{\mathrm{CBL}}}\pi\right)\right)^{-1/2} dt =$$

$$= \frac{1}{2\rho\, c_p}\frac{\left(\Delta_{\mathrm{CBL}}\right)^{1/2}}{\sqrt{2} h_{\mathrm{PBL}}^{\mathrm{noon}}}\int_0^{\Delta_{\mathrm{CBL}}}\left(t \sin\left(\frac{t}{\Delta_{\mathrm{CBL}}}\pi\right)\right)^{-1/2} dt. \tag{8a}$$

Where $\Delta_{\mathrm{CBL}}$is the duration of the convective heating, which is in the most cases just elapsed time from sunrise. This integral diverges as it possesses a logarithmic singularity at sunrise $t = 0$. In reality, the CBL growth should be considered from the moment when the internal CBL becomes deeper than the previously existing SBL. Taken the SBL depth of 100 m, it implies a time shift of about 1 hour. Thus Eq. (8a) can be integrated from $t = 0.1\Delta_{\mathrm{CBL}}$. The integral is taken numerically. Taking the duration of the day of 12 hours and typical h_{PBL} of (500; 1500; 5000 m) corresponding to shallow, mid-latitude and deep tropical convection without significant release of the latent heat, the CBL feedback factor in Eq. (5b) would be (14; 47; 144 $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$) which makes it insignificant in comparison to other feedbacks including the commonly quoted the total climate feedback for doubling of CO_2 of 2.7 $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$ (Colman, 2003; Miura et al., 2005). This gives the feedback gain $g_{CBL} = \lambda/\lambda_{Plank}$ of (4.4; 14.7; 45.0). Hence the climate should be insensitive to the CBL-feedback as its feedback gain is at least an order of magnitude larger than the gains of the traditionally quoted feedbacks. In the absence of other feedbacks, the CBL-feedback could be observed as damping effect on variations of T_{max}. Interesting that the Eq. (8a) could be simplified assuming that the heat flux divergence is constant and constitutes a half of its noon maximum. Then

$$\frac{\delta\theta}{\delta Q} = \frac{1}{2\rho c_p}\int_0^{\Delta_{CBL}} h_{PBL}^{-1}(t)dt = \frac{1}{2\rho c_p}\frac{t_{noon}^{1/2}}{(C'Q_{noon}t_{noon}/2)^{1/2}N^{-1}}\int_0^{\Delta_{CBL}} t^{-1/2}dt =$$

$$= \frac{1}{\rho c_p}\frac{(\Delta_{CBL})^{1/2}}{h_{PBL}^{noon}}(\Delta_{CBL})^{1/2} = \frac{1}{\rho c_p}\frac{\Delta_{CBL}}{h_{PBL}^{noon}} \tag{8b}$$

and the feedback parameter values are (15; 44; 148 $W \cdot m^{-2} \cdot K^{-1}$) thus being insignificantly different from those computed after Eq. (8a).

B. SBL-feedback. In the SBL, the situation is fundamentally different as the PBL heat divergence is not a source but one of many sinks of the turbulent kinetic energy. Therefore, the SBL depth stabilizes rather quickly as large-eddy simulations (e. g. Esau and Zilitinkevich, 2006) shows. The equilibrium SBL depth was obtained by Zilitinkevich and Esau (2003). Following Zilitinkevich et al. (2007) it reads

$$h_{PBL} = \frac{u_*}{f^{1/2}}\left(\frac{f}{C_R^2} + \frac{N}{C_{CN}^2} + \frac{\beta}{C_{NS}^2\rho c_p u_*^2}Q\right)^{-1/2} = A(B+CQ)^{-1/2}, \tag{9a}$$

where

$$A = \frac{u_*}{f^{1/2}}, B = \frac{f}{C_R^2} + \frac{N}{C_{CN}^2}, C = \frac{\beta}{C_{NS}^2\rho c_p u_*^2}.$$

Here, f [s^{-1}] is the absolute value of the Coriolis parameter; N [s^{-1}] is the Brunt—Vaisala frequency of the free atmosphere above the PBL; u_* [$m \cdot s^{-2}$] is the friction velocity; $\beta = 1/30$ [$m \cdot s^{-2} \cdot K^{-1}$] is the constant air thermal expansion coefficient multiplied by the earth's gravity acceleration. Constants $C_R = 0.65$, $C_{CN} = 1.36$ and $C_{NS} = 0.51$ were empirically fitted to the large-eddy simulations. We will also use the inverse dependence

$$Q = C^{-1}\left(\left(\frac{A}{h_{PBL}}\right)^2 - B\right). \tag{9b}$$

These dependences are implicit as u_* is a function of both the mean geostrophic wind U_g and N as well as Q and h_{PBL}. In turn, Q is a function of u_* and hence of the mean wind U_g and N. Such implicit dependences make Eq. (9a) different from the simpler daytime dependence in Eq. (5). To obtain analytical equations, we assume that u_* is independent on Q and h_{PBL}. The assumption is partially justified through analysis of the large-eddy simulations where dependences on U_g and N are considerably stronger (Esau, 2004; Esau and Zilitinkevich, 2006) than others. Section 3 will also justify this assumption. With such a simplification, one can write

$$\frac{\delta h_{PBL}}{\delta Q} = -\frac{1}{2}A(B+CQ)^{-3/2}C = -\frac{C}{2A^2}h_{PBL}^3. \tag{10}$$

Substitution of Eq. (10) into Eq. (3) gives

$$\frac{\delta\dot{\theta}}{\delta Q} = \frac{1}{\rho\, c_p}\left(h_{\text{PBL}}^{-1} - Qh_{\text{PBL}}^{-2}\frac{\delta h_{\text{PBL}}}{\delta Q}\right) = \frac{1}{\rho\, c_p}\left(h_{\text{PBL}}^{-1} + \frac{C}{2A^2}Qh_{\text{PBL}}\right). \tag{11}$$

Now taking into account Eq. (9b), one can obtain

$$\frac{\delta\dot{\theta}}{\delta Q} = \frac{1}{\rho\, c_p}\left(h_{\text{PBL}}^{-1} + \frac{C}{2A^2}\frac{\left((A/h_{\text{PBL}})^2 - B\right)}{C}h_{\text{PBL}}\right) = \frac{1}{2\rho\, c_p}\left(3h_{\text{PBL}}^{-1} - \frac{B}{A^2}h_{\text{PBL}}\right). \tag{12}$$

Eq. (12) is intriguing. Recall that $B > 0$, it suggests a possibility for sign alternation of the climate sensitivity in response to particular changes in meteorological conditions. It requires realizable meteorological situation with $h_{\text{PBL}}^0 = \sqrt{3}A/B^{1/2}$. Taken $f = 1.2\cdot10^{-4}$ s^{-1}, $u_* = 2.5\cdot10^{-2}$ m $\cdot$ s^{-1}, $A/B^{1/2}$ becomes equal to 30 in long-lived SBL with $N \sim 10^{-2}$ s^{-1}, and to 135 in the nocturnal SBL with $N \sim 0$ s^{-1}. Then the expected critical long-lived SBL depth becomes $h_{\text{PBL}}^0 = 55$ m and the expected critical nocturnal SBL depth becomes $h_{\text{PBL}}^0 = 235$ m. These numbers are frequently observed under the assumed conditions (e. g. Steenveld et al., 2007; Esau and Grachev, 2007). If the parameter values are hold, the climate feedback in shallower layers $h_{\text{PBL}} < h_{\text{PBL}}^0$ will be positive and in deeper $h_{\text{PBL}} > h_{\text{PBL}}^0$ — negative. The positive feedback denotes here temperature increase in response on reduction of the negative heat flux divergence that is in response on global warming. The negative feedback denotes paradoxical temperature decrease in response on the global warming if it is strong enough to overcome the Plank (always positive) response. Whether the strong negative feedback is observed on the climate time scales accounting for the realistic probability distribution of SBL conditions is unclear now and needs further investigations.

Thus, in the observable SBL, the climate sensitivity may change sign. Finally, the sensitivity can be obtained analytically assuming time independence of h_{PBL}:

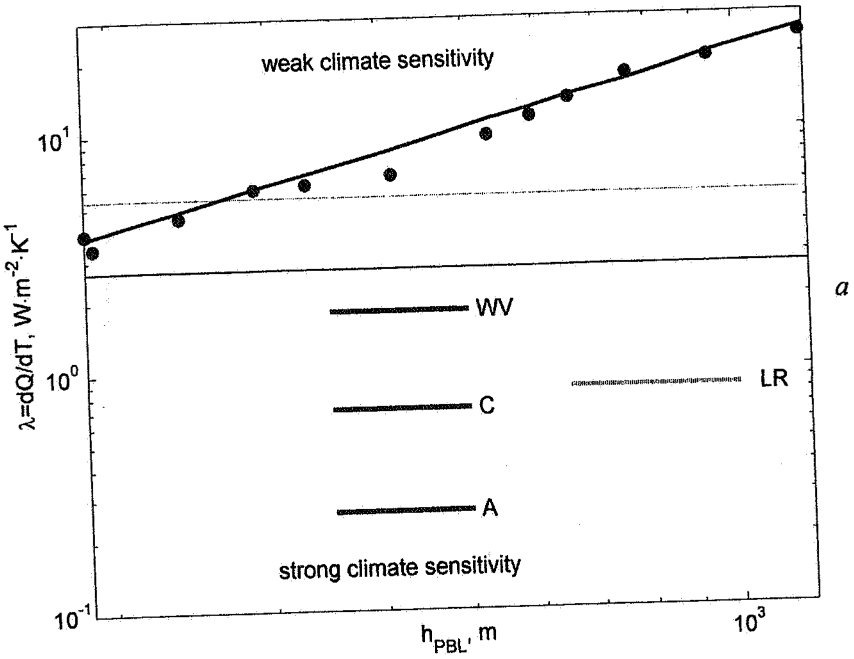
$$\frac{\delta\theta}{\delta Q} = \frac{1}{2\rho\, c_p}\left(3h_{\text{PBL}}^{-1} - \frac{B}{A^2}h_{\text{PBL}}\right)\Delta_{\text{SBL}}, \tag{13}$$

where Δ_{SBL} is the duration of cooling, which can be of several days or even months during high latitude winters. In the SBL, g_{SBL} may become infinitely large virtually meaning no PBL feedback. At the same time whatever small the difference $\left(3h_{\text{PBL}}^{-1} - h_{\text{PBL}}\cdot B/A^2\right)$ is it could result in observable climate feedback given sufficiently long time Δ_{SBL} with negative heat flux divergence. Such conditions are likely to be found in polar and wintertime continental climates.

3. Falsification of the PBL-feedback against Large-Eddy Simulations

Using equations (8a, b) and (13), the PBL-feedback strength can be computed from ERA-40 re-analysis or other data. This is an exercise for the future. Here, the falsification of the obtained analytical formula will be done with an available large-eddy simulation data base, referred to as DATABASE64 (Esau and Zilitinkevich, 2006).

The SBL-feedback is presented in Fig. 2 for the nocturnal, $N \sim 0$ s^{-1} and $U_g = 10$ m $\cdot$ s^{-1} (a), and long-lived, $N \sim 0.04$ s^{-1} and $U_g = 5$ m $\cdot$ s^{-1} (b), SBL runs. Observe that in these calculations, $A/B^{1/2}$ is not a constant as it has been discussed above. It results



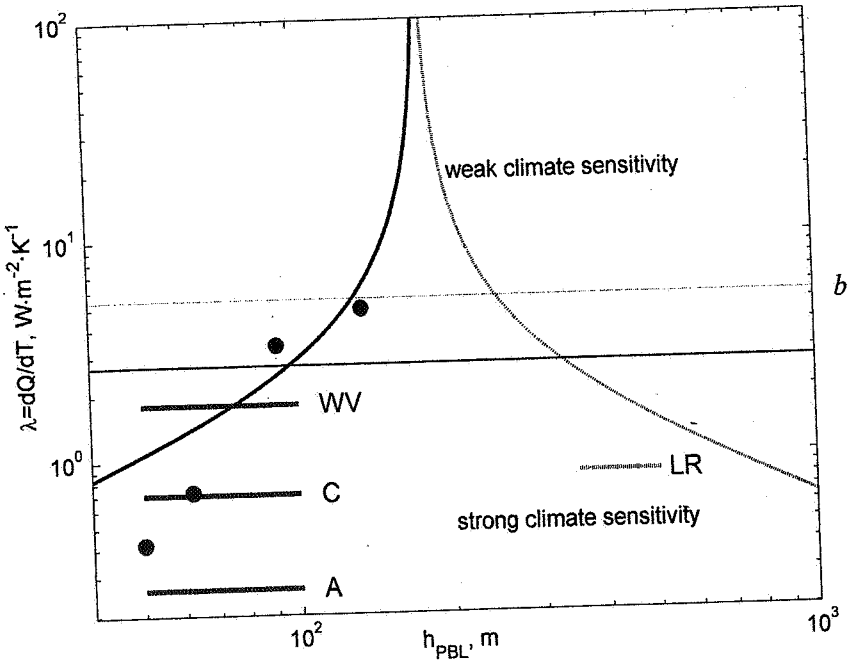


Fig. 2. Falsification of the analytically obtained PBL feedback parameter in the nocturnal SBL (*a*) and the long-lived SBL (*b*) with large-eddy simulations from DATABASE64. The bold solid curve represents the SBL-feedback from Eq. (13) with the PBL depth and $A/B^{1/2}$ parameter taken from DATABASE64 runs and piecewise (extra-) interpolated to cover the interval of the PBL depth variability. The dotted curve is the analytical extrapolation of the feedback in the parameter interval of the negative feedbacks with $A/B^{1/2}$ kept constant. Bold dots represent the SBL feedback parameter directly computed from DATABASE64 data on the surface heat flux divergence and the cooling / heating rate of the aero-dynamical surface air temperature. The negative PBL-feedback has not been captured in DATABASE64 runs. Thin horizontal line represents the commonly quoted total climate sensitivity of 2.7 $\mathrm{W \cdot m^{-2} \cdot K^{-1}}$ (solid line) and 50 % of that value (dashed line). Values of other feedback parameters from Bony et al. (2006) are given for comparisons for: A — the ice-albedo feedback; C — the cloud feedback; WV — the water vapor feedback; and LR — the lapse rate feedback

greenhouse experiments by Kurklu et al. (2003) confirm the sensitivity of the temperature extremes to restrictions on the turbulence convection. Moreover, Kurklu et al. showed that impediments on the air mixing during clear-sky nights result in additional air cooling and the excessive temperature drop inside the greenhouse, thus overwhelming the radiation absorption by the glass roof.

Second, the analysis paradoxically discovers that SBL-feedback may also be negative. That is the reduction (strengthen) of the surface negative heat flux divergence (cooling) can result in the decrease (increase) of the SAT under certain realizable meteorological conditions. I leave for the future studies the question whether the observed decrease of the SAT in some high-latitude regions could be partially explained by this negative SBL-feedback. The discovery of the negative SBL-feedback is also important as it undermines the basis under the climate change skeptics conjuncture "observed local air cooling — no global warming".

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References

[1] IPCC Fourth Assessment Report (AR4), Working Group I Report "The Physical Science Basis", 2007, available online on http://ipcc-wg1.ucar.edu/wg1/wg1-report.html

[2] Rahmstorf, S. et al., 2007: Recent Climate Observations Compared to Projections, Science, 316, 709.

[3] ACIA, 2004: Impacts of a Warming Arctic — Arctic Climate Impact Assessment, Cambridge Univ. Pres., pp. 144.

[4] Stroeve, J., M.M. Holland, W. Meier et al., 2007: Arctic sea ice decline: Faster than forecast, Geophys. Res. Lett., 34, L09501.

[5] Chapman, W.L., J.E. Walsh, 2007: Simulations of Arctic Temperature and Pressure by Global Coupled Models, Journal of Climate, 20(4), 609.

[6] Miura, H., H. Tomita, T. Nasuno et al., 2005: A climate sensitivity test using a global cloud resolving model under an aqua planet condition, Geophys. Res. Lett., 32, L19717.

[7] Wyant et al. 2006: Climate sensitivity and cloud response of a GCM with a uper parameterization, Geophys. Res. Lett., 33, L06714.

[8] Oppenheimer, M., B.C. O'Neill, M. Webster, S. Agrawala, 2007: The limits of consensus, Science, 317, 1505–1506.

[9] Manabe, S., R.F. Strickler, 1964: Thermal Equilibrium of the Atmosphere with a Convective Adjustment, J. Atmos. Sci., 21(4), 361–385.

[10] Manabe, S., R.T. Wetherald, 1967: Thermal Equilibrium of the Atmosphere with a Given Distribution of Relative Humidity, J. Atmos. Sci., 24(3), 241–259.

[11] Hall, M., D.G. Cacuci, M.E. Schlesinger, 1982: Sensitivity analysis of a radiative-convective model by the adjoint method. J. Atmos. Sci., 39, 2038–2050.

[12] Cunnington, W.M., J. F. B. Mitchell, 1990: On the dependence of climate sensitivity on convective parametrization, Climate Dynamics, 4(2), 85–90.

[13] MORAES, E., S.H. FRANCHITO, V.B. RAO, 2005: Evaluation of surface air temperature change due to the greenhouse gases increase with a statistical-dynamical model, J. Geophys. Res., 110, D24109.

[14] MEDEIROS, B., A. HALL, B. STEVENS, 2005: What controls PBL depth? J. Clim., 18(16), 3157–3172.

[15] MAHRT, L., 1998: Stratified atmospheric boundary layers and breakdown of models. J. Theor. Comp. Fluid Dyn., 11, 263–280.

[16] WANG, X., J.R. KEY, 2003: Recent trends in Arctic surface, cloud and radiation properties from space, Science, 299, 1725.

[17] COMISO, J.C., 2003: Warming Trends in the Arctic, J. Clim., 16(21), 3498–3510.

[18] HOLLAND, M.M., C.M. BITZ, 2003: Polar amplification of climate change in coupled models, Climate Dynamics, 21, 221–232.

[19] ZILITINKEVICH, S.S., I. ESAU, 2002: On integral measures of the neutral barotropic planetary boundary layers, Boundary Layer Meteorol., 104(3), 371–379.

[20] ZILITINKEVICH, S.S., I. ESAU, 2003: The effect of baroclinicity on the depth of neutral and stable planetary boundary layers, Quarterly. J. Roy. Meteorol. Soc., 129, 3339–3356.

[21] ZILITINKEVICH, S.S., I. ESAU, 2005: Resistance and Heat Transfer Laws for Stable and Neutral Planetary Boundary Layers: Old Theory, Advanced and Re-evaluated, Quarterly J. Royal Meteorol. Soc., 131, 1863–1892.

[22] ESAU, I., 2004: An improved parameterization of turbulent exchange coefficients accounting for the non-local effect of large eddies, Annales Geophysicae, 22, 3353–3362.

[23] ESAU, I., S.S. ZILITINKEVICH, 2006: Universal Dependences between Turbulent and Mean Flow Parameters in Stably and Neutrally Stratified Planetary Boundary Layers, Nonlinear Processes in Geophys., 13, 122–144.

[24] BEARE, R.J. ET AL., 2006: An intercomparison of large-eddy simulations of the stable boundary layer, Boundary Layer Meteorol., 118(2), 2, 247–272.

[25] CUXART, J. ET AL., 2006: Single-Column Model Intercomparison for a Stably Stratified Atmospheric Boundary Layer, Boundary Layer Meteorol., 118, 273–303.

[26] ZILITINKEVICH, S., I. ESAU, A. BAKLANOV, 2007: Further comments on the equilibrium height of neutral and stable planetary boundary layers, Quarterly J. Royal Meteorol. Soc., 133, 265–271.

[27] HAUF, T., H. HÖLLER, 1987: Entropy and potential temperature, J. Atmos. Sci., 44, 2887–2901.

[28] TJERNSTROEM, M., 2005: The summer Arctic boundary layer during the Arctic Ocean Experiment 2001 (AOE-2001), Boundary-Layer Meteorology, 117, 5–36.

[29] BEESLEY, J.A., C.S. BRETHERTON, C. JAKOB ET AL., 2000: A comparison of cloud and boundary layer variables in the ECMWF forecast model with observations at Surface Heat Budget of the Arctic Ocean (SHEBA) ice camp, J. Geophys. Res., 105(D10), 12,337–12,350.

[30] ZILITINKEVICH, S.S., Turbulent Penetrative Convection. Aldershot, Brookfield, USA, 1991.

[31] FEDOROVICH ET AL., 2004: Entrainment into sheared convective boundary layers as predicted by different large eddy simulation codes, 16th AMS Symposium on Boundary Layers and Turbulence, August, Portland, Main, USA.

[32] WALTERS, J., R. MCNIDER, X. SHI ET AL., 2007: Positive temperature feedback in the stable boundary layer, Geophys. Res. Lett., 34.

[33] EASTERLING ET AL., 1997: Maximum and minimum temperature trends for the globe, Science, 277, 364–367.

[34] VOSE ET AL., 2005: Maximum and minimum temperature trends for the globe: An update through 2004, Geophys. Res. Lett., 32, L23822.

[35] STONE, D.A., WEAVER, A.J., 2002: Daily maximum and minimum temperature trends in a climate model, Geophys. Res. Lett., 29(9), L014556.

[36] ROECKNER, E. ET AL., 2006: Sensitivity of simulated climate to horizontal and vertical resolution in the ECHAM5 atmosphere model, J. Clim., 19, 3771–3791.

[37] BYRKJEDAL, O., I. ESAU, N.-G. KVAMSTO, Sensitivity of simulated wintertime Arctic atmosphere to vertical resolution in the ARPEGE/IFS model, Climate Dynamics, published online-first 16 October, DOI 10.1007/s00382-007-0316-z.

[38] MAURITSEN, T., G. SVENSSON, S.S. ZILITINKEVICH ET AL., 2007: A total turbulent energy closure model for neutral and stably stratified atmospheric boundary layers, J. Atmos. Sci., 64(11), 4117–4130.

[39] ESAU, I., O. BYRKJEDAL, 2007: Application of large eddy simulation database to optimization of first order closures for neutral and stably stratified boundary layers, Boundary Layer Meteorol., 125(2), 207–225.

[40] ZILITINKEVICH, S.S., I. ESAU, 2007: Similarity theory and calculation of turbulent fluxes at the surface for the stably stratified atmospheric boundary layers, Boundary Layer Meteorol., 125(2), 193–205.

[41] KING, J.C., A. JRRAR, W.M. CONNOLLEY, 2007: Sensitivity of modeled atmospheric circulation to the representation of stable boundary layer processes, Geophys. Res. Lett., 34, L06708.

[42] DETHLOFF, K., C. ABEGG, A. RINKE, ET AL., 2001: Sensitivity of Arctic climate simulations to different boundary-layer parameterizations in a regional climate model, Tellus A, 53(1), 1–26.

[43] HANSEN, J., A. LACIS, D. RINDET ET AL., 1984: Climate sensitivity: Analysis of feedback mechanisms. In *Climate Processes and Climate Sensitivity*, AGU-Geophysical Monograph 29, Maurice Ewing Vol. 5., Eds. J.E. Hansen and T. Takahashi, American Geophysical Union, 130–163.

[44] BONY, S., R. COLMAN, V.M. KATTSOVET AL., 2006: How well do we understand and evaluate climate change feedback processes? J. Climate, 19, 3445–3482.

[45] HUANG, Y., V. RAMASWAMY, 2007: Effect of the temperature dependence of gas absorption in climate feedback, J. Geophys. Res., 112, D07101.

[46] SAVIJÄRVI, H, 2006: Radiative and turbulent heating rates in the clear-air boundary layer, Quarterly J. Roy. Meteorol. Soc., 132, 147–161.

[47] COLMAN, R.A., 2003: A comparison of climate feedbacks in general circulation models. Climate Dynamics, 20, 865–873.

[48] STEENEVELD, G.J., B.J.H. VAN DE WIEL, A.A.M. HOLTSLAG, 2007: Diagnostic Equations for the Stable Boundary-Layer Height: Evaluation and Dimensional Analysis, J. Appl. Meteorol. Clim., 46, 212–225.

[49] ESAU, I., A. GRACHEV, 2007: Turbulent Prandtl Number in Stably Stratified Atmospheric Boundary Layer: Intercomparison between LES and SHEBA Data, e-WINDENG journal, 006.

[50] WOOD, R.W., 1909: XXIV. Note on the Theory of the Greenhouse, London, Edinborough and Dublin Philosophical Magazine, 109, vol. 17, p. 319–320. Cambridge UL shelfmark p.340.1. c.95.

[51] KURKLU, A., S. BILGIN, B. OZKAN, 2003: A study on the solar energy storing rock-bed to heat a polyethylene tunnel type greenhouse, Renewable Energy, 28, 683–697.

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