Supersymmetry with Trilinear R-Parity Violation – Implications for Cosmology and the LHC

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Scientific environment

This work has been carried out in the small but efficient theoretical particle physics group, which is part of the subatomic section at the Department of physics and technology at the University of Bergen.

Due to the small size of the group, the work has been performed in collaboration with scientists from the University of Patras, the University of Delhi, the University of Cambridge and Stockholm University.

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Preface

We are presently living in a potentially historical era within physics. After decades of attempts, the hopes to finally be able to go beyond the Standard Model of particle physics has never been greater. The LHC is producing results at amazing speed and quality and high precision satellite measurements of cosmic rays seem to be closing in on the nature of Dark Matter.

If one is an optimistic person, one might say that we soon will have a revolution in our understanding of the universe, including a theory of quantum gravity and possibly a deeper understanding of quantum theory. Or at least we might get closer to such a goal.

However, despite numerous tantalizing anomalies, there are yet no clear results revealing in which direction this revolution lies. In fact, some of our best guesses are being squeezed into small corners a bit faster than feels comfortable. A pessimistic person might say we are getting nowhere.

This work is leaning towards the optimistic side, studying a possible direction physics might take in the not so distant future.

Abstract

The current Standard Model of particle physics is sometimes referred to as one of the most successful theories ever constructed. When looking at the precision with which its predictions are being confirmed, it is easy to agree.

Yet no theoretical particle physicist is content with the state of affairs. Some of the reasons for the dissatisfaction is the non-observation of the Higgs boson, a vital part of the Standard Model, as well as the theoretical problems associated with it. Most obvious, though, is the fact that most of the matter density of the universe comes in a form that cannot be associated with any part of the Standard Model. The nature of this dark matter is one of the most discussed questions in the field.

Many of the speculations beyond the Standard Model are going in the direction of supersymmetry, a symmetry that mixes internal symmetries with spacetime symmetries in a non-trivial way and in the process introduces a symmetry between fermions and bosons. Due to problems with too fast proton decay and possible presence of other exotic processes, one usually assumes that R-parity is conserved when studying supersymmetric models.

This, however, is not necessary and the inclusion of R-parity violating couplings have interesting implications for the phenomenology both within cosmology and LHC physics. If R-parity is violated, one has to abandon the neutralino as the dark matter candidate, but there is a perfectly valid alternative in the gravitino.

Gravitino dark matter that (very slowly) decays, might be a source of measurable cosmic rays. In fact, the recent anomalies in cosmic ray positrons and electrons, seen by PAMELA and Fermi LAT, can be well described by decay products from gravitino dark matter. Unfortunately, the corresponding gamma ray signal was not seen by the Fermi LAT telescope and although this does not exclude gravitinos as the source of the cosmic ray anomalies, it makes it less likely to be the correct answer.

It is also worth noticing that the gravitational nature of the gravitino interactions suppresses the decay rate enough to, in some cases, allow R-parity violating couplings in the neighborhood of the experimental limits, without coming in conflict with cosmic ray measurements. This, however, requires a light (GeV scale) gravitino and a coupling that does not allow a loop decay.

For LHC physics, the implications from violating R-parity can be paramount. However, the most likely scenario might still be pair production of squarks and gluinos, followed by a cascade decay down to the neutralino. If the R-parity violating couplings are large enough, the neutralino will then decay to three Standard Model particles.

This gives rise to multi-jet and/or multi-lepton finals states. This is in general promising for detection since the detection of especially leptons is much easier than measuring missing transverse energy as is required in R-parity conserving supersymmetric models. The large number of invariant mass distributions are also very useful in determining the couplings and measure the neutralino mass. All relevant invariant mass distributions can be calculated theoretically, at least to a good approximation, and then compared to data.

For trilinear R-parity violating couplings, it is shown that, due to our good knowledge of the expected invariant mass distributions, it is clearly possible to identify most couplings responsible for the decay of the neutralinos. Some problematic cases remain due to combinatorial jet background, the presence (and associated information-loss to neutrinos) of taus in the final state as well as phase-space suppression of channels with top quarks.

List of papers

1. N.-E. Bomark, S. Lola, P. Osland, A.R. Raklev, *Gravitino dark matter and the flavour structure of R-violating operators*, Physics Letters B, **677**, 62-70 (2009)
2. N.-E. Bomark, S. Lola, P. Osland, A.R. Raklev, *Photon, neutrino and charged particle spectra from R-violating gravitino decays*, Physics Letters B, **686**, 152-161 (2010)
3. N.-E. Bomark, C. Debajyoti, S. Lola, P. Osland, *Flavour structure of R-violating neutralino decays at the LHC*, JHEP, **7**, 70 (2011)

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Chapter 1

Introduction

What is Particle Physics?

In order to understand the value of a work like this thesis, it is important to understand the role of Particle Physics in modern science and therefore we have to answer the question above.

The theoretical foundation of Particle Physics can be traced back to the theory of special relativity as well as quantum mechanics. However, to understand the role of Particle Physics in the history of science, we have to go even further back.

At all times people have been wondering about the true nature of the universe. This curiosity is the foundation of physics research and it has lead to numerous deep insights about the world we live in. One of the most important moments in the history of physics was when Isaac Newton published his Principia Mathematica. This was the first time some mathematical laws were presented that were supposed to be universally valid and applicable to (more or less) all kinds of phenomena.

Since the days of Newton we have continued the quest to describe all phenomena found in nature in a single mathematical framework. Along the path we have invented quantum mechanics, special and general relativity and quantum field theory. However, despite the success, we know we are not done yet; there are several problems that still lack satisfying descriptions.

These problems include the nature of Dark Matter and Dark Energy, a consistent theory of quantum gravity and the hierarchy problem. The purpose of Particle Physics research today is to address these problem and hence continue the path from Newton and others towards an all unifying mathematical description of the universe.

One of the most promising paths towards a deeper understanding of the universe is offered by supersymmetry. It can resolve the hierarchy problem, improve on the unification of the forces and it provides excellent dark matter candidates. This is in fact true also if the peculiar R-parity is not taken to be exactly conserved.

In this thesis, a scenario where R-parity is violated by trilinear couplings, is investigated.

Chapter 2 is setting the stage by introducing the most important concepts behind the current Standard Model of particle physics. This is followed by some discussion of the shortcomings of this model in chapter 3.

In chapter 4 supersymmetry is introduced, starting with some mathematical basics and then introducing the minimal supersymmetric standard model and finally discussing the implications of R-parity violation in that framework.

Chapter 5 is devoted to dark matter, experimental efforts to detect dark matter are discussed leading up to the discussion of searches for decay products from gravitinos in cosmic rays.

Returning to Earth, chapter 6 is devoted to LHC physics. General properties of supersymmetric theories and searches at the LHC are discussed before we dive into the exploration of three-body neutralino decays through R-parity violating interactions. This is followed by a summary of the thesis in chapter 7.

To make the text comprehensible and to maintain a flow, many technical details have been omitted or just commented on in a footnote. Some notational conventions are collected in appendix A so if the reader is unsure about some notation, appendix A would be the first place to look. Finally some notes on probability distributions, needed for invariant mass calculations, are given in appendix B.

Chapter 2

The Standard Model

At the end of the 19th century it was sometimes thought that the physical laws of the universe were more or less written down and that physics would soon be a dead science. This idea was shattered completely during and after the turn of the century.

Among the new revolutionary discoveries at that time, were special relativity [1] and quantum mechanics [2].

From Maxwell's equations [3] we can deduce the existence of electromagnetic waves and calculate their propagation speed in vacuum. Since the result is independent of any background objects, one might conclude that the speed of light is independent of the velocity of the observer. The alternative conclusion, that the vacuum is filled with some kind of fluid (ether) that allows us to define a preferred reference system, was excluded by interferometry experiments [4].

The assumption that the speed of light is independent of the velocity of the observer may seem innocent, but as Einstein proved, the consequences are far reaching and somewhat counterintuitive. Perhaps the most discussed effect is that time no longer flows at the same rate for all observers [1]. However, for particle physics, the most important consequences are the new relations between energy, momentum and mass.

Quantum mechanics on the other hand, is rather mind-boggling from the start. At first sight it might look like ordinary wave mechanics, but something very surprising happens whenever we attempt a measurement. In general, all measurable quantities are related to operators. These operators have eigenvalues and eigenfunctions and any state of a system can be written as a superposition of such eigenstates. The strange thing is that a measurement of the quantity related to an operator, always gives one of the eigenvalues of that operator and hence the wavefunction of the measured system will collapse to the eigenfunction related to the measured eigenvalue.

How this collapse happens and what exactly a measurement is, are questions yet to be answered. The resulting theory plays a crucial role in particle physics, as a matter of fact, the very existence of particles can be seen as a consequence of the quantization of the underlying matter fields.

2.1 Quantum Field Theory

The best known method of constructing a quantum theory that is consistent with special relativity is through a quantum field theory. The idea is to use fields rather than individual particles as the fundamental constituents of the theory.

Let us briefly illustrate the main procedures in the construction of quantum field theories, for a more thorough elaboration see e.g. [5, 6]. To keep the mathematics simple we use the simplest possible example, a real scalar field ϕ. The dynamics of this field can be described by a lagrangian density like:

$$\mathcal{L} = \frac{1}{2}(\partial_\nu \phi)(\partial^\nu \phi) - \frac{1}{2}\mu^2\phi^2 + interactions, \tag{2.1}$$

where μ is a mass parameter and *interactions* means terms of order 3 or higher in the field (in this simple case we have only one field but in general this will include interactions between different fields). If we now ignore the interactions, we arrive at the free field equation of motion

$$(\partial_\nu \partial^\nu - \mu^2)\phi = 0, \tag{2.2}$$

which we recognize as the Klein-Gordon equation. The solutions of eq. (2.2) can then be expanded as a Fourier series as follows:

$$\phi(x) = \sum_{\mathbf{p}} \sqrt{\frac{1}{2VE_{\mathbf{p}}}} \left(a(\mathbf{p})e^{-ipx} + b(\mathbf{p})e^{ipx} \right), \tag{2.3}$$

where V is a volume which is taken as the region of validity for the solution.

The actual quantization is performed by promoting the Fourier coefficients $a(\mathbf{p})$ and $b(\mathbf{p})$ to operators. In this process the coefficients $b(\mathbf{p})$ will have to be replaced by the hermitian conjugate of the operators $a(\mathbf{p})$, i.e. by $a^\dagger(\mathbf{p})$. We then impose the following commutation relations on them[1]:

$$[a(\mathbf{p}), a(\mathbf{p}')] = [a^\dagger(\mathbf{p}), a^\dagger(\mathbf{p}')] = 0, \qquad [a(\mathbf{p}), a^\dagger(\mathbf{p}')] = \delta_{\mathbf{pp}'}. \tag{2.4}$$

To see the effect of the operators $a(\mathbf{p})$ and $a^\dagger(\mathbf{p})$ on some state $|n\rangle$ we assume that there is a vacuum state $|0\rangle$ and that the operators $a(\mathbf{p})$ annihilate this vacuum, i.e. $a(\mathbf{p})|0\rangle = 0$.[2]

Let us now look at what the operator $a(\mathbf{p})$ does to the state $a^\dagger(\mathbf{p})|0\rangle$:

$$a(\mathbf{p})a^\dagger(\mathbf{p})|0\rangle = \left(a^\dagger(\mathbf{p})a(\mathbf{p}) + 1\right)|0\rangle = |0\rangle, \tag{2.5}$$

i.e. it lowers the state into the vacuum state. Since $a^\dagger(\mathbf{p})|0\rangle$ should be some state with higher energy and we have seen that $a(\mathbf{p})$ lowers it exactly to the vacuum state we may guess that $a^\dagger(\mathbf{p})$ is in fact raising the vacuum state into the next lowest state, let us call it $|1\rangle$. This can be generalized to an arbitrary number of $a^\dagger(\mathbf{p})$ operators:

$$a(\mathbf{p})(a^\dagger(\mathbf{p}))^n|0\rangle = \left((a^\dagger(\mathbf{p}))^n a(\mathbf{p}) + n(a^\dagger(\mathbf{p}))^{n-1}\right)|0\rangle = n(a^\dagger(\mathbf{p}))^{n-1}|0\rangle, \tag{2.6}$$

which clearly indicates that the $a^\dagger(\mathbf{p})$ operators are creation operators that excite the system to a higher state while $a(\mathbf{p})$ annihilates these higher states, for a more thorough derivation of this conclusion, see [5].

[1] Strictly speaking this should be done in the opposite order, the field ϕ and its hermitian conjugate should be promoted to operators and commutation relations should be imposed on them. For simplicity we here use a more direct approach that gives the same result.

[2] One can prove that $a(\mathbf{p})|0\rangle = 0$ is a necessary condition for $|0\rangle$ to be the state of lowest energy, i.e. the vacuum state.

This means we can interpret the operators $a(\mathbf{p})$ and $a^\dagger(\mathbf{p})$ as creation and annihilation operators that create or annihilate excitations of the field. These excitations correspond to what we normally call particles.

The next step is to see how interactions can bring one state into another state. To calculate this is in general very difficult but there are possible simplifications. The standard solution is to use a perturbation expansion in order to break the calculation down to manageable pieces.

Let us say we start with some state $|i\rangle$ and want to know how likely it is to evolve into some other state $|f\rangle$.[3] The probability of $|i\rangle$ turning into $|f\rangle$ can be written $|\langle f|S|i\rangle|^2$ where S is a transition matrix, usually called the S-matrix.

The S-matrix can be written as [5],

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int \ldots \int d^4x_1 d^4x_2 \ldots d^4x_n T\{\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\ldots\mathcal{H}_I(x_n)\}, \tag{2.7}$$

where T means time ordering and $\mathcal{H}_I$ is the interaction lagrangian, i.e. $\mathcal{H}_I = -\mathcal{L}_I$ where $\mathcal{L}_I$ is the part of eq. (2.1) that says *interactions*.

The first thing to note from eq. (2.7) is that the S-matrix contains some product of annihilation and creation operators. This means that we can do some perturbation series expansion and then discard all terms that do not exactly annihilate all particles in $|i\rangle$ and create all particles in $|f\rangle$.

The result is an infinite series of terms for each type of interaction. It was realized that these terms can be graphically represented by Feynman diagrams [7] and that there are some rather simple (Feynman) rules relating the diagrams with the terms in the perturbation expansion. The beauty of this is that, as long as the interactions are weak enough, the terms corresponding to the simplest diagrams are the most important ones. This means we can reverse the work order; we start by drawing all the simplest diagrams that turn our initial state into our final state, using the rules that are specified by our interaction terms in the lagrangian. Simplest here refers to the number of loops in the graph, in the first step we use diagrams without loops and in the second step only one loop etc. until we reach our desired accuracy (or run out of time or computer power...).

Although we have only talked about a scalar field here, the methods described are applicable to much more complicated quantum field theories as well. Feynman diagrams are routinely used to describe and discuss possible interactions. Given the illustrative nature of the diagrams, it is easy to even start to think of them as a "true" description of how interactions happen in nature. This, however, would be a mistake.

To illustrate why this would be a problematic association, let us look at one diagram describing electron-electron scattering by the electromagnetic force. This is described by the exchange of a photon as shown in figure 2.1[4]. To see how the kinematics work we can use the rest frame of one of the incoming electrons, this allows us to use the momentum four-vectors as defined in the right part of figure 2.1. With this notation we get from conservation of energy-momentum:

$$m = E + E_\gamma, \qquad |\mathbf{p}| = |\mathbf{p}_\gamma|, \tag{2.8}$$

[3] The states $|i\rangle$ and $|f\rangle$ are defined at time equal to $-\infty$ and ∞ respectively. This is just another way of saying that they are separated by much longer time than the timescale of the interaction.

[4] Note that this is just one diagram out of three needed for the lowest order calculation.

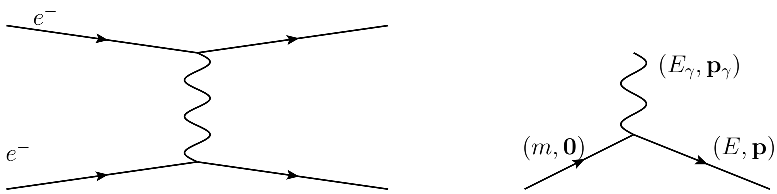

Figure 2.1: Feynman diagram describing electron-electron scattering by a photon exchange. To the right we show the notation used in the text for the momentum four-vectors in the lower vertex of the diagram, given in the rest-frame of the lower incoming electron.

and since we know that the outgoing electron has the same mass as the incoming one, we get:

$$E^2 = m^2 + |\mathbf{p}|^2. \tag{2.9}$$

Now we can use eqs. (2.8) and (2.9) to calculate the mass of the photon:

$$m_\gamma^2 = E_\gamma^2 - |\mathbf{p}_\gamma|^2 = (E-m)^2 - E^2 + m^2 = 2m^2 - 2Em. \tag{2.10}$$

Since we know from eq. (2.9) that $E > m$ eq. (2.10) means that the photon has an imaginary mass in this reference frame!

This is a clear violation of the physical laws as we know them and it shows that the diagram of figure 2.1 does not show the dynamics of the interaction, it merely represents one part of the calculation of the probability to go from the initial to the final state.

Finally some remarks on the validity of these procedures. The idea of studying the probability of going from some state $|i\rangle$ to some state $|f\rangle$, is essential to study scattering processes but for the study of bound states and strongly coupled theories we need other tools. However, since scattering of high energy particles in large accelerator experiments like the LHC is the main experimental arena in particle physics, these methods are of great importance.

One should also note that the procedure used above describes a perturbation expansion of the theory, the expansion of the field according to eq. (2.3) is done by only using the field equations from terms of order two in the fields and higher order terms are regarded as interactions and are added afterwards as perturbations. It should be remembered that this approach only works as long as the fields only take small values, i.e. the second order terms have to be much larger than the higher order ones. This is not usually a big problem but it is important when choosing the point around which to expand the field in a spontaneously broken gauge theory.

2.2 Renormalization

One problematic feature with the approach to quantum field theory described above is that when we try to calculate the amplitude corresponding to diagrams with loops, the result often turns out to diverge to infinity.

One might think that a theory that gives infinite results from standard calculations, is completely unphysical and belongs in the garbage. However, there are other ways forward, if we accept that couplings and parameters entering the original lagrangian are merely unphysical (bare) parameters and not observable quantities, we can let the observable quantities absorb the infinities and the theory works fine again, at least from a mathematical point of view [6, 8].

From a philosophical point of view, though, this might appear more problematic and there are many important physicists who do not feel comfortable with the state of affairs [9, 10].

Another way of looking at this is to consider the theory under discussion an effective theory rather than a fundamental one and then one would expect some more fundamental theory to become important at higher energy. Since the problematic divergences appear when the momentum integral is taken to infinity, they can be considered artifacts of our ignorance about the deeper theory.

To illustrate the technical procedure, let is look at the mass term for a scalar field ϕ. This mass term takes the form

$$-\frac{1}{2}m^2\phi^2, \tag{2.11}$$

where m is the bare mass of the field ϕ. This term can be split into two terms

$$-\frac{1}{2}m_p^2\phi^2 - \frac{1}{2}\delta_m^2\phi^2, \tag{2.12}$$

where δ_m is called a counter term and m_p is to be considered the physical mass. The trick is now to regard the counter term as an interaction and adjust it so that the Feynman rules it introduces, exactly cancel the divergences from loop diagrams. We can always adjust the counter term so that m_p represents the physical mass, which means that a tree level calculation will give a more exact result than in the unrenormalized theory, where a larger part of the amplitude will come from loop diagrams.

In general, we expect counter terms for all terms in the lagrangian and some of them might take divergent values. This is not to be considered a problem since the term itself is not an observable and the result of calculations where it takes part always give finite results.[5]

Since the determination of counter terms is done by looking at some diagram, the split into counter terms and physical terms might depend on the momentum of the incoming particles. This scale dependence is quantified in the Renormalization Group Equations (RGE) that describe the scale dependence (running) of the parameteres of the theory.

2.3 Gauge Theory

As long as we have studied the universe, we have used symmetries to understand it. Modern Particle Physics is no exception, on the contrary; through the invention of gauge theory, symmetries have taken the lead role in the quest for a deeper understanding of nature.

The idea of gauge symmetry can be summarized as the process of introducing interactions through localized symmetries. If a theory is invariant under some global

[5] All divergences have to be regularized before the renormalization procedure is used, so technically speaking we never have divergences in the calculations.

symmetry, we may promote the symmetry to a local symmetry and to do that we need to introduce a new field that will act as a force mediator.

The result is a theory including interactions. This way of introducing interactions is now fundamental to physics; it is the fundamental idea behind the construction of the Standard Model of particle physics. As a matter of fact, also general relativity can be deduced by promoting the Lorentz symmetry to a local symmetry.

To introduce the concept we start with the simplest gauge theory, Quantum Electro-Dynamics (QED) [7, 11].

2.3.1 Quantum ElectroDynamics

Let us start with a spinor[6] ψ with a lagrangian density

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi, \tag{2.13}$$

where m is a mass parameter. We notice that eq. (2.13) has a symmetry of type $U(1)$, i.e. the transformation $\psi \to e^{i\theta}\psi$, where θ is a constant angle, leaves the lagrangian unchanged.

If we now allow θ to vary with space-time coordinates, i.e. $\theta \to \theta(x)$, we see that the derivative of eq. (2.13) will act on $\theta(x)$ and thus render the lagrangian non-invariant under this new transformation. More precisely the lagrangian now transforms as

$$\mathcal{L} \to \bar{\psi}(i\partial\!\!\!/ - e^{-i\theta(x)}(\partial\!\!\!/\theta(x))e^{i\theta(x)} - m)\psi = \bar{\psi}(i\partial\!\!\!/ - \partial\!\!\!/\theta(x) - m)\psi. \tag{2.14}$$

In order to restore invariance, we have to introduce a new field A_μ that transforms according to $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\theta(x)$. If we now change the partial derivative of (2.13) into a covariant derivative, $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$ we get an extra term from the field A_μ after the transformation that exactly cancels the problematic term $-\partial\!\!\!/\theta(x)$ and therefore the invariance is restored.

This might seem like a rather strange thing to do, why should we require invariance under this local version of the symmetry?

The simplest answer lies in the result: the introduction of the covariant derivative $D_\mu = \partial_\mu + eA_\mu$ will give a term of type $e\bar{\psi}A\!\!\!/\psi$ in the lagrangian, i.e. an interaction term between the field ψ and our new field A_μ. On closer inspection it turns out that this interaction looks exactly like an electron ψ interacting electromagnetically through the photon field A_μ, in other words, it looks like we have reinvented electrodynamics.

To properly formulate the theory we have arrived at, we need to introduce the kinetic terms for the field A_μ, which takes the form $F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This gives us the lagrangian of QED;

$$\mathcal{L} = \bar{\psi}(iD\!\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{2.15}$$

The lagrangian (2.15) can now be used to study electromagnetic scattering of charged fermions and the field equations of the field A_μ take the form:

$$\partial^\mu F_{\mu\nu} = -e\bar{\psi}\gamma_\nu\psi, \tag{2.16}$$

[6] A spinor is a representation of the infinitesimal Lorentz group that correspond to spin-$\frac{1}{2}$ particles.

which, after some mathematics, can be recognized as equivalent to the Maxwell equations.

The above procedure can be applied to other symmetries as well, rendering field theories with certain interactions. The advantage of this approach to constructing field theories is that the resulting theory will remain renormalizable after quantization, as long as the symmetry is maintained.

If the rather pragmatic argument used above to motivate making the symmetry local seems inadequate, one could argue that we do not expect space-like separated parts of the universe to be correlated in any way and a global symmetry would represent some sort of such correlation. However, this is a rather *ad hoc* argument since we could view these symmetries as mathematical constructions rather than any real property of nature and then this argument would be invalid. One may also note that there are indeed examples of well established correlations over space-like distances within quantum entangled states so this question is far from resolved [12].

2.3.2 General gauge theory

Let us now look at the more general form of the gauge theory. We start with a group G of continuous symmetries on some vector space V. This is by definition a Lie group.

For a general Lie group we can define its generators X^a that correspond to infinitesimal group transformations. A general group transformation of some object $v \in V$ can then be written:

$$v \to e^{ia^a X^a} v, \tag{2.17}$$

where a^a are some real parameters. The group generators form an algebra called a Lie algebra:

$$[X^a, X^b] = ic^{abc} X^c \tag{2.18}$$

where c^{abc} are real numbers called structure constants. Since the group generators are enough to specify any group transformation, the generators and their algebra is all we need to completely specify the group.

Let us now see what happens if we try to make a consistent theory around this symmetry, starting with some fermion field ψ that is expected to transform under the symmetry group G as vectors in V, i.e. $\psi \to e^{ia^a X^a} \psi$. Being fermionic, ψ has to obey the Dirac equation and therefore we need a lagrangian like eq. (2.13).

Let us now make the symmetry local. Similarly to the case in QED that means new terms are showing up in the derivative. To cancel them is a little more tricky in general because the generators of the symmetry group do not necessarily commute. However, we still introduce a covariant derivative $D_\mu = \partial_\mu + igX^a A^a{}_\mu$ where the gauge field $A^a{}_\mu$ now transforms according to

$$A^a{}_\mu \to A^a{}_\mu + \partial_\mu a^a - gc^{abc} A^b{}_\mu a^c, \tag{2.19}$$

under an infinitesimal gauge transformation. The parameter g, is a number called the coupling constant that determines the strength of the particular interaction. This ensures the invariance of the fermionic part of the lagrangian. In addition we need to include kinetic terms for the gauge fields $A^a{}_\mu$. This is done by including the field strength tensors

$$F^a_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + gc^{abc} A^b{}_\mu A^c{}_\nu. \tag{2.20}$$

The kinetic term now takes the form $F^a_{\mu\nu}F^{a\mu\nu}$ and we can write down the lagrangian for a general gauge theory;

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}. \tag{2.21}$$

This seemingly simple lagrangian is the starting point for all of particle physics.

It is worth reflecting a bit on the properties of the particles in this lagrangian. The field ψ is a fermion field by assumption and it has to transform according to eq. (2.17) under the gauge transformations. These particles are said to transform as the fundamental representation of the symmetry group, the fundamental representation being the group representation of lowest dimension.

For the covariant derivative to make sense, the gauge field has to be a vector field; this is seen by the fact that it has to have the same Lorentz indices as the partial derivative. As can be seen in the above calculation, we need as many gauge fields as there are generators of the symmetry group and they have to transform according to the adjoint representation of the group.

2.3.3 Colour symmetry

The strong nuclear force of the Standard Model is described by an $SU(3)$ gauge symmetry and as such it follows the main lines of the previous section closely.

Since we have the symmetry $SU(3)$, i.e. complex 3×3 matrices with determinant equal to 1, the fundamental representation will basically be the group itself and therefore we have to put our fermions in triplets. The strong force only affects quarks and is only important on very small scales inside the atomic nucleus and in mesons. It does not mix with the other forces in any non-trivial way and therefore we do not have to deal with the components of the triplet separately but it suffices to add an index to the quark, stating in which position in the triplet it is located.

Due to the presence of exactly three possible positions in the triplet, it has become common to speak of this index as the colour of the quark, where colour refers to one of the three basic colours red, green and blue. One additional reason for the colour analogy is that quarks are never seen as free particles but only as $SU(3)$ singlet combinations of quarks and this can be well described by the colour analogy; one colour (quark) and its anti-colour (anti-quark) give a colour neutral object and all three colours (three quarks) combined gives white, i.e. colour neutrality. So we see that the only quark objects we expect to see are mesons (quark anti-quark pairs) and baryons (three-quark combinations) and that is consistent with the mathematics and experiments.

The $SU(3)$ symmetry needs a total of 8 generators, which e.g. can be taken to be the Gell-Mann matrices. The presence of 8 generators means we will have 8 gauge fields. These are usually called gluons and are massless vector bosons that have never been seen as free particles but only act within the mesons and baryons. This last point is in contrast to the photon of QED and is due to the colour charge of the gluon, since only colour neutral objects are allowed to be free particles, the gluon can only act within colour neutral quark combinations.

Due to the colour analogy, the theory of the strong interaction is called Quantum ChromoDynamics or QCD and as we have seen, it is a rather clean gauge theory in its fundamental construction. In that sense one might regard it as a relatively easy theory to work with compared to the electroweak theory that we will discuss later.

However, the strong nature of the theory means that calculations are notoriously difficult. As mentioned above, we know that the strong force has the property called confinement that makes sure all particles with colour charge (i.e. quarks and gluons) only appear in colour singlets i.e. configurations with total colour charge zero. The paradox is that although this is well known and one of the most important properties of QCD, we do not have a full mathematical understanding of this feature. Qualitatively the confinement arises due to a quark antiquark force that grows with distance, which means that at some point the potential energy between the two particles will be so great that new quark-antiquark pairs will be pulled out of the vacuum and allow the formation of colour neutral hadrons [6].

The calculational problems in QCD are of vital importance if one wants to understand the physics at a proton-proton collider like the LHC. The first problem is to calculate which of the constituents of the protons will collide and how much energy they will have. This problem is handled by parton distributions functions, pdfs, that parameterize the probabilities for the various constituents, called partons, to interact at a given energy [13]. These pdfs cannot in general be fully calculated but have to be measured.

The next problem is that the final state of most interactions will contain quarks and they will somehow form colour-neutral hadrons long before we can detect them. This is again a process that cannot be calculated exactly. There are, however, approximation schemes that can deal with this. One of the most important is the Lund string model [14] that exploits the flux-tube picture of colour interactions. The colour flow in this model is represented by a string that then will be cut into pieces that are seen as mesons. This model is the basis for the hadronization scheme in the event generator PYTHIA [15] that has been extensively used throughout this thesis.

2.3.4 The electroweak symmetry

The electroweak theory [16] is a more complicated application of gauge theory due to the presence of two symmetry groups that mix in a non-trivial way as well as the massive nature of the gauge bosons of the weak force. The symmetry group to discuss is $SU(2) \times U(1)$ which means we have to place the fermions in complex doublets.

The group $SU(2)$ is the same group that describes the spin of fermions and therefore we know that we can use the Pauli spin matrices as generators of the symmetry. There are three such matrices and hence we need three gauge fields associated with the $SU(2)$ part of the symmetry, let us call these fields W^1_μ,W^2_μ and W^3_μ. We also need a gauge field for the $U(1)$ part of the symmetry and that we shall call B_μ.

In order to make contact with interactions we observe in nature, these gauge fields have to mix in the following way:

$$W_\mu^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, \quad (2.22)$$

$$W_\mu^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}, \quad (2.23)$$

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu, \quad (2.24)$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3, \quad (2.25)$$

where θ_W is the Weinberg mixing angle. The new fields are the ones observed in nature; the A_μ being the electromagnetic, or photon field and $W^\pm{}_\mu$ and Z_μ are responsible for the weak interaction.

Since we have two symmetry groups, we need two gauge field terms in the covariant derivative, $D_\mu = \partial_\mu + igX^aW^a{}_\mu + ig'B_\mu$ where X^a are half times the Pauli matrices and g and g' are the coupling constants. To make sure the electromagnetic interaction works as we are used to we have to require

$$g\sin\theta_W = g'\cos\theta_W = e, \quad (2.26)$$

where e is the electromagnetic coupling constant.

To make the picture more complicated we also need to take into account that different particles interact differently under these symmetries. For the $SU(2)$ this is not so difficult, there are singlet fields that do not transform under the symmetry and there are doublets that do. Let us introduce a charge I_3^W that represents the eigenvalue of a field under transformation by X^3.[7] This means that I_3^W will be $\frac{1}{2}$ for the upper component and $-\frac{1}{2}$ for the lower component of a doublet. In slight abuse of the formalism we assign $I_3^W = 0$ for singlets.[8] Note that the eigenvalues $\pm\frac{1}{2}$ are the same as the eigenvalues for the third spin component for fermions, as we would expect.

The $U(1)$ symmetry has an infinite number of ways particles can transform under it. These can be summarized by introducing the hypercharge Y by saying that a field ψ transforms under $U(1)$ as

$$\psi \to e^{iY\theta(x)}\psi, \quad (2.27)$$

where $\theta(x)$ is some arbitrary function. If there is only one field ψ in the theory we could absorb the charge Y into the function $\theta(x)$ but since we have several fields and need them to transform differently under the same $U(1)$ transformation, we need to keep Y and assign field-specific values to it. To maintain gauge invariance, this also requires us to include the hypercharge Y in the covariant derivative $D_\mu = \partial_\mu + igX^aW^a{}_\mu + ig'YB_\mu$. These charges can in principle take any real values but are chosen so as to reproduce the physics we see.

The charges I_3^W and Y can be used to find an expression for the electric charge Q. To do this we use the mixing of $W^3{}_\mu$ and B_μ into the fields A_μ and Z_μ together with

[7] These charge assignments are representation dependent and the values given here are only true if the Pauli matrices are used.

[8] This is abuse of the formalism because 0 is not an eigenvalue of I_3^W, but this assignment simplifies the further discussion.

eq. (2.26). The last two terms of our covariant derivative can be written

$$igI_3^W W^3{}_\mu + ig'YB_\mu = ie\left(\frac{I_3^W W^3{}_\mu}{\sin\theta_W} + \frac{YB_\mu}{\cos\theta_W}\right) \tag{2.28}$$

$$= ie\left(\frac{I_3^W \sin^2\theta_W - Y\cos^2\theta_W}{\sin\theta_W\cos\theta_W}Z_\mu + (I_3^W + Y)A_\mu\right). \tag{2.29}$$

The first term is the term for the Z_μ field which does not interest us at the moment while the second term gives the electromagnetic interaction and we can see from this that the electric charge Q should be given by

$$Q = I_3^W + Y. \tag{2.30}$$

Note that Q is defined such that the physical charge is Qe where e is the electromagnetical coupling constant which also denotes the fundamental electric charge. From eq. (2.30) we can now assign values of Y such that the particles have the observed electric charges.

Noting that the value of I_3^W differs by 1 between the upper and lower component of an $SU(2)$ doublet, we see that since the whole doublet has to have the same hypercharge, there will be a difference of 1 in electric charge between the two components as well. This means that the two fields of the doublet will behave very differently, in stark contrast to the components of the colour triplets which we can regard as essentially the same particle but with different colour charge.

The last point to note about the electroweak interaction is perhaps also the strangest. It turns out that only left-chiral fields have non-zero I_3^W, no right-chiral fermions transform under the $SU(2)$ symmetry. This is an experimental fact that can relatively easily be included in the theory but we do not have any theoretical reason why this is the case.

2.4 Spontaneous symmetry breaking

The proof that a gauge theory is renormalizable as long as the gauge symmetry is maintained [17], is a crucial step towards the Standard Model of particle physics. However, it also comes with a problem; we know that the particles we see have masses, but if mass terms are added to the lagrangian, they break the gauge symmetry.

More specifically, the mass term for a gauge boson Z_μ is of the type:

$$\frac{M_Z^2}{2}Z_\mu Z^\mu \tag{2.31}$$

and it is clear that this is not invariant under a gauge transformation. This mass term is therefore forbidden by the gauge symmetry. Since we know that the weak bosons have masses, it seems that the whole idea of gauge theory has reached a dead end.

The way around this is called spontaneous symmetry breaking. The idea is that the symmetry is left intact in the lagrangian but the vacuum state is allowed to break it. In order to achieve this, we need the vacuum state to be non-trivial, i.e. we need some field that takes a non-zero value in the vacuum state. Since we do not want to give the

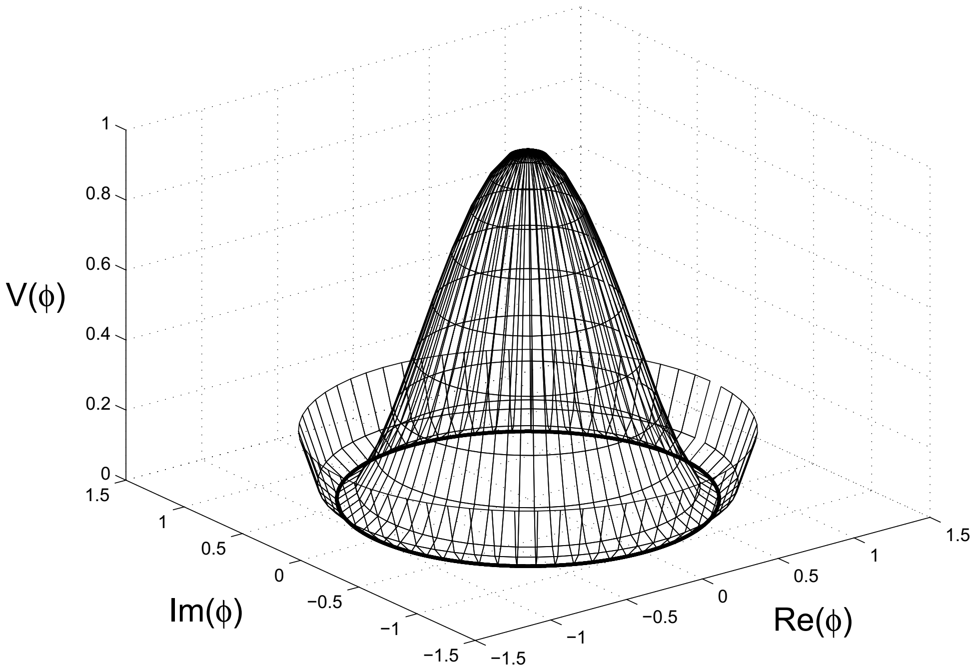

Figure 2.2: The Mexican hat potential.

vacuum properties like spin or electric charge, the field responsible for the non-trivial vacuum has to be a neutral scalar field.

The simplest implementation of this idea introduces one new fundamental scalar field that does the symmetry breaking. To introduce the main ideas we start by breaking a $U(1)$ symmetry. Although the $U(1)$ symmetry of the SM is in fact not broken (the photon is indeed massless), this is a good way of describing the idea and the generalization to a more realistic scenario is more or less straight forward. In order for our scalar field to be invariant under a $U(1)$ or in other words, a phase rotation, we need to work with a complex scalar field $\Phi(x)$.

To achieve a non-trivial vacuum state we need the field to obtain its lowest energy for a non-zero field value. This can e.g. be achieved by introducing a potential of the kind:

$$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2 \tag{2.32}$$

where v will be shown to be the vacuum expectation value (VEV) and λ is a coupling constant. The potential (2.32) is often referred to as the Mexican hat potential for reasons that are obvious from figure 2.2.

The vacuum state of Φ should have the lowest possible energy and will therefore be somewhere along the ring with $|\Phi| = v$, hence the name VEV for v. We have no way of telling where on the circle the field is but what is clear is that the vacuum state is no longer invariant under the $U(1)$ symmetry; a $U(1)$ transformation will move the vacuum state around on the circle.

We now use this to rotate the field so that $\langle 0|\Phi|0\rangle = v$, i.e. we rotate away the phase of the VEV (just for simplicity).

Now we want to quantize this theory. Normally, quantization is done by a Fourier expansion around 0; here, however, we need to expand around the VEV, v, instead. This means that we write our field Φ as

$$\Phi = v + \frac{\phi + i\eta}{\sqrt{2}}, \tag{2.33}$$

where ϕ and η are real scalar fields. As expected from a complex scalar field, we get two degrees of freedom: ϕ and η.

In order to see the effects of this symmetry breaking we need to expand the original lagrangian in terms of ϕ and η, we therefore have to specify our theory more precisely. Let us look at some examples.

2.4.1 Broken Global symmetry

Let us start with the simple but perhaps not so useful case of a global $U(1)$ symmetry. Although this does not solve any problems with introducing mass terms, it does enlighten some important aspects of the breaking of symmetries.

The lagrangian will here only contain the complex scalar field Φ (with complex conjugate $\bar{\Phi}$) and can be written:

$$\mathcal{L} = (\partial_\mu \bar{\Phi})(\partial^\mu \Phi) - \frac{\lambda}{4}(|\Phi|^2 - v^2)^2. \tag{2.34}$$

The result from replacing (2.33) into the lagrangian (2.34) is:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\eta)^2 - \frac{\lambda}{4}\left(\left(v + \frac{\phi}{\sqrt{2}}\right)^2 + \frac{1}{2}\eta^2 - v^2\right)^2 \tag{2.35}$$

$$= \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\eta)^2 - \frac{\lambda}{4}\left(\sqrt{2}v\phi + \frac{1}{2}\phi^2 + \frac{1}{2}\eta^2\right)^2. \tag{2.36}$$

Since the field η does not have any mass term in this lagrangian, it has to be a massless scalar field. This is a general consequence of breaking a global symmetry, we get massless scalars, usually called Goldstone bosons, that can be proven to always show up in these types of models [18]. Since no such scalars are observed in nature, this is an important point to remember when dealing with spontaneously broken symmetries.

The field ϕ on the other hand, has acquired a mass term, but it is worth noticing that this is not of direct interest for the sake of mass generation in general; for scalar fields there is no problem in introducing mass terms; mass terms for scalar fields do in general obey all symmetries of the theory. In addition, we have so far no evidence for the presence of any massive or massless fundamental scalar field in nature, so whether or not they can be given mass terms is not a big issue at this stage.

2.4.2 The Higgs mechanism

As soon as the symmetry that is to be broken is a gauge symmetry rather than a global symmetry, things become more interesting [19, 20]. The general discussion on spontaneous breaking of symmetries already indicated that we need to break the $SU(2)\times U(1)$ of the electroweak model.

Again we start with the simplified case of a $U(1)$ symmetry; this allows us to illustrate the idea without the mathematical complexity of the electroweak model. It should be noted though that our interest in breaking a $U(1)$ gauge symmetry is very limited since we want the electromagnetic $U(1)$ to remain unbroken in order to keep the photon massless.

We now need two fields, the complex scalar Φ and the gauge field A_μ. The starting point for this discussion is the lagrangian:

$$\mathcal{L} = (\bar{D}_\mu\bar{\Phi})(D^\mu\Phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}(|\Phi|^2 - v^2)^2, \tag{2.37}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu + ieA_\mu$. Again we should insert the expansion of Φ from eq. (2.33) into the lagrangian (2.37) and the result is the following:

$$\mathcal{L} = \frac{1}{2}(\bar{D}_\mu\phi)(D^\mu\phi) + \frac{1}{2}(\bar{D}_\mu\eta)(D^\mu\eta) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.38}$$

$$+ e^2v^2A_\mu A^\mu - \sqrt{2}evA_\mu\partial^\mu\eta - \frac{\lambda}{2}v^2\phi^2, \tag{2.39}$$

where all terms of order > 2 have been omitted since they are to be considered interactions and should therefore not be included in the discussion of the free fields of the theory.

Although we have used our $U(1)$ symmetry to rotate away the phase of the VEV of Φ, we still have the gauge invariance that can be used to simplify this expression. This is very fortunate since we have the term $-\sqrt{2}evA_\mu\partial^\mu\eta$ that does not look like anything we know from before; given that it is a second order term we expect it to be part of the free field solutions but since it contains two different fields it clearly induces interactions. One way of looking at this term is in the form of a mixing between the two involved fields and then it is clear that we should use a gauge rotation to remove it if we want to study the real particle content of the theory.

To resolve the issue with the term $-\sqrt{2}evA_\mu\partial^\mu\eta$ we make a gauge rotation of the form $\Phi \to e^{i\arctan\left(\frac{\eta}{\sqrt{2}v+\phi}\right)}\Phi$. This removes the complex phase of Φ and the lagrangian now takes the form[9]:

$$\mathcal{L} = \frac{1}{2}(\bar{D}_\mu\phi)(D^\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2v^2A_\mu A^\mu - \frac{\lambda}{2}v^2\phi^2. \tag{2.40}$$

The field η has now disappeared and we may conclude that it can be regarded a gauge artifact. The vector field A_μ on the other hand has gained a mass $M_A = \sqrt{2}ev$ and this is in fact what we wanted when we started trying to spontaneously break symmetries.

It is important to note that the field η is no longer present in the final theory. This was the problematic Goldstone boson from the spontaneously broken global symmetry. Since the vector field A_μ gained mass in the process we say that the Goldstone boson

[9]The gauge rotation will of course also change the field ϕ but for the sake of simplicity we keep the same name for the new field.

η has been eaten by the gauge boson and hence given it mass. It is also worth noting that there is no problem with the degrees of freedom; the Goldstone degree of freedom has been transferred to the vector boson; remember that a massive vector field has three degrees of freedom as compared to the two degrees of a massless vector field.

2.4.3 Electroweak symmetry breaking

If the symmetry to be broken is a more complicated one like e.g. $SU(2)\times U(1)$ [21], we first need a slightly more complicated scalar field. Since the scalar field needs to transform under $SU(2)\times U(1)$ transformations we need a complex scalar doublet

$$\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \tag{2.41}$$

where Φ_1 and Φ_2 are complex scalar fields. In a similar fashion as for the $U(1)$ symmetry we can rotate the VEV of the Higgs as we wish and in order to keep the electromagnetic physics formulated the way we are used to, it is convenient to choose

$$\Phi_0 = \langle 0|\Phi|0\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{2.42}$$

We then need to expand the doublet Φ around Φ_0 and that gives us

$$\Phi = \begin{pmatrix} \frac{\eta_1+i\eta_2}{\sqrt{2}} \\ v+\frac{\phi+i\eta_3}{\sqrt{2}} \end{pmatrix}, \tag{2.43}$$

where η_{1-3} and ϕ are real scalar fields.

It is now possible to do a gauge transformation so that the Higgs field takes the form

$$\Phi = \begin{pmatrix} 0 \\ v+\frac{\phi}{\sqrt{2}} \end{pmatrix}. \tag{2.44}$$

This is referred to as the unitary gauge and is convenient since it does not contain any unphysical degrees of freedom. To see why this transformation is always possible we note that we can use an $SU(2)$ transformation to remove the upper component of the doublet and then we can use a $U(1)$ rotation to remove the complex phase as we did for the local $U(1)$ symmetry.

Since we want the electromagnetic $U(1)$ symmetry to remain unbroken, it is necessary that the VEV v is electrically neutral. This can be achieved by an appropriate choice of hypercharge for the Higgs doublet. Since the VEV is in the lower component of an $SU(2)$ doublet, it has $I_3^W = -\frac{1}{2}$ and since the electric charge $Q = I_3^W + Y$, we need hypercharge $Y = \frac{1}{2}$ for the Higgs doublet.

With these charge assignments we see that we have spontaneously broken the electroweak $SU(2)\times U(1)$ symmetry down to the electromagnetic $U(1)$ symmetry. In the process three Goldstone degrees of freedom of the Higgs doublet, η_{1-3}, have been eaten by the weak gauge bosons. In addition the gauge bosons have gained masses of the type

$$M_{W^\pm} = \frac{gv}{\sqrt{2}}, \tag{2.45}$$

$$M_Z = \frac{gv}{\sqrt{2}\cos\theta_W}. \tag{2.46}$$

The first thing we notice is that the masses are related as $M_{W^\pm} = M_Z \cos\theta_W$. We also see that the VEV of the Higgs field can be determined from the strength of the weak interaction and the mass of the weak bosons.

Also the Higgs field will gain a mass in this process and the value of that mass is given by

$$M_H = \sqrt{\lambda}v. \tag{2.47}$$

The presence of the coupling constant λ in the Higgs mass means that, despite our knowledge of the Higgs VEV, we do not know the Higgs mass.

2.4.4 Yukawa couplings

After having successfully introduced masses to the vector bosons in a gauge invariant manner, we turn our attention to the fermions. Fermions can be given two kinds of mass terms[10], the Dirac mass terms:

$$\psi_L^\dagger m \psi_R + \psi_R^\dagger m \psi_L, \tag{2.48}$$

where ψ is the fermion field and the L and R indices refer to the component with left- and right-chirality, respectively. The other possibility would be a Majorana mass term which for a left-chiral neutrino looks like

$$\psi_L^\dagger m \psi_L. \tag{2.49}$$

The Majorana mass terms describe fermions that are their own antiparticles (called Majorana fermions) and are therefore only allowed for neutral particles. Since we are here concerned with mass terms for charged fermions[11] we only look at the Dirac mass terms for the moment.

It might seem like the terms of eq. (2.48) are in fact already gauge invariant so that adding masses to the charged fermions can be done without using the Higgs mechanism in any way. This assumption, however, turns out to be a bit naive.

If the left- and right-chiral components of the fermions would transform the same way under the gauge transformations, eq. (2.48) would indeed be gauge invariant. However, it is experimentally well established that this is not the case, it is only the left-chiral components that transform under the $SU(2)$ symmetry of the standard model. Why this is the case is a mystery.

Regardless of the reason for this strange fact, eq. (2.48) is clearly not gauge invariant so we need the Higgs also here. The solution is to use Yukawa couplings, i.e. couplings between one scalar field and two fermion fields written as

$$\psi_L^\dagger \Phi \psi_R + \psi_R^\dagger \Phi^\dagger \psi_L, \tag{2.50}$$

[10] The nature of the mass terms depends on the representation used for the gamma matrices.

[11] The only known neutral fermions, the neutrinos, are massless in the Standard Model. We will discuss neutrino masses later.

where Φ is the Higgs doublet. We see that the transformation of the Higgs field will combine with the transformation of the left-chiral fermion field to make a gauge invariant quantity. When the Higgs field acquires a VEV as described in section 2.4, we can again transform into the unitary gauge and we see that the fermion will have a mass term of the form

$$\psi_L^\dagger v \psi_R + \psi_R^\dagger v \psi_L. \tag{2.51}$$

We see that due to the parity violation in the weak sector, the Higgs mechanism is crucial also in order to give masses to the charged fermions.

2.5 The Standard Model

Let us now summarize the Standard Model (SM) of particle physics.

What we need to do is to assign all particles the right charges for the symmetries. For the gauge bosons we are essentially done; we have the massless vector field A_μ for the electromagnetic force and 8 massless gluons for the strong force. In addition we have the weak massive bosons $W^\pm_\mu$ and Z_μ.

Responsible for the electroweak symmetry breaking $SU(2) \times U(1) \to U(1)$ is the massive scalar field ϕ commonly called the Higgs field. It is important to note that this field has not been detected yet but the question of its existence is soon to be determined by the LHC. If the Higgs is found to not exist we have a big problem as the Standard Model as formulated today would be excluded due to lack of a mechanism to spontaneously break the electroweak symmetry.

Turning now to the fermions, we have two groups that differ by the presence of strong interactions, quarks do interact strongly while leptons do not. This means that quarks have to be placed in $SU(3)$ triplets. The left-chiral quarks also interact weakly so they have to be placed in $SU(2)$ doublets as well. The upper components of these doublets are referred to as up-type quarks and the lower components are referred to as down-type quarks. The right-chiral quarks do not interact weakly and therefore we need to place them in $SU(2)$ singlets and there will be one singlet for each of the two components of the left-chiral doublet.

In order for the electric charge of the $SU(3)$ singlet quark combinations that we observe in nature, to take integer values, we need the up-type quarks to have $Q = \frac{2}{3}$ and the down-type quarks need $Q = -\frac{1}{3}$. This can be achieved by the right assignments of hypercharge.

For the leptons we have no strong interaction but again we need to assign the left-chiral fields to $SU(2)$ doublets. This time the upper components are the neutrinos and the lower components are the charged leptons. Since we have not observed any right-chiral neutrinos, they are not included in the SM, but one may note that if such neutrinos would exist and in line with all right-chiral fields we know about, would be $SU(2)$ singlets, they would have Q, I_3^W and Y all zero and therefore would not have any interactions we can observe. For the right-chiral charged leptons on the other hand we need to include $SU(2)$ singlets.

The fermionic content of the SM is summarized in table 2.1. The electric charge can be calculated from this by the relation $Q = I_3^W + Y$ where I_3^W is zero for all right-chiral fields and $\pm\frac{1}{2}$ for the components of the left-chiral doublets. It is interesting to note

	$SU(3)$	$SU(2)$	Y
Q_L	✓	✓	$\frac{1}{6}$
U_R	✓		$\frac{2}{3}$
D_R	✓		$-\frac{1}{3}$
L_L		✓	$-\frac{1}{2}$
E_R			-1

Table 2.1: Fermion content of the SM, where the check marks denote whether a field is transforming under the respective symmetry. Also given is the hypercharge Y of all the fields. Q_L refers to a left-chiral quark doublet while U_R and D_R refer to right-chiral up and down quark $SU(2)$ singlets respectively. L_L denotes a left-chiral lepton doublet while E_R is a right-chiral charged lepton $SU(2)$ singlet.

that the hypercharge is different for all five fields in table 2.1 and there is no obvious pattern among the numbers.

Finally we should mention that the fields of table 2.1 all come in three copies called generations. These generations are identical apart from their masses. We say that the particles appear in three flavours, e.g. up quarks come in the three flavours (u,c,t) and down type quarks have the flavours (d,s,b) while the charged leptons include the flavours (e,μ,τ) with the corresponding neutrino flavours (ν_e,ν_μ,ν_τ).

Chapter 3

The need for New Physics

Despite being one of the most successful theories ever constructed, the Standard Model does have a lot of shortcomings. Apart from the fact that the theory might seem a bit *ad hoc* (why a $SU(3) \times SU(2) \times U(1)$ symmetry? Why does the $SU(2)$ only couple to left-handed particles? etc.) and the 20 parameters that have no theoretically preferred values; there are a number of issues that are more or less difficult to incorporate into the theory.

For the discussion of supersymmetry the most important of these problems are the hierarchy problem, dark matter and grand unification, but also the problem of neutrino masses as well as quantum gravity, might be of importance. These will also be the problems discussed here.

3.1 The hierarchy problem

One of the most discussed problems in the standard model regards the presence of a fundamental scalar field, namely the Higgs field, in the theory.

When a fundamental scalar field is present in a theory, it will receive radiative corrections to its mass. Since the Higgs couples to all heavy particles, the heaviest of these will give the largest contribution and therefore the Higgs mass should be pushed towards the highest scale of the theory. Since this is inconsistent with observations we seem to have a problem.

Of course, we are dealing with a renormalizable theory where the Higgs mass can be considered a free parameter so it seems we need a better understanding of this problem.

The problem is a consequence of the presence of quadratic divergences in the diagrams containing loops. When these divergences are removed through renormalization, the result is a contribution to the Higgs mass term that is of the scale of the particle in the loop.

This means that we expect the Higgs mass to be of the order of the heaviest particle in the theory, in the standard model that would be the top quark which seems just fine. However, we expect new physics to show up at higher scales, probably at least at some GUT scale but certainly at the Planck scale. If this new physics contains some particle, we would expect the Higgs mass to be of the scale of this new physics, i.e. far above the allowed range.

It is still possible to make the Higgs mass small by setting the bare mass parameter almost equal to the contribution from new physics, but now the theory starts to look a bit unlikely; at some high scale we have a Higgs mass parameter that is very large but just happens to cancel almost exactly with some radiative corrections so that in the low scale effective theory, the Higgs mass is suddenly many orders of magnitude smaller.

3.2 Dark matter

Over the last century the evidence for large amounts of nonbaryonic matter in the universe has been mounting. It started with kinematical observations of galaxies and galaxy clusters that showed a clear discrepancy between the amount of matter needed to explain the observed motion and the matter seen through optical observations [22].

Of course, these observations alone do not prove the nonbaryonic nature of the required extra matter. However, more recent measurements do point strongly in that direction.

First of all, the possibility of the extra matter being in the form of dead stars, free-floating planets and black holes, is now weakened by the absence of large numbers of such objects in microlensing surveys [23]. With the detailed studies of the Cosmic Microwave Background as performed by WMAP, it has become possible to measure both the matter density and the baryonic matter density of the universe independently of each other. The result clearly suggests a large nonbaryonic component to the total matter density [24].

Do we have any nonbaryonic matter in the Standard Model? Yes, we do, but only the neutrinos could possibly be many enough to be dark matter and in order to explain the formation of galaxies and galaxy clusters in the early universe the neutrinos turn out to be too light. The problem with light dark matter particles is that they were relativistic during structure formation and therefore traveled with almost the speed of light. Something that is traveling with close to the speed of light will not form structures due to gravitational collapse but rather erase such structures.

Before we assume the existence of a new kind of matter with properties different from all known matter, we might consider the possibility that there is some problem with our understanding of gravity at large distances. There have been numerous attempts in this direction, [25], but the large variety of measurements supporting dark matter is very hard to accommodate with a modification of the gravitational interaction.

Perhaps the biggest problem to explain is the bullet cluster measurements [26]. This concerns two galaxy clusters that have recently collided. For the baryonic matter in the cluster we know that the bulk is in the form of clouds of hydrogen and the location of these clouds can be traced by X-ray spectroscopy. At the same time we can measure the total matter distribution through gravitational lensing techniques. The result shows a clear separation between the two; the intergalactic gas clouds show clear shock fronts from the collision caused by electromagnetic interactions between the gas molecules. These interactions have also slowed the clouds down so that they lag behind the galaxies as well as the total mass of the clusters. This spatial separation between the total gravitating mass and the total visible mass, is perfectly consistent with the expectation if most of the mass is in the form of noninteracting dark matter and is very hard to explain by other means.

3.3 Grand unification

It is not very difficult to see the history of physics as a history of unification. Many of the ground-breaking discoveries have been achieved by unifying seemingly different phenomena, Newton unified the motion of the planets and the physics of falling apples in his law of gravity, Faraday and Maxwell unified electricity with magnetism to get electromagnetism, the elements of the periodic table were unified by the theory of the atom, Gell-Mann unified the zoo of mesons and baryons by the theory of the quarks, electromagnetism and the weak interaction were unified into the electroweak theory.

Now we have a model that is in its entirety constructed from the idea of gauge symmetry, the complete gauge group being $SU(3) \times SU(2) \times U(1)$, describing three different forces. Surely there must be a way of unifying these force into one? In other words, there should be some way of unifying these symmetries so that we have only one symmetry group rather than a product of three, should it not?

The idea of a Grand Unified Theory or GUT that unifies the forces of the SM, is almost as old as the SM itself [27]. What we need to realize this idea is a symmetry group that can be spontaneously broken to the SM group. The simplest such group is the group $SU(5)$. However, this model leads to too fast proton decay [28].

Despite the problems with proton decay, the idea of a GUT is thriving. As of today there is no concrete evidence in favour of unification of the forces but such a unification is usually assumed to exist anyway.

There is in fact one more argument in favour of the GUT idea, namely the running couplings as mentioned in section 2.2. If one extrapolates the values of the coupling constants for the three forces in the SM, one finds that they seem to go toward a similar value at high energy.[1] The unification is not perfect but close enough to believe that some new physics somewhere along the way can make sure they coincide perfectly, as a matter of fact, supersymmetry is known to improve the situation significantly in this respect. This would then be strong evidence in favour of a GUT.

However, it might be worth remembering that we do not know whether Grand Unification is part of nature or not.

3.4 Neutrino masses

From a number of neutrino experiments it is now clear that neutrinos change flavour as they travel through space [29]. Since massless particles do not experience time-evolution, a change of flavour over time indicates that neutrinos do indeed have mass.

In principle it is not so difficult to add mass terms for the neutrinos to the SM. The perhaps most obvious way of doing it would be to add Dirac masses similar to the masses of the charged leptons and the quarks. That would, however, require the introduction of right-chiral neutrinos. Such neutrinos are not observed in nature but, if we follow the structure of the weak interaction with respect to the leptons and quarks, we would not expect these right-chiral neutrinos to interact weakly and since they do not posses electric nor color charge, they would be invisible to us.

Since neutrinos are electrically neutral, it is also possible to add Majorana mass

[1] A similar value is actually a bit of an exaggeration, but at least they go towards the same region.

terms to them, in that case the need for right-chiral neutrinos is circumvented[2], but this introduces a number of new phenomenological features. The result of Majorana mass terms is that the neutrinos would be their own antiparticle and that would lead to lepton number violation in terms of e.g. neutrinoless double beta decay [30]. It also opens the door to produce the observed baryon asymmetry of the universe through these lepton number violating processes in combination with sphaleron processes during the electroweak phase transition [31].

Regardless of which type of neutrino masses we introduce we are stuck with a question: why are the neutrino masses so much smaller than the masses of all other known massive particles?[3]

The perhaps most popular way of answering this question is by the so-called seesaw mechanism [35]. The idea is to introduce some new neutrinos that are very heavy. These new neutrinos will mix with the known ones and the mass matrix can be given the form:

$$\begin{pmatrix} M & m \\ m & 0 \end{pmatrix}, \tag{3.1}$$

where M is some high (possibly GUT) scale and m is some electroweak scale. When the mass matrix (3.1) is diagonalized, one finds two mass eigenstates. One eigenstate has mass around M and one around $\frac{m^2}{M}$. The point is that $\frac{m^2}{M}$ is in fact much smaller than m and hence, from the presence of two scales we have constructed a third that is much smaller than the smallest scale we started with.

3.5 Quantum gravity

Let us take a look at the Einstein field equations for general relativity [36]. The most compact way of writing them would be:

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{3.2}$$

where $T_{\mu\nu}$ is the energy-momentum tensor, $G_{\mu\nu}$ is the Einstein tensor and κ is a constant.

Quantum mechanics tells us that the energy-momentum tensor, $T_{\mu\nu}$, is quantized. If we now assume that $T_{\mu\nu}$ is in a superposition of eigenstates to some operator, from eq. (3.2) we see that also $G_{\mu\nu}$ has to be in a similar superposition. A measurement related to the operator under discussion will now collapse the wavefunction of $T_{\mu\nu}$ and hence must also collapse the wavefunction of $G_{\mu\nu}$. The inevitable conclusion is that $G_{\mu\nu}$ behaves like a quantum system and must be quantized.

The conclusion of the above discussion is that the gravitational interaction needs to be quantized. The problem is that when attempting to do this along the most straight forward lines, one runs into problems with renormalizability. The gravitational interaction contains terms that give rise to infinities that cannot be canceled with counter

[2] There will still be right-chiral neutrinos in the theory, but they will just be what used to be anti-neutrinos.

[3] Recent measurements of the muon-neutrino propagation speed may even indicate imaginary masses for the neutrinos [32]. However, experimental errors [33] or perhaps violation of Lorentz invariance [34] are probably better ideas in this respect.

terms similar to the terms already in the theory [8]. If we were to follow this path anyway and include the required terms, we would end up including ever more new terms to cancel infinities but we would never achieve that goal.

This means that the quantum gravity theory is nonrenormalizable and cannot be used to make reliable predictions.

One less technical way of looking at this problem would be to think of a Feynman-diagram with a loop in it. Since the energy of the particle in the loop is unconstrained by the overall energy conservation, it can take infinitely high values. This is just the reason the momentum integrals have to be taken to infinity and thus can cause infinities in the first place.

Now imagine gravity is part of our theory; a particle of energy larger than the Planck mass would according to general relativity form a black hole. How do we interpret a particle in a loop that has to form a black hole?

The reason we end up at such a peculiar question is that our procedure of constructing a quantum field theory is done through a perturbative expansion of the fields on a fixed background. This must fail if gravity is fully included. The gravitational interaction works by changing the background; if a particle approaches infinite energy, the gravitational modification would go to infinity and we have no reason to expect that our approach has any validity.

Chapter 4

Supersymmetry

Since symmetries have had such an important role in physics so far, what would happen if we try to invoke the most general symmetry we can find in our theories?

This approach has been tried in a more limited sense; relativity theory tells us that space-time is invariant under the Lorentz symmetry group. What is then the most general nontrivial extension of this group?

The answer turns out to be supersymmetry (SUSY).

4.1 Structure and field content of supersymmetric theories

Starting with the Lorentz group, or more precisely the Poincaré group[1], we want to combine it with some other symmetry in a non-trivial way, that is, not merely as a direct product.

One of the first steps towards an understanding of this problem, was the proof by Coleman and Mandula [37], basically stating that it is impossible. They proved that the most general symmetry of the S-matrix is a direct product of the Poincaré group and some internal group. This means that the only symmetries allowed apart from the Poincaré group are symmetries that only affect the fields of the theory and do not affect spacetime and which are Lorentz invariant. The $SU(3) \times SU(2) \times U(1)$ symmetry of the SM is a good example of such internal symmetries[2].

However, it was later realized that by including anti-commuting operators in the algebra of the symmetries one can get around this [38]. It is in fact possible to include a symmetry with spinor generators. Let us call these generators Q_α and $\bar{Q}_{\dot{\beta}}$. Being spinors, they are not Lorentz invariant and must therefore have some non-trivial mixing with the Poincaré group. In fact we can see this from their algebra,

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu, \tag{4.1}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \tag{4.2}$$

$$[Q_\alpha, P^\mu] = [\bar{Q}_{\dot{\alpha}}, P^\mu] = 0, \tag{4.3}$$

where P_μ is the generator of spacetime translations and $\sigma^\mu_{\alpha\dot{\beta}}$ are the Pauli spin matrices.

[1] The Poincaré group is the group of all spacetime symmetries, i.e. the Lorentz group combined with spacetime translations.

[2] Note that it is the symmetry generators that need to be Lorentz invariant; the gauge fields being vectors is not any problem with respect to this theorem.

Due to the spinor nature of the operators Q_α and $\bar{Q}_{\dot{\beta}}$, they have a rather unusual effect when applied on fields; they change their spin by half a unit, turning bosons into fermions and vise versa!

The result is a symmetry between fermions and bosons and in order to make the theory consistent we need to place all particles in supermultiplets where the total number of bosonic and fermionic degrees of freedom are the same.

The assumption of supersymmetry only means that we need to place all particles in supermultiplets. It does not tell us how to do this and how many new particles we need. The simplest approach would be to try to add as few new particles as possible, this leads to the Minimal Supersymmetric Standard Model or MSSM. For some introductions to supersymmetric theory, see [39–41].

4.1.1 Superfields

The most elegant formalism for supersymmetric models is the superfield formalism [42]. We first introduce the anticommuting (Grassmann) variables θ^α and $\bar{\theta}^{\dot{\beta}}$. The superalgebra can be written in terms of these variables as:

$$\{\theta Q, \bar{Q}\bar{\theta}\} = 2\theta\sigma^\mu\bar{\theta}P_\mu, \tag{4.4}$$

$$\{\theta Q, \theta Q\} = \{\bar{Q}\bar{\theta}, \bar{Q}\bar{\theta}\} = 0. \tag{4.5}$$

Note that $\theta Q = \theta^\alpha Q_\alpha = \theta^\alpha Q^\beta \varepsilon_{\alpha\beta}$, where $\varepsilon_{\alpha\beta}$ is totally antisymmetric and $\varepsilon_{12} = 1$.

We now define the superspace as ordinary spacetime extended by these variables θ_α and $\bar{\theta}_{\dot{\beta}}$. We can then write Supersymmetry transformations as transformations in superspace [40]:

$$S(x_\mu, \theta, \bar{\theta}) = e^{i(\theta Q + \bar{Q}\bar{\theta} - x_\mu P^\mu)}. \tag{4.6}$$

Since generators of the supersymmetry should take the form of eq. (4.6), we expect fields in superspace, which we call superfields $\Phi(x_\mu, \theta, \bar{\theta})$, to transform according to [40]:

$$S(y_\mu, \alpha, \bar{\alpha})[\Phi(x_\mu, \theta, \bar{\theta})] = \Phi(x_\mu + y_\mu - i\alpha\sigma_\mu\bar{\theta} + i\theta\sigma_\mu\bar{\alpha}, \theta + \alpha, \bar{\theta} + \bar{\alpha}). \tag{4.7}$$

The particle content of these superfields can be seen by expanding them in power series of the variables θ_α and $\bar{\theta}_{\dot{\beta}}$. If we take a superfield Φ_L to be independent of $\bar{\theta}_{\dot{\beta}}$, we have a left-chiral superfield, sometimes Φ_L is also called a scalar superfield. Similarly, if a superfield Φ_R is independent of θ_α, it is called a right-chiral superfield. The left-chiral field can be written:

$$\Phi_L(x_\mu, \theta) = \phi + \theta^\alpha \psi_\alpha + \theta^\alpha \theta^\beta \varepsilon_{\alpha\beta} F, \tag{4.8}$$

where ϕ and F are complex scalar fields and ψ is a left-chiral Weyl fermion field.

If we instead expand in both θ_α and $\bar{\theta}_{\dot{\beta}}$ we get a representation that can be reduced to [40]:

$$V(x_\mu, \theta, \bar{\theta}) = -\theta\sigma_\mu\theta\bar{V}^\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D, \tag{4.9}$$

where V^μ is a spin-1 field, λ is a Weyl fermion and D is a real scalar field. This superfield is denoted a vector superfield or gauge superfield.

The scalar field F in left-chiral superfields and the scalar D in the vector superfields are auxiliary fields that do not propagate. This will be seen in the next section where they both will be removed from the lagrangian of the theory.

4.1.2 Supersymmetric lagrangian

In order to construct a supersymmetric lagrangian we start by including the kinetic terms associated with the matter fields[3], i.e. the chiral supermultiplets[4]. Denoting the Weyl fermions by ψ^i and the complex scalars by ϕ^i and F^i, we have[5]:

$$\mathcal{L}_{\text{free}} = -\partial^\mu \bar{\phi}^i \partial_\mu \phi^i - i\psi^{\dagger i}\sigma^\mu \partial_\mu \psi^i + \bar{F}^i F^i. \tag{4.10}$$

In order to determine the most general interaction potential it is convenient to introduce the superpotential W as an analytic function of the fields ϕ^i,

$$W = \frac{1}{2}M^{ij}\phi^i\phi^j + \frac{1}{6}y^{ijk}\phi^i\phi^j\phi^k, \tag{4.11}$$

where M^{ij} is a mass matrix for the fermions and y^{ijk} are Yukawa couplings. The only allowed terms in the lagrangian containing the fields F^i and $\bar{F}^i$ turn out to be $\bar{F}^i F^i + W^i F^i + \bar{W}^i \bar{F}^i$, where $W^i = \frac{\partial W}{\partial \phi^i}$ [41]. This leads to the equations of motion for the fields F^i:

$$F^i = -\bar{W}^i, \qquad \bar{F}^i = -W^i. \tag{4.12}$$

This can be used to substitute for the fields F^i in the lagrangian.

With $W^{ij} = \frac{\partial^2 W}{\partial \phi^i \partial \phi^j}$, the total lagrangian for the chiral supermultiplets becomes:

$$\mathcal{L}_{\text{chiral}} = -\partial^\mu \bar{\phi}^i \partial_\mu \phi^i - i\psi^{\dagger i}\sigma^\mu \partial_\mu \psi^i - \frac{1}{2}\left(W^{ij}\psi^i\psi^j + \bar{W}^{ij}\psi^{\dagger i}\psi^{\dagger j}\right) - \bar{W}^i W^i. \tag{4.13}$$

We see that the superpotential summarizes all interactions among the chiral superfields. If we look at the definition of W^{ij} we see that it is constructed by removing all combinations of two scalar fields in the superpotential and in the lagrangian they are replaced by their fermionic partners. This means that the M^{ij} gives mass terms for the fermions while the y^{ijk} give all possible Yukava couplings. The terms $\bar{W}^i W^i$ give purely scalar terms that include scalar masses and higher order interactions; in a spontaneously broken gauge theory, this is where the Higgs potential will be.

We now include terms involving the gauge supermultiplets. In this case we work with the gauge fields A^a_μ with associated Weyl fermions λ^a and real scalar fields D^a. The free-field part of the lagrangian for the gauge supermultiplets is:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^{\dagger a}\sigma^\mu D_\mu \lambda^a + \frac{1}{2}D^a D^a, \tag{4.14}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - igf^{abc}A^b_\mu A^c_\nu$ and $D_\mu \lambda^a = \partial_\mu \lambda^a + gA^b_\mu X^b \lambda^a$, X^b being the gauge symmetry generators. The replacement $\partial_\mu \to D_\mu = \partial_\mu + gA^a_\mu X^a$ in the lagrangian of the chiral supermultiplets gives the gauge interactions [43].

[3] A more stringent way of doing this would be to integrate the superfields over the parameters θ_α and $\bar{\theta}_{\dot{\beta}}$.

[4] A supermultiplet denotes the collection of fields in a superfield.

[5] Note that the Pauli matrices have replaced the Dirac gamma matrices due to the use of two-component (Weyl) spinors rather than four-component spinors.

Above we used the vector fields from the vector supermultiplets to introduce the gauge interactions. This, however, is not the complete story, the superpartners of the vector fields, the fermions λ^a and the scalars D^a, will obtain similar interactions with the components of the chiral multiplets. These interactions can be written $(\bar{\phi}X^a\psi)\lambda^a$, $\lambda^{\dagger a}(\psi^\dagger X^a\phi)$ and $(\bar{\phi}X^a\phi)D^a$. We do not calculate the coefficients for these terms here but state the result from [41], $-\sqrt{2}g(\bar{\phi}X^a\psi)\lambda^a$, $-\sqrt{2}g\lambda^{\dagger a}(\psi^\dagger X^a\phi)$ and $g(\bar{\phi}X^a\phi)D^a$.

The equations of motion for the fields D^a are now $D^a = -g(\bar{\phi}X^a\phi)$, and therefore D^a can be eliminated, which gives the full lagrangian [41]:

$$\begin{aligned}
\mathcal{L} \;=\; & -\bar{D}^\mu\bar{\phi}^i D_\mu\phi^i - i\psi^{\dagger i}\sigma^\mu D_\mu\psi^i \\
& -\frac{1}{2}\left(W^{ij}\psi^i\psi^j + \bar{W}^{ij}\psi^{\dagger i}\psi^{\dagger j}\right) - \bar{W}^i W^i \\
& -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - i\lambda^{\dagger a}\sigma^\mu D_\mu\lambda^a \\
& -(\sqrt{2}g\bar{\phi}X^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger X^a\phi) - \frac{g^2}{2}(\bar{\phi}X^a\phi)^2.
\end{aligned} \tag{4.15}$$

4.1.3 The Particle content of the MSSM

When placing particles in supermultiplets, we need to respect their quantum numbers. This means that none of the known particles can be placed in the same multiplet[6] and therefore we have to introduce scalar partners to all fermions and fermionic partners to all gauge bosons.

Of course we could ask if it is possible to introduce partners with higher rather than lower spin as compared to our SM particles. Although this might be possible; since we are interested in the simplest supersymmetric model we do not pursue such possibilities further.

To give these new particles names, a standardized approach is used; the scalar partners of the fermions are given the same name as their fermionic partners but with an “s” for “scalar” in front. So we have e.g. selectron, sneutrino, stop, sup and the unpronounceable sstrange. The same approach is sometimes used also for the whole groups of particles, i.e. sleptons, squarks and sfermions which if read out “scalar fermions” is a self-contradiction.

For the fermionic partners to the gauge bosons we again use the same names but with an “ino” extension in the end, giving us e.g. the gluino, Wino and photino.

Then we come to the Higgs sector. In the quark sector we need to give masses to both the up-type and down-type quarks (in the lepton sector, the neutrinos are massless and therefore only the leptons need masses). The down sector can be handled just like it is done in the SM with Yukawa terms:

$$\bar{\Psi}_L\Phi\psi_{d\,R} + \bar{\psi}_{d\,R}\Phi^\dagger\Psi_L, \tag{4.16}$$

where Ψ_L is a left-chiral $SU(2)$ doublet, Φ is the Higgs doublet and $\psi_{d\,R}$ is a down-type right-chiral $SU(2)$ singlet. For the up sector, though, there is a problem. In the SM one uses terms like,

$$\bar{\Psi}_L\widetilde{\Phi}\psi_{u\,R} + \bar{\psi}_{u\,R}\widetilde{\Phi}^\dagger\Psi_L, \tag{4.17}$$

[6] This is most easily seen by the fact that all vector bosons have to be in adjoint representations of the gauge groups while the fermions have to be in the fundamental representations.

SM particle	spin	Supersymmetric partner	spin
quark	$\frac{1}{2}$	squark	0
lepton	$\frac{1}{2}$	slepton	0
neutrino	$\frac{1}{2}$	sneutrino	0
$W_0, W^{\pm}, B_0$	1	Wino, Bino	$\frac{1}{2}$
Higgs	0	Higgsino	$\frac{1}{2}$
gluon	1	gluino	$\frac{1}{2}$

Table 4.1: Particle content of the MSSM.

where $\psi_{u\,R}$ is a up-type right-chiral $SU(2)$ singlet and

$$\widetilde{\Phi} = \begin{pmatrix} \bar{\Phi}_2 \\ -\bar{\Phi}_1 \end{pmatrix}. \tag{4.18}$$

This, however, does no longer work when supersymmetry is included. The most obvious reason is that all these Yukawa couplings have to come from the superpotential which only contains combinations of the scalar fields and no constructions like $\widetilde{\Phi}$.

The result is that we need to have two Higgs superfields, one being the SM Higgs (usually called H_1 or H_d) and one with hypercharge $-\frac{1}{2}$ (called H_2 or H_u). This gives us a total of 8 Higgs degrees of freedom. Similarly to the SM case, 3 of these will give masses to the weak bosons and we are therefore left with 5 Higgses.

The Higgs doublets also need superpartners and since they are scalars, we have no other choice than to give them fermionic partners. These are named Higgsinos, in line with fermionic partners to the gauge bosons.

The particle content of the MSSM is summarized in table 4.1.

In a similar fashion as the W^3_μ and B_μ mix in the SM, the neutral higgsinos, winos and binos as well as the charged higgsinos and winos will mix in the MSSM. The result of this mixing depends on the parameters of the SUSY model, but the notation is the same: the neutral fermions are mixed into four neutralinos χ^0_1, χ^0_2, χ^0_3 and χ^0_4 where the numbering is done according to the mass of the neutralinos, χ^0_1 being the lightest.

The charged higgsinos and winos are mixed into two charginos $\chi^\pm_1$, $\chi^\pm_2$ again with $\chi^\pm_1$ being the lighter one.

4.1.4 Benefits from including Supersymmetry

So how many of the shortcomings of the Standard Model can supersymmetry fix?

One of the most quoted reasons for considering TeV scale SUSY, is actually the hierarchy problem, mentioned in section 3.1. The point is that the loop contributions to the Higgs mass term from fermions and bosons have opposite sign and since supersymmetry can be seen as a symmetry between fermions and boson, it forces all these contributions to cancel. However, we know that SUSY has to be broken at some scale and that will destroy our cancellation of loop contributions to the Higgs mass term.

If the breaking of SUSY happens at the TeV scale, the cancellation of the quadratic divergences might still be good enough but if the scale of the breaking is pushed much higher the fine tuning issue will reappear. There are already claims that the μ parameter, which describes the coupling between the two Higgs doublets, $\mu H_1 H_2$, is more than acceptably fine tuned, leading to the invention of the NMSSM [44].

Another "problem" that supersymmetry can do something about is that of grand unification; it is known that supersymmetry significantly improves the convergence of the coupling constants at some GUT scale energy. A tantalizing consequence of this might be that we no longer seem to need any more new physics apart from SUSY before the GUT scale and that means we might be much closer to a theory of everything than one might think.

As will be discussed later also dark matter might be found among the new SUSY particles.

4.1.5 Supergravity and the Gravitino

In the case of gauge theories, the process of making the symmetries local gave unexpected and very useful results. What happens if we do the same with supersymmetry?

At first this seems to be an even better idea, the result is nothing less than general relativity!

However, after a bit of thought, the result is not so surprising; supersymmetry contains the Lorentz group and we know that gauging the Lorentz group is one way of deducing general relativity. We would have been surprised if we did not get general relativity also when supersymmetry is added to the game.

Due to the presence of gravity, local supersymmetry is usually referred to as supergravity. Unfortunately, the addition of supersymmetry does not fix the renormalization problems that arise when the theory is quantized.

In a similar fashion to the gauge theories, localizing supersymmetry forces us to introduce a new supermultiplet. This is the gravity supermultiplet and contains the spin-2 graviton and its superpartner, the spin-3/2 gravitino.

Although the MSSM in principle does not have local supersymmetry, one often adds the gravitino to the phenomenological discussion.

4.1.6 Breaking supersymmetry

The biggest problem with introducing supersymmetry, is how to break it. This paradox is not so different from the case with gauge theories; the need for mass terms forces us to introduce the Higgs mechanism to spontaneously break the gauge symmetry.

The reason we have to break supersymmetry is the rather obvious fact that we have not seen the superpartners to the SM particles. If supersymmetry was an exact symmetry of the universe we live in, these particles would have the same masses as their SM partners and would have been discovered long ago.

A general consequence of spontaneously breaking a symmetry, is massless Goldstone bosons. In the case of supersymmetry, we get a massless fermion usually called a goldstino. If the broken symmetry is local, the Goldstone boson is eaten by the gauge field which then acquires a mass. For local supersymmetry the goldstino is eaten by the gravitino and thus gives it a mass.

A consequence of this is that the mass of the gravitino is closely related to the scale at which supersymmetry is broken. For a vanishing cosmological constant we have [40],

$$m_{3/2} = \frac{M_S^2}{\sqrt{3}M}, \tag{4.19}$$

where M_S is the SUSY breaking scale, M is the Planck scale and $m_{3/2}$ is the gravitino mass. This means that a large range of masses are possible for the gravitino, from eV scale all the way up to the Planck scale.

There are many proposals for how to break supersymmetry, e.g. Gravity mediated breaking, Gauge Mediated SUSY Breaking (GMSB) and Anomaly Mediated SUSY Breaking (AMSB). All these ideas are merely speculation at this stage, so we must admit that we do not know how SUSY, if present, is broken.

Given the large uncertainty in the way to break supersymmetry, it is good to make some general points that apply to all such methods. The first is that the breaking should not reintroduce the fine-tuning of the hierarchy problem. This can be achieved if the breaking of SUSY only introduces so-called soft terms. These terms include mass terms for the sfermions and gauginos

$$\frac{1}{2}M^a\lambda^a\lambda^a, \qquad \frac{1}{2}(m^2)^{ij}\bar{\phi}^i\phi^j, \tag{4.20}$$

where M^a are gaugino masses and m^{ij} is a sfermion mass matrix. We also have some scalar couplings:

$$\frac{1}{2}b^{ij}\phi^i\phi^j, \qquad \frac{1}{6}a^{ijk}\phi^i\phi^j\phi^k, \tag{4.21}$$

where b^{ij} and a^{ijk} are coupling constants.

It is worth reflecting for a moment on the implications of introducing the mass terms of eq. (4.20). In the SM we needed the Higgs mechanism to allow masses, but there seems to be no such need here, how is that possible?

The answer is that the sfermions are scalars and mass terms to scalar particles do not violate any gauge invariance, this is actually the reason we are allowed to add the appropriate terms to the Higgs potential to make the Higgs mechanism work. For the gauginos there is also nothing that prevents us from adding mass terms; the only reason we needed the Higgs for the SM fermions to acquire mass, was that only the left-chiral component of the fermion fields interact weakly, but for the gauginos the left- and right-chiral components interact just the same.

This is in fact an interesting point; one could argue that we should not be surprised that the sparticles are heavier than the known particles. If we adopt the attitude that everything that is not explicitly forbidden will happen, we would expect these mass terms for the sparticles to be non-zero and hence that the sparticles would be heavy.

That said, it would be more comforting to have some idea where these masses come from and what values we should give them.

4.2 R-parity

When constructing a supersymmetric lagrangian there are some couplings that can be included, which are rather special. First of all it turns out that the Higgs superfield H_2 and the Lepton superfields have the same quantum numbers. That means that we can construct new couplings by replacing H_2 with L_i in the Yukawa couplings as well as the μ term. The resulting couplings [45]:

$$\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \mu_i H_1 L_i, \tag{4.22}$$

where L_i, Q_i, H_1 are left-chiral lepton, quark and Higgs superfields and E_i, D_i are right-chiral lepton and down quark superfields, all violate lepton number ($L = 1$ for leptons and sleptons and 0 for other particles) and may therefore induce dangerous interactions. In addition we can write down the terms:

$$\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k, \tag{4.23}$$

where D_i, U_i are right-chiral down and up quark superfields, which violate baryon number ($B = \frac{1}{3}$ for quarks and squarks and 0 for other particles).

Both baryon and lepton number are conserved by accident in the SM at least at a perturbative level. The reason is that it simply turns out to be impossible to write down renormalizable terms that violate B or L.

Violating B and L is problematic for many reasons, first of all, violating both at the same time induces proton decay and we have very strict limits on the proton lifetime. Other problems include double nucleon decay that only requires B violation, neutron-antineutron oscillation and flavour changing neutral currents.

The most common way to deal with these problematic couplings is to introduce a discrete $\mathbb{Z}_2$ symmetry called R-parity (R_P), defined as:

$$R_P = (-1)^{3B-L+2S} \tag{4.24}$$

where B is baryon number, L is lepton number and S is the spin of the particle, and require it to be conserved. This has the effect that all SM particles have $R_P = 1$ and all their partners have $R_P = -1$. All the problematic couplings of eqs. (4.22) and (4.23) are then forbidden. The impact of including R-parity on the phenomenology is paramount; first of all it requires supersymmetric partners to be produced in pairs and secondly it forbids partners to decay to only standard model particles.

This last point is very important for cosmology because it means that any super-partners produced in the early universe cannot decay to SM particles and the Lightest SuperPartner (LSP) will therefore constitute a dark matter candidate.

There are, however, benefits from violating R-parity[7], in general one has to remember that we do not have any theoretical reason why R-parity should be exactly conserved. Moreover, the tightest constraints on the couplings (4.22) and (4.23) only constrain products of couplings, most importantly $\lambda'_{11k}\lambda''_{11k} < 10^{-22}$ [50] and $\lambda'_{ijk}\lambda''_{ijk} < 10^{-11}$ [51] from proton decay, in addition some individual couplings are tightly constrained, like $\lambda''_{112} < 10^{-6}$ [52] from double nucleon decay, but it is still possible to have quite large amount of R-parity violation without coming into conflict with experiments [53].

The lepton number violating terms of eq. (4.22) can also induce neutrino masses [54].

[7]For some early discussions on R-parity violating SUSY, see. [46–49].

4.3 Supersymmetric Dark Matter

Given the requirement for heavier partners to all SM particles, one might expect to find plenty of good dark matter candidates and as a matter of fact some of the most studied dark matter candidates are supersymmetric partners. However, all these new particles tend to decay and therefore the requirement of a massive neutral and most importantly, sufficiently stable particle that can be produced to the right amount in the early universe puts tight constraints on the model.

The first question is the stability of our dark matter candidate. This is most easily solved by assuming exact R-parity conservation. However, we must also require that the lightest superpartner is neutral in order for it to be a valid candidate. Fortunately it is often possible for the lightest neutralino to be the LSP.

The neutralino then belongs to the most popular group of dark matter candidates, called Weakly Interacting Massive Particles or WIMPs, and has in fact become somewhat of a favorite dark matter candidate among particle theorists.

There are a couple of reasons why WIMPs are such popular dark matter candidates; first of all the problem with generating masses as well as the following hierarchy problem if the Higgs field is used, strongly suggests that there is some new physics at a scale of $O(100\text{ GeV})$, so we expect new particles with masses of this scale. If we further assume that the annihilation cross-section, averaged over particle velocity v, of these particles is of the order of a weak interaction i.e. $\langle\sigma v\rangle \approx 10^{-25} - 10^{-26}\text{cm}^2$, the expected abundance in the universe fits with what is measured by experiments [55].

The problem with the WIMPs is that they all require some exactly preserved discrete symmetry to prevent them from decaying too fast. This symmetry is often put in by hand for no other reason than to make the dark matter stable and does therefore appear rather *ad hoc*.

The situation in supersymmetry is somewhat different in that the discrete symmetry is not introduced for the sole purpose of stabilizing the dark matter, but to prevent too fast proton decay. Having said that, R-parity still has no theoretical motivation and therefore bears a bit of an *ad hoc* mark as well.

4.4 The Gravitino

Although the neutralino is by far the most studied supersymmetric dark matter candidate, it is not the only one. An interesting alternative is the gravitino [56, 57].

As mentioned before, gauging supersymmetry gives a theory of gravity called supergravity and despite it being nonrenormalizable, it is a popular theory to discuss and there is hope that it is a low energy approximation of the real quantum gravity theory (possibly coming from string theory).

The gauge superfield of supergravity will be the graviton superfield and the spontaneous breaking of supersymmetry will give mass to the gravitino in a way similar to the way the electroweak symmetry breaking generates mass to the weak vector boson fields. The result is that the mass of the gravitino is intimately connected to the scale at which supersymmetry is broken.

The predictions for the gravitino mass vary wildly between different schemes for supersymmetry breaking, from very heavy in AMSB via GeV to TeV scale in gravity mediated breaking to very light (few keV) in GMSB. The important lesson here is that the gravitino might very well be the lightest superpartner and therefore our dark matter candidate.

Adding the gravitino into the picture, does have some important consequences for the early universe. If the gravitino is the LSP, then the smallness of the couplings involving the gravitino potentially makes the decay of the NLSP (Next to Lightest SuperPartner) to the gravitino very slow and potentially slow enough for the NLSP to survive until the Big Bang Nucleosynthesis (BBN) [58]. This might then cause problems since a charged NLSP could act as a catalyzer for various BBN reactions while a neutral NLSP might give rise to high energy particles during its decay and those decay products might destroy nuclei. In either case the predictions from BBN will be altered and there are very strict limits on such modifications. If on the other hand the gravitino is not the LSP, then the gravitino might survive until the BBN before decaying to the LSP and thereby creating high energy particles that might upset the BBN predictions.

The above problems are collectively referred to as the gravitino problem [58]. One solution to this can be found by simply not require exact R-parity conservation in combination with a gravitino LSP. The result is that the NLSP will decay much faster to SM particles and will be gone long before BBN [59].

The idea is then that the gravitino, having only gravitational interactions, can live long enough to be dark matter despite the presence of R-parity violating decay modes [60, 61].

4.5 Gravitino Dark Matter

Gravitino dark matter in an R-parity conserving setting might be one of the most problematic dark matter candidates imaginable. The reason being the gravitational nature of the gravitino interactions.

The weakness of these couplings means that we do not expect any signals in direct detection experiments, the cross-section would be too low for that. Also the annihilation cross-section is expected to be too low for any measurable production of cosmic rays for indirect detection.

This would leave us with a scenario where the nature of dark matter would have to be inferred by measurements of other aspects of new physics, while the particle itself would be practically impossible to detect.

This situation is slightly improved if R-parity is not exactly preserved, since then the gravitino will decay and the decay-products might show up in cosmic ray measurements. However, unless we can infer the R-parity violating couplings as well as the gravitino mass from theory and/or other experiments, we do not know what kind of cosmic ray signals to expect.

The best we can do is then to compare possible signals with measurements.

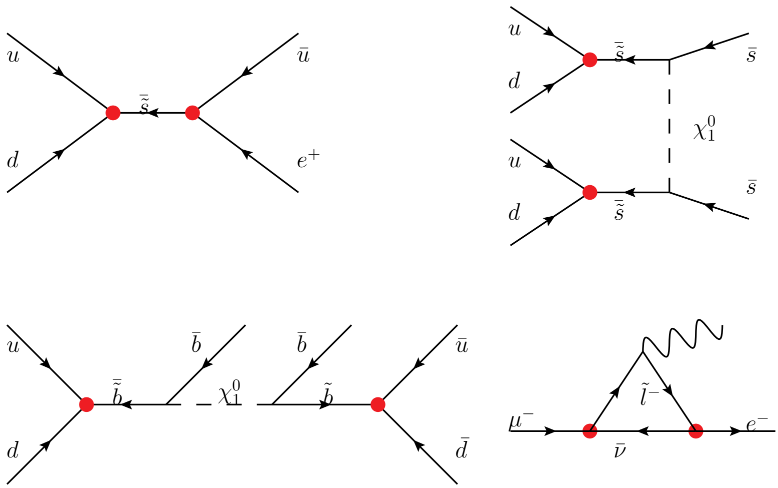

Figure 4.1: Some problematic processes due to the presence of trilinear R-parity violating interactions (mark red in the diagrams). The most important one is the proton decay in the upper left corner. Also double nucleon decay (upper right corner) and neutron-antineutron oscillations (lower left corner) as well as flavour changing neutral currents such as $\mu \to e\gamma$ (lower right corner) have to be considered.

4.6 Trilinear R-parity violation

In this thesis we are mainly concerned with supersymmetric models with trilinear R-parity violating couplings. In other words we are interested in the 45 operators:

$$\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k. \tag{4.25}$$

The reason these only add up to 45 operators is that gauge invariance requires $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$, reducing the allowed independent $LL\bar{E}$ and $\bar{U}\bar{D}\bar{D}$ type operators to 9 each.

One should remember that these couplings should go into the superpotential of the theory and therefore the terms entering the lagrangian will be all possible Yukawa couplings including the superfields of the coupling, see [45] for details.

These couplings can induce a number of problematic processes, some of which are shown in figure 4.1. It is worth noting that many of the most important processes (including proton decay and flavour changing neutral currents) require a combination of at least two different couplings and it is therefore enough to forbid one of the couplings to avoid the constraint. For proton decay, we need both B and L violating couplings and therefore it is enough to forbid either L violation or B violation.

There are constraints also on individual operators but apart from constraints from double nucleon decay and neutron-antineutron oscillations on some $\bar{U}\bar{D}\bar{D}$ operators, these tend to be rather weak [53].

As mentioned before, if R-parity is violated, all sparticles would decay. For the lightest neutralino, trilinear R-parity violation would yield a three-body decay as depicted in figure 4.2. It is important to note that this decay will be possible through any

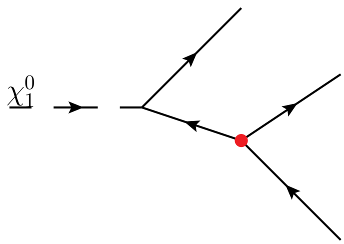

Figure 4.2: The R-parity violating three-body decay of the neutralino. Note that the R-parity violating interaction (in red) can be any of the 45 allowed couplings (for $\bar{U}\bar{D}\bar{D}$ operators all three arrows from the red vertex would go outwards.).

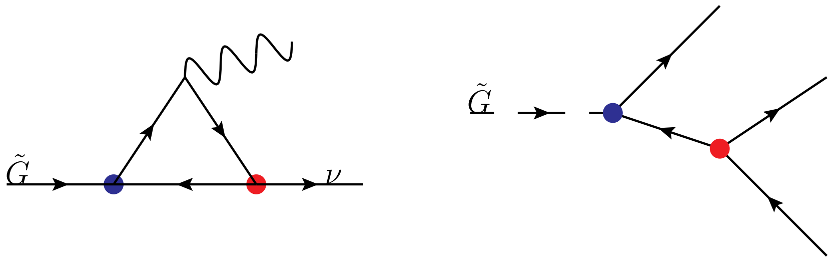

Figure 4.3: Gravitino decay channels through trilinear R-parity violating couplings. The left diagram is one of three loop decays to neutrino and photon while the right is a general three-body decay. All these diagrams are suppressed by the smallness of the gravitational vertex (in blue). The loop diagram requires the R-parity violating vertex (in red) to have two similar flavours while the three-body channel is open for all couplings.

of the 45 allowed operators.

Also the gravitino will decay if R-parity is conserved, but it will do so much slower than the neutralino. For small gravitino masses ($\lesssim$ 40 GeV) there is a loop decay that is open for L violating operators with two similar flavours [62]. One of the three diagrams important for this channel is shown to the left in figure 4.3. For higher masses this channel is less important than the three-body decay shown to the right in figure 4.3. This channel is very similar to the neutralino decay shown in figure 4.2, but the weak vertex in figure 4.2 has been replaced by a gravitational vertex (blue vertex in figure 4.3) and hence giving a much longer lifetime for the gravitino.

Chapter 5

Detecting Dark Matter

If we accept that dark matter has to be some kind of new particle that has little or no interactions apart from gravitational, we are facing a bit of a problem: How should we find out what kind of particle we are dealing with?

This is indeed a critical question, we have to remember that for all we know dark matter may not interact with ordinary matter in any other way than gravitationally and if that is the case, finding the properties of that particle and even to conclusively demonstrate that it exists, may be almost impossible.

However, there are much more favourable scenarios. All the experimental efforts to detect dark matter particles[1] are based on the idea that the dark matter belongs to a class of particles called WIMPs (Weakly Interacting Massive Particles) that in fact offers several ways to detection. It is therefore necessary to discuss these WIMPs a bit before looking closer at how to detect dark matter.

5.1 The WIMP miracle

If we take the hierarchy problem seriously, we expect some kind of new physics to show up in the 100 GeV to 1 TeV range. That means we expect new particles with masses around a few hundred GeV. Since we are trying to solve a problem in the weak sector we would expect these particles to have weak interactions.

If we now take a particle with mass around 100 GeV and only weak (and gravitational) interactions, we generically expect these particles to have an annihilation cross-section of the order $10^{-25} - 10^{-26}\text{cm}^2$.

Let us imagine that such a particle is in thermal equilibrium in the early universe. Since the universe is expanding and cooling, there will be a point where the thermal equilibrium breaks down. The reason is that at some point the expansion of the universe will make the particles so far apart that they cannot find each other and annihilate fast enough to keep up with the decreasing temperature. At this point there will be no significant annihilation and the particles that are left will remain as a relic density in the universe for all times. This is the simplest way of producing a relic density and is commonly referred to as thermal freezeout [63].

[1] From now on, when detecting dark matter is mentioned, it exclusively means a non-gravitational detection. We already have plenty of gravitational dark matter detections but to extract the precise properties of the particle (and completely close the window to explain all data by modifying gravity) we need at least one non-gravitational observation too.

When this happens depends on the annihilation cross-section, which therefore will control how many of our particles there will be left in the universe. So what value should the cross-section have to give the observed amount of dark matter?

The answer depends on the mass of the particle, but for a 100 GeV particle the answer is around $3 \times 10^{-26}\text{cm}^2$.

This is what is referred to as the WIMP miracle; a weakly interacting particle with mass around 100 GeV, naturally gives the right amount of dark matter in the universe. Even better, due to their weak interactions we can actually detect them!

This is surely too good to be true, is it not?

Well, why should this new particle not just decay like most other very heavy particles we know about?

This is in fact a big problem. We need to invent some method of making the particle stable.

It is actually rather easy to find such a method, we just need to introduce some discrete symmetry such as a $\mathbb{Z}_2$ symmetry or parity, where the SM particles are given parity 1 and some of the new particles have parity -1. If we then assume the parity to be conserved, the new particles can never decay into SM particles only, and the lightest new particle will be stable.

Although the above method is simple and has been used in several dark matter models, it is hard to accept such a parity that is introduced for no other reason than to make the dark matter particle stable. We should note that for supersymmetry, the parity used for this is R-parity that was introduced to avoid proton decay. Even though R-parity was not introduced to make the dark matter candidate stable, dark matter is the only reason (apart from simplicity) to keep it exactly conserved.

Despite this stability problem, the WIMP is today the most studied dark matter particle and among all WIMPs, the supersymmetric neutralino is probably the most studied. In fact, the WIMP idea is more or less behind all attempts to detect dark matter.

While the neutralino is probably the most discussed WIMP it is far from the only one. WIMPs show up in virtually every theory of new physics at the electroweak scale, some examples are scalar dark matter in inert Higgs models [64, 65] and Kaluza-Klein excitations in models with extra dimensions [66, 67].

It is important to remember that not all particle dark matter candidates are WIMPs, one obvious example in this thesis being the gravitino, but we also have e.g. the axion [68] and sterile neutrinos [69]. One of course has to ask, how do we get the right amount of these particles when thermal freezeout does not work?

The first part of the answer is to realize that we do not even know how the right amount of the matter we are familiar with, came to be. We have many theories like baryogenesis through sphaleron processes in the electroweak phase transition [70] and thermal leptogenesis through the decay of some heavy neutrinos [31], but we are far from any real understanding.

This means that any discussion on production mechanisms of dark matter necessarily has to be rather speculative and it might be dangerous to exclude dark matter candidates only due to lack of any obvious production mechanism.

That said we should at least speculate a little on possible production mechanisms for gravitinos. The small annihilation cross-section rules out the thermal freezeout that worked for WIMPs, the reason is that if the gravitinos at some point were in thermal equilibrium, the small cross-section would make them freeze out very early and leave a much too high relic density. This means that the gravitinos cannot have been in thermal equilibrium in the early universe.

This sounds a bit problematic. The higher the temperature, the more gravitinos will be produced from collisions in the thermal plasma and it seems impossible to avoid overproduction of gravitinos if the temperature of the early universe approached infinity. Luckily we are rescued in a somewhat surprising way; if we assume that the early universe went through a period of exponential expansion, called inflation [71], we see that any gravitino or other particle production that happened before inflation would be so diluted by the expansion that it is of no interest to us today. This means that all that matters is what happened after inflation [58].

It is expected that the field that caused inflation (usually called the inflaton field, although we do not have any established theory for it yet), after the end of inflation decayed to other particles and thus released a lot of energy which increased the temperature of the universe. This process is called reheating and the temperature reached, the reheating temperature, is what is important to gravitino production. It turns out that the resulting relic abundance of gravitinos is essentially linearly dependent on the reheating temperature and it is indeed possible that the gravitino density is appropriate to account for dark matter [72].

If R-parity is conserved, additional gravitinos may be produced by the decay of other, heavier sparticles but if R-parity is violated, the other sparticles would decay directly to SM particles instead, leaving no impact on the gravitino abundance.

5.2 Direct detection

The presence of weak interactions opens the possibility to detect interactions between dark matter and ordinary matter. In order to do this one needs a very sensitive experiment due to the smallness of the studied interactions.

Since we already have seen that we know the annihilation cross-section of WIMP dark matter, one might think that also the cross-section with nuclear matter is known, but that is not the case and hence it is not possible to know exactly how hard it is to see such a signal.

There are several experiments running and they are presenting limits on the cross-section that more and more approach the interesting regions. Worth mentioning are EDELWEISS [73] and CDMS II [74] and especially XENON100 [75] is now producing important constraints on WIMP models. So far none of these has seen a clear signal though.

Among all negative results from direct detection experiments, there is actually a very clear signal in the DAMA experiment [76]. In contrast to all the other experiments, DAMA is not simply counting events and comparing to expectations, they are instead trying to detect an annual modulation in the signal.

The idea is to exploit the motion of the Earth around the Sun. We expect the dark matter halo of our galaxy to on average be at rest, i.e. it should not have a net rotation like the disk of stars have. This comes from not having strong enough self-interaction to end up in a uniform motion. Since the Earth does rotate around the galaxy, we expect the Earth to have a velocity compared to the dark matter halo and that velocity should impact on the number of events in the experiments.

Since the velocity of the Earth compared to the halo can be seen as the sum of the Sun's motion around the galaxy and the Earth's motion around the Sun, the event rate in the experiments should oscillate with the Earth's rotation around the Sun.

This annual modulation is what DAMA is looking for and they have indeed found it!

It is today quite clear that they are seing a modulation that is consistent with the expected annual modulation of a dark matter signal. That, however, is the end of the good news. Since the other experiments are not seing anything, it is becoming increasingly difficult to reconcile a dark matter explanation of the DAMA signal with the limits set by other experiments.

Lately there is another experiment, also looking for the annual modulation, called CoGeNT [77] and they are also seing a signal although they do not have enough data to be certain on their own. The problem is that it seems almost impossible to construct a dark matter model that can explain both DAMA and CoGeNT without being excluded by other experiments [78].

It is still a mystery what causes the signal seen by DAMA, but it does not seem to be dark matter.

If the dark matter is in fact not a WIMP but e.g. a gravitino, one would normally not expect any signal in direct detection experiments. This is simply due to the gravitational nature of the gravitino interactions, it will suppress the gravitino-matter cross-section to undetectable values.

5.3 Detecting Dark Matter at the LHC

Since the LHC is in fact built in order to study electroweak physics, it should also be capable of producing WIMPs if they exist. Since the WIMP scenario is partially motivated by the need for new physics at the electroweak scale, it is rather natural to expect evidence of WIMPs at the LHC.

Of course, detecting dark matter at the LHC, even if it is produced, sounds like an impossible task. If one only considers detecting the particles directly that would be correct, however, if everything that is detectable is detected, it is possible to look for momentum imbalances. A WIMP escaping the detector in one direction means that something has to carry the same amount of momentum in the opposite direction and if that something is detectable it might be possible to deduce the presence of the WIMP from that.

The variable to use is the E_T^{MISS} that basically measures the imbalance in energy[2] in the plane transverse to the beam and it is in fact already in use to look for neutrinos. This variable is rather difficult to use since it requires an extremely good understanding of everything that happens in the detector, but wise from experiences at the Tevatron, the experimentalists at the LHC seem to have this under control [79].

Now one should remember that the requirement of a stabilizing discrete symmetry makes sure that WIMPs are always produced in pairs at the LHC. This might at first

[2]The observant reader may object that the transverse energy, opposed to the momentum, is not necessarily conserved. However, the detector measures energy and not momentum and since all particles we detect are practically massless compared to the beam energy, the difference between E_T^{MISS} and p_T^{MISS} is small.

look like a problem because then the something that carries the momentum in the opposite direction from the first WIMP, could in fact be the second WIMP and E_T^{MISS} will tell us nothing. The rescue is the fact that in some fraction of the events there should be something else like e.g. a photon produced somewhere in the interaction and an event with a single photon in one direction and nothing else would be a rather spectacular indication of dark matter.

In general, though, the situation is far more complicated; in most models the WIMPs will not be directly produced but will only appear as end-stages in cascade decays of particles from some new physics theory. This means there will be a lot more particles accompanying the E_T^{MISS}. This is the case for supersymmetry and this will be discussed more thoroughly in section 6.2.

One should, however, be a bit careful with WIMP detections at the LHC. The only thing the LHC can ever say is that there seems to be a particle that is long-lived enough to leave the detector. Of course, if we find a particle that is neutral, has a mass around 100 GeV and escapes detection, it would be tempting to proclaim the discovery of Dark matter.

This might, however, be a mistake; it is fully possible that such a particle exists but is not the dark matter of the universe; just because the lifetime is long enough to escape detection at the LHC (which means longer than a few μs) does not mean it is long enough to be dark matter.

There are many possibilities within R-parity violating SUSY for this to happen. If the R-parity violating decay of the neutralino is suppressed for some reason, e.g. through small couplings or through phase-space due to top quarks in the final state, the lifetime might very well be long enough to escape detection while still be well below cosmological bounds.

5.4 Cosmic rays from Dark Matter

Under the WIMP assumption, we know the expected annihilation cross-section from the requirement of thermal production of the right dark matter density in the early universe. It turns out that this cross-section is large enough to give a chance to see cosmic rays from WIMPs that annihilate today. This is the theoretical motivation for all indirect attempts to find dark matter, i.e. attempts to see dark matter signatures in cosmic rays.

These searches are not easy to perform, there are a number of uncertainties to overcome and the requirements for the detectors are very difficult to meet, which is why the data only in the last few years have become good enough to study this in detail. To this comes a large number of other sources of cosmic rays and one of the biggest problems is in fact to disentangle the data to see which features are from which source.

Additional difficulties come from the fact that we do not know the mass of the WIMP and we do not know into which channels it annihilates. If we are really lucky it annihilates into a pair of photons and then momentum conservation assures us a monochromatic line in the gamma ray spectrum. This is relatively easy to see and is unlikely to come from anything else than dark matter. In more realistic scenarios, the dark matter particles will annihilate into a number of different channels and the resulting decay products will decay in several steps before reaching us. The result is spectra that in many cases look problematically similar to possible astrophysical cosmic ray sources.

For charged cosmic rays, there are also significant propagation effects that we have to take into account. The galaxy is filled with magnetic fields that impact on the propagation of charged particles. There are also plasma effects, possible collisions with interstellar gas as well as with photons. All this combines into a complicated propagation that makes sure that we cannot identify point sources by looking at the incident direction. The energy loss due to these processes also means that we only see charged cosmic rays from our galactic neighborhood and spectral peaks are washed out so that pair production of electrons and positrons is significantly less useful as compared to a di-photon channel.

In order to take these propagation effects into account we need to solve the diffusion equation [80]:

$$\begin{aligned}\frac{\partial \psi(\mathbf{r},p,t)}{\partial t} = \quad & q(\mathbf{r},p,t) + \nabla \cdot (D_{xx}\nabla\psi - \mathbf{V}\psi) \\ & + \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2}\psi - \frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\nabla\cdot\mathbf{V})\psi\right] - \frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi \qquad (5.1)\end{aligned}$$

where $\psi(\mathbf{r},p,t)$ is the distribution of the cosmic ray species of interest[3], $q(\mathbf{r},p,t)$ is the source term including all types of production. D_{xx} and D_{pp} are diffusion coefficients in physical space and phase-space, respectively, and $\mathbf{V}$ is a convective velocity field. τ_f and τ_r are time scales for fragmentation and radioactive decay, respectively. One should remember that there are large uncertainties in what values these parameters should have, adding to the already large uncertainty in these calculations.

Eq. (5.1) can be solved semi-analytically but there are also codes for a fully numerical treatment of the problem. The most used package is the GALPROP code [81] that solves eq. (5.1) numerically for all cosmic ray species simultaneously to get a consistent picture to compare with observations. One advantage with GALPROP is that it also calculates and propagates the known astrophysical background, otherwise one has to be careful to make sure the used background is calculated with the same propagation parameters as is used for the dark matter annihilation products.

We also need to know the distribution of dark matter; we have only measured the gravity from very large amounts of dark matter and it is very hard to see the detailed structure of the dark matter. Progress within gravitational lensing can help to some degree [82], but these are very difficult measurements.

One other source of input is structure formation simulations [83, 84]. These are large simulations that try to simulate the formation of galaxy clusters and also galaxies through the process of gravitational collapse. The picture from the simulations is in good agreement with what we see and they do in fact give important information about dark matter as well since they can say something about how much dark matter they need to reproduce the universe we see. These simulations were also responsible for ruling out neutrinos as dark matter since they showed that neutrinos would be too relativistic for structures to form as we know they did [85].

However, the simulations still do not have the resolution to answer all our questions. For the overall structure of the dark matter halo around a galaxy we have a model

[3] Note that it is the distribution in space as well as momentum space.

derived from simulations that works pretty fine. It is the Navarro-Frenk-White halo profile that takes the rather simple form [86]:

$$\rho_{\text{halo}}(r) = \frac{\rho_0}{(r/r_c)(1+r/r_c)^2}, \tag{5.2}$$

where r denotes the distance to the center of the galaxy, $r_c = 20$ kpc and $\rho_0 = 0.33$ GeV cm^{-3}. This tells us what the overall halo looks like but we want to know something about the small-scale structure as well, i.e. how many smaller clumps of dark matter are there?

This is important because the presence of many small overdensities of dark matter would increase the annihilation rate significantly, but the simulations do not have the resolution to tell us this yet. This is usually handled by introducing a boost factor which is a simple numerical factor that is multiplied into the spectrum to parameterize the clumpiness.

5.5 Decaying Gravitino Dark Matter

If nature has chosen a supersymmetric scenario where R-parity is in fact broken, we are left with a rather limited choice of dark matter candidates. Since the interactions of most sparticles will cause them to decay much too fast, we need to find a sparticle with very weak interactions, so that the lifetime can be sufficiently long despite the inevitable decay due to the R-parity violation.

The most obvious option would be the gravitino, it interacts only through the weakest known interaction; gravity. Indeed it turns out that it is fully possible to achieve lifetimes much longer than the current age of the universe even for R-parity violating couplings close to the experimental upper limits, at least if the gravitino is light enough [87].

The fact that the gravitino after all does decay gives us some hope to detect the decay products in the cosmic rays. For a number of trilinear R-parity violating operators[4] it is possible for the gravitino to decay to a photon and a neutrino, giving a monochromatic line in the cosmic gamma ray spectrum. The decay width of this channel was studied in [62]. There are also three-body decay modes that tend to dominate at higher gravitino masses, these give more complicated spectra caused by internal bremsstrahlung off the leptons in the final state and in the case of quarks and taus, π^0 decay.

The implications of this for the possible magnitudes of the R-parity violating couplings was the subject of the first paper [87]:

Paper I: *Gravitino dark matter and the flavour structure of R-violating operators*

The goal of this paper is to study the constraints on R-parity violating operators, imposed by cosmic gamma ray measurements, if gravitinos are assumed to constitute all the dark matter.

The approach is to simulate the decay of the gravitinos and the following fragmentation and hadronization (when applicable), both through the two body loop decay

[4] For studies of cosmic rays from decaying gravitinos due to bilinear operators see e.g. [88–94].

channel [62] and the three-body channel (with width taken from [58]), using the Monte Carlo event generator PYTHIA 6.4 [15]. The result is then converted into an expectation on the flux of gamma rays on Earth, which is compared to the EGRET [95] data on the diffuse isotropic gamma ray background and thus constraints on the R-parity violating couplings are derived.

It is clear from these results that the three-body mode dominates for gravitino masses above ≈ 100 GeV. This is true for all couplings while at low energy, the loop decay may dominate for the couplings where such a channel is possible.

For low gravitino masses it is clearly possible to have R-parity violating couplings larger than 10^{-6}, for GeV scale gravitino masses and operators without loop decays, it is even possible to approach the experimental upper limits.

Of course, gamma rays are not the only cosmic ray species of interest for dark matter searches, perhaps even more important are e.g. positrons and antiprotons since those are produced less abundantly by astrophysical sources. Recently there has been a lot of interest in especially positrons, due to the publication of the PAMELA data on the positron fraction [96], which shows a clear excess at higher energy.

Although there are claims that e.g. solar modulation [97] or misidentification of protons [98] can explain the data, most studies conclude that a so far unknown source of electron-positron pair production is needed [99, 100]. Although supernova remnants [101] and especially pulsars [102] are good candidates to account for this production, there has been a considerable effort to explain the data with dark matter [103].

There are many problems when attempting an explanation of the PAMELA data by means of dark matter annihilation. The first is that the annihilation cross-section needed to achieve the correct relic abundance through thermal freezeout, perhaps the main motivation for the WIMP scenario, is too small to explain the observed excess. This can be solved by invoking e.g. Sommerfeld enhancements [104] of the cross-section or adopt a very high boost factor but it means that the models have to be finely tuned to get it right. One can also assume that we happen to live close to a clump of dark matter that enhances the annihilation locally [105], but this seems very unlikely from structure formation simulations [106].

The second problem comes from another set of data from PAMELA, this time measuring antiprotons [107]. In this data no excess over the expected background was seen. This means that if dark matter is responsible for the production of electron-positron pairs, it does not decay to anything that can give rise to antiprotons, in other words; the dark matter has to annihilate to leptons but not to quarks. This is very unnatural for most dark matter models.

Finally, if we also look at the electron plus positron spectrum from Fermi LAT [108], we see that in order to explain both that data and the PAMELA positron fraction data, we need a new source of electron-positron pairs up to 1 TeV [109] and for dark matter that means that we need a mass of at least 1 TeV, which is significantly more than one normally hopes for.

Many of these problems are eased in a scenario with decaying gravitino dark matter, first of all we do not know the lifetime of the gravitino and can set it to the value that reproduces the data. Moreover, we have the freedom to add whichever operator we need to fit the data and can therefore easily make the gravitino decay only to leptons.

The problem of high dark matter mass is still present though, and one should remember that the reason we can fit the data so easily is that we have a large number of parameters to adjust to our liking.

The comparison of the above-mentioned cosmic ray data with our models of decaying gravitino dark matter was the main topic of the second paper [110] (see also [111]):

Paper II: *Photon, neutrino and charged particle spectra from R-violating gravitino decays*

In this paper we studied all possible cosmic rays from decaying gravitinos, including positrons, electrons, antiprotons, gamma rays and neutrinos. This is done by again simulating the gravitino decay, including fragmentation and hadronization of the decay products, in PYTHIA 6.4 [15] and then propagate the charged particle spectra with GALPROP [81] from which we also get expected astrophysical backgrounds.

Given the recent excesses seen in electrons and positrons, the main emphasis is on how to accommodate these measurements with decaying gravitinos, without violating the antiproton measurements. It is found to be rather straightforward to do so, although at the cost of pushing the gravitino mass up to at least around 2 TeV, practically excluding any chance of seing SUSY at the LHC and potentially reintroducing problematic fine-tuning.

In addition to the study of charged cosmic rays, we look at neutrino and gamma ray signals and specifically the expected signals from the models preferred by the charged cosmic ray measurements. The conclusion is that the scenarios capable of explaining the electron/positron anomalies are not yet excluded by other experiments. (However, see comment on this issue below.)

After the publication of the above paper, Fermi LAT has published its data on the diffuse isotropic gamma ray emission [112]. The data shows that the excess earlier seen by EGRET [95] is no longer there and we may conclude that it was an experimental issue (there were speculations about a calibration error [113]).

The Fermi LAT data does not show any evidence of dark matter and may therefore be used to constrain models trying to explain the cosmic ray anomalies. The Fermi LAT data is shown in figure 5.1 together with the expected gamma ray fluxes from those two of our models that best fit the PAMELA positron anomaly.

When looking at the comparison in figure 5.1 it is important to understand how much of our flux we would expect to see in the isotropic component of the diffuse emission. The only component that is truly isotropic is the red-shifted extragalactic part (the red curve in figure 5.1) and that is clearly not enough for an exclusion although there is some tension (notice that we expect an exponentially falling background as well although we do not know how big it is) especially due to the lack of any indication of a change of slope in the data where our gravitino component is getting important.

Furthermore, depending on exactly how the isotropic component is extracted, we would expect at least some of our halo emission to also be included in the isotropic data and therefore the tension is even larger than if only the extragalactic contribution is taken into account. However, it is very hard to know how much larger and given the

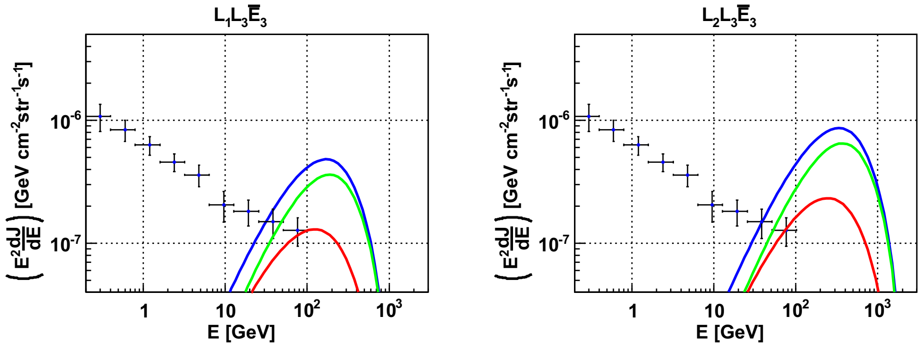

Figure 5.1: Gamma ray fluxes for the best fit models from [110], compared with the Fermi LAT isotropic diffuse emission [112]. The red curves show the red-shifted extragalactic flux while the green curves show the flux from the galactic halo averaged over all directions; the blue curves show the sum of the other two.

uncertainties in backgrounds and the sizeable error bars on the Fermi LAT data, it is not possible to exclude our explanation for the PAMELA anomaly. It is, though, fair to say that our models are looking rather unlikely to be the answer, due to the Fermi LAT isotropic diffuse spectrum.

The Fermi LAT collaboration also looked at annihilating dark matter and concluded that the existing dark matter explanations for the electron/positron anomalies are excluded by their gamma ray measurements [114].

The new data on isotropic diffuse emission will improve the constraints on the R-parity violating couplings. The improvement as compared to the older EGRET data can be seen in figure 5.2[5]. The improvement might seem small but this is partially due to the logarithmic axes of the plot; it is an improvement by a factor 2 at the higher energies. The bounds in figure 5.2 are produced with only the extragalactic component of the gravitino gamma ray emission but it is still clear that there is plenty of room for non-zero R-parity violating couplings without coming into conflict with data, at least for small gravitino masses.

It is also interesting to note that the Fermi LAT collaboration has started to exclude thermally produced WIMPs [115], at least for small WIMP masses.

[5]The change of slope in the curves is due to the transition from the radiative two-body decay to three-body decay of the gravitino.

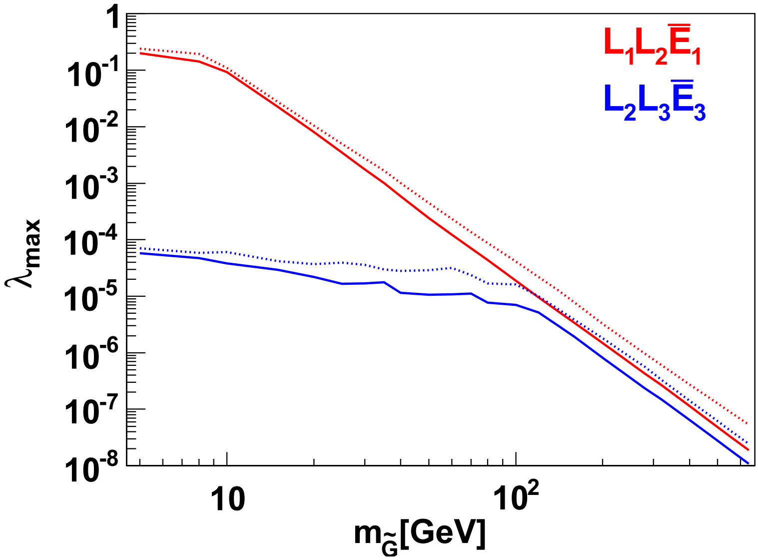

Figure 5.2: The constraints on trilinear R-parity violating couplings from measurements of cosmic gamma rays. The dotted lines are due to the EGRET data [95] while the solid lines are due to the more recent Fermi LAT data [112].

Chapter 6

Supersymmetric phenomenology at the LHC

Supersymmetry is possibly the most discussed new physics that the LHC is looking for. One problem though, is that the lack of knowledge about the precise SUSY model to look for makes it hard to say what the LHC might actually see.

We need to narrow down the possibilities and to do that we use simplified SUSY models and benchmark points. Most phenomenological studies assume the simplest supersymmetric extension of the SM; the MSSM.

6.1 The MSSM at the LHC

Although the MSSM is the simplest supersymmetric model we can look for, it still has more than 100 free parameters. This is a parameter space far too large to explore in its entirety.

The solution is to make some further simplifying assumptions. Most of these new parameters are soft SUSY breaking couplings, i.e. mass terms to the SUSY-partners.

If we assume that there is some kind of unification at some GUT scale, typically around 10^{16} GeV, we can assume that these parameters have common values at this scale. Running the renormalization group equations between this GUT scale and the electroweak scale will then give us a mass spectrum for LHC physics. The most common unification assumption is the CMSSM or Constrained MSSM, that uses one common parameter m_0 for all scalar masses, one parameter $m_{1/2}$ for all fermions (only the superpartners), one parameter A_0 for the trilinear couplings and one parameter called $\tan(\beta)$ that gives the ratio between the VEVs of the two Higgs doublets. In addition there is the sign of the parameter μ (coupling constant for the H_1H_2 term).

This provides a parameter space that can be handled and still contains rich phenomenological possibilities.

To structure the search strategies further, there has been a number of benchmark points defined. The idea is to pick some representative points in parameter space so that studying them should give most of the possible phenomenologies. There are many sets of such points, but the ones used in this work are the SPS points [116].

6.2 SUSY cascade chains

If we assume that R-parity is conserved, all production of sparticles at the LHC will be in pairs. The produced sparticles will then decay through a cascade down to the neutralino that will escape the detector.

The details of these events will depend strongly on the exact SUSY model, especially the mass hierarchy among the sparticles. This hierarchy will determine which sparticles show up where in the cascade chain and that will then determine the structure of the final state of the event.

Common for almost all chains is that they start with squarks or gluino and end with a neutralino. Ending with a neutralino is necessary if we adopt R-parity conservation and neutralino dark matter. Starting with squarks or gluinos on the other hand, is not necessary but is usually what happens since they can be produced through strong interactions at the LHC. On an electron-positron collider like the ILC one would not expect the same dominance by squark and gluino production.

Somewhere between the squark/gluino and the lightest neutralino, we often find one other neutralino or a chargino and the intermediate steps typically contain some squarks and/or sleptons. Especially the presence of sleptons in the chain is interesting since that implies leptons in the final state of the event.

One group of chains that have been very thoroughly studied are the ones with a χ_2^0 that decays to a lepton and a slepton which due to lack of other options decays to a lepton and a χ_1^0. The advantage of this chain is that we get two leptons in the final state that we can look for. As will be discussed in the next section, the presence of these leptons allows us to form a number of invariant mass distribution that can be very helpful in determining the mass spectrum of our SUSY model.

If we are lucky enough to find such a scenario, we can say a lot about the spectrum of sparticles, however, this it quite often not the case. If the χ_2^0 prefers to decay to a tau pair instead of light leptons (i.e. electrons and muons) things are more complicated and if we are even more unlucky, the χ_2^0 could decay via a squark, giving just two quarks.

Another rather common possibility is that the decay chain goes through a chargino instead of the χ_2^0. In that case we could still get leptons but only one charged lepton and one neutrino and that would be significantly more difficult to use to determine the precise underlying model.

6.3 Brief introduction to LHC physics

Before moving on to the main topic of this chapter, it might be good to introduce some concepts regarding LHC physics.

As is widely known, the LHC is colliding protons on protons at very high energy[1].

Using proton-proton collisions has both advantages and disadvantages over other types of collisions. Compared to proton-antiproton collisions as was used at the Tevatron, one can reach higher luminosity due to the easier production of protons. Compared to electron-positron collisions one can reach higher energy due to smaller energy-loss in the form of bremsstrahlung.

[1]We can here ignore the heavy ion program since it does not have any impact on SUSY searches.

However, the high energy and high luminosity come at a heavy price; measurements on proton-proton machines are very difficult. There are many reasons for this, the most important ones being listed below:

- We are colliding composite objects, which means a lot of uninteresting remnants from the parts of the proton that did not take part in the interaction we are interested in.
- The protons are made of particles with strong interactions and that means that the by far most common type of collision at the LHC is a QCD interaction. That is not what we are interested in so we need to separate out the interesting events from this huge background and hope nothing was drowned in it.
- Since the protons are composite, we do not know the energy of the particles that actually interact.

In order to deal with these problems we need very good detectors, advanced algorithms for dealing with the data from those detectors, simulations of what we expect the detectors to see and enough computer power to run all this code.

What we really want to study is the primary interaction that takes place between some partons in each of the protons. In the interesting collisions, the result should be some particle with high transverse momentum (hard particles); particles with low transverse momentum (soft particles) are more likely to be remnants of the protons or from uninteresting QCD events.

6.3.1 Objects to look for in the detectors

The particles actually detected by the detectors are mostly various mesons as well as electrons, muons, photons and some protons and neutrons (including antiparticles where applicable). However, in order to reconstruct what happened in the interaction we need to use the distributions of these particles and form new objects that can tell us more [117, 118].

The first thing to remember is that when strongly interacting particles, i.e. quarks and gluons, are produced in the primary interaction, they will show up in the detector not as one particle but as a group of hadrons. In addition, we expect the production of many unstable particles that will decay long before we can detect them and they will also show up as a group of particles in the detector.

The above issues are dealt with through the construction of jets. The idea is to group particles that are close in the detector together and hope that those particles are decay and hadronisation products from the same particle. This approach is useful for all quarks except the top quarks and for taus. One has to keep in mind though, that the jet is not the same as the particle it is coming from and, as will be discussed in the next section, there are many different ways of doing the grouping of detected particles into jets.

In the case that some particle decayed into some other particles that then formed a jet, it would be good to be able to determine what the decaying particle was. In the case of decaying b quarks and taus, that can be done, at least with some efficiency.

For detecting b quarks the trick is to look for leptons inside the jet as well as looking for a secondary vertex; the b quark has a lifetime that allows it to travel a short distance in the detector before decaying and that distance can be measured.

For taus the idea is to look for narrow jets and also to look at how many charged particles there are in the jet. Since the tau is a lepton and does not have colour charge, there is no hadronisation to worry about and therefore we know the branching ratios of all decay channels very well.

In addition to jets, we also look for leptons in the detector. This is much easier than the jets because leptons (in this case electrons and muons) can be measured directly. Since a large fraction of the leptons produced, will be decay products from other particles like mesons and b quarks, they will be part of jets. This is actually a good thing that can be used in order to find the more interesting leptons that stem from the primary interaction of the event, we just need to look for the leptons that are not inside any jet.

Finally we would like to be able to say something about neutrinos, however, their lack of both strong and electromagnetic interactions make them impossible to detect at the LHC[2]. The only thing we can do is to use the E_T^{MISS} discussed in relation with dark matter, to detect a momentum imbalance that could be caused by a neutrino.

6.3.2 Jet algorithms

How should one construct these jets that are to represent the products of the primary interaction?

This question is not easy to answer, there are several ways of doing this and which one is best depends on what one is looking for.

The simplest approach is to find the hardest particle[3] of the event, i.e. the particle with the highest transverse momentum, and make a cone around it and say that everything inside that cone is one jet. The procedure is repeated until all particles are assigned to jets or the remaining particles are so soft that one does not care about them. This method is called the cone-algorithm.

A more sophisticated approach would be to look at which particles are closest in the detector and merge them first and then continue until some criterion to stop is met. The stopping criterion is something like we do not join particles that are above a certain distance from each other; without the stopping criterion we would end up joining everything into one jet. The method described here is a clustering algorithm and it comes with a problem; in the most naive implementation, for each merge one has to compare N^2 values, where N is the total number of particles. Since we want to merge (almost) all particles, we need to do this N times and hence the complexity of this algorithm will be N^3. Since the multiplicity expected at the LHC is of the scale $N = \mathcal{O}(1000)$, an $\mathcal{O}(N^3)$ algorithm is too slow.

The solution came in the form of improved implementations of the cluster algorithm that made it faster than any jet algorithm in use [119]!

One bonus with the cluster algorithm is that it in fact contains a number of algorithms; there is no reason why one has to only use geometrical distance when deciding

[2]Although there are neutrino detectors in the world, they rely on detecting a very small fraction of a very large flux and are therefore of no interest for the LHC.

[3]In reality it may not be actual particles that are being used here, but e.g. energy deposited in calorimeter cells. This does not affect the general discussion.

which particles to merge, it might make more sense to use something that takes into account the transverse momentum of the particle, since the hardest particles are usually the most interesting and the soft ones are likely to be background or even just detector noise.

An important group of cluster algorithms can be constructed from distance measures of the general type,

$$d_{ij} = \min\left(k_{ti}^{2p}, k_{tj}^{2p}\right)\frac{\Delta R_{ij}^2}{R^2}, \qquad d_{iB} = k_{ti}^{2p}, \tag{6.1}$$

where k_{ti} is the transverse momentum of particle i and p and R are constants defining the algorithm. Note that $2p$ is an exponent and not an index and the index B in d_{iB} refers to "beam", so d_{iB} is loosely speaking distance to the beam. ΔR_{ij} is a geometrical distance measure defined as $\Delta R_{ij}^2 = \Delta\phi_{ij}^2 + \Delta y_{ij}^2$, where ϕ_i is the azimuth angle of particle i and y_i is the rapidity.

The rapidity is a distance measure that can be written

$$y = \frac{1}{2}\ln\frac{E + p_z}{E - p_z}, \tag{6.2}$$

where E is the energy of the particle and p_z is the momentum component in the beam direction.

Differences in rapidity are preserved by Lorentz boosts along the beam direction and as a result particle production at hadron colliders will be more or less uniformly distributed in y. The reason for the last point is that in general, the colliding partons will have a total boost in the direction of the beam and hence the final state particles of different events will display a similar pattern in y rather than the angle to the beam. This is why the rapidity is used instead of the simpler angle. Sometimes one instead uses the pseudorapidity η,

$$\eta = -\ln\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\ln\frac{|\mathbf{p}| + p_z}{|\mathbf{p}| - p_z}, \tag{6.3}$$

which only requires the angle θ to the beam axes and is therefore easy to measure. In the limit of massless particles the rapidity and pseudorapidity are the same.

Let us now go back to the distance measure (6.1) and see how to make a jet algorithm out of that. The way to do this is to take the smallest of all d_{ij} and d_{iB} and if it is one of the d_{ij} we merge those two particles and if it is one of the d_{iB} we call particle i a jet and remove it. This continues until we do not have any more particles.

By changing the parameter p of eq. (6.1) we can achieve a number of qualitatively different algorithms, the most common choices are the kt-algorithm with $p = 1$ [120], the Cambridge-Aachen algorithm [121] with $p = 0$, i.e. a purely geometrical measure, and anti-kt [122] with $p = -1$ which is similar in behavior to the cone algorithm. Especially the kt-algorithm is often used because the merging of particles happens in approximately the opposite order as the branching off of radiation. This means that tracing back in the sequences of mergers, can give additional information about the event.

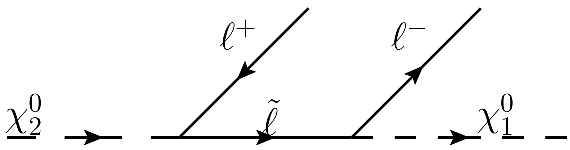

Figure 6.1: A SUSY decay chain including leptons.

6.3.3 Invariant mass distributions

Many of the particles we have discovered, as well as most of the particles we want to discover, are heavy and will decay almost immediately after production. For heavy particles we may not be lucky enough to find all decay products in a jet and therefore we need some other method of identifying them and study them.

The most useful tool for this task is invariant mass distributions. Let us illustrate the idea with the simplest possible example; a Z^0 that decays to an electron positron pair. Conservation of energy and momentum together with special relativity then give for the energy momentum vectors of the electron (p_-), positron (p_+) and Z^0 (p_Z):

$$(p_- + p_+)^2 = p_Z^2 = M_Z^2, \tag{6.4}$$

where M_Z is the Z^0 mass, i.e. we can reconstruct the Z^0 mass from the (measurable) energy and momentum for the electron and the positron. That way we can be confident that the electron and positron indeed come from a decaying Z^0 and we can measure the mass of the Z^0.

Many scenarios, including SUSY scenarios, do not have this simple topology where one can construct a mass peak from invariant mass measurements. In most SUSY models we have a decay chain that ends with a neutralino that escapes detection. Furthermore, there are two such chains in each event that makes it harder to know which particles come from the same chain. The use of invariant mass distributions will therefore be much more complicated, but it is nevertheless crucial in order to understand the data [123].

To illustrate the use of invariant masses in a SUSY scenario let us look at the chain of figure 6.1 where we have a decay chain including a lepton pair. Let us first assume that all the sparticles in the chain are on mass-shell, although this is rather likely to be the case it is in no way necessary and we will comment on the case when this is not true below. For simplicity we assume that the slepton has negative charge but the charge inverted chain can of course be treated identically.

The best starting point for this calculation is in the rest frame of the slepton, i.e. where the slepton energy-momentum four-vector takes the form $p_{\tilde{\ell}} = (M_{\tilde{\ell}}, \mathbf{0})$. Note that the Lorentz invariance of the invariant mass allows us to do the calculation in one reference frame and the measurement in another. Let us then look at the invariant mass of the energy-momentum four-vector, p_{ℓ^+}, of the first lepton combined with the $p_{\tilde{\ell}}$. This reads:

$$(p_{\ell^+} + p_{\tilde{\ell}})^2 = M_{\tilde{\chi}_2^0}^2, \tag{6.5}$$

which can be expanded,

$$M_{\tilde{\chi}_2^0}^2 = p_{\ell^+}^2 + p_{\tilde{\ell}}^2 + 2p_{\ell^+}p_{\tilde{\ell}} = M_{\ell^+}^2 + M_{\tilde{\ell}}^2 + 2M_{\tilde{\ell}}E_{\ell^+}. \tag{6.6}$$

This can be simplified if we assume that the leptons are massless, although we know that this is not true, the masses of the leptons are so small compared to the center of mass energy of the LHC that they can be neglected here. This leads to an expression for the energy of the lepton:

$$E_{\ell^+} = \frac{M^2_{\tilde{\chi}^0_2} - M^2_{\tilde{\ell}}}{2M_{\tilde{\ell}}}. \tag{6.7}$$

To find an expression for the energy of the second lepton, we look at the conservation of energy in the decay of the slepton:

$$E_{\ell^-} + E_{\tilde{\chi}^0_1} = M_{\tilde{\ell}}. \tag{6.8}$$

We know that $E_{\tilde{\chi}^0_1} = \sqrt{M^2_{\tilde{\chi}^0_1} + \mathbf{p}^2_{\tilde{\chi}^0_1}}$ and the assumption of massless leptons together with conservation of momentum, gives us that $E_{\ell^-} = |\mathbf{p}_{\ell^-}| = |\mathbf{p}_{\tilde{\chi}^0_1}|$. This allows us to solve eq. (6.8) for E_{ℓ^-} and we get:

$$E_{\ell^-} = \frac{M^2_{\tilde{\ell}} - M^2_{\tilde{\chi}^0_1}}{2M_{\tilde{\ell}}}. \tag{6.9}$$

We are now ready to put everything together in order to calculate the invariant mass of the lepton pair:

$$\begin{aligned} M^2_{\ell^+\ell^-} &= (p_{\ell^+} + p_{\ell^-})^2 = 2p_{\ell^+}p_{\ell^-} = 2E_{\ell^+}E_{\ell^-}(1+\cos\theta) \\ &= \frac{\left(M^2_{\tilde{\chi}^0_2} - M^2_{\tilde{\ell}}\right)\left(M^2_{\tilde{\ell}} - M^2_{\tilde{\chi}^0_1}\right)}{M^2_{\tilde{\ell}}} \frac{1+\cos\theta}{2}, \end{aligned} \tag{6.10}$$

where the first step comes from the assumption of massless leptons and θ is the angle between the two leptons. Since the slepton is a scalar particle it does not carry any information about angular momentum and hence the second lepton will decay in a random direction uniformly distributed on the sphere. This means that the factor $1 + \cos\theta$ has a flat distribution and from that one can deduce that the distribution of $M_{\ell^+\ell^-}$ has a triangular shape with a clear cutoff at the value

$$M_{\ell^+\ell^-\text{max}} = \frac{\sqrt{\left(M^2_{\tilde{\chi}^0_2} - M^2_{\tilde{\ell}}\right)\left(M^2_{\tilde{\ell}} - M^2_{\tilde{\chi}^0_1}\right)}}{M_{\tilde{\ell}}}. \tag{6.11}$$

This cutoff is easy to look for and as can be seen from eq. (6.11) it gives us some information about the sparticle masses. There are of course also other invariant mass distributions that one can search for cutoffs like this and in total it might be possible to find enough information to disentangle all the sparticle masses. It should though be noted that in many cases we need detailed knowledge about the expected shape of the distributions to use them effectively, just knowledge about where they should have a cutoff is not enough [124].

Let us so finally return to the question about an off-shell slepton in the chain of figure 6.1. In this case we have to look at the decay as a three-body decay $\chi^0_2 \to \ell^+\ell^-\chi^0_1$.

Since this calculation would be in close analogy with the calculations done in [125], except that the neutrino should be replaced by the massive neutralino, we do not repeat it here. We just point out that in SPS6 there is indeed an important contribution of this sort which, as one can see from plots in [125], looks very similar to the triangular distribution obtained for an on-shell slepton. It is, though, important to note that the distribution is different (to a good approximation it should be a third degree polynomial) but the suppression of the low end of the distribution imposed by the event selection, makes it look more triangular. It is also important to note that the cutoff no longer occurs at the value given by eq. (6.11) but by $M_{\tilde{\chi}_2^0} - M_{\tilde{\chi}_1^0}$.

This last point also emphasizes the importance of knowing the invariant mass distributions one is dealing with, if the distribution mentioned from SPS6 is interpreted as a triangular distribution from a chain with an on-shell slepton, we would end up assuming a sparticle mass hierarchy where the sleptons have masses between the two lightest neutralinos, while in reality the sleptons are heavier than the χ_2^0.

6.4 Neutralino decays at the LHC

If R-parity is violated, the expected phenomenology of supersymmetric models is radically modified. It is for instance possible to produce single sparticles at colliders, which then allows e.g. sfermions to appear as resonances, mediating a number of otherwise unlikely or forbidden processes [126–128].

However, since there are limits on how large the R-parity violating couplings can be, it might be more likely that the sparticle production is still dominated by pair production. Even so, the fact that all sparticles will eventually decay to SM particles, significantly modifies the strategy for SUSY searches.

With R-parity violated, there is in principle no reason to require the neutralino to be the lightest of the sparticles[4]. However, after running the renormalization group equations, the neutralino naturally turns out light due to lack of color and electric charge. Moreover, the neutralino couples to all of the 45 trilinear R-parity violating operators and hence the decay of the neutralino provides an opportunity to study hierarchies and flavour structures among them.

The decay of neutralinos through trilinear R-parity violating couplings at the LHC and the possibility to determine operator hierarchies and flavour structures among the couplings, is the topic of the third paper [125]:

Paper III: *Flavour structure of R-violating neutralino decays at the LHC*

This paper is meant as an exhaustive investigation of all possible signals that might arise from neutralino decays through trilinear R-parity violating couplings at the LHC. This is done through extensive simulations using the Monte Carlo event generator PYTHIA 8 [129].

It is first demonstrated that with a neutralino LSP, it is likely that pair produced

[4]We ignore the gravitino in this discussion since its small couplings will make sure it plays no role in LHC physics.

squarks and gluinos will cascade down to the neutralino[5], which will then decay into three SM particles.

Due to the decay of the neutralino, the final state will no longer be characterised by large missing E_T but by a large multiplicity of leptons and/or jets (and possibly some missing E_T due to final-state neutrinos). Due to the large jet background from QCD processes, leptons are by far the most useful particles when it comes to identifying new physics scenarios. So also in this case, where the purely leptonic operators can be well identified while the purely hadronic ones are more difficult.

The large lepton multiplicity arising from the purely leptonic $LL\bar{E}$ operators makes the background suppression more or less trivial, however, identifying the flavours of the coupling, especially in the presence of more than one large coupling, requires some more work. This problem is enhanced by the loss of information carried by the neutrino that is always present in the final state of the decay, as well as the presence of taus in the final state. Taus are problematic since they cannot be directly observed but have to be inferred by their decay products (light leptons and tau-jets) and in both cases we lose one or two neutrinos.

To resolve these problems we make theoretical predictions of the expected invariant mass distributions from the final state particles and show that it is possible to reconstruct the flavour structure of the operator (or operators if more than one is large) by fitting the theoretical expectations to the data.

In the semileptonic $LQ\bar{D}$ operators, we have the advantage of being able to identify all decay products from the neutralino if the decay goes to a charged lepton and two jets. This allows easy identification due to the mass peak we then expect in the appropriate invariant mass distribution. However, taus again turn out to be a problem since the loss of neutrinos will destroy the mass peak and here the problem is even worse than for the $LL\bar{E}$ type operators since the large combinatorial background in all distributions including jets is hard to reduce enough to see a continuous signal. The presence of b-jets in the final state would, though, significantly improve our chances.

The most problematic set of $LQ\bar{D}$ operators is, however, the ones with a third generation Q operator, i.e., $LQ_3\bar{D}$. In this case the presence of charged leptons in the final state would require a top quark to be produced and unless the neutralino is extremely heavy, this channel will be suppressed. The result is only decays to neutrinos and jets, and although the missing E_T from the neutrinos might be enough to see a deviation from SM physics, it will be hard to identify the scenario and especially the lepton flavour of the operator cannot be measured at the LHC.

Purely hadronic operators, $\bar{U}\bar{D}\bar{D}$ will in general produce jets and hence the signal is likely to drown in the QCD background. In order to see these signals one has to resort to a more advanced treatment of the jets involved [130]. It is also worth noticing that there are operators of type $\bar{U}_3\bar{D}\bar{D}$ that behave very differently from the others. In fact, due to the presence of a top quark in the final state, a light neutralino might actually be stable on LHC detector scales and we might face a fake MSSM scenario. Alternatively, if the neutralino is heavy enough it will decay to top quarks and we will see events with same-sign top pairs.

[5]Though it is possible for some sparticles higher up the decay chain to decay directly through some RPV coupling, this coupling would have to be problematically large.

Chapter 7

Summary and conclusions

Despite the success of the current Standard Model of particle physics, there are plenty of phenomena still to be explained. There is no lack of ideas for how to move forward and one of the most interesting ideas for the future is supersymmetry.

In this thesis, the phenomenological implications of introducing trilinear R-parity violating couplings to a supersymmetric scenario were studied. It is first of all important to note that the gravitino is a perfectly valid dark matter candidate also if R-parity is violated.

The decay of gravitino dark matter can potentially give rise to detectable cosmic rays and it is fully possible to explain the recent cosmic ray anomalies in electrons and positrons, seen by PAMELA and Fermi LAT, with the help of decaying gravitinos. However, the corresponding gamma ray signal has not been seen. This does not exclude gravitinos as the source of the cosmic ray anomalies but it makes it less likely to be the correct answer.

If the gravitino is light enough, it is possible to have relatively large R-parity violating couplings without violating any cosmic ray measurements. In some cases the couplings can even approach the experimental upper bounds.

Adding trilinear R-parity violating terms also has large implications for LHC physics. The usual search strategies for supersymmetry, based on searches for missing transverse energy, have to be replaced by searches for multi-jet and/or multi-lepton finals states.

If the R-parity violating couplings are too small to affect the initial part of the event, i.e. squark and gluino pair production followed by cascade decay to the neutralino, they will still make the neutralinos decay to standard model particles. In such a scenario, there is good hope to determine which R-parity violating operators are important and what the neutralino mass is.

Appendix A

Notational conventions

Throughout the thesis natural units have been used, i.e. we have $c = \hbar = 1$.

Square brackets are used for commutators, i.e. $[X,Y] = XY - YX$ and curly brackets are used for anti-commutators, i.e. $\{X,Y\} = XY + YX$.

The Greek indices μ, ν refer to spacetime indices and the Einstein summation is used i.e. $A^\mu B_\mu = \sum_{\mu=0}^{3}(A^\mu B_\mu)$. We also employ the Feynman slash notation $\not{A} = \gamma_\mu A^\mu$ for any four vector A^μ; γ_μ denotes the Dirac gamma matrices.

The Greek indices α, β, γ (running from 1 to 2) are used for two-component spinor indices (used in chapter 4) and also here Einstein summation is assumed, the method of raising and lowering indices is by the totally antisymmetric $\varepsilon_{\alpha\beta}$ ($\varepsilon_{12} = 1$). For the conjugated spinors we use dotted indices, i.e. $\dot{\alpha}$ and $\dot{\beta}$.

Latin indices are used for collections of fields and symmetry generators. To not confuse them with indices where upper and lower indices differ, all these indices are up. When applicable, summation is also assumed over these indices, it should be evident from the context where that is the case.

When a metric is needed we use the standard Minkovski metric as used in particle physics:

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.1}$$

The representation used for the $SU(2)$ symmetry of the Standard Model is $\frac{1}{2}$ times the Pauli matrices:

$$X^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad X^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad X^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.2}$$

In the discussion of two-component (Weyl) spinors in chapter 4 the Pauli matrices (without the factor $\frac{1}{2}$) are denoted σ^i and also used is the four-vector $\sigma^\mu = (\mathbf{1}, \sigma^i)$.

Appendix B

Some words on probability distributions

When discussing invariant mass distributions it is good to repeat some basics on probability distributions, especially about how to combine the distributions of several random parameters and how to change variables [131].

Let us start with the change of variables. Suppose we have a continuous variable x that has a probability distribution $p_x(x)$ and we want to know the probability distribution of a variable $y = f(x)$ where $f(x)$ is some smooth, invertible function.[1]

Denoting the distribution of y, $p_y(y)$, we start by looking at the probability of x having a value in some small interval dx. This is by definition of the probability distribution equal to $p_x(x)dx$. This probability has to be the same as the probability of y having a value in some small interval dy around $f(x)$, i.e it has to be equal to $p_y(y)dy$ for $y = f(x)$.

In other words, we have:

$$p_x(x)dx = p_y(y)dy \Rightarrow p_y(y) = \frac{dx}{dy}p_x(x) = \frac{df^{-1}(y)}{dy}p_x(f^{-1}(y)). \tag{B.1}$$

As an example we can look at the case where we know the distribution $p_x(x)$ of $x = y^2$ and we want the distribution of y. In this case the easiest way forward is to use that $\frac{dx}{dy} = 2y$ and the result is:

$$p_y(y) = 2yp_x(y^2). \tag{B.2}$$

The next calculation we need is how to combine two random variables. More precisely, if we have two independent random variables x and y with distributions $p_x(x)$ and $p_y(y)$, what is the distribution of $z = xy$?

The answer necessarily includes an integral over one of the variables x or y. The easiest, although not very rigorous (for more general formulas see [131]), way of finding the appropriate integral is to set up an ansatz like

$$p_z(z) = \int p_x(x)p_y\left(\frac{z}{x}\right)g(x)dx \tag{B.3}$$

where $g(x)$ is a function to be determined. The unknown function $g(x)$ can be determined by the fact that z has to be normalized correctly, i.e. $\int p_z(z)dz = 1$ if the integral

[1] In the calculation we assume that $f(x)$ is increasing, if it is decreasing, replace $\frac{df^{-1}(y)}{dy}$ with $|\frac{df^{-1}(y)}{dy}|$ in eq. (B.1).

is taken over the whole distribution. This gives us

$$\int p_z(z)dz = \int \int p_x(x) p_y\left(\frac{z}{x}\right) g(x) dx dz = \int p_x(x) x g(x) dx = 1, \tag{B.4}$$

and from this it follows that $g(x) = 1/x$.

Finally we need to look at the intervals of validity for the variables. This means that the probability distribution is zero outside of some interval. The most convenient way of including this is by stating an interval within which a variable is valid, i.e. saying that $x \in [x_{\min}, x_{\max}]$ and similar for the other variables.

With the above formalism it is clear that $z = xy \in [x_{\min} y_{\min}, x_{\max} y_{\max}]$ as long as all limits are positive. It is also important to find the right limits for the integral in eq. (B.3). This is done by comparing the arguments to the distributions with the ranges of validity of the same distributions and for simplicity we assume that all limits are positive. This gives us the final formula for combining the two distributions:

$$p_z(z) = \int_{\max\left(x_{\min}, \frac{z}{y_{\max}}\right)}^{\min\left(x_{\max}, \frac{z}{y_{\min}}\right)} \frac{p_x(x)}{x} p_y\left(\frac{z}{x}\right) dx. \tag{B.5}$$

The formula (B.5) is extensively used in the calculation of theoretical invariant mass distributions in [125].

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