Estimating fundamental equilibrium exchange rates: how related are these rates to the actual exchange rate changes during the last decade?

by

Torkil Bårdsgjerde

Master's thesis

This master's thesis is submitted for completion of the following degree

Master of Economics

University of Bergen, Department of Economics

December 2011

UNIVERSITETET I BERGEN

Preface

This thesis marks the end of my 5 years spent at the University of Bergen. I would like to thank Professor Erling Vårdal for his help while writing this thesis. His door was always open, and his feedback has been invaluable.

I would also like to thank my fellow students for our time spent together. I would not have enjoyed it as much without any of you. A special thanks goes to Teis, Stian, Kaspar, Jonas and Steven, both for listening to me when I had something to say and for guiding me when I had nothing.

Torkil Bårdsgjerde, Bergen, 1st of December 2011

Abstract

Estimating fundamental equilibrium exchange rates: how related are these rates to the actual exchange rate changes during the last decade?

by

Torkil Bårdsgjerde, Master of Economics

University of Bergen, 2011

Teaching supervisor: Erling Vårdal

This thesis will estimate fundamental equilibrium exchange rates (FEERs) for a set of currencies during the last decade, using a five-country framework. The aim is to see if the results from FEER estimation can explain medium-term real and nominal exchange rate changes. I start off by defining the concept of the FEER, and proceed to explain the underlying mechanisms at play. The framework of the model is then presented, along with my application of the model. I will then present the results from my analysis, both in real and nominal terms. The results are followed by an analysis of the problems encountered while using the model. The result of the USD/CNY rate is then compared with other similar studies. My results show that FEER estimation provides a good indicator for two of the three bilateral relationships I have studied in the medium term.

Table of contents

Appendixes and references

Tables

Figures

1: Introduction

"But if the renminbi isn't deeply undervalued, why has China had to buy around $1 billion a day of foreign currency to keep it from rising?"

-Paul Krugman, 2011.

During the last decade, the major currencies in the world experienced large changes in both their real and nominal exchange rates. During the latter part of this decade, significant political pressure, especially from the USA, called for an appreciation of the renminbi[1]. Inspired by these events, this thesis aims to explain the exchange rate changes that occurred during the previous decade. To do this, I will use a framework earlier presented by Williamson and Cline (2008), and tailor it to fit my purpose.

This thesis is divided into three parts. The first part is purely theoretical, providing an introduction into the concepts incorporated in the framework by Williamson and Cline (2008). Firstly, the concept of the FEER is defined. I then provide an introduction to the concept of real effective exchange rates. As FEER estimation demands normative choices, the reasons behind these choices are explained in chapter 5. Chapter 6 and 7 also provides an introduction to the parameters needed in FEER estimation, and where I obtain the necessary data to construct these.

The second part of this thesis is model specific. It will firsty present the model in a three-country framework. This framework is the same as the five-country model I will use in my analysis, and therefore provides insight into the process of creating FEERs. The three-country model will be presented with an example. During this example I will provide tables with the information necessary to estimate FEERs, and link it to the presented framework. I will then present the five-country model and the application of this in my analysis.

[1] The renminbi is the official currency of China. Its primary unit is the Chinese yuan, denoted CNY.
[2] This interpretation is also found in Williamson and Cline (2008:p.4)

The third and final part presents the results. Firstly, it presents the results of my analysis as real and nominal bilateral exchange rates, all with regard to the U.S. dollar. As my data contain an inconsistency problem and I use a set of equations that are over determined, chapter 13 measures the consequence of these problems. I compare the results from my analysis to the results presented in Williamson and Cline (2008), which is useful to see the different results produced by two almost identical models. The thesis ends with a comparison of my CNY/USD results with regard to other studies, before the final conclusions are presented in chapter 16. Appendixes and references are presented after the conclusion.

2: The concept of the FEER

A fundamental equilibrium exchange rate (FEER) is the real exchange rate in fundamental equilibrium. This chapter will define the concept of the FEER and fundamental equilibrium. Moreover, it will provide a general overview over the key questions regarding such an analysis.

Williamson (1994, p.179) defines the FEER as “the exchange rate that is consistent with macroeconomic balance, meaning the simultaneous achievement of internal and external balance”. This statement demands further definitions of internal and external balance. In the same paper, Williamson defines internal balance as “acceptance of the historically determined wage rate and achievement of a level of effective demand such as to sustain the highest level of activity consistent with the control of inflation” (Williamson, 1994, p.179). He then goes on to define external balance as “in terms of a current account target rather than overall balance. A minimum criterion is to require that the current account outcome is sustainable. This rules out the possibility of very large current account deficits financed by massive inflows attracted by exceptionally high interest rates” (Williamson, 1994, p.180). As these definitions are not entirely crystal clear, the definition and application of internal and external balance will be discussed more thoroughly in chapter 4 and 5.

The term “fundamental equilibrium” has a historical pretext. Member countries in the Bretton Woods agreement were allowed by the IMF to change their par values if their balance of payments was in a fundamental disequilibrium. If a country’s balance of payment stayed within certain sustainable targets, it was defined as in fundamental equilibrium. If the selected country’s balance of payment exceeded these targets, the country would be defined as in disequilibrium and allowed to change their par values. This statement did not imply that their balance of payments should equal zero. Economic theory predicts increased global growth if capital is allowed to flow from capital-intensive to capital-scarce countries. This point of view is taken into consideration when estimating FEERs.

More recently, Williamson and Cline (2008, p.1) defines FEERs as the "real, effective exchange rates that will achieve specified medium-term objectives for the economy". This is the definition I will base my work on. As clear from the definition, FEERs are expressed in terms of real exchange rates. A FEER is also effective, implying that it is expressed as the currency's value in terms of more than one other currency. This is achieved by expressing the exchange rate as a weighted average of more than one bilateral exchange rate. Chapter 3 will deal with the measurement of real effective exchange rates.

Estimating FEERs requires an empirical model which quantifies how the real exchange rate affects macroeconomic variables. To achieve a FEER the country needs to be in both internal and external equilibrium. Advanced studies use a general macroeconomic model to estimate FEERs. For simplicity, I have chosen a partial model which solely focuses on the external balance of the country. This makes the estimation of FEERs both easier to obtain and update, a reflection also made by Akram, Brunvatne and Lokshall (2003). FEER estimations are presented from earlier mentioned economists John Williamson and William R. Cline on a yearly basis for the Peterson Institute of International Economics, and are also used by numerous other economists. As the concept of the FEER is a relatively new concept, it is still under development.

When determining if a country is in external balance, the key question is how large a current account balance that is sustainable for each specific country. To determine this, one has to set a current account balance threshold that cannot be exceeded. When setting these thresholds, I will rely on conventional economic theory and an individual assessment of the countries in question. This will be discussed further in subchapter 5.3.

FEER estimations are always within a time frame. As the changes needed to realign a currency within FEER cannot be done instantly, this modeling allows potential changes to happen over time. By choosing a benchmark year (e.g. 2008) and a base year (e.g. 2007), the results predict a real exchange rate that would occur if the country in question changed certain macroeconomic values within the defined period. I will discuss my time frame and its implications in chapter 10.

The concept of FEERs is criticized by some economists. Partial models which only include external balance do not account for spillover effects resulting from the real exchange rate changes produced in the model. Critics claim that these effects are substantial, and therefore reduce the credibility of FEERs. The idea itself is valid: if a real exchange rate change affects GDP, that particular change in GDP can again affect the real exchange rate. However, as Driver, Power and Ramsay (2001) concluded, studies on spillover effects tend to show insignificant changes on the real exchange rate.

Another point which has received some criticism is the current account targets set. Some critics claim that the thresholds set in FEER estimation are too wide, and that only very small current account balances should be regarded as sustainable. Other critics claim that any level of capital inflow and outflow can be sustainable, as “the market cannot fail”. If the latter statement is true, it will be meaningless to estimate FEERs. I agree with the view expressed by Williamson and Cline (2008, p.2) that “one can still identify dangerously large capital inflows and economically unproductive capital outflows”. I will determine which levels of capital outflow and inflow I regard as sustainable in subchapter 5.3.

To explain to usefulness of FEERs in exchange rate theory, I will first present the concept of purchasing power parity (PPP). To explain PPP, it is useful to be familiar with the Law of one price. The Law of one price states that in competitive markets free of transportation costs and official barriers to trade, identical goods sold in different countries must sell for the same price when their prices are expressed in terms of the same currency (Krugman & Obstfeld, 2009). An example can be useful to clarify this further: assume that the exchange rate for U.S dollars to euros is 1EUR = 1.25USD. If a product is sold for 30 euros in Europe, the corresponding price must be 30 x 1.25 = 37.5USD in USA. If the price deviates from this, e.g. 40USD, U.S. importers would have an incentive to buy the product in Europe for 30EUR and sell it for a price lower than 40USD to capture significant market shares. These market forces will therefore ensure that the price will be equal in both countries.

The theory of PPP states that the exchange rate between two countries' currencies equals the ratio of the countries' price levels (Krugman & Obstfeld, 2009). It was developed into its modern form by the Swedish economist Gustav Cassel, who stated: "As long as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries takes place; the actual rate of exchange cannot deviate very much from this PPP" (Cassel, 1918, p.413). This can be interpreted as an aggregated version of the law of one price: if the price level in one country rises more than in another, the bilateral exchange between the currencies rate should reflect this by a corresponding change. However, as the law of one price applies for a specific good, PPP applies for the general price levels in countries. If PPP holds, nominal exchange rates will always reflect the price levels between countries. A country's price level is measured in a basket of goods, usually a selection of consumer goods and services.

It should be noted that PPP is further divided into relative and absolute PPP. Absolute PPP states that a basket of goods should cost the same in two different countries. Relative PPP states that the inflation rates of the countries in question, measured by the price level of a basket of goods, should change by the same rate or trigger an exchange rate depreciation or appreciation. The percentage change in the value of the currency should then equal the difference in the inflation rates between the two countries.

A natural question to ask at this point is: "How well does PPP explain changes in exchange rates?". The answers found for this question are very diverse, ranging from very pessimistic to very optimistic under certain conditions. To quote maybe the most pessimistic, Paul Krugman (Krugman & Obstfeld, 2009, p.392) claims: "All versions of PPP does badly in explaining the facts". On the more optimistic side, Rogoff and Obstfeldt (1999) found that relative PPP has held in the long run for a sample of 20 countries and a time period from 1870 to 1990. As clear from these differing opinions, whether PPP is a good estimator for exchange rates is a question of substantial controversy. Since the analysis I preform is a medium-term model, I will assume in my analysis that PPP does not hold. This can be assumed without too much controversy as very few (if any) serious studies conclude that PPP holds in anything other than in the long run.

FEER estimation uses a framework designed to do what it appears that PPP-based analyses cannot: explain exchange rate changes in the medium term. This is an important question for economists, which seems to spark substantial controversy. Numerous frameworks are designed for this purpose, all predicting different results. As of today, there does not seem to be consensus about which framework one should apply. My CNY/USD exchange rate results will be compared with other studies using both FEER estimation and other frameworks in chapter 15.

3: Real effective exchange rates and how they are calculated

Starting with nominal exchange rates, a nominal exchange rate is what we observe in practice every day. A short look in the financial pages of a serious newspaper would easily inform that e.g. 1 USD is worth 6 NOK, or 1 EUR is worth 1.25 USD.

If we assume PPP, the nominal exchange rate is explained as:

$$E_{USD/EUR} = \frac{P_{US}}{P_E} \qquad (1)$$

$E_{USD/EUR}$ is the exchange rate of U.S. dollars to euros, i.e. the number of U.S. dollars one needs to buy 1 euro, P_{US} is the price level in the USA and P_E is the European price level. The price level is measured by the price level of a basket of goods, assumed to be equal in the USA and Europe. As mentioned earlier, the analysis in this paper is medium-term and I will therefore assume that PPP does not hold. For my purpose it is more useful to express exchange rates in terms of real exchange rates, defined as:

$$R_{USD/EUR} = \frac{E_{USD/EUR} * P_E}{P_{US}} \qquad (2)$$

Per definition, the nominal exchange rate is then expressed as:

$$E_{USD/EUR} = \frac{R_{USD/EUR} * P_{US}}{P_E} \qquad (3)$$

$R_{USD/EUR}$ is the bilateral real exchange rate between U.S. dollars and euros. If we assume that PPP holds, the real exchange rate stays constant. Likewise, as clear from the expression of $E_{USD/EUR}$, the nominal exchange rate will change only according to changes in price levels. This relationship can be clarified further by a numerical example:

$$R_{USD/EUR} = \frac{\text{1.25 per euro x (EUR 100 per Goods Basket}_E)}{\text{(USD 125 per Goods Basket}_{US})}$$

$$R_{USD/EUR} = 1 Goods\ Basket_E\ per\ 1 Goods\ Basket_{US}$$

In this example I have assumed that the price levels are actually reflected in the nominal exchange rates, i.e. PPP holds. That might not always be the case. If the price of the European basket increases but the nominal exchange rates stays constant, $R_{USD/EUR}$ rises and we have a real depreciation of the U.S. dollar against the euro. A real depreciation means that the U.S. dollar's purchasing power in Europe falls (Krugman & Obstfeld, 2009).

While a real bilateral exchange rate involves two countries, a real effective exchange rate is a weighted average of more than one foreign currency. In a hypothetical situation where we have three countries, we can express one country's real effective exchange rate as:

$$R_1 = {R_{1,2}}^{\varphi_{1,2}} + {R_{1,3}}^{\varphi_{1,3}} \quad (4)$$

where the real effective exchange rate of country 1, R_1, is expressed as a weighted average of the two bilateral exchange rates between country 1 and 2 and country 1 and 3. The parameter $R_{1,2}$ implies the bilateral real exchange rate between country 1 and 2. The subscript defines firstly the domestic country, and secondly the foreign country. The parameter $\varphi_{1,2}$ is the bilateral trade weight between country 1 and 2. The idea is that one country's real effective exchange rate should reflect on the real exchange rate of the countries it trades more with. This method prevents insignificant bilateral real exchange rates to affect the real effective exchange rate much. Chapter 7 will explain how trade weights are equated.

By taking the logarithm of equation (1), one obtains:

$$ln\, R_1 = \varphi_{1,2} * ln\, R_{1,2} + \varphi_{1,3} * ln\, R_{1,3} \quad (5)$$

And taking the derivative, one obtains:

$$\frac{d\, R_1}{R_1} = \varphi_{1,2} * \frac{d\, R_{1,2}}{R_{1,2}} + \varphi_{1,3} * \frac{d\, R_{1,3}}{R_{1,3}} \quad (6)$$

This relationship is vital in our model. The left side of equation (6) measure change in percent of R_1. The right hand side shows the corresponding percentage change in $R_{1,2}$ and $R_{1,3}$, weighted by the trade weights $\varphi_{1,2}$ and $\varphi_{1,3}$. This relationship is used to estimate the necessary change in the bilateral real exchange rates, $R_{1,2}$ and $R_{1,3}$, to follow from the desired change in the real effective exchange rate, R_1. This necessary change, $d\, {R_1}^*$, is then expressed as percent of total R_1, $\hat{R}_1 = \frac{d\, {R_1}^*}{R_1}$. The logic behind it goes as follows: one identifies the desired change, $\hat{R}_1$, needed to restore equilibrium. This desired change leads to a change in the bilateral real exchange rates, weighted by their importance. When using (6) to calculate for desired change in real effective exchange rates one obtains:

$$\frac{d\,R_1{}^*}{R_1} = \varphi_{1,2} * \frac{d\,R_{1,2}{}^*}{R_{1,2}} + \varphi_{1,3} * \frac{d\,R_{1,3}{}^*}{R_{1,3}}$$

or,

$$\hat{R}_1 = \varphi_{1,2} * \frac{d\,R_{1,2}{}^*}{R_{1,2}} + \varphi_{1,3} * \frac{d\,R_{1,3}{}^*}{R_{1,3}} \tag{7}$$

In equation (7), a real depreciation of currency 1 will cause $\hat{R}_1$ to rise, and trigger a corresponding rise in $\frac{d\,R_{1,2}{}^*}{R_{1,2}}$ and $\frac{d\,R_{1,3}{}^*}{R_{1,3}}$, weighted by the trade weights.

Knowing how to express real effective exchange rates, I turn my attention to how one can express bilateral and triangular relationships. In a free market like the exchange rate market, the relationship between currencies (in nominal terms) will always be arbitrage-free. If arbitrage opportunities arise in such a market, they will be exploited until the resulting supply and demand shifts eliminates them. The relationship between real exchange rates is also arbitrage-free, given the definition of real exchange rates. In a two-currency example, the relationship between two real exchange rates can therefore be expressed as:

$$R_{1,2} = \frac{1}{R_{2,1}} \tag{8}$$

By taking the logarithm:

$$ln\,R_{1,2} = -\,ln\,R_{2,1} \tag{9}$$

And the derivative, and therefore measuring rate of change, it can be expressed as:

$$\frac{dR_{1,2}}{R_{1,2}} = -\frac{dR_{2,1}}{R_{2,1}} \tag{10}$$

Hence, a real appreciation of one currency creates a corresponding real depreciation of the other country's currency. When three currencies are involved, a bilateral exchange rate can always be expressed as the product of two others. Assuming no arbitrage opportunities, this can be modeled as:

$R_{1,3} = R_{1,2} * R_{2,3}$, or

$$R_{2,3} = \frac{R_{1,3}}{R_{1,2}} \quad (11)$$

Taking the logarithm gives us:

$$\ln R_{2,3} = \ln R_{1,3} - \ln R_{1,2} \quad (12)$$

Taking the derivative, and therefore measuring rate of change, it can be expressed like:

$$\frac{d\,R_{2,3}}{R_{2,3}} = \frac{d\,R_{1,3}}{R_{1,3}} - \frac{d\,R_{1,2}}{R_{1,2}} \quad (13)$$

Now that these relationships are in place, I will construct the equations which will be used later in my analysis. The first necessary step is to express (7) in negative terms. In the model I use in my analysis, I need a parameter that indicates a real effective depreciation when negative. By changing the prefix in front of each exchange rate, (7) can be expressed as:

$$\check{R}_1 = -\varphi_{1,2} * \frac{d\,R_{1,2}{}^*}{R_{1,2}} - \varphi_{1,3} * \frac{d\,R_{1,3}{}^*}{R_{1,3}} \quad (14)$$

where $\check{R}_1 = -\hat{R}_1$. A real effective depreciation of currency 1 will therefore cause a fall in $\check{R}_1$, followed by a corresponding fall in the weighted bilateral real exchange rates as their prefixes are negative.

Using equation (14), (10) and (13), we can construct the necessary relationship for country 2:

$$\check{R}_2 = \varphi_{2,1} * \frac{d\,R_{1,2}{}^*}{R_{1,2}} + \varphi_{2,3}\left(\frac{d\,R_{1,2}{}^*}{R_{1,2}} - \frac{d\,R_{1,3}{}^*}{R_{1,3}}\right) \quad (15)$$

Based on this line of reasoning, we can also express $\check{R}_3$ as:

$$\check{R}_3 = \varphi_{3,1} * \frac{d\,R_{1,3}{}^*}{R_{1,3}} + \varphi_{3,2}\left(\frac{d\,R_{1,3}{}^*}{R_{1,3}} - \frac{d\,R_{1,2}{}^*}{R_{1,2}}\right) \quad (16)$$

This modeling allows us to estimate the three relationships through two bilateral exchange rates. Equation (14), (15) and (16) will be used when estimating FEERs in my analysis.

4: Internal balance

Internal balance is defined as "acceptance of the historically determined wage rate and achievement of a level of effective demand such as to sustain the highest level of activity consistent with the control of inflation" (Williamson, 1994, p.179). Since I will be using a partial model only dealing with the external balance, I assume that all countries are in internal balance. By avoiding the question of internal balance, I also avoid a lot of normative questions which would have to be addressed. Determining e.g. which level of effective demand that sustains the highest level of activity consistent with the control of inflation, is a time-consuming task which would require a thorough review.

It should be noted that most recent FEER estimations tend to solely focus on external balance: even Williamson and Cline (2008) choose to ignore internal balance. This is despite that Williamson was central in developing the concept of FEERs himself. One should be careful to conclude too much from this, but it might seem like the focus in FEER estimations are shifting to external balance.

From the national accounts identity it will also be necessary that the trade deficit (goods and services) equals the excess of investment over domestic saving (including saving by the government). In this general equilibrium system it will be necessary by implication that changes in domestic absorption occur in parallel to the changes directly predicted from the export and import equations in response to exchange rate and activity changes (Cline, 2008).

5: External balance

This chapter will deal with the decisions I have to make when determining whether a country is in external balance or not. First, I will describe the relationship between the current account deficit of a country and its real exchange rate in subchapter 5.1. Second, I will take a look at the current account balances of 2009 and explain the reasons behind the situation in subchapter 5.2. I will proceed to set a general current account balance threshold, choose which countries to include in my analysis and evaluate them individually in chapter 5.3. Furthermore, the next two chapters will deal with the more data-related aspect of the calculations. Chapter 6 will deal solely with the elasticity parameter which defines the relationship between a current account balance change and the corresponding real exchange rate movement. In chapter 7 the necessary data to perform FEER estimation is presented along with the sources of data.

5.1: The relationship between the current account deficit and the real exchange rate

To explain the underlying mechanisms at play, I will present a stylized partial model for the balance of payments and how it is used to estimate FEERs. It is not identical to the model later used to estimate FEERs, but does a fine job in showing how a bilateral relationship can be expressed. The model I present in this chapter is published earlier in among others Akram, Brunvatne and Lokshall (2003).

First, it is assumed that the volume of import (B) is determined by the income level (Y) and the real exchange rate (R) of the country in question. The volume of import is measured in the domestic country's product units. For export (A), the same unit is used. An increase in the income level will affect the import positively, implying that at least a part of the increased income will be spent on goods that are imported. An increase in the real exchange rate will affect the import negatively, as imports will become relatively more expensive than goods produced domestically. This relationship gives us the import function:

$$B = B\left(\underset{+}{Y}, \underset{-}{R}\right) \tag{17}$$

Symmetrically, one can express the export volume of the home country as a function of the level of income in foreign countries (Y_f) and the real exchange rate. It should be noted that in this simple two-country model, one assumes a symmetrical relationship with no consistency problems. This will not be the case in the later part of my analysis. It is however useful here, as it allows us to construct the entire relationship from one country's viewpoint. Putting this point aside, when the real exchange rate increases it will affect export positively since it will make export relatively cheaper. The export function is expressed as:

$$A = A\left(\underset{+}{Y_f}, \underset{+}{R}\right) \tag{18}$$

The balance of trade is defined as the value of exports minus the value of imports of goods and services. A positive balance of trade means that a country is running an export surplus. The balance of trade is one of the three components of the current account balance, which is the sum of the balance of trade, the net factor income and the net transfer payments. I will denote the balance of trade deficit as TD, which is a negative unit. Since we have functions which express import (17) and export (18), one can express the TD:

$$TD = B(Y, R) - A\left(Y_f, R\right) \tag{19}$$

$$TD = TD\left(\underset{+}{Y}, \underset{-}{Y_f}, \underset{-}{R}\right) \tag{20}$$

where one can see that a country's TD is affected positively by an increase in the domestic income level. An increase in the foreign income level and real exchange rate depreciation (an increase in R) will reduce the deficit. Solving this equation with respect to the real exchange rate one obtains:

$$R = R\left(\underset{+}{Y}, \underset{-}{Y_f}, \underset{-}{TD}\right) \tag{21}$$

The derivation of this solution is presented fully in appendix A. The end result is:

$$\frac{dR}{R} = \frac{\varepsilon(TD,Y)}{\varepsilon(TD,R)} * \frac{dY}{Y} - \frac{\varepsilon\left(TD,Y_f\right)}{\varepsilon(TD,R)} * \frac{dY_f}{Y_f} - \frac{1}{\varepsilon(TD,R)} * \frac{dTD}{TD} \tag{22}$$

$\varepsilon(TD, Y)$ is the income elasticity for the balance of trade deficit. The other elasticities are to be understood the same way.

At this point my modeling differs from the one presented by Akram, Brunvatne and Lokshall (2003). The model I later use in my analysis focuses on the current account balance, not on the balance of trade. However, the relationship I have just presented can also express the effects a current account balance has on the real exchange rate. By assuming that the other components of the current account, the net factor income and the net transfer payments, have the same effect on the real exchange rate as the balance of trade, one can rewrite 16 so it contains the current account balance (CB) instead:

$$R = R\left(\underset{+}{Y}, \underset{-}{Y_f}, \underset{+}{CB}\right), \quad (23)$$

Or as:

$$\frac{dR}{R} = \frac{\varepsilon(CB,Y)}{\varepsilon(CB,R)} * \frac{dY}{Y} - \frac{\varepsilon(CB,Y_f)}{\varepsilon(CB,R)} * \frac{dY_f}{Y_f} + \frac{1}{\varepsilon(CB,R)} * \frac{dCB}{CB} \quad (24)$$

The CB is a positive unit, but should except from that be interpreted as the TD. Since the CB is a positive unit, it is given the opposite prefix. Remembering the definition of Williamson and Cline (2008), a real effective exchange rate is a FEER if its host country is in external and internal balance. To express the fundamental real exchange rate through equation (23), one obtains:

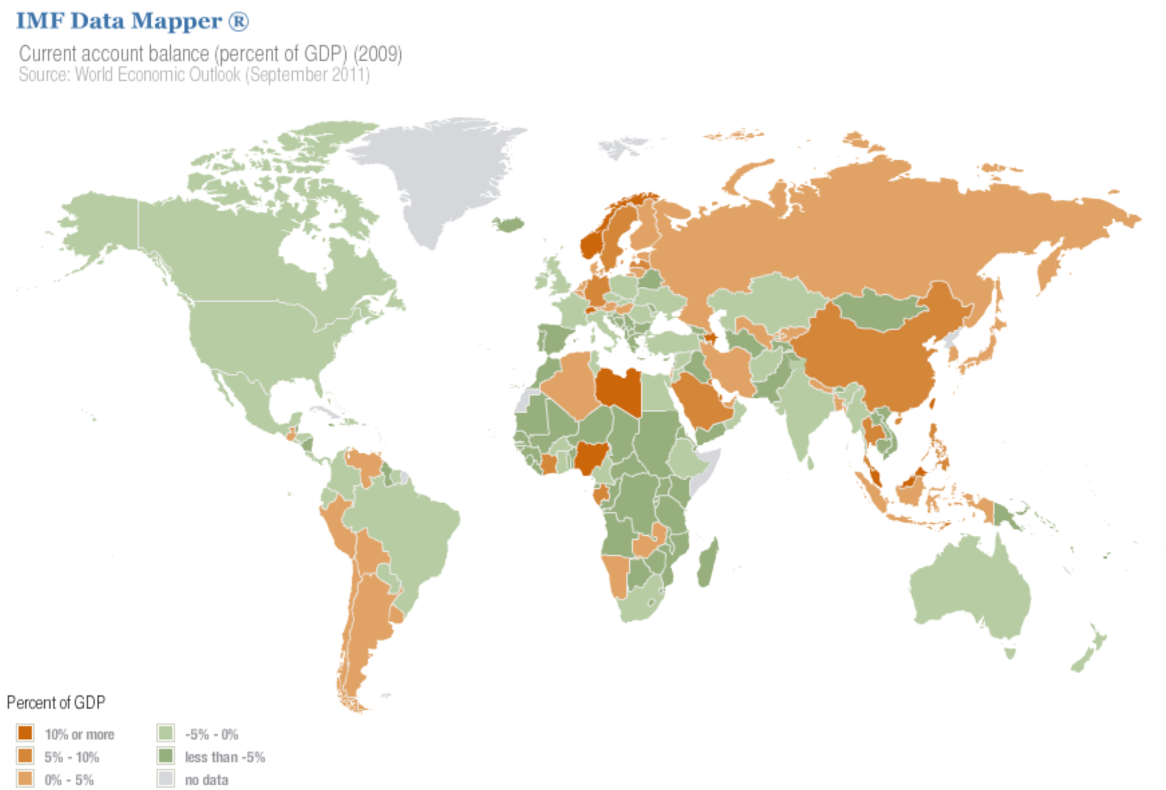
$$R^* = R(\left(\underset{+}{Y^*}, \underset{-}{Y_f^*}, \underset{+}{CB^*}\right) \quad (25)$$

Where Y^*, Y_f^* and CB^* are the equilibrium levels for domestic income level, foreign income level and current account balance. The equilibrium level for Y^* and Y_f^* can be set as equal to the potential levels of GDP in their respective countries, a situation which describes a country in internal balance trading with a foreign country in internal balance. Since I have assumed that all the countries in question are in internal balance, Y and Y_f will not be included in my analysis. As defined earlier, a country in external balance has a CB within certain values which are still to be defined. Bypassing the discussion about what these values are, one can still assume that at some levels the current account balance is in equilibrium, CB^*. R^* in equation (25) is therefore a real exchange rate when a country is in internal balance and external balance, a FEER.

5.2: A short overview of the current account imbalances in 2009

When using a partial model focusing on external balance, there is only one normative choice left: how large a CB is a country allowed without being classified as out of external equilibrium? This chapter will give a general idea of the current account imbalances in 2009, as an illustration of the size of the problem. I start by looking at the CBs divided on GDP ratios of the world in 2009:

Figure 1: CBs divided on GDPs, 2009



(Source: IMF World Economic Outlook, 2011)

In this figure, the CB/GDP ratios are presented for all the countries in the world. The darkest type of brown represents the countries with a CB divided on GDP ratio equal to or more than +10% (e.g. Libya). The lighter brown represents the countries with a ratio between +5% to +10% (e.g. China), and the lightest type of brown represents countries with a 0% to +5% ratio (e.g. Russia). The darkest type of green represents countries with a CB/GDP ratio of -5% or less (e.g. USA). The lighter type of green represents countries with a CB/GDP ratio of -5% to 0% (e.g. Brazil), and the gray countries do not have a ratio as there is not sufficient data.

As apparent from figure 1, there was a certain imbalance in the flow of capital and a resulting imbalance in the world economy in 2009. Contrary to what economic theory predicts, it is evident that the capital flow was not solely going from capital-intensive to capital-scarce countries. Countries like China and Nigeria were exporting large amounts of capital, and countries like USA, the U.K. and France were importing capital. If capital was exported solely from capital-intensive countries to capital-scarce countries, the relationship would be the other way around. The situation observed in 2009 is representative for the situation during the last decade. It is widely claimed that this is the result of mainly two causes: First, the increase in oil prices during the last decade which have led to large positive CBs for oil-exporting countries like among others Saudi-Arabia, Nigeria and Norway. Second, a number of East-Asian countries are exporting large amounts of capital to most notably USA and some countries in Europe. As much as I regard oil-exporting countries as cases of great interest, my analysis will not include the currency of an oil-exporting country. I will return to why later in subchapter 5.3.

The imbalance just discussed is at the core of FEER estimation. As is clear from subchapter 5.1, a currency's real exchange rate will be affected by its country's CB. Since we have a large imbalance in the CBs in the world today, an imbalance in the world currencies will by definition also be apparent. FEERs aim to estimate the fundamental exchange rate, i.e. the exchange rate which would occur if the CB imbalance was corrected.

5.3: Determining CB thresholds, currencies chosen and individual assessment of the countries in question.

To determine a CB threshold that a country cannot exceed is, quoting Williamson (1994, p.182): "The most controversial issue to arise at a conceptual level in defining the FEER". The threshold will always be determined as an interval. An important feature when setting a threshold as an interval is that it allows a country to aim for a specific target and miss modestly without necessarily being classified as out of equilibrium. If e.g. a shock of small or moderate size emerges, it is unlikely to bring the country outside its threshold if the country originally was pursuing sound politics. If e.g. a large shock emerges, it is natural to assume that it does affect the short-term exchange rate, but not the fundamental exchange rate. We will then have a situation where the FEER differs from the actual real exchange rate until the country is in external balance again. In other words, if the shock is big enough to cause the current account balance to diverge outside its threshold, it will affect a currency's FEER value. The thresholds should therefore be set so that a country can absorb shocks of a moderate size without being classified as out of equilibrium.

When setting the negative CB threshold, the question of sustainability is important. Discussing sustainable imbalances, Reinhart, Rogoff and Savastano (2003) argue that emerging-market economies need a CB that prevents a gradual buildup of debt. They calculate that to avoid an increase in debt, emerging-market economies require a CB/GDP ratio which does not exceed −3%[2]. Turning our attention to industrialized countries, there is a number of different estimates to consider. Freund (2000) argues that the critical threshold lies around a CB of -5% of GDP, while Mussa (2005) argues that USA (which at the time his paper was written had a CB/GDP ratio at approx. -5.9%) cannot sustain such a deficit. He estimates that a necessary 3% CB of GDP reduction is necessary, with a CB/GDP ratio threshold between -2.5% and -3% regarded as sustainable. Cline (2005) argues that for USA the critical CB/GDP threshold is at -3%.

Considering these studies, I set a general negative CB/GDP ratio threshold of 3%. Assuming this, it makes sense to adopt the same general threshold when it comes to current account surpluses. To solely burden the deficit countries with the task of correcting the capital flow imbalance would be unfair, as it can hardly be claimed that any level of current account surplus is sustainable either in the medium-term run. As briefly mentioned before, a current account surplus will affect the exchange rate as much as a current account deficit. To allow countries to run unlimited current account surpluses would also give them a huge advantage in the exporting market, as they can use their resulting capital to intervene in the currency market. This would give these countries' exporters an unfair advantage if exploited. To set an

[2] This interpretation is also found in Williamson and Cline (2008:p.4)

equal current account surplus threshold also ensures symmetry. I have therefore set a general rule that the CB/GDP ratio should not exceed $\pm 3\%$. This will be the general rule for all countries when they are individually assessed. I now turn to specify the countries that will be involved in my analysis, and I will then discuss if the general threshold should be applied to each country.

When preforming my analysis, I will be focusing on three countries and one economic area with a common currency: USA, China, Japan and the Eurozone. I will also include a “Rest of the World” (RoW) category, which is solely a residual country, unfit for interpretation. Counting the Eurozone as a country, it has the second highest GDP in the world according to the World Bank (2010), only surpassed by USA. China and Japan follow suit on third and fourth. Their currencies are therefore arguably the most important ones in the world as well, which makes them the most interesting to study. When dealing with the euro, I will regard the Eurozone as a country equal to any other country. This causes a few complications which are dealt with in the next chapter. These economies and their currencies are also interesting because of the diversity of their CBs. When the data is presented, it will be clear that these countries have very different CBs and therefore require different CB adjustments. The different CBs ensure interesting results when equating the bilateral real exchange rate changes.

When choosing these countries, I have not included any oil-exporting countries. It would be interesting to include e.g. Nigeria or Norway in my analysis, as these countries had a CB/GDP ratio of more than 10%. If they were required to realign their CBs within the same thresholds as the other countries, their real exchange rates would surely be affected greatly. However, setting a sensible threshold for an oil-exporting country is not a simple task. For some countries, e.g. Norway, it is part of a long-term policy to save all the direct income from oil-exports. To convert a nation’s exhaustible resource into foreign assets which will provide long-term income can hardly be regarded as unsustainable. To which degree the oil-exporting countries do this is varying, as e.g. Ecuador and Russia use most of their oil income. One should also consider the oil price development. During the previous decade, oil prices increased from an initial price of around 25USD in year 2000 to a top point of over 100USD. As oil production is a relatively fixed amount, it is sensible for oil-exporting countries to use the high oil price as an opportunity to acquire foreign assets. To realign itself within normal thresholds, an oil-exporting country would either have to halt production, increase domestic oil consumption or import more as the oil price increases. Neither of these options seems sustainable in the long run. It should also be noted that most oil-exporting countries are smaller countries, with the exception being Russia. Smaller countries will typically trade a lot relative to their size with larger countries. However, since the larger countries' trade share with smaller countries is relatively small, a large country would be relatively unaffected by the changes in the small country. When picking the largest economies in the world, I ensure that my analysis only contains countries with a significant mutual influence. With these arguments in mind, I avoid using an oil-exporting economy in my analysis.

Now that a general CB/GDP threshold has been established and the countries included in the analysis are presented, it is necessary to determine whether the countries in question should have similar thresholds. There are a few arguments that can be made when considering each country.

An important consideration is the demand for investment in each specific country. As fully industrialized economies have large amounts of capital already, it is assumed that the marginal benefit of capital is small. This would imply that it can be reasonable for such a country to export part of its capital to less industrialized countries, as the investment opportunities there will yield higher profits. This will also allow less industrialized countries to import capital for investments they cannot finance themselves. An industrialized country should therefore be expected to export capital, and less industrialized countries should be expected to import capital.

Another consideration is the demographics of a country. If the country in question has a rapidly aging population, their inhabitants would wish to save money for their retirement. In this regard, it seems wise to acquire foreign assets. Countries with young populations and a high average life expectancy will demand investment, as the workforce of these counties will grow. This implies that countries with an aging populations should be allowed higher

CB/GDP thresholds, and vice versa with regards to the negative CB/GDP threshold for countries with a young population.

One should also consider the GDP trend growth of a country. Countries with periods of close to zero or negative GDP growth will usually counter this by increased government spending in an attempt to promote growth. This implies that countries experiencing close to zero or negative GDP trend growth over a longer period of time should be allowed run a larger negative CB than countries with solid GDP trend growth. It should be noted that I do not regard a temporary recession as a valid reason to run a larger current account deficit. Since a temporary recession will affect an exchange rate for a short period of time, it is natural to assume that it does not affect the fundamental exchange rate of a country. A longer period of recession will however do just that, which explains the different treatment of the recession types.

Starting with the Eurozone, I choose to keep the positive CB/GDP threshold at 3%, but reduce the negative CB/GDP threshold. This might seem odd given that I have quoted papers stating that fully industrialized economies like the Eurozone can sustain a current account deficit as large as 5% of GDP. However, if the Eurozone should have a negative CB/GDP ratio, that would imply that capital goes “uphill”, in other words from capital-scarce countries to capital-abundant countries. This argues in favour of a lower negative CB threshold. It is difficult to conclude anything regarding the demographics argument for the Eurozone. The new member countries of the Eurozone have younger populations than the old, which combined with worker immigration results in a complex demography. I therefore construct no argument from the demographics of the Eurozone. The Eurozone experienced high GDP growth during most of this decade, and the recession starting in 2008 is regarded as a temporary recession. Their GDP growth is therefore not regarded as an argument in any favour. Considering these arguments, I set a CB/GDP ratio threshold from 0 to 3% as acceptable.

For the case of USA, one can easily conclude that it is an industrialized country, as it has the highest GDP per capita of the world. However, as a vast country with a lower population density than Europe, one can claim that there is still significant demand for investment. That the USA has a young and growing population, mostly due to immigration, also calls for a lower negative CB/GDP ratio threshold. Their GDP growth has like Europe's been solid until 2008, which calls for no argument in that favour. I will be following the advice of Cline (2005), and set a CB/GDP ratio threshold of ±3%. As this analysis is USA specific, I regard this evaluation as a strong argument. He argues that both the unique position of the US economy and the high returns to equity investment in USA "is cause for prudence when setting an acceptable level of net US liabilities" (Cline, 2005, p.172–74).

Japan and China are both capital exporters, with generally large surpluses on their CBs. It stands to reason that Japan with a GDP per capita at approx. 39 thousand U.S. dollars, around the same as Germany, should have capital needs equal to the Eurozone. Considering the demographics, one can construct an argument that Japan needs to export capital because of their ageing population. This is a valid argument in my opinion, but Japan has also experienced longer periods of deflation and negative GDP growth during the last decade. In such a situation, to export much capital in a period where the normal approach taken is to spend more seems unsustainable. Japan is therefore given CB/GDP ratio thresholds of 0% to +3%, the same as the Eurozone.

China is a significantly less developed country than the others mentioned so far. With a GDP per capita at approx. 7.5 thousand U.S. dollars it cannot be said to be fully industrialized. One would think that a country like China, with a low GDP per capita, tremendously high GDP growth and a young population, would demand capital. However, it has been and is still exporting large amounts of capital. As none of my arguments speaks in favour of a higher CB/GDP ratio threshold, I do not create an argument that they should be allowed to export more capital than the other countries in question. Their CB/GDP ratio is therefore set at ±3%.

6: The elasticity parameter γ

As will be clear when the model is presented in chapter 8 and 9, my model operates with solely one elasticity per country, γ. γ is defined as “a parameter that indicates the change in a country’s CB as a percent of GDP that takes place in response to a change in the country’s real effective exchange rate by 1 percent” (Cline, 2008, p.17). γ is therefore by definition the same as $\varepsilon(CB, R)$ from equation (24), but estimates the change in CB in percent of GDP instead of in percent of total CB. To explain this parameter in terms of what has already been presented, it can be useful to present equation (24) from chapter 5.1 again:

$$\frac{dR}{R} = \frac{\varepsilon(CB,Y)}{\varepsilon(CB,R)} * \frac{dY}{Y} - \frac{\varepsilon(CB,Y_f)}{\varepsilon(CB,R)} * \frac{dY_f}{Y_f} + \frac{1}{\varepsilon(CB,R)} * \frac{dCB}{CB} \tag{24}$$

It should be stressed that these models both estimate FEERs, but focuses on different aspects of the concept. While Akram, Brunvatne and Lokshall (2003) estimates for a bilateral relationship using the change in CB as a percent of total CB, my model estimates for several countries using the change in CB as a percent of GDP. Their model also includes internal balance, allowing the real effective exchange rate to change in accordance with change the three endogenous variables from year to year. My analysis uses a partial model, focusing solely on external balance. The two elements concerning internal balance, Y and Y_f, are as mentioned before not included in my model.

Another important simplification regards the fraction $\frac{1}{\varepsilon(CB,R)} * \frac{dCB}{CB}$. $\varepsilon(CB, R)$ includes by definition both the elasticity between import and the real exchange rate, and the elasticity between export and the real exchange rate. Cline (2008) assumes that the price elasticity for imports is equal to unity. By assuming this, the amount of money spent on imports, as well as the import measured in the domestic country’s product units, is constant. Doing this allows us to consider the whole adjustment process on the export side, which is a practical simplification. However, the assumption of a unitary elasticity is not necessary. The important thing is that the elasticity is negative, which will be the case if the Marshall-Lerner condition is met[3].

Having taken these assumptions, the model is stripped down to one relationship. A change in the CB will through $\varepsilon(CB, R)$ result in a corresponding change in R. This is modeled as the country specific:

$$\frac{d\,CB_1^*}{GDP_1} = \gamma_1\left(-\frac{d\,R_1{}^*}{R_1}\right) \qquad (26)$$

where $\gamma_1 = \varepsilon(CB, R)$. The model is therefore expressing the change in the real effective exchange rate as a product of γ multiplied with the change in the CB/GDP ratio required to restore equilibrium. Equation (26) will be dealt with in greater detail in chapter 8 when it is presented as part of the model.

To determine the size of real exchange rate movements resulting from a change in the CB, one needs a set of values for γ. Ideally, they would be estimated for every specific country for every year for maximal accuracy. However, as this would be a sizeable task, I will follow the parameter values set by Cline (2008) in his normal-elasticity model, with one value per country. For later studies it would be interesting to estimate γ for each year in question.

Cline (2008) estimates γ as the product of two factors. They are the price elasticity of exports, and the share of exports of goods and services in GDP. He proceeds to set the export price elasticity equal to unity for a relatively closed economy, falling towards 0.5 for a relatively open economy. An open economy is defined as an economy where the exports of goods and services are close to 100 percent of GDP. For the important case of the United States, the impact parameter also incorporates the effect of changes in valuation of international assets and liabilities from an exchange rate change, and consequential effects on subsequent capital service payments (Cline, 2005). This modeling of γ gives high γ values for open economies, and lower for more closed economies. The underlying concept is that an open economy will

[3]The Marshall-Lerner condition says that the sum of the (absolute value) price elasticity for imports and exports is larger than 1. A sufficient condition for this to be met is that the unitary elasticity of imports equal 1.

be more affected by a real exchange rate change than a less open economy. This implies that the necessary CB change will result in a smaller real exchange rate for a more open economy than a less open economy.

The γ values in my analysis are:

Table 1: The γ values for the countries in my analysis

2000-2010	USA	China	Japan	Eurozone
γ	-0.16	-0.30	-0.12	-0.14

(Source: Cline 2008)

The first row presents the time period and the countries in question. The second row presents the different γ values. As presented in table 1, China has the largest γ followed by USA, the Eurozone and Japan. When presenting the three-country example in chapter 8, I will show how the impact parameter is used together with the necessary CB adjustments to estimate the corresponding change in the real effective exchange rate.

7: The data to use and where to obtain them

In addition to the parameter γ discussed in the previous chapter, I need three more parameters to estimate FEERs: GDP, CB and trade weights (φ).

I will be estimating FEERs ex post[4]. Unlike solely focusing on one year, I will be performing my estimates on a year-to-year basis for an entire decade starting from 2000. The consequences of this are discussed further in chapter 10. As data for GDP and CB, I will use actual observed data. This is unlike ex ante[5] FEER estimations, which bases its results on predicted data. This data in my analysis is gathered from the IMF World Economic Outlook, April 2011. As the IMF also provides GDP and CB data for the Eurozone, the data can be used in the form it is presented. The GDP and CB data are among other data presented in appendix E.

Remembering equation (4), one needs trade weights (φ) to construct real effective exchange rates. As trade weights, I will use data from the Direction of Trade Statistics (DoTS) from IMF. The DoTS measures the value of export from one country to another country, hence effectively showing the importance of bilateral trade. To construct a trade weight between country 1 and 2, one calculates:

$$\varphi_{1,2} = \frac{export_{1,2} + export_{2,1}}{total\ export_1 + total\ import_1} \qquad (27)$$

and obtains country 1's equally weighted share of exports and imports with regard to country 2. Since the dataset only measures exports, import is estimated by summarizing the total amount of export to the country in question. In an example with two countries, the trade weights will equal 1. This is not surprising as they only can trade with each other, and therefore just has one other currency to appreciate/depreciate against. When e.g. three countries are in question, the value of each trade weight will be less than 1 if all three countries trade together. The trade weights of one country with respect to all its trading

4 Ex post is latin and means "after the event". In this case it means that we estimate for a time period that has already passed.

5 Ex ante is latin and means "before the event" In this case it implies the opposite of ex post, that we estimate for a time period that has not yet happened.

partners will always sum up to one. As the value of export is measured in U.S. dollars for all countries, it is unproblematic to compare this across borders. I will calculate the trade weights for each year in question, and use the trade weights for the base year when equating FEERs for the benchmark year.

When gathering DoTS data, problems arise when dealing with the Eurozone. Since I will treat the Eurozone equally to the other countries, I need export data from and to the Eurozone as any other country. This is not available as I need it in the DoTS dataset. The Eurozone as a region exists in the dataset as a sum of its member countries. That implies that the Eurozone exports to itself, as the member countries exports and imports from each other. This problem would be straightforward to eliminate if the Eurozone existed as a recipient of exports. One would simply have to remove the Eurozone's export to the Eurozone from the total export and then estimate the trade weights afterwards. However, the Eurozone does not exist as a recipient of exports in this dataset. This implies two problems: Firstly, the Eurozone would have too much export as the data includes exports within the Eurozone. Secondly, I would have to manually sum the export to the Eurozone.

My solution to this is to use the trade weights for the European Union. The EU exists as a sum of its member countries like the Eurozone, but curiously also exists as a recipient of exports unlike the Eurozone. When estimating the trade weights for the EU, I remove its export to itself from the total export, and then equate the trade weights. Using the EU instead of the Eurozone will increase the trade weights of the other countries toward the Eurozone, since the export and import of the EU is bigger than the export and import of the Eurozone. However, the euro is connected to the countries in the EU who does not use the euro. They influence the value of the euro through e.g. the inner market for goods and services and the institutions of the EU. I reckon using the EU trade weights therefore is an equally good representative of the euros actual importance. The estimation of the trade weights is presented in Appendix D.

It should be noticed that the DoTS definition of the EU is rigid and defined in 2011. This implies that countries which joined the EU in e.g. 2004 will be included as a part of the EU during all the years in question. The countries which have joined the EU since 2000 are

Bulgaria, Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Romania Slovakia, and Slovenia. Since the export of these countries summarized are a very small part of the EUs total export, I regard this problem as insignificant due to its small size.

To standardize the consumer prices indexes, which are set to equal 100 in different years across countries, I have set them all equal to 100 in 2000. Then, using IMF World Economic Outlook data, I have constructed CPIs for the remaining years adding their end of the year growth. Since the CPIs are equated from data at the end of the year, the same is done for the nominal exchange rates. The CPIs are presented in appendix H.

8. The three-country version of the model

This chapter will present the model as a three-country model. After presenting the model, I will use an example to illustrate the method. The results from this example will not be used further; they are only calculated for illustrative reasons. I will then expand it to the final five-country model in chapter 9, which has the same interpretation as the presented model. The presentation of the three-country model will be following the footsteps of Cline (2008).

Assume we have three countries: country 1 has an excessive negative CB, country 2 has and excessive positive CB, and country 3 has a CB close to 0. If these countries exceed the CB thresholds I determine, they will be adjusted to stay just within these thresholds with a corresponding change in their real effective exchange rate. The real effective exchange rates I end up with afterwards will be their FEERs.

We can express the relationship for our three countries as:

$$\widehat{C_1} = \gamma_1 \widetilde{R_1} \Leftrightarrow \frac{d\,CB_1^*}{GDP_1} = \gamma_1(-\frac{d\,R_1{}^*}{R_1}) \tag{28}$$

$$\widehat{C_2} = \gamma_2 \widetilde{R_2} \Leftrightarrow \frac{d\,CB_2^*}{GDP_2} = \gamma_2(-\frac{d\,R_2{}^*}{R_2}) \tag{29}$$

$$\widehat{C_3} = \gamma_3 \widetilde{R_3} \Leftrightarrow \frac{d\,CB_3^*}{GDP_3} = \gamma_3(-\frac{d\,R_3{}^*}{R_3}) \tag{30}$$

Where $\widehat{C_1}$ is the desired change in the CB as a percent of GDP for country 1. By desired change, I mean the change in CB as a percent of GDP needed to restore fundamental equilibrium. If a country is running a negative CB of 4 percent of GDP, we would need to reduce that deficit by 1 percent to bring the CB within our set boundaries of 3%. γ is the current account parameter discussed in chapter 6. We here see how the size of the country specific γ will determine the real exchange rate changes resulting from a CB change. A higher γ value implies that the resulting change in $\breve{R}$ is smaller than with a lower γ value, given the same $\hat{C}$. $\widetilde{R_1}$ is the corresponding change in the real effective exchange rate $(-\frac{dR_1{}^*}{R_1})$, resulting from the values of $\widehat{C_1}$ and γ_1. Since γ is always negative by definition, and $\hat{C}$ positive when a country is in need of a depreciation, $\breve{R}$ is used as it represents a real depreciation by definition when negative.

The thresholds I will use in my later analysis are already given in subchapter 5.3. For this specific example however, country 1 must stay within a negative CB/GDP ratio threshold of 3% to GDP, country 2 must stay within a CB/GDP ratio of +3%, and country 3 must have a CB equal to zero after they have all adjusted their CBs. An inconsistency problem then arises: this will only sum up to zero if country 1 and 2 have equally large GDPs. To solve this, a standard method is to relax the criteria for the surplus country, allowing it to run a surplus equal to the sum that changes the global sum to zero. In our example I will leave the inconsistency problem, knowing that the results produced will not give precisely the CB changes needed to realign all countries within their thresholds. Chapter 13 will estimate the size of this inconsistency problem in my analysis.

It should be noted that when equating FEERs in the actual analysis, no country will ever be required to have a CB equal to zero. All countries will be given an interval, I.e. the Eurozone will be forced not to run negative CB, but allowed a positive 3% CB/GDP threshold.

The next step is to use $\breve{R_1}$, $\breve{R_2}$ and $\breve{R_3}$ to equate the corresponding bilateral exchange rate changes needed. This system must be able to express the bilateral real exchange rate changes and the real effective exchange rate changes. Remembering equations (14), (15) and (16) from chapter 3, this relationship between effective and bilateral real exchange rates can here be defined as:

$$\breve{R_1} = -\varphi_{1,2} z_2 - \varphi_{1,3} z_3 \tag{31}$$

$$\breve{R_2} = \varphi_{2,1} z_2 + \varphi_{2,3}(z_2 - z_3) \tag{32}$$

$$\breve{R_3} = \varphi_{3,1} z_3 + \varphi_{3,2}(z_3 - z_2) \tag{33}$$

where z is:

$z_2 = \frac{d\,R_{1,2}}{R_{1,2}}$

$z_3 = \frac{d\,R_{1,3}}{R_{1,3}}$

In words, z_2 is the percentage-wise rise in the bilateral exchange rate of country 1's currency against the currency of country 2. A positive z means an appreciation of country 2's currency, and a corresponding depreciation of country 1's currency. There is no z_1 as the currency of country 1 cannot appreciate or depreciate against its own currency. Equation (31)-(33) uses the same two unknowns, z_2 and z_3.

By substituting equations (28)-(30) into equations (31)-(33), we get the final three equations in our three-country model:

$$\widetilde{R_1} = \frac{\widehat{C_1}}{\gamma_1} = -\varphi_{1,2} z_2 - \varphi_{1,3} z_3 \qquad (34)$$

$$\widetilde{R_2} = \frac{\widehat{C_2}}{\gamma_2} = \varphi_{2,1} z_2 + \varphi_{2,3}(z_2 - z_3) = z_2 - \varphi_{2,3} z_3 \qquad (35)$$

$$\widetilde{R_3} = \frac{\widehat{C_3}}{\gamma_3} = \varphi_{3,1} z_3 + \varphi_{3,2}(z_3 - z_2) = z_3 - \varphi_{3,2} z_2 \qquad (36)$$

As discussed in chapter 7, the trade shares of a country with its trading partners' equal unity. That is why the right-hand side of equations (35) and (36) can be reduced to their final form.

To further explain the three-country model, I will clarify by using an example. I assume that country 1, the one having an excessive negative CB, is USA. The country having an excessive positive CB is China, and country 3, the one having a CB close to zero, is the Eurozone. Firstly, I need to choose a base and benchmark year. To illustrate the mechanisms in play most effectively, I have chosen 2007 as my base year and 2008 as my benchmark year. When reading the z values, they should be read as e.g. "considering the predicted GDP and CB of 2008, the U.S. dollar should be depreciated "x" percent against the renminbi from 2007 to 2008 to realign its currency within FEER". As will be clear from table 1, all three countries will need a CB adjustment 2008, making it an interesting year for FEER estimation. For the prediction of 2008 I will use the actual CBs of 2008, as I will in my later analysis. The trade weights are equated from 2007. This is in accordance with ex ante FEER estimation.

Table 2: The GDPs, CBs, CB thresholds, required change in CB, impact parameters and corresponding change in $\breve{R}$ for the selected countries in 2008

2008	GDP	CB		CB threshold	Required change in CB		Impact parameter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	In real effective exchange rates ($\breve{R}$)
USA	14369075	-668856	-4,6548 %	±3%	237780	1,6548 %	-0,16	-0,103425
China	4519950	436107	9,6485 %	±3%	-300509	-6,6485 %	-0,3	0,2216167
Eurozone	13615861	-100834	-0,7406 %	0%-3%	100839	0,7406 %	-0,14	-0,0529

(Source: IMF World Economic Outlook, 2011)

Appendix E will present this information from 2001 to 2010 for all countries included in the analysis. The top row of table 2 describes the data presented in its respective columns and the year in question. The first left column presents the countries in question. In this simple example, only the data for USA, China and the Eurozone is included. The GDPs and CBs presented are measured in millions of U.S. dollars, and then each CB is presented as percent of GDP. Comparing the CB/GDP ratio in the fourth column to the required CB/GDP threshold, described in the fifth column, the targeted changes in each CB is presented as U.S. dollars in column 6 and as percent of GDP in column 7. Column 7 presents the parameter previously described as $\hat{C}$ for each country. The eight column presents the elasticity parameter γ for each country, and with these two parameters one can equate each corresponding change in the real effective exchange rate necessary to achieve FEER, $\breve{R}$.

Using $\hat{C}$ and γ from table 2, we can manually estimate $\widetilde{R_1}, \widetilde{R_2}$ and $\widetilde{R_3}$ using equations (28), (29) and (30):

$$\widetilde{R_1} = \frac{\widehat{C_1}}{\gamma_1} <=> 1.65 = -0.16 * \widetilde{R_1} \leq> \widetilde{R_1} = \frac{1.65}{-0.16} = -10.30$$

$$\breve{R_2} = \frac{\widehat{C_2}}{\gamma_2} <=> -6.65 = -0.3 * \breve{R_2} <=> \breve{R_2} = \frac{-6.65}{-0.3} = 22.16$$

$$\breve{R_3} = \frac{\widehat{C_3}}{\gamma_3} <=> 0.74 = -0.14 * \breve{R_3} <=> \breve{R_3} = \frac{0.74}{-0.14} = -5.29$$

These results are presented for each country in question in the ninth column in table 1. As the equation in table 2 uses more decimals, their predictions are more accurate than the manual two decimal equations above. As the necessary changes in the real effective exchange rate for each country are found, the next step is to find the trade weights for the three countries. Presenting the export of each country in question as an export matrix:

Table 3: Export matrix for the countries in the example and the resulting trade weights

2007	USA	China	EU	Total
USA	0.0	65238.4	247788.0	313026.4
China	233181.0	0.0	245429.0	478610.0
EU	358636.0	98841.2	0.0	457477.2
Total	591817.0	164079.6	493217.0	1249113.6

$\varphi_{1,2} =$	0.3298	$\varphi_{2,1} =$	0.4643	$\varphi_{3,1} =$	0.6379
$\varphi_{1,3} =$	0.6702	$\varphi_{2,3} =$	0.5357	$\varphi_{3,2} =$	0.3621

This matrix measures the value of the exports in millions of U.S. dollars for each country in question with regard to each other. The left column shows the exporting country with the year in question in top left corner. The top row shows the importing country. The fifth column measures the total export for each country, while the bottom row measures the total export to each country. The last two independent rows present the resulting trade weights. All exports to other countries are in this simple example excluded. A good indicator to see if the trade weights are estimated correctly is to see whether one country's trade weights sum up to one, which it does in this example. Remembering equation (27), the trade weight $\varphi_{1,2}$ is equated as:

$$\varphi_{1,2} = \frac{export_{1,2} + export_{2,1}}{total\ export_1 + total\ import_1} \quad (27)$$

Using the data from Table 3, one can estimate $\varphi_{1,2}$:

$$\varphi_{1,2} = \frac{65238.40 + 233181.00}{313026.40 + 591817.00} = 0.3298$$

This gives us a trade weight measuring the importance of country 1s trade with country 2. These trade weights are presented for each year in appendix D.

Now that the values of the real effective exchange rates and the trade weights are calculated, the next step is to equate the resulting bilateral real exchange rate changes. Equations (34)-(36) can therefore now be used to calculate the bilateral relationship:

$$\widetilde{R_1} = \frac{\widehat{C_1}}{\gamma_1} = -\varphi_{1,2}z_2 - \varphi_{1,3}z_3 \ll=\gg -10.30 = -0.33z_2 - 0.67z_3$$

$$\widetilde{R_2} = \frac{\widehat{C_2}}{\gamma_2} = z_2 - \varphi_{2,3}z_3 \ll=\gg 22.16 = z_2 - 0.54z_3$$

$$\widetilde{R_3} = \frac{\widehat{C_3}}{\gamma_3} = z_3 - \varphi_{3,2}z_2 \ll=\gg -5.29 = z_3 - 0.36z_2$$

Combining three equations with two unknowns gives us three different z values. The system is over determined, which means that each possible combination that can be used to equate the unknowns will provide a different result. The dataset is also inconsistent, as the required changes do not sum up to zero. Since the required CB changes do not sum up to zero, it means that we cannot perfectly adjust all three countries' CBs. As mentioned before, this problem is solved by taking the average of these results. This ensures that the solution is a close but not perfect fit for all countries involved. The entire example is equated in Appendix I. Presenting here only the results:

$$z_2 = \frac{23.5291+11.8272+33.8928}{3} = 23.0830$$

$$z_3 = \frac{3.8354-2.7696+ 3.3361}{3} = 1.4673$$

We can here see how different results an over determined set of equations with inconsistency problem produces. In chapter 13 I will analyze how far away from the optimal CB threshold the average solution is. In our simple example, the bilateral real exchange rate $z_2 = \frac{d\,R_{1,2}}{R_{1,2}}$, which is the real exchange rate of the U.S. dollars to the renminbi, should rise by 23.0830%. The bilateral rate $z_3 = \frac{d\,R_{1,3}}{R_{1,3}}$, which is the real exchange rate of U.S. dollars to euros, should rise by 1.4673%. Both exchange rate changes imply a real depreciation of the dollar.

Remembering that FEERs are always estimated within a time range, it should be read as the necessary real depreciation needed from the base year 2007 to the benchmark year 2008.

9: The five-country model

When my model is expanded to include five countries, it is practical to express the equations in terms of vectors and matrixes. Let me now express the desired set of real effective exchange rate changes ($\breve{R}$) and the bilateral exchange rate changes (z) as vectors, and the trade weights (φ) as a matrix. These are denoted $\dot{R}$, $\dot{z}$ and $\dot{\varphi}$. The relationship from equation (34) - (36) is here described as:

$$\dot{R} = \dot{z} * \dot{\varphi} \tag{37}$$

This system incorporates five equations (three countries, the Eurozone and the residual country), but the number of unknowns are four. So the system has five different solutions. Each solution is estimated by using four of the five equations, and solving for each possible combination of countries. The reason one equation has to be removed when equating, is because the U.S. dollar cannot appreciate against itself. This is the over determining problem I have discussed earlier. When equating the solutions for each bilateral real exchange rate change, I firstly present the 4 equations in play, then invert the matrix and solve for $\dot{z}$. The equations are presented in appendix B, with a short summary of the end results presented in appendix C.

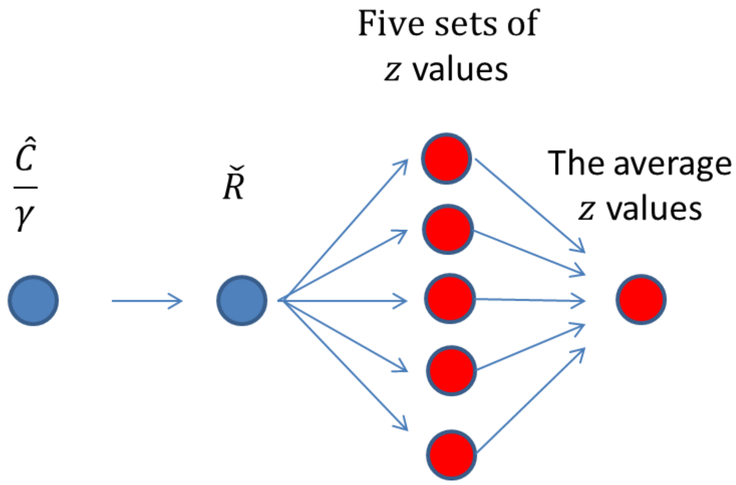
10: Application of this model compared to other FEER estimations

The main goal of the FEER model I use is to produce estimates of the fundamental equilibrium exchange rates (FEERs) for my chosen set of currencies. The number of currencies used in FEER estimation varies. In his 1989 study, Williamson (1989) estimates the FEERs for 7 currencies, while Williamson and Cline (2008) estimates as much as 35. Both studies estimate FEERs ex ante. As mentioned in chapter 5.3, I will estimate FEERs ex post for 4 currencies with a fifth residual country.

Despite the fact that my model itself is identical to the model used by Williamson and Cline (2008), I will use it differently. Starting in 2000, I will estimate FEERs on a year-to-year basis until 2010. To estimate FEERs on a year-to-year basis is a frequently used method, found in among others Williamson (1994) and Cline and Williamson (2008). Some studies prefer a longer time horizon, e.g. Williamson (2010) uses 2012 as his benchmark. However, while these studies estimate for just one year, I will estimate for ten years in succession. During this period all 4 economies in my analysis will have CBs that both stay within and exceed the thresholds set. In an ex ante FEER estimation, predictions for the next years CBs and GDPs are used to estimate FEERs. I will estimate ex post and use actual data. When using predicted data like in ex ante FEER estimation, there is at least some deviation between the predicted data and actual data. A FEER estimation based on incorrect data, like Williamson and Cline (2008), is bound to produce different results than a FEER prediction based on correct data, like mine. This does not mean that my method is superior; it is merely a consequence of estimating ex post. Chapter 14 analyzes the difference between the two results and the reasons behind it. Now, given these correct data, will FEER estimation explain the real and nominal real exchange rate fluctuations during the last decade? If a country actually realigns its CB within the thresholds set, will its real and nominal exchange rate move towards the predicted FEER?

It should be stressed that the analysis has four steps: Firstly, I identify the desired CBs for the countries in question. This is already done in subchapter 5.3. Secondly, using the CB targets and import elasticity parameters, I will equate how much the real effective exchange rate has to change to become a FEER. These data are presented in Appendix E. By using the real effective exchange rates and the trade weights, this leads to a set of different bilateral real exchange rates. The average value of these bilateral real exchange values are then presented as the solution. The trade weights are presented in appendix D, and the equation of the bilateral exchange rates and their average values are presented in Appendix B. The bilateral real and nominal exchange rates are the results that I present in chapter 11 and 12. These steps can be presented graphically:

Figure 2: A graphical interpretation of the steps in my FEER analysis



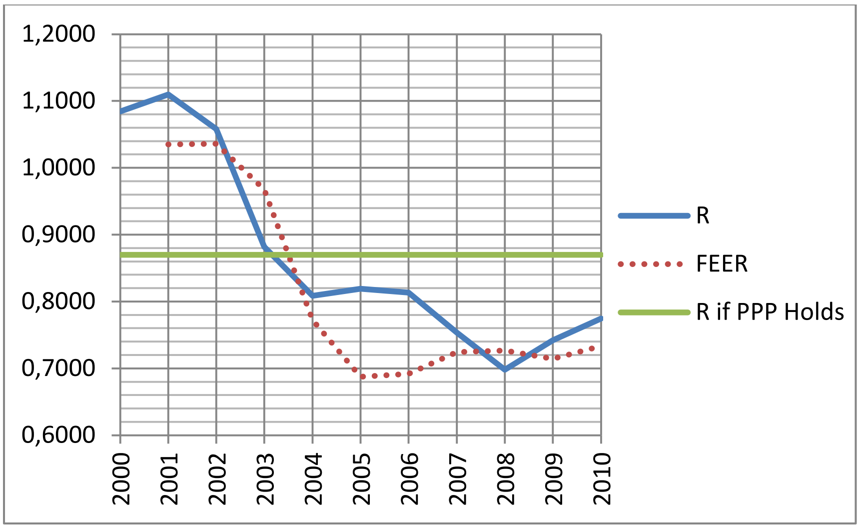
(Source: Own modeling)

This graphical interpretation presents the four steps in the analysis, with the blue dots representing ideal values and the red representing the imperfect values resulting from the over determination and inconsistency problems. The results presented in chapter 11 are the bilateral real exchange rate estimations made for 2001-2010, here modeled as the average z values. Appendix C presents a summary of the average z values for this time period. From the z values it is unproblematic to equate the bilateral real exchange rates between all countries. However, the U.S. dollar is both the largest currency in the world and the one going through the largest nominal change from 2000 to 2010. I therefore choose to only present the bilateral relationships with regards to the U.S. dollar.

11: The real exchange rates

The results of the EUR/USD bilateral real exchange rates are:

Figure 3: $R_{EUR/USD}$, $R^*{}_{EUR/USD}$ *and* $R_{EUR/USD}$ *if PPP holds*



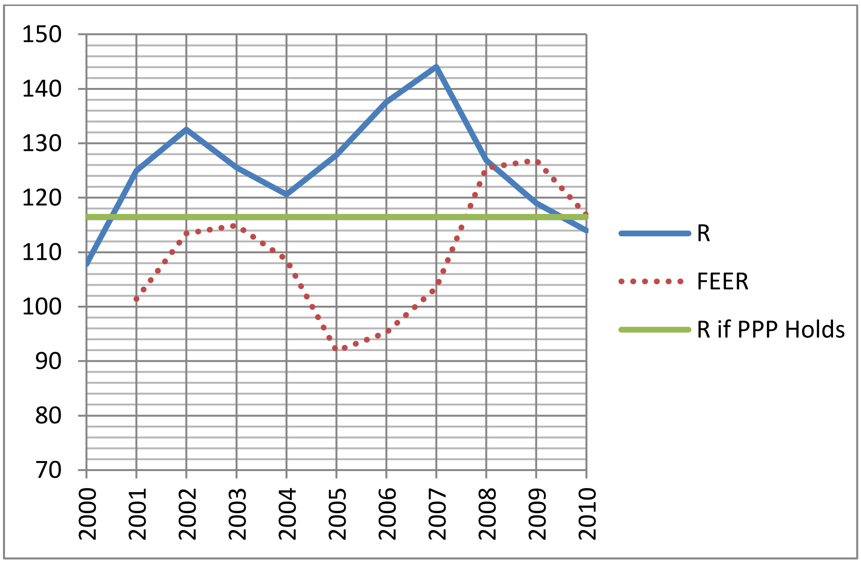
This graph measures the real exchange rate changes for the EUR/USD relationship during the last decade. The vertical axis measures the real exchange rate, the horizontal measures the year in question. All data are updated yearly. The blue line, R, measures the actual development of the real exchange rate. As I have set the CPI-indexes equal to 100 in 2000, the real exchange rate in year 2000 is the same as the nominal exchange rate. The red dotted line is the estimated FEER. Bear in mind that FEERs are estimated on a year-to-year basis. For each year, the FEER line measures the estimated FEER calculated from actual data, and how it deviates from the actual real exchange rate. It should be read as the predicted real EUR/USD rate that would occur if the model's required CB/GDP changes were made. Since 2001 is my first benchmark year, this is the first year where I can estimate FEERs during this decade. Year 2000 does therefore not have a FEER value. The green line is the predicted real EUR/USD change that would happen if we assumed that relative PPP would hold. The relative PPP line is equated from the average real exchange rate during the last two decades, or the last 13 years in the case of the euro. These values are presented fully in appendix H. This line is relatively uninteresting in real terms as it predicts a constant real exchange rate by definition. It is included to measure how the real exchange rate deviates from its average value. As I present the nominal exchange rate in the next chapter, the PPP line will be mainly commented during that chapter. The next two figures in this chapter should be read the same way as figure 3.

Analyzing figure 3, one can here see that the U.S. dollar has had a real depreciation with regard to the euro during this period. The real EUR/USD rate has changed a lot and rapidly during this decade, starting at a rate of 1.08 and ending at 0.77. A year of U.S. dollar appreciation lead the EUR/USD rate to its highest actual rate of 1.15 in 2001, and it was followed by a steady decrease until 2004. It stayed almost constant until 2006, depreciated until 2008 and then appreciated until 2010. The biggest difference between the real exchange rate and the FEER is found in 2005. FEER estimation then predicted a real exchange rate of 0.68, but the actual real exchange rate was 0.82. The FEER estimation has largely followed the actual movement of the real exchange rate. During the last decade it has predicted both slightly higher and lower real exchange rates than the actual development of the real exchange rate. The PPP line does not predict the actual real exchange rate changes well in this period, and is a worse estimator than FEER estimation.

That the FEER estimated rates are so closely connected to the actual real exchange rate is curious. The FEER values are estimated from year to year, and represent the resulting real exchange rate that would occur if the necessary changes with regard to the CB/GDP ratio were made. In this relationship, depreciations predicted by the FEER occur even if the CB/GDP ratios are nearly identical afterwards. This might imply that the EUR/USD relationship is strongly dependent on either country’s foreign liabilities.

The results of the JPY/USD bilateral real exchange rates are:

Figure 4: $R_{JPY/USD}$, $R^{*}{}_{JPY/USD}$ *and* $R_{JPY/USD}$ *if PPP holds*



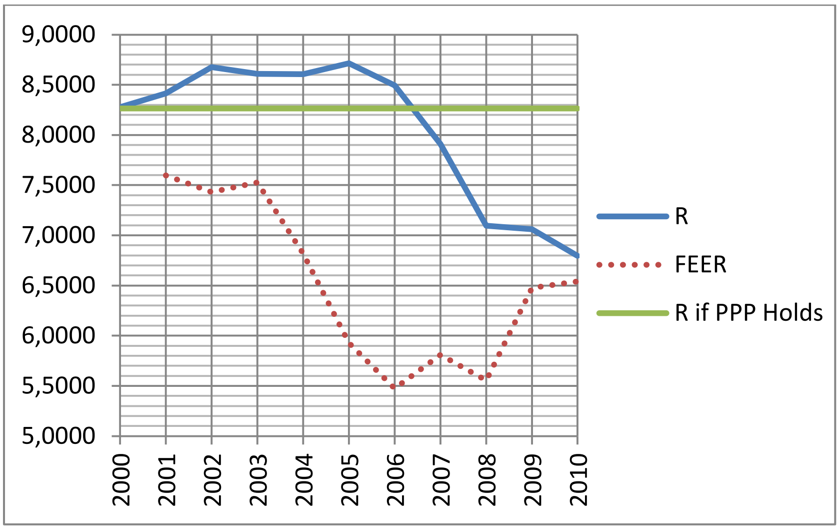
One can here see that the U.S. dollar has experienced significant fluctuations with regards to the Japanese yen in real terms during this decade. The actual real JPY/USD exchange rate started at 109.78 in 2000, increased to a top level of 144.04 in 2007, it then gradually decreases to its final 113.97 in 2010. The FEER estimates predict a rather different course, estimating a weaker U.S. dollar than the actual development of the real exchange rate for most of the decade. From 2004 to 2006 the FEER estimates predict a depreciation of the U.S. dollar in real terms, from a real JPY/USD rate of 108.65 to a 95.22 rate. This is a consequence of the increased Japanese CB and the increased negative American CB. However, the real JPY/USD rate rose from an actual rate of 120.63 in 2004 to a 137.60 rate in 2006. That the Japanese yen strongly depreciates in real terms to the U.S. dollar, during a period when their CB predicts the opposite, implies that FEER estimations does not explain this relationship very well. The difference in 2006 just described is the largest difference between the FEER and actual real exchange rate during this decade. In 2008 and 2009 the relationship changes: the FEERs estimate a real appreciation of the U.S. dollar with regard to the Japanese yen, but the actual real exchange decreases. They reach roughly the same value in 2010, with FEERs predicting a weaker yen with regard to the dollar than the actual development during the last two years of the decade.

The PPP estimated real exchange rate is in this relationship a better estimator than FEERs, with smaller average deviation from the real exchange rate than FEERs. From the results presented here, it is clear that FEER estimation does not explain the real exchange rate changes for the JPY/USD relationship during this decade.

As the American and Japanese financial markets are less integrated than the European and American markets, one could speculate that the changes predicted by FEER analysis before 2008 will occur with a time lag. If that is the case, a further appreciation of the Japanese yen is in order. To back up this claim one would have to estimates FEERs for the current decade, which would be interesting for later studies. Viewing the results as they are, one could also speculate that the CB thresholds set for Japan are incorrect. As there is no evidence of the large yen appreciation predicted by FEER estimation during this period, one could argue that Japan shows no sign of being out of external equilibrium. From this line of reasoning, one can construct an argument that Japans CB/GDP threshold should have had a higher top threshold. A larger CB/GDP top threshold would allow them to export larger amounts of capital without being classified as out of equilibrium. This would result in a higher JPY/USD FEER from 2004 to 2008, when Japans CB exceed the thresholds set in this analysis.

The results of the CNY/USD bilateral real exchange rates are:

Figure 5: $R_{CNY/USD}$, $R^*_{CNY/USD}$ *and* $R_{CNY/USD}$ *if PPP holds*



This final real exchange rate figure measure how the U.S. dollar has depreciated against the renminbi in real terms. This figure is very different compared to the other two. The actual U.S dollar with regard to the renminbi depreciation is of a more steady type than figure 3 and 4, mostly due to the fact that the renminbi is not a free-floating currency like the other 3 in question. The FEER is in this case always predicting a stronger renminbi than the actual development of the real exchange rate. The largest difference is between the two is measured in 2006, when FEER estimation predicts a CNY/USD real exchange rate of 5.47 while the actual observed rate was 8.49. The difference between the FEER and the actual real exchange also fluctuates a lot, ending up at almost equal values in 2010.

With FEER estimation providing these results, it is natural to question if anything in the estimation has gone wrong. Remembering that γ was estimated for 2008, there is one point to be made with regard to γ. As presented in chapter 7, γ is the product of two factors: the export price elasticity and the share of exports of goods and services in GDP. I do not have data providing the share of exports of both goods and services, but estimating the goods export/GDP ratio will give a good indicator of its development.

The total export of China has increased from 249,223 million U.S. dollars in 2000 to 1429,340 million U.S. dollars in 2008. In the same period the GDP increased from 1324,814 million U.S. dollars to 4519,950 million U.S. dollars, implying that the export/GDP ratio has increased from 249223/1324814 = 0.188 to 1429340/4519950 = 0.316 during this period. In 2009 the export/GDP ratio equals 1203420/4990528 = 0.241, and for 2010 there is no available export data yet. Since an open economy requires less real exchange rate adjustments to realign its CB, it implies that FEER estimation which equated γ for each year would predict an even stronger renminbi before 2008, as the openness of China increased steadily during this period. In 2009 the export/GDP ratio decreased, implying that the FEER for the renminbi would be stronger again after 2008. Estimating γ for each year would therefore very likely give the FEER line a more horizontal development.

Putting this point aside, as FEERs predict a stronger renminbi than the actual development for the whole decade, it is natural to raise the question of whether the renminbi is undervalued.

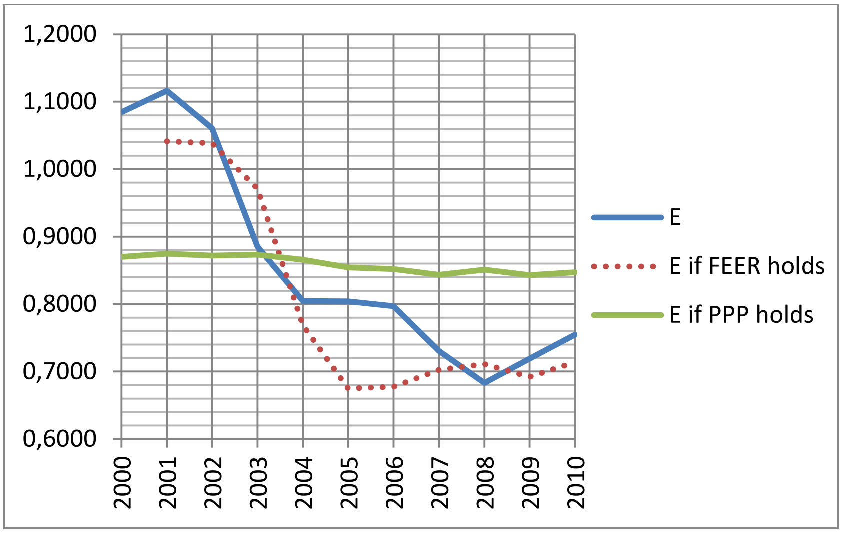
Such a claim does however require stronger evidence than provided in this thesis. Since the renminbi is constantly undervalued according to FEER estimation, I will compare my results regarding this relationship to other similar studies. This will give a general idea if my analysis estimate a too large renminbi appreciation, or if they are in accordance with other economists. This comparison is presented in chapter 15.

The PPP line is not in accordance with the observed real exchange rate. The PPP line, equated as the average real exchange rate during the last two decades, is 8.26. The actual real exchange rate has depreciated from a top point of 8.49 to its 6.79 in 2010. The PPP theory does i.e. not explain the real exchange rate changes during this decade.

12: The nominal exchange rates

Presenting the results for the EUR/USD nominal bilateral exchange rate:

Figure 6: $E_{EUR/USD}$, $E^{*}{}_{EUR/USD}$ *and* $E_{EUR/USD}$ $if\ PPP\ holds$

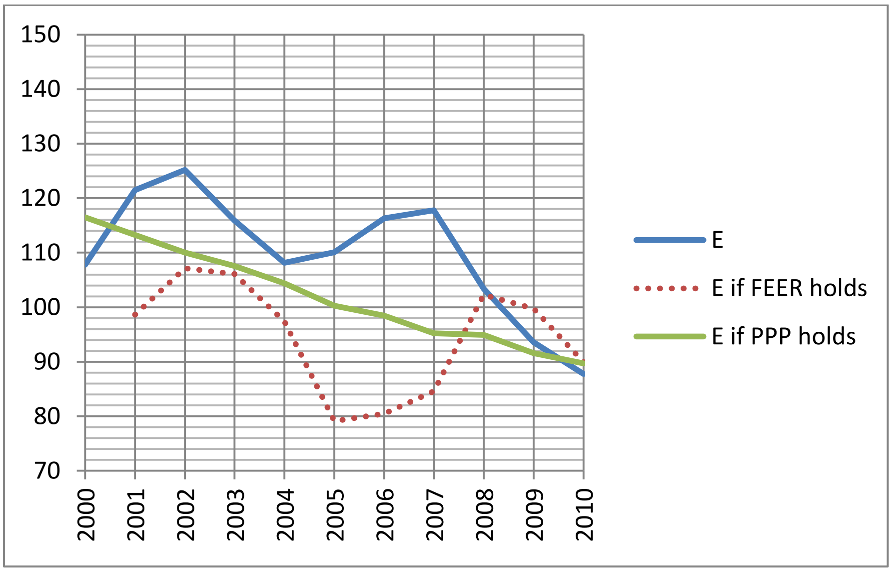


This graph measures the nominal exchange rate changes for the EUR/USD relationship for the period in question. The vertical axis measures the nominal exchange rate, the horizontal measures the year in question. The blue line, E, measures the actual nominal exchange rate and its development during this decade. The red dotted line, E if FEER holds, is equated by using the definition of the nominal exchange rate, the real exchange rate values from the FEER estimation and the CPI values for each respective year. The actual real exchange rate presented in the previous chapter was equated using the nominal exchange rate. As the FEER estimated nominal exchange rate in this chapter is a direct consequence of the FEER estimation in the previous, the relationships between these two lines are the same in real and nominal terms. That makes the relationship between the two relatively uninteresting, and will not be commented extensively. The green line is the nominal exchange rates that would occur if relative PPP holds. This line is equated by using the definition of the nominal exchange rates and assuming the real exchange rate is equal to the PPP estimated value from chapter 11. In this chapter this line will be interpreted, to get an idea of how well the theory of relative PPP predicts nominal exchange rate changes. Figure 7 and 8 should be read the same way as figure 6.

As apparent from figure 6, the nominal exchange rates between the U.S. dollar and the euro are largely following the values of the real exchange rate in numerical terms. The reason is that the CPI index in America and the Eurozone were both set to equal 100 in 2000, and have developed largely the same way, as also evident from the almost horizontal green line. The nominal EUR/USD rate has decreased from 1.08 in 2000 to 0.75 in 2010 during this decade, compared to the steady approx. 0.87 rate predicted by PPP. As in real terms, PPP does not explain the EUR/USD rate movements well in nominal terms.

Presenting the results for the JPY/USD nominal bilateral exchange rate:

Figure 7: $E_{JPY/USD}$, ${E^*}_{JPY/USD}$ *and* $E_{JPY/USD}$ $if\ PPP\ holds$



One can here see the nominal depreciation of the U.S. dollar against the Japanese yen. The most striking feature here is the steady nominal depreciation of the U.S. dollar predicted by the PPP theory, a consequence of the approx. 2.5% Japanese deflation compared to the approx. 25% inflation in the USA during this decade. The nominal exchange rate has decreased from 109.78 in 2000 to 82.54 in 2010, which corresponds almost fully to the inflation difference in this period. The relationship between the Japanese yen and the U.S. dollar in both real and nominal terms is therefore better explained by PPP theory than FEER estimation.

Presenting the results for the CNY/USD nominal bilateral exchange rate:

Figure 8: $E_{CNY/USD}$, ${E^*}_{CNY/USD}$ *and* $E_{CNY/USD}$ *if PPP holds*

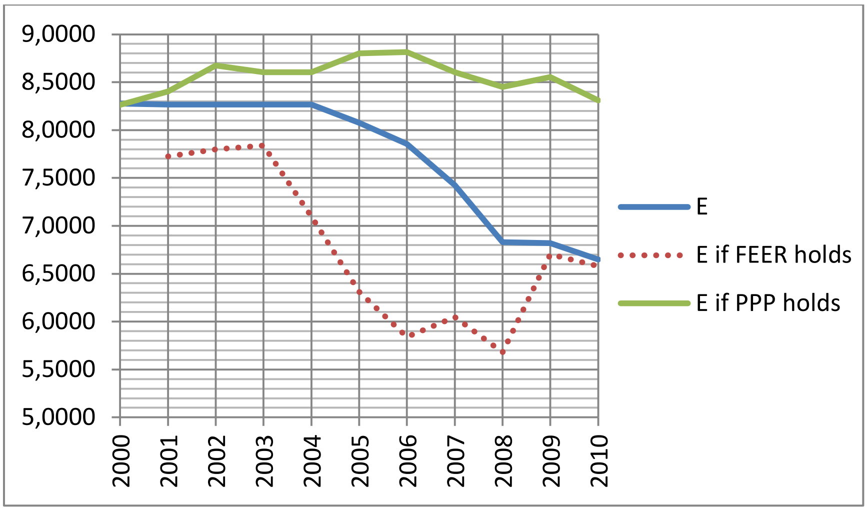
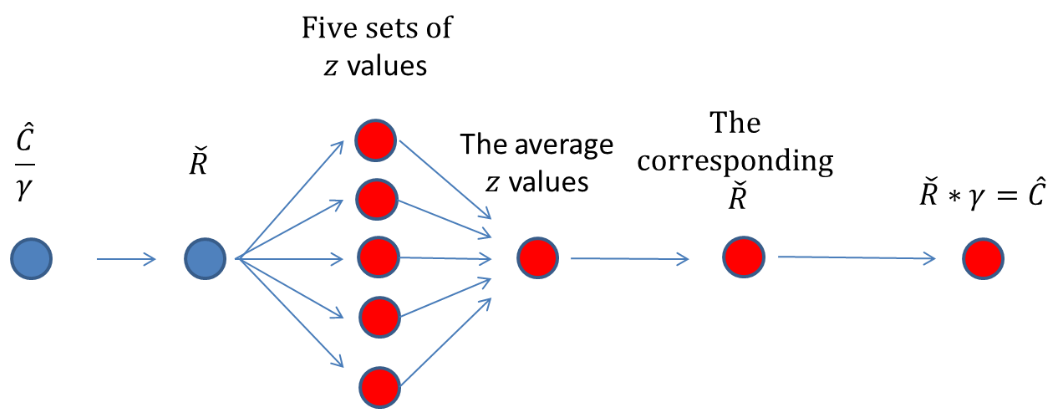


Figure 9 measures the gradual nominal exchange rate decrease between the U.S. dollar and the renminbi. As shown in the figure, the CNY/USD relationship is a partly rigid relationship. From 2000 to 2004 the Chinese government pegged its currency solely to the U.S. dollar, with a desired nominal exchange rate equal to approx.. 8.26. During 2005 they pegged the renminbi against a basket of currencies, allowing its exchange rate to the USD to float. It should be noted that they still intervene in the exchange rate market to prevent large deviations from their desired bilateral exchange rates, which are changed marginally on a year-to-year basis. After 2004 we can see a steady depreciation of the U.S. dollar to the renminbi, with FEER estimation predicting even larger depreciations during the entire period like in figure 5. The green PPP line predicts an almost rigid and stable course between 8.2 and 8.8 during this decade. As the nominal CNY/USD rate has either stayed constant or decreased during the entire decade, one can safely say that the PPP theory does not explain the nominal CNY/USD rate changes. It does however look like FEER estimation can explain at least a part of it, and that the renminbis depreciation against the U.S. dollar is partly driven by the current account balances of these countries.

13: Analyzing the size of the over determination and inconsistency problems

As mentioned earlier, there are two problems in my analysis: that the CB changes does not sum up to zero, and that the over determined system produces different bilateral real exchange rates depending on which equations one uses when solving for the unknowns. The solution to this in my analysis is to solve for all the equations and take the average of those results. This solution to the problem is also found in among others Williamson and Cline (2008). Other studies, like Cline (2007) solved the problem by placing USAs threshold as supreme, and the other countries as residuals. This solution ensured that USA would always correct its CB perfectly, while the resulting CBs of the other countries would be imperfectly corrected. Knowing that there is more than one solution to this problem, it is useful to look at how large deviations the results of my analysis produce. To do this, I have to use the equated average z values, equate the corresponding real effective exchange rate ($\breve{R}$) and then the change in the CB/GDP ratio ($\hat{C}$) that would occur if the real effective exchange rate changes ($\breve{R}$) were effected. The result from this process will tell me how far away from the CB/GDP threshold each country will be if the bilateral real exchange rate changes I have estimated are effected. Remembering figure 2, this process can be also presented graphically:

Figure 9: A graphical interpretation of the steps when estimating the size of the over determination and inconsistency problem



(Source: Own modeling)

This figure presents the six steps necessary to do analyze the size of the over determination and inconsistency problems. As figure 2, the blue dots represents ideal values and the red dots represents the imperfect values resulting from the over determination and inconsistency problems. It should be noted that the export elasticity parameter γ used in the final step is unaltered.

When equating the CB/GDP ratio threshold difference, I have chosen the year 2008. This is the same year that I used to show how different results the inconsistency problem combined with the over determination problem causes in the 3-country example. As I have already shown how large the different results are during this year, it is interesting to see how well the solution works. The equations are presented in appendix G, presenting here only the results:

Table 4: The CB/GDP ratio a country would end up with if the estimated real bilateral exchange rate changes from my results were effected:

2008	USA	China	Japan	Eurozone
CB/GDP ratio realignment required by the original thresholds	1.6548%	-6.6485%	-0.2143%	0.7406%
CB/GDP ratio realignment result from the imperfect solution	1.5938%	-6.8399%	-0.3378%	0.5209%
CB/GDP ratio difference between the original thresholds and the imperfect results	0.0610%	0.1914%	0.1235%	0.2197%
CB/GDP ratio result from the imperfect solution	-3.0610%	3.1914%	3.1235%	-0.2197%

In this table, the top row presents the year and the countries in my analysis. The second row measures the required CB/GDP realignment (the blue $\hat{C}$ in figure 9) if the countries were to align themselves within the original CB/GDP thresholds set. The third row measures the realignment the imperfect solution demands (the red $\hat{C}$ in figure 9), with the fourth measuring the difference between row two and three. Finally, the fourth row measures the estimated CB/GDP ratios each country would end up with in 2008 if the estimated imperfect bilateral real exchange rate were effected and the CB/GDP ratios reacted correspondingly.

As can be read from table 4, the effect of the inconsistency problems is apparent but not very troublesome. The largest difference between the original thresholds set and thresholds from the imperfect solution is in China, where their imperfect solution is a CB/GDP ratio of 3.1914%. As all countries are treated equally by the averaging solution, all countries are given a proportionate residual depending on their original CB/GDP realignment. As China is the country requiring the largest original CB/GDP realignment, the averaging solution provides the largest difference with regard to this country. I regard the averaging solution to over determination and inconsistency problems as a satisfying, since that the results are still very close to the original CB/GDP ratios.

14: Result comparison to Williamson and Cline (2008)

I now compare my results to the results from Cline (2008) and Williamson and Cline (2008). The main results in these two papers are the same, and will be referred to as the results in Williamson and Cline. To compare my analysis with their can also answer another question: how much do the results differ when the model is reduced from a 35-country model to a five-country model?

Williamson and Cline uses the exact same model with the same CB thresholds for the 4 countries I analyze. As their model also chooses to ignore internal balance, the CB threshold is the only determinant when defining if a country is in equilibrium or not. They estimate for 35 countries and ex ante, and one would still expect relatively moderate differences in results. However, looking at the predicted data for 2008 and 2009 used in their analysis, compared to the actual data used in my analysis, it is clear that the results will differ significantly:

Table 5: Predicted and actual data for 2008 and 2009 and the CB/GDP thresholds

2008	USA	China	Japan	Eurozone
Predicted GDP	14195032	3941536	4866992	13521197
Actual GDP	14061800	4519950	4886952	13615861
Predicted CB	-614703	385855	193322	-98046
Actual CB	-718094	436107	157079	-100834
Predicted CB/GDP	-4.33 %	9.79 %	3.97 %	-0.73 %
Actual CB/GDP	-5.11 %	9.65 %	3.21 %	-0.74 %
2009				
Predicted GDP	14533167	4430118	5026525	13978475
Actual GDP	14119050	4984731	5068894	12483643
Predicted CB	-605494	442774	198484	-121076
Actual CB	-378434	297100	141750	-50664
Predicted CB/GDP	-4.17 %	9.99 %	3.95 %	-0.87 %
Actual CB/GDP	-2.68 %	5.96 %	2.80 %	-0.41 %
CB/GDP Thresholds	±3%	±3%	0% - 3%	0% - 3%

(Source: IMF World Economic Outlook, April 2008 and April 2011)

This table presents the difference between the predicted and actual GDP and CB data for 2008 and 2009. The predicted data is the data Williamson and Cline uses in their analysis, gathered from IMF World Economic Outlook, April 2008. The actual data is the actual observed data used in my analysis, gathered from IMF World Economic Outlook, April 2011. The CB/GDP thresholds are the same in the two analyses, and are presented in the bottom row. A quick analysis of this data predicts a large divergence in the results for 2009. Both USA and Japan had actual CB/GDP ratios inside their respective thresholds, the opposite of what was predicted. The entire 2009 adjustment in my analysis will therefore result from adjustments in China and the Eurozone. If I had done my analysis ex ante and used predicted data like Williamson and Cline, all four currencies would be predicted to be in disequilibrium and need adjustments to restore equilibrium. When choosing to benchmark their FEER estimation to 2009, Williamson and Cline assumed that there would be no further significant changes in current account positions on the basis of existing policies. Considering that the actual CB/GDP ratios for 2009 changed significantly compared to 2008, it is clear that this assumption was wrong. Comparing the bilateral real exchange rate changes for the two analyses gives us the following result:

Table 6: Estimates of bilateral real exchange rate changes

2009	CNY/USD real appreciation	JPY/USD real appreciation	EUR/USD real appreciation
FEER analysis as done by Williamson and Cline	31.50 %	19.00%	-0.2%
FEER analysis as done in this paper	8.74 %	0.01 %	-2.32 %

The first row presents the year in question and the bilateral real exchange relationships. The second row is the estimates done by Williamson and Cline, using predicted data in a 35-country model. The bottom row presents the results from my analysis, using actual data in a five-country model. Since the fifth country in my analysis is solely a residual country, its bilateral real exchange rate change is not included.

It is clear from table 6 that the FEER estimates for 2009 differs significantly. There are two possible causes of these differences: the different data used, or the remodeling from a 35-country model to a five-country model. Since the difference in the two analyses is so large, it is interesting to determine which of the two causes that produces most of the difference. To measure the difference in results produced from the remodeling, I will estimate FEERs for 2009 based on the data used by Williamson and Cline. By using their data, the size of the difference resulting from the remodeling will be estimated. As the trade weights, CB thresholds and impact parameter is already equal, using their data leaves the remodeling as the only reason for the difference in results. Comparing the results of Williamson and Cline to my results using their data:

Table 7: Estimates of 2009 by a 35-country model and a five-country model, using the same data

2009	CNY/USD real appreciation	JPY/USD real appreciation	EUR/USD real appreciation
FEER analysis as done by Williamson and Cline	31.50 %	19.00%	-0.2%
FEER analysis with the data used in Williamson and Cline	27.63 %	14.07%	1.17 %

Where the second column reports the estimates from Williamson and Cline again, and the bottom column reports the estimates using their data in a five-country model. The FEER estimation of 2009 using predicted data is presented in appendix I. When comparing a five-country model with a 35-country model there will almost always be different results. This is because a 35-country model is likely to predict a number of CB changes required for countries that are not included in a five-country analysis. Unless these countries have equally large trade weights to the countries included in five-country model, the results will be different.

The largest difference between the analyses presented in table 7 is measured in the JPY/USD rate. There is also a small difference in the CNY/USD rate, and an almost insignificant difference in the EUR/USD rate. In this case, the analysis by Williamson and Cline required e.g. a number of East-Asian economies to realign their currencies. As these countries trade more with Japan and China than USA, it leads to a stronger appreciation of the Japanese yen and the renminbi with regard to the U.S. dollar. However, as I have included the arguably most important countries and our analysis seems to yield roughly the same results given roughly the same data, I regard my simplification as satisfying.

15: USD/CNY result comparison to other studies

As the CNY/USD exchange rate has been and still is frequently debated, the literature regarding this topic is massive. I will present my estimates, a range of estimates from other kinds of estimations, as well as other FEER estimations with different assumptions than in my analysis.

Table 8: Estimates of the CNY/USD real and nominal exchange rate from other studies

Author	Method	Type of exchange rate	Year	Underevaluation
Bårdsgjerde (2011)	FEER	Real and nominal CNY/USD	2003	13%
M.Funke & J.Rahn (2004)	BEER	Bilateral nominal CNY/USD	2003	11%
M.Funke & J.Rahn (2004)	PEER	Bilateral nominal CNY/USD	2003	12%
Bénassy-Quéré et. al. (2004)	BEER	Bilateral nominal CNY/USD	2003	47% 44%
Goldstein (2003)	Simplified FEER, with a CAD threshold of ±1%	Bilateral nominal CNY/USD	2003	15-25%
Wang (2004)	BEER	REER CNY/USD	2003	Near 0
Bårdsgjerde (2011)	FEER	Real and nominal CNY/USD	2005	31%
J.Frankel (2005)	Balassa-Samuelsson estimation	Bilateral real CNY/USD	2005	>35%
V.Coudert & C.Couharde (2005)	FEER with a CAD threshold of ±1.5%	REER CNY/USD	2005	23% 44%
V.Coudert & C.Couharde (2005)	BEER	Bilateral nominal CNY/USD	2005	18%

Table 8 summarizes several studies done on the CNY/USD exchange rate for the years 2003 and 2005. The table provides a quick overview of the crucial information in the studies. The first column presents the author(s) of the article and the year it was published. The second column gives a short introduction about the method used, and the third shows how the exchange rate changes are measured. The fourth presents the benchmark year in the analyses, with the first six analyses estimating for 2003, and the last four for 2005. The final fifth column measures the amount of depreciation required of the renminbi for it to be defined as fundamental, permanent or optimal by the authors.

As these studies are using different methods, I will give a short introduction of these without going into too much detail. A behavioral equilibrium exchange rate (BEER) uses net exports as a proportion of GDP, a real interest differential, terms of trade differential and GDP per capita differentials to estimate a fundamental real exchange rate. As BEER estimation contains more variables than FEER estimations, fewer countries are normally included in such studies. As FEER estimation, BEER estimation requires normative choices when determining levels of internal and external equilibrium. A permanent equilibrium exchange rate (PEER) is equated by estimating the "fundamental determinants" of an exchange rate and then removing the deviations made by the current business cycle. The idea is that there are factors that determine the real exchange rate that are both permanent and temporary, and by identifying and removing the temporary factors one can obtain a PEER. I have deliberately also included two FEER estimations done by other authors with other CB/GDP thresholds and parameter values. Balassa-Samuelsson estimation is a model based on PPP, but includes that some goods and services are untradeable. A Balassa-Samuelsson estimation gives an indicator of what the real exchange rate should be, considering the price levels of tradeable and untradeable goods and services types.

As my goal is to see how my results compare to other studies, I have picked two different years and gathered a range of results for the years in question. Since these are two years when I estimate a large renminbi depreciation, comparing my results these years will give a rough idea if my analysis estimates larger or smaller exchange rate changes than other similar studies. If I were to choose e.g. one or two studies for each year, I would have a less diverse range of estimates and less an idea of the extreme values that studies on this exchange rate contain.

The five studies of 2003 present a range from 0% to 47% depreciation of the renminbi. My FEER analysis for 2003 estimates that the renminbi is undervalued by 13% in real terms. In my analysis, undervalued means how much the renminbi must appreciate with regard to the U.S. dollar to reach FEER. The difference in nominal terms is the same, as explained in chapter 13. As three of the studies, the FEER estimation by Goldstein (2003) and the BEER and PEER estimation of M.Funke and J.Rahn (2003) estimates a 15-25%, 11% and 12% depreciation in nominal terms, I find my results in a middle segment of the range presented.

In 2005, three studies present a range from 18% to 44% depreciation of the renminbi. My analysis estimates that the CNY/USD rate is undervalued by 31%. I once again find that my estimations are in the middle segment of the range, roughly in the middle of the FEER results of V.Coudert and C.Couharde (2005) which predicts a 23% or 44% real depreciation of the renminbi with regard to the U.S. dollar, depending on the size of the impact parameter. Their BEER estimation predicted a lower 18% nominal depreciation, while Frankel (2005) estimates that the Renminbi is overvalued by at least 35% in real terms with regard to the U.S. dollar.

Since both my results show a medium-sized depreciation of the renminbi compared to other studies, it seems like the renminbi is and has been undervalued for some time. My FEER estimates predict that unless China adjust their CB/GDP ratio, their currency is likely to steadily appreciate in the years to come as well. The size of the current undervaluation is uncertain. During 2009 and 2010, USAs and Chinas GDP/CB ratios were heavily reduced. USA had a GDP/CB ratio of -3.2% in 2010, very close to their $\pm$3% threshold. China had CB/GDP ratio of 5.2%, closer to their $\pm$3% ratio than during most of the decade, but still some way off. These reductions have brought FEER estimation closer to their actual bilateral exchange rate. However, the U.S. dollar’s effective exchange rate has depreciated a lot during USAs period of disequilibrium. This is as a result of the large depreciation with regard to the euro, and the medium-sized depreciation with regard to the renminbi. The renminbi have not appreciated very much during their period of disequilibrium. Their currency was firstly pegged to the U.S. dollar, and since to a basket of currencies with small appreciations each year. Because the renminbi has appreciated so little compared to what has been predicted, it could still be in need even further appreciation in the years to come in addition to the appreciation created by their current CB.

16: Conclusion

The objective of this thesis is to see how well FEERs do in predicting real and nominal exchange rate changes when estimating ex post in a five-country framework. Estimating FEERs during the last decade, we can see that FEER estimation provides a good indicator in the medium term for two of the three bilateral real and nominal exchange rates studied. The FEER estimation results are particularly promising when looking at the real and nominal exchange rate between the U.S. dollar and the euro. FEER estimation predicts the exchange rate changes in this relationship quite accurately during the entire decade. The relationship between the U.S. dollar and the renminbi is partly explained by FEER estimation, and in both cases FEER estimation explains exchange rate changes better than the PPP theory. The relationship between the U.S. dollar and the Japanese yen is not explained by FEER estimation, and the development of this exchange rate is better explained by the PPP theory. As briefly discussed, it could imply that the CB/GDP thresholds set in this analysis for Japan are incorrect.

In two of three cases, FEER estimation provides more accurate results than estimation using the relative PPP theory. If asked to make a prediction about the real or nominal exchange rate changes for the short-term future, it seems like the wiser move to include the current account balances of the countries in question as well as studying the changes in inflation.

When estimating the size of the over determination and inconsistency problems, it is clear that the problems are apparent but not very troublesome. By using the averaging solution, all countries are treated equally and their imperfect results are nearly identical to their original thresholds. The remodeling of the framework for it to contain five instead of 35 countries does cause less precision when estimating FEERs. However, as the differences are small and the modeling allows much easier estimation of 10 years in succession, I regard the remodeling as successful.

The most curious result from my analysis is the CNY/USD exchange rate results. FEER estimation predicts a stronger renminbi during the entire decade. The difference between the FEER and the actual real exchange also changes a lot during the decade. I have discussed two possible causes for this. The first probable cause is that the γ value would change significantly if estimated on a year-to-year basis. This would predict a more horizontal FEER line. Another possibility is that since the renminbi is not a free floating currency, it could be kept artificially weaker than its actual value. My studies on the CNY/USD exchange rate predict a relatively medium-sized depreciation compared to studies using both similar and different frameworks. This might imply that the renminbi has been and still is significantly undervalued.

Appendix A: The detailed calculation of equation (20)→(22)

Starting from equation (20):

$$TD = TD\left(\underset{+}{Y}, \underset{-}{Y_f}, \underset{-}{R}\right)$$

Deriving this, one obtains:

$$\partial TD = \frac{\partial TD}{\partial Y} * \partial Y - \frac{\partial TD}{\partial Y_f} * \partial Y_f - \frac{\partial TD}{\partial R} * \partial R$$

Which I want to express as a function of R:

$$+\frac{\partial TD}{\partial R} * \partial R = \frac{\partial TD}{\partial Y} * \partial Y - \frac{\partial TD}{\partial Y_f} * \partial Y_f - \partial TD$$

Divide on $\frac{1}{\frac{\partial TD}{R}}$:

$$\partial R = \left[\left(\frac{\frac{\partial TD}{\partial Y}}{\frac{\partial TD}{\partial R}}\right) * \partial Y\right] - \left[\left(\frac{\frac{\partial TD}{\partial Y_f}}{\frac{\partial TD}{\partial R}}\right) * \partial Y_f\right] - \left[\left(\frac{1}{\frac{\partial TD}{\partial R}}\right) * \partial TD\right]$$

Divide on R:

$$\frac{\partial R}{R} = \left[\left(\frac{\frac{\partial TD}{\partial Y}}{\frac{\partial TD}{\partial R}}\right) * \left(\frac{\partial Y}{R}\right)\right] - \left[\left(\frac{\frac{\partial TD}{\partial Y_f}}{\frac{\partial TD}{\partial R}}\right) * \left(\frac{\partial Y_f}{R}\right)\right] - \left[\left(\frac{1}{\frac{\partial TD}{\partial R}}\right) * \left(\frac{\partial TD}{R}\right)\right]$$

And reorganize the fractions on the right hand side so they express elasticities:

$$\frac{\partial R}{R} = \left[\left(\frac{\frac{\partial TD}{\partial Y}\frac{Y}{TD}}{\frac{\partial TD}{\partial R}\frac{R}{TD}}\right) * \left(\frac{\partial Y}{R} * \frac{R}{Y}\right)\right] - \left[\left(\frac{\frac{\partial TD}{\partial Y_f}\frac{Y_f}{TD}}{\frac{\partial TD}{\partial R}\frac{R}{TD}}\right) * \left(\frac{\partial Y_f}{R} * \frac{R}{Y_f}\right)\right] - \left[\left(\frac{1}{\frac{\partial TD}{\partial R}\frac{R}{TD}}\right) * \left(\frac{\partial TD}{R} * \frac{R}{BT}\right)\right]$$

$$\frac{\partial R}{R} = \left[\left(\frac{\varepsilon(TD,Y)}{\varepsilon(TD,R)}\right) * \left(\frac{\partial Y}{Y}\right)\right] - \left[\left(\frac{\varepsilon(TD,Y_f)}{\varepsilon(TD,R)}\right) * \left(\frac{\partial Y_f}{Y_f}\right)\right] - \left[\left(\frac{1}{\varepsilon(TD,R)}\right) * \left(\frac{\partial TD}{TD}\right)\right]$$

Where $\varepsilon(TD,Y) = \frac{\partial TD}{\partial Y} * \frac{Y}{TD}$,

$\varepsilon(TD,Y_f) = \frac{\partial TD}{\partial Y_f} * \frac{Y_f}{TD}$, and

$\varepsilon(TD,R) = \frac{\partial TD}{\partial R} * \frac{R}{TD}$

Appendix B: The calculation of effective to bilateral real exchange rates for 2001-2010

For each year, there will be a total of 5 equations with 4 unknowns. The solution to this problem is to firstly remove one equation, secondly to solve for each possible combination and finally equate the average z values for each year. For each possible combination of equations and trade weights, the trade weights are firstly presented, then inverted and multiplied with the real exchange rate changes. The resulting matrix presents the bilateral real exchange rate changes resulting from this particular combination of equations.

The estimates for 2001 are based on the following equations;

$$
\begin{aligned}
&(1): -5.381 = -z2 * 0.0348 - z3 * 0.1067 - z4 * 0.1989 - z5 * 0.6596 \\
&(2): 0 = z2 - z3 * 0.1562 - z4 * 0.1495 - z5 * 0.5465 \\
&(3): 0 = z3 - z2 * 0.0879 - z4 * 0.1485 - z5 * 0.5087 \\
&(4): 0 = z4 - z2 * 0.0429 - z3 * 0.0756 - z5 * 0.6396 \\
&(5): 0 = z5 - z2 * 0.0844 - z3 * 0.1394 - z4 * 0.3444
\end{aligned}
$$

Using 1,2,3 and 4;

$$
\begin{matrix}
-0.0348 & -0.1067 & -0.1989 & -0.6596 \\
1 & -0.1562 & -0.1495 & -0.5465 \\
-0.0879 & 1 & -0.1485 & -0.5087 \\
-0.0429 & -0.0756 & 1 & -0.6396
\end{matrix}
\text{, inverse:}
$$

$$
\begin{matrix}
-0.84325 & 0.97551 & 6.1435\times10^{-2} & -0.01276 \\
-0.75653 & 5.5009\times10^{-2} & 0.92753 & -4.5128\times10^{-3} \\
-0.80170 & 6.2020\times10^{-3} & -2.1195\times10^{-2} & 0.83832 \\
-1.1075 & -6.2236\times10^{-2} & -0.14689 & -0.25139
\end{matrix}
*
\begin{matrix}
-5.381 \\ 0 \\ 0 \\ 0
\end{matrix}
=
$$

$$
\begin{matrix}
4.5375 \\ 4.0709 \\ 4.3139 \\ 5.9595
\end{matrix}
$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0348 & -0.1067 & -0.1989 & -0.6596 \\ 1 & -0.1562 & -0.1495 & -0.5465 \\ -0.0879 & 1 & -0.1485 & -0.5087 \\ -0.0844 & -0.1394 & -0.3444 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.82773 & 0.97917 & 6.7931 \times 10^{-2} & 0.0237 \\ -0.75105 & 5.6303 \times 10^{-2} & 0.92983 & 8.3818 \times 10^{-3} \\ -1.8211 & -0.23417 & -0.44794 & -1.5571 \\ -0.80175 & 9.8438 \times 10^{-3} & -1.8921 \times 10^{-2} & 0.46692 \end{matrix} * \begin{matrix} -5.381 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 4.454 \\ 4.0414 \\ 9.7993 \\ 4.3142 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0348 & -0.1067 & -0.1989 & -0.6596 \\ 1 & -0.1562 & -0.1495 & -0.5465 \\ -0.0429 & -0.0756 & 1 & -0.6396 \\ -0.0844 & -0.1394 & -0.3444 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.99001 & 0.9409 & -0.13345 & -0.22416 \\ -2.9722 & -0.46742 & -1.8266 & -3.3842 \\ -0.75106 & 0.01814 & 0.87996 & 7.7333 \times 10^{-2} \\ -0.75655 & 2.0501 \times 10^{-2} & 3.7169 \times 10^{-2} & 0.53595 \end{matrix} * \begin{matrix} -5.381 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 5.3272 \\ 15.993 \\ 4.0415 \\ 4.0710 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0348 & -0.1067 & -0.1989 & -0.6596 \\ -0.0879 & 1 & -0.1485 & -0.5087 \\ -0.0429 & -0.0756 & 1 & -0.6396 \\ -0.0844 & -0.1394 & -0.3444 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -4.9805 & -1.6705 & -3.415 & -6.3192 \\ -0.98983 & 0.82987 & -0.19637 & -0.35634 \\ -0.82800 & -3.2206 \times 10^{-2} & 0.81669 & -4.0175 \times 10^{-2} \\ -0.8435 & -3.6398 \times 10^{-2} & -3.4332 \times 10^{-2} & 0.40315 \end{matrix} * \begin{matrix} -5.381 \\ 0 \\ 0 \\ 0 \end{matrix} =$$

$$\begin{matrix} 26.8 \\ 5.3263 \\ 4.4555 \\ 4.5389 \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.1562 & -0.1495 & -0.5465 \\ -0.0879 & 1 & -0.1485 & -0.5087 \\ -0.0429 & -0.0756 & 1 & -0.6396 \\ -0.0844 & -0.1394 & -0.3444 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} 1.1743 & 0.41443 & 0.68068 & 1.2880 \\ 0.23339 & 1.2442 & 0.61762 & 1.1555 \\ 0.19523 & 0.31441 & 1.4976 & 1.2245 \\ 0.19889 & 0.3167 & 0.65932 & 1.6915 \end{matrix} * \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{matrix}$$

Which gives us the following average values of z;

$$z2 = \frac{4.5375+4.454+5.3272+26.8+0.0}{5} = 8.2237$$

$$z3 = \frac{4.0709+4.0414+15.993+5.3263+0.0}{5} = 5.8863$$

$$z4 = \frac{4.3139+9.7993+4.0415+4.4555+0.0}{5} = 4.522$$

$$z5 = \frac{5.9595+4.3142+4.0710+4.5389+0.0}{5} = 3.7767$$

The estimates for 2002 are based on the following equations;

$$(1):\ -8.1513 = -z2 * 0.0402 - z3 * 0.0985 - z4 * 0.2162 - z5 * 0.6451$$
$$(2):\ 0 = z2 - z3 * 0.1559 - z4 * 0.1600 - z5 * 0.5331$$
$$(3):\ 0 = z3 - z2 * 0.1056 - z4 * 0.1492 - z5 * 0.4947$$
$$(4):\ 0 = z4 - z2 * 0.0450 - z3 * 0.0619 - z5 * 0.6646$$
$$(5):\ 0 = z5 - z2 * 0.0881 - z3 * 0.1207 - z4 * 0.3906$$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0402 & -0.0985 & -0.2162 & -0.6451 \\ 1 & -0.1559 & -0.1600 & -0.5331 \\ -0.1056 & 1 & -0.1492 & -0.4947 \\ -0.0450 & -0.0619 & 1 & -0.6646 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.8397 & 0.97271 & 6.7959\times10^{-2} & -0.01577 \\ -0.75872 & 6.7614\times10^{-2} & 0.93496 & -1.3721\times10^{-2} \\ -0.82046 & 6.6330\times10^{-4} & -3.0063\times10^{-2} & 0.81824 \\ -1.107 & -7.1162\times10^{-2} & -0.13692 & -0.27115 \end{matrix} * \begin{matrix} -8.1513 \\ 0 \\ 0 \\ 0 \end{matrix} =$$

$$\begin{matrix} 6.8446 \\ 6.1846 \\ 6.6878 \\ 9.0235 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0402 & -0.0985 & -0.2162 & -0.6451 \\ 1 & -0.1559 & -0.1600 & -0.5331 \\ -0.1056 & 1 & -0.1492 & -0.4947 \\ -0.0881 & -0.1207 & -0.3906 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.82304 & 0.97715 & 7.4507\times10^{-2} & 2.6833\times10^{-2} \\ -0.74422 & 7.1472\times10^{-2} & 0.94065 & 2.3347\times10^{-2} \\ -1.685 & -0.22945 & -0.36979 & -1.3923 \\ -0.8205 & 5.0919\times10^{-3} & -2.4339\times10^{-2} & 0.46137 \end{matrix} * \begin{matrix} -8.1513 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 6.7088 \\ 6.0664 \\ 13.735 \\ 6.6881 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0402 & -0.0985 & -0.2162 & -0.6451 \\ 1 & -0.1559 & -0.1600 & -0.5331 \\ -0.0450 & -0.0619 & 1 & -0.6646 \\ -0.0881 & -0.1207 & -0.3906 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -1.0126 & 0.92668 & -0.17945 & -0.27851 \\ -3.138 & -0.56567 & -2.2656 & -3.8316 \\ -0.74396 & 2.1026\times10^{-2} & 0.89064 & 0.1232 \\ -0.75857 & 2.1578\times10^{-2} & 5.8621\times10^{-2} & 0.56111 \end{matrix} * \begin{matrix} -8.1513 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 8.254 \\ 25.579 \\ 6.0642 \\ 6.1833 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0402 & -0.0985 & -0.2162 & -0.6451 \\ -0.1056 & 1 & -0.1492 & -0.4947 \\ -0.0450 & -0.0619 & 1 & -0.6646 \\ -0.0881 & -0.1207 & -0.3906 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -4.4943 & -1.3681 & -3.4746 & -5.8853 \\ -1.0128 & 0.83513 & -0.25415 & -0.40909 \\ -0.82296 & -3.1043\times10^{-2} & 0.81588 & -4.0132\times10^{-3} \\ -0.83964 & -3.1857\times10^{-2} & -1.8106\times10^{-2} & 0.43056 \end{matrix} * \begin{matrix} -8.1513 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 36.634 \\ 8.2556 \\ 6.7082 \\ 6.8442 \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.1559 & -0.1600 & -0.5331 \\ -0.1056 & 1 & -0.1492 & -0.4947 \\ -0.0450 & -0.0619 & 1 & -0.6646 \\ -0.0881 & -0.1207 & -0.3906 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} 1.1962 & 0.39792 & 0.77895 & 1.3522 \\ 0.26955 & 1.2331 & 0.70435 & 1.2218 \\ 0.21904 & 0.29234 & 1.5947 & 1.3213 \\ 0.22348 & 0.29808 & 0.77655 & 1.7827 \end{matrix} * \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

Which gives us the following average values of z;

z2; $\frac{6.8446+6.7088+8.254+36.634+0.0}{5} = 11.688$

z3; $\frac{6.1846+6.0664+25.579+8.2556+0.0}{5} = 9.2171$

z4; $\frac{6.6878+13.735+6.0642+6.7082+0.0}{5} = 6.639$

z5; $\frac{9.0235+6.6881+6.1833+6.8442+0.0}{5} = 5.7478$

The estimates for 2003 are based on the following equations;

$(1): -10.4560 = -z2 * 0.0506 - z3 * 0.0943 - z4 * 0.2090 - z5 * 0.6459$
$(2): 0 = z2 - z3 * 0.1482 - z4 * 0.1447 - z5 * 0.5526$
$(3): 1.8450 = z3 - z2 * 0.1224 - z4 * 0.1446 - z5 * 0.4954$
$(4): 0 = z4 - z2 * 0.0530 - z3 * 0.0642 - z5 * 0.6493$
$(5): 0 = z5 - z2 * 0.1129 - z3 * 0.1226 - z4 * 0.3621$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0506 & -0.0943 & -0.2090 & -0.6459 \\ 1 & -0.1482 & -0.1447 & -0.5526 \\ -0.1224 & 1 & -0.1446 & -0.4954 \\ -0.0530 & -0.0642 & 1 & -0.6493 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.84325 & 0.96337 & 6.1459 \times 10^{-2} & -2.7953 \times 10^{-2} \\ -0.76909 & 7.5100 \times 10^{-2} & 0.93769 & -1.4284 \times 10^{-2} \\ -0.81277 & -2.0023 \times 10^{-4} & -0.0236 & 0.82669 \\ -1.1069 & -8.6371 \times 10^{-2} & -0.13408 & -0.26322 \end{matrix} * \begin{matrix} -10.4560 \\ 0 \\ 1.8450 \\ 0 \end{matrix} =$$

$$\begin{matrix} 8.9304 \\ 9.7716 \\ 8.4548 \\ 11.326 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0506 & -0.0943 & -0.2090 & -0.6459 \\ 1 & -0.1482 & -0.1447 & -0.5526 \\ -0.1224 & 1 & -0.1446 & -0.4954 \\ -0.1129 & -0.1226 & -0.3621 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.81202 & 0.97361 & 7.3861 \times 10^{-2} & 5.0125 \times 10^{-2} \\ -0.75313 & 8.0332 \times 10^{-2} & 0.94403 & 2.5613 \times 10^{-2} \\ -1.7364 & -0.30301 & -0.39040 & -1.4824 \\ -0.81278 & 1.0048 \times 10^{-2} & -1.7287 \times 10^{-2} & 0.47201 \end{matrix} * \begin{matrix} -10.4560 \\ 0 \\ 1.8450 \\ 0 \end{matrix} =$$

$$\begin{matrix} 8.6268 \\ 9.6165 \\ 17.436 \\ 8.4665 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{pmatrix} -0.0506 & -0.0943 & -0.2090 & -0.6459 \\ 1 & -0.1482 & -0.1447 & -0.5526 \\ -0.0530 & -0.0642 & 1 & -0.6493 \\ -0.1129 & -0.1226 & -0.3621 & 1 \end{pmatrix}, \text{ inverse:}$$

$$\begin{pmatrix} -0.99802 & 0.91263 & -0.16647 & -0.24839 \\ -3.1304 & -0.69902 & -2.1276 & -3.7897 \\ -0.75334 & 1.9283\times10^{-2} & 0.87988 & 9.5381\times10^{-2} \\ -0.76925 & 2.4319\times10^{-2} & 3.8961\times10^{-2} & 0.54188 \end{pmatrix} * \begin{pmatrix} -10.4560 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 10.435 \\ 32.731 \\ 7.8769 \\ 8.0433 \end{pmatrix}$$

Using 1,3,4 and 5;

$$\begin{pmatrix} -0.0506 & -0.0943 & -0.2090 & -0.6459 \\ -0.1224 & 1 & -0.1446 & -0.4954 \\ -0.0530 & -0.0642 & 1 & -0.6493 \\ -0.1129 & -0.1226 & -0.3621 & 1 \end{pmatrix}, \text{ inverse:}$$

$$\begin{pmatrix} -3.7818 & -1.1055 & -2.6580 & -4.7162 \\ -0.99817 & 0.84672 & -0.21931 & -0.36765 \\ -0.81216 & -2.3358\times10^{-2} & 0.82724 & 9.8020\times10^{-4} \\ -0.84343 & -2.9458\times10^{-2} & -2.7431\times10^{-2} & 0.42283 \end{pmatrix} * \begin{pmatrix} -10.4560 \\ 1.8450 \\ 0 \\ 0 \end{pmatrix} =$$

$$\begin{matrix} 37.503 \\ 11.999 \\ 8.4488 \\ 8.7646 \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{pmatrix} 1 & -0.1482 & -0.1447 & -0.5526 \\ -0.1224 & 1 & -0.1446 & -0.4954 \\ -0.0530 & -0.0642 & 1 & -0.6493 \\ -0.1129 & -0.1226 & -0.3621 & 1 \end{pmatrix}, \text{ inverse:}$$

$$\begin{pmatrix} 1.2398 & 0.39632 & 0.72676 & 1.3533 \\ 0.32723 & 1.2431 & 0.67405 & 1.2343 \\ 0.26625 & 0.29915 & 1.5541 & 1.3044 \\ 0.27651 & 0.30547 & 0.72743 & 1.7765 \end{pmatrix} * \begin{pmatrix} 0 \\ 1.8450 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.73121 \\ 2.2935 \\ 0.55193 \\ 0.56359 \end{pmatrix}$$

Which gives us the following average values of z;

$$z2; = \frac{8.9304+8.6268+10.435+37.503+0.73121}{5} = 13.245$$

$$z3; = \frac{9.7716+9.6165+32.731+11.999+2.2935}{5} = 13.282$$

$$z4; = \frac{8.4548+17.436+7.8769+8.4488+0.55193}{5} = 8.5537$$

$$z5; = \frac{11.326+8.4665+8.0433+8.7646+0.56359}{5} = 7.4328$$

The estimates for 2004 are based on the following equations;

$$\begin{aligned} &(1) : -14.4540 = -z2 * 0.0624 - z3 * 0.0874 - z4 * 0.2132 - z5 * 0.6371 \\ &(2) : 1.8480 = z2 - z3 * 0.1435 - z4 * 0.1544 - z5 * 0.5535 \\ &(3) : 6.1317 = z3 - z2 * 0.1429 - z4 * 0.1488 - z5 * 0.5009 \\ &(4) : 0 = z4 - z2 * 0.0659 - z3 * 0.0638 - z5 * 0.6533 \\ &(5) : 0 = z5 - z2 * 0.1349 - z3 * 0.1226 - z4 * 0.3727 \end{aligned}$$

Using 1,2,3 and 4;

$$\begin{pmatrix} -0.0624 & -0.0874 & -0.2132 & -0.6371 \\ 1 & -0.1435 & -0.1544 & -0.5535 \\ 0.1429 & 1 & 0.1488 & -0.5009 \\ -0.0659 & -0.0638 & 1 & -0.6533 \end{pmatrix}$$, inverse:

$$\begin{pmatrix} -0.82151 & 0.93754 & 6.0226 \times 10^{-2} & -3.9352 \times 10^{-2} \\ -0.34073 & -0.1699 & 0.93070 & -0.23736 \\ -0.83554 & 5.0721 \times 10^{-3} & -1.9627 \times 10^{-2} & 0.82557 \\ -1.1628 & -7.0216 \times 10^{-2} & -0.12701 & -0.23985 \end{pmatrix} * \begin{pmatrix} -14.4540 \\ 1.8480 \\ 6.1317 \\ 0 \end{pmatrix} =$$

$$\begin{matrix} 13.976 \\ 10.318 \\ 11.966 \\ 15.899 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0624 & -0.0874 & -0.2132 & -0.6371 \\ 1 & -0.1435 & -0.1544 & -0.5535 \\ 0.1429 & 1 & 0.1488 & -0.5009 \\ -0.1349 & -0.1226 & -0.3927 & 1 \end{matrix}, \text{ inverse:}$$

$$\begin{matrix} -0.77083 & 0.95075 & 7.8171\times 10^{-2} & 7.4299\times 10^{-2} \\ -3.5042\times 10^{-2} & -9.0195\times 10^{-2} & 1.0389 & 0.44816 \\ -1.8987 & -0.27215 & -0.3961 & -1.5587 \\ -0.85391 & 1.0325\times 10^{-2} & -0.01763 & 0.45286 \end{matrix} * \begin{matrix} -14.4540 \\ 1.8480 \\ 6.1317 \\ 0 \end{matrix} =$$

$$\begin{matrix} 13.378 \\ 6.71 \\ 24.512 \\ 12.253 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0624 & -0.0874 & -0.2132 & -0.6371 \\ 1 & -0.1435 & -0.1544 & -0.5535 \\ -0.0659 & -0.0638 & 1 & -0.6533 \\ -0.1349 & -0.1226 & -0.3927 & 1 \end{matrix}, \text{ inverse:}$$

$$\begin{matrix} -0.99159 & 0.89319 & -0.17142 & -0.24935 \\ -2.9691 & -0.85523 & -2.2783 & -3.8534 \\ -0.78011 & 1.9525\times 10^{-2} & 0.86861 & 8.1261\times 10^{-2} \\ -0.80412 & 2.3308\times 10^{-2} & 3.8661\times 10^{-2} & 0.52585 \end{matrix} * \begin{matrix} -14.4540 \\ 1.8480 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 15.983 \\ 41.335 \\ 11.312 \\ 11.666 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0624 & -0.0874 & -0.2132 & -0.6371 \\ 0.1429 & 1 & 0.1488 & -0.5009 \\ -0.0659 & -0.0638 & 1 & -0.6533 \\ -0.1349 & -0.1226 & -0.3927 & 1 \end{matrix}, \text{ inverse:}$$

$$\begin{pmatrix} -4.4171 & -1.2130 & -2.8313 & -5.2715 \\ 0.31087 & 1.1614 & 0.26860 & 0.95529 \\ -0.85499 & -2.6515\times10^{-2} & 0.81046 & -2.8519\times10^{-2} \\ -0.89351 & -3.1652\times10^{-2} & -3.0749\times10^{-2} & 0.39480 \end{pmatrix} * \begin{pmatrix} -14.4540 \\ 6.1317 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 56.407 \\ 2.628 \\ 12.195 \\ 12.721 \end{pmatrix}$$

Using 2,3,4 and 5;

$$\begin{pmatrix} 1 & -0.1435 & -0.1544 & -0.5535 \\ 0.1429 & 1 & 0.1488 & -0.5009 \\ -0.0659 & -0.0638 & 1 & -0.6533 \\ -0.1349 & -0.1226 & -0.3927 & 1 \end{pmatrix}$$, inverse:

$$\begin{pmatrix} 1.1517 & 0.35112 & 0.59855 & 1.2044 \\ -8.1058\times10^{-2} & 1.0513 & 2.7210\times10^{-2} & 0.49953 \\ 0.22293 & 0.27623 & 1.4744 & 1.2250 \\ 0.23298 & 0.28474 & 0.66306 & 1.7048 \end{pmatrix} * \begin{pmatrix} 1.8480 \\ 6.1317 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4.2813 \\ 6.2965 \\ 2.1057 \\ 2.1765 \end{pmatrix}$$

Which gives us the following average values of z;

$$z2 = \frac{13.976+13.378+15.983+56.407+4.2813}{5} = 20.805$$

$$z3 = \frac{10.318+6.71+41.335+2.628+6.2965}{5} = 13.458$$

$$z4 = \frac{11.966+24.512+11.312+12.195+2.1057}{5} = 12.418$$

$$z5 = \frac{15.899+12.253+11.666+12.721+2.1765}{5} = 10.943$$

The estimates for 2005 are based on the following equations;

$$(1) : -18.2200 = -z2 * 0.0715 - z3 * 0.0818 - z4 * 0.2082 - z5 * 0.6374$$
$$(2) : 13.7529 = z2 - z3 * 0.1359 - z4 * 0.1555 - z5 * 0.5612$$
$$(3) : 5.3317 = z3 - z2 * 0.1511 - z4 * 0.1468 - z5 * 0.5146$$
$$(4) : 0 = z4 - z2 * 0.0723 - z3 * 0.0615 - z5 * 0.6654$$

$(5):\ 0 = z5 - z2 * 0.1489 - z3 * 0.1229 - z4 * 0.3794$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0715 & -0.0818 & -0.2082 & -0.6374 \\ 1 & -0.1359 & -0.1555 & -0.5612 \\ -0.1511 & 1 & -0.1468 & -0.5146 \\ -0.0723 & -0.0615 & 1 & -0.6654 \end{matrix}\text{, inverse:}$$

$$\begin{matrix} -0.85534 & 0.94585 & 5.7184\times10^{-2} & -2.2608\times10^{-2} \\ -0.81548 & 8.2916\times10^{-2} & 0.94343 & -1.8393\times10^{-2} \\ -0.83989 & -3.4462\times10^{-3} & -1.8627\times10^{-2} & 0.82186 \\ -1.0939 & -0.11562 & -0.1214 & -0.26356 \end{matrix} * \begin{matrix} -18.2200 \\ 13.7529 \\ 5.3317 \\ 0 \end{matrix} =$$

$$\begin{matrix} 28.897 \\ 21.028 \\ 15.156 \\ 17.693 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0715 & -0.0818 & -0.2082 & -0.6374 \\ 1 & -0.1359 & -0.1555 & -0.5612 \\ -0.1511 & 1 & -0.1468 & -0.5146 \\ -0.1489 & -0.1229 & -0.3794 & 1 \end{matrix}\text{, inverse:}$$

$$\begin{matrix} -0.83361 & 0.95638 & 6.6659\times10^{-2} & 3.9681\times10^{-2} \\ -0.79779 & 9.1482\times10^{-2} & 0.95114 & 3.2282\times10^{-2} \\ -1.6299 & -0.38620 & -0.36310 & -1.4425 \\ -0.84058 & 7.1253\times10^{-3} & -1.0939\times10^{-2} & 0.46259 \end{matrix} * \begin{matrix} -18.2200 \\ 13.7529 \\ 5.3317 \\ 0 \end{matrix} =$$

$$\begin{matrix} 28.697 \\ 20.865 \\ 22.449 \\ 15.355 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0715 & -0.0818 & -0.2082 & -0.6374 \\ 1 & -0.1359 & -0.1555 & -0.5612 \\ -0.0723 & -0.0615 & 1 & -0.6654 \\ -0.1489 & -0.1229 & -0.3794 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.98649 & 0.88231 & -0.15904 & -0.23946 \\ -2.9793 & -0.96535 & -2.2693 & -3.9507 \\ -0.79717 & 1.7251\times10^{-2} & 0.86631 & 7.8003\times10^{-2} \\ -0.81549 & 1.9280\times10^{-2} & 2.6098\times10^{-2} & 0.50839 \end{matrix} * \begin{matrix} -18.2200 \\ 13.7529 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 30.108 \\ 41.006 \\ 14.762 \\ 15.123 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0715 & -0.0818 & -0.2082 & -0.6374 \\ -0.1511 & 1 & -0.1468 & -0.5146 \\ -0.0723 & -0.0615 & 1 & -0.6654 \\ -0.1489 & -0.1229 & -0.3794 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -2.8077 & -0.79407 & -2.0536 & -3.5647 \\ -0.98663 & 0.86881 & -0.19644 & -0.31250 \\ -0.83278 & -1.5525\times10^{-2} & 0.82926 & 1.2988\times10^{-2} \\ -0.85528 & -1.7351\times10^{-2} & -1.5300\times10^{-2} & 0.43573 \end{matrix} * \begin{matrix} -18.2200 \\ 5.3317 \\ 0 \\ 0 \end{matrix} =$$

$$\begin{matrix} 46.923 \\ 22.609 \\ 15.09 \\ 15.491 \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.1359 & -0.1555 & -0.5612 \\ -0.1511 & 1 & -0.1468 & -0.5146 \\ -0.0723 & -0.0615 & 1 & -0.6654 \\ -0.1489 & -0.1229 & -0.3794 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} 1.3602 & 0.43012 & 0.86717 & 1.5617 \\ 0.47798 & 1.2990 & 0.82992 & 1.4889 \\ 0.40345 & 0.34758 & 1.6956 & 1.5335 \\ 0.41435 & 0.35556 & 0.87442 & 1.9973 \end{matrix} * \begin{matrix} 13.7529 \\ 5.3317 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 21.000 \\ 13.499 \\ 7.4018 \\ 7.5943 \end{matrix}$$

Which gives us the following average values of z;

z2= $\frac{28.897+28.697+30.108+46.923+21.000}{5} = 31.125$

z3= $\frac{21.028+20.865+41.006+22.609+13.499}{5} = 23.801$

z4= $\frac{15.156+22.449+14.762+15.09+7.4018}{5} = 14.972$

z5= $\frac{17.693+15.355+15.123+15.123+7.5943}{5} = 14.178$

The estimates for 2006 are based on the following equations;

$$
\begin{aligned}
&(1): -18.689 = -z2 * 0.0816 - z3 * 0.0762 - z4 * 0.1999 - z5 * 0.6423 \\
&(2): 21.1187 = z2 - z3 * 0.1214 - z4 * 0.1555 - z5 * 0.5714 \\
&(3): 7.5567 = z3 - z2 * 0.1547 - z4 * 0.1335 - z5 * 0.5313 \\
&(4): 0 = z4 - z2 * 0.0789 - z3 * 0.0532 - z5 * 0.6792 \\
&(5): 0 = z5 - z2 * 0.1623 - z3 * 0.1184 - z4 * 0.3800
\end{aligned}
$$

Using 1,2,3 and 4;

$$
\begin{pmatrix} -0.0816 & -0.0762 & -0.1999 & -0.6423 \\ 1 & -0.1214 & -0.1555 & -0.5714 \\ -0.1547 & 1 & -0.1335 & -0.5313 \\ -0.0789 & -0.0532 & 1 & -0.6792 \end{pmatrix}, \text{ inverse:}
$$

$$
\begin{pmatrix} -0.85284 & 0.93633 & 4.7698 \times 10^{-2} & -1.8517 \times 10^{-2} \\ -0.82265 & 7.7031 \times 10^{-2} & 0.94527 & -2.6277 \times 10^{-2} \\ -0.84913 & -7.4513 \times 10^{-3} & -2.1654 \times 10^{-2} & 0.82621 \\ -1.0867 & -0.12577 & -0.11146 & -0.25167 \end{pmatrix} * \begin{pmatrix} -18.6890 \\ 21.1187 \\ 7.5567 \\ 0 \end{pmatrix} =
$$

$$
\begin{pmatrix} 36.073 \\ 24.144 \\ 15.548 \\ 16.811 \end{pmatrix}
$$

Using 1,2,3, and 5;

$$\begin{matrix} -0.0816 & -0.0762 & -0.1999 & -0.6423 \\ 1 & -0.1214 & -0.1555 & -0.5714 \\ -0.1547 & 1 & -0.1335 & -0.5313 \\ -0.1623 & -0.1184 & -0.3800 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.83536 & 0.94573 & 5.5075 \times 10^{-2} & 3.3095 \times 10^{-2} \\ -0.79785 & 0.09037 & 0.95574 & 4.6963 \times 10^{-2} \\ -1.6291 & -0.42687 & -0.35080 & -1.4767 \\ -0.8491 & 1.9821 \times 10^{-3} & -1.1205 \times 10^{-2} & 0.44980 \end{matrix} * \begin{matrix} -18.6890 \\ 21.1187 \\ 7.5567 \\ 0 \end{matrix} =$$

$$\begin{matrix} 36.001 \\ 24.042 \\ 18.78 \\ 15.826 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0816 & -0.0762 & -0.1999 & -0.6423 \\ 1 & -0.1214 & -0.1555 & -0.5714 \\ -0.0789 & -0.0532 & 1 & -0.6792 \\ -0.1623 & -0.1184 & -0.3800 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.96587 & 0.87555 & -0.13825 & -0.21399 \\ -3.0627 & -1.1275 & -2.3991 & -4.2409 \\ -0.79782 & 2.0142 \times 10^{-2} & 0.88056 & 9.7150 \times 10^{-2} \\ -0.82255 & 0.01626 & 2.8126 \times 10^{-2} & 0.50007 \end{matrix} * \begin{matrix} -18.6890 \\ 21.1187 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 36.542 \\ 33.427 \\ 15.336 \\ 15.716 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0816 & -0.0762 & -0.1999 & -0.6423 \\ -0.1547 & 1 & -0.1335 & -0.5313 \\ -0.0789 & -0.0532 & 1 & -0.6792 \\ -0.1623 & -0.1184 & -0.3800 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -2.5941 & -0.68710 & -1.8630 & -3.2966 \\ -0.9659 & 0.88482 & -0.17802 & -0.27121 \\ -0.83528 & -1.5807 \times 10^{-2} & 0.84089 & 2.6234 \times 10^{-2} \\ -0.85279 & -0.01276 & -3.9045 \times 10^{-3} & 0.44282 \end{matrix} * \begin{matrix} -18.6890 \\ 7.5567 \\ 0 \\ 0 \end{matrix} =$$

$$\begin{matrix} 43.289 \\ 24.738 \\ 15.491 \\ 15.841 \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.1214 & -0.1555 & -0.5714 \\ -0.1547 & 1 & -0.1335 & -0.5313 \\ -0.0789 & -0.0532 & 1 & -0.6792 \\ -0.1623 & -0.1184 & -0.3800 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} 1.3949 & 0.40759 & 0.88488 & 1.6146 \\ 0.51940 & 1.2924 & 0.84514 & 1.5575 \\ 0.44915 & 0.33667 & 1.7257 & 1.6076 \\ 0.45857 & 0.34711 & 0.89944 & 2.0573 \end{matrix} * \begin{matrix} 22.1187 \\ 7.5567 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 33.933 \\ 21.255 \\ 12.479 \\ 12.766 \end{matrix}$$

Which gives us the following average values of z;

z2$= \frac{36.073+36.001+36.542+43.289+33.933}{5} = 37.168$

z3$= \frac{24.144+24.042+33.427+24.738+21.255}{5} = 25.521$

z4$= \frac{15.548+18.78+15.336+15.491+12.479}{5} = 15.527$

z5$= \frac{16.811+15.826+15.716+15.841+12.766}{5} = 15.392$

The estimates for 2007 are based on the following equations;

$$\begin{aligned} &(1) : -13.1670 = -z2 * 0.0916 - z3 * 0.0732 - z4 * 0.1958 - z5 * 0.6393 \\ &(2) : 25.4710 = z2 - z3 * 0.1097 - z4 * 0.1606 - z5 * 0.5757 \\ &(3) : 15.1600 = z3 - z2 * 0.1577 - z4 * 0.1284 - z5 * 0.5372 \\ &(4) : 0 = z4 - z2 * 0.0884 - z3 * 0.0491 - z5 * 0.6814 \\ &(5) : 0 = z5 - z2 * 0.1765 - z3 * 0.1146 - z4 * 0.3797 \end{aligned}$$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0916 & -0.0732 & -0.1958 & -0.6393 \\ 1 & -0.1097 & -0.1606 & -0.5757 \\ -0.1577 & 1 & -0.1284 & -0.5372 \\ -0.0884 & -0.0491 & 1 & -0.6814 \end{matrix}$$, inverse:

$$\begin{matrix} -0.85293 & 0.92679 & 3.8585\times10^{-2} & -1.3208\times10^{-2} \\ -0.82735 & 7.0782\times10^{-2} & 0.94577 & -2.9191\times10^{-2} \\ -0.85551 & -8.7725\times10^{-3} & -2.2924\times10^{-2} & 0.82814 \\ -1.0852 & -0.13821 & -0.10680 & -0.2484 \end{matrix} \quad \begin{matrix} -13.1670 \\ 22.4710 \\ 15.1600 \\ 0 \end{matrix} \; : \; \begin{matrix} 32.641 \\ 26.822 \\ 10.720 \\ 9.564 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0916 & -0.0732 & -0.1958 & -0.6393 \\ 1 & -0.1097 & -0.1606 & -0.5757 \\ -0.1577 & 1 & -0.1284 & -0.5372 \\ -0.1765 & -0.1146 & -0.3797 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.84072 & 0.93406 & 4.3642\times10^{-2} & 2.3705\times10^{-2} \\ -0.80037 & 8.6844\times10^{-2} & 0.95694 & 5.2392\times10^{-2} \\ -1.6211 & -0.46444 & -0.33994 & -1.4863 \\ -0.85562 & -1.5330\times10^{-3} & -1.1708\times10^{-2} & 0.44583 \end{matrix} \quad \begin{matrix} -13.1670 \\ 22.4710 \\ 15.16 \\ 0 \end{matrix} \; : \; \begin{matrix} 32.721 \\ 26.997 \\ 5.7551 \\ 11.054 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0916 & -0.0732 & -0.1958 & -0.6393 \\ 1 & -0.1097 & -0.1606 & -0.5757 \\ -0.0884 & -0.0491 & 1 & -0.6814 \\ -0.1765 & -0.1146 & -0.3797 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.94611 & 0.87133 & -0.114 & -0.18091 \\ -3.1112 & -1.2886 & -2.4998 & -4.4342 \\ -0.80015 & 2.4177\times10^{-2} & 0.88802 & 0.10748 \\ -0.82735 & 1.5295\times10^{-2} & 3.0584\times10^{-2} & 0.50072 \end{matrix} * \begin{matrix} -13.1670 \\ 22.4710 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 32.037 \\ 12.009 \\ 11.079 \\ 11.237 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0916 & -0.0732 & -0.1958 & -0.6393 \\ -0.1577 & 1 & -0.1284 & -0.5372 \\ -0.0884 & -0.0491 & 1 & -0.6814 \\ -0.1765 & -0.1146 & -0.3797 & 1 \end{matrix}$$, inverse:

$$\begin{pmatrix} -2.4100 & -0.60621 & -1.6976 & -3.0231 \\ -0.94627 & 0.89652 & -0.15783 & -0.23088 \\ -0.84077 & -1.6821\times10^{-2} & 0.84408 & 2.8615\times10^{-2} \\ -0.85305 & -1.0641\times10^{-2} & 2.7861\times10^{-3} & 0.45083 \end{pmatrix} * \begin{pmatrix} -13.1670 \\ 15.1600 \\ 0 \\ 0 \end{pmatrix} : \begin{pmatrix} 22.542 \\ 26.051 \\ 10.815 \\ 11.071 \end{pmatrix}$$

Using 2,3,4 and 5;

$$\begin{pmatrix} 1 & -0.1097 & -0.1606 & -0.5757 \\ -0.1577 & 1 & -0.1284 & -0.5372 \\ -0.0884 & -0.0491 & 1 & -0.6814 \\ -0.1765 & -0.1146 & -0.3797 & 1 \end{pmatrix}$$, inverse:

$$\begin{pmatrix} 1.4345 & 0.39179 & 0.90947 & 1.656 \\ 0.56323 & 1.2884 & 0.86581 & 1.6063 \\ 0.50044 & 0.33135 & 1.7536 & 1.661 \\ 0.50775 & 0.34261 & 0.92559 & 2.1071 \end{pmatrix} * \begin{pmatrix} 22.4710 \\ 15.1600 \\ 0 \\ 0 \end{pmatrix} : \begin{pmatrix} 38.174 \\ 32.188 \\ 16.269 \\ 16.604 \end{pmatrix}$$

Which gives us the following average values of z;

$$z2=\frac{32.641+32.721+32.037+22.542+38.174}{5}=31.623$$

$$z3=\frac{26.822+26.997+12.009+26.051+32.188}{5}=24.813$$

$$z4=\frac{10.720+5.7551+11.079+11.079+16.269}{5}=10.98$$

$$z5=\frac{9.564+11.054+11.237+11.071+16.604}{5}=11.906$$

The estimates for 2008 are based on the following equations;

$$(1):\ -10.310 = -z2*0.0978 - z3*0.0682 - z4*0.1987 - z5*0.6353$$
$$(2):\ 22.161 = z2 - z3*0.1019 - z4*0.1661 - z5*0.5880$$
$$(3):\ 1.785 = z3 - z2*0.1653 - z4*0.1295 - z5*0.5424$$
$$(4):\ -5.290 = z4 - z2*0.0971 - z3*0.0467 - z5*0.6850$$
$$(5):\ 0 = z5 - z2*0.1941 - z3*0.1105 - z4*0.3866$$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0978 & -0.0682 & -0.1987 & -0.6353 \\ 1 & -0.1019 & -0.1661 & -0.5880 \\ -0.1653 & 1 & -0.1295 & -0.5424 \\ -0.0971 & -0.0467 & 1 & -0.6850 \end{matrix}$$, inverse:

$$\begin{matrix} -0.86452 & 0.91966 & 3.4071 \times 10^{-2} & -1.4613 \times 10^{-2} \\ -0.84087 & 0.07139 & 0.94841 & -3.2402 \times 10^{-2} \\ -0.86346 & -7.9027 \times 10^{-3} & -2.1195 \times 10^{-2} & 0.82437 \\ -1.0806 & -0.14677 & -0.10043 & -0.25211 \end{matrix} \quad \begin{matrix} -10.343 \\ 22.162 \\ 1.786 \\ -5.290 \end{matrix} = \begin{matrix} 29.461 \\ 12.145 \\ 4.3569 \\ 9.0782 \end{matrix}$$

Using 1,2,3,5;

$$\begin{matrix} -0.0978 & -0.0682 & -0.1987 & -0.6353 \\ 1 & -0.1019 & -0.1661 & -0.5880 \\ -0.1653 & 1 & -0.1295 & -0.5424 \\ -0.1941 & -0.1105 & -0.3866 & 1 \end{matrix}, inverse: \begin{matrix} -0.85194 & 0.92821 & 3.9343 \times 10^{-2} & 2.58 \\ -0.81296 & 9.0342 \times 10^{-2} & 0.96011 & 5.74 \\ -1.5735 & -0.49007 & -0.31865 & -1 \\ -0.8635 & 6.873 \times 10^{-4} & -9.4620 \times 10^{-3} & 0. \end{matrix}$$

$$\begin{matrix} -10.343 \\ 22.162 \\ 1.786 \\ 0 \end{matrix} = \begin{matrix} 29.453 \\ 12.125 \\ 4.8447 \\ 8.9295 \end{matrix}$$

Using 1,2,4,5;

$$\begin{matrix} -0.0978 & -0.0682 & -0.1987 & -0.6353 \\ 1 & -0.1019 & -0.1661 & -0.5880 \\ -0.0971 & -0.0467 & 1 & -0.5424 \\ -0.1941 & -0.1105 & -0.3866 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.95888 & 0.86468 & -0.10863 & -0.15966 \\ -3.4226 & -1.4599 & -2.6509 & -4.4707 \\ -0.70736 & 2.4443 \times 10^{-2} & 0.87982 & 4.2203 \times 10^{-2} \\ -0.83779 & 1.5965 \times 10^{-2} & 2.6125 \times 10^{-2} & 0.49131 \end{matrix} \quad \begin{matrix} -10.343 \\ 22.162 \\ -5.290 \\ 0 \end{matrix} : \begin{matrix} 29.655 \\ 17.069 \\ 3.2037 \\ 8.8809 \end{matrix}$$

Using 1,3,4,5;

$$\begin{matrix} -0.0978 & -0.0682 & -0.1987 & -0.6353 \\ -0.1653 & 1 & -0.1295 & -0.5424 \\ -0.0971 & -0.0467 & 1 & -0.5424 \\ -0.1941 & -0.1105 & -0.3866 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -2.4145 & -0.53552 & -1.5873 & -2.6853 & -10.343 & & 32.414 \\ -0.96504 & 0.90416 & -0.15449 & -0.20647 & 1.786 & : & 12.413 \\ -0.74851 & -1.5138\times10^{-2} & 0.83803 & -2.9192\times10^{-2} & -5.290 & & 3.2816 \\ -0.86466 & -9.8876\times10^{-3} & -1.1753\times10^{-3} & 0.44468 & 0 & & 8.9317 \end{matrix}$$

Using 2,3,4,5;

$$\begin{matrix} 1 & -0.1019 & -0.1661 & -0.5880 \\ -0.1653 & 1 & -0.1295 & -0.5424 \\ -0.0971 & -0.0467 & 1 & -0.5424 \\ -0.1941 & -0.1105 & -0.3866 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} 1.4343 & 0.35277 & 0.86541 & 1.5041 & 22.162 & & 27.839 \\ 0.57327 & 1.2592 & 0.82581 & 1.4680 & 1.786 & : & 10.585 \\ 0.44464 & 0.26024 & 1.5984 & 1.2696 & -5.290 & & 1.8634 \\ 0.51364 & 0.30822 & 0.87715 & 1.9450 & 0 & & 7.2936 \end{matrix}$$

Which gives us the following average values of z;

z2; $\frac{29.461+29.453+29.655+32.414+27.839}{5} = 29.764$

z3; $\frac{12.145+12.125+17.069+12.413+10.585}{5} = 12.867$

z4; $\frac{4.3569+4.8447+3.2037+3.2816+1.8634}{5} = 3.5101$

z5; $\frac{9.0782+8.9295+8.8809+8.9317+7.2936}{5} = 8.6228$

The estimates for 2009 are based on the following equations;

$$(1) : 0 = -z2 * 0.0972 - z3 * 0.0616 - z4 * 0.1929 - z5 * 0.6482$$
$$(2) : 9.8673 = z2 - z3 * 0.0996 - z4 * 0.1688 - z5 * 0.5978$$
$$(3) : 0 = z3 - z2 * 0.1653 - z4 * 0.0118 - z5 * 0.5755$$

$(4): -2.8990 = z4 - z2 * 0.0988 - z3 * 0.0417 - z5 * 0.7039$
$(5): 0 = z5 - z2 * 0.1966 - z3 * 0.1141 - z4 * 0.3956$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \end{matrix}$$, inverse:

$$\begin{matrix} -0.86946 & 0.92053 & 3.7631 \times 10^{-2} & -1.1888 \times 10^{-2} & 0 \\ -0.77435 & 6.9950 \times 10^{-2} & 0.95400 & -0.12631 & 9.8673 \\ -0.87687 & -6.5963 \times 10^{-3} & -2.0082 \times 10^{-2} & 0.8295 & 0 \\ -1.0778 & -0.14272 & -9.0328 \times 10^{-2} & -0.23307 & -2.8990 \end{matrix}$$:

$$\begin{matrix} 9.1176 \\ 1.0564 \\ -2.4698 \\ -0.73259 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} -0.85916 & 0.92772 & 4.1968 \times 10^{-2} & 2.1835 \times 10^{-2} & 0 \\ -0.66494 & 0.14629 & 1.0001 & 0.23198 & 9.8673 \\ -1.5954 & -0.50794 & -0.32270 & -1.5235 & 0 \\ -0.87593 & -1.8586 \times 10^{-3} & -5.3000 \times 10^{-3} & 0.42806 & 0 \end{matrix}$$:

$$\begin{matrix} 9.1541 \\ 1.4435 \\ -5.0120 \\ -1.8339 \times 10^{-2} \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.95881 & 0.85819 & -0.11504 & -0.18945 \\ -3.0396 & -1.5105 & -2.7413 & -4.8028 \\ -0.82919 & 2.6673\times 10^{-2} & 0.88455 & 0.1011 \\ -0.86334 & 6.9218\times 10^{-3} & 1.4528\times 10^{-2} & 0.45475 \end{matrix} \begin{matrix} 0 \\ 9.8673 \\ -2.8990 \\ 0 \end{matrix} : \begin{matrix} 8.8015 \\ -6.9575 \\ -2.3011 \\ 2.6183\times 10^{-2} \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -2.1888 & -0.51802 & -1.5350 & -2.7974 \\ -0.87461 & 0.91178 & -0.24204 & -0.21257 \\ -0.86741 & -0.0161 & 0.84042 & 2.0045\times 10^{-2} \\ -0.87326 & -4.1781\times 10^{-3} & 3.0751\times 10^{-3} & 0.43371 \end{matrix} \begin{matrix} 0 \\ 0 \\ -2.8990 \\ 0 \end{matrix} =$$

$$\begin{matrix} 4.4500 \\ 0.70167 \\ -2.4364 \\ -8.9147\times 10^{-3} \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} 1.5272 & 0.40381 & 0.99184 & 1.8435 \\ 0.61023 & 1.2801 & 0.76762 & 1.6418 \\ 0.60521 & 0.34922 & 1.8418 & 1.8592 \\ 0.60929 & 0.3636 & 1.0112 & 2.2853 \end{matrix} \begin{matrix} 9.8673 \\ 0 \\ -2.8990 \\ 0 \end{matrix} : \begin{matrix} 12.194 \\ 3.7960 \\ 0.63241 \\ 3.0806 \end{matrix}$$

Which gives us the following average values of z;

z2= $\frac{9.1176+9.1541+8.8015+4.4500+12.194}{5} = 8.7434$

z3= $\frac{1.0564+1.4435-6.9575+0.70167+3.7960}{5} = 8.014 \times 10^{-3}$

z4= $\frac{-2.4698-5.0120-2.3011-2.4364+0.63241}{5} = -2.3174$

z5= $\frac{-0.73259-1.8339\times10^{-2}+2.6183\times10^{-2}-8.9147\times10^{-3}+3.0806}{5} = 0.46939$

The estimates for 2010 are based on the following equations;

$$(1) : -1.3010 = -z2 * 0.0972 - z3 * 0.0616 - z4 * 0.1929 - z5 * 0.6482$$
$$(2) : 7.3633 = z2 - z3 * 0.0996 - z4 * 0.1688 - z5 * 0.5977$$
$$(3) : 4.7308 = z3 - z2 * 0.1653 - z4 * 0.1183 - z5 * 0.5755$$
$$(4) : 0 = z4 - z2 * 0.0988 - z3 * 0.0417 - z5 * 0.7039$$
$$(5) : 0 = z5 - z2 * 0.1966 - z3 * 0.1141 - z4 * 0.3956$$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.8682 \\ 1 & -0.0996 & -0.1688 & -0.5977 \\ -0.1653 & 1 & -0.1183 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \end{matrix}$$, inverse:

$$\begin{matrix} -0.70662 & 0.94270 & 5.1571\times10^{-2} & 2.8921\times10^{-2} \\ -0.69883 & 9.1215\times10^{-2} & 0.96582 & -5.1505\times10^{-3} \\ -0.70834 & 1.5649\times10^{-2} & -5.9926\times10^{-3} & 0.86529 \\ -0.86573 & -0.11549 & -7.2969\times10^{-2} & -0.19513 \end{matrix} * \begin{matrix} -1.3010 \\ 7.3633 \\ 4.7308 \\ 0 \end{matrix} =$$

$$\begin{matrix} 8.1047 \\ 6.1499 \\ 1.0084 \\ -6.9275\times10^{-2} \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.8682 \\ 1 & -0.0996 & -0.1688 & -0.5977 \\ -0.1653 & 1 & -0.1183 & -0.5755 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix}$$, inverse:

$$\begin{pmatrix} -0.72618 & 0.92578 & 4.1393\times10^{-2} & -5.3308\times10^{-2} \\ -0.69535 & 9.4229\times10^{-2} & 0.96763 & 9.4934\times10^{-3} \\ -1.2934 & -0.49061 & -0.31052 & -1.5949 \\ -0.73379 & -1.3261\times10^{-3} & -4.2967\times10^{-3} & 0.35966 \end{pmatrix} * \begin{pmatrix} -1.3010 \\ 7.3633 \\ 4.7308 \\ 0 \end{pmatrix} = \begin{pmatrix} 7.9574 \\ 6.1762 \\ -3.3988 \\ 0.92457 \end{pmatrix}$$

Using 1,2,4 and 5;

$$\begin{pmatrix} -0.0972 & -0.0616 & -0.1929 & -0.8682 \\ 1 & -0.0996 & -0.1688 & -0.5977 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{pmatrix}$$,inverse:

$$\begin{pmatrix} -0.80571 & 0.85696 & -0.11761 & -0.27009 \\ -2.5545 & -1.5144 & -2.7495 & -5.0583 \\ -0.69683 & 2.5612\times10^{-2} & 0.88232 & 3.1385\times10^{-2} \\ -0.72554 & 5.8169\times10^{-3} & 1.2209\times10^{-2} & 0.38216 \end{pmatrix} * \begin{pmatrix} -1.3010 \\ 7.3633 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 7.3583 \\ -7.8276 \\ 1.0952 \\ 0.98676 \end{pmatrix}$$

Using 1,3,4 and 5;

$$\begin{pmatrix} -0.0972 & -0.0616 & -0.1929 & -0.8682 \\ -0.1653 & 1 & -0.1183 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{pmatrix}$$, inverse:

$$\begin{pmatrix} -1.7961 & -0.51548 & -1.5823 & -2.9699 \\ -0.80425 & 0.91095 & -0.16105 & -0.28736 \\ -0.72643 & -1.5406\times10^{-2} & 0.83855 & -4.9301\times10^{-2} \\ -0.73226 & -3.499\times10^{-3} & 2.2666\times10^{-3} & 0.36384 \end{pmatrix} * \begin{pmatrix} -1.3010 \\ 4.7308 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.10191 \\ 5.3559 \\ 0.8722 \\ 0.93612 \end{pmatrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.0996 & -0.1688 & -0.5977 \\ -0.1653 & 1 & -0.1183 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix}$$, inverse:

$$\begin{matrix} 1.5541 & 0.41935 & 1.0739 & 1.9262 \\ 0.69588 & 1.3295 & 1.0283 & 1.9049 \\ 0.62855 & 0.36268 & 1.9129 & 1.9309 \\ 0.63359 & 0.37762 & 1.0852 & 2.3599 \end{matrix} * \begin{matrix} 7.3633 \\ 4.7308 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 13.427 \\ 11.414 \\ 6.3440 \\ 6.4518 \end{matrix}$$

Which gives us the following average values of z;

$$z2 = \frac{8.1047 + 7.9574 + 7.3583 + -0.10191 + 13.427}{5} = 7.3491$$

$$z3 = \frac{6.1499 + 6.1762 + -7.8276 + 5.3559 + 11.414}{5} = 4.2537$$

$$z4 = \frac{1.0084 + -3.3988 + 1.0952 + 0.8722 + 6.3440}{5} = 1.1842$$

$$z5 = \frac{-6.9275 \times 10^{-2} + 0.92457 + 0.98676 + 0.93612 + 6.4518}{5} = 1.8460$$

Appendix C: A short summary of the bilateral real exchange rate changes required

The table below provides the z values for each year in my analysis. They are the results of the estimations in appendix B. Z values should be read as explained in chapter 8.

Table C1: A short summary of the bilateral real exchange rate changes required

	z2	z3	z4	z5
2001	8.2237	5.8863	4.522	3,7767
2002	11.688	9.2171	6.639	5,7478
2003	13.245	13.282	8,5537	7,4328
2004	20.805	13.458	12,418	10,943
2005	31.125	23.801	14,972	14,178
2006	37.168	25.521	15,527	15,392
2007	31.623	24.813	10,98	11,906
2008	29.764	12.867	3,5101	8,6228
2009	8.7434	0.008	-2,3174	0,46939
2010	7.3491	4.2537	1,1842	1,846

Appendix D: Calculation of the trade weights

Each matrix firstly presents the export from and to each country and economic area. The export is measured in millions of U.S. dollars. All matrixes should be read as explained in chapter 7. The export from the EU to the EU and from the ROW to ROW is then removed in the following matrix, so these economic areas don't export to themselves. The matrixes are then used to equate trade weights as presented in chapter 8.

Table D1: Export from and to each country and economic area in question

2000	USA	China	Japan	EU	ROW	Total
USA	0.0	15963.7	64537.6	168199.0	523579.7	772280.0
China	52161.7	0.0	41654.0	41056.2	114351.1	249223.0
Japan	144009.0	30356.2	0.0	80611.1	223565.7	478542.0
EU	220793.0	27864.6	40978.3	1649890.0	513114.1	2452640.0
ROW	766056.3	137591.5	192823.1	515243.7	823930.4	2435645.0
Total	1183020.0	211776.0	339993.0	2455000.0	2198541.0	6388330.0

2000	USA	China	Japan	EU	ROW	Total
USA	0.0	15963.7	64537.6	168199.0	523579.7	772280.0
China	52161.7	0.0	41654.0	41056.2	114351.1	249223.0
Japan	144009.0	30356.2	0.0	80611.1	223565.7	478542.0
EU	220793.0	27864.6	40978.3	0.0	513114.1	802750.0
ROW	766056.3	137591.5	192823.1	515243.7	0.0	1611714.6
Total	1183020.0	211776.0	339993.0	805110.0	1374610.6	3914509.6

$\varphi_{1,2} =$	0.03484141				
$\varphi_{1,3} =$	0.10665709	$\varphi_{2,3} =$	0.15620468	$\varphi_{3,2} =$	0.08797449
$\varphi_{1,4} =$	0.19894236	$\varphi_{2,4} =$	0.14950314	$\varphi_{3,4} =$	0.14854514
$\varphi_{1,5} =$	0.65955915	$\varphi_{2,5} =$	0.54651442	$\varphi_{3,5} =$	0.50870006

$\varphi_{4,2} =$	0.04286493	$\varphi_{5,2} =$	0.08436543
$\varphi_{4,3} =$	0.07562188	$\varphi_{5,3} =$	0.13943183
$\varphi_{4,5} =$	0.4390539	$\varphi_{5,4} =$	0.3443556

2001	USA	China	Japan	EU	ROW	Total
USA	0.0	19234.9	57639.1	163062.0	491186.0	731122.0
China	54395.1	0.0	45078.2	44643.3	122606.4	266723.0
Japan	122701.0	30948.2	0.0	66736.1	183266.7	403652.0
EU	232952.0	33421.1	40668.1	1649890.0	658158.8	2615090.0

ROW	690491.9	137436.8	172829.6	494148.6	630476.1	2125383.0
Total	1100540.0	221041.0	316215.0	2418480.0	2085694.0	6141970.0

2001	USA	China	Japan	EU	ROW	Total
USA	0.0	19234.9	57639.1	163062.0	491186.0	731122.0
China	54395.1	0.0	45078.2	44643.3	122606.4	266723.0
Japan	122701.0	30948.2	0.0	66736.1	183266.7	403652.0
EU	232952.0	33421.1	40668.1	0.0	658158.8	965200.0
ROW	690491.9	137436.8	172829.6	494148.6	0.0	1494906.9
Total	1100540.0	221041.0	316215.0	768590.0	1455217.9	3861603.9

$\varphi_{1,2} =$	0.04019846				
$\varphi_{1,3} =$	0.09845708	$\varphi_{2,3} =$	0.15586718	$\varphi_{3,2} =$	0.10561173
$\varphi_{1,4} =$	0.21620474	$\varphi_{2,4} =$	0.16004543	$\varphi_{3,4} =$	0.14920006
$\varphi_{1,5} =$	0.64513971	$\varphi_{2,5} =$	0.53313324	$\varphi_{3,5} =$	0.49466957

$\varphi_{4,2} =$	0.04502529	$\varphi_{5,2} =$	0.08814651
$\varphi_{4,3} =$	0.06194764	$\varphi_{5,3} =$	0.1207055
$\varphi_{4,5} =$	0.47929011	$\varphi_{5,4} =$	0.39059615

2002	USA	China	Japan	EU	ROW	Total
USA	0.0	22052.7	51439.6	147367.0	472497.7	693357.0
China	70063.8	0.0	48483.0	52946.9	154289.3	325783.0
Japan	120198.0	39957.6	0.0	63812.4	193001.0	416969.0
EU	232952.0	33421.1	40668.1	1762640.0	545408.8	2615090.0
ROW	702716.2	175441.6	164937.3	512113.7	826562.2	2381771.0
Total	1125930.0	270873.0	305528.0	2538880.0	2191759.0	6432970.0

2002	USA	China	Japan	EU	ROW	Total
USA	0.0	22052.7	51439.6	147367.0	472497.7	693357.0
China	70063.8	0.0	48483.0	52946.9	154289.3	325783.0
Japan	120198.0	39957.6	0.0	63812.4	193001.0	416969.0
EU	232952.0	33421.1	40668.1	0.0	545408.8	852450.0
ROW	702716.2	175441.6	164937.3	512113.7	0.0	1555208.8
Total	1125930.0	270873.0	305528.0	776240.0	1365196.8	3843767.8

$\varphi_{1,2} =$	0.0506333				
$\varphi_{1,3} =$	0.09434333	$\varphi_{2,3} =$	0.14822712	$\varphi_{3,2} =$	0.12240964
$\varphi_{1,4} =$	0.20904838	$\varphi_{2,4} =$	0.14475343	$\varphi_{3,4} =$	0.14461029
$\varphi_{1,5} =$	0.64597499	$\varphi_{2,5} =$	0.5526315	$\varphi_{3,5} =$	0.49541839

$\varphi_{4,2} =$	0.05302912	$\varphi_{5,2} =$	0.11290586
$\varphi_{4,3} =$	0.06415002	$\varphi_{5,3} =$	0.12256458
$\varphi_{4,5} =$	0.43614568	$\varphi_{5,4} =$	0.36211494

2003	USA	China	Japan	EU	ROW	Total
USA	0.0	28418.5	52063.7	154963.0	488439.8	723885.0
China	92633.2	0.0	59422.6	79144.8	207285.4	438486.0
Japan	117384.0	57479.8	0.0	75562.8	221636.4	472063.0
EU	258493.0	46671.9	46122.6	2141950.0	640142.5	3133380.0
ROW	747289.8	243667.8	187948.1	605219.4	965940.9	2750066.0
Total	1215800.0	376238.0	345557.0	3056840.0	2523445.0	7517880.0

2003	USA	China	Japan	EU	ROW	Total
USA	0.0	28418.5	52063.7	154963.0	488439.8	723885.0
China	92633.2	0.0	59422.6	79144.8	207285.4	438486.0
Japan	117384.0	57479.8	0.0	75562.8	221636.4	472063.0
EU	258493.0	46671.9	46122.6	0.0	640142.5	991430.0
ROW	747289.8	243667.8	187948.1	605219.4	0.0	1784125.1
Total	1215800.0	376238.0	345557.0	914890.0	1557504.1	4409989.1

$\varphi_{1,2} =$	0.06240792				
$\varphi_{1,3} =$	0.08735836	$\varphi_{2,3} =$	0.14348712	$\varphi_{3,2} =$	0.14297889
$\varphi_{1,4} =$	0.21315626	$\varphi_{2,4} =$	0.15442862	$\varphi_{3,4} =$	0.1488288
$\varphi_{1,5} =$	0.63707746	$\varphi_{2,5} =$	0.55350425	$\varphi_{3,5} =$	0.50094726

$\varphi_{4,2} =$	0.06599978	$\varphi_{5,2} =$	0.1349501
$\varphi_{4,3} =$	0.06383262	$\varphi_{5,3} =$	0.1225703
$\varphi_{4,5} =$	0.43439223	$\varphi_{5,4} =$	0.37268106

2004	USA	China	Japan	EU	ROW	Total
USA	0.0	34721.0	54400.0	173554.0	553955.0	816630.0
China	125155.0	0.0	73514.3	108661.0	286164.7	593495.0
Japan	128606.0	73917.2	0.0	89412.2	274201.6	566137.0
EU	294331.0	60028.0	53808.0	2553640.0	786703.0	3748510.0
ROW	871348.0	322646.8	227884.7	763912.8	1220435.7	3406228.0
Total	1419440.0	491313.0	409607.0	3689180.0	3121460.0	9131000.0

2004	USA	China	Japan	EU	ROW	Total
USA	0.0	34721.0	54400.0	173554.0	553955.0	816630.0
China	125155.0	0.0	73514.3	108661.0	286164.7	593495.0
Japan	128606.0	73917.2	0.0	89412.2	274201.6	566137.0
EU	294331.0	60028.0	53808.0	0.0	786703.0	1194870.0
ROW	871348.0	322646.8	227884.7	763912.8	0.0	2185792.3
Total	1419440.0	491313.0	409607.0	1135540.0	1901024.3	5356924.3

$\varphi_{1,2} =$	0.07149866				
$\varphi_{1,3} =$	0.0818427	$\varphi_{2,3} =$	0.13590562	$\varphi_{3,2} =$	0.1510965
$\varphi_{1,4} =$	0.20924434	$\varphi_{2,4} =$	0.15550125	$\varphi_{3,4} =$	0.14678051
$\varphi_{1,5} =$	0.6374143	$\varphi_{2,5} =$	0.5612159	$\varphi_{3,5} =$	0.51456765

$\varphi_{4,2} =$	0.07238598	$\varphi_{5,2} =$	0.14896962
$\varphi_{4,3} =$	0.06145708	$\varphi_{5,3} =$	0.12285511
$\varphi_{4,5} =$	0.43536875	$\varphi_{5,4} =$	0.37941898

2005	USA	China	Japan	EU	ROW	Total
USA	0.0	41836.7	55409.6	187408.0	619776.7	904431.0
China	163348.0	0.0	84097.2	145664.0	369538.8	762648.0
Japan	136002.0	80004.7	0.0	87037.4	292093.9	595138.0
EU	315077.0	64483.1	54518.9	2733940.0	887861.0	4055880.0
ROW	994523.0	402881.5	271396.3	920810.6	1470191.6	4059803.0
Total	1608950.0	589206.0	465422.0	4074860.0	3639462.0	10377900.0

2005	USA	China	Japan	EU	ROW	Total
USA	0.0	41836.7	55409.6	187408.0	619776.7	904431.0
China	163348.0	0.0	84097.2	145664.0	369538.8	762648.0
Japan	136002.0	80004.7	0.0	87037.4	292093.9	595138.0
EU	315077.0	64483.1	54518.9	0.0	887861.0	1321940.0
ROW	994523.0	402881.5	271396.3	920810.6	0.0	2589611.4
Total	1608950.0	589206.0	465422.0	1340920.0	2169270.4	6173768.4

$\varphi_{1,2} =$	0.08163693				
$\varphi_{1,3} =$	0.07615702	$\varphi_{2,3} =$	0.12139025	$\varphi_{3,2} =$	0.15473137
$\varphi_{1,4} =$	0.19992393	$\varphi_{2,4} =$	0.15545103	$\varphi_{3,4} =$	0.13347317
$\varphi_{1,5} =$	0.64228213	$\varphi_{2,5} =$	0.57137849	$\varphi_{3,5} =$	0.53131383

$\varphi_{4,2} =$	0.07891782	$\varphi_{5,2} =$	0.1623113
$\varphi_{4,3} =$	0.0531595	$\varphi_{5,3} =$	0.11840811
$\varphi_{4,5} =$	0.43534294	$\varphi_{5,4} =$	0.38006231

2006	USA	China	Japan	EU	ROW	Total
USA	0.0	55224.0	59649.3	214990.0	707236.7	1037100.0
China	203898.0	0.0	91772.5	189926.0	484101.5	969698.0
Japan	147230.0	92789.1	0.0	94082.5	313080.4	647182.0
EU	338641.0	80295.5	56186.1	3091790.0	1001137.4	4568050.0
ROW	1100011.0	484487.4	315737.1	1082601.5	1784933.0	4767770.0
Total	1789780.0	712796.0	523345.0	4673390.0	4290489.0	11989800.0

2006	USA	China	Japan	EU	ROW	Total
USA	0.0	55224.0	59649.3	214990.0	707236.7	1037100.0
China	203898.0	0.0	91772.5	189926.0	484101.5	969698.0
Japan	147230.0	92789.1	0.0	94082.5	313080.4	647182.0
EU	338641.0	80295.5	56186.1	0.0	1001137.4	1476260.0
ROW	1100011.0	484487.4	315737.1	1082601.5	0.0	2982837.0
Total	1789780.0	712796.0	523345.0	1581600.0	2505556.0	7113077.0

$\varphi_{1,2} =$	0.0916636				
$\varphi_{1,3} =$	0.07318291	$\varphi_{2,3} =$	0.10969525	$\varphi_{3,2} =$	0.15767394
$\varphi_{1,4} =$	0.19584524	$\varphi_{2,4} =$	0.1606077	$\varphi_{3,4} =$	0.12837688
$\varphi_{1,5} =$	0.63930825	$\varphi_{2,5} =$	0.57568639	$\varphi_{3,5} =$	0.53720888

$\varphi_{4,2} =$	0.08836948	$\varphi_{5,2} =$	0.17647951
$\varphi_{4,3} =$	0.04914175	$\varphi_{5,3} =$	0.11457224
$\varphi_{4,5} =$	0.43065232	$\varphi_{5,4} =$	0.37966284

2007	USA	China	Japan	EU	ROW	Total
USA	0.0	65238.4	62664.9	247788.0	787018.7	1162710.0
China	233181.0	0.0	102116.0	245429.0	637974.0	1218700.0
Japan	145575.0	109297.0	0.0	105716.0	354295.0	714883.0
EU	358636.0	98841.2	59910.1	3637840.0	1197582.7	5352810.0
ROW	1151978.0	580994.4	339486.0	1230377.0	2140961.6	5443797.0
Total	1889370.0	854371.0	564177.0	5467150.0	5117832.0	13892900.0

2007	USA	China	Japan	EU	ROW	Total
USA	0.0	65238.4	62664.9	247788.0	787018.7	1162710.0
China	233181.0	0.0	102116.0	245429.0	637974.0	1218700.0
Japan	145575.0	109297.0	0.0	105716.0	354295.0	714883.0
EU	358636.0	98841.2	59910.1	0.0	1197582.7	1714970.0
ROW	1151978.0	580994.4	339486.0	1230377.0	0.0	3302835.4
Total	1889370.0	854371.0	564177.0	1829310.0	2976870.4	8114098.4

$\varphi_{1,2} =$	0.09777575				
$\varphi_{1,3} =$	0.06822885	$\varphi_{2,3} =$	0.10198059	$\varphi_{3,2} =$	0.16528779
$\varphi_{1,4} =$	0.19869204	$\varphi_{2,4} =$	0.16606773	$\varphi_{3,4} =$	0.12949049
$\varphi_{1,5} =$	0.63530337	$\varphi_{2,5} =$	0.58800128	$\varphi_{3,5} =$	0.54241474

$\varphi_{4,2} =$	0.09713403	$\varphi_{5,2} =$	0.19411234
$\varphi_{4,3} =$	0.04673053	$\varphi_{5,3} =$	0.11047986
$\varphi_{4,5} =$	0.43367587	$\varphi_{5,4} =$	0.3866359

2008	USA	China	Japan	EU	ROW	Total
USA	0.0	71457.0	66579.2	275290.0	886863.8	1300190.0
China	252786.0	0.0	116176.0	293176.0	767202.0	1429340.0
Japan	138932.0	124969.0	0.0	110446.0	408512.0	782859.0
EU	368477.0	115571.0	62210.2	3991090.0	1393891.8	5931240.0
ROW	1275285.0	680290.0	431235.6	1518938.0	2707922.4	6613671.0
Total	2035480.0	992287.0	676201.0	6188940.0	6164392.0	16057300.0

2008	USA	China	Japan	EU	ROW	Total
USA	0.0	71457.0	66579.2	275290.0	886863.8	1300190.0

China	252786.0	0.0	116176.0	293176.0	767202.0	1429340.0
Japan	138932.0	124969.0	0.0	110446.0	408512.0	782859.0
EU	368477.0	115571.0	62210.2	0.0	1393891.8	1940150.0
ROW	1275285.0	680290.0	431235.6	1518938.0	0.0	3905748.6
Total	2035480.0	992287.0	676201.0	2197850.0	3456469.6	9358287.6

$\varphi_{1,2} =$	0.09720476				
$\varphi_{1,3} =$	0.06161017	$\varphi_{2,3} =$	0.09957975	$\varphi_{3,2} =$	0.16527422
$\varphi_{1,4} =$	0.19299481	$\varphi_{2,4} =$	0.16879024	$\varphi_{3,4} =$	0.11833386
$\varphi_{1,5} =$	0.64819026	$\varphi_{2,5} =$	0.59773532	$\varphi_{3,5} =$	0.57554014

$\varphi_{4,2} =$	0.09877888	$\varphi_{5,2} =$	0.19661085
$\varphi_{4,3} =$	0.04172455	$\varphi_{5,3} =$	0.11406176
$\varphi_{4,5} =$	0.4410651	$\varphi_{5,4} =$	0.39564568

2009	USA	China	Japan	EU	ROW	Total
USA	0.0	69576.0	51179.7	221314.0	714980.3	1057050.0
China	221384.0	0.0	98044.9	236511.0	647480.1	1203420.0
Japan	95343.1	109632.0	0.0	72404.6	304199.3	581579.0
EU	286616.0	115394.0	50279.9	3054630.0	1089010.1	4595930.0
ROW	882176.9	614461.0	293221.5	1016690.4	2112771.2	4919321.0
Total	1485520.0	909063.0	492726.0	4601550.0	4868441.0	12357300.0

2009	USA	China	Japan	EU	ROW	Total
USA	0.0	69576.0	51179.7	221314.0	714980.3	1057050.0
China	221384.0	0.0	98044.9	236511.0	647480.1	1203420.0
Japan	95343.1	109632.0	0.0	72404.6	304199.3	581579.0
EU	286616.0	115394.0	50279.9	0.0	1089010.1	1541300.0
ROW	882176.9	614461.0	293221.5	1016690.4	0.0	2806549.8
Total	1485520.0	909063.0	492726.0	1546920.0	2755669.8	7189898.8

$\varphi_{1,2} =$	0.11443539				
$\varphi_{1,3} =$	0.05762783	$\varphi_{2,3} =$	0.09830938	$\varphi_{3,2} =$	0.19331279
$\varphi_{1,4} =$	0.19977031	$\varphi_{2,4} =$	0.16658359	$\varphi_{3,4} =$	0.11419895
$\varphi_{1,5} =$	0.62816646	$\varphi_{2,5} =$	0.59737338	$\varphi_{3,5} =$	0.5560998

$\varphi_{4,2} =$	0.11395075	$\varphi_{5,2} =$	0.22687725
$\varphi_{4,3} =$	0.03972661	$\varphi_{5,3} =$	0.10740691
$\varphi_{4,5} =$	0.44758197	$\varphi_{5,4} =$	0.37857198

Appendix E: The GDPs, CBs and changes needed to restore FEER for 2001-2010

These tables present the GDPs, CBs, CB thresholds, required changes in the CBs, impact parameter and the corresponding change in the real, effective exchange rate. These tables should be read as explained in chapter 8.

Table E1: The GDPs, CBs, CB thresholds, required change in CB, impact parameters and corresponding change in $\check{R}$ from 2001-2010

2001	GDP	CB		CB threshold	Required change in CB		Impact parame-ter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\check{R}$
USA	10286175	-397154	-3.8610 %	±3%	88564	0.8610 %	-0.16	-0.0538125
China	1324814	17405	1.3138 %	±3%	0	0.0000 %	-0.3	0
Japan	4095483	87794	2.1437 %	0%-3%	0	0.0000 %	-0.12	0
Eurozone	6341013	3894	0.0614 %	0%-3%	0	0.0000 %	-0.14	0
ROW	10101117	107460	1.0638 %	No target	0	0.0000 %	0	0
Total	32148602	-180601	-0.5618 %	No target	0	0.0000 %	0	0

2002	GDP	CB		CB threshold	Required change in CB		Impact parame-ter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\check{R}$
USA	10642300	-458066	-4.3042 %	±3%	138797	1.3042 %	-0.16	-0.0815125
China	1453833	35422	2.4365 %	±3%	0	0.0000 %	-0.3	0
Japan	3918334	112607	2.8738 %	0%-3%	0	0.0000 %	-0.12	0
Eurozone	6921837	45864	0.6626 %	0%-3%	0	0.0000 %	-0.14	0
ROW	10307594	127795	1.2398 %	No target	0	0.0000 %	0	0
Total	33243898	-136378	-0.4102 %	No target	0	0.0000 %	0	0

2003	GDP	CB		CB threshold	Required change in CB		Impact parameter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	11142175	-520675	-4.6730 %	±3%	186409	1.6730 %	-0.16	-0.1045625
China	1640961	45875	2.7956 %	±3%	0	0.0000 %	-0.3	0
Japan	4229098	136238	3.2214 %	0%-3%	-9363	-0.2214 %	-0.12	0.01845
Eurozone	8538539	40696	0.4766 %	0%-3%	0	0.0000 %	-0.14	0
ROW	11824998	224825	1.9013 %	No target	0	0.0000 %	0	0
Total	37375771	-73041	-0.1954 %	No target	0	0.0000 %	0	0

2004	GDP	CB		CB threshold	Required change in CB		Impact parameter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	11867750	-630491	-5.3126 %	±3%	274454	2.3126 %	-0.16	-0.1445375
China	1931646	68659	3.5544 %	±3%	-10709	-0.5544 %	-0.3	0.01848
Japan	4605939	172070	3.7358 %	0%-3%	-33890	-0.7358 %	-0.12	0.061316667
Eurozone	9772495	115295	1.1798 %	0%-3%	0	0.0000 %	-0.14	0
ROW	13893268	274219	1.9738 %	No target	0	0.0000 %	0	0
Total	42071098	-248	-0.0006 %	No target	0	0.0000 %	0	0

2005	GDP	CB		CB threshold	Required change in CB		Impact parameter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	12638375	-747590	-5.9152 %	±3%	368434	2.9152 %	-0.16	-0.1822
China	2256919	160818	7.1256 %	±3%	-93111	-4.1256 %	-0.3	0.13752
Japan	4552192	165690	3.6398 %	0%-3%	-29125	-0.6398 %	-0.12	0.053316667
Eurozone	10153900	44618	0.4394 %	0%-3%	0	0.0000 %	-0.14	0
ROW	15913483	411224	2.5841 %	No target	0	0.0000 %	0	0
Total	45514869	34760	0.0764 %	No target	0	0.0000 %	0	0

2006	GDP	CB		CB threshold	Required change in CB		Impact parame-ter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	13398925	-802637	-5.9903 %	±3%	400668	2.9903 %	-0.16	-0.18689375
China	2712917	253268	9.3356 %	±3%	-171880	-6.3356 %	-0.3	0.211186667
Japan	4362577	170437	3.9068 %	0%-3%	-39560	-0.9068 %	-0.12	0.075566667
Eurozone	10749104	45683	0.4250 %	0%-3%	0	0.0000 %	-0.14	0
ROW	18071916	543041	3.0049 %	No target	0	0.0000 %	0	0
Total	49295439	209792	0.4256 %	No target	0	0.0000 %	0	0

2007	GDP	CB		CB threshold	Required change in CB		Impact parame-ter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	14061800	-718094	-5.1067 %	±3%	296240	2.1067 %	-0.16	-0.13166875
China	3494235	371833	10.6413 %	±3%	-267005	-7.6413 %	-0.3	0.25471
Japan	4377961	210967	4.8188 %	0%-3%	-79626	-1.8188 %	-0.12	0.151566667
Eurozone	12362037	46679	0.3776 %	0%-3%	0	0.0000 %	-0.14	0
ROW	21319441	399384	1.8733 %	No target	0	0.0000 %	0	0
Total	55615474	310769	0.5588 %	No target	0	0.0000 %	0	0

2008	GDP	CB		CB threshold	Required change in CB		Impact parame-ter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	14369075	-668856	-4.6548 %	±3%	237779	1.6548 %	-0.16	-0.103425
China	4519950	436107	9.6485 %	±3%	-300509	-6.6485 %	-0.3	0.221616667
Japan	4886952	157079	3.2143 %	0%-3%	-10473	-0.2143 %	-0.12	0.017858333
Eurozone	13615861	-100834	-0.7406 %	0%-3%	100839	0.7406 %	-0.14	-0.0529
ROW	23795325	387243	1.6274 %	No target	0	0.0000 %	0	0
Total	61187163	210739	0.3444 %	No target	0	0.0000 %	0	0

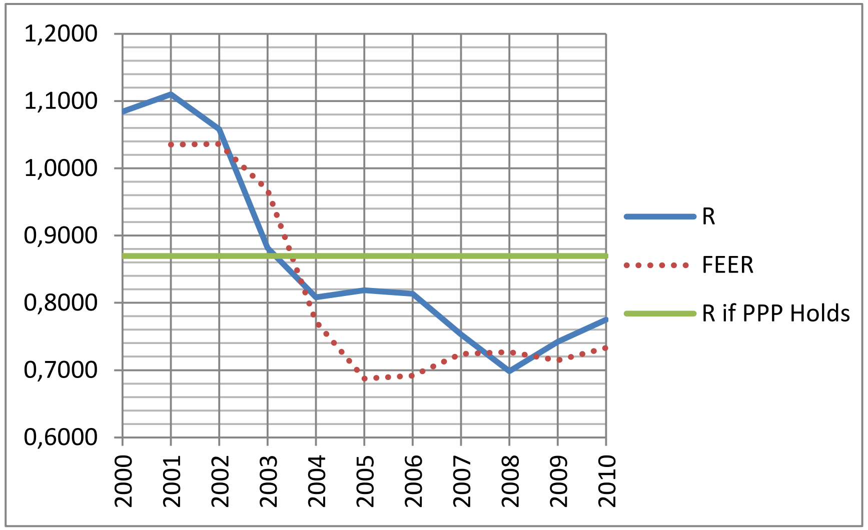
2009	GDP	CB		CB threshold	Required change in CB		Impact parameter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	14119050	-378434	-2.6803 %	±3%	0	0.0000 %	-0.16	0
China	4984731	297100	5.9602 %	±3%	-147558	-2.9602 %	-0.3	0.098673333
Japan	5068894	141751	2.7965 %	0%-3%	0	0.0000 %	-0.12	0
Eurozone	12483643	-50664	-0.4058 %	0%-3%	50659	0.4058 %	-0.14	-0.028985714
ROW	21187058	206826	0.9762 %	No target	0	0.0000 %	0	0
Total	57843376	216579	0.3744 %	No target	0	0.0000 %	0	0

2010	GDP	CB		CB threshold	Required change in CB		Impact parameter	Corresponding change
Country	Millions of U.S. dollars	Millions of U.S. dollars	As percent of GDP	CB/GDP	Millions of U.S. dollars	As percent of GDP	γ	$\breve{R}$
USA	14657800	-470244	-3.2081 %	±3%	30503	0.2081 %	-0.16	-0.01300625
China	5878257	306200	5.2090 %	±3%	-129851	-2.2090 %	-0.3	0.073633333
Japan	5458872	194754	3.5677 %	0%-3%	-30990	-0.5677 %	-0.12	0.047308333
Eurozone	12192829	11589	0.0950 %	0%-3%	0	0.0000 %	-0.14	0
ROW	24721516	240291	0.9720 %	No target	0	0.0000 %	0	0
Total	62909274	282590	0.4492 %	No target	0	0.0000 %	0	0

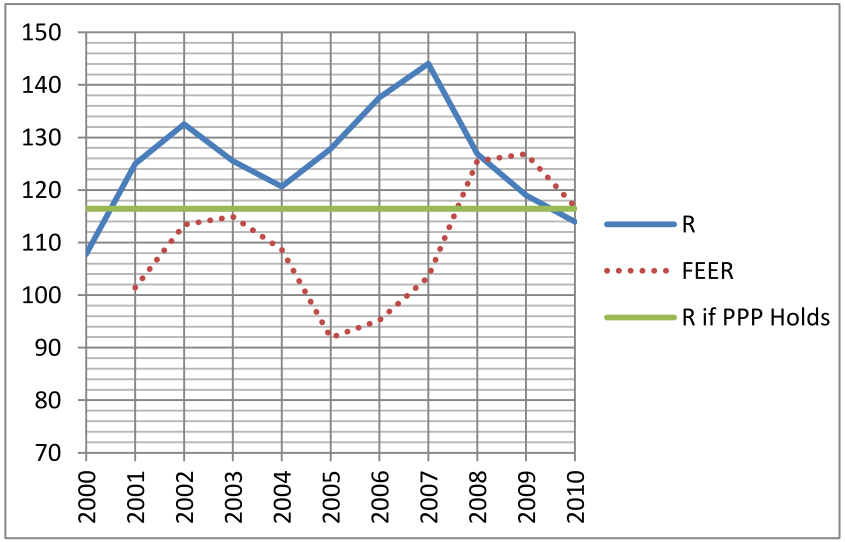
Appendix F: The calculation of real and nominal exchange rates, FEER and rates if PPP holds

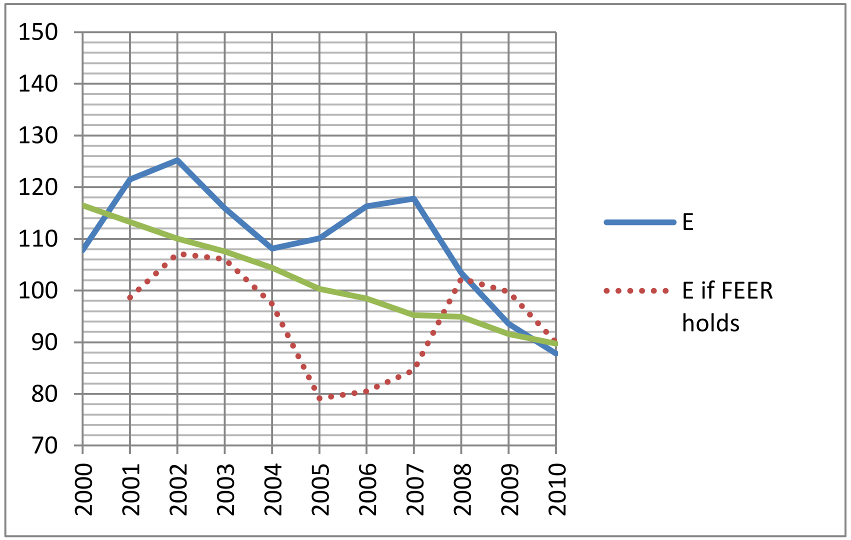
This appendix presents all the necessary information to estimate the figures presented in chapter 11 and 12. Firstly the data will be presented for the EUR/USD relationship, secondly the JPY/USD relationship and then the final CNY/USD relationship is presented. The first column presents the year in question. The second column presents the nominal exchange rate at the end of the year. The third and fourth column presents the inflation, presented as a consumer price index (CPI) for each country. These indexes are calculated by their end of the year inflation rate, implying that both the nominal exchange rate and CPIs are estimated at the end of the year. The resulting exchange rates are therefore also end of the year values. The fifth column presents the real exchange rate, calculated by using its definition, the CPIs and the nominal exchange rate. The sixth column presents the real exchange rate change necessary for the real exchange rate to realign itself within FEER. This column is year specific for the benchmark year. The FEER adjustment needed for e.g. 2001 is therefore calculated with 2000 as the base year, and 2001 as the benchmark year. The seventh column is the resulting FEER, estimated from the base years real exchange rate value and the benchmark year reduction. The eight column, R if PPP holds, is the average real exchange rate value during the last 20 years for the JPY/USD and CNY/USD relationship, and the last 13 years for the USD/EUR relationship. It is equated fully in appendix H. The last two columns are the nominal counterparts to the real exchange rate values. The ninth column is the nominal exchange rate equated using the FEER as the real exchange rate. This is equated using the definition of the nominal exchange rate, the FEER values as the real exchange rate and the CPIs. The tenth and final column presents the nominal exchange rate that would occur if the real exchange rate was equal to its average value during the last 20 years (or 13 if the euro is involved). It is equated using the definition of the nominal exchange rate, the average real exchange rate values as the real exchange rate values, and the CPIs.

EUR/USD	E	CPI EUR	CPI USA	R	FEER adjustment required	FEER	R if PPP Holds	E if FEER holds	E if PPP holds
2000	1.0843	100.00	100.00	1.0843	0.00 %		0.8699		0.869894346
2001	1.1163	102.14	101.55	1.1099	4.52 %	1.035268	0.8699	1.041272492	0.87493972
2002	1.0606	104.46	104.21	1.0581	6.64 %	1.036179	0.8699	1.038664641	0.871981077
2003	0.8851	106.60	106.20	0.8818	8.55 %	0.967558	0.8699	0.971211854	0.873178982
2004	0.8048	109.10	109.61	0.8085	12.42 %	0.772272	0.8699	0.768736454	0.865911586
2005	0.8043	111.61	113.64	0.8190	14.97 %	0.687453	0.8699	0.675151177	0.854328015
2006	0.7968	113.74	116.13	0.8136	15.53 %	0.691796	0.8699	0.677528185	0.851953767
2007	0.7306	117.24	120.88	0.7533	10.98 %	0.724248	0.8699	0.702428547	0.843686821
2008	0.6832	119.11	121.73	0.6982	3.51 %	0.726853	0.8699	0.71124864	0.851218767
2009	0.7190	120.22	124.07	0.7420	-2.32 %	0.714369	0.8699	0.692201407	0.842900505
2010	0.7546	122.88	126.16	0.7748	1.18 %	0.733239	0.8699	0.714132701	0.847227322

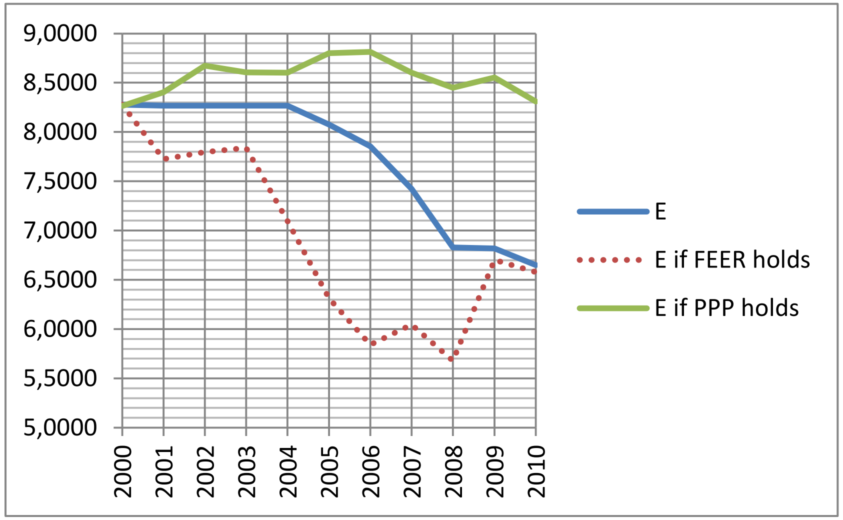


JPY/USD	E	CPI JPY	CPI USA	R	FEER adjustment required	FEER	R if PPP Holds	E if FEER holds	E if PPP holds
2000	107.78	100	100	107.78	0.00 %		116.47282		116.4728207
2001	121.48	98.732	101.552	124.9497	5.89 %	101.4357	116.47282	98.61897412	113.238484
2002	125.18	98.439753	104.2086	132.5159	9.22 %	113.433	116.47282	107.1534936	110.025043
2003	115.92	98.049932	106.1979	125.553	13.28 %	114.9151	116.47282	106.0983036	107.5364725
2004	108.13	98.245051	109.6058	120.6339	13.46 %	108.6561	116.47282	97.39375653	104.4002656
2005	110.09	97.855018	113.6426	127.8515	23.80 %	91.92179	116.47282	79.15171687	100.2920407
2006	116.29	98.147605	116.1337	137.6007	25.52 %	95.22256	116.47282	80.47507922	98.43423545
2007	117.77	98.830712	120.8789	144.0433	24.81 %	103.4579	116.47282	84.58727364	95.22830902
2008	103.42	99.221094	121.7262	126.8775	12.87 %	125.5093	116.47282	102.3047134	94.93893888
2009	93.58	97.562117	124.0658	119.0019	0.01 %	126.8674	116.47282	99.76519528	91.59117739
2010	87.78	97.171868	126.1625	113.9687	1.84 %	116.8099	116.47282	89.96836599	89.70873506





CNY/USD	E	CPI CNY	CPI USA	R	FEER adjustment required	FEER	R if PPP Holds	E if FEER holds	E if PPP holds
2000	8.2777	100	100	8.2784	0.00 %	8.2777	8.2641	8.2777	8.264065032
2001	8.2670	99.879	101.552	8.4129	8.22 %	7.597609	8.2641	7.724871208	8.402490334
2002	8.2671	99.29171	104.2086	8.6763	11.69 %	7.429597	8.2641	7.797508285	8.673298476
2003	8.2669	101.9924	106.1979	8.6080	13.25 %	7.527101	8.2641	7.837469428	8.604820623
2004	8.2663	105.2837	109.6058	8.6058	20.81 %	6.81709	8.2641	7.0969445	8.603320175
2005	8.0746	106.723	113.6426	8.7144	31.13 %	5.927211	8.2641	6.311516305	8.799886015
2006	7.8538	108.8926	116.1337	8.4942	37.17 %	5.475442	8.2641	5.839541852	8.813599048
2007	7.4234	116.1155	120.8789	7.9089	31.62 %	5.808093	8.2641	6.046357613	8.603080383
2008	6.8279	119.0567	121.7262	7.0959	29.76 %	5.554866	8.2641	5.679419626	8.449365548
2009	6.8184	119.8508	124.0658	7.0611	8.74 %	6.475494	8.2641	6.703229449	8.554702488
2010	6.6484	125.4838	126.1625	6.7971	7.35 %	6.542167	8.2641	6.577553059	8.308765005



Appendix G: Equating the size of the inconsistency problem

To estimate the size of the inconsistency problems, I will firstly present the results for 2008 from appendix C in table G1. I also need trade weights for 2007 from appendix D, presented in Table G2. I will then proceed to show how the results lead to imperfect current account adjustments.

Table G1: The bilateral real exchange rate changes required to reach FEER levels in 2008

Year	z_2	z_3	z_4	z_5
2008	29.764	12.867	3.5101	8.6228

Table G2: The trade weights for 2007

2007	USA	China	Japan	EU	ROW	Total
USA	0.0	65238.4	62664.9	247788.0	787018.7	1162710.0
China	233181.0	0.0	102116.0	245429.0	637974.0	1218700.0
Japan	145575.0	109297.0	0.0	105716.0	354295.0	714883.0
EU	358636.0	98841.2	59910.1	3637840.0	1197582.7	5352810.0
ROW	1151978.0	580994.4	339486.0	1230377.0	2140961.6	5443797.0
Total	1889370.0	854371.0	564177.0	5467150.0	5117832.0	13892900.0

2007	USA	China	Japan	EU	ROW	Total
USA	0.0	65238.4	62664.9	247788.0	787018.7	1162710.0
China	233181.0	0.0	102116.0	245429.0	637974.0	1218700.0
Japan	145575.0	109297.0	0.0	105716.0	354295.0	714883.0
EU	358636.0	98841.2	59910.1	0.0	1197582.7	1714970.0
ROW	1151978.0	580994.4	339486.0	1230377.0	0.0	3302835.4
Total	1889370.0	854371.0	564177.0	1829310.0	2976870.4	8114098.4

$\varphi_{1,2} =$	0.09777575				
$\varphi_{1,3} =$	0.06822885	$\varphi_{2,3} =$	0.10198059	$\varphi_{3,2} =$	0.16528779
$\varphi_{1,4} =$	0.19869204	$\varphi_{2,4} =$	0.16606773	$\varphi_{3,4} =$	0.12949049
$\varphi_{1,5} =$	0.63530337	$\varphi_{2,5} =$	0.58800128	$\varphi_{3,5} =$	0.54241474

$\varphi_{4,2} =$	0.09713403	$\varphi_{5,2} =$	0.19411234
$\varphi_{4,3} =$	0.04673053	$\varphi_{5,3} =$	0.11047986
$\varphi_{4,5} =$	0.43367587	$\varphi_{5,4} =$	0.3866359

Using the values found in table G1 and G2, one can equate the real, effective exchange rate changes. Using equations (34)-(36), but expanding the equations to include 5 countries, one can calculate the resulting CB adjustment for country 1, USA:

$$\widetilde{R_1} = \frac{\widehat{C_1}}{\gamma_1} = -\varphi_{1,2}z_2 - \varphi_{1,3}z_3 - \varphi_{1,4}z_4 - \varphi_{1,5}z_5$$

$$\widetilde{R_1} = 0.0977 * 29.764 - 0.0682 * 12.867 - 0.1987 * 3.5101 - 0.6353 * 8.6228$$

$$\widetilde{R_1} = 2.9079 - 0.8775 - 0.6975 - 5.4781 = 9.9610$$

$$\widehat{C_1} = \widetilde{R_1} * \gamma_1 = -9.9610 * -0.16 = 1.5938$$

For country 2, China, the results are:

$$\widetilde{R_2} = \frac{\widehat{C_2}}{\gamma_2} = z_2 - \varphi_{2,3}z_3 - \varphi_{2,4}z_4 - \varphi_{2,5}z_5$$

$$\widetilde{R_2} = 29.764 - 0.1019 * 12.867 - 0.1661 * 3.5101 - 0.5880 * 8.6228$$

$$\widetilde{R_2} = 29.764 - 1.3111 - 0.5830 - 5.0702 = 22.7997$$

$$\widehat{C_2} = \widetilde{R_2} * \gamma_2 = 22.7997 * -0.3 = -6.8399$$

For country 3, Japan, the results are:

$$\widetilde{R_3} = \frac{\widehat{C_3}}{\gamma_3} = z_3 - \varphi_{3,2}z_2 - \varphi_{3,4}z_4 - \varphi_{3,5}z_5$$

$$\widetilde{R_3} = 12.867 - 0.1653 * 29.764 - 0.1295 * 3.5101 - 0.5424 * 8.6228$$

$$\widetilde{R_3} = 12.867 - 4.9199 - 0.4546 - 4.6770 = 2.8155$$

$$\widehat{C_3} = \widetilde{R_3} * \gamma_3 = 2.8155 * -0.12 = -0.3378$$

For the final country, the Eurozone, the results are:

$$\widetilde{R_4} = \frac{\widehat{C_4}}{\gamma_4} = z_4 - \varphi_{4,2} z_2 - \varphi_{4,3} z_3 - \varphi_{4,5} z_5$$

$$\widetilde{R_4} = 3.5101 - 0.0971 * 29.764 - 0.0467 * 12.867 - 0.4337 * 8.6228$$

$$\widetilde{R_4} = 3.5101 - 2.8901 - 0.6009 - 3.7397 = -3.7206$$

$$\widehat{C_4} = \widetilde{R_4} * \gamma_4 = -3.7206 * -0.14 = 0.5209$$

As the fifth country is solely a residual country, it will not be equated. Comparing these results to the required CB changes, we get table 4, presented in chapter 13:

Table 4: The CB/GDP ratio a country would end up with if the estimated real bilateral exchange rate changes from my results were effectuated

2008	USA	China	Japan	Eurozone
CB/GDP ratio realignment required by the original thresholds	1.6548%	-6.6485%	-0.2143%	0.7406%
CB/GDP ratio realignment result from the imperfect solution	1.5938%	-6.8399%	-0.3378%	0.5209%
CB/GDP ratio difference between the original thresholds and the imperfect results	0.0610%	0.1914%	0.1235%	0.2197%
CB/GDP ratio result from the imperfect solution	-3.0610%	3.1914%	3.1235%	-0.2197%

Appendix H: The average real exchange rate value used for PPP-estimation

Year	CPI USA	CPI JPY	CPI CNY	CPI EUR	E JPY/USD	E CNY/USD	E EUR/USD	R JPY/USD	R CNY/USD	R EUR/USD
1990	75.00	89.89	48.77		144.7200	5.2352		120.7439964	8,0505	
1991	79.32	93.20	50.87		134.4800	5.4478		114.4590722	8,4953	
1992	81.65	95.72	53.16	84.55	126.6100	5.7662		107.989449	8,8565	
1993	84.22	96.79	55.45	87.24	111.1000	5.8145		96.66383269	8,8315	
1994	86.45	97.86	68.71	90.01	102.1700	8.4662		90.25699787	10,6524	
1995	88.80	98.45	86.23	92.31	94.0000	8.3374		84.79458099	8,5863	
1996	91.21	98.06	94.94	94.51	108.7500	8.3284		101.1532772	8,0011	
1997	94.02	98.64	101.58	95.90	120.9700	8.3100		115.3081732	7,6914	
1998	95.61	100.49	101.99	97.43	130.8100	8.2789	0.8535	124.4599689	7,7610	0,8375
1999	97.14	101.07	100.92	98.29	113.7400	8.2761	0.9387	109.3236099	7,9666	0,9278
2000	100.00	100.00	100.00	100.00	107.7800	8.2784	1.0843	107.78	8,2784	1,0843
2001	101.55	98.73	99.88	102.14	121.4800	8.2743	1.1163	124.9497322	8,4129	1,1099
2002	104.21	98.44	99.29	104.46	125.1800	8.2669	1.0606	132.5159009	8,6763	1,0581
2003	106.20	98.05	101.99	106.60	115.9200	8.2671	0.8851	125.5530245	8,6080	0,8818
2004	109.61	98.25	105.28	109.10	108.1300	8.2664	0.8048	120.6338511	8,6058	0,8085
2005	113.64	97.86	106.72	111.61	110.0900	8.1838	0.8043	127.8515497	8,7144	0,8190
2006	116.13	98.15	108.89	113.74	116.2900	7.9646	0.7968	137.6007468	8,4942	0,8136
2007	120.88	98.83	116.12	117.24	117.7700	7.5972	0.7306	144.0433441	7,9089	0,7533
2008	121.73	99.22	119.06	119.11	103.4200	6.9403	0.6832	126.877541	7,0959	0,6982
2009	124.07	97.56	119.85	120.22	93.5800	6.8212	0.719	119.0019265	7,0611	0,7420
2010	126.16	97.17	125.48	122.88	87.7800	6.7605	0.7546	113.9686586	6,7971	0,7748
					Total years measured			21	21	13
					Average R value of the period in question:			116.4728207	8.2641	0.8699

(source: IMF World Economic Outlook April 2011, x-rates.com)

Appendix I: The calculation of the simple example

Starting from equation (34), (35) and (36):

$$\widehat{R_1} = \frac{\widehat{C_1}}{\gamma_1} = -\varphi_{1,2}z_2 - \varphi_{1,3}z_3 \lll=\ggg -10.30 = -0.33z_2 - 0.67z_3 \quad (38)$$

$$\widehat{R_2} = \frac{\widehat{C_2}}{\gamma_2} = z_2 - \varphi_{2,3}z_3 \lll=\ggg 22.16 = z_2 - 0.54z_3 \quad (39)$$

$$\widehat{R_3} = \frac{\widehat{C_3}}{\gamma_3} = z_3 - \varphi_{3,2}z_2 \lll=\ggg -5.29 = z_3 - 0.36z_2 \quad (40)$$

From (39) one can then express z_2 and z_3:

$$z_2 = 0.54z_3 + 22.16 \quad (39a)$$

$$z_3 = \frac{z_2 - 22.16}{0.54} \quad (39b)$$

From (40) one can express z_2 and z_3 again:

$$z_2 = \frac{z_3 + 5.29}{0.36} \quad (40a)$$

$$z_3 = -5.29 + 0.36z_2 \quad (40b)$$

Now we have as anticipated a set of three equations and two unknowns. Equation (39a), (39b), (40a) and (40b) comes directly from equations (39) and (40), but are rearranged to express z_2 and z_3. Since this set does have certain consistency problems, differing results occur depending on which equations used. To deal with this problem, I solve for all three possible combinations ((38) and (39), (38) and (40) and (39) and (40)) and let the end result be the average of the results found.

When inserting equation (39a) into equation (38) one gets:

$$-10.30 = -0.33z_2 - 0.67z_3$$
$$-10.30 = -0.33(0.54z_3 + 22.16) - 0.67z_3$$

$$-10.30 = -0.1782z_3 - 7.3128 - 0.67z_3$$

$$-10.30 + 7.3084 = -0.8482z_3$$

$$z_3 = \frac{-10.30 + 7.3128}{-0.8482} = 3{,}8354$$

When inserting equation (39b) into equation (38) one gets:

$$-10.30 = -0.33z_2 - 0.67z_3$$
$$-10.30 = -0.33z_2 - 0.67\left(\frac{z_2 - 22.16}{0.54}\right)$$

$$-10.30 * 0.54 = -0.33z_2 * 0.54 - z_2 + 22.16$$

$$-5.562 - 22.16 = -0.1782z_2 - z_2$$

$$z_2 = \frac{-22.16 - 5.562}{-1.1782} = 23.5291$$

This first set of equations gives us the first values of z_2 and z_3. This exercise is then repeated with different equations. Firstly, by inserting equation (40a) into equation (38):

$$-10.30 = -0.33z_2 - 0.67z_3$$
$$-10.30 = -0.33\left(\frac{z_3 + 5.29}{0.36}\right) - 0.67z_3$$

$$-10.30 * 0.36 = -0.33z_3 - 5.29 - 0.67z_3 * 0.36$$

$$-3.708 + 5.29 = -0.33z_3 - 0.2412z_3$$

$$z_3 = \frac{-3.708 + 5.29}{-0.5712} = -2.7696$$

When inserting (40b) into equation (38) one gets:

$$-10.30 = -0.33z_2 - 0.67z_3$$
$$-10.30 = -0.33z_2 - 0.67(-5.29 + 0.36z_2)$$
$$-10.30 = -0.33z_2 - 3.5443 - 0.2412z_2$$

$$-10.30 + 3.5443 = -0.5712z_2$$

$$z_2 = \frac{-6.7557}{-0.5712} = 11.8272$$

Which is the second set of solutions. Progressing to the final set of solutions, one solves for z_3 by inserting (40a) into equation (39):

$$22.16 = z_2 - 0.54z_3$$

$$22.16 = \frac{z_3 + 5.29}{0.36} - 0.54z_3$$

$$22.16 * 0.36 = z_3 + 5.29 - 0.54z_3 * 0.36$$

$$7.9776 = z_3 + 5.29 - 0.1944z_3$$

$$z_3 = \frac{7.9776 - 5.29}{0.8056} = 3.3361$$

And solves for z_2 by inserting (40b) into equation (39):

$$22.16 = z_2 - 0.54z_3$$
$$22.16 = z_2 - 0.54(-5.12 + 0.49z_2)$$

$$22.16 = z_2 - 2.7648 - 0.2646z_2$$

$$22.16 + 2.7648 = 0.7354z_2$$

$$z_2 = \frac{22.16 + 2.7648}{0.7354} = 33.8928$$

Solving for all three possible solutions gives us three z values:

$$z_2 = 23.5291, 11.8272 \text{ and } 33.8928$$

$$z_3 = 3.8354, -2.7696 \text{ and } 3.3361$$

Which provides an average of:

$$z_2 = \frac{23.5291 + 11.8272 + 33.8928}{3} = 23.0830$$

$$z_3 = \frac{3.8354 - 2.7696 + \ 3.3361}{3} = 1.4673$$

Appendix J: The FEER estimation for 2009, using predicted data.

This appendix should be read as appendix B. The prediction for 2009, using the data from Williamson and Cline (2008), is based on the following data;

$$(1): -7.289 = -z2 * 0.0972 - z3 * 0.0616 - z4 * 0.1929 - z5 * 0.6482$$
$$(2): 23.3153 = z2 - z3 * 0.0996 - z4 * 0.1688 - z5 * 0.5978$$
$$(3): 7.9058 = z3 - z2 * 0.1653 - z4 * 0.0118 - z5 * 0.5755$$
$$(4): -6.187 = z4 - z2 * 0.0988 - z3 * 0.0417 - z5 * 0.7039$$
$$(5): 0 = z5 - z2 * 0.1966 - z3 * 0.1141 - z4 * 0.3956$$

Using 1,2,3 and 4;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.86946 & 0.92053 & 3.7631 \times 10^{-2} & -1.1888 \times 10^{-2} \\ -0.77435 & 6.9950 \times 10^{-2} & 0.95400 & -0.12631 \\ -0.87687 & -6.5963 \times 10^{-3} & -2.0082 \times 10^{-2} & 0.8295 \\ -1.0778 & -0.14272 & -9.0328 \times 10^{-2} & -0.23307 \end{matrix} * \begin{matrix} -7.289 \\ 23.3153 \\ 7.9058 \\ -6.187 \end{matrix} =$$

$$\begin{matrix} 28.171 \\ 15.599 \\ 0.94683 \\ 5.2564 \end{matrix}$$

Using 1,2,3 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.85916 & 0.92772 & 4.1968 \times 10^{-2} & 2.1835 \times 10^{-2} \\ -0.66494 & 0.14629 & 1.0001 & 0.23198 \\ -1.5954 & -0.50794 & -0.32270 & -1.5235 \\ -0.87593 & -1.8586 \times 10^{-3} & -5.3000 \times 10^{-3} & 0.42806 \end{matrix} * \begin{matrix} -7.289 \\ 23.3153 \\ 7.9058 \\ 0 \end{matrix} =$$

$$\begin{matrix} 28.224 \\ 16.164 \\ -2.7651 \\ 6.2994 \end{matrix}$$

Using 1,2,4 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -0.95881 & 0.85819 & -0.11504 & -0.18945 \\ -3.0396 & -1.5105 & -2.7413 & -4.8028 \\ -0.82919 & 2.6673\times10^{-2} & 0.88455 & 0.1011 \\ -0.86334 & 6.9218\times10^{-3} & 1.4528\times10^{-2} & 0.45475 \end{matrix} * \begin{matrix} -7.289 \\ 23.3153 \\ -6.187 \\ 0 \end{matrix} : \begin{matrix} 27.709 \\ 3.8983 \\ 1.1931 \\ 6.3644 \end{matrix}$$

Using 1,3,4 and 5;

$$\begin{matrix} -0.0972 & -0.0616 & -0.1929 & -0.6482 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} -2.1888 & -0.51802 & -1.5350 & -2.7974 \\ -0.87461 & 0.91178 & -0.24204 & -0.21257 \\ -0.86741 & -0.0161 & 0.84042 & 2.0045\times10^{-2} \\ -0.87326 & -4.1781\times10^{-3} & 3.0751\times10^{-3} & 0.43371 \end{matrix} * \begin{matrix} -7.289 \\ 7.9058 \\ -6.187 \\ 0 \end{matrix} : \begin{matrix} 21.356 \\ 15.081 \\ 0.99559 \\ 6.3131 \end{matrix}$$

Using 2,3,4 and 5;

$$\begin{matrix} 1 & -0.0996 & -0.1688 & -0.5978 \\ -0.1653 & 1 & -0.0118 & -0.5755 \\ -0.0988 & -0.0417 & 1 & -0.7039 \\ -0.1966 & -0.1141 & -0.3956 & 1 \end{matrix} \text{, inverse:}$$

$$\begin{matrix} 1.5272 & 0.40381 & 0.99184 & 1.8435 \\ 0.61023 & 1.2801 & 0.76762 & 1.6418 \\ 0.60521 & 0.34922 & 1.8418 & 1.8592 \\ 0.60929 & 0.3636 & 1.0112 & 2.2853 \end{matrix} * \begin{matrix} 23.3153 \\ 7.9058 \\ -6.187 \\ 0 \end{matrix} = \begin{matrix} 32.663 \\ 19.599 \\ 5.4763 \\ 10.824 \end{matrix}$$

Which give us the average z values;

$$z2=\frac{28.171+28.224+27.709+21.356+32.663}{5}=27.625$$

$$z3=\frac{15.599+16.164+3.8983+15.081+19.599}{5}:14.068$$

$$z4=\frac{0.94683-2.765+1.1931+0.99559+5.4763}{5}=1.1694$$

$$z5=\frac{5.2564+6.2994+6.3644+6.3131+10.824}{5}=7.0115$$

References

Akram, Q.F. (2003) *Reel likevektsvalutakurs for Norge.* Oslo: Occasional Papers nr. 32. URL: <https://docs.google.com/viewer?url=http%3A%2F%2Fwww.norges-bank.no%2FUpload%2Fimport%2Fpublikasjoner%2Fskriftserie%2Fskriftserie-31%2Fkap4.pdf> [Read 4th of May 2011].

Akram, Q.F., Brunvatne, K.M. & Lokshall, R. (2003) *Real Equilibrium Exchange Rates.* Oslo: Occasional Papers nr. 32. URL: <http://www.norges-bank.no/Upload/import/publikasjoner/skriftserie/skriftserie-32/akram.pdf> [Read 4th of May 2011].

Bénassy-Quéré, A., Lahrèche-Révil, A. & Mignon, V. (2004). *World consistent equilibrium exchange rates.* CEPII, Working Paper No 2006-20. URL: <http://www.iie.com/publications/pb/pb07-4/benassy.pdf> [Read 10th of October 2011].

Cassel, G. (1918). *Abnormal Deviations in International Exchanges*. The Economic Journal, Vol.28. URL: <http://www.jstor.org/stable/2223329> [Read 4th of April, 2011]s

Cline, W. R, (2008). *Estimating Consistent Fundamental Equilibrium Exchange Rates.* Working paper. Washington: Peterson Institute for International Economics. URL: <http://www.www.petersoninstitute.org/publications/wp/wp08-6.pdf> [Read 20th of February 2011].

Cline, W. R, (2005). *The United States as a Debtor Nation*, Washington: Institute for International Economics. URL:

<http://www.piie.com/publications/chapters_preview/3993/05iie3993.pdf> [Read 25th of March 2011].

Cline, W. R. (2007). *Estimating Reference Exchange Rates*. Washington: Paper prepared for the Workshop on Policies to Reduce the Global Imbalances, sponsored by Bruegel, the Korea Institute for International Economic Policy, and the Peterson Institute for International Economics, February 8–9. URL: < http://www.iie.com/publications/pb/pb07-4/cline.pdf> [Read 25th of April 2011].

Cline, W. R., & Williamson, J. (2010). *Notes on Equilibrium Exchange Rates January 2010*. Washington: Institute for International Economics, URL: <http://www.iie.com/publications/pb/pb10-02.pdf> [Read 25th of April 2011].

Cline, William R., & Williamson, John (2008). *New Estimates of Fundamental Euilibrium Exchange Rates*. Washington: Peterson Institute for International Economics. URL: <http://www.bookstore.iie.com/publications/pb/pb08-7.pdf> [Read 25th of March 2011].

Coudert ,V. & Couharde, C. (2005). *Real Equilibrium Exchange Rate in China*. CEPII, Working Paper No 2005 – 01. URL: <http://www.cepii.fr/anglaisgraph/workpap/pdf/2005/wp05-01.pdf> [Read 20th of October 2011].

Driver R.L., Power J. og Ramsay G. (2001). *Fundamental equilibrium exchange rates: an empirical investigation*. Working Paper, Bank of England.

Frankel, J. (2005). *On the Renminbi: The choice between adjustment under a fixed exchange rate and adjustment under a flexible rate.* NBER Working Paper Series. Massachusetts: National Bureau of Economic Research. URL: <https://docs.google.com/viewer?url=http%3A%2F%2Fmirror.nber.org%2Fcgi-bin%2FsendWP.cgi%2F69167606%2Fw11274.pdf> [Read 22nd of October 2011].

Freund, C. L. (2000). *Current account adjustments in industrialized countries*. International Finance Discussion Papers, number 692. URL: <http://www.federalreserve.gov/pubs/ifdp/2000/692/ifdp692.pdf> [Read 22nd of October 2011].

Funke, M. & Rahn, J (2004). *Just How Undervalued is the Chinese Renminbi?* URL: <http://www.hkimr.org/cms/upload/seminar_app/sem_paper_0_140_Funke-paper050302.pdf> [Read 24th of October 2011].

Goldstein, M. (2003). *China's Exchange Rate Regime*. Testimony before the Subcommittee on Domestic and International Monetary Policy, Trade, and Technology Committee on Financial Services. URL: <http://www.iie.com/publications/testimony/testimony.cfm?ResearchID=266> [Read 25th of October 2011].

International Monetary Fund (2009). Direction of Trade Statistics. URL: <http://www.imf.org/external/pubs/ft/weo/2010/02/weodata/index.aspx> [Read 5th of February 2011].

International Monetary Fund (2010). World Economic Outlook October 2010. URL: <http://www.imf.org/external/pubs/ft/weo/2010/02/weodata/index.aspx> [Read 20th of February 2010].

International Monetary Fund (2011). IMF Data Mapper. URL: <http://www.imf.org/external/datamapper/index.php> [Read 11th of May 2011].

Krugman, P. (2011). *The Renminbi Runaround*. Article published for the N.Y. Times. URL: <http://www.nytimes.com/2010/06/25/opinion/25krugman.html> [Read 5th of May 2011].

Krugman, P., & Obstfeld, M. (2009). *International Economics. Theory & Policy*. Boston: Pearson Addison Wesley.

Mussa, M. (2005). *Sustaining Global Growth While Reducing External Imbalances*. Washington: Institute for International Economics. URL: <http://www.piie.com/publications/chapters_preview/3802/6iie3802.pdf> [Read 28th of October 2011].

Obstfeld, M., & Rogoff, K. (1999). *Foundations of International Macroeconomics.* Massachusetts: The MIT Press.

Reinhart, C. M,, Rogoff, K. S. & Savastano, M. A. (2003). *Debt Intolerance*. Cambridge: National Bureau of Economic Research. URL: <http://www.nber.org/papers/w9908.pdf> [Read 1st of April 2011].

Wang, T. (2004). *Exchange Rate Dynamics*. Washington D.C: International monetary fund. URL:<https://docs.google.com/viewer?url=http%3A%2F%2Fwww.imf.org%2Fexternal%2Fpubs%2Fft%2Fop%2F232%2Fop232.pdf> [Read 20th of October 2011].

Williamson, J. (1994). *Estimating Equilibrium Exchange Rates*. Washington: Institute for International Economics.

World Bank (2010). *Gross domestic product of 2011*. URL: <http://siteresources.worldbank.org/DATASTATISTICS/Resources/GDP.pdf> [Read 5th of February 2011]