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A diagnosis of warm-core and cold-core extratropical cyclone development using the Zwack–Okossi equation

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Abstract

In this study, the development of a warm-core and cold-core extratropical cyclone over North Atlantic is examined. The geostrophic relative vorticity tendency used to diagnose the development is calculated utilizing the so-called extended form of the Zwack–Okossi development equation. In both cases, the cyclonic vorticity advection acted to develop the system, but warm-air advection (diabatic heating) made the largest contribution to explosive development in the cold-core (warm-core) case. Further, a vertical cross section of the temperature advection in the warm-core case reveals that the largest values of this contributor are located far and ahead of the cyclone center. Copyright © 2009 Royal Meteorological Society

Keywords: **extratropical cyclone; warm-core and cold-core cyclones; Zwack-Okossi equation; geostrophic vorticity tendency**

1. Introduction

The development of extratropical cyclones has been studied in many papers during past decades (Newton and Holopainen, 1990; Bosart, 1999). In spite of different analyses and classifications of the dynamics of cyclones in mid-latitude, less attention has been paid to the comparison of the dynamics of cold-core and warm-core cyclones. Shapiro and Keyser (1990) present a conceptual model to describe the process by which marine extratropical frontal cyclones evolve toward a warm-core structure. The evolution of a tropical cyclone into an extratropical cyclone is a common occurrence in the North Atlantic in which a warm-core ex-tropical structure is maintained during the transition and development at midlatitudes (Bosart and Lackmann, 1995; Atallah and Bosart, 2003; Evans and Hart, 2003). This study aims to examine how different synoptic-scale forcing terms contributed to the development of a warm-core and cold-core extratropical cyclone, which intensified over the North Atlantic Ocean. The warm-core case is an ex-tropical cyclone that crossed the British Isles with a structure broadly resembling the final mature state of Shapiro and Keyser's (1990) life-cycle model (Browning *et al.*, 1998) and the cold core is a typical midlatitude cyclone.

To investigate the dynamics of cyclones with different thermal-vertical structures, one needs to employ a suitable diagnostic tool that permits the explicit consideration of all dynamic and thermal forcing mechanisms at all atmospheric levels including troposphere and lower stratosphere. In this study, the Zwack–Okossi equation, originally developed by Zwack and Okossi (1986) is used to include all forcing mechanisms in the troposphere and the lower stratosphere. There is a large body of literature published earlier that used the Zwack–Okossi equation to study the dynamics of extratropical cyclones (Lupo *et al.*, 1992; Rolfson and Smith, 1996; Lupo 2002; Parsons and Smith, 2004). The present study is structured as follows. In Section 2, the data and the methodology used to calculate the diagnostic equation and the criterion adopted to select the two mentioned cases are described. Section 3 describes the synoptic situation of both cases especially at the mature stage when the strong warm-core and cold-core structure is present. The examination of the diagnostic equation in both cyclones and contribution of all synoptic-scale forcing terms is presented in Section 4. Finally, Section 5 contains a summary of results for both warm-core and cold-core cases.

2. Data and methodology

The data used in this investigation is the National Centers for Environmental Prediction-National center for Atmospheric Research (NCEP/NCAR) reanalysis with a horizontal resolution of 2.5°latitude × 2.5°longitude on mandatory 14 levels from 1000 to 100 hPa, including standard atmospheric variables at the surface like sea level pressure and at pressure levels such as geopotential height, temperature, vertical motion and u and v wind components (Kalnay *et al.*, 1996). The diagnostic tool to investigate the role of dynamic and thermodynamic forcing terms during the development of cyclones is an extended form of the Zwack–Okossi equation suggested by Lupo *et al.* (1992) and applicable for synoptic-scale motions. This equation explicitly

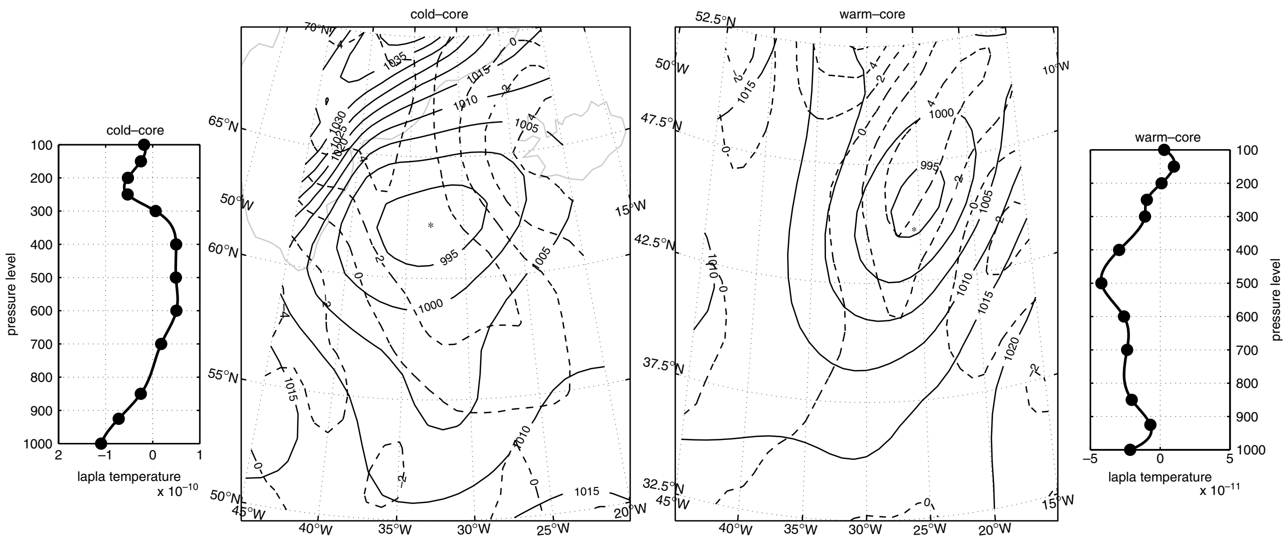

Figure 1. Synoptic maps of sea level pressure (solid line, mb) and Laplacian of temperature (dashed line, 10^{-11} K m^{-2}) at 500 hPa for 06 : 00 UTC 9 November 2000 (cold core) and for 06 : 00 UTC 27 October 1996 (warm core). Vertical profiles show Laplacian of temperature at the cyclone center at the same time for each cyclone. The '*' denotes the grid point with minimum sea level pressure.

couples surface development with dynamic and thermodynamic forcing at all levels above the surface. The extended Zwack–Okossi (Z–O) equation is given by

$$\underset{\text{(A)}}{\frac{\partial \varsigma_{g_s}}{\partial t}} = \frac{1}{(p_s - p_t)} \int_{p_t}^{p_s} \Bigg[\underset{\text{(B)}}{-\mathbf{V} \cdot \nabla_p (\varsigma + f)} - \frac{R_d}{f} \int_{p}^{p_s} \nabla_p^2 \Bigg(\underset{\text{(C)}}{-\mathbf{V} \cdot \nabla_p T} + \underset{\text{(D)}}{\frac{\dot{Q}}{c_p}} \underset{\text{(E)}}{+ S\omega} \Bigg) \frac{\mathrm{d}p}{p} \underset{\text{(I)}}{+ \mathbf{k} \cdot \nabla_p \times \mathbf{F}} \Bigg] \mathrm{d}p \qquad (1)$$

where all symbols have their conventional meteorological meanings. This equation shows that the near-surface geostrophic vorticity tendency (A) is forced by vertically integrated horizontal vorticity advection (B), horizontal temperature advection (C), diabatic heating/cooling (D), adiabatic cooling/heating (E) and friction (I), while P_t (100 hPa) and P_s (925 hPa) are the upper and the near-surface pressure levels respectively. This equation explicitly models the geostrophic vorticity tendency at surface using thermal and dynamic forcing at all levels over the surface and includes the non-quasi-geostrophic forcing.

The observed 12-h finite difference 925-mb geostrophic vorticity tendency contains both propagation and development components. In this study, the development component was isolated by a nine-point average method suggested by Lupo *et al.* (1992). Horizontal and vertical derivatives in the Z–O equation were calculated using second-order finite differencing, and the trapezoidal rule was used for estimating vertical integrals. A simple smoothing described in Lupo *et al.* (1992) was applied to the thermodynamic terms in order to filter computational noises induced by applying the Laplacian operator. The NCEP/NCAR reanalysis omega was used when solving the thermodynamic equation to calculate the diabatic heating/cooling term. The frictional term, which is normally a small contributor (Lupo *et al.*, 1992; Rolfson and Smith, 1996; Parsons and Smith, 2004), was computed using the following balance equation, which assumes that the pressure gradient and the frictional and Coriolis forces are in equilibrium in the boundary layer (below 850 hPa):

$$f\mathbf{k} \times \mathbf{V} + \nabla\varphi - \mathbf{F} = 0 \qquad (2)$$

where $\mathbf{V}$ is the horizontal wind vector, φ is the height geopotential and $\mathbf{F}$ is the frictional force.

The vertical profile of Laplacian of temperature is used to select a warm-core and cold-core cyclone over North Atlantic. A warm-core (cold-core) cyclone is characterized by warmer (colder) air near its center than around and thus by negative (positive) values of the Laplacian of temperature. It has been taken into account that, because of a certain tilt that is present during development of cyclones at mid-latitude, vertical profiles of the Laplacian of temperature are not performed exactly over the cyclone center.

3. Synoptic situation

The first case appeared as a closed cyclone near the southern coast of Greenland and west of Iceland and

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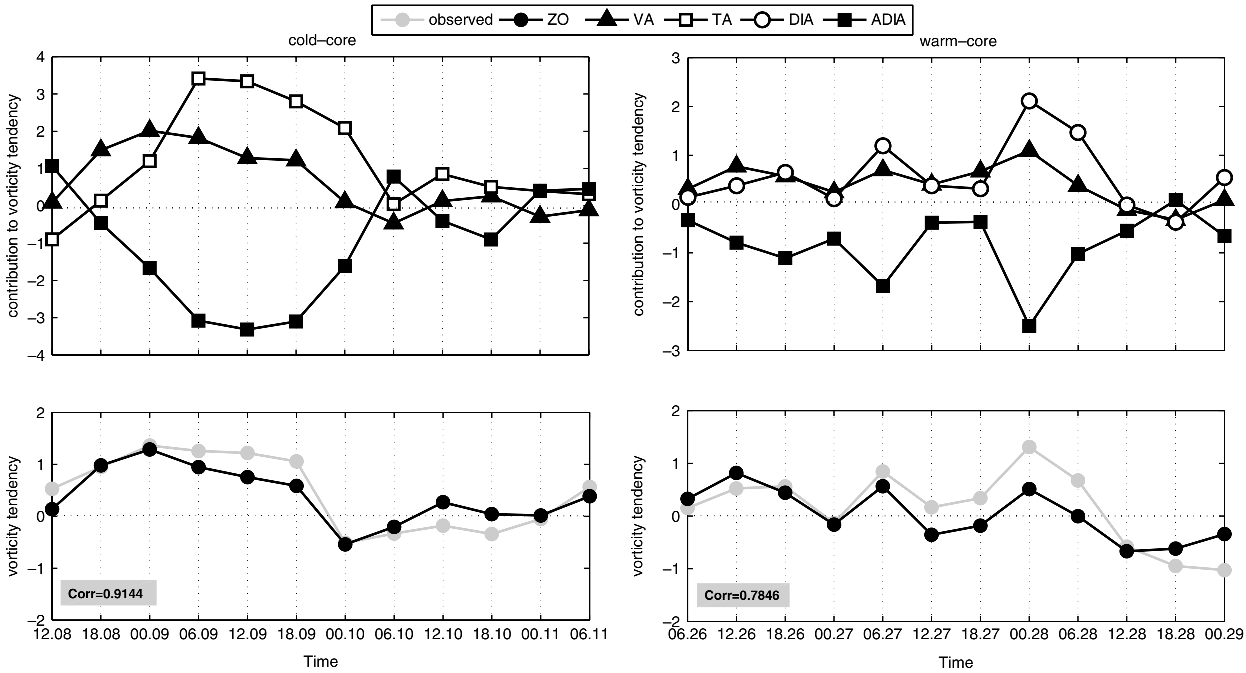

Figure 2. Time series of observed and diagnosed 925-hPa geostrophic vorticity tendency (10^{-9} s^{-2}) for the cold-core case (left), warm-core case (right) and individual 925-hPa forcing contributions (10^{-9} s^{-2}) to the total calculated geostrophic vorticity tendency for largest terms during the development stage.

reached its maximum intensity (considering relative vorticity at 850 hPa) at 06 : 00 UTC 10 November 2000 with a sea level pressure of 981 mb while moving to lower latitudes toward the eastern coast of British Islands. The second case used in this study was an ex-tropical cyclone named 'Lili' studied exhaustively by Browning *et al*. (1998). This cyclone traveled slowly while it was over the warm waters of the western North Atlantic, but then it moved quite rapidly after 26 October 1996 as it gradually reached an increasingly cool sea surface. Figure 1 shows the sea level pressure pattern, the Laplacian of temperature at mid-troposphere (500 hPa) and the vertical profiles of Laplacian of temperature of both cases over the cyclone area as described in the previous section. Both cases exhibit nearly the same vertical profiles (as depicted in Figure 1) during the development stage; while the first case is a cold-core cyclone at mid- and upper troposphere, the second case is a warm-core cyclone with negative values of Laplacian of temperature in the whole troposphere. The vertical profile of the cold-core case shows a shallow warm core at lower troposphere, which disappears at some stage of the development. The symbol '*' displays the minimum of mean sea level pressure (MSLP), which is located in the area of positive (negative) values of the Laplacian of temperature for the cold-core (warm-core) case at mid-troposphere.

4. Results and discussion

Figure 2 depicts the accuracy of the 925-hPa calculated geostrophic vorticity tendency by comparing with the observed values at the cyclone center. In addition, the contributions of forcing terms with the largest values at the cyclone center are shown for both cases. Time series of diagnosed and observed geostrophic vorticity tendency (bottom plots in Figure 2) and also the contributions (upper plots in Figure 2) are shown for the evolution period of both cases before reaching the maximum value of the relative vorticity at 850 hPa. There are quite high and acceptable correlations between the calculated and observed geostrophic vorticity tendency for both cases. However, at times, the Z–O equation underestimates or overestimates the observed values especially for the warm-core case with lower correlation coefficient than the cold-core case. The forcing mechanisms with the larger contributions for the cold-core case are the temperature advection, adiabatic cooling and vorticity advection, while in the warm-core case the diabatic heating is substituted by temperature advection. Patterns of the precipitation rate (not shown here) for both cases justify the higher contribution of diabatic heating in the warm-core case. The adiabatic term shows negative contributions for both cases and, as mentioned by Lupo *et al*. (1992), this term almost always operates in the opposite sense to the sum of the other forcing terms. The vorticity advection term shows a large contribution for both cases, especially for the cold-core case, and therefore underscores the importance of this synoptic-scale mechanism in the rapid development. In the cold-core case, this rapid development was mainly a result of cyclonic vorticity advection and temperature advection, while in the warm-core case the cyclonic vorticity and diabatic heating combined to initiate the development.

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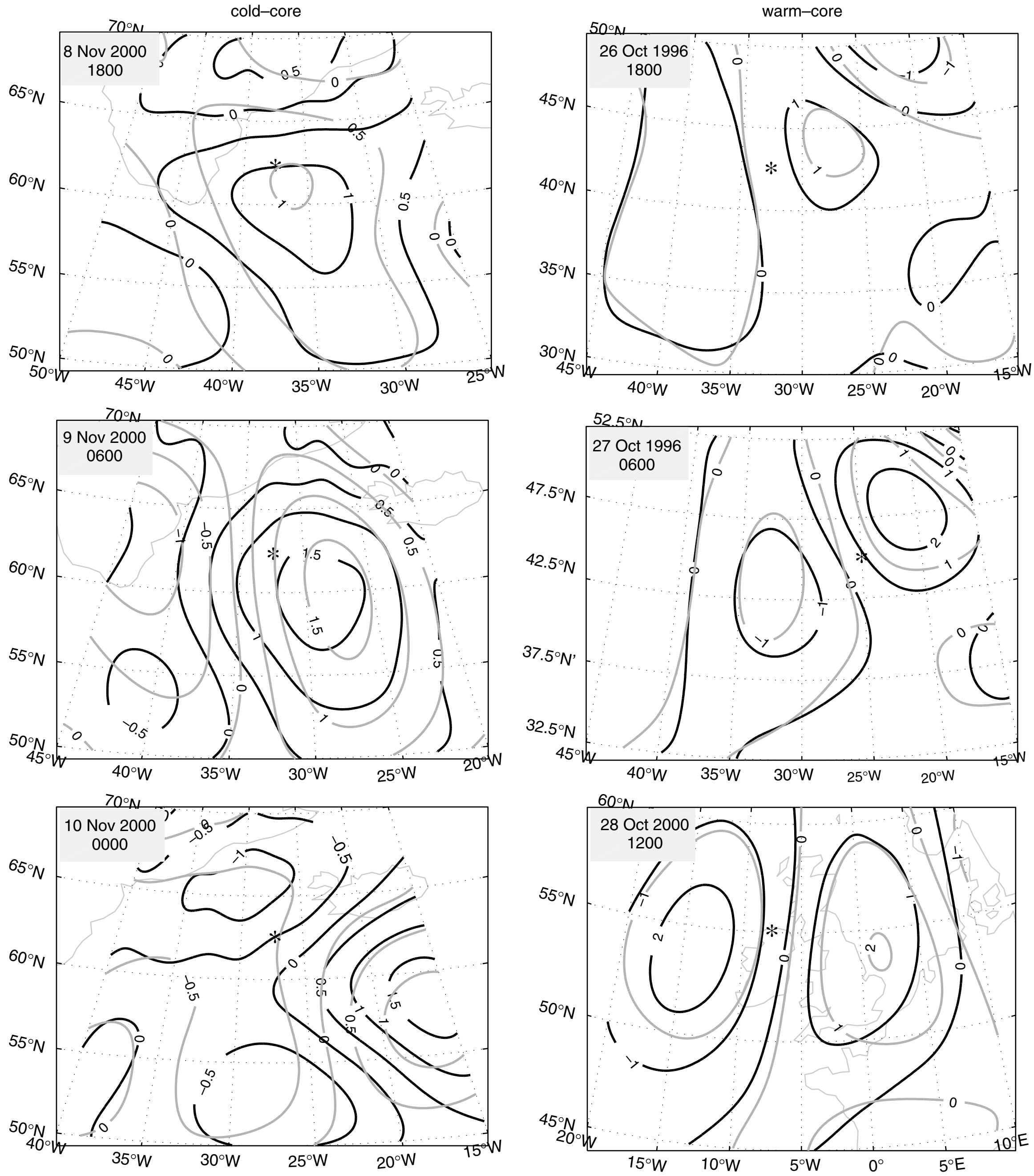

Figure 3. Observed (light-gray) and diagnosed (black) 925-hPa geostrophic vorticity tendency (10^{-9} s^{-2}) for the cold-core (left) and warm-core (right) cases. The '*' denotes the grid point with minimum sea level pressure.

As the geostrophic vorticity tendency gives the pressure tendency via a Laplacian operator, the sea level pressure falls (rises) when the geostrophic vorticity tendency is positive (negative) at the cyclone center. Furthermore, the pattern of near-surface geostrophic vorticity tendency in the cyclone area shows the track of the cyclone center and its displacement. Therefore, a larger value of the geostrophic vorticity tendency at the cyclone center and in the cyclone area can be applied as a measure of SLP deepening at the cyclone center and the cyclone center displacement. Figure 3 displays the pattern of the observed and calculated 925-hPa geostrophic vorticity tendency at three different times in the period shown in Figure 2 for both the warm-core and cold-core cases. Whereas there is a better pattern comparability between the observed and calculated fields in the warm-core case (Figure 3), the Z–O time series correlate better with the observed one in the cold core. At the beginning of the development (18 : 00 8 November 2000 and 18 : 00 26 October 1996) in both cases, the cyclone center at the surface was located near the observed and calculated cyclonic (positive) vorticity tendency maximum. By the middle maptime (06 : 00 9 November 20 : 00 and 06 : 00 27 October 1996), the cyclonic vorticity tendency increased in the cyclone center, while at the final maptime (00 : 00 10 November 2000 and 06 : 00 28 October 1996) the anticyclonic vorticity tendency

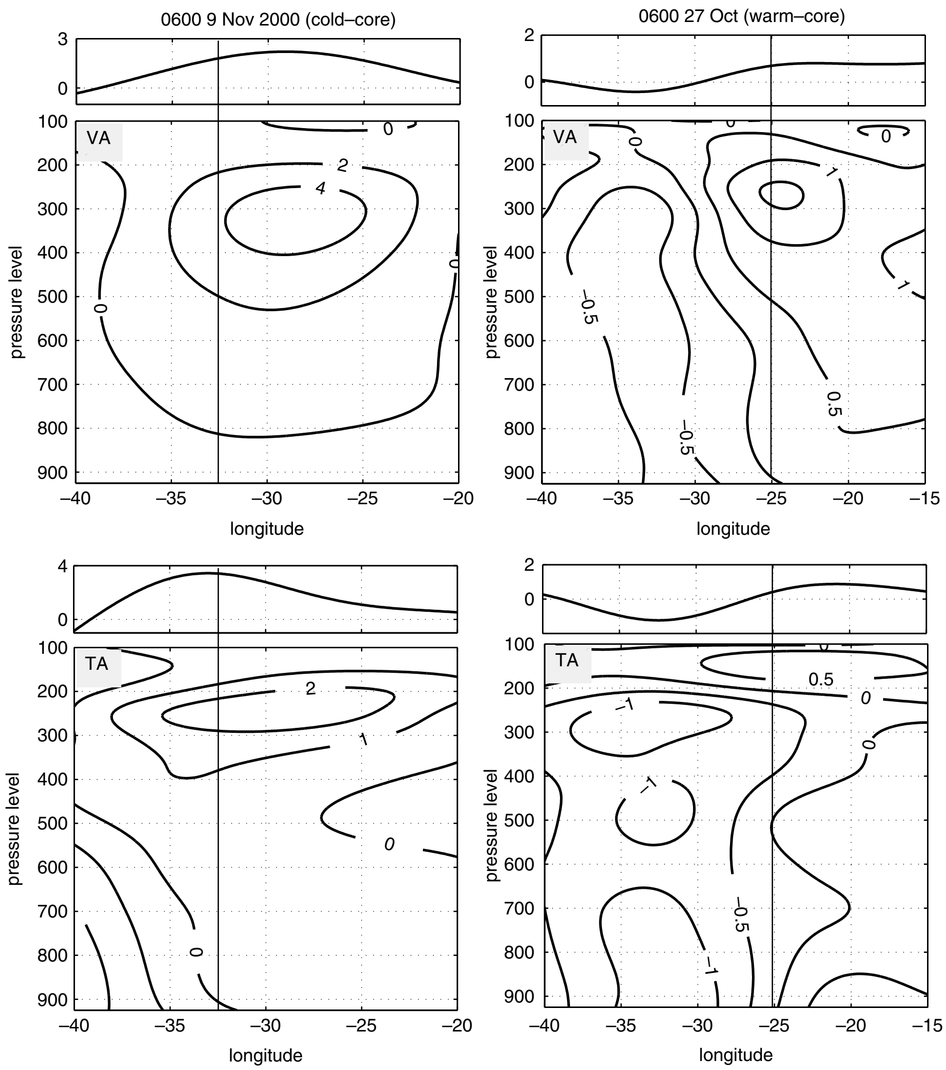

Figure 4. Vertical cross sections of vorticity advection (upper, 10^{-9} s^{-2}) and temperature advection (bottom, 10^{-4} K s^{-1}) for warm-core and cold-core cases along the latitude passing the cyclone center. The rectangular at the top of each cross section shows contribution to the total geostrophic vorticity tendency for the same forcing term at the same latitude. The vertical black line passes the grid point with minimum sea level pressure at the surface.

was seen at the cyclone center, which raised the pressure and initiated the decaying stage. As observed in the cold-core (warm core) case, the maximum cyclonic vorticity tendency is located ahead and south (north) of the cyclone center and implies that the cyclone center is traveling toward lower (higher) latitudes.

The vorticity advection contributes to the development in both cases, but the temperature advection contribution is small in the warm-core case. To clarify these features, cross sections of vorticity advection and temperature advection along the latitude passing the cyclone center are shown in Figure 4. Also, the vertical integrals are shown inside a rectangular area at the top of each cross section. These cross sections are qualitatively similar to the results of Lupo *et al.* (1992) and show larger values in the upper troposphere/lower stratosphere. However, these maximums are smaller in the warm-core case. The maximum cyclonic vorticity advection is located upstream the cyclone center. This is due to the westward tilt of both systems with height, which seems to be smaller in the warm-core case. Also, these vertical integrals show that the warm-air advection is larger over the cyclone center in the cold-core system. Furthermore, the temperature advection in the warm-core case has a large contribution ahead of the cyclone center, while this contribution is small at the cyclone center and has a small contribution to the near-surface vorticity tendency.

5. Summary

A diagnosis of two evolving cyclones with warm and cold cores characterized by the vertical profile of Laplacian of temperature at the cyclone center was performed using an extended form of the Z–O equation. In both the cases, the diagnosed geostrophic vorticity tendency confirms the importance of forcing at all levels in the troposphere and lower stratosphere. The time series of forcing terms with the largest contributions for both cases demonstrate that cyclonic vorticity advection and temperature advection constitute a substantial contribution to the development of the cold-core case. In the warm-core case, cyclonic vorticity advection and diabatic heating were the most significant contributors. The cold-core cyclone needs strong quasi-geostrophic forcing in order to obtain explosive development, while the warm-core cyclone benefits from the addition of the latent heating.

The geostrophic vorticity tendency in the development stage is dominated more by the temperature advection (diabatic heating) at the cyclone center for the cold-core (warm core) case. When comparing these two major forcing mechanisms with the adiabatic cooling term (Figure 2), we find them to be the main drivers of upward vertical motion. On the basis of the patterns of observed and calculated Z–O geostrophic vorticity tendency at 925 hPa, we reach the conclusion that the position of maximum vorticity tendency near the cyclone center determines the development of the cyclone center and cyclone track. Vertical cross sections demonstrate that temperature advection is a small contributor in the warm-core case and has a larger contribution ahead of the cyclone center.

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